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F. A. Tsander

PROBLEMS OF FLIGHT BY JET PROPULSION

(Problema poleta pri pomoshchi reaktivnykh apparatov)

INTERPLANETARY FLIGHTS

(Mezhplanetnye polety)

Collection of articles

Second edition, enlarged

L. K. Korneev, Editor

Gosudarstvennoe
Nauchno - tekhnicheskoe Izdatel'stvo
Oborongiz
Moskva 1961

Translated from Russian

Israel Program for Scientific Translations
Jerusalem 1964

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CONTENTS

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EXPLANATORY LIST OF ABBREVIATIONS OF U.S.S.R. INSTITUTIONS AND ORGANIZATIONS APPEARING IN THIS TEXT

Abbreviation	Full name (transliterated)	Translation
Aviatrest	Gosudarstvennyi trest aviatsionnoi promyshlennosti	State Trust for Air Industry
Glavnauka	Glavnoe upravlenie nauchnymi muzeinymi i nauchno-khudozhestvennymi uchrezhdeniyami	The Main Administration of Scientific, Museum and Scientific-Artistic Organizations
Lenoszhatgaz	Leningradskii zavod szhatykh gazov	Leningrad Factory for Compressed Gases
MGU	Moskovskii gosudarstvennyi universitet	Moscow State University
NARKOMPROS	Narodnyi komissariat prosveshcheniya	People's Commissariat of Education
ONTI NKTP SSSR	Ob"edinennoe nauchno-tekhnicheskoe izdatel'stvo Narodnogo komissariata tyazheloi promyshlennosti	United Scientific and Technical Publishing House of the People's Commissariat of Heavy Industry of the U.S.S.R.
Osoaviakhim	Obshchestvo sodeistviya oborone i aviatsionno-khimicheskomu stroitel'stvu SSSR	Society for the Defence of the Soviet Union and for the Development of its Aviation and Chemical Industries
RSFSR	Rossiiskaya Sovetskaya Federativnaya Sotsialisticheskaya Respublica	Russian Soviet Federative Socialist Republic
TsAGI	Tsentral'nyi aerogidrodinamicheskii institut	Central Aero-Hydrodynamic Institute
TsGIRD	Tsentral'naya gruppa po izucheniyu reaktivnogo dvizheniya	Central Group for the Study of Jet Propulsion

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In the book of the eminent Soviet scientist and rocket enthusiast F. A. Tsander, questions of long-distance rocket flight, problems of interplanetary flight, and problems dealing with the preparations for such flights are discussed. The new edition includes one biographical and seven scientific papers published for the first time. The book is for engineers and technicians associated with the rocket industry, and also for the general public.

Translation Editor's Note

The editor of this translation endeavoured to render the author's ideas faithfully. Some obvious errors appearing in the formulas or in the text of the Russian original have, however, been corrected.

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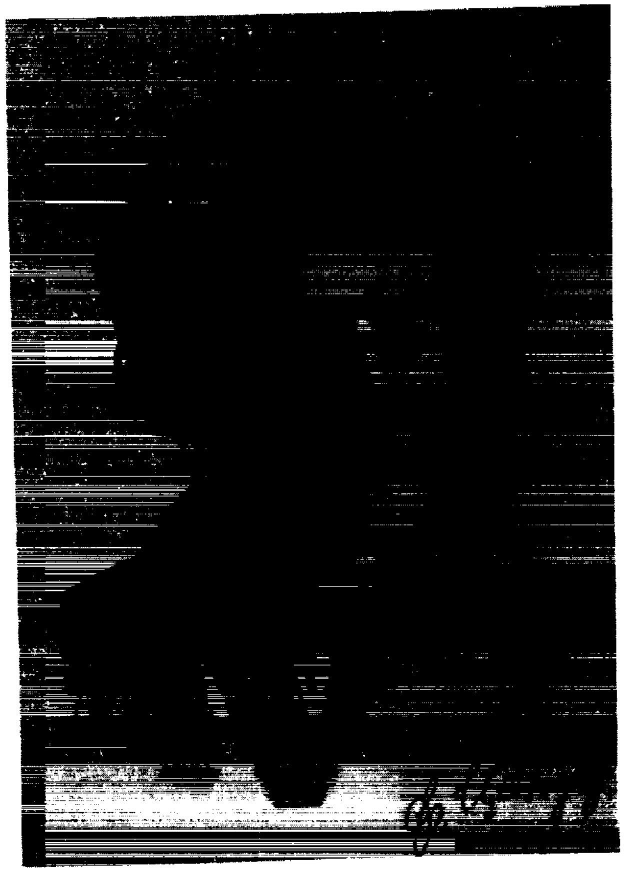

F. A. TSANDER
(1887–1933)

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"Is there anyone who, looking at the sky on a clear spring night and seeing the twinkling stars, has not thought of the possibility that the distant planets may be inhabited by intelligent creatures thousands of years ahead of ourselves in their civilization? What incalculable cultural wealth may be brought back to Earth through science should man be capable of travelling there, and how cheap is the cost of such a tremendously important achievement in comparison to the vast amounts that are wasted by humanity."

F. A. Tsander

INTRODUCTION

Nineteen hundred and sixty-two will be the seventy-fifth year since the birth of the great Soviet research scientist and talented engineer Fridrikh Arturovich Tsander [Canders], who devoted his entire life to solving difficulties associated with interplanetary flight. He pursued throughout his life the slogan "Forward to Mars". In the history of science, the name Tsander will, along with our remarkable scientist K. E. Tsiolkovskii, remain associated with the development of rocket technology.

The first edition of Tsander's book "Problems of Flight with the Aid of Jet Propulsion Machines" was published by the government aviation and auto-tractor publishing house ONTI NKTP SSSR in 1932 and edited by Tsander himself.

A collection of papers written by Tsander and edited by M.K. Tikhonravov was published in 1947 by the Oborongiz publishing house.

The purpose of the present, second edition of papers by Tsander, published on his seventy-fifth birthday, is to present in a single book all his main papers on rocket technology and interplanetary flight; however, this is not a complete collection of all his works.

An edition including all Tsander's works will become possible only after the decoding and studying of all his scientific papers. This is an extremely difficult task, because Tsander used shorthand in his notes in order to save time; this shorthand code was known only to him. Decoding his shorthand is particularly difficult, since Tsander could use the Latvian, Russian and German languages, which he knew fluently.

In addition to articles already published in 1932 and 1947, the present edition contains for the first time the following papers:

Design of the ER-1 engine;

Flights to Other Planets (The Theory of Interplanetary Travel);

The Use of Light Pressure for Flight in Interplanetary Space. Light Pressure on a Combination of Mirrors;

Calculations of Spaceship Flight in the Earth's Atmosphere (gliding descent);

Calculations of Spaceship Flight in the Earth's Atmosphere (ascent);

Spaceship Temperature in Glide Landing on Earth;

Deflection and Repulsion of Meteors by Electrostatic Charges Emitted by the Spaceship.

These articles contain the present editor's notes. The notes in other articles were reproduced from the first edition almost without alterations.

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As a supplement we have an article "The Life and Work of F. A. Tsander" (Zhizn', tvorchestvo i deyatel'nost' F. A. Tsandera), based on information from archives and personal reminiscences of his students and friends.

Some of the articles being now published for the first time in the present edition were originally scattered among various notes in a state unsuitable for publication. In them, we find new and extremely interesting ideas for anyone working in the field of space travel.

It is important to point out that 30 to 40 years ago Tsander was capable of foreseeing many of the problems of space travel, to develop them, and to show the technical methods for their solution.

The variety of problems covered by Tsander's articles that are included in the present publication (heat technology, aerodynamics, celestial mechanics, electrical engineering, physics and other subjects) show the high level of his engineering and technical ability and his broad erudition in the field of rocket technology and interplanetary flight of space vehicles.

The editor would like to thank all the friends and pupils of Tsander who took part in the preparation of the present edition as well as his wife, who was kind enough to supply us with material from his personal archives.

L. Korneev

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L.K. Korneev

LIFE AND WORK OF F.A. TSANDER

(On the 75th anniversary of his birth)

On 4 October 1957 Soviet scientist took the first step on the road to realizing man's ancient and daring dream of conquering space. They launched the first artificial satellite weighing 83.6 kgs. After this first firing, others followed.

On 3 November 1957 the second Soviet Satellite, carrying the dog Laika, was launched. The third Soviet satellite launched on 15 May 1958 was a well-equipped scientific laboratory. The first space rocket to reach the second cosmic speed was "Mechta", fired on 2 January 1959.

On 12 September 1959 the second space rocket was launched, and reached the Moon's surface; a Soviet pennant was planted there. It was a triumphant day in the annals of Soviet rocket technology.

Within a month, on 4 October 1959, the third Soviet space rocket was fired. It carried an automatic interplanetary station which photographed the face of the Moon, hitherto unseen by man, and relayed the pictures to Earth. The last stage of the rocket weighed 1553 kgs (without fuel), and the weight of the automatic station including the scientific and radio-technological equipment was 278.5 kgs. Humanity welcomed our scientific achievement.

This, however, was not the end; the Soviet people continued to develop spaceships further and to prepare them for man's flight into space. It meant more sleepless nights, research, strenuous, selfless and inventive labor.

On 15 May 1960 the first spaceship, weighing 4540 kgs, began to orbit the Earth (without a rocket launcher); it was fifty-two times heavier than the first Earth satellite fired on 4 October 1957. The capsule of the first spaceship was airtight, and was loaded with a weight similar to that of a human being. The capsule was also fitted with various types of instruments, and other installations necessary for future manned flight; these, together with the food supply, weighed 1447 kgs. Thus a reliable foundation, guaranteeing the safety of future manned space probes was laid.

Three months later, on 19 August 1960, the USSR launched a second spaceship around the Earth. The cabin carried all the equipment necessary for future manned flights. The capsule also carried two dogs, Belka and Strelka; in addition there were rats, white mice, flies, mushrooms and seeds of various plants. A radio-television network within the capsule enabled close observation of the animals' behaviour pattern and relayed to Earth all physiological symptoms. The capsule covered a distance of 700,000 kms around the Earth, and a signal on its eighteenth orbit started its

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descent. Immediately special equipment started to relay the performance of the braking and control systems, keeping a record of the parameters as the vehicle passed through the dense layers of atmosphere back to Earth. Both the control and braking systems worked with extraordinary precision, and guaranteed the descent of the spaceship within the designated area. The deviation of the actual landing place from the estimated one was only 10 kms.

The spaceship passed successfully through the Earth's atmosphere, and the capsule with the test animals descended smoothly and safely after separation. After the flight, on 21 August 1960, Strelka and Belka and all the other animals were delivered to Moscow in perfect health.

For the first time in world history, living creatures returned safely to Earth after completing a space trip. In the Soviet Union the life of a man is valued above all else; consequently, one successful trip with living creatures was insufficient proof for a manned orbital flight. Therefore, on 1 December 1960, a third spaceship weighing 4563 kgs was launched. Within the capsule were the dogs Pchelka and Mushka and other animals. The spaceship's booster-rocket was not included in the above weight. More work was carried out on the spaceship's systems and also on the medico-biological data collected from observations of the animals.

Two months after the third spaceship had been fired, on 4 February 1961, the heavy artificial Earth satellite (AES) was launched. All the satellite's controls functioned normally, both during the ascent and during its orbital flight. The weight of the (AES) was 6483 kgs without the booster rocket.

The next stage was to check the course of the interplanetary flight, the launching, the radio communications and the controls of the cosmic station. Thus on 12 February 1961 a guided space rocket was fired from the heavy Earth satellite. The space rocket then placed the automatic interplanetary station on a trajectory to Venus. The weight of the automatic interplanetary station (AIS) was 643.5 kgs. Gradually, the components were developed, information from space was analyzed, and the performance of the rocket-assembly system was checked.

In the conclusive stages of construction of the spaceship, March 1961 was the most decisive month. The fourth and fifth spaceships, were fired on 9 and 25 March 1961 respectively. The fourth spaceship weighed 4700 kg without the booster rocket, and carried the dog Chermushka. The fifth space ship weighed 4695 kgs, and carried the dog Zvezdochka. In both cases all the equipment within the satellites worked normally, and Chernushka and Zvezdochka descended within the required areas of the USSR; both animals were fit.

At 0907 hours (Moscow time) 1961 the great event took place: Yuri Alekseevich Gagarin, a Soviet pilot, orbited the Earth once, and landed successfully within the USSR. This was man's first successful attempt at orbital flight.

It was a triumph of man over nature, a great conquest of science and technology, and a victory of the human mind. It was the beginning of man's flight into space.

The second Soviet cosmonaut German Stepanovich Titov was launched in the "Vostok 2" on 6 August 1961. The "Vostok 2" orbited the Earth seventeen times, spending over 25 hours in space.

A period of three and a half years elapsed between the launching of the first satellite and man's first orbital flight. It was during those years that

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a gigantic task was accomplished on the way to man's mastery of space. The following paragraphs will answer the questions as to where these uninterrupted space flights had their beginnings, and who started this movement in the USSR.

The history of rocketry in the USSR is an interesting narrative of rocket-technology enthusiasts, of selfless and strong-willed people, manfully laboring and laying the foundations for today's space flights. We dedicate this book to Fridrikh Arturovich Tsander, an important Soviet scientist and research-worker and a talented engineer, whose whole life was devoted to the development of interplanetary flight. Tsander was a close friend and follower of Konstantin Eduardovich Tsiolkovskii.

In the history of science the names of Fridrikh Arturovich Tsander and Konstantin Eduardovich Tsiolkovskii will remain connected with Soviet rocket technology.

Tsander's father, a medical doctor and a lover of natural science, was employed at the Zoological Museum in Riga. Tsander was born there on 11 August 1887. His life was many-sided and academic; he was humble, shy, affable, kind and richly-endowed*.

Fridrikh's mother died when the boy was two years old. The family had five children, three boys and two girls. Fridrikh was an extremely sensitive child. Frequent visits to the Museum, stories related by his father about other planets which could be inhabited by unknown creatures and the first flights of Lilienthal awoke in Tsander during his childhood the urge ... "to try and succeed in reaching other planets. This thought, later, never left me", wrote Tsander in his autobiography.

He had an uneventful childhood and began his studies at the Riga High School from which he graduated in 1905 at the top of his class. During his last year at school the astronomy teacher introduced his pupils to the article by K.E. Tsiolkovskii "Space Research with the Aid of Jet Propulsion Machines" (Issledovanie mirovykh prostranstv reaktivnymi priborami) written in 1903. This article created a great impression on the young dreamer. Thinking of distant worlds and endless space, he dreamed of using the wealth of the universe for the good of mankind. This idea was the source of his inexhaustible and endless enthusiasm throughout his life.

Tsander knew that only science could help him realize his dream of interplanetary flight. Consequently in 1907 when he enrolled in the mechanical department of the Riga Polytechnic Institute, he displayed the same interest and zest for his studies as he had previously when attending High School.

In 1908, Tsander as a student, invested his first savings in a small telescope, 1.5 m long and 4 inches in diameter, and carried out observations of the Moon and planets.

Observations alone did not satisfy Tsander, as already then he conceived the idea of organizing a group or a society dedicated to the problems of interplanetary flight. He spoke to the students of the necessity of research

* Biographical and other data about Tsander were taken from his autobiography written on 12 March 1927, from the synopsis of his popular science essay "Flights to other Planets and the Moon" (Polety na drugie planety i na lunu), 2 August 1925, and are also based on additional information from Tsander's archives. It was compiled for the contemplated edition of this book.

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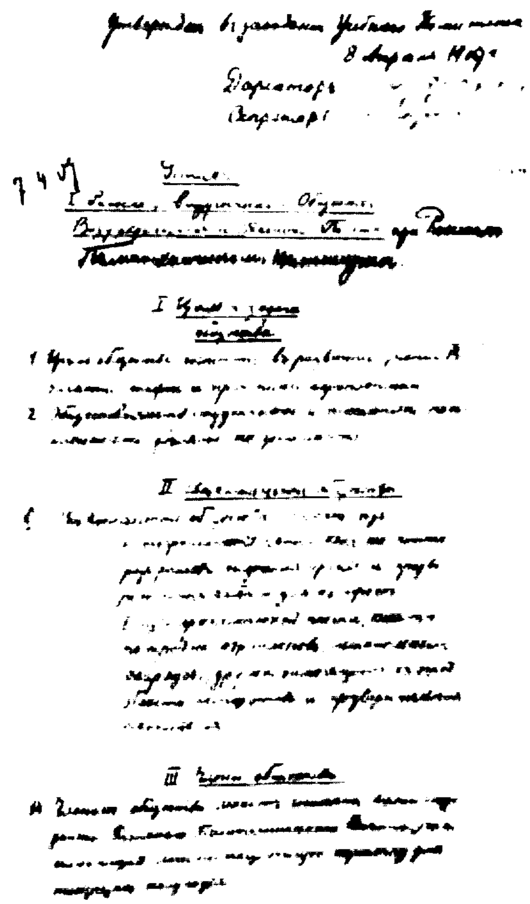

Facsimile of the first page of the statute

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in the field of interplanetary flight. Having roused the interest of his comrades, Tsander, together with other students, addressed the following request to "His Excellency, the Director of the Riga Polytechnic Institute":*

"Herewith is the project for the First Riga Students' Society of Air Travel and Flight Technology, and a list of those willing to organize this Society. We are honored to ask Your Excellency to place this petition for the introduction of this Society before the Committee of the Riga Polytechnic Institute." This request was discussed later by the educational committee, and on 8 April 1909 the statutes of the Society were confirmed.

Tsander as a student – 1908

This was one of the first students' aeronautical societies, and therefore its statutes are of particular interest. The full text and the facsimile of the first page follows.

* The copy has been taken from the original preserved in the Riga archives.

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Approved by the session of the Educational Committee on 8 April 1909.
Director: Prof. Dr. V. F. Knirim
Secretary: Prof. B. Vodzinskii

STATUTE
FIRST STUDENTS' SOCIETY OF AIR TRAVEL AND FLIGHT TECHNOLOGY AT THE RIGA POLYTECHNIC INSTITUTE

I. AIM AND PURPOSE OF THE SOCIETY

§ 1. To develop knowledge in the field of theoretical and practical aeronautics.

§ 2. To remain purely a student body and not to participate in politics.

II. SOCIETY ACTIVITIES

§ 3. The activities of the Society consist of:

a) The theoretical section, i.e., the reading of papers, scientific discussions and other preliminary work.

b) On the practical side: the building of airplanes, flying projectiles, other related machines and their preliminary testing.

III. SOCIETY MEMBERSHIP

§ 4. Membership is open to all students of the Riga Polytechnic Institute who own a legitimate student's card for the current half-year. The candidate must, however, receive a two-thirds majority in a secret ballot. Should admission be refused, no reasons are given.

§ 5. Honorary members of the Society may be only those who have previously held membership in the First Students Society of Air Travel and Flight Technology, and who have made important contributions to the Society or to air travel. They enjoy full membership rights but cannot have the deciding vote.

§ 6. Members may be expelled for unseemly behavior, failure to pay membership dues and insubordination to other Society rules. Members are expelled by majority vote. Those expelled may reapply for membership on the same basis as any new member.

§ 7. Every member has a right to vote at meetings and all members are permitted to enter the workshops and participate actively or passively in the work.

§ 8. Members must pay monthly fees in accordance with the decisions of the general assembly. New members are liable to entrance fees which are also set at the general meeting.

§ 9. Entrance fees are paid on the first of every month except during holidays; these are determined at a meeting.

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IV. DISTRIBUTION OF DUTIES

§ 10. Elections are held biannually at the General Meeting for:
a) the Presidium;
b) the Officers;
c) the Technical Committee.

a) The Presidium is the body responsible for the activities of the Society before the Educational Committee of the R. P. I. and the police. This body shall hand to the Director of the R. P. I. at the beginning of every semester a list of Society members, members of the Presidium, and the Society's address.
The Presidium is composed of:
1) The President of the Society who is the official representative of the Society in all business; he also presides at all meetings.
2) The Vice President who helps the President in his duties and is his deputy.
3) The Secretary who is in charge of the Society's protocols and all the correspondence.

b) The Officers responsible to the General Meeting:
1) Librarian;
2) Cashier controlled by the Presidium.

c) The Technical Committee is responsible to the General Meeting. This Committee in turn chooses the technical representative. All flight experiments are made under the supervision of the technical representative or of his assistant.

Note. The Society is not responsible for any mishaps to its members. Under special circumstances a meeting may be convened and may decide to aid the victim financially according to available means, if approached by said victim.

V. MEETINGS

§ 11. Meetings will be held outside the R. P. I., and are divided into three categories:
a) General Meeting;
b) Extraordinary Meeting;
c) Regular Meeting.

All meetings are convened by the Presidium. All meetings of a higher order include the rights of the lower ones with the exception of conditions under which the meeting may be convened and under which it can make decisions.

a) The General Meeting is convened at the beginning and at the end of every semester. All current members must attend, and should have one week's notice and the agenda. The General Meeting chooses the new Presidium, receives the Cashier's report, decides on questions of payment of fees, and inspects the activities of the functionaries. It requires a quorum of three-quarters of the members and may make decisions with a two-thirds majority.

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Note. If owing to the absence of the necessary number of members the meeting does not take place, the Presidium has the right at some future time to convene a postponed General Meeting. It does not require a quorum, and carries out decisions in a similar manner to an Extraordinary Meeting. It is compulsory for all members.

b) An Extraordinary Meeting may be convened at any time. All members should be informed of the agenda not later than three days before the meeting. It may accept new members whose names must be handed in to the Presidium. It may expel members and may make decisions on important affairs of the Society, financial and otherwise. A quorum of two-thirds of the members is necessary, and requires a two-thirds majority of all present.

c) The Regular Meeting takes place at least once every two weeks, and all members of the Society must be informed one day in advance. It may make use of the workshop and the office. It may decide, and pass judgement, on essays. It requires a 50% quorum, and its decisions are carried out by a simple majority. In case of a tie, the deciding vote is cast by the President.

VI. SOCIETY RESOURCES

§ 12. The resources for the Society's organization are:

1) Monthly dues determined at the General Meeting (see § 8).
2) Entrance fees set at the General Meeting.
3) Voluntary contributions.
4) Charity.
5) Fines for breaking any of the Society's laws.

Note. Money is not returnable.

VII. EXPENDITURE

§ 13. Expenditure is permissable for ends not opposed to those of the Society, i.e.,

1) To cover building costs, rent and purchase of equipment;
2) To cover office expenses;
3) To cover library costs;
4) Any other expenses approved at a General Meeting.

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VIII. LIQUIDATION OF THE SOCIETY

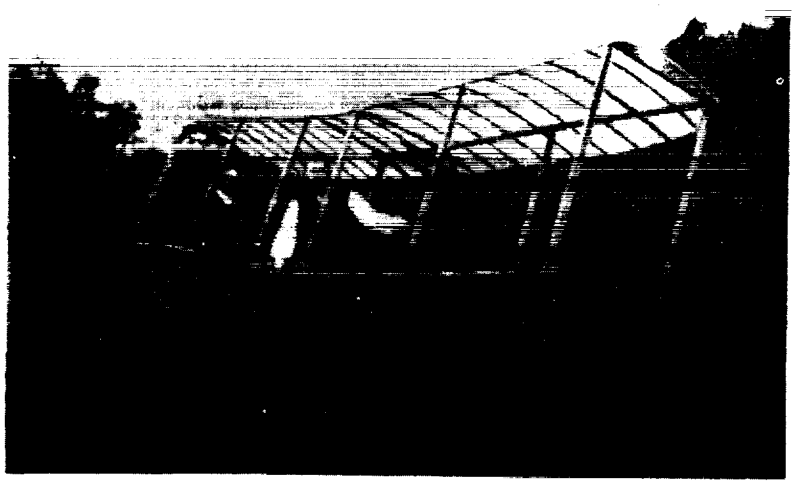
§ 14. Should the number of members be fewer than eight, the Society will be dissolved. The disposition of property is to be decided by the last General Meeting.

IX. STATUTE CHANGE

§ 15. Any changes in the statutes may be introduced only during a General Meeting. All changes must be approved by the Educational Committee of the R. P. I., before they may be enforced.*

In 1908 Tsander began work on various problems of interplanetary communications. He made several calculations connected with the problem of gas escaping from containers, and studied the possibilities of overcoming the force of gravity and acquainted himself with various other problems.

The opposition of Mars on 24 September 1909 at a minimum distance of 56 million kms (maximum distance 400 million kms) was of particular interest to Tsander and other young enthusiasts of the Society. The same year, they began the construction of a glider, but as Tsander wrote in his autobiography, the Society never progressed further than the initial stage.



Glider built in 1909. To the left – Tsander

* Unfortunately, no evidence is available of the activities of the First Riga Students' Society of Air Travel and Flight Technology.

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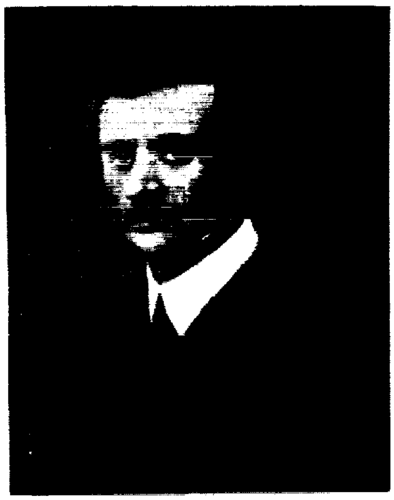
In 1914 Tsander received an honors degree in "Technological Engineering" from the Riga Polytechnic Institute. The Institute had a wide curriculum, and the knowledge acquired was of great value in Tsander's subsequent scientific and engineering work. He was the first engineer in our country to devote himself to the practical solution of problems connected with interplanetary flight and rocket technology. "Flight to Mars" was his slogan throughout life.

The purely scientific study, and the first fundamental theoretical works in the field of interplanetary flight belong to K. E. Tsiolkovskii who showed a great perspective in rocket technology and in the realization of space flight. The preliminary technical work on rockets in the USSR, and the creative theoretical research which made possible the charting of paths for interplanetary flights, belong to Tsander.



Tsander in 1913

F. A. Tsander's obituary in the newspaper "Tekhnika" on 30 March 1933 read as follows: "To him belongs a series of theoretical works containing the world's only calculations in the field of jet propulsion from which he created his own school of theory and jet engine construction."

Throughout his life he worked intensively in the field of theoretical interplanetary flights, and as a result he developed a new thermal cycle for rocket engines. He was the first to propose the use of metals, including components of the spaceship itself, as fuel for propulsion. He made this public for the first time in December 1923 during a lecture to the theoretical section of the Moscow Society of Astronomic Amateurs, and also in the Journal: "Tekhnika i Zhizn", No. 13. July 1924. Discussing his project for the construction of a spaceship, he pointed to the possibility of using solid structural materials as rocket fuel. Tsander first mentioned this in a series of lectures in several towns in the years 1924 to 1925.

In the years prior to 1927, Tsander wrote the theoretical work "The Use of Metal Fuel in Rocket Engines" (Primenenie metallicheskogo topliva v raketnykh dvigazvaniem); however, it was published only in 1936, in the Journal: "Raketnaya Tekhnika", No. 1. In 1937, his second article: "Problems in the Construction of a Rocket Using Metallic Fuel" (Voprosy konstruirovaniya rakety ispol'zuyushchei metallicheskoe toplivo) was published in the Journal: Raketnaya Tekhnika, No. 5.

Two of his articles, under the heading "Thermal Estimates of a Liquid Rocket Engine" (Teplovoi raschet raketnogo dvigatelya na zhidkon toplive), are of particular interest. They present for the first time estimates of the combustion chamber wall temperature, and of the chamber's capacity required for full and effective combustion of all fuel components. In addition to a series of theoretical deductions it contains estimates of heat transmission

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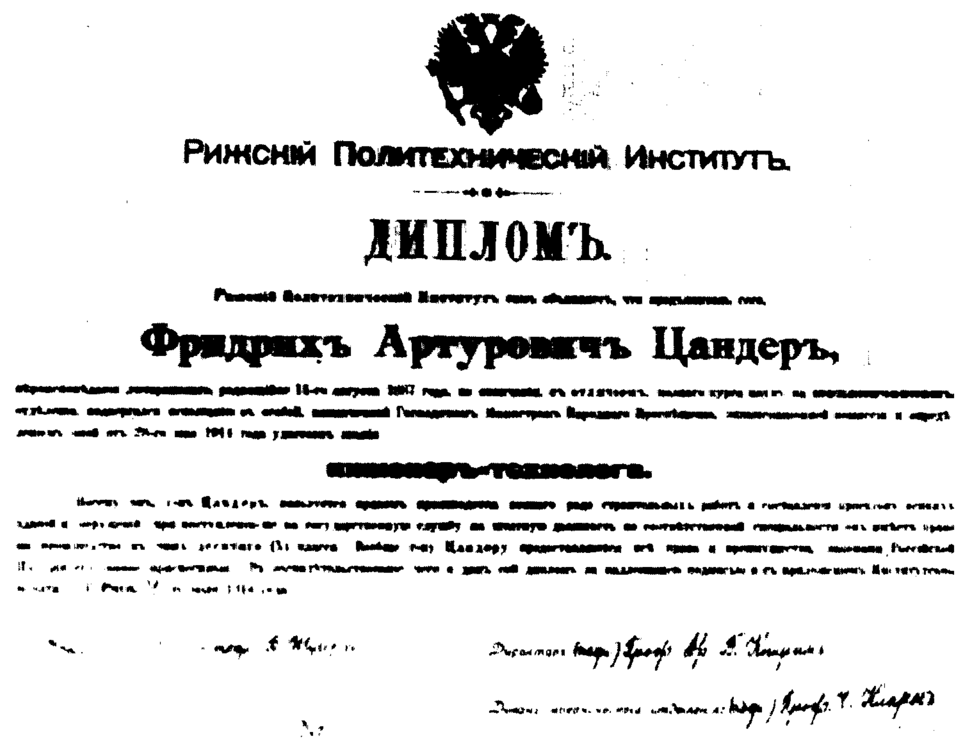
Рижскій Политехническій Институтъ.

ДИПЛОМЪ.

Фридрихъ Артуровичъ Цандеръ,

Tsander's diploma after graduating from the Institute

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indicating the possibility of an all-metal rocket engine without the use of ceramics; it also gives rocket engine estimates from entropy diagrams.

On 16 February 1926, at the "Motor" factory imeni M. V. Frunze, Tsander demonstrated with the help of diagrams the engine he proposed to use for launching a spaceship*; he spent much time over the construction of aircraft engines too. In one case he considered the use of a high-pressure piston engine without carburetor running on liquid oxygen and gasoline fed into the cylinders by pumps. The above high-pressure piston engine was to serve as a spaceship launcher.

At the same time, Tsander worked out important problems of astronautics, some of which still exist today. In papers on interplanetary travel Tsander discussed the problem of choosing interplanetary trajectories for maximum fuel conservation. He determined the launching time and the time spent on the journey and discussed trajectory corrections of interplanetary rockets to guarantee safe landing on the planet. He gave, in particular, a detailed trajectory description of the flight to Mars.

The idea of orbiting the Moon or other planets in order to accelerate or decelerate the spaceship, was put forward by Tsander in 1924. He also developed a method for calculating the acceleration**.

He also devised a method for calculating a spaceship's orbit, based on the determination of the planet's angular distance from some fixed direction in space, and its angular diameter.

In the years 1920-1930 several authors devoted their books to the research in interplanetary trajectories, yet not a single one touched on the question of corrected trajectories of the spaceships and the disturbance of the heliocentric orbit of the rocket by planets or satellites.

Tsander paid particular attention to the return of the spaceship to Earth. His paper "Calculations of Space-Vehicle Flight in the Earth's Atmosphere" (Raschet poleta mezhplanetnogo korablya v atmosfere Zemli), considered the ballistic and aerodynamic problems of ascent and descent from space to Earth. He set forth the idea of a gliding descent from space with deceleration within the Earth's atmosphere; this idea is integrated into his project for a spaceship. In 1920 he discussed the above project in a lecture, and in 1924 he published an article in the Journal: "Tekhnika i Zhizn'", No. 13.

Tsander showed an acute awareness of the problems of interplanetary flight. In particular he realized that the return journey to Earth demanded the solution of complicated scientific problems, one of which is the protection of the spaceship from the heat generated by friction, when traveling at supersonic speeds within the Earth's atmosphere. In his paper "Space Vehicle Temperature on Re-entry into the Earth's Atmosphere" (O temperature, kotoruyu primet mezhplanetnyi korabl' pri planiruyushchem spuske na Zemlya) one method of coping with the aerodynamic heat of the spaceship is discussed. In 1925 Tsander suggested the use of static

* The manuscript of this lecture is in Tsander's personal archives.

** Problems connected with acceleration when orbiting the Moon were investigated by V. A. Egorov in 1957; Egorov's method of approximation is very closely related to Tsander's. (Egorov, V. A. O nekotorykh zadachakh dinamiki poleta k Lune (Some Problems of the Dynamics of Flight to the Moon)), – Uspekhi fizicheskikh nauk, Vol. 13, No. 1a. 1957. [English translation published by Israel Program for Scientific Translations. IPST Cat. No. 112. 1961.])

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electricity for the deflection of meteors from the spaceship thus avoiding any possible collisions*.

Time and again in his works Tsander proved himself to be one of the founders of applied space technology i. e., the study of rocket movement in space. He never overlooked any of the practical problems of interplanetary travel, paying particular attention to the life and diet of the future cosmonauts. In 1915 Tsander began a series of experiments, which lasted a number of years, on the construction of a lightweight hothouse supplying fresh vegetables, and absorbing the carbon dioxide exhaled by the astronauts; he cultivated beans and cabbage in flower pots filled with crushed charcoal. Thus patiently and painstakingly he achieved success.

In his autobiography compiled on 12 March 1927, he wrote: "To the best of my knowledge, I am the originator of the following**:

1. Supplying the rocket with wings for flight within the atmosphere and for attaining a cosmic speed of 8 kms per second in its upper layers; also a gliding descent to Earth and other planets possessing an atmosphere.

2. Equipping the rocket with airplane engines***, for flight in the lower layers of the atmosphere where the rocket efficiency is very small owing to the slow speed. The engines should be of a special type preferably working for about half an hour without danger of failure.

3. Proposals for the simultaneous combustion of rocket propellants yielding both solid and gaseous combustion products. We may use (particulary as the method proposed by others of inserting one rocket inside the other demands enormous weights and is therefore not economical, and because of insufficient study of booster-rocket construction, it is more dangerous than my method of flight with the aid of an airplane) parts of the spaceship as rocket fuel, i. e., rods etc., made from various alloys of aluminum, magnesium, lithium, etc. When the spaceship becomes lighter due to fuel combustion, these parts become unnecessary and may therefore themselves be used as fuel. The main advantage is that we may build reliable spaceships and still be able to carry enough fuel.

4. A proposal for the use of an arrangement of rockets with convex mirrors to concentrate sunlight within the spaceship in order to increase the speed of the escaping gases thereby increasing the speed of the rocket for flights in space.

5. The use of coils carrying an electric current; the pressure of the Sun's rays on a cloud of iron dust held within the coil by electromagnetic forces, produces a thrust on the spaceship traveling through space. The advantage is that meteors passing through the cloud cause practically no interference with the flight.

6. A proposal for the concentration of the Sun's rays with the help of a system of huge convex and concave mirrors in parallel clusters, as in No. 4, for the sole purpose of attaining great speeds for flights to other solar systems (this is the only feasible method which even today may realize the hope for travel to other solar systems).

* Tsander wrote all his works in shorthand which he later decoded. Therefore we consider 1925 as the date when he decoded this article.

** Typed strictly in accordance with Tsander's manuscript.

*** Piston engines are implied.

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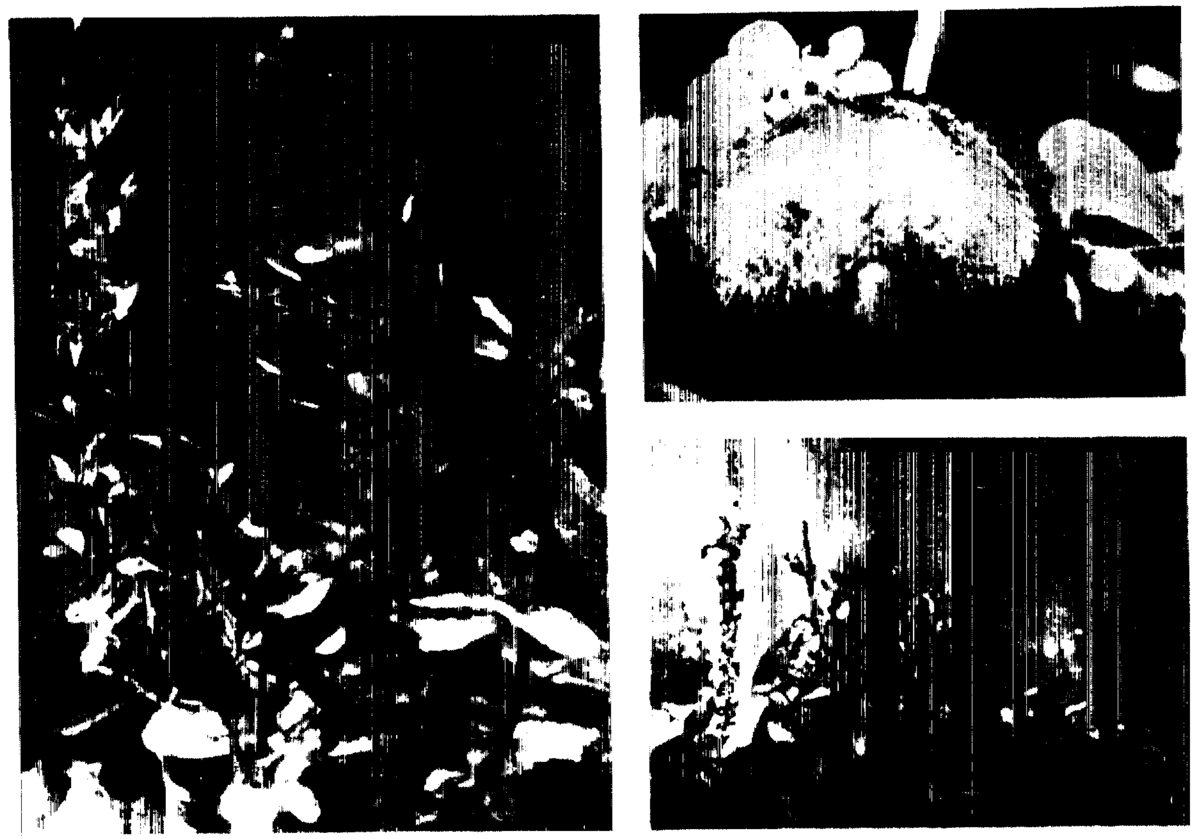

Beans and cabbage cultivated by Tsander in his own greenhouse

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7. The use of a sphere made of extremely thin metal plates, electrified by a charge from Earth, and pushing itself away through electrostatic forces for space flights. This is possible only if the Earth contains a charge.

8. A proposal for flight around planets, in their atmospheres, or out of them, in order to increase the speed of flight (thus gaining energy during interplanetary flights). A further proposal to accelerate the spaceship at the point of the trajectory while its velocity is the highest (for the same purpose).

9. A proposal for the deflection of meteors by using static electricity emitted against them in the form of cathode rays. The rays emitted by the spaceship are also placed within an electrified sphere.

I have another series of suggestions connected with the construction of the spaceship and its components. I also have other unfinished proposals."

After his graduation from the Institute, Tsander worked at the rubber plant "Provodnik"; here he familiarized himself with the rubber industry, for rubber could be used as insulating material in the vacuum of space. In 1915, during World War I, the factory was moved to Moscow, and when the factory closed down in 1917, Tsander devoted himself to the theoretical calculations of interplanetary flight. In the initial stages he worked from estimates of an airplane flying at high altitudes in the Earth's atmosphere by means of a propeller-driven engine; later, attaching a rocket to the engine, he made calculations connected with interplanetary travel.

In February 1919, he started work at the aviation factory No. 4 "Motor" in order to realize his plans for interplanetary flight. He devoted all his spare time to the construction of an airplane fitted with an engine which would enable it to fly out of the Earth's atmosphere. He also spent much of his time on the problem of cosmic speed.

In 1920 he made a detailed speech about his spaceship at the Provincial conference of inventors in Moscow. V. I. Lenin was present and promised to help the scientist. Although Tsander described his meeting with Lenin modestly and briefly, he remembered it throughout his lifetime.

This is what he told me when we were working together in the years 1932-1933: "Before the lecture I was informed that Lenin would be in the audience. This upset me and I became nervous. However, noticing later how intently Lenin was listening to my speech, I calmed down and lectured with enthusiasm on the construction of my spaceship, on the possibility of man's flight to other planets, introducing my calculations. After the speech I was invited to meet Lenin; this made me confused. Lenin was greatly interested in my work and my plans for the future; he spoke with such simplicity and cordiality that I am afraid I took advantage of his time by relating to him in great detail my work and my determination to build a rocket spaceship.

"I also told Lenin that I was working on the problems of man's flight to Mars: the construction of a suitable spaceship, which method to devise in order to assist man to overcome acceleration, and also the questions of suitable clothing and diet.

"Lenin asked me: 'Will you be the first to fly?' I answered that I had to set an example, and that I never thought possible to do otherwise. I was sure that others would subsequently fly after me. At the end of our conversation, Lenin shook my hand strongly, wished me success in my work, and promised support.

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"Lenin made a tremendous impression on me: that night I could not sleep. Pacing up and down in my small room, I thought of the greatness of this man – our country is ravaged by war, there is a lack of bread, of coal, and the factories are at a standstill, but this man who controls this huge country finds time to listen to space flights. It means my wishes will come true, I thought".

Having completed his story, Tsander stopped speaking and began to pace the room excitedly; he was thinking, and one could feel him going through his meeting with Lenin once again.

After his encounter with Lenin, Tsander devoted himself with renewed energy to the construction of a spaceship.

In those days the Soviet Union had not yet overcome the ravages of the civil war, and it was impossible to apportion large sums for the purchase of equipment and special materials needed for the solution of the rocket problem; still the conditions contrasted greatly with those under which Tsander worked before the revolution.

Before the revolution, Tsander as well as Tsiolkovskii were on their own. Now Tsander felt the support of the Soviet society; this increased his strength and greatly helped him in his work.

From 15 June 1922 to 15 July 1923 Tsander devoted himself entirely to the construction of his spaceship and consequently had to leave his work at the "Motor" factory. The workers at the factory helped him a great deal financially. In April 1923 during a speech at the factory, Tsander stated: "I hope sincerely that at a later stage, the possibility of handing over the execution of my project to our factory will arise."

We found notes of this speech in the archives. Reading the speech, it was difficult to remain unaffected by this lucid mind and by the romantic and great personality of this talented space flight enthusiast.

"Who knows? The inhabitants of other planets may be more intelligent and better organized than the inhabitants of Earth. Their discoveries, inventions and achievements could contribute much to man's welfare. Or if we were to populate the other planets, it would be possible to extend man's life to 100 or 120 years."

The theoretical calculations of this scientist-inventor are of particular interest*:

"My spaceship consists of an airplane fitted out with a high-pressure aviation engine. The engine would use liquid oxygen, and gasoline or ethylene or hydrogen, depending on which of these would be found more suitable during experiments.

"This engine would start the propellers and the plane would leave Earth. The speed would increase proportionally to the height. At a height of 28 kms the aviation engine would be cut out and the rocket engine would take over with a force of 1500 kgs. Then, with the aid of a special mechanism, we would transfer parts of the airplane to the boiler. We would then get liquid aluminum, which combined with hydrogen and oxygen furnishes us with an excellent fuel. The speed of flight would increase constantly as a result of rocket engine thrust, also increasing the height. At approximately 85 kms above the Earth the plane would have vanished. It would have melted in the

* Style changes have been made in the excerpts from the speech.

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boiler and the molten metal would be used as fuel leaving only the rocket with small wings and rudders and the manned cabin.

"From these calculations we would have attained sufficient speed to leave Earth and to fly over to other planets. The speed necessary before rotation around the Earth may be achieved should be 8 km/sec. In order to leave Earth and enter space the necessary speed would have to be 11.3 km/sec, and in order to reach another planet – Mars – 14 km/sec are required. It is much easier to attain these speeds in space than it is within the Earth's atmosphere.

Tsander in 1922

"In order to return to Earth, it is necessary to reverse the rocket motor, thus slowing the flight speed. It is possible to achieve a gliding descent or else to make use of a small engine".

Tsander has expressed his belief that "humanity will fly out of its childhood nest, into the huge world", and that "astronomy as no other science beckons humanity to unite for a longer and happier life".

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On 20 January 1924 he lectured at the theoretical section of the Moscow Society of Amateur Astronomers. In his personal archives we have found the résumé of the above lecture.

These documents are of great interest and show the wide scope of problems he touched upon when working towards the solution of problems of interplanetary travel, 35 years ago. The complete résumé follows.

RÉSUMÉ OF TSANDER'S SPEECH*

This speech about his spaceship and flights to other planets was read before the theoretical section of the Moscow Society of Amateur Astronomers on 20 January 1924.

At the beginning Tsander read the article entitled "Flights to Other Planets" (Perelety na drugie planety) which he wrote for the magazine "Samolet".

In this article he put forward a series of proposals constituting a new method for interplanetary travel: a combination of airplane and rocket, and the use of solid components of the ship's structure as fuel. He explained his sketch of such a spaceship and demonstrated the advantages of this method of flight as compared to that of rockets: the rockets are smaller, the material stress is lessened, experimental flights are easy, the acceleration is low, and a gliding descent is feasible.

He then gave an estimate of the rocket performance and of the determination of temperature, pressure, velocities and the rocket's internal friction**; he presented diagrams of these values and efficiency formulas characterizing the transfer of energy during flight. He showed that the ratio of the outflow velocity to the maximum velocity attainable was approximately 0.8; the percentage loss resulting from the friction of gases against the rocket walls was rather small.

He then moved on to estimates of velocity, time and flight altitude, and discussed the influence of friction and the lifting-performance; these values were shown on a diagram as functions of the rocket's decreasing weight. A table of mean flight efficiency was shown, and an area where a test of flight without oxygen is desirable, was pointed out.

The efficiency of certain metals and their rate of discharge when used as fuel were discussed. In the second part of his speech, he presented briefly his estimates of the minimum and total additional velocities required by the spaceship to overcome gravity and to reach the closest and more distant planets. It was proved that this additional energy is unimportant for reaching the planets Mars and Venus, and that in the instance of other planets this small additional energy would suffice when orbiting Mars and Venus in the upper layers of their atmospheres. The speech included formulas and tables shown with the aid of a slide-projector. Towards the end a recent book by Obert was shown, and in addition the results of experiments by Professor Goddard on the escape velocities of gases were

* The résumé is printed strictly in accordance with the original without any stylistic corrections.

** Reference being made to a rocket engine.

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given*, confirming the estimates reported in the relevant part of the speech."

In his summary of the speech, ** Tsander puts forth once again his idea of the possibility of using mirrors and screens instead of rocket engines during flight in space.

At this meeting, the section upheld a proposal by Tsander to organize within the USSR a "Society for the Study of Interplanetary Travel", and within a short while it was created. F. E. Dzeržhinskii, [Dzierżynski], K. E. Tsiolkovskii, Ya. I. Perel'man et al., were honorary members of this society. The chairman of the society was the pamphleteer G. M. Kramarov. This society existed for almost a year, and Tsander played a very active part in it.***

Eager to see as early a start as possible on work related to the various practical problems of spaceships, Tsander proposed, on 15 July 1924 in a speech before the society's research section, to organize the section's activities under the following divisions:

1. theoretical research;
2. construction work;
3. production and laboratory work.

He recommended the following plan****:

1. Testing small rockets powered by different fuels.
2. Testing rockets inserted one into another.
3. Constructing and testing folding and nonfolding aircraft models of various types, propelled by rockets and engines or by rockets only.
4. Testing high-acceleration effects in specially-constructed centrifugal machines.
5. Constructing and testing engines running on liquid oxygen or by means of solar energy.
6. Testing diving suits and safety devices for high-altitude flights.
7. Testing an apparatus for regenerating the exhaled air.
8. Investigating lightweight hothouses.
9. Testing, with the aid of a wind tunnel consisting of two rockets joined together at their wide section, parts of spaceships under the effect of low pressure and high velocities, determining resistance, lift, and heating.
10. Investigating the upper atmosphere by rockets, free and sounding balloons, and photometric observations of twilight.
11. Testing very thin sheets for screens.
12. Testing coils carrying an electric current and enclosing iron filings.

In 1924 a report was received that on 4 August of that year, Professor Goddard in America had sent a missile to the Moon*****. A debate on this news was arranged by the Society for the Study of Interplanetary Travel, and was scheduled to take place on 1 October 1924 in the large auditorium of the Physical Institute of the First University (in the MGU Building on Mokhovaya Street). Tsander later reminisced that the number of people

* The speed of gas ejected from rocket engines is considered. – Editor's note.

** The summary may be found in Appendix 1 to this collection.

*** Material on the work of this society was recently discovered, and will be prepared for publication.

**** In abridged form. For the full plan, see Appendix 2.

***** It was learned later that this was a false report.

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who came to hear the "truth" about Professor Goddard's dispatch of a missile to the Moon was so great that they could not all be accomodated; the horse militia had to be called out to keep order, and the debate had to be repeated twice more – on 4 and 5 October.

Participating in the debate, Tsander stated that he had constructed a spaceship which consisted of two aircrafts with a jet engine. He discussed the spaceship's capabilities in take-off and gliding re-entry, its flight safety, and the possibility of starting and shutting off the jet engine at will. He also presented his ideas on the creation of interplanetary stations, which could receive spaceships from Earth and dispatch them for further flights or back to Earth; he then went on to speak of flights to Mars and Venus. He pointed out that three kinds of engines could be employed for distant interplanetary travel:

a) metal-fueled jet engines;
b) mirrors utilizing solar energy;
c) coils containing electrified iron filings.

Tsander also spoke about lightweight (from the point of view of aviation) hothouses which could supply fresh vegetables on cosmic flights. In addition, he explained the significance of interplanetary flights for science, particularly for astronomy. His talk was illustrated by slides. Finally, he discussed the aims of the Society for the Study of Interplanetary Travel.

Tsander remained an indefatigable promoter of the idea of interplanetary travel throughout his life, and strove to attract as many people as possible to developing this field. In this he was successful. He made speeches in Moscow, Leningrad (here the famous astronomer, Professor C. P. Glazenap presided), Kharkov, Saratov, Tula, Ryazan, and other cities.

In 1924 Tsander began to write actively for publication, and his first article, "Flights to Other Planets" (somewhat abridged) appeared in the July 1924 issue of the Journal "Tekhnika i Zhizn' ".

At the beginning of this article, Tsander wrote:

"As I am interested in mathematical and structural investigations related to interplanetary travel, I have for some years been doing some computations and have come to the conclusion that *flights to other planets will become feasible, in all probability, within the next few years*".

This article contained the first formulation of the idea of utilizing parts of a rocket spaceship as fuel. It should be noted that this idea had already been mentioned in a manuscript dated 11 March 1909, and that in 1917 Tsander designed and had constructed a crucible for use in experiments in burning molten metal. *Thus, it is Tsander who must be credited with the concept of using metal as fuel.*

The idea of using separate parts of a rocket ship as a form of fuel was one which Tsander persistently strove to realize throughout his life. The first rocket he constructed to run on liquid fuel, in 1932, was originally designed also to run on metal fuel.

In 1924 he also wrote an article entitled, "Description of Tsander's Spaceship" (Opisanie mezhplanetnogo korablya sistemy F.A. Tsandera); on 8 April of that year Tsander sent it to the Committee on Inventions, as a statement. The statement was not published until 1937, when it was

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Poster announcing the debate of 4 October 1924

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included in the Journal "Raketnaya Tekhnika" No. 5, in slightly abridged form*.

In this article Tsander developed further the idea of using metallic fuel. He considered that several parts of a spaceship should be made of various materials such as aluminum, magnesium, and various plastic substances, that give great heat when burned in rocket engines. As the spaceship ascended, these parts (tanks, wings, etc.,) would become superfluous. The idea was that they should be retracted by a special device into a special compartment of the spaceship, ground, transferred to a boiler, melted, and fed in molten form to the jet engine as fuel.

The above-mentioned papers also dealt with a number of problems related to the practical realization of interplanetary flights. Tsander's idea of using wings on spaceships is presented for the first time, and the superiority of wings over parachutes, for descent to Earth or to other planets possessing an atmosphere, is established. He also mentions the superiority of winged rockets over wingless types for ascent in the Earth's atmosphere or in that of some other planet. Tsander's idea was that a winged cosmic rocket should have a compound engine consisting of a piston engine with propellers, or a jet engine for flight in the atmosphere, and a rocket engine for use beyond its limits.

The article also contained a description of the construction of a spaceship, which represented a combination of two aircraft, the one large, the other small with the latter mounted inside the former. The body of the rocket serves both aircraft, while the wings, tail fins, and propeller engine belonging to the larger aircraft are of foldable construction, so that in flights in outer space they might be used as fuel. For such flights, the length of the rocket's body is similarly reduced so that at the end of the flight, before re-entry into the atmosphere, we are left with the smaller aircraft only, which can glide easily to Earth.

According to Tsander's calculations, the use of wings would allow a 15- to 20-fold reduction in the weight of the rocket; it would reduce sharply the dangers connected with re-entry from outer space, and would free the crew from the effects of high acceleration, etc.

A model of Tsander's spaceship was shown at the interplanetary flight exhibition held in Moscow in 1927.

In 1924 Tsander conceived the idea of bringing out a popular-science book on "Flights to Other Planets and the Moon" (Polety na drugie planety i na Lunu). In the book's preface, he states:

"The idea of producing a popular-science book on flights to other planets came to me when, at the invitation of the Art-Lectures Bureau of the Political Directorate of the Leningrad Military District, I began giving lectures on the theme of flights to other worlds. At one of these lectures a representative of the Ryazan Provincial Administration of Literature and Publishing Houses proposed that I write a book on this theme.

"The main purpose of this book should be: 1) to acquaint a wide group of readers with the achievements, to date, of a theoretical and practical character, implying the great probability that interplanetary travel will become a reality in the near future; and 2) to prepare scientific workers by

* See below, "Description of Tsander's spaceship" (Opisanie mezhplanetnogo korablya sistemy F. A. Tsandera).

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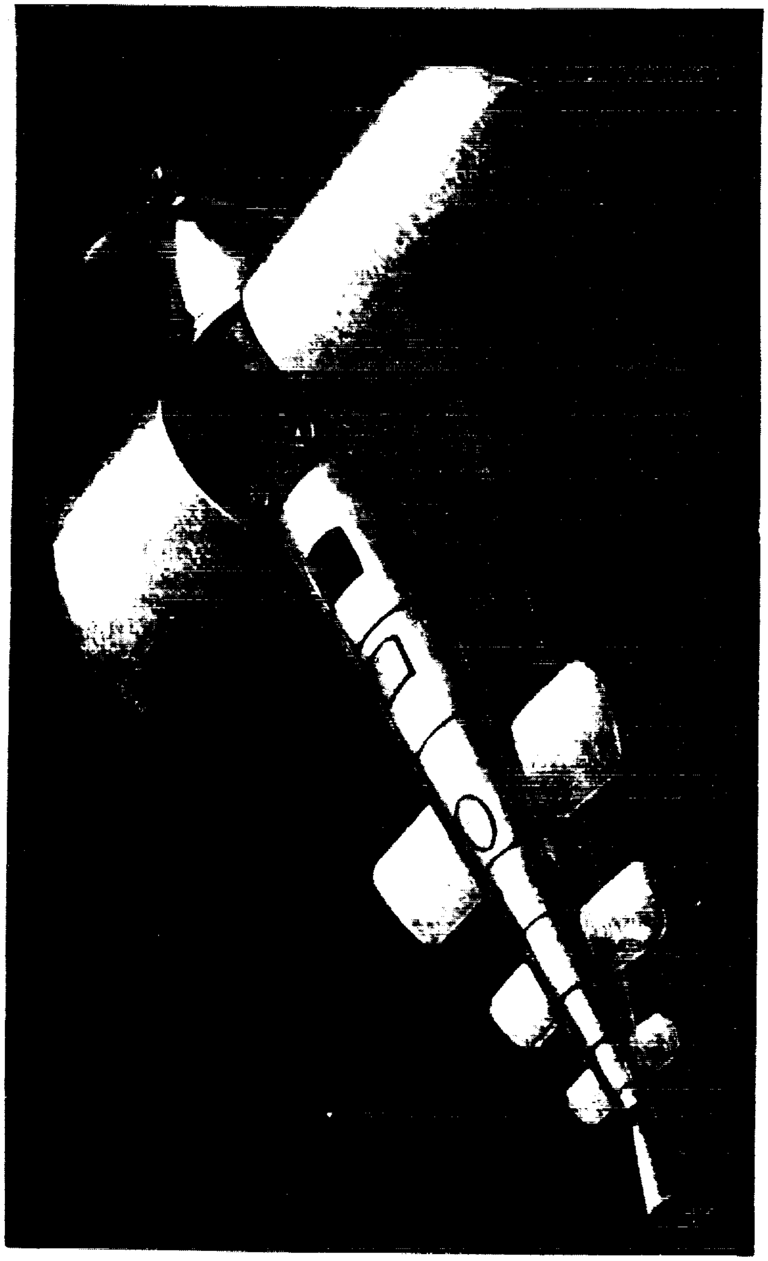

Tsander's model of a spaceship

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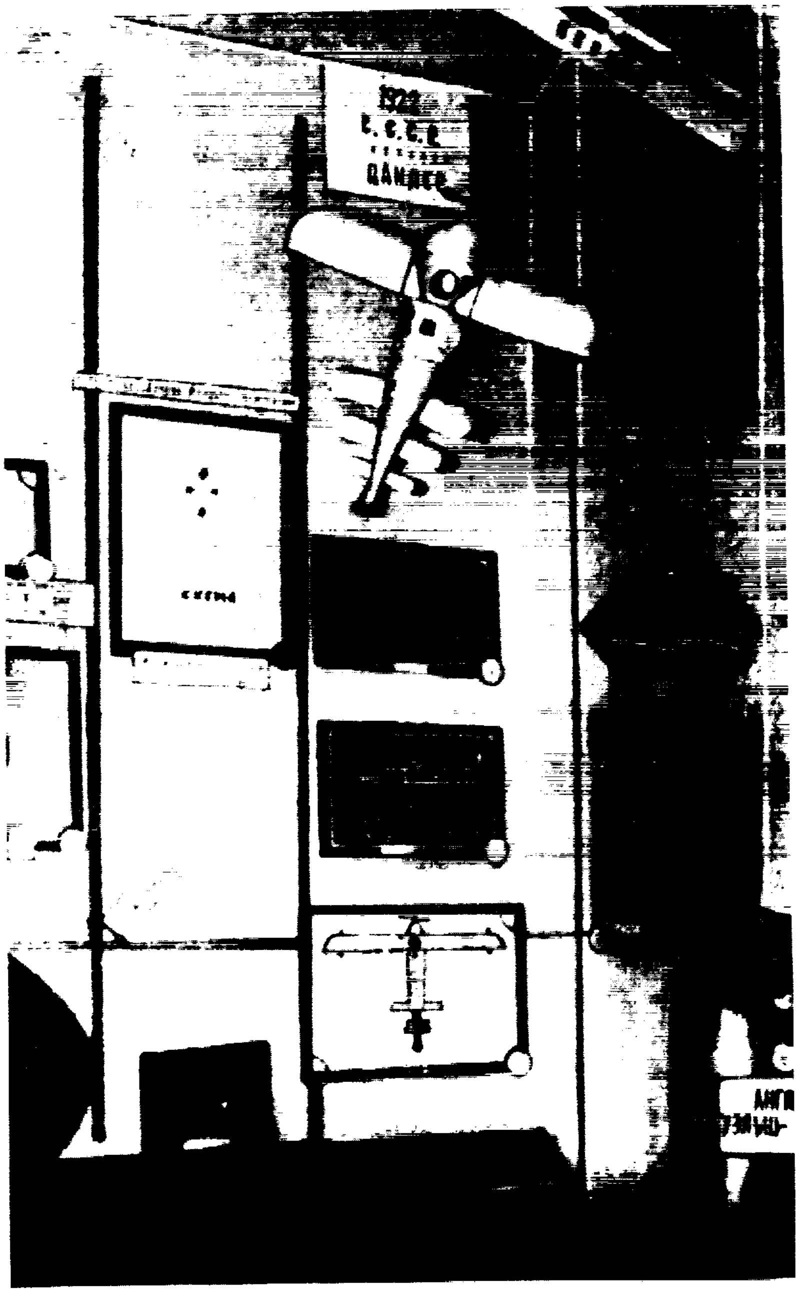

Tsander's corner at the interplanetary flight exhibition held in Moscow in 1927

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transmitting to them, in the form of a survey, several results of my computations. My scientific works will be published somewhat later."

He stated further: "I have in draft form a total of approximately 650 large-format pages of various scientific and technical computations in the field concerned; all this material is written in shorthand, whereby it is contracted four- or five-fold. In addition, there are 27 structural sketches.

"I will give the results of my scientific investigations the utmost attention. I hope that the thoughts I have expressed will attract to this sphere new investigators and engineers.

"This is a vast and rich field but man has as yet scarcely touched it by means of scientific investigations and technically developed constructions and experiments."

In 1926 Tsander proposed to publish a book on "Flights to Other Planets; the First Step into the Vast Universe (Theory of Interplanetary Flight)" (Perilety na drugie planety; pervyi shag b neob' yatnoe mirovoe prostranstvo; teoriya mezhplanetnykh soobshchenii). A ten-page list of the subdivisions of this book was found in Tsander's archives. The main subdivisions were the following*:

I. Preface. The investigator's and inventor's path.

II. Introduction. Subject of the book. Outline of the problem.

III. Calculations for the design of rockets for spaceships.

IV. Theory of interplanetary flight.

V. Calculations for rocket flights through outer space around a large part of the terrestrial globe. Minimum velocities to be given to a rocket to ensure a given range of flight. Numerical data.

VI. Calculations for flight in outer space with the aid of light pressure.

VII. Apparatus for the conversion of solar rays into low-velocity cathode rays. Flight with the aid of cathode-ray pressure.

VIII. Calculations of charged spheres repelled by the planets and the Sun. Stresses in the material of these spheres. Repulsion and attraction forces. Attainable velocities. Methods of charging the spheres.

IX. Design and calculation of a spaceship and of its engine.

X. Use of lightweight hothouses and the closed cycle necessary to sustain life in airtight quarters in a spaceship, in an interplanetary station, on the Moon, and on other planets possessing an atmosphere.

XI. Approximate outline of theoretical and experimental investigations of materials and structures stuitable for interplanetary flight. Conditions of life in interplanetary space.

XII. Brief survey of results achieved.

XIII. Prospects for the future. 1) The near future. 2) The distant future.

Some of the author's calculations, namely:

1) On the advantage of accelerating flight by means of a rocket, when the rocket-flight speed is high;

2) Calculations for the design of rockets for spaceships;

3) Flights to other planets;

4) Calculations for the flight of a spaceship in the atmosphere,

and several other computations were sent by Tsander to the Scientific

* A full list of subheadings will be found in Appendix 3 to this collection.

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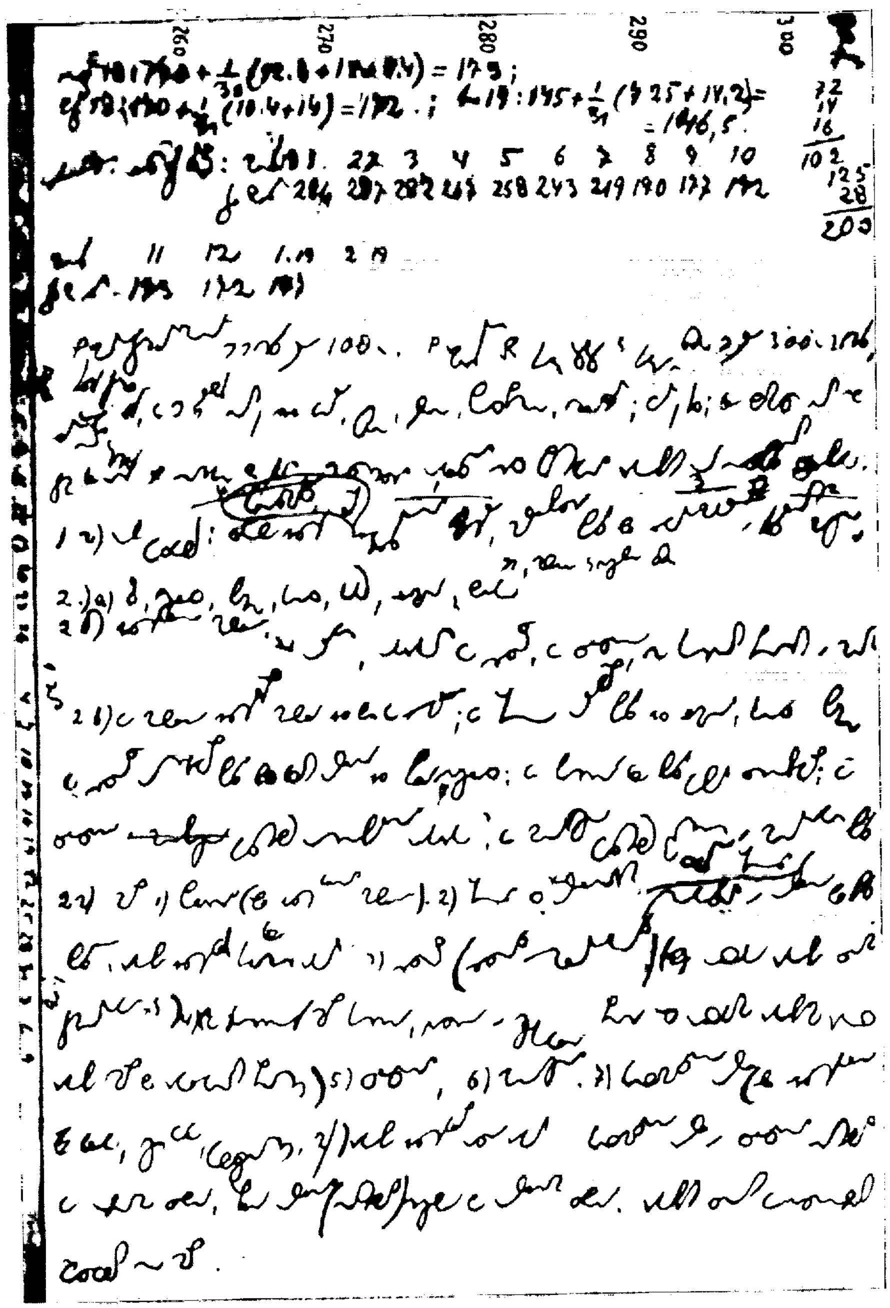

Sample of Tsander's stenographic script

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Council of the NARKOMPROS of the RSFSR which on 15 October 1926 transmitted the material, under file No. 141,930, to Professor V. P. Vetchinkin for his conclusions.

On 8 February 1927 Vetchinkin addressed the following conclusions to The Main Administration of Scientific Institutions:

"Tsander's calculations of interplanetary travel and the design of a spaceship rank undoubtedly as foremost contributions to the literature on this subject.

"K. E. Tsiolkovskii was the first scientist, as early as 24 years ago, to point out that rockets were the only possible means of reaching outer space. He also demonstrated the possibility of reaching, with their aid, cosmic velocities of over 11 km/sec, and the possibility of breaking out of the atmosphere employing only existing fuels. However, he provided no engineering solution to the problems of the rocket, and his proposed way of take-off overcoming the force of gravity by a reaction force is not entirely sound.

"Foreign scientists – Esnaud-Pelterie, Goddard, Oberth, and Vallier – followed the path of Tsiolkovskii, repeating his work and advancing it theoretically (Oberth) and experimentally (Goddard).

"An essentially new contribution to this difficult problem was made by Tsander in his three proposals:

1. To provide the rocket with wings for flight in the atmosphere and for gliding descent. This would make it possible to use a lighter rocket using low acceleration ($j<g/2$), in place of the high acceleration required in Tsiolkovskii's method ($j>3g$), and to achieve considerable economy in fuel, braking the rocket down to 8 km/sec only and not to zero.

2. Flight with the aid of engines of a specially light type capable of only a half hour's continuous operation in the atmosphere's lower layers, where the rocket's efficiency is negligible and resumption of rocket flight occurs only after reaching rarefied air layers.

3. The use of solid fuel in the form of unnecessary components of the rocket itself as a supplement to ordinary fuel in order to raise the combustion temperature.

"In addition, he engaged in flight and descent calculations and in work on fundamental engineering problems, e.g., calculations on the design of the nozzle and its cooling system, which appears to be the chief obstacle in rocket flight.

"Unfortunately Tsander only delivered his reports orally but did not publish them. Meanwhile, W. Hohman published in 1925 a study in which he, too, proposed flight with the aid of wings and gliding descent. Possibly, Tsander's work, and his reports on it in the winter of 1924-25, influenced the appearance of this study.

"Thus, due to the lack of a possibility of publishing our work, we lose the credit for discoveries even in those instances where it undoubtedly belongs to the USSR.

"In the light of the above considerations, I consider it entirely essential that Tsander be given the possibility, as soon as possible, to prepare for publication, and to have published the works whose several headings he presented to Glavnauka.

8 February 1927. V. Vetchinkin."

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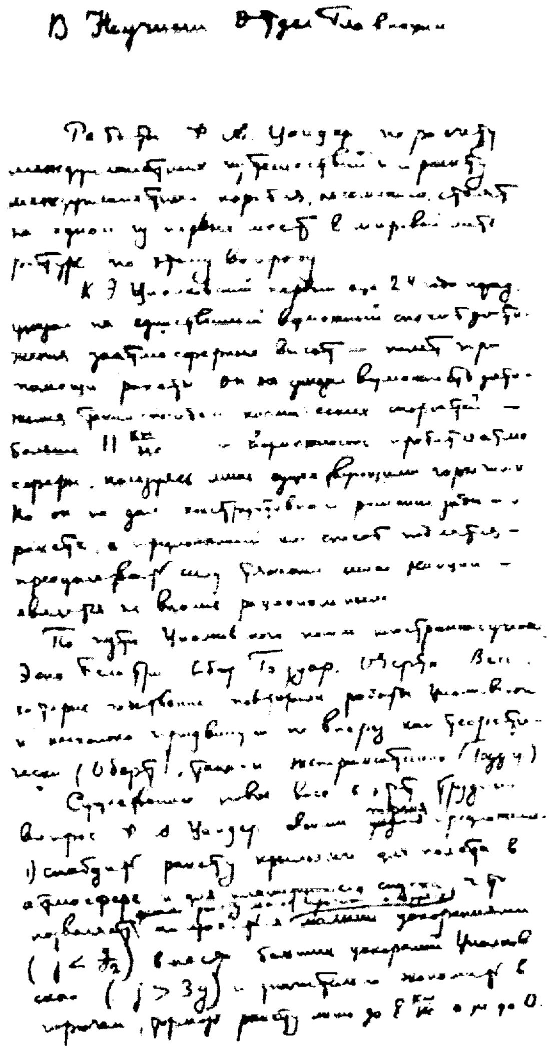

Facsimile of Vetchinkin's letter

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Facsimile of Vetchinkin's letter (continued)

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Several days after the submission of this evaluation, Tsander presented to the Science Department of Glavnauka a ten-page conspectus on the advice of Professor Vetchinkin, and on 23 February 1927 he requested authorization to receive, either from TsAGI or from Aviatrest facilities to work exclusively in the field of interplanetary travel, or to receive permission to prepare a book for publication on interplanetary travel and to submit regular reports on the progress of his investigations.

At the end of his application, he added: "Approximately 5000 to 6000 individuals are employed in the USSR in the aviation field. If an opportunity is provided even to one person, for the time being, to work exclusively in the field of high-altitude and high-velocity and interplanetary flights, this research would constitute only 1/5000th of the effort devoted to the field of conventional aviation which may be considered entirely permissable."

After four months, Tsander received Glavnauka's response (letter dated 7 July 1927, File No. 76,990), as follows: "It is not considered possible to satisfy your petition for assistance in completing your work on interplanetary travel."

Thus, Tsander's proposed books - "Flights to Other Planets and to the Moon" and "Theory of Interplanetary Travel" - were not written at that time.

In order to unify his work on the development of a spaceship, Tsander joined, in October 1926, the staff of Aviatrest's Central Bureau of Construction as a senior engineer.

On 30 November 1928, he presented a paper on "Preliminary Work on the Construction of a Jet Propelled Machine" at the XVth session of the Commission for Scientific Aeronautics of the Moscow Aerological Observatory, held at the First Meteorological Observatory of the Moscow State University. Tsander systematized all his reports with a view to writing a book on rocket technology in the near future.

At the end of 1929, Aviatrest, where Tsander was [still] employed, received an invitation from the Dutch Royal Club to participate in the Vth International Congress on Air Travel, and to present a paper. Tsander, among others, was asked to prepare a report for the Congress scheduled to open on 1 September 1930 at The Hague.

This proposal appealed to Tsander. He postponed the preparation of his book, and by 25 January 1930 he had already prepared a paper entitled, "Problems of High-Altitude Flight. Jet Engines Improved by Closed-Cycle Operation. Preparatory Tasks for Interplanetary Travel" (Problemy sverkhaviatsii. Reaktivnye dvigateli, uluchshennye rabochimi krugovymi protsessami. Ocherednye zadachi po podgatovke k mezhplanetnym puteshestviyam).

The paper was discussed at a technical meeting on 23 February 1930, then underwent editing and revision, and, on 5 April 1930 was submitted to Aviatrest under the title, "Problems of High-Altitude Flight and Preparatory Tasks for Interplanetary Travel" (Problemy sverkhaviatsii i ocherednye zadachi po podgotovke k mezhplanetnym puteshestviyam)*. For this paper,

* The original copy is dated 25 March 1930.

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based entirely on original material, Tsander utilized the results of past years of work and the article, "Flight of Long-Distance Rockets Beyond the Atmosphere" (Polet daleko letayushchikh raket vne atmosfery), which he had wanted to publish in the Journal "Tekhnika Vozdushnogo Flota"*.

Six months later, by 6 May 1930, the paper had been translated into French for sending to The Hague and had been inspected by Professor Vetchinkin who evaluated it highly.

However, it was decided to send no one to the Vth International Congress on Air Travel, for the reason that neither Aviatrest nor TsAGI were engaged in any work on interplanetary travel.

It may be considered certain that when he wrote the paper for the Hague Congress, Tsander knew only Tsiolkovskii's work and, of foreign literature, only Oberth's book in its first edition. Furthermore, Tsander took a position completely contrary to that of Oberth who opposed the use of rockets in combination with aircraft.

Concerning the theme of his paper, Tsander stated, in a letter dated 6 February 1930: "Unfortunately, I have not had an opportunity to acquaint myself with the newest German books (and books appearing recently in other languages) in the field of interplanetary travel."

After the decision not to send anyone to The Hague, Tsander wanted to have the unrevised version of his paper published under its former title in the Journal "Tekhnika Vozdushnogo Flota" **, but later revised it and changed the title to "Problems of Flight with Jet Propulsion Machines and Problems of High-Altitude Flight in General" (Problemy poleta pri pomoshchi reaktivnykh apparatov i problemy sverkhaviatsii voobshche). This was not completed until 20 July 1930. However, Tsander decided later against publication in the form of a journal article and reverted to the plan of publishing a book. He worked on the material once more and then completed the book. On receipt of consent for its publication, he submitted part of the manuscript on 9 May 1931 and the remainder on 15 May 1931; the book turned out to be twice the length of the paper.

The book came off the press in 1932 under the title, "Problems of Flight with the Aid of Jet Propulsion Machines". It dealt in detail (for that time) with problems of interplanetary flight.

Tsander states in the preface:

"...the purpose of the present book is to popularize the ideas of interplanetary travel. The author addresses himself to inventors in general, students, engineers, and astronomers, with an appeal to work in this field..."

It is to be noted that this was one of the first books ever to provide a comprehensive discussion of flight with the aid of rocket engines – comprehensive not only from the theoretical point of view but also in its presentation of the practical engineering aspects of the subject. One of the special features of this book was the fact that the author did not confine himself to an examination of the liquid jet engine, which proves unsatisfactory for use at low altitudes and velocities, but pointed out once more

* Tsander's letters of 3 December 1929 and 5 January 1930.
** Letter of 30 May 1930.

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the advantage of a jet or piston-type engine running on a mixture of pure oxygen and gasoline, for use in ascent within the limits of the atmosphere.

In this book Tsander proposed to use a new kind of thermodynamic cycle in order to increase the exhaust velocity; he also expounded the principles of practical thermodynamic and thermal calculations for the liquid jet engine. A considerable part of the book is devoted to questions of dynamics and flight conditions for a winged rocket in the atmosphere and in outer space, and also to determining the most advantageous trajectories for spaceship flights. Several of the calculations are given in outline only, together with the final results.

In 1932 Tsander proposed publishing a book under the title, "Calculations of Jet Engines and Their Combinations with Other Engines" (Raschet reaktivnykh dvigatelei i ikh kombinatsii s dvigatelyami drugikh vidov), and reached an agreement with the publishing house. Death, however, prevented him from realizing this plan; after his death only the detailed table of contents remained*. His more detailed calculations on the liquid jet engine were published posthumously in 1936-1937, in the Journal "Raketnaya tekhnika".

Simultaneously with the theoretical work on interplanetary flight with its long-term aspects, Tsander tirelessly worked for the practical solution of technological problems connected with the creation and testing of several components of the rocket and with the design of a spaceship.

On 20 December 1930 he began working at the Central Institute for Aircraft Motor-Construction, where he embarked on experiments with the first jet engine, known as ER-1**, fueled by gasoline and gaseous air. It is interesting to note that the ER-1 engine contained all the basic elements of present day liquid jet engines: a combustion chamber with a conical nozzle which was cooled by components of the fuel mixture, a feed system for the mixture components, electric ignition, etc.

At that time the testing station for rocket engines was only rudimentary and had very limited material facilities. Consequently, the dimensions of the (ER-1 type) engine were very small by today's standards; total fuel and oxidizer consumption per second was only 1.69 g. At the same time the diagram method of calculation and engine testing that Tsander worked out is the same as that employed with modern liquid jet engines. As may be seen from the calculations the thrust was determined by the formula

$$P = (m_{\text{ox}} + m_{\text{fuel}}) W_2,$$

and not by the formula corresponding to the jet engine

$$P = (m_{\text{ox}} + m_{p\text{fuel}}) W_2 - m_{\text{ox}} W_0.$$

Subsequently, in developing the idea of using atmospheric air to improve the thermodynamic cycle of the liquid jet engine, Tsander arrived at an

* See Appendix 4 at the end of this book.

** [In Russian OR-1 Opytnaya raketa (experimental rocket) number 1.]

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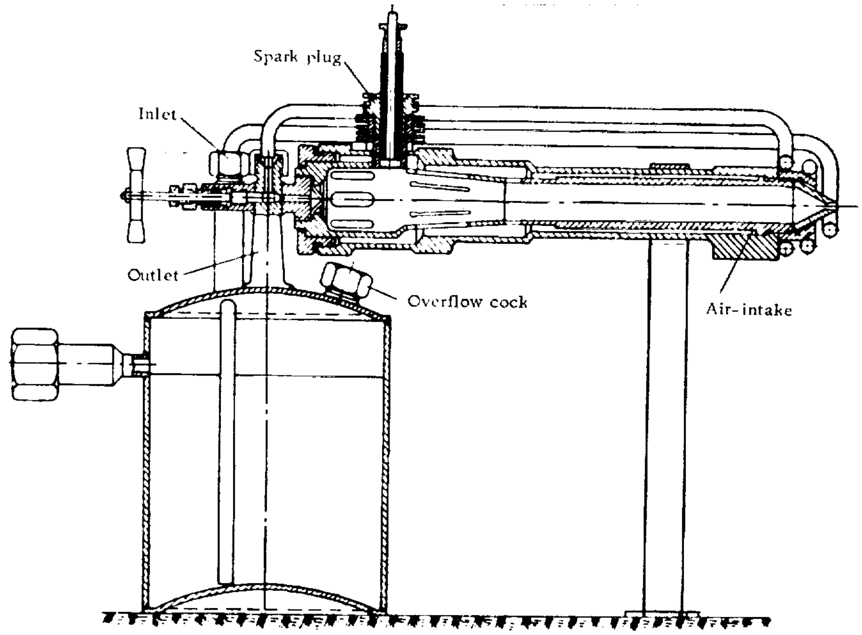
engine design with a direct and an inverse cone. This is how he described his first rocket engine, ER-1, in notes dated 30 September 1929:

"When I had completed all the theoretical calculations, I had to subject my method and calculations to a practical test, and obtain the initial experimental results essential for the creation of a second, more powerful engine. In addition, I had to use this engine for tests and initial results on the combustion of metallic fuel.



Schematic diagram of the ER-1 Engine

"Because of the shortage of funds, I unexpectedly conceived the idea of transforming a blowtorch into the first jet engine. I made this idea a reality. My first engine consisted of a transformed blowtorch, of the Lenoszhatgaz Trust Factory imeni Matveev in Leningrad. The gasoline tank was of one-liter capacity, the air pump diameter including the piston was 16 mm, the piston stroke was 107 mm, the inner outlet diameter of the torch nozzle was 22 mm. I reconstructed the nozzle and encased it in a jacket into which air was forced under pressure. There was a combustion chamber inside the jacket arranged with the aid of a special pipe. A disposable conic nozzle was attached to the end of the pipe; this ensured exhaust velocities exceeding the speed of sound.

"The copper pipe for liquid gasoline was replaced by a longer one coiled around the conic nozzle for heating the gasoline. In addition, the tank was equipped with a pressure-gauge to control the feeding of the gasoline and a nipple for air release. A thermometer for measuring the temperature of the tank lid was attached to the tank. There was a special gauge-cock for the control of fuel consumption.

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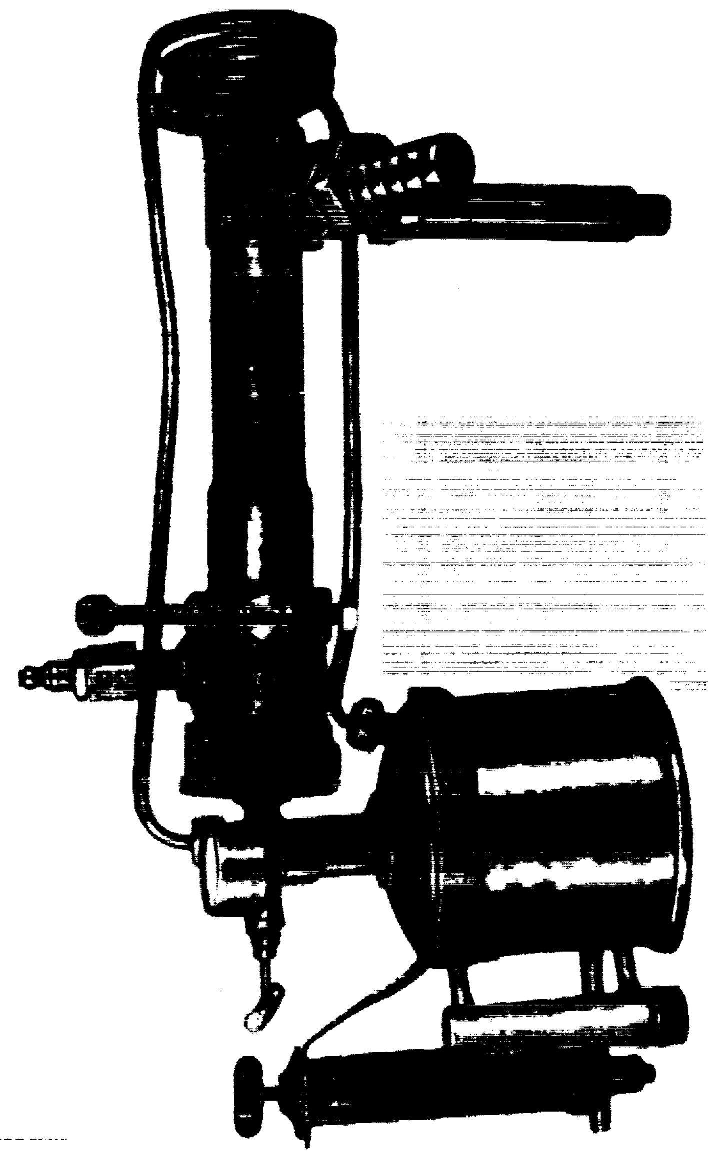

General View of the ER-1 Engine

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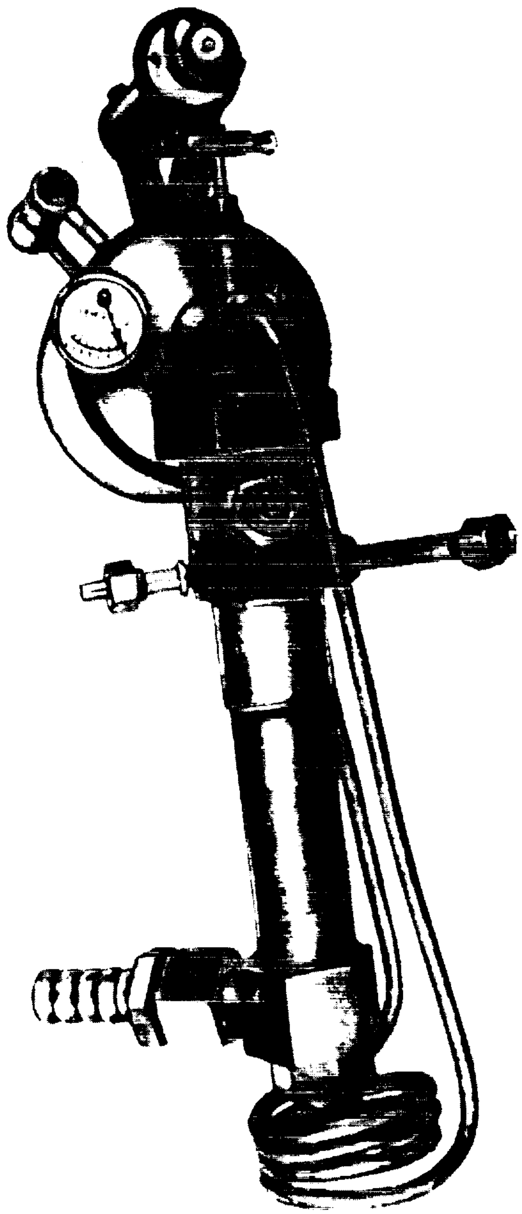

ER-1 Engine (view from above)

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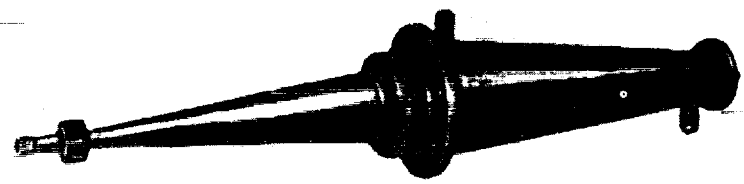
"Compressed air for combustion and cooling of the combustion chamber, was conducted into a cooling section via a connecting pipe attached to the casing in front of the nozzle.

"Ignition of the mixture was accomplished by means of an electric sparkplug screwed into a socket; I called this engine ER-1."

By July 1932 Tsander had conducted more than 50 combustion experiments with the ER-1 and numerous cold experiments. He developed a detailed method for successive experiments and, characteristically, his experiments with new structures were a gradual transition to more complex experimental conditions. He was capable of repeating an experiment countless times in order to ensure reliability of the required results.



Direct and inverse cone for the ER-1 engine

Individual stages of the combustion experiments with the ER-1 are of interest. When it became necessary to measure the thrust of this unique engine, it became apparent that there did not exist a special dynamometer or other measuring apparatus for this purpose, nor were funds for the construction or purchase of such equipment available. Tsander resorted to his native common sense. He suspended the small engine on metal wires and started it. The jet pressed upon a small metal disc attached to calibrated scales on whose other side were placed the usual weights; a home-made pointer indicated the thrust of the ER-1.

Tsander encountered a variety of other difficulties, both in constructing and testing the ER-1, but managed to overcome all of them.

He worked intensively, produced theoretical calculations, and went on designing and building his first jet engine, the ER-1. During the same period he designed a special building for an experimental station, hoping to have it erected in the near future. The design included space for the installation of machines for the production of liquid oxygen and other liquefied gases and for their storage, laboratories of all kinds for the testing of materials at low and high temperature, separate compartments for experiments in the combustion of metals and preparation of new alloys. The station would be used for testing both liquid rockets on vertical and horizontal stands, and compound piston and rocket engines. Provision was included for separate quarters for testing piston engines operating on liquid oxygen and in combination with liquid jet engines.

A testing stand was developed for mechanisms designed to retract metal components of aircraft components to be melted in special boilers. In addition compartments were reserved for developing gyroscopic instruments for stabilizing rocket flights, a start-control device, etc.

It was Tsander's plan to develop, test and produce in this building all parts and components of the rocket and of the spaceship. However, he was

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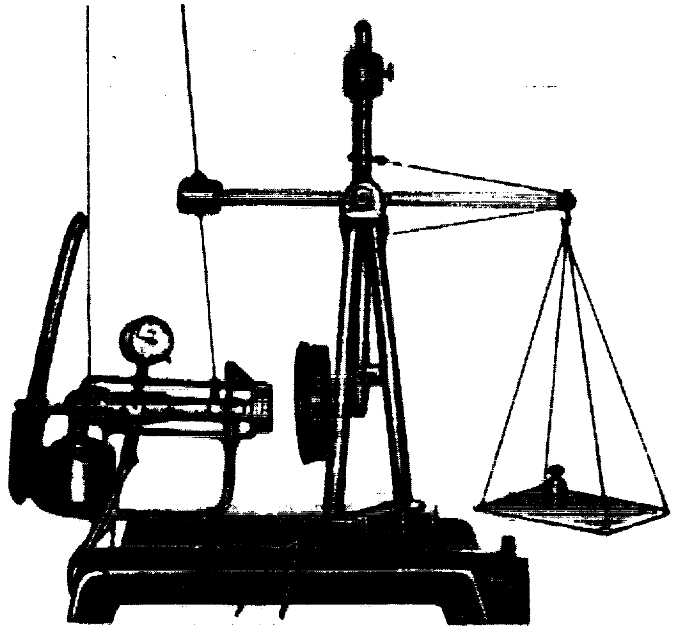

The ER-1 engine suspended on two wires, and the weights for measuring the thrust

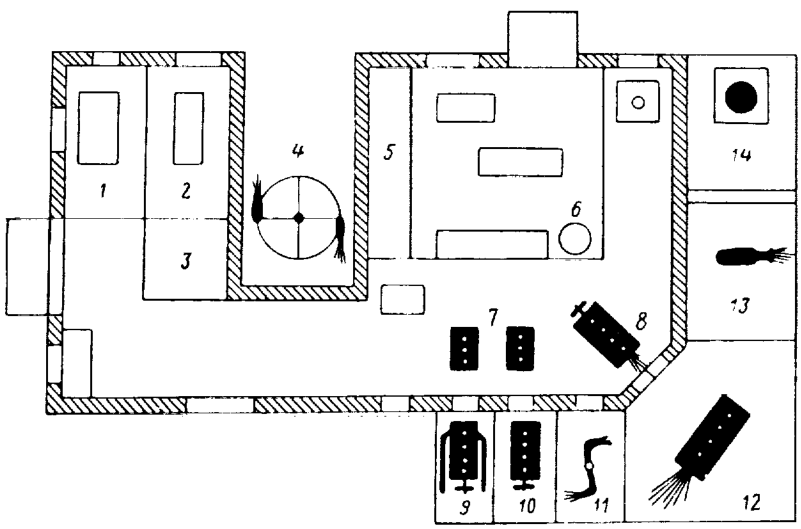

Diagram of the Testing Station and Laboratory Designed by Tsander

1-Setup for liquefying gases; 2-setup for liquefying oxygen; 3-storage reservoir for liquid gases; 4-testing turntable; 5-exhaust hood; 6-machine for mechanical testing of materials; 7-piston engines; 8-compound engine; 9-compound jet-and-piston engine; 10-piston engine operating on carbureted oxygen; 11-jet rotor; 12-compound piston and liquid rocket engine; 13-horizontal stand for liquid jet engine; 14-vertical stand for liquid jet engine.

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not destined to realize personally his big dream. A jet testing station, a powerful one (for those days), comprising many buildings and compartments, was built only in 1935 by one of Tsander's former students.

In the thirties the Osoaviakhim, a voluntary organization, was performing many useful services in the Soviet Union, including provision of financial assistance to many individuals, such as inventors engaged in developing new technologies.

At the beginning of 1931 a jet-engine section was established within the Central Council of Osoaviakhim, and Tsander was appointed its Director. The members were specialists in different fields of science and technology.

In the second half of 1931 the section was reorganized into TsGIRD; a technical council under Tsander's chairmanship was put in charge of the group.

Combining the task of conducting the first Soviet experiments with the liquid jet engine and the duties of chairman of the Technical Council of TsGIRD, this tireless scientist and thinker, engineer and inventor, managed not only to be everywhere and help everyone, but to be connected in numerous ways with scientific and technical circles, as well as with the youth, in whom he placed great hope. Tsander understood well that the practical development of rocket technology required the participation of adequate numbers of engineers and technicians, and he therefore made an effort to involve as many students of the Moscow Aviation Institute as possible by giving them lectures, conducting group study courses, and entrusting them with certain technical calculations and with the construction of various rocket components, etc.

No matter with whom he spoke he constantly turned to problems of cosmic flight, especially flight to Mars.

In 1932, courses on jet propulsion were organized under the auspices of TsGIRD, and Tsander drew up a detailed study program. Many leading specialists were enlisted as lecturers: Vetchinkin, B.C. Stechkin, V. V. Uvarov, and others; Tsander constantly prepared and renewed his lectures.

At this time a number of enthusiasts in jet technology formed a group and decided to start practical work in the construction of rocket engines and rockets. This, in turn, resulted in the establishment in April 1932, by decision of the Central Council of Osoaviakhim, of a production group named GIRD.

The Moscow Group for the Study of Jet Propulsion (GIRD) played a significant role in the practical development of rocket technology in the Soviet Union.

However, this work did not begin smoothly. The sums allocated by Osoaviakhim were very limited, and the lack of a building in which to perform the work of production and construction was an even greater problem. In those days the attitude to rocket enthusiasts was one of scepticism. They were jokingly referred to as "lunatics", and the response to their application for a small building was a smile and the expression of a sincere wish that they might fly off to the moon as quickly as possible and not bother "serious" people. But the "GIRD-ers" did not despair, and continued energetically to seek quarters. Finally, they were lucky: an unoccupied basement was discovered at No. 19 Sadovo-Spasskaya Street, which, while somewhat dark and damp, was nevertheless spacious. Tsander's exultation at finding these quarters cannot be described.

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The basement was quickly leased, cleaned, relieved of its rubbish, and whitewashed; it was wired for electricity, and equipped with two half-broken machine-tools; soon the premises began to hum with activity.

Thus began GIRD's work. Many years have passed. Much is forgotten. What will not be forgotten is the enthusiasm, love, and faith in their work evinced by Tsander and his young colleagues when they began working in that basement. Many were the sleepless nights they spent, often on an empty stomach, in search of solutions of rocket technology problems.

One technical problem ran against the next, and at times everything was so confused that there seemed no way out. There were weeks and even months of successive failures. In the combustion experiments, rocket engines were burned up in one instant, since the temperature inside the combustion chamber reached 3000°, and no one knew how and with what to cool them. When a satisfactorily operating engine was at last achieved, it was discovered that the exhaust-speed from the nozzle should be 2500 to 3000 m/sec while only one-third of this speed was obtained. Often there were failures in the fuel-feeding system, and even some of the simplest mechanisms, such as, for example, the armature and the reducing valves, caused trouble and failed to operate at liquid oxygen temperature. There was also some difficulty with the ignition. In addition, there was a constant shortage of necessary metal and cutting instruments, not to speak of the absence of needed machine-tools.

The situation was particularly difficult as regards measuring instruments. These were not produced in the Soviet Union at the time, and GIRD had to design and manufacture their own instruments, inventing various ways of testing them.

In addition to everything else, there was an extreme shortage of funds: available funds did not even meet production costs, not to speak of the miserly salaries received by Tsander and his colleagues. GIRD was humorously referred to as the "Gruppa inzhenerov, rabotayushchaya darom" (Group of Engineers Working Gratis).

It was the solidarity of the staff of GIRD, their love of rocket technology, and their deep faith in what they were doing that overcame many obstacles.

Tsander went over to full-time work for GIRD in April 1932. Most of the other rocket enthusiasts worked for GIRD only in the evening and in their spare time.

Tsander's work in designing rocket engines and rockets attained full stride. He continued experiments with the ER-1 engine and began designing and later constructing a new engine, subsequently called ER-2.

This was a particularly fruitful period in Tsander's life. He found he had a following of students to whom he lovingly and patiently elucidated complex problems of rocket technology. By nature very modest, good-natured, and somewhat shy, Tsander, both at work and in his personal life, was inseparable from his idea of interplanetary flight. He was cheerful, loved jokes and laughter. When something went wrong, or when testing had to be interrupted for one reason or another, Tsander became very distressed but would then say, with a smile, "Just the same, we'll fly to Mars"; whereupon he would carry on. He was endowed with an inexhaustible store of creative imagination and with a gift for fostering creative enthusiasm among his students and followers. He loved people and believed in them.

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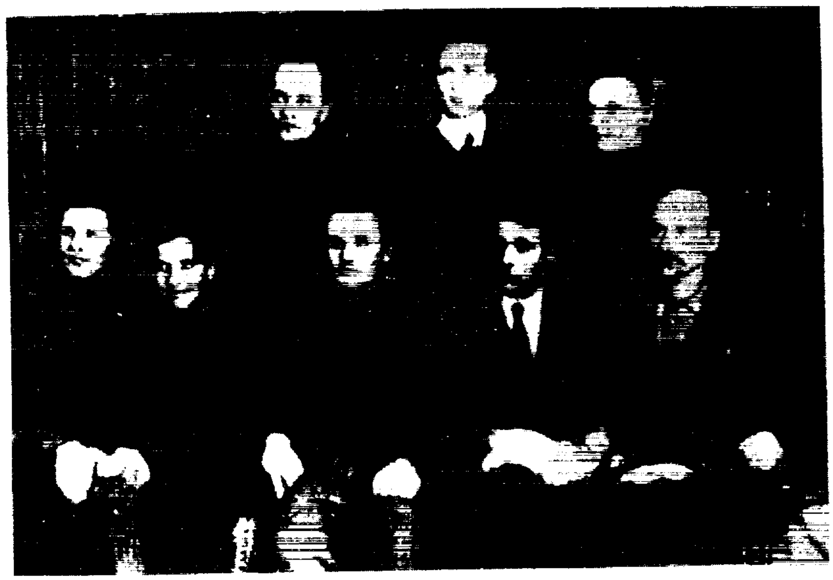
Tsander deeply respected the founder of rocket technology, K. E. Tsiolkovskii; at the latter's request, Tsander edited his works*. These two Soviet patriots and life-long devotees of the most complex technical subject – rocket technology – enjoyed an exceptionally warm and sincere mutual friendship.

On his seventy-fifth birthday, Tsiolkovskii received as a gift from Tsander a copy of the latter's book, "Problems of Flight with the Aid of Jet Propulsion Machines", together with the following letter: **

"Highly esteemed Konstantin Eduardovich!

"On your seventy-fifth birthday I send you warm greetings and sincere congratulations! My wish is that you may still be present at the time of the first flights into interplanetary space and to the nearest heavenly bodies.



Group of rocket enthusiasts, taken in 1931 (Tsander sits on the extreme right)

"The enthusiasm that is apparent in your books has inspired me from childhood, and we in GIRD, through the friendly collaboration of a number of inspired people, will continue research in the successful field of stellar flight, a field in which your works broke the eternal ice that had been obstructing the path to this objective.

"I have undertaken to edit and prepare for republication a number of your books on astronautics and rocket flight, and I am convinced that their widest dissemination will bring us a large number of new workers.

"The most important thing at the moment is the full development, testing and practical application of all proposed methods of jet flight.

* They were published in 1934, after Tsander's death.

** A copy of this letter is kept in the archives of the Academy of Sciences of the USSR.

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"When successful flights have occurred, the necessary funds for the widest possible development of this matter will, I am sure, be available, and it is desirable that you too, despite your advanced age, should still directly participate in the development of contemporary questions.

"With the present letter, I send you a copy of my book, "Problems of Flight with the Aid of Jet Propulsion Machines", in which I have presented my views on projects whose development will lead to flights to other planets.

"I also express my thanks for having sent me all your books, of which I acknowledge receipt.

"Long live he whose birthday we celebrate today!

"Long live work on interplanetary travel, to the benefit of all mankind!

Engineer F.A. Tsander
17 September 1932
Moscow."

Tsander began designing the ER-2 engine in September-October 1931 when he had not yet transferred to GIRD. The ER-2 engine constituted the first liquid jet engine, earmarked for installation as an independent engine, in the GL-1 glider of the B.I. Cheranovskii make.

This was a deliberate decision, as it was Tsander's belief that one of the steps towards flight in outer space was the development of the combined use of rocket and aircraft.

In designing the ER-2, provision was made to have the thrust controlled by regulating the feeding of the fuel mixture. Gasoline was chosen as fuel and the oxidizer was liquid oxygen. Fuel was fed from tanks with the aid of a pressure accumulator and liquid nitrogen. Cooling of the combustion chamber was accomplished through the use of oxygen, which was then burned, and the engine nozzle was cooled by water circulating through a closed-circuit system.

The engine system consisted of one tank for gasoline and two tanks for oxygen, all pear-shaped and equipped with a device for discarding them through round openings in the wing of the GL-1 glider. The oxygen tanks were double-walled, like Dewar flasks.

The system also contained a small water tank, a centrifugal water pump, a pressure accumulator with liquid nitrogen, a compensator for tank pressure control, two heat-exchange evaporators for the liquid oxygen, and the engine itself.

This system operated as follows: the engine was fed gasoline and liquid oxygen with the aid of nitrogen under pressure; this was pipe-conducted to each tank. The liquid oxygen first entered the evaporators, from which it traveled in gaseous form to the engine jacket, and thence, through special outlets, to the combustion chamber. The gasoline was fed directly to the spray nozzles, which were mounted inside the combustion chamber.

The tanks were kept at a constant pressure by using the nitrogen compensator; this consisted of a small tank containing liquid nitrogen and a charged cylindrical vessel through which hot water supplied from the small water tank circulated. The quantity of nitrogen evaporated depended on the extent to which the vessel containing hot water was immersed in the compensator. If, for example, the pressure in the system dropped, a small piston in the nitrogen compensator rose, lowered the hot-water

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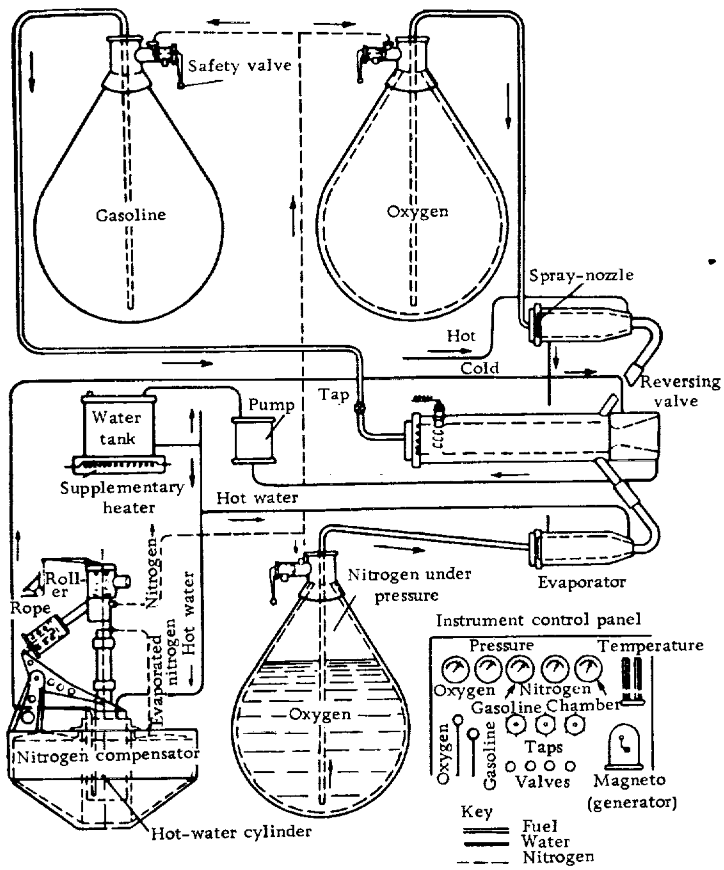
vessel more deeply in the liquid nitrogen, and thus increased the pressure. When the pressure became greater than necessary, the hot-water vessel rose (as if emerging from the nitrogen), gasification was reduced, and the pressure dropped*.



Assembly diagram of the ER-2 engine

The pressure accumulator and the tanks were equipped with safety valves, and the oxygen evaporators were doubled-walled cylindrical vessels between which hot water circulated. Liquid oxygen passing through the evaporators was gasified by heat exchange with the hot water and passed on to the cooling section of the combustion chamber.

The GL-1 glider, specially designed for the ER-2 jet engine and produced by Osoaviakhim in 1932, was of the "Flying Wing" type, triangular in shape. The wings, made of wood, consisted of three parts with the pilot's seat in one of them.

* The nitrogen compensator was not used for combustion experiments, and the pressure was regulated manually.

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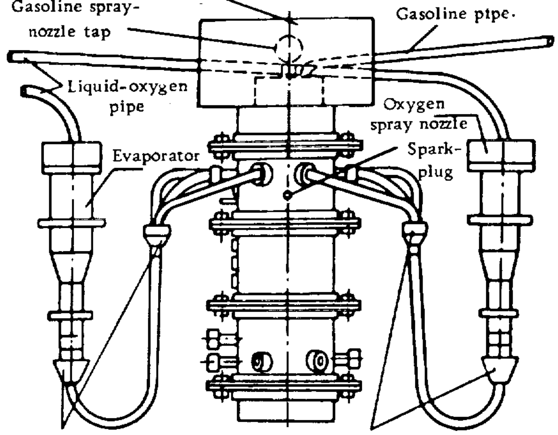
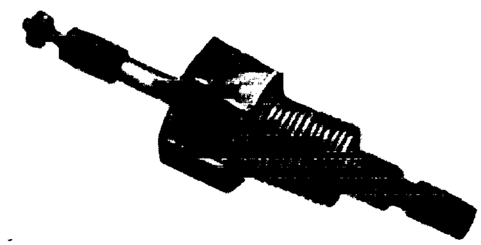
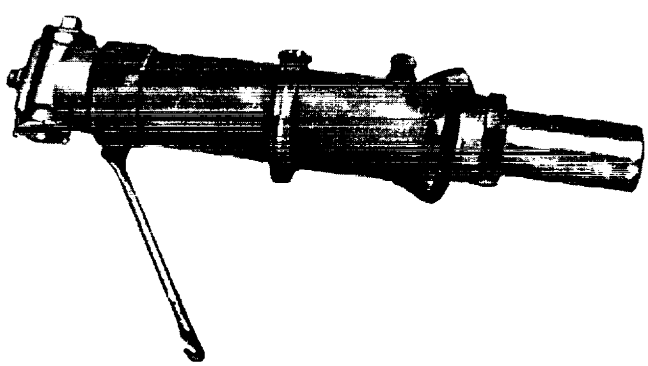


Layout of the fuel-combustion section of the ER-2 engine



Starting spark-plug of ER-2



Combustion chambers of ER-2, with nozzle and spray-valve

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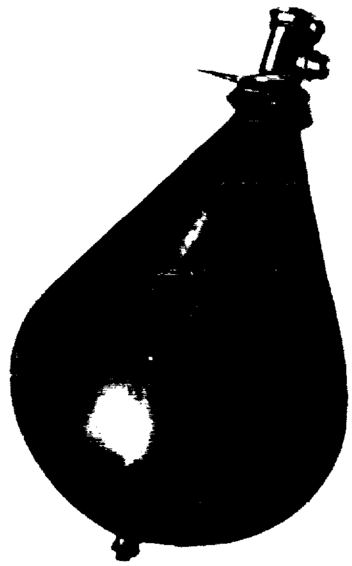
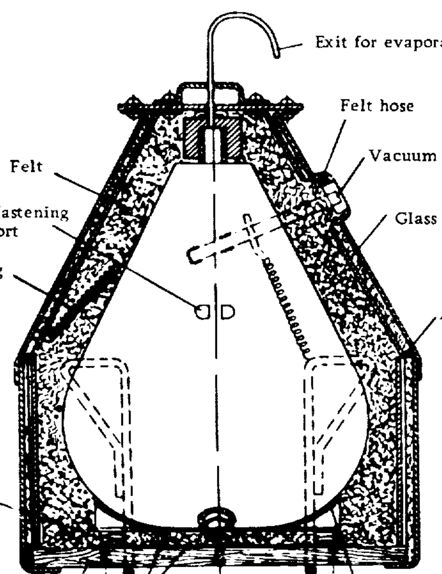


Gasoline tank with discarding device



Liquid-oxygen tank

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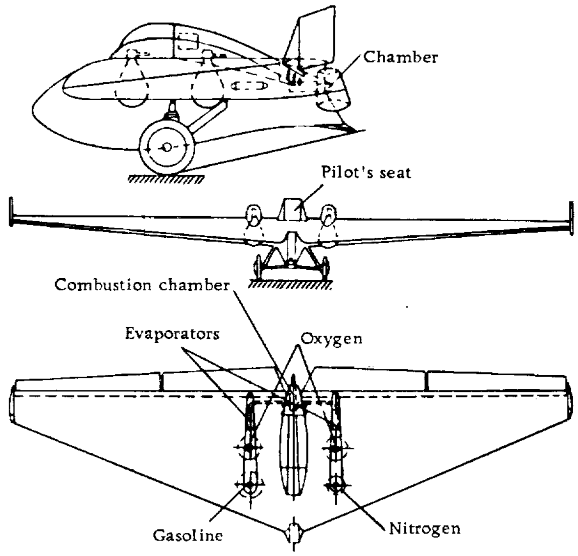
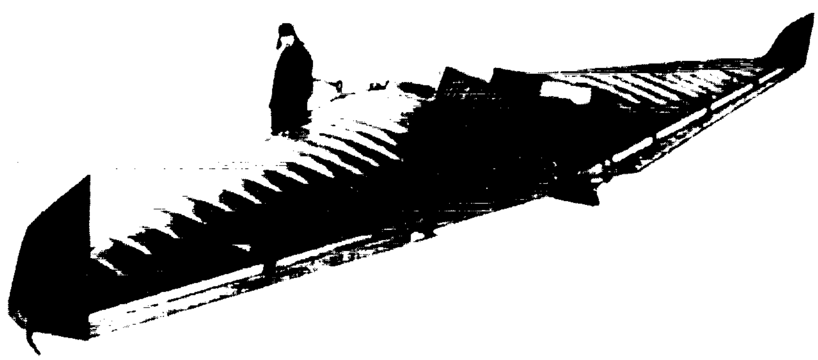
Duralumin beams were placed between the longerons to secure the tanks to the wings, while the tanks could be discarded at will. Steering was done manually and by pedals, while the control sticks and keels were located at the tips of the wing. By pressing the pedal the control deviated only in one direction (the direction in which the vehicle was being turned). The altitude sticks were attached by brackets to the rear center edge of the wing; the ailerons were similarly attached at the end of the wing.



The GL-1 glider, "Flying Wing", designed for use with the ER-2 engine



General view of the GL-1 glider

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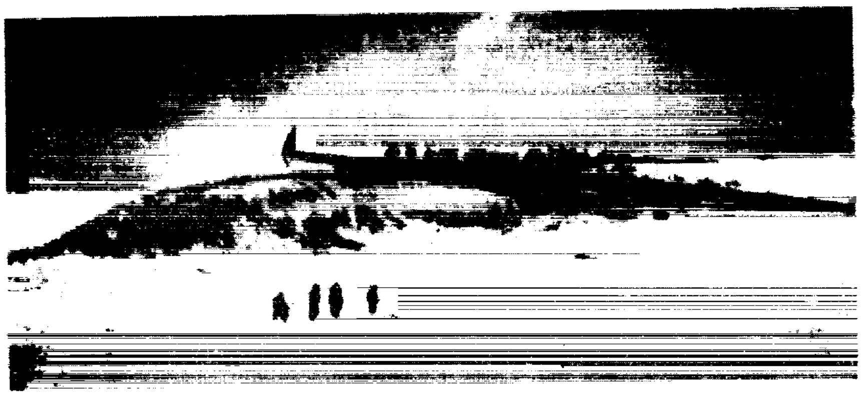
Dimensions of the GL-1:

Span	12.1 m
Length	3.09 m
Height	1.25 m
Wing area	20 m^2
Elongation	7.3 m
Weight exclusive of rocket engine	200 kg
Maximum quality	16

The design of the whole system and the ER-2 engine were completed in August-September 1932. Everyone was anxious to see the whole thing assembled as soon as possible, and to see the "live" flying rocket.



Take-off of the GL-1 glider

Despite the great enthusiasm and dedication of the GIRD staff, progress was not rapid enough. For this reason, the party bureau of GIRD decided in December 1932 to declare a week of 'storming'. The entire staff wholeheartedly supported this measure.

A committee of three, one of whom was Tsander, prepared the 'storming' plan. After its approval at a general meeting, the work assumed an extraordinary pace. Tsander seemed to have grown younger; he could be seen everywhere, helping everyone. Boundless was the joy when the ER-2 stood fully assembled on its stand in the GIRD building in the shape required for use on the GL-1 glider.

On 23 December the engine received the approval of a special committee, and several days later its builders were rewarded with a certificate of honor.

Tsander was not destined to see this engine in action. The combustion tests of the ER-2 engine, which began on 18 March 1933 in the vicinity of Moscow, took place in his absence, since he had gone to Kislovodsk to undergo medical treatment.

At the end of 1932, on completing the assembly of the ER-2 engine, Tsander began to design a new, more powerful liquid engine with a thrust

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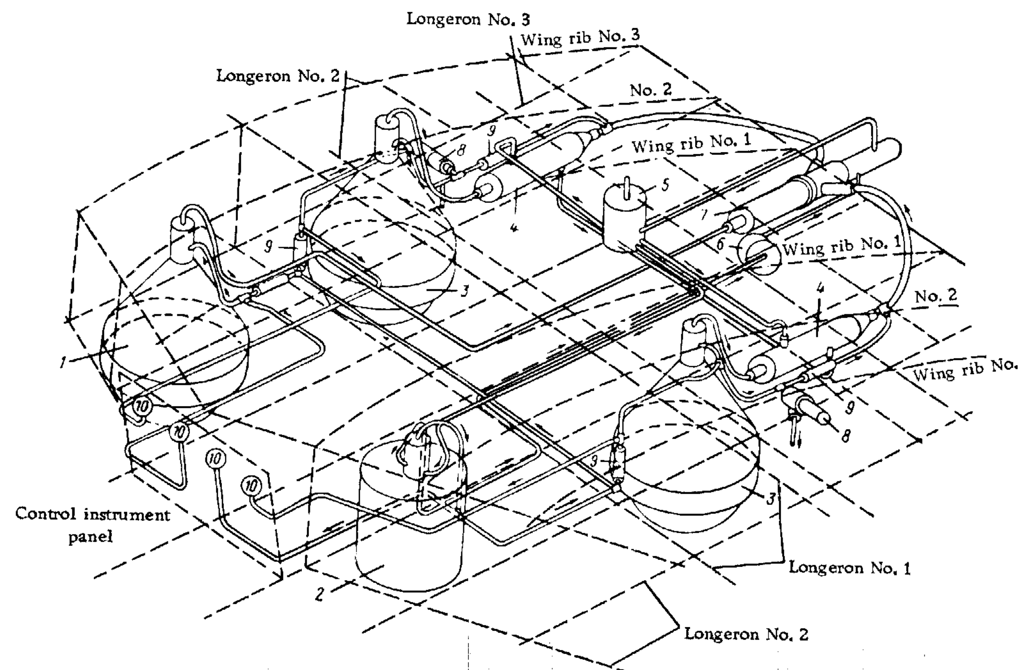

The ER-2 engine installed in the GL-1 glider

1-gasoline tank; 2-nitrogen tank; 3-oxygen tank; 4-evaporator; 5-expansion vessel; 6-pump; 7-combustion chamber; 8-safety valve; 9-reducing valve; 10-manometer.

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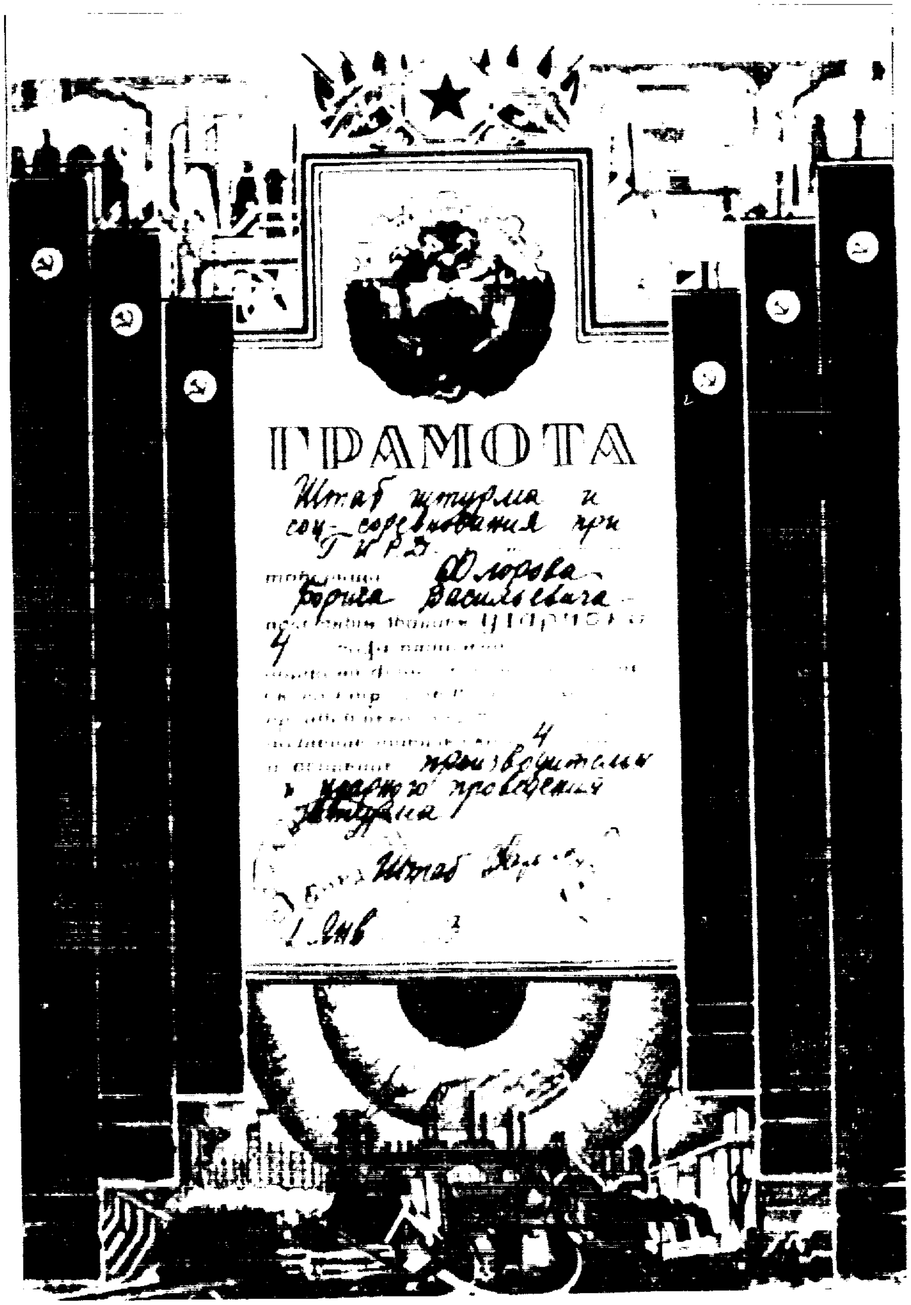

Diploma granted to B. V. Florov, mechanic on the staff of GIRD

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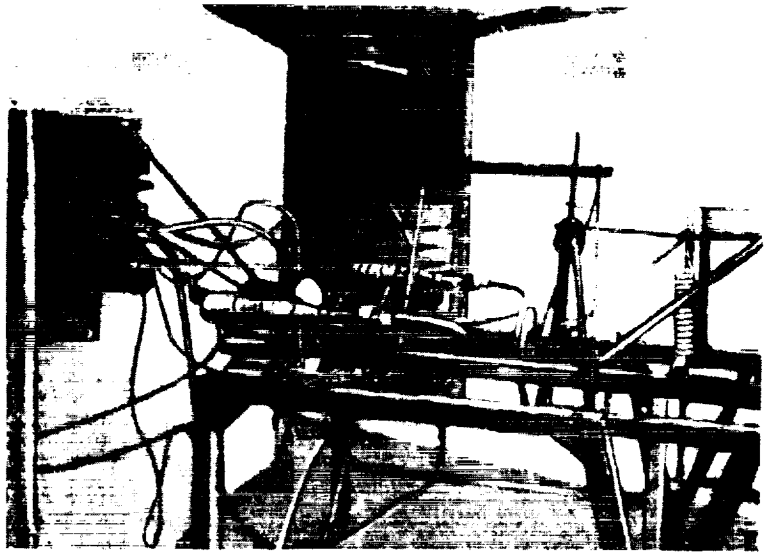

The ER-2 engine on the test stand

Testing the ER-2 engine

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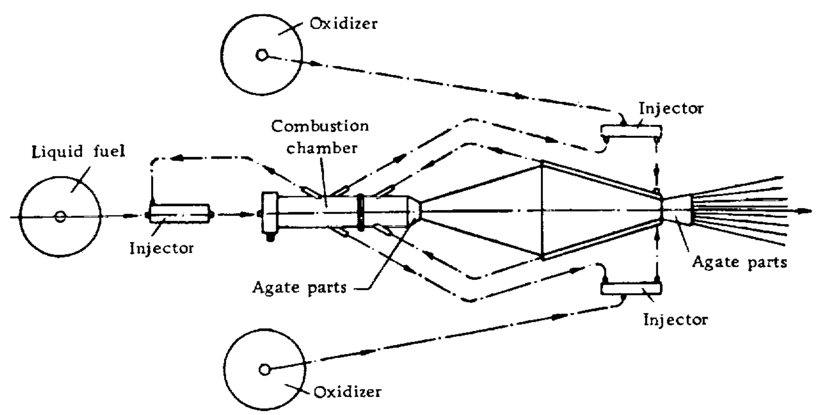
of 600 kg, as well as three different versions of an engine with a thrust of 5 tons.

In the first version of the 5 ton engine the combustion chamber was designed on the same principle as in the ER-2 engine but the whole layout was significantly simplified*.

The fuel tanks operated without surplus pressure, fuel being fed with the aid of special injectors. The system was started with the aid of a separate, small, liquid-oxygen tank (not shown in the diagram) or by manual pumps.



First version of the 5 ton jet engine

The engine nozzle was constructed in the form of a double (a direct and an inverse) cone to ensure rapid exhaust. Oxygen, cooling the inverted cone, proceeded to the cooling section of the combustion chamber and then reached the engine sprayer. Fuel supplied by the injectors went directly to the fuel sprayers. The critical section of the nozzle was regulated to maintain low pressure, with the aid of a movable core and an automatic device in the form of bellows or a Bourdon tube. The system was capable of operating without the double cones if more powerful injectors were used.

In the second version of this engine Tsander retained the same water-cooling system and oxygen evaporators as in the ER-2 engine. In this version the inverse cone was water-cooled, and four injectors were employed, one of which was for water, two for the oxygen tanks, and one for fuel.

The water injector was operated by a jet of combustion products. After cooling the section of the inverted cone, the cooling water entered a three-branched pipe and was utilized for heating two oxygen evaporators. From there, already cooled, it traveled to the water compensator, and then, with the aid of an injector, it was employed for the cooling of the inverted cone.

The injectors that fed oxygen to the evaporators operated by issuing a jet of evaporated oxygen from the cooling section to the combustion chamber. The oxidizer then returned from the evaporators to cool the combustion

* The operating principle of the engine is explained in Tsander's technical descriptions, dated 2 January 1933.

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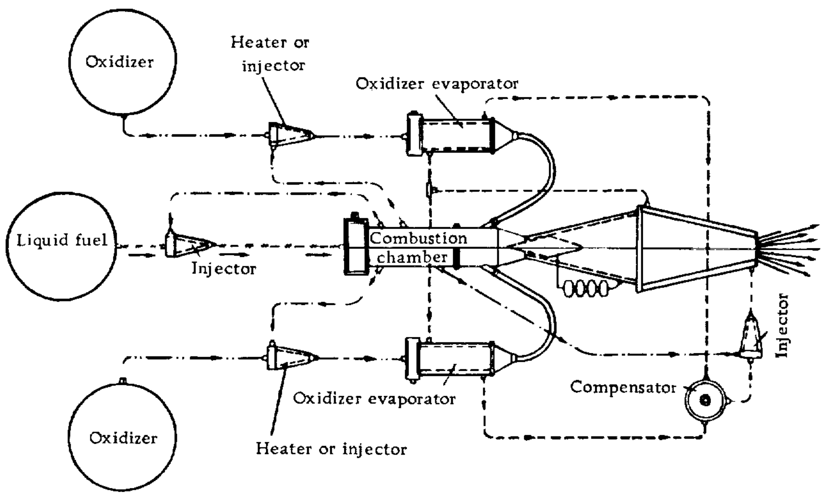
chamber, and was then conveyed to the combustion chamber for combustion, the remainder being used again as the active jet from the injector. The liquid fuel was fed by the injector directly to the combustion chamber. Here too, as in the water injector, the combustion products formed the active jet.

The third version of the engine, again with a 5 ton thrust, more closely resembled the ER-2 engine. There were no injectors and fuel was supplied instead by centrifugal pumps, started by a small gas turbine. The gas turbine, in turn, ran on selected combustion products from the combustion chamber.

As in the two previous versions, cooling of the combustion chamber and heating of the evaporator were effected in a closed cycle employing the same principle as in the ER-2, but with the aid of pumps.



Second version of the 5 ton jet engine

Tsander started to design his next engine – the 600 kilogram thrust one – with the idea of using metallic fuel in combination with liquid fuel.

He regarded this engine as a step toward realizing the idea of employing rocket components as fuel for the rocket engine. He believed that he could solve the problem of constructing a rocket by using a large part of itself as fuel.

It was proposed to first test in this engine certain powdered materials – magnesium, boron, beryllium and aluminum, and then to test pieces of different alloys that might be used for the construction of rocket components. It was also planned to test certain liquid fuels whose combustion gives the bulk of gaseous products, namely, kerosene, gasoline, alcohol, etc., and such oxidizers as liquid oxygen, nitrogen tetroxide, nitric acid, fluorine, and chlorine.

Tsander's idea was that the layout of this engine should consist of three tanks, with a tank containing oxidizer inside another containing powdered

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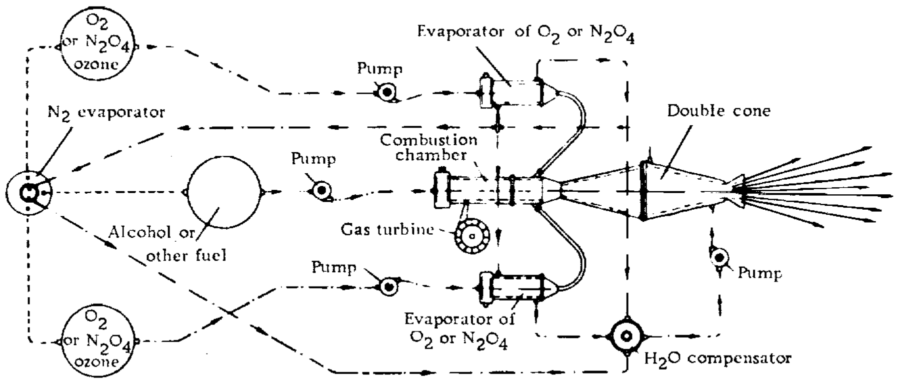
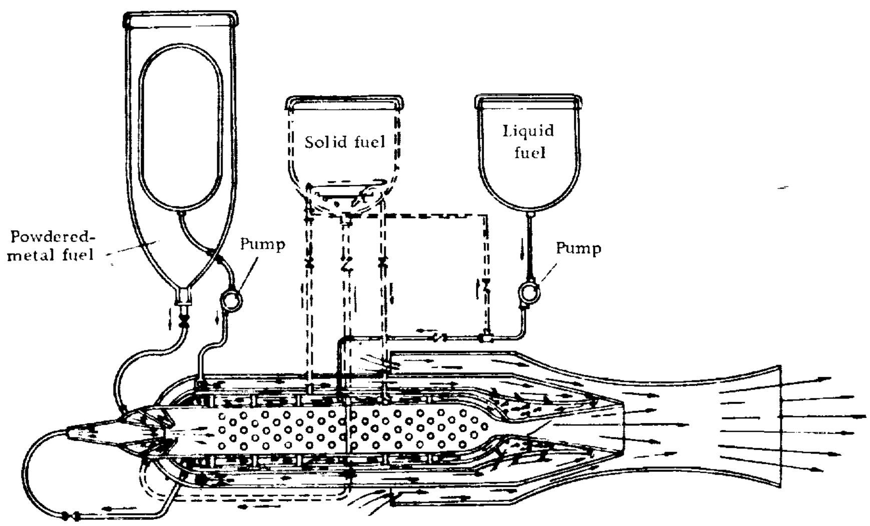
solid fuel. This arrangement was designed to utilize the heat-insulating qualities of the powdered material.

In the second tank (boiler), there was solid fuel in pig form which would be melted by heat from a tubular coil within which the fuel components would be burned. The third tank was a reservoir for the liquid fuel, which would be fed with the aid of pumps turned on by a small gas turbine or by a windmill.



Third version of the 5 ton jet engine



Schematic diagram of a solid-fuel engine

In January 1933, the first GIRD brigade began work, under Tsander's immediate supervision, on a liquid rocket, subsequently named GIRD-X.

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Developing the idea that future spaceships would be fueled by components of the spaceship itself, as components lose their usefulness after a certain stage of the flight, Tsander designed the first rocket using metal as its basic fuel and gasoline as a secondary one. The rocket was designed to use liquid oxygen as oxidizer*.

There were two kinds of metal in the rocket that could be used as fuel – powdered metal conveyable to the combustion chamber by injectors, and components of the rocket which had to be melted first in a special boiler and then fed to the combustion chamber by a special injector.

At that time the problem of employing metallic fuel was far from any practical solution. For this reason, simultaneously with work on the rocket structure and the theoretical calculations, the brigade started research on a feed system for metallic fuel and on a method of burning such fuel. The following equipment was designed and produced: a tank in which metal could be melted; a counter-pressure chamber; a filings-settling cyclone; and an injector for conveying powdered magnesium. The ER-1 engine was used for testing the metallic fuel.

Soon after the first tests of the injector and of metallic fuel combustion**, it became apparent that using metallic fuel in rockets presented enormous difficulties both in evolving a feed system and in elucidating its effect on the jet engine's basic parameters. Experiments on burning rocket components showed the need for many years of determined effort, and problems even more difficult than those involved in the use of powdered magnesium would have to be solved.

Tsander strove hard to see the rockets in flight, but at that time the Soviet Union had not yet flown liquid rockets and no one knew what they would be like in flight, how the automatic fuel-feeding device and the jet engine would work, etc. These considerations constantly worried Tsander and he tried to simplify the rocket's construction. Another version was developed without provision for burning components of the structure. Use of metallic fuel was retained, but only in the form of powdered metal fed by a special injector; gasoline and liquid oxygen were similarly fed by injectors, which needed perfecting and preliminary investigation.

Tests revealed that it was impossible to eject the metallic fuel into the engine's combustion chamber because the powdered metal was baked into a hard mass. Furthermore, the GIRD production facilities, being old and deficient, failed to turn out injectors of adequate quality – they usually failed after a short period of operation.

Because of all these difficulties substantial modifications were made in the second version and a third version, the GIRD-X, evolved which was fully developed later and withstood test flights successfully.

In this model the liquid fuel and the oxidizer were no longer fed by injectors, but by pressure from a small balloon with the aid of a special reduction valve. The use of solid fuel was temporarily excluded pending laboratory tests using better equipment.

* This rocket design was unfortunately not preserved in the archives.

** Combustion experiments in burning fuel, including all related work, were carried out by L. K. Korneev and A. I. Polyarnyi.

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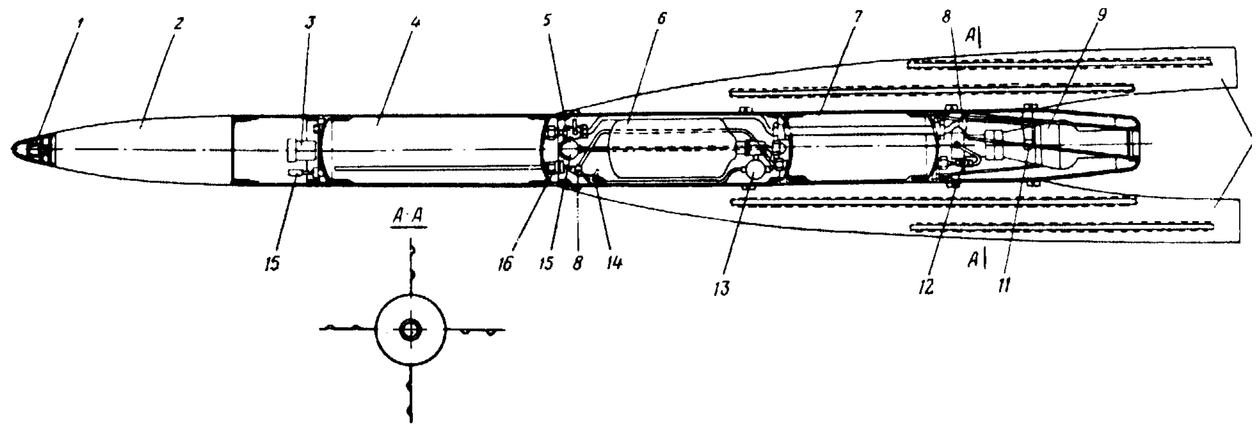
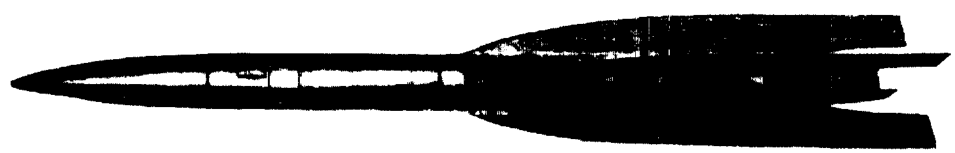


Diagram of the GIRD-X rocket.

1-Ejector; 2-parachute; 3-safety valve; 4-oxygen tank; 5-oxygen tap; 6-compressed air; 7 - gasoline tank; 8 - T-tubes; 9 - jet engine; 10-stabilizers; 11-spark plug; 12-gasoline tap; 13-reduction valve; 14-tap; 15-pressure gauge; 16-check valve.



General view of the GIRD-X rocket

The rocket, with a parachute equipped with an ejector, had the following characteristics:

Length	220 cm
Weight	29. 5 kg (of which 8.3 kg w[illegible]el)
Diameter	14 cm
Engine thrust	70 kg
Duration of combustion	22 sec
Pressure accumulator	special balloon, 150 atm, 2 liter capacity
Payload	2 kg
Calculated height of flight . . .	5.5 km

Tsander was not destined to realize his dream of seeing the rockets in flight. Despite his heavy duties in connection with the construction and testing of rockets, Tsander continued his systematic scientific and theoretical research. In addition he was working at new constructions, writing a book, being active in public affairs, and tending his hothouse at home. The extent of his accomplishments was amazing. Tireless, sweeping everyone along with him by the strength of his faith in the possibility of interplanetary flight, Tsander began his day early and filled it to capacity. The painstaking experiments on the combustion of powdered solid fuel and the development of an injector system would go on late into the night. Tsander brushed aside with a joke all suggestions that he take more rest and sleep, and was in the habit of staying at his work until satisfied that some progress had been achieved.

As a result, he began to show alarming signs of overwork. After a great deal of persuasion he agreed to go to Kislovodsk for rest and treatment. On arrival he was found to be running a very high temperature; he had apparently contracted typhoid fever during his journey to the sanatorium.

From his sickbed he sent a letter to the GIRD staff in which he shared with them his latest thoughts and future plans. He dreamed of a quick recovery and of resuming work with renewed energy. The letter closed with a general exhortation:

–"Forward, comrades, and only forward!
Raise the rockets ever higher, higher and higher,
closer to the stars!"

At 6 a.m. on 28 March 1933, shortly after this letter was written, Tsander succumbed to the typhoid fever. This extraordinary and brilliant man's life came to an abrupt end in his 46th year.

An obituary appeared in the newspaper "Tekhnika", No. 30, of 30 March 1933 (see page 57).

The following are the recollections of a former GIRD mechanic, N. N. Krasnukhin:

"When, in 1932, I was employed by GIRD as a fitter, my job was to mount and assemble structures for the first jet engine designed by Tsander.

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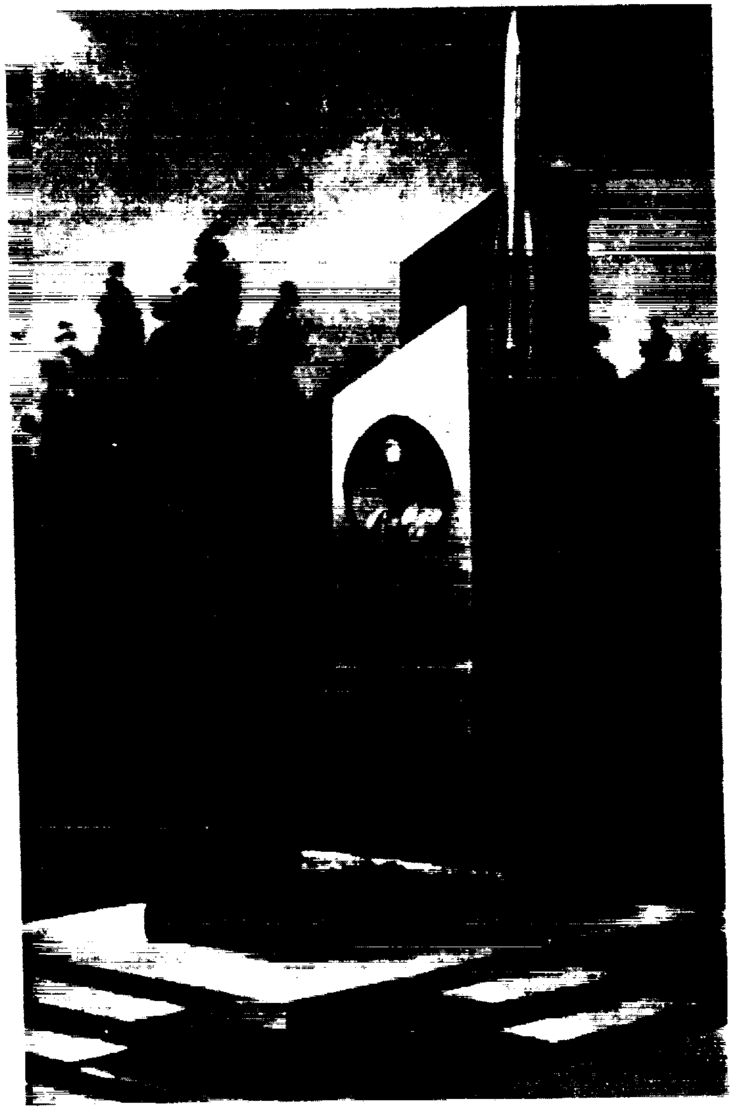

Granite tombstone on Tsander's grave at Kislovodsk

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"After each successful experiment, Tsander's confidence in the work he had begun mounted. At the end of each talk with us he would say: "Forward to Mars". We so loved and respected him that we wanted to complete the mounting and assembly ahead of schedule, often paying no heed to time and remaining at his side until we had fully completed an experiment.

"So deep was his belief in the reality of his ideas, and so convincingly did he imbue us with that faith, that, as we worked, we envisaged the contours of the future and took joy in our success.

"Fridrikh Arturovich was often to be found in the workshops, and the construction workers were in the habit of coming to him there to ask him how to calculate this or that detail. He would promptly indicate the relevant book, page and formula, and, to our wonder, was never wrong. He would calculate on the spot, in his head, the required material thickness, and tell us the best way to manufacture or assemble the part in question.

"At that time we were working by artificial light in the basement quarters of GIRD. Carried away, Tsander would work late into the night, and would not leave unless led away by force. His eating habits were haphazard, and he remained constantly preoccupied by his constructions, sketches, descriptions and calculations.

"Very often we traveled with him to Osoaviakhim to attend study courses. Tsander used the time of our journey, or any free moment that we might have, to talk to us of jet-aircraft rockets and rocket flights from Earth and back, and expounded his idea on the possibility of using aircraft components as fuel. He was lucid and convincing and was pleased if we understood him well.

"As soon as an engine began working, Tsander would become a different man. His spirits would rise and infect all those present, thus easing our work and enabling us to overcome difficulties more readily.

"On learning of his death we deeply mourn the loss of such a wonderful man."

Obituary appearing in "Tekhnika", No. 30, 30 March 1933.

F. A. Tsander – Engineer

The eminent jet-propulsion theoretician, and engineer-inventor, Fridrikh Arturovich Tsander, passed away in Kislovodsk at 6 a. m. on 28 March.

Tsander began in the new, then only rudimentary, field of applied-jet propulsion in 1908. After the October Revolution he was one of the first engineers to go to work in a Soviet factory and for many years he was the most eminent specialist devoting himself to the work of socialism in the aviation industry. With extraordinary persistance and enthusiasm he conducted theoretical research in the technology of jet propulsion, and, on the basis of these theoretical and practical activities he created his own school of jet-engine theory and construction.

Fridrikh Arturovich devoted the whole of the last year of his life to practical jet-engine construction. He took an active role in establishing TsGIRD, The Central Group for the Study of Jet Engines, of the Central Council of Osoaviakhim of the USSR. Despite poor health, Tsander displayed on many occasions genuine bolshevist zeal and heroic enthusiasm.

Tsander was the author of a series of theoretical works providing the only existing calculations in the field of jet propulsion.

EIDEMAN, BELITSKII, BELOTSKII, MALINOVSKII, NOVIKOV, TERENT'EV, KONONENKO, TSIOLKOVSKII, IL'IN, NIKITFOROV, PETROV, RASKIN, KOROLEV, FORTIKOV, BULANOV, PARAEV, TIKHOMIROV, POBEDONOSTSEV, KORNEEV, EFREMOV.

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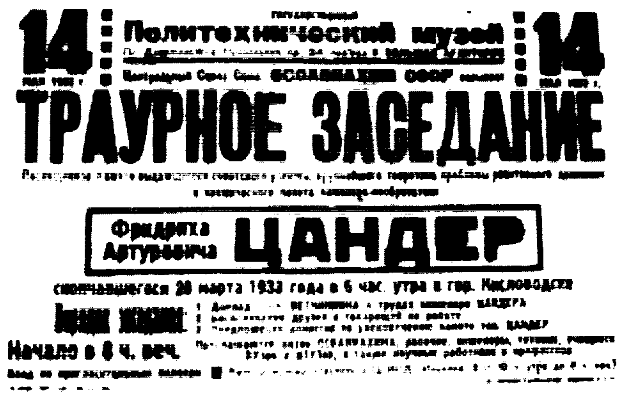
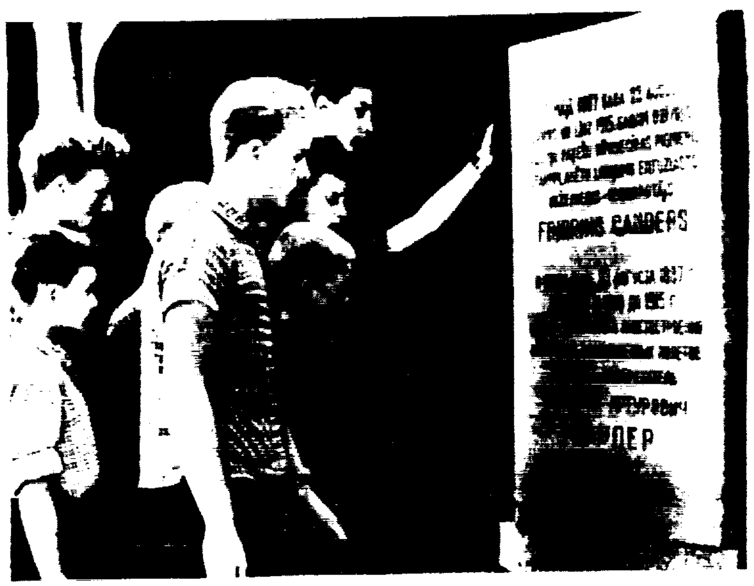
A memorial meeting was convened by the Central Council of Osoaviakhim on 14 May 1933. Professor Vetchinkin delivered an address on the significant life work of the great scientist and great man.



Poster of the memorial meeting



The memorial plaque at Tsander's house in Riga

Later, at the initiative of his friends and students, a granite tombstone bearing an exact replica of the GIRD-X rocket was placed on Tsander's grave at Kislovodsk. In Riga, his birthplace, a commemorative plaque was placed at the house on Bartas Street in which Tsander lived.

The brigade continued working on Tsander's GIRD-X rocket after his death, and, on 25 November 1933, one of the Soviet Union's first liquid rockets stood ready on its launching platform on the outskirts of Moscow.

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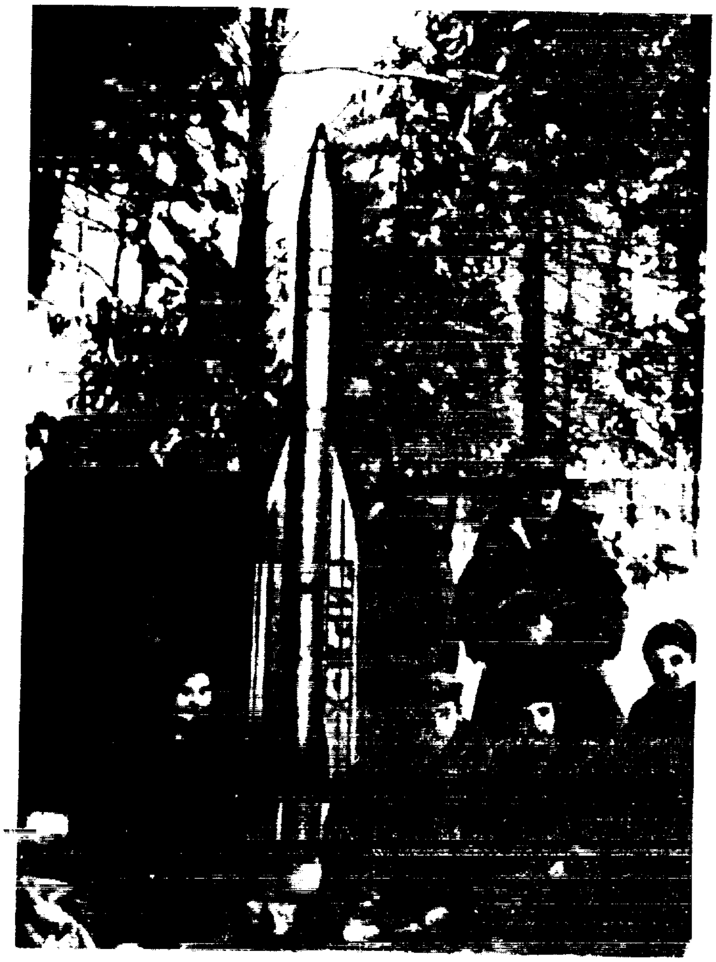
When the final preparations had been made, the tanks filled with fuel and oxidizer, and the hatch closed, the pressure in the tanks began to rise, and with it the tension of the launching crew. At last, the pressure reached the required level, and an excited voice ordered: "Fire!"



Rocket enthusiasts just prior to the launching of GIRD-X, on 25 November 1933

Ignition was instant, the jet engine went into action, and the rocket bearing the inscription "GIRD-X" flew off. Gathering speed, it disappeared into the clear blue sky, thus glorifying the name of the great scientist, the ardent Soviet patriot, the pioneer of rocket construction in the USSR – Fridrikh Arturovich Tsander.

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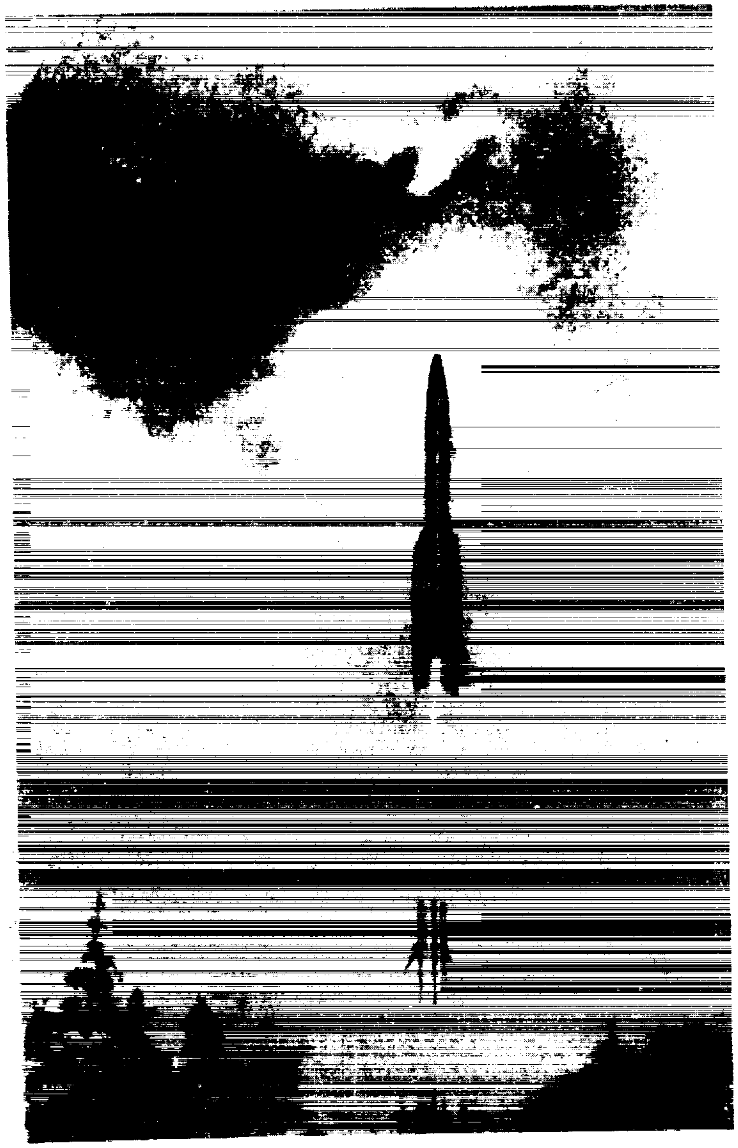

GIRD-X rocket at moment of take-off

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PROBLEMS OF FLIGHT WITH THE AID OF JET PROPULSION MACHINES*

PREFACE

This book has been written to popularize principles of interplanetary communications.

Calculations of the theoretical compression diagram in a jet supercharger and calculations of the flights of long-range rockets outside the atmosphere are presented. However, in view of the limited size of the book, some calculations are omitted, e.g., the jet supercharger's practical calculation and its air compression diagram; detailed calculations of a rocket carrying with it the oxygen for combustion; original formulas for rocket efficiency; determination of heat loss through the nozzle walls; all detailed theoretical and practical calculations of various forms of jet engines; the theory of ascent of a rocket airplane into interplanetary space and its descent back to the Earth; and, finally, the theory of flight of long-range rockets, taking into consideration the rotation of the Earth about its axis and about the Sun, including in particular, the most suitable flight trajectories.

I hope that this book will arouse interest in the problem of interplanetary communications in a wide circle of aviation workers and in general among all those following the many-sided development of aviation engineering research. The author appeals to inventors in general, to students, engineers and astronomers to work in this field in view of its importance for superaviation.

For high-velocity rockets, one arrives at rather high power, simple design and high efficiency. To overcome terrestrial gravitation, the rocket must overcome a sort of obstacle, present at velocities of about 400-700 m/sec**, after which further acceleration is already easier.

For astronomers the future spaceship will serve as a flying astronomical observatory, since, during the flight to another planet, astronomical instruments must be used for observations and so astronomers will naturally take an active part in its construction.

F. A. Tsander

* Published as a separate book: Problema poleta pri pomoshchi reaktivnykh apparatov. – Moskva, ONTI. 1932.

** [The physical nature of this "obstacle" is by no means clear, since the velocities mentioned are quite above the "sound barrier", where difficulties were expected 30 years ago.]

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INTRODUCTION

Theoretical and practical studies in the field of jet engines are now carried out in two directions. The first is the development of an independent rocket which could serve for research in the upper layers of the atmosphere, the second is the design of an airplane which would be able to fly away from the terrestrial atmosphere using its engines and a rocket, and then be further accelerated by jet engines outside the atmosphere. In order to reach a height of 2200 km (or 20° of the Earth circumference), a velocity of 4.3 km/sec has to be given to the airplane inside the atmosphere. The duration of stay outside the atmosphere would then be 12.6 min which is sufficient for further acceleration to a velocity of 7.9 km/sec, necessary for free flight around the Earth.

From this example we see that the velocity which has to be developed inside the atmosphere is considerably lower than the velocity (11.2 km/sec) necessary for complete escape from the Earth. Jet engines (rockets) for super-aviation, as well as for flight to other planets, can work quite successfully at high velocities in airless space. This is due to the fact that their axial thrust is increased while air resistance falls off. This simplifies considerably the problems of super-aviation and interplanetary communications.

In the following the author acquaints the reader with a new type of rocket whose theory he adequately developed in recent years. These rockets are particularly safe since they do not use liquid oxygen with its explosion hazards. For flights through the lower layers of the atmosphere, the absence of liquid oxygen creates particularly favorable conditions due to the weight reduction. We will also examine rockets that use part of their structure as fuel. In these rockets the limit of usable fuel material is so near 100% of the rocket's total weight that it is quite probable they will be used in the first flight to cosmic space with complete escape from Earth.

1. THE HEIGHT OF THE EARTH'S ATMOSPHERE

The pressure, temperature, density and composition of the atmosphere at high altitudes play a primary role for flights through its upper layers and for short duration flights beyond it.

In Figure 1 we show curves of these quantities as functions of the altitude, according to the data of the international standard atmosphere. These curves will be used for future reference.

The international standard atmosphere, for which the standard atmosphere of France was adopted in 1920, is shown up to an altitude of 20 km; p_{is}, T_{is} and γ_{is} denote the pressure, temperature and density of the atmosphere.

Figure 2 shows $\lg p_{is}$ and $\lg \gamma_{is}$ up to 300 km and Figure 3 gives T_{is}, assuming a constant temperature (T_{is} = -56.5°C = 216.5° abs) from 11 km upwards.

For comparison, we show in Figure 3 temperature curves according to various sources, and also in Figure 2 the logarithms of the corresponding density and pressure. Curve T_1 represents an adiabatic variation of the temperature at great heights; curve T_2 is drawn assuming that for heights above 200 km the temperature is close to the absolute zero; curve T_3 is an approximate average between curves T_2 and T_4, the latter assuming a constant

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temperature of 180° abs above 33 km. From curves T_2, T_3 and T_4 I have calculated the logarithms of the density and of the pressure shown in Figure 2. Finally, curve T_5 gives the temperature according to Andoier; this is close to the values of curve T_1.

The temperatures and the logarithms of the density for three constituents of the atmosphere, O_2, N_2 and H_2, are also shown in Figure 3. These values are derived from a paper by Professor V. G. Fesenkov*, who conducted observations on the duration of the evening-glow in 1915-1916 in Kharkov. His purpose was to determine the atmosphere's height and temperature. The curves have been calculated on the basis of these observations by V. P. Vetchinkin in Moscow by a special method, resulting in coincidence of the initial temperatures with those calculated**.

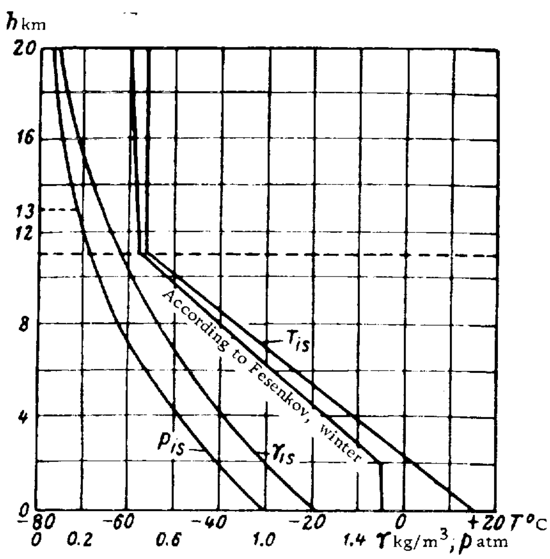

FIGURE 1. Temperature, pressure and density of the atmosphere up to a height of 20 km

The result of Professor Fesenkov's observations is represented by the curve At', in quite good agreement with his theoretical curve At. The density curve, obtained by Professor V. P. Vetchinkin, practically coincides with the curve At'.

Logarithms of the pressure of individual constituents of the atmosphere, according to Fesenkov, are given in Figure 4, which shows also, for comparison, a pressure curve of Wegener. It is seen that the form of both curves is in quite good agreement with each other and that hydrogen and the hypothetical element geocoronium introduced by Wegener in his theory

* Trudy Glavnoi Rossiiskoi Astrofizicheskoi Observatorii, Vol. II, pp. 6-113. 1923. (I used these data to calculate the absolute values of pressure and density).

** Russkii Astronomicheskii Zhurnal, Vol. IV, No. 4. 1927.

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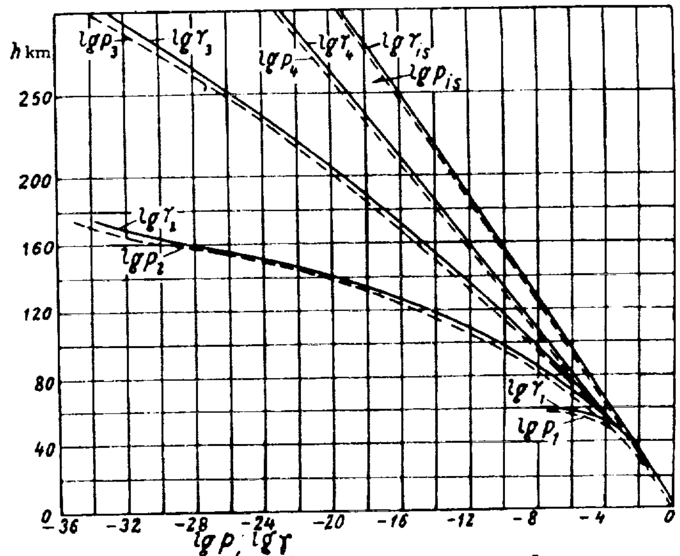

FIGURE 2. Logarithms of the atmospheric density (γ kg/m^3) and pressure (P atm) as function of height up to 300 km

play a decisive role in determining the density and pressure of the atmosphere at heights above 80 km.

It should be noted, however, that the geocoronium, introduced by Wegener, very probably does not exist, since its spectrum coincides with the nitrogen spectrum at the extremely low pressures prevailing at such great heights and nothing is known on the disintegration of nitrogen molecules at these pressures.

If we assume a homogeneous composition, the atmospheric pressure, according to Fesenkov, is given by the curve At'' (see Figure 4).

The volume percentage content of the individual components of the atmosphere is given, according to Fesenkov, Vetchinkin, and Humphreys in Figure 5, and according to Wegener, in Figure 6.

2. INFLUENCE OF COMPOSITION, DENSITY, PRESSURE AND TEMPERATURE OF THE ATMOSPHERE ON TECHNICAL FLIGHT PROBLEMS*

From the above figures, we see that at a height of 55-60 km above the surface of the Earth the volume of hydrogen is approximately twice the volume of oxygen, so that the mixture constitutes a detonating gas to which nitrogen has been added. If we are able, by means of the special jet superchargers which will be discussed below, to compress this mixture to sufficiently high pressures, it would be possible to use it as a fuel, eliminating the necessity of liquid fuel for high velocity flights at this height and also for further acceleration to the first interplanetary velocity of 8 km/sec.

* [It is interesting to remark that although the concept of the atmosphere's structure prevailing at the time has been found inaccurate, some of Tsander's conclusions and proposals have turned out to be of great practical significance (e. g., the importance of aerodynamic heating and the solution of this problem by ablative cooling).]

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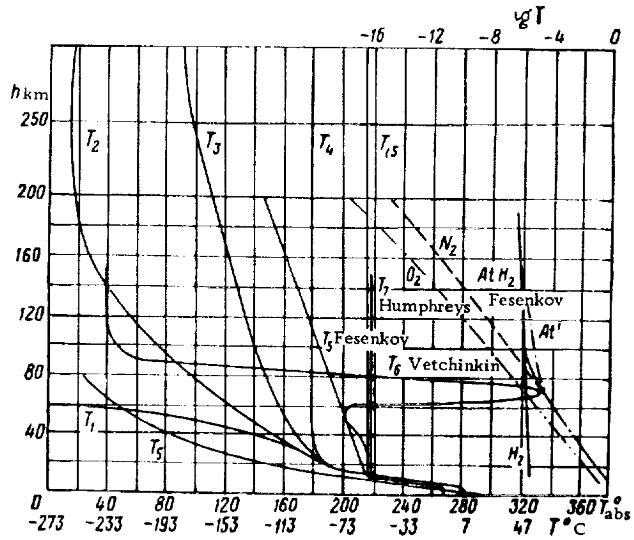

FIGURE 3. Atmospheric temperature up to 300 km and logarithms of atmospheric density (γ kg/m^3) up to 200 km, according to Fesenkov

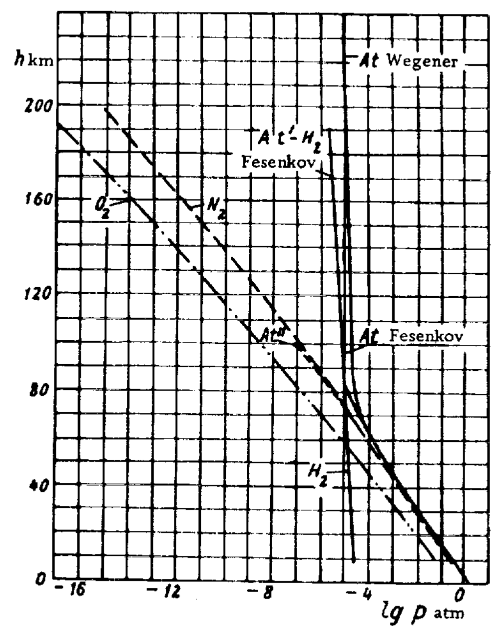

FIGURE 4. Logarithms of the pressure of the atmosphere and its constituents

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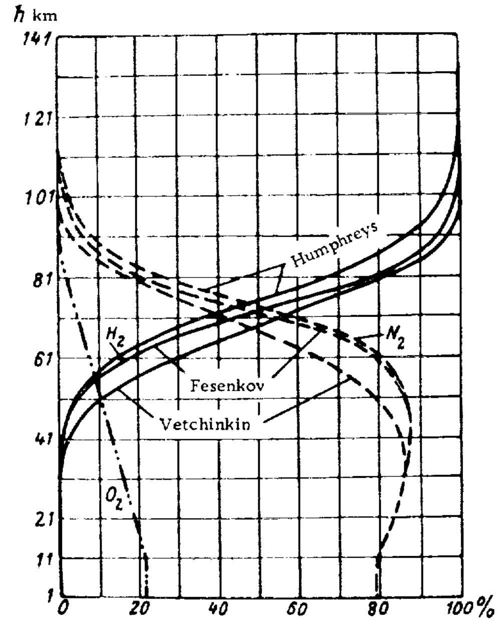

FIGURE 5. The composition of the atmosphere according to Fesenkov, Vetchinkin and Humphreys

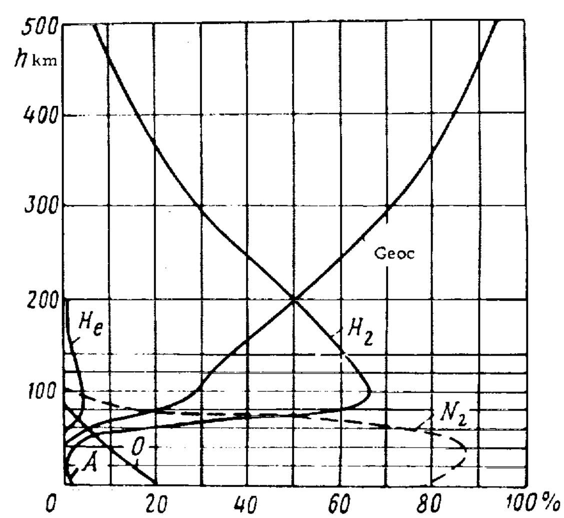

FIGURE 6. Composition of the atmosphere according to Wegener

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The atmospheric density at this height is approximately 0.0001 kg/m^3, which corresponds to a flight velocity of $\sim$3500 m/sec for an airplane brought into motion by a rocket*. In this case it is preferable to fly slower, since then the velocity is lower and the amount of propellant consumed for acceleration up to the flight height of 55-60 km is smaller.

However, in our case the necessary amount of propellant is considerably reduced, due to the fact that at velocities larger than 3500 m/sec no propellant is used. Using, for example, an ordinary jet engine working on gasoline and liquid oxygen, and assuming an ejection velocity of the combustion products of gasoline equal to 3340 m/sec, we need in our case (in order to reach a height, attainable for airplanes, of 60 km) an amount of gasoline and liquid oxygen constituting 65% of the total weight of the ship. If we do not use the detonating gas, then the amount of propellant required for flying around the Earth should constitute 91% of the total weight of the ship, neglecting in both cases air friction and the Earth's attraction; for an inclined flight and sufficiently high accelerations, the actual amount of propellant does not exceed the above-indicated by much.

In order to support the full weight of the airplane up to flight velocities of about 11,000 m/sec, we need an atmospheric density of about 10^{-5} kg/m^3, which exists at a height of 89-90 km over the surface of the Earth. At a height of 200 km over the surface of the Earth, the atmospheric density, even taking into account its heterogeneous composition, is equal to 1:330 only of the above-indicated figure; its resistance is reduced by the same amount for that flight velocity. It follows therefore that for a complete departure from the atmosphere, the atmospheric layer above 100-120 km does not play a great role. The situation is different if we desire to fly in the extremely rarefied atmosphere. It can be easily seen from Figure 2 that for a homogeneous composition of the atmosphere its resistance at heights greater than 120 km can be completely neglected since here $\lg \gamma_{is} = -7.5$, i.e., $\gamma_{is} = 3.10^{-8}$ kg/m^3.

The atmospheric pressure is of interest to us because the efficiency of jet engines increases with height over the Earth's surface, since the gases can expand more; it is assumed that the nozzle is constructed so that its exit cross section can vary in such a way that for constant initial pressure the final pressure is always equal to atmospheric. We shall further see that a pressure of 0.001 atm is completely sufficient for obtaining thermal efficiencies of 80 to 90%. This pressure exists at a height of 45-50 km.

In the presence of external lubricated parts or open vessels for melting metals or other types of propellants as, for example, naphthalene or celluloid, these may evaporate rapidly in the almost absolute vacuum. It should be remembered, however, that due to the strong cooling the external surface freezes. The partial pressure at the moment of freezing is extremely small—for metals it is of the order of 10^{-6} atm; by covering the metals by a layer of nonmelting material or constructing a jacket, inside which a gas under rather low internal pressure is held, it will be possible to reduce the evaporation considerably.

At pressures lower than $\approx$150 mm of mercury man needs either a hermetic cabin or a completely closed suit.

* [The way Tsander finds a correspondence between density and flight velocity is not clear.]

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Air temperature at great heights plays a rather important role in determining the heating of the frontal parts of the ship. Assuming adiabatic compression of the air at the vehicle's front, a rather strong air heating may be obtained. This will happen in particular if the air temperature, calculated by Professor V. P. Vetchinkin for heights of 60-80 km (up to +60°C) where strong temperature inversion occurs, will be found by further research to be correct.

Assuming adiabatic compression of the air, its temperature can increase considerably at these heights in particular, and it may turn out that all parts would melt, making flight beyond the limits of the terrestrial atmosphere very difficult. A calculation of the heat developed per unit time based on the power corresponding to the airplane's total resistance indicates, however, that it is quite possible to remove it. This can be achieved either by making the frontal parts of refractories or by passing through them a sliding surface which would also come into contact with cold air. It will also be possible to utilize the heated parts as heaters for the air, for the evaporation of the fuel and also for melting of the more fusible metals. If local heating turns out to be too great, the flight through the air layer from 60 to 80 km must be comparatively slow and one then accelerates above 80 km (for example, with a velocity of 4.3 km/sec and at an angle of 40° to the horizontal).

In vertical ascent the velocity becomes zero at a height of 80 km if the rocket's velocity at the Earth's surface was somewhat above 1.3 km/sec, and flight continues by inertia; if, however, ascent is effected by acceleration, the amount of propellant for the entire flight up to velocities of 5-12 km/sec will be larger.

In order to counteract the extreme cooling of some parts in flights outside the atmosphere it is possible to pass an electric current through them; other parts can be made tubular and one could pass heated air through them or use them as radiators. Certain components could constitute a part of the jet engine, as will be explained below. Due to the fact that at low temperatures heat capacity and radiation are low, a small amount of heat should be sufficient for a quite considerable amount of heating. Such metals as tin and certain kinds of steel, possessing high strength at low temperatures, can be used in very hot places together with materials possessing great strength at ordinary temperatures.

At great heights the pilot's window will not be cooled but at low altitudes the glass has to be heated either by warm air from outside or by warm water, circulating between double windows, from inside. One can also conceive a rotating brush which wipes the glass automatically and is then dried when passing near very hot surfaces such as the exhaust tubes of the engine. It is also possible to build the external windows as a moving transparent band, which is dried inside the ship on part of its path.

3. REVIEW OF METHODS FOR ATTAINING GREAT HEIGHTS AND HIGH FLIGHT VELOCITIES

In passing to the basic problem of super-aviation, the methods for attaining great flight velocities and heights, it has to be noted that research and experiments have brought, in recent years, such advances in rockets and jet engines in general, that their use in super-aviation is no more in doubt.

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Centrifugal superchargers take first place among the methods used for increasing the power of aviation engines. It is possible to increase their efficiency to approximately 80% by giving the inlet and outlet suitable form and dimensions; the degree of supercharging can also be raised by increasing the number of rotations and the diameter of the wheel. In this respect, however, there is a limit to the peripheral velocity suitable for super-aviation purposes; it should not exceed approximately 500 m/sec. This corresponds to a pressure increase up to $h=\eta\frac{\gamma u^2}{g}=0.5\frac{500^2}{8}=15{,}600$ mm of water. or approximately 1.5 atm for one stage.

When two stages are used, there is a temperature increase which causes certain difficulties. It is very likely that a further increase of the engine's power may be achieved by placing the carburetor between the first two stages; in this method, no special heater for the carburetor is required in winter. In the jet engines examined below, it is possible to transform heat into useful flow velocity producing a thrust; this feature does not exist in superchargers.

With two stages, the pressure obtained is equal to about $(1+1.56)^2=6.5$ times the initial pressure; this will provide normal engine power up to a height of about 13 km (see Figure 1). The free atmosphere pressure at this height is about 1:6.5 kg/cm^2.

With ten stages, a compression factor of $2.56^{10}=1.2\cdot 10^4$ is obtained which would give, under complete cooling of the air, normal power at a height of approximately 85 km, although the power required at this height is many times larger. However, due to the large supercharger, the weight of the engine is almost doubled and the air temperature is raised by a factor of $(2.56^{10})^{0.4/1.4}=2.56^{2.86}=14.7$, or approximately to $285\cdot 14.7=4200°$ abs. Even with complete cooling between each couple of neighboring stages (which is not quite possible) the amount of heat, uselessly removed, is great and the power necessary for operating the supercharger is considerable.

It is possible, nevertheless, that a combination of such an engine with a jet engine in whose channels the compressed air is cooled, will allow us to attain a height of about 60 km, as mentioned earlier.

At peripheral velocities larger than 500 m/sec, the weight of the supercharger's wheel becomes too great. In order to reduce the gyroscopic effect on the airplane, one part of the supercharger must be operated in one direction and another in the opposite one; this may be accomplished by a design with two parallel axes.

Next, it is possible to increase the engine's power by increasing the area of the piston for constant dimensions of the crank mechanism. This measure, combined with an increase in compression ratio in usual engines, gives rather good results; the increase in compression ratio economizes the fuel. Under super-aviation conditions, an increase in the piston area by a factor of nine trebles its circumference, and also the weight of the cylinders by approximately the same amount. For cylinders arranged in series, this increases the length of the shaft, while in a star-engine this requires, in general, removal of the cylinders further from the center. This method may, nevertheless, find some application in combination with the previous one.

Another method for increasing the power of the engine is the addition of oxygen to the air. By adding pure oxygen to the fuel, the calorific capacity is increased by a factor of 4-5, and by adding 15% of oxygen, it goes up by 50%.

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The liquid oxygen weighs approximately four times as much as the fuel, but in vertical flight great heights are rapidly attained. Therefore, the method of adding atomized liquid air in the space between the stages of the supercharger can be regarded as important. Complications arise due to the explosiveness of a mixture containing oxygen. However, the methods for reducing detonation eliminate these difficulties, especially when operating on heavy fuel.

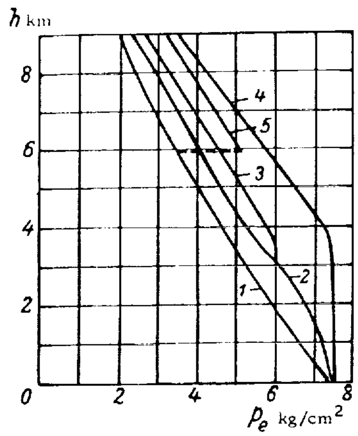

FIGURE 7. Average effective pressure of high-altitude engines up to a height of 9 km

In engines working on liquid oxygen, it would be possible to allow high friction on the shaft journal (larger than kv) if the shaft and the bearings are cooled by liquid oxygen. Liquid oxygen may also be used for partial cooling of the cylinders, which may therefor have larger dimensions.

The reduction of the average effective engine pressure with increasing flight height is shown in Figure 7 for low altitudes (up to 9 km): curve 1 refers to an ordinary engine: 2–to an engine with an enlarged compression ratio; 3–to an engine with an enlarged compression ratio and enlarged piston dimensions; 4–to an engine equipped with a supercharger; 5–to the use of additional oxygen.

Engines working on pure oxygen instead of air have not yet been studied extensively. I have designed schematically two such engines. In the first one, liquid oxygen is sprayed directly into the cylinder by a special pump together with liquid fuel. In this design a very high pressure, of approximately 200 atm, is envisaged; the engine's weight is very small and it may work with a variable volumetric efficiency. In combination with a jet engine, it would weigh, for a large volumetric efficiency, less than 0.2 kg/hp. In this design, the following points have not yet been investigated:

1) the possibility of injecting liquid oxygen at high pressures;

2) the possibility of ignition with liquid fuel, which may be established experimentally in the combustion chamber of a jet engine. If this is successful, liquid oxygen may be used in aviation engines;

3) the possibility of cooling the walls of the cylinder, which, however, does not cause any complications for small diameter cylinders;

4) the possibility of constructing a system of leakproof piston rings for the combustion products at such a high pressure.

The low weight of an engine operating on liquid oxygen allows 2-3 hours of flight, and the total weight of the engine, fuel and oxygen is lower than that of an ordinary engine with its fuel. By varying the pressure and the volumetric efficiency, it is possible to vary the power within rather wide limits. Furthermore, the airplane is light due to the low weight of the engine and this enables it to ascend higher than with an ordinary engine.

The second design is of an engine with external combustion of oxygen with fuel; this has the advantage that at low altitudes, the combustion

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products can be used in the engine, and at high altitudes – in the rocket. The combustion process can take place in the combustion chamber of the rocket. However, in this connection there are some difficulties with the cooling of the inlet valve by liquid oxygen, a possibility which should be tested first.

4. JET COMPRESSORS. THEORETICAL COMPRESSION DIAGRAM. COMBINATIONS OF JET COMPRESSORS WITH JET AND PISTON ENGINES

I shall now examine types of jet compressors in which air, or generally any gas, after being heated under a given pressure and expanded adiabatically, polytropically or according to any other law, accelerates in a nozzle where a rather low pressure is attained. The gases are then compressed again, slowing down in another chamber. The process takes place again according to some law. This can be, in particular, either isothermal or almost adiabatic. The important point is that during the compression a rather large amount of heat should be removed from the gas.

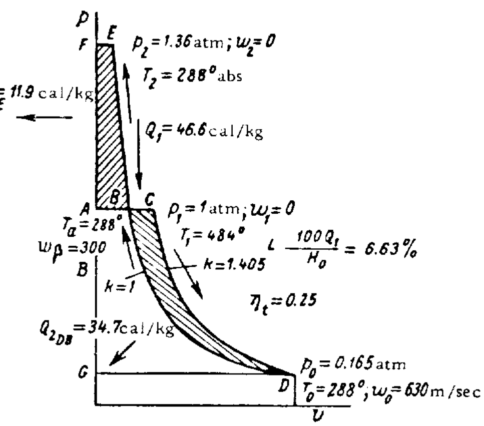

FIGURE 8. *pv* diagram of a jet compressor; air compression to $p_2 = 1.86$ atm

The theoretical *pv* diagram (p–pressure, v–volume) of such a device is shown in Figures 8 and 9. Figure 8 refers to a compression from 1 to 1.86 atm, and Figure 9 to a compression from 1 to 10 atm. The initial state of the gas is represented by point B (see Figure 8); the gas is heated at constant pressure, its final stage being point C; then it expands down to point D and is compressed again along the curve DBE.

The velocity is given by the formula $w^2/2g = \int v\,dp$. Thus, the velocity at point D, for example, is obtained from $w_D^2/2g$ = area of $ACDG$; the velocity at point B in the reverse compression is $w_B^2/2g$ = area of BCD; the velocity at point E is equal to zero if the area of BCD is equal to the area of $FEBA$.

The practical diagram will differ somewhat from the theoretical one and will give a somewhat lower pressure. The pressure p, the temperature

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T° abs and the velocity w m/sec are given in the diagrams as well as the amount of heat which has to be supplied or removed from the air.

The scheme of the apparatus for this cyclic process is shown in Figure 10. High pressure gas (combustion products from the engine or fuel specially burned under high pressure, and so on) flows through the chamber H; it expands and is recompressed to the pressure of the external air in the inverted cone L, and streams out through the opening O. External air is sucked in between the external jacket A and the chamber H; from B to C it is heated by ribs. The chamber H may also have the form of a heating spiral or of a radiator. From C to D the air expands and is then compressed again, leaving for further use through pipe E. K is a small pipe instead of which an opening in the wall of the internal pipe can also be used. This pipe serves as an air pump for establishing the low air pressure in the cavity D during the starting period; P are cooling ribs or a cooling medium, e. g., liquid air, rather cold fuel, or cooling water.

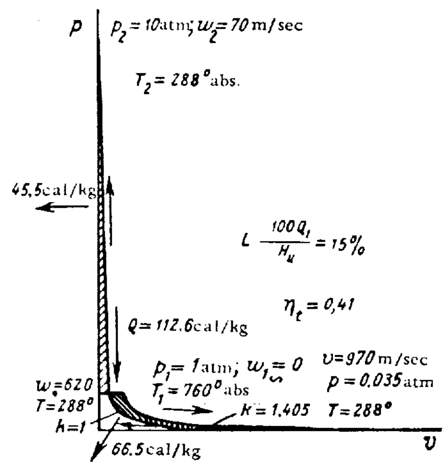

FIGURE 9. pv diagram of a jet compressor; compression of air to p_2=10 atm

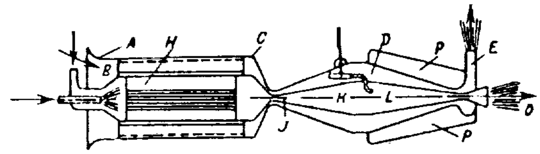

FIGURE 10. Jet compressor for compressed air

The high pressure which may be expected from these devices, makes it quite probable that they will replace the engine supercharger. The pressures which may be attained under intensified cooling are so high that

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especially strong engines with thick parts will be required and the power will increase very much even at great heights.

In combinations of engines with these devices the heating from B to C (see Figure 8) can be secured partly from the air, which cools the cylinders, and partly from the exhaust gas. In engines with air cooling, 58% of the entire heat contained in the fuel is used for cooling the cylinder and in the exhaust; this ensures strong air compression.

We can also go further and let the air, sucked in at B, pass with a certain velocity through pipe E and use its thrust, by pointing the pipe in a direction opposite to the airplane's flight direction.

It is also possible to compress part of the air to a pressure larger than one atmosphere and the rest, only to a pressure of one atmosphere. The first part may be used for combustion and the remaining, for production.

In one variant, the entire device may be placed at the end of the propeller blade, bringing there the exhaust gases of the engine. The high peripheral velocity of the blades enables one to use, to some extent, the kinetic energy of the air reaching the device at B.

The combination of an internal combustion engine with a jet engine is very advantageous, since the first has a high efficiency at altitudes attained already now, and the second, at altitudes at which the propeller unit cannot be used.

Theoretical diagram of atmospheric air compression by a jet compressor

Let the initial pressure of the air or gas we want to compress be p_1 kg/m^2, its initial temperature T°_a abs. Let us also denote by v_1 m^3/kg the specific volume of the gas and by R, the gas constant (for air $R=29.26$).

Then the specific volume in the initial state, corresponding to point B in the pv diagram (see Figure 8), is obtained from the equation of state

$$v_a = \frac{RT_a}{p_1}. \qquad (1)$$

Heating the air under constant pressure p_1 to temperature T_1, we obtain for the specific volume

$$v_1 = \frac{RT_1}{p_1}. \qquad (2)$$

From this we find point C of the diagram.

If the velocity of the gas at point C is low, its kinetic energy can be neglected. In the case of a fast airplane or rocket, we have two possibilities:

1. We may take for the velocity at points B and C of the diagram approximately the flight velocity; then the pressure p_1 will be equal to the pressure of the surrounding atmospheric air.

2. We may compress the air at the entry to the device by a special nozzle, as in the Loren engines; then, p_1 will be greater than the pressure of the surrounding atmospheric air, and the velocity for points B and C of the diagram will be lower than the flight velocity.

Let us denote in the general case the velocity at point C by w_1. Expanding the air adiabatically to a low pressure p_0 corresponding to the

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temperature of the surrounding medium or to a lower temperature T_0 the kinetic energy increase of the air is given by

$$\frac{w_0^2 - w_1^2}{2g} = \int_{p_0}^{p_1} v\,dp, \tag{3}$$

or, for an adiabatic expansion

$$\frac{w_0^2 - w_1^2}{2g} = \frac{c_p}{A}(T_1 - T_0) = \frac{k}{k-1} RT_1 \left[1 - \left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}\right], \tag{3a}$$

where c_p is the heat capacity at constant pressure;

$A = 1:427$ is the mechanical equivalent of heat;

$k = \frac{c_p}{c_v}$ is the polytropic exponent.

From this expression we may determine the velocity w_0 at point D.

In the last equation RT_1 can also be replaced by p_1v_1.

Formula (3a) is easily obtained by integration, if we remember that

$$\frac{c_p}{A} = \frac{k}{k-1} R. \tag{4}$$

The pressure p_0 at point D is obtained from

$$p_0 = p_1 \left(\frac{T_0}{T_1}\right)^{\frac{k}{k-1}}. \tag{5}$$

The specific volume at point D is equal to

$$v = \frac{RT_0}{p_0}. \tag{6}$$

The amount of heat, transmitted to the air under constant pressure along BC, is equal to

$$Q_1 = c_p(T_1 - T_0) = \frac{k}{k-1} AR(T_1 - T_a). \tag{7}$$

In a theoretical cycle, the greatest final compression pressure is obtained by carrying out the compression at the lowest temperature. Taking an isotherm for the compression, we obtain for the velocity in the inverted cone (in which the compression takes place under atmospheric pressure)

$$\frac{w_0^2 - w_B^2}{2g} = \int_{p_0}^{p_1} v\,dp, \tag{8}$$

where w_B is the air velocity at point B in the reverse compression.

After substituting v from the equation of state of the gas we obtain for an isothermal compression

$$\frac{w_0^2 - w_B^2}{2g} = RT_0 \ln \frac{p_1}{p_0}. \tag{8a}$$

The velocity w_B enables us to compress the air to a pressure p_2 higher than the pressure p_1.

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We shall examine here only the case in which the entrance air velocity w_1 can be neglected.

In this case, the air loses again all its velocity at a point for which the following condition is satisfied

$$\int_{p_0}^{p_2} v\,dp = \int_{p_0}^{p_2} v\,dp. \tag{9}$$

However, compressing the air according to an isotherm, we obtain

$$\int_{p_0}^{p_2} v\,dp = RT_0 \ln \frac{p_2}{p_0}, \tag{10}$$

and, taking into consideration that from (3) and (3a)

$$\int_{p_0}^{p_1} v\,dp = \frac{c_p}{A}(T_1 - T_0), \tag{11}$$

we obtain by substituting (10) and (11) in (9)

$$RT_0 \ln \frac{p_2}{p_0} = \frac{c_p}{A}(T_1 - T_0). \tag{9a}$$

Further, using equations (5) and (4), we obtain for $p_2 : p_1$

$$\frac{p_2}{p_1} = \frac{p_2}{p_0}\frac{p_0}{p_1} = \left(\frac{T_0}{T_1}\right)^{\frac{k}{k-1}} e^{\frac{k}{k-1}\left(\frac{T_1}{T_0}-1\right)}. \tag{12}$$

By giving, for example, the ratio $T_0 : T_1$, we can calculate the pressure increase $p_2 : p_1$ for this theoretical cycle.

The solution of the reverse problem is not so convenient. However, it is possible to determine the curve $p_2 : p_1$ vs $T_1 : T_0$ for a given k and then to calculate the required $T_1 : T_0$ for a given $p_2 : p_1$.

Let us also determine the amount of heat which has to be removed from the air during its compression in the inverted cone. From the lowest pressure p_0 to the pressure p_1 (path DB in Figure 8), the following amount of heat has to be removed per kg of air

$$Q_{2DB} = A \int_{p_0}^{p_1} v\,dp = A_D RT_0 \ln \frac{p_1}{p_0}; \tag{13}$$

this follows from the general equation of thermodynamics

$$-dQ = di - Av\,dp, \tag{14}$$

where dQ is the heat removed, and di is the enthalpy increase. However, in view of the fact that the temperature is constant, we have $di=0$; in this case, we obtain equation (13) by integration of (14).

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Comparing (13) with (8a), we also find

$$Q_{2DB}=\frac{A}{2g}(w_0^2-w_B^2), \qquad (13a)$$

i.e., under isothermal compression, the entire heat, corresponding to the reduction of the kinetic energy of the jet, should be removed through the walls.

In a similar way we obtain for the amount of heat which has to be removed from the air along BE (see Figure 8) for a pressure variation from p_1 to p_2

$$Q_{2BE}=ART_0 \ln\frac{p_2}{p_1}=\frac{A\cdot w_B^2}{2g}. \qquad (15)$$

We divided the compression curve into these two sections in view of the fact that in the jet engines examined below, the compression is partly carried out to the pressure p_1 only, letting the air with the velocity w_B into the atmosphere in order to obtain an axial thrust from the atmospheric air entrained in this way.

The total heat removed from the air during its compression in the inverted cone is equal to

$$Q_2=Q_{2DB}+Q_{2BE}=ART_0 \ln\frac{p_2}{p_1}\frac{p_1}{p_0}=ART_0 \ln\frac{p_2}{p_0}=\frac{Aw_0^2}{2g}. \qquad (16)$$

In our case, the initial temperature T_a of the air sucked in was taken equal to the temperature T_0 of the isothermal compression. In this case we find easily from (7) and (3a), assuming $T_0=T_a$ and $w_1=0$

$$Q_1=\frac{Aw_0^2}{2g}. \qquad (17a)$$

Comparing (17a) and (16), we find that in our case $Q_1=Q_2$, i.e., in the diagram of Figure 8, the whole heat which was given to the air under constant pressure p_1 should be again removed from it in the reverse isothermal compression.

However, the air temperature varies during the heating from T_0 to T_1 and in the reverse compression it was equal to the lower value T_0, so that heat passes from a higher source to a lower one.

We shall see nevertheless in the following, that the heat Q_2 which must be removed from the air can be used to heat the fresh air. The cycle, however, must then be different.

Let us also determine the efficiency of the working cycle considered.

The thermal efficiency of the cycle BCD, in which air is compressed only to the pressure p_1, is equal to

$$\eta_{t_1}=1-\frac{Q_{2DB}}{Q_1}, \qquad (17)$$

since Q_1-Q_{2DB} is the useful kinetic energy obtained.

But, in our case

$$Q_1-Q_{2DB}=Q_{2BE}. \qquad (18)$$

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Using (15) and (7) and taking $T_a = T_0$, we obtain

$$\eta_{t1} = \frac{Q_{2BE}}{Q_1} = \frac{(k-1)T_0 \ln \frac{p_2}{p_1}}{k(T_1 - T_0)} = \frac{\ln \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}}{\frac{T_1}{T_0} - 1}. \tag{19}$$

We can look upon the second cycle $ABEF$ (see Figure 8) as the isothermal compression cycle with which other cycles are usually compared. The kinetic energy spent here is equal to (see 15)

$$\frac{w_B^2}{2g} = \frac{Q_{2BE}}{A}.$$

The useful compression work obtained, equal to the area $ABEF$, is given for an isotherm by

$$RT_0 \ln \frac{p_2}{p_1},$$

i.e., it is also equal to $Q_{2BE}:A$.

Therefore, the thermal efficiency of the cycle $ABEF$ is

$$\eta_{t2} = \frac{R \cdot T_0 \ln \frac{p_2}{p_1}}{\frac{w_B^2}{2g}} = 1$$

and the total thermal efficiency η_t is

$$\eta_t = \eta_{t1}\eta_{t2} = \eta_{t1} = \frac{\ln \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}}{\frac{T_1}{T_0} - 1}. \tag{19a}$$

Example. The theoretical diagram of air compression from $p_1 = 1$ atm to $p_2 = 1.86$ atm (see Figure 8).

We take for air

$$k = 1.405;\ c_p = 0.238;\ T_0 = 288° \text{ abs}.$$

Let us determine initially the values of $p_2:p_1$ for a series of values $T_1:T_0$ according to formula (12) (see Figure 20 below); we find $\frac{T_1}{T_0} = 1.68$ for $\frac{p_2}{p_1} = \frac{1.86}{1}$, and therefore, $T_1 = 1.68 \cdot 288 = 484°$ abs.

The pressures and specific volumes are now obtained for points B, C, D and E.

For point B

$$p_1 = 1 \text{ atm};\ T_a = T_0 = 288°;\ v = \frac{RT_0}{p_1} = \frac{29.26 \cdot 288}{1 \cdot 10^4} = 0.844 \text{ m}^3/\text{kg};$$

for point C

$$p_1 = 1 \text{ atm};\ T_1 = 484°;\ v = \frac{29.26 \cdot 484}{1 \cdot 10^4} = 1.416 \text{ m}^3/\text{kg};$$

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for point D

$$p_0 = p_1\left(\frac{T_0}{T_1}\right)^{\frac{k}{k-1}} = 1\left(\frac{288}{484}\right)^{\frac{1.405}{0.405}} = 0.165\,\text{atm};$$

$$T_0 = 288°;\; v = \frac{29.28 \cdot 288}{0.165 \cdot 10^4} = 5.11\ \text{m}^3/\text{kg};$$

for point E

$$p_2 = 1.86\,\text{atm};\, T_0 = 288°;\; v = \frac{29.26 \cdot 288}{1.86 \cdot 10^4} = 0.453\,\text{m}^3/\text{kg}.$$

The amount of heat transmitted to the air along BC is, by (7)

$$Q_1 = \frac{k}{k-1} AR(T_1 - T_0) = \frac{1.405 \cdot 29.26}{0.405 \cdot 427}\,(484 - 288) = 46.6\ \text{cal/kg};$$

the amount of heat, removed from the air along DB, is by (13)

$$Q_{2DB} = ART_0 \ln\frac{p_1}{p_0} = \frac{29.26 \cdot 288}{427}\ln\frac{1}{0.165} = 34.7\,\text{cal/kg};$$

and that along BE, by (15)

$$Q_{2BE} = \frac{29.26 \cdot 288}{427}\ln 1.86 = 12.3\,\text{cal/kg};\ 34.7 + 12.3 = 47.0\,\text{cal/kg}.$$

By (17) the thermal efficiency is

$$\eta_t = \eta_{t1} = 1 - \frac{Q_{2DB}}{Q_1} = 1 - \frac{34.7}{46.6} = 0.255.$$

If the compressed air has to serve for burning gasoline, giving $i = \frac{H_u}{L} = \frac{10\,500}{14.9} = 705$ cal/kg (where $H_u = 10{,}500$ cal/kg is the calorific value of gasoline and $L = 14.9$ is the theoretical amount of air required for the combustion of 1 kg of gasoline), then the theoretical fraction of this amount of heat that must be spent on air compression, is

$$\frac{100L \cdot Q_1}{H_u} = \frac{100 \cdot 14.9 \cdot 46.6}{10\,500} = 6.63\%.$$

The theoretical air velocities at points C, D, B, and E are the following: at point C $w_1 = 0$; at point D, according to (7a)

$$w_0 = \sqrt{\frac{2g}{A}Q_1} = 91.5\sqrt{46.6} = 630\ \text{m/sec},$$

where

$$\sqrt{\frac{2g}{A}} = \sqrt{2 \cdot 9.81 \cdot 427} = 91.5.$$

According to equation (15), for reverse compression at point B we have

$$w_B = 91.5\sqrt{Q_{2BE}} = 91.5\sqrt{12.0} = 317\,\text{m/sec}$$

and, finally, at point E, $w_2 = 0$.

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5. DIRECT-ACTION JET ENGINES WITH CONSTANT FLOW VELOCITY. CURVES OF STATE OF GASES VELOCITY. AXIAL THRUST. HEAT TRANSFER THROUGH THE WALLS. FRICTION WITH THE WALLS

Let us now examine pure jet engines. If the engines are mounted on an airplane the engine nozzle is placed with its axis more or less horizontal. If these engines are built into the structure, they may constitute an independent flight device – a rocket.

By the term pure jet engines, we mean those carrying with them the oxygen for combustion.

Most jet engines described in literature are of the direct-action type, i. e., the propellant is burned under high pressure and then expands in a nozzle of suitable form. The combustion products do not participate in a cyclic process, but are ejected directly from the nozzle with a velocity which is constant or almost constant with respect to it during the entire flight.

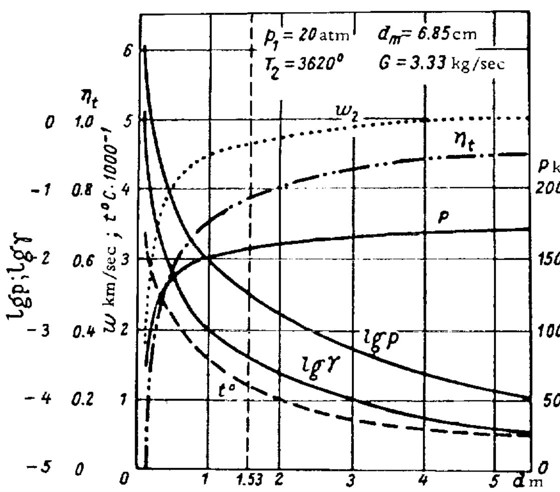

FIGURE 11. Diagram of state and flow of the gases in a hydrogen-oxygen rocket

I performed a calculation of the phenomena taking place inside the nozzle of such a rocket for a large hydrogen-oxygen rocket with a thrust of approximately 1500 kg; the calculation is analogous to that of steam turbine injectors, where a specially constructed *iS* diagram (*i*–enthalpy, *S*–entropy) and Pir's* formula for the heat content of water vapor are used. When the dimensions of the nozzle are large, the amount of heat passing through the walls represents a small percentage of the total heat and I have, therefore, assumed an adiabatic variation of the state of the gas (Figure 11).

* [The name has been transliterated back from the Russian.]

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Basic data of the rocket

Combustion pressure	20 atm
Combustion temperature	3620° abs
Hydrogen flow rate	0.37 kg/sec
Oxygen flow rate	2.96 kg/sec
Minimum diameter of the nozzle	6.85 cm
Angle between the axis and the generatrix of the nozzle cone	10° and 10° and 12°10'

In Figure 11 the following quantities are given as functions of the diameter d of the cross section considered (this may be taken in particular as the exit cross section):

w_2 the velocity of combustion products, km/sec;
$t°$ the temperature of gases, °C;
$\lg p_{atm}$ the logarithm of gas pressure;
$\lg \gamma$ (kg/m³) the logarithm of specific weight of gases;
P the resulting thrust, kg;
η_t the thermal efficiency up to the cross section considered, assuming the heat of the exhaust gases as lost.

We can see from the diagram that the curves for P, η_t and w_2 change rather sharply from a steep ascent to a gentle slope at a pressure of about 0.01 atm, which corresponds to a diameter of the exit section of approximately 1 m, $t° \approx 1500$ °C, $P \approx 1500$ kg, $w_2 = 4.4$ km/sec, and $\eta_t = 0.71$.

We may therefore draw the conclusion that direct-action jet engines, developing tremendous power at high flight velocities, need a comparatively small cross section. The length of the nozzle is approximately 2.6 m.

A free atmospheric pressure of 0.01 atm exists at a height of approximately 28 km. For operating the engine at low altitudes, it is possible to construct a nozzle with streamlined folding walls, though, due to the resulting impact with the air sucked in, it is expedient to use in this case one of the jet engine designs considered below, which suck in external air.

Regulation is also possible by varying the initial gas pressure, by varying the minimum cross section area of the nozzle or by varying the exit cross section by means of a flexible tube or flap.

Figure 12 has been composed with the purpose of showing the influence of the initial pressure p_1 on the final diameter of the nozzle for a given final pressure and given minimum diameter. We see the linear dependance of the thrust P on p_1, while the final nozzle diameter d_2 depends also, but not so strongly, on p_1. We can therefore conclude that the use of high combustion pressures (about 100 atm) is possible.

The combustion chamber. Assuming for it a diameter of 0.5 m, a length of 1 m, and a wall thickness of 4 mm, and using the Nusselt–Nernst formula for heat transmission through walls, we find for a cooling water temperature of 100 °C, a heat loss of approximately 0.4 % of the entire heat contained in the fuel; the difference between the temperatures of the external and internal wall surfaces is only 2°C. We see, therefore, that for a combustion chamber of large diameter the heat loss is negligible.

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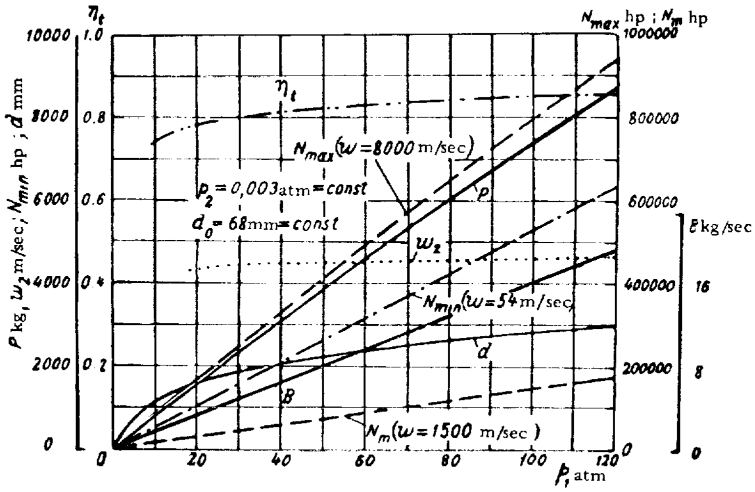

FIGURE 12. Rocket parameters as a function of initial pressure

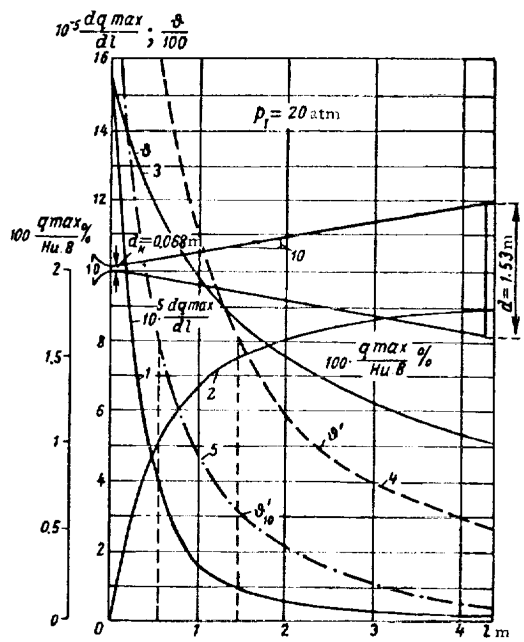

FIGURE 13. Determination of the heat transfer through the rocket walls

The heat loss through the walls of the nozzle was determined for various diameters, starting with the smallest (Figure 13). Curve 1 represents the heat loss through the walls in cal/hour for unit length

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of the tube (1 m). Curve 2 represents the percentage fraction of heat loss starting with the smallest diameter. We see that the total loss in this section is equal to approximately only 2 % of the total heat contained in the fuel. Curve 3 represents that temperature ϑ of the walls (the least suitable one) at which the heat flow rate is highest ($\vartheta = \frac{t}{2} - \frac{1}{2\beta}$) where t is the gas temperature, and β is a temperature dependent coefficient in the heat conductivity of water vapor). The entire calculation of the heat transfer through the walls was performed according to Nusselt's formula

$$\alpha = 19.23 \frac{\lambda_w}{d^{1.786}} \left(\frac{Bc_p}{\lambda}\right)^{0.786},$$

where α is a coefficient of heat transfer from the gases to the nozzle walls;
λ and λ_w are the heat conductivity coefficients of the gases at the average temperature inside the nozzle, and at the wall temperature;
c_p is the specific heat of the gases at constant pressure;
d is the nozzle diameter;
B is the fuel flow rate.

In this formula, the heat transfer coefficient α and the percent of heat lost through the walls, are inversely proportional to $d^{0.214}$, i. e., by reducing the diameter by a factor of ten, the percent of heat lost is increased by a factor of $10^{0.214} = 1.64$; in other words, in small rockets the percent of heat lost is larger than in big rockets. This conclusion is applicable to nozzles having similar forms and identical initial state of the gases; Nusselt's formula is applicable if the temperature of the walls is higher than the liquefaction temperature of the gases or part of them, i. e., in the absence of condensation. In this respect rockets are more advantageous for use than aviation engines, where much heat is lost due to the fact that part of the gases is liquefied near the walls of the cylinders which are considerably colder than the walls of properly cooled rockets.

By increasing the initial pressure of the gases, the coefficient α increases proportionally to $B^{0.786}$, and the percent of heat lost is reduced proportionally to $B^{-0.214}$ for a given nozzle.

Curves 4 and 5 show the temperatures of the nozzle walls due to heat radiation; it is assumed either that the walls are bare (ϑ' – curve 4), or that their radiation surface is increased by a factor of ten using ribs. It is seen that in the first case the temperature is higher than 800° over a length of approximately 1.5 m and in the second case that temperature is established over a length of approximately 0.6 m.

The nozzle walls will have the temperatures indicated above only if we wish to pass through them the greatest amount of heat without the appearance of condensation. We can, of course, maintain any temperature of the walls, cooling them, for example, by water or by liquid oxygen.

If the walls are thick, heat propagation along them is rather important for successful heat removal and the wall temperature falls off steeply at the nozzle throat. A sufficiently low temperature can be easily established by heat radiation; from Curves 4 and 5 (see Figure 13) it is seen that quite close to strongly heated places of small diameter there are places requiring comparatively little heat removal.

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The heat generated by friction with the walls can be expressed by the formula

$$Z = \frac{A\xi}{2g_0} \int \frac{w^2}{d} dl \text{ cal/kg.}$$

where $\xi \cong 0.02$ is the friction coefficient, $A = 1/427$, $g_0 = 9.81$ m/sec².

For the above-mentioned nozzle, the curve of w^2/d is shown in Figure 14. The total loss constitutes 12.3% of the entire heat contained in the fuel. For similar nozzles, the percent lost is a constant quantity, if the final velocity w_2 is the same in both cases, i.e., if the initial and final gas pressures are the same. The greatest loss occurs somewhat behind the throat; this place should be especially smooth. The pressure loss in the nozzle due to friction is given by the expression

$$\Delta p = \frac{\xi}{2g_0} \int \gamma \frac{w^2}{d} dl.$$

The curve $\gamma \frac{w^2}{d}$ is also given in Figure 14; the greatest loss is near the throat; the total loss is 3.5% of the initial pressure.

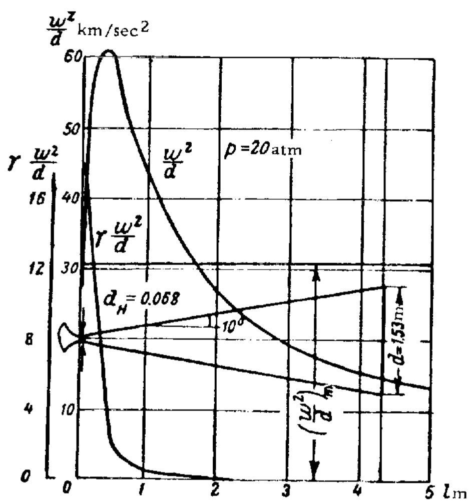

FIGURE 14. Friction of $H_2 + O$ gases in a rocket

The increase in thermal efficiency with height for various initial pressures p_1, assuming adiabatic expansion after combustion, is shown in Figure 15; we find e.g., that an initial pressure $p_1 = 1$ atm, gives $\eta_t = 0.20$ at a height of 16 km. For such a low initial pressure, the walls of the nozzle may be very thin.

Instead of the above formula for Z it is possible to use more exact formulas, taking then $w^{1.7}$ instead of w^2. This, however, changes the picture very little.

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6. JET ENGINES WITH CLOSED WORKING CYCLE

The use of a closed working cycle in a jet engine with constant exhaust velocity has a great effect in flights in a not too rarefied atmosphere.

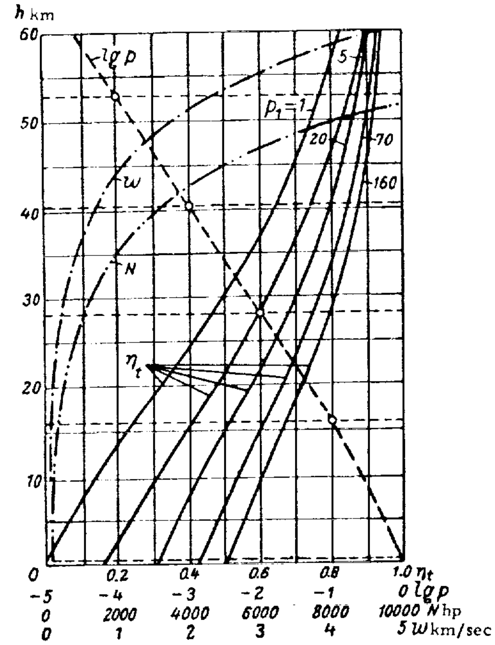

FIGURE 15. Thermal efficiency of a rocket, flight velocity and power of a jet engine as functions of flight altitude

Indeed, examining the diagram in Figure 16, where p is the pressure, and v is the specific volume of the gas expanding in the rocket nozzle, we find that the velocity attained by the gases leaving the nozzle is given by

$$\frac{w^2}{2g}=\int_{p_z}^{p_a} v\cdot dp,$$

where p_z is the combustion pressure and p_a the pressure of the free atmosphere at the given altitude above the Earth.

However, $\int_{p_z}^{p_a} v\,dp$ is the area of $ABCD$. If we continue the expansion curve (adiabatic, polytropic, or any other curve) beyond point C and then compress the gas isothermally along EF at a temperature as low as possible, then the total area, corresponding to the kinetic energy, $w^2/2g$, of the gas obtained will be equal to $ABEFD$; this area is considerably larger than the area

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$ABCD$ and, therefore, the exhaust velocity W_2, the reaction force P and the total thermal efficiency η_t are increased.

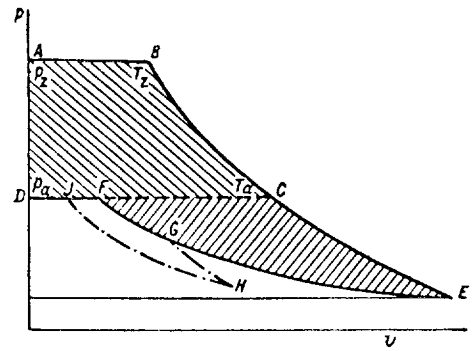

FIGURE 16. pv diagram of a jet engine improved by a working cycle

The gas can be cooled strongly in the reverse compression, it can also be cooled initially at the lowest pressure since here the friction loss is small (see Figure 14), and compressed later, while cooling, to the pressure p_a.

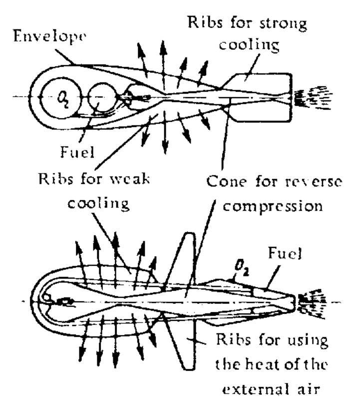

FIGURE 17. Two schematic diagrams of jet engines improved by a working cycle

A scheme of such a rocket is shown in Figure 17. The cooling can be performed either by air (with longitudinal ribs) or by water; if liquefied gases (O_2, H_2 or others) are available they may also be utilized for cooling. Another possibility is to cool or freeze the usual fuel (gasoline, toluene) before the flight, in order to obtain an area $\int v\,dp$, as large as possible i.e., a thrust as large as possible.

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For a low energy fuel it is possible to extend the expansion curve to temperatures lower than that of the external air. When cooling by liquid air or by liquid hydrogen it is possible to use some of the heat of the external air, i. e., using the energy of the atmospheric air: this is possible since on Earth we have done research on the cooling and liquefying of fuel.

A detailed investigation of the conditions under which one could regain part of the rather great amount of energy spent on the liquefaction of oxygen and hydrogen would be desirable; the entire working cycle should then be carried out accordingly. In this case hydrogen will become an even more powerful fuel.

It is also possible to start a secondary expansion, e.g., a polytropic expansion with subsequent compression (see curve *GHI* in Figure 16) at a particular point on the compression curve.

This method can give good results if the cooling on curve *EG* is effected at a higher temperature than on curve *HI*.

7. EFFICIENCIES OF PURE JET ENGINES USING COMBUSTION PRODUCTS EXCLUSIVELY

While the thermal efficiency in the method described above can be increased already for low altitude flights, the mechanical efficiency of the jet engines considered here remains low at low flight velocities since the combustion products carry with them a large amount of kinetic energy (corresponding to their absolute velocity).

This follows also from the following considerations.

At flight velocities v, small compared with the exhaust velocity w_2, it is sufficient to consider the efficiency η_i, which is equal to the ratio of the useful work during a short time interval, to the total heat content of the fuel consumed during the same period of time

$$\eta_i = \eta_m \varphi^2 \cdot \eta_t,$$

where $\eta_m = \frac{2vw_2}{w_2^2} = \frac{2v}{w_2}$ is the ratio of the useful work to the energy corresponding to the velocity w_2;

$\varphi = \frac{w_2}{w_{2\,max}}$ is the velocity reduction coefficient due to friction;

$\eta_t = \frac{i_1 - i_2}{i_1 - i_0}$ is the thermal efficiency, where i_1 is the initial enthalpy, i_2 the effective final enthalpy, and i_0 the greatest possible final enthalpy without friction.

We may also write

$$\eta_i = \frac{2w_2 v}{w_{2\,max}^2}\,*,$$

It is seen, therefore, that for a constant exhaust velocity, η_i is proportional to v, i. e., at low flight velocities the efficiency is proportional to the flight velocity (Figure 18).

If the flight velocity is increased so much that the kinetic energy of the propellant in the tanks of the ship (airplane or rocket) starts to increase

* [The assumption $\eta_t = 1$ is implicitly made here.]

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noticeably as compared with the thermal energy, then we have to take into account the sum of the absolute kinetic energy and the thermal energy of the propellant exhausted at that instant. This efficiency is denoted by η_{i+E}. We can then write the formula (which I derived in 1918)

$$\eta_{i+E} = \frac{\varphi^2 \frac{i_1 - i_2 - q_s}{A} + \frac{v^2 - v_2^2}{2g}}{\frac{i_1 - i_0}{A} + \frac{v^2}{2g}} = \frac{2w_2 v}{w_{2\,max}^2 + v^2} = \frac{\eta_i}{1 + \frac{\eta_i \eta_m}{4}} = \frac{\eta_i}{1 + \frac{\eta_i^2}{4\eta_t \varphi^2}};$$

where v_2 denotes the absolute exhaust velocity after ejection from the nozzle, and q_s is the heat escaping to the outside in cal/kg.

Curves showing $\eta_i + E$ vs v/w_2 and vs the flight velocity v for an exhaust velocity w_2 of 4000 m/sec are also given in Figure 18 for $\eta_t \cdot \varphi^2 = 1$, $\eta_t \cdot \varphi^2 = 0.75$ and $\eta_t \cdot \varphi^2 = 0.5$.

Thus, the theoretical maximum of $\eta_{i+E} = 1$ is obtained for $v = w_2$, i.e., when the absolute velocity v_2 of the gases ejected from the nozzle is zero; in other words, when there is no loss of kinetic energy. We see that for flight velocities from 1500 m/sec to 8000 m/sec, of interest in the field of superaviation, the efficiency in interplanetary space will be high with $w_2 = 4000$ m/sec.

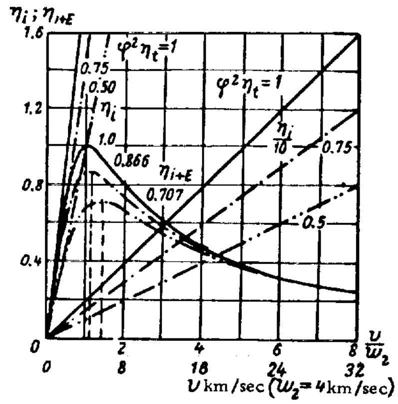

FIGURE 18. Instantaneous efficiency of a jet engine operating on the combustion product alone versus flight velocity

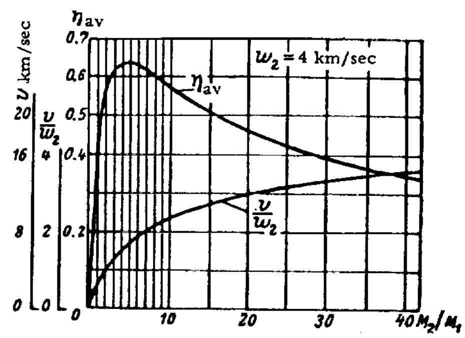

FIGURE 19. Average efficiency and flight velocity of a rocket versus mass ratio, neglecting gravity

If the flight conditions do not permit a rapid transition from low to high velocities, then the air breathing jet engines examined below, utilizing partly the external air to produce the reaction force, should be used for flight velocities lower than 1500 m/sec, while the aviation engines described above will be needed in the lowest layers of the atmosphere.

I have derived, following the Russian scientist K. E. Tsiolkovskii, the average efficiency η_{av} during the entire flight (Figure 19) as a function of the ratio of propellant consumed M_2 to the final mass M_1 of the rocket. The maximum average efficiency $\eta_{av\,max} = 0.65$ is obtained for

$$\frac{M_2}{M_1} = 4.$$

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The curve v/w_2 for w_2=4 km/sec is also shown. We see that initially the velocity grows rapidly and it increases little subsequently, i.e., it is a logarithmic curve. The entire diagram refers to flight in a medium without gravitational forces; attraction by the Earth and friction with the air reduce the values of η_{av} and v. To attain a flight velocity of 8 km/sec, neglecting gravity and friction with the atmosphere, a value of M_2/M_1=6.5 or a propellant consumption of $\frac{6.5}{6.5+1}$ = 0.965 of the initial weight of the rocket would be needed for an exhaust velocity of w_2=4 km/sec, whereas for a flight velocity of 3.5 km/sec, a M_2/M_1=1.41 or a propellant consumption of 59% of the initial weight of the rocket would be required.

8. AIR-BREATHING JET ENGINES. THEORETICAL AIR COMPRESSION PRESSURES. SECONDARY USE OF THE OUTGOING HEAT. VARIOUS CYCLES. SCHEMATIC DRAWINGS. WEAK IMPACT MIXING OF AIR AND COMBUSTION PRODUCTS AND INDIVIDUAL CYCLES

We have already examined, to a certain extent (see Figure 10), one of the forms of air-breathing jet engines; the theoretical working cycle of the air sucked in is given in Figures 8 and 9. In the case of an adiabatic expansion from atmospheric pressure p_1 and temperature T_1 to $p_0<p_1$ and an isothermal recompression to a final pressure p_2 and temperature T, we may express all the quantities referring to the closed cycle as functions of T_1/T (Figure 20). In this figure $\frac{p_0}{p_1}$; $\frac{p_2}{p_1}$; $\lg\frac{p_2}{p_1}$ and η_t are given for air (with adiabatic exponent k=1.405); η_t is the theoretical thermal efficiency of the part $BCDB$ of the process (see Figure 8), defined as $\eta_t=1-\frac{Q_{2DB}}{Q_1}$, where Q_1 is the amount of heat transferred to the air under atmospheric pressure along the section BC, and Q_{2DB} the amount of heat removed along the section DB. Consequently, the final pressure p_0 should not be too low even under strong heating and the theoretically obtained pressure p_2 increases quite steeply. Increasing the temperature by a factor of four (to approximately 900°C for T=300° abs) gives a pressure of about 275 atm and a temperature increase by a factor of five (to 1200°C) yields a pressure of 4000 atm. Consequently, by means of such a device, a jet engine could compress air for combustion to rather high pressures and, in addition, up to rather high flight altitudes. Assuming the air temperature at high altitudes to be -60°C or 273-60=213° abs, we obtain for T_1/T=5, a temperature of 5×213=1065° abs, or 792°C. It is quite probable that it will be possible to compress the mixture of hydrogen, oxygen and nitrogen, existing at altitudes of about 60 km, to such an extent that it will be ignited and could serve as a fuel giving a high reaction force P. Even considerably higher final pressures may be obtained when cooling by liquid oxygen, which can also be injected for cooling into the inverted cone. The only difference in the case of weak cooling is that lower pressures are obtained. The thermal efficiency of such an engine increases with temperature (see Figure 20), reaching up to 60% for T_1/T=5.

If we use the external air for heat removal, we may increase the thermal efficiency by utilizing it as the air sucked in; since it is already slightly

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heated, it reduces the heat flow out of the engine. For this purpose it is possible to put behind the engine streamlined pipes around which the hot gases coming out of the engine would stream; at the same time these pipes would suck in fresh air for the jet engine. The schematic design of such an apparatus is shown in Figure 21.

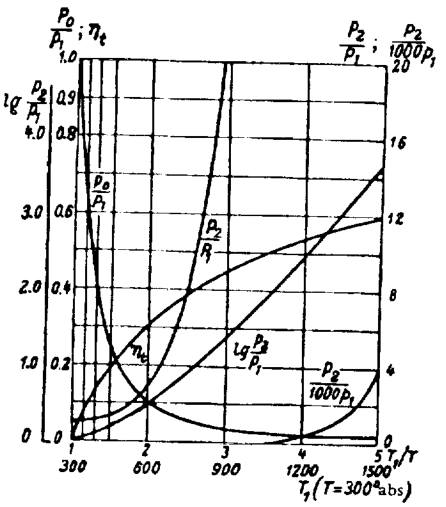

FIGURE 20. Theoretical air compression curves of jet compressors

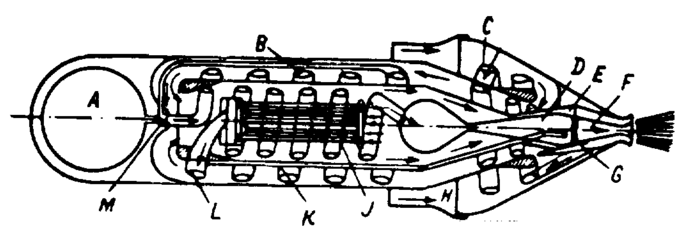

FIGURE 21. Schematic diagram of an air-breathing jet engine with separate cycles

A-fuel tank; *B*-reverse compression of air for combustion; *C*-reverse compression of the air sucked in; *D*-expansion of the combustion products; *E*-expansion of the air sucked in; *F*-reverse compression of the combustion products; *G*-connecting openings for sucking in air to obtain low pressure; *H*-space for increasing air pressure; *I*-heating of air for combustion; *K*-connecting pipe to the air used for combustion, for obtaining low pressure; *L*-expansion of air for combustion; *M*-location of flame.

It should be noted that in this design the length of the pipe conductors does not correspond to the actual dimensions. Should the calculations show that the pipes of the inverted cones should be short, they may be built along a straight line and not along a spiral.

The device itself compresses all the air necessary for the combustion and thus constitutes an independent engine without continuously moving parts. Only control means are required: valves, folding flaps and other shutters. The efficiency of such a device is considerably higher than that of the

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previous one, since the heat removed from the air during compression is transferred to air which is going to participate in the closed process instead of being lost. The amount of heat which has to be removed from the combustion products for compressing the air necessary for the combustion is small. For the theoretical closed process, shown in Figures 8 and 9, it is equal to $6^2/_3$% of the entire heat contained in the fuel for a compression factor of 1.86, and it is 15% for a ten-fold compression.

Cooling the air in the reverse compression by the fresh air sucked in may further reduce the heat loss. The limit for the possible cooling is imposed by the weight of the entire device and the problem consists in designing the hardware which will give best results under given flight conditions.

If the air sucked in does not serve for combustion, and is only accelerated inside the engine to increase the reaction force, it can be mixed with the combustion products at the place of lowest pressure. This place should be chosen so that the velocity of the heated air is equal to the velocity of the combustion products, or is somewhat lower (Figure 22). In this case, there is no impact between the air and the combustion products. This impact exists in ordinary injectors and constitutes the main reason for their low efficiency. Therefore in the device considered, the efficiency is considerably higher than in ordinary injectors. If the working conditions vary a certain impact loss may be encountered. This may be avoided by using a design (see Figure 21), in which the velocities of the air and of the combustion products may be different in the connecting slots or pipes between the air and the combustion products.

In order to reduce the friction, the walls of the cones, in particular near the place of the critical velocity, have to be ground; the nozzles of the direct cones have to be made short, choosing a design which minimizes the sum of the losses due to friction and to partial jet separation from the walls.

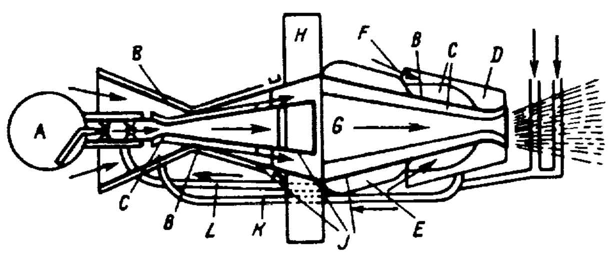

FIGURE 22. Schematic diagram of an air-breathing jet engine, in which the combustion products are mixed smoothly with air

A-fuel; *B*-isolating wall; *C*-ribs; *D*-envelope; *E*-liquid oxygen for cooling; *F*-air; *G*-mixing chamber; *H*-ribs for external heat inflow; *J*-walls of the cones; *K*-pipe for sucking in the heated air; *L*-pipe for carrying off O_2.

There may be different closed cycles and it is possible to find the most advantageous cycle for a given case by a variational calculation. The amount of air sucked in may be very large in these designs. The amount of heat which must be removed from the combustion products in order to equalize the velocities, has been determined for the device shown in Figure 22 for various ratios of air to combustion products, (it is assumed that combustion

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takes place between gasoline and the theoretical amount of air). Further, the velocity w, common to both gases at their mixing place, has been determined. From Figure 23 is seen that if, for example, the amount of air is ten times larger than the amount of combustion products (M=10), then for the equalization of the velocities it is necessary to transfer to the air x=82.3% of the total heat contained in the fuel, and the common velocity at the mixing place will be w=1000 m/sec. The diagram has been drawn assuming that the initial air temperature is T=300° abs, and the final temperature, identical for the air and for the combustion products, is 50° abs [sic*], at the point of lowest pressure, i.e., at the mixing place. For large M, the velocity w and to some extent x, too, depend on the difference between these two temperatures. For M=100, the pressure at the mixing place is approximately p=0.0033 atm if the heat is removed at constant pressure from the combustion products, and only then they are expanded adiabatically. The largest amount of heat can be removed if the pressure of the combustion products is not varied, i.e., if the heat is removed at the highest pressure of the combustion products. This is important for the design.

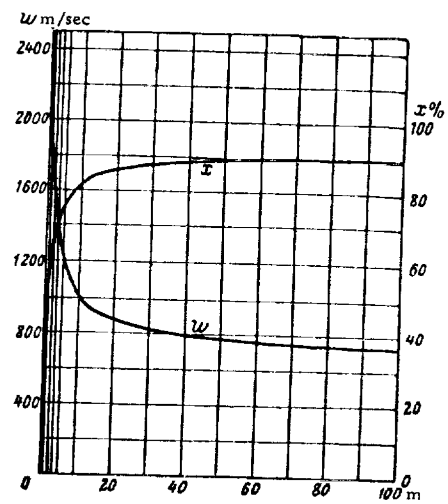

FIGURE 23. Diagram for an air-breathing jet engine w-common velocity; x-fraction of heat transferred to the air (percent).

The diagram in Figure 24 is obtained for the following conditions: cooling to an external air temperature T_a of 288° abs during the reverse compression; initial combustion products pressure of 1.86 atm; an external air pressure of one atmosphere; an expansion of the air and the combustion products to the same pressure p_4 with a temperature decrease to the same value T_4; heat transfer at a pressure p_2=1.86 atm followed by a separate compression. The diagram shows also the velocities w_3 and w_3' of the air and the gases at the

* [This is well below the condensation temperature of air.]

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exit from the device, its total thermal efficiency, and its total reaction force P; without air sucking this force is equal to 160 g for the given fuel consumption. From Figure 24 we see the great increase of the effect due to the air sucking. Other quantities plotted are the temperature T'_a of the gases leaving the device when the temperature of the gases at the place of lowest pressure p_4 is T'_E, and the percent of the entire heat contained in the combustion products which should be transferred to the air. If we adopt the design of Figure 21, then the axial forces obtained will be even higher, in particular for a large air surplus M. From Figure 21 we see that without secondary use of the heat removed from the air during the compression period, it is possible to reach M=30. Further, the reaction force P increases only slightly. For M=30, the reaction force increases by a factor of 6.2 with respect to an ordinary rocket. It is interesting that the reaction force increases with M, whereas the thermal efficiency has its optimum value ($\eta_{t\,max}$=0.42) at M=2; this is due to the fact that at the high exhaust velocities which occur for small M, the reaction force is comparatively small. M increases faster than w_3 diminishes, and the reaction force, being proportional to Mw_3 increases. The product Mw_3^2, which is approximately proportional to η_t, however, decreases. Complete cooling is, of course, impossible, and therefore, the performance of the device will be somewhat lower. The efficiency η_t will, however, be considerably higher in the device of Figure 21 and, therefore, the reaction force will also increase considerably. Only the arrangement of the engine's separate parts brings about certain difficulties; it is necessary that the air sucked in, in particular at high flight velocities, should not change its direction of motion sharply. Taking into consideration all these losses, it is possible to develop the best design. It is possible, for example, to reduce the air velocity at the inlet to the apparatus, making the inlet section of variable cross section in the form of an inverted cone. This method has the following advantages:

1) at high air pressures, the efficiency of the apparatus increases and
2) with low gas velocity inside the device, the losses due to variations of jet direction will be small.

Till the present time, designs of jet engines use impact mixing like ordinary injectors, which gives low efficiency, or the outgoing gases have a high temperature, which also lowers the performance. If the air sucked in serves at the same time for combustion, there exists an upper limit to the air surplus equal approximately to twice the theoretical amount, since in a large amount of air the mixture does not ignite. At rather high flight velocities, the following simple design may find application (Figure 25). The incoming air is compressed in the cone AB, it is heated under high pressure (see pv diagram) from B to B' and then expands from B' to C; the area $ABB'C$ corresponds in the pv diagram to the kinetic energy obtained $\frac{w_2^2 - v^2}{2g}$*. In this case also it is better, however, to add an inverted cone DE in order to obtain a closed process, (see the dotted section). We obtain then a higher exhaust velocity. The increase in kinetic energy corresponds to the area $ABB'C+CDE$ in the pv diagram, if DE is cooled.

* One should not confuse the specific volume v in the pv diagram with the flight velocity v in the formula $\frac{w_2^2 - v^2}{2g}$.

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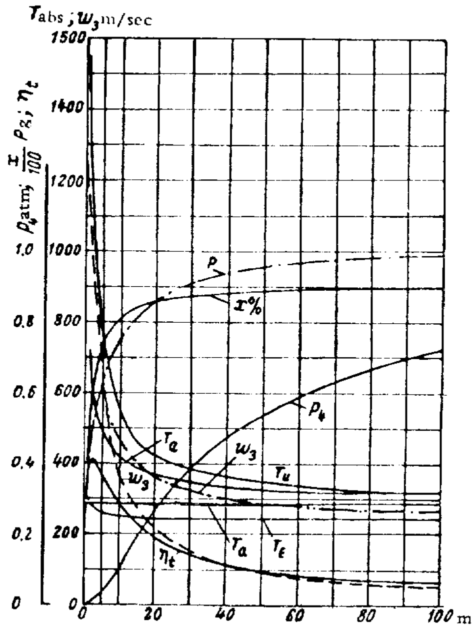

FIGURE 24. Diagrams for an air-breathing jet engine with separate cycles

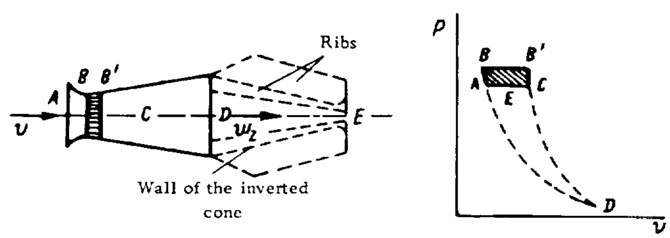

FIGURE 25. Schematic diagram of an ordinary air-breathing jet engine

9. EFFICIENCY OF AIR-BREATHING JET ENGINES. COMPARISON WITH PISTON ENGINES

The problem of the effective efficiency η_e of an air-breathing jet engine at high flight velocities is rather important. B. M. Stechkin* derives the formula

$$\eta_e = \frac{2\eta_t}{1+\sqrt{1+\eta_t/y}} = \frac{2v}{v+w_2}\eta_t.$$

* Stechkin, B. M. – Tekhnika vozdushnogo flota, No. 2. 1929.

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Applied to our case, we have

$$y = \frac{MAv^2}{2gq};$$

here q is the amount of heat removed to the air per kg of combustion products, and $\frac{2v}{v+w_2}$ is the efficiency of the propeller, where v is the air velocity in front of the propeller, which is equal to the flight velocity, and w_2 is the air velocity behind it. Comparing the jet engine with an ordinary internal combustion engine, consuming 240 g per hp per hour of fuel with an enthalpy of 10,800 cal per kg and having a propeller efficiency of 0.75, we obtain for the internal combustion engine

$$\eta_e = \frac{632 \cdot 0.75}{0.240 \cdot 10\,800} = 0.244 \cdot 0.75 = 0.183, \text{ i. e., } 18.3\%.$$

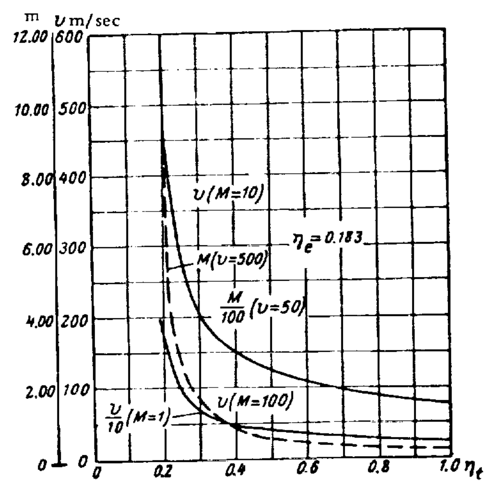

FIGURE 26. Comparison of an air-breathing jet engine with an internal combustion engine

For gasoline we may also write

$$y = \frac{v^2 M}{5.68 \cdot 10^6 x},$$

where x is the percentage of heat removed to the air. The results for the case in which the total effective efficiency of the jet engine is also equal to $\eta_e = 0.183$ i. e., for $x = 1.0$, are shown in Figure 26. For points lying above the curves, the jet engine is superior to the internal combustion engine, and for points lying below, it is inferior. An increase in flight velocity and the amount of air sucked in, improves the performance of the jet engine. In the above formula, the thermal efficiency is referred to the amount of heat contained in the fuel and not to the sum of the heat and of the kinetic energy

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accumulated during acceleration in the fuel situated in the tank. For the jet engine shown in Figures 22 and 25, this may mean at high flight velocities $\eta_t > 1$, similarly η_i in Figure 18. For the design of Figure 21 it is as if we had two jet engines: an air-breathing one, in which the heat is transferred to the air, and one working with the combustion products only; a special study has to be made on the most favorable conditions for the operation of the latter type at high flight velocities. According to the formula of Stechkin, at high flight velocities v and also for large amounts of air sucked in (large M) the value of η_e approaches from below that of η_t.

10. FUEL FOR JET ENGINES

We will now examine various types of fuel which might have an application in super-aviation. It can be shown that while our aviation engines can carry a sufficient supply of fuel but are unable to lift the airplane above a certain ceiling due to insufficient engine power, in jet engines, as well as in rockets, the power increase with higher altitudes and flight velocities is adequate but it is difficult to store enough fuel in the vehicle. It will, therefore, be rather expedient to use part of the fuel for jet engines in solid form and to prepare from it beams and surfaces for the flying apparatus (using, e.g., celluloid). It is also worthwhile to undertake experimental research for extruded materials, used nowadays in almost all fields of chemical engineering, which may be suitable for our purposes. Naphthalene engines already exist; one may conceive materials containing naphthalene or some other fuel mixed with such a substance, which melt by heating and then proceed from a special melting chamber as a liquid fuel to the injectors of the rocket. Instead of heating, a solution process may be used in certain cases. For example, cellulose, of which papier-maché is made, may be dissolved in nitric acid. The oxygen of the latter may serve in this case to replace partially the liquid oxygen, which will then be required in smaller amounts. Some metals as, e.g., lithium, magnesium, aluminum, contain a tremendous amount of heat; they may find successful applications in air-breathing jet engines, but particularly in such engines in which liquid oxygen for combustion is taken along for flight. For air-breathing jet engines, those materials requiring much oxygen are most advantageous. The above-described closed processes can be carried out not only in rockets with liquid propellant, but also in all thermochemical or solid propellant rockets as well as in such working with liquid metals.

However, in all the cases in which the combustion products are obtained partly in the form of solid particles, it is necessary to use a fuel which gives at the same time gaseous combustion products, or air containing inert nitrogen, so that the heat of the solid particles might pass in the expansion to the gaseous combustion products. For instance, black powder, which contains KNO_3, produces K_2CO_3, K_2SO_4, and K_2S, and from these heat passes to the carbon dioxide and to other gases which are formed at the same time.

Furthermore, the inside surface of the walls should not be dirtied by the solid combustion products; for that purpose I have tested a very simple design (Figure 27) of a double cone (the internal one is perforated); a film of

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magnesium was burnt under it and gaseous bunsen burners were placed in the space between the cones. As a result of the entrance of gas through the openings of the internal cone, it remained completely clean in a certain section. The total deposit on it was reduced from 23 % of the total amount of combustion products to 13 %. If a rocket is constructed with perforated internal walls, on whose exterior we maintain a pressure somewhat higher than the internal pressure, then, in my opinion, the rocket will be almost completely protected against contamination. Schematic diagrams of rockets and airplanes, which may use these structures, will be shown below.

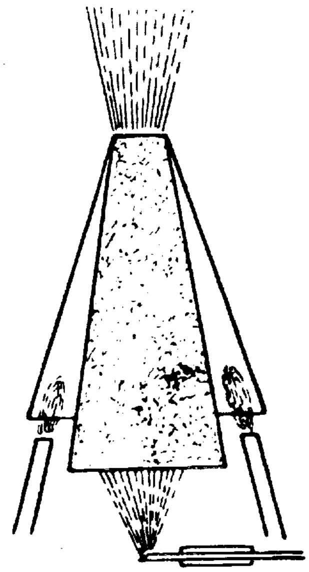

FIGURE 27. Experiments with cones; the internal cone is perforated

It is possible to perform, as I did, the following experiment with magnesium: evaporate it in a jet of very hot hydrogen in a closed vessel and then ignite the hydrogen jet. A uniform brilliant flame is obtained. All the metal may be evaporated already below the boiling temperature if it is atomized by a suitable atomizer. The boiling temperature of several metals is the following: Cd–770°; Zn–907°; Mg–1120°; Sb–1140°; Bi–1420°; Pb–1525°; Al–1800°; Mn–1900°; Cr–2200°; Sn–2270°; Cu–2310°; Fe–2450°C.

I have experimented in 1928-1929 with the possibility of igniting in air alloys containing magnesium; it turned out that alloys of magnesium with zinc containing also other impurities were ignited on a wire loop; for a magnesium content of 5-10 % and more, the alloys ignited better; although alloys containing much Al did not burn well. Probably only good atomization of alloys containing Al will make it possible to burn the latter fully, since Al is easily covered with a film of oxide which protects it against further oxidation. Alloys of copper and iron with magnesium burned well. Probably all mixtures of metals which are not covered by a protective oxide film and which combine readily with oxygen will burn well with Mg. Experiments with the eutectic alloys of magnesium with zinc: 5 % Mg, 95 % Zn and 40 % Mg, 60 % Zn are rather advantageous; they melt at temperatures of about 305 and 330°C. The metals used as fuel should have sufficient viscosity and strength and be easy to melt. The melting vessels should have a rate of heat transfer sufficient for melting and using the materials employed as fuel during the flights; this may be achieved by using fire-tubes or strongly corrugated walls, although, in certain cases, the flame may impinge directly on the metal.

Several metals and other compounds are listed in order of increasing fuel and oxygen consumption in Table 1 below. Column 1 gives the compounds formed in the combustion; column 2 the required ratio of initial to final rocket weight in order to obtain a flight velocity of 7.55 km/sec, assuming that the exhaust velocity amounts to 75 % of the theoretical one (the energy loss is $1-0.75^2=0.44$ or 44 %); column 3 gives the ratio of the weight of the metal fuel to the final weight of the rocket; column 4, the order of consumption of melted metal; column 5, the calorific value per kg of

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combustion products; column 6, the percent of O_2 in the compound and column 7, the theoretical exhaust velocity. We see that together with oxygen compounds, fluorine (F) compounds may also find applications; the latter has the advantage that fluorine is easier to liquefy than oxygen. Only the price and the effect on the breathing organs may limit its application.

The lowest total weight would be obtained by the use of Li_2O and the smallest amount of solid material would have to be burned when using B_2O_3. However, it will be probably possible to use boron only in the form of insulation powder (amorphous boron) or in the form of compressed rods (crystalline boron). It might also be possible to carry along liquid boron hydride in a rather cold state.

TABLE 1

Compound	Total weight φ 0.75, v = 7.55 km/sec	Weight of combustible metal	Order of metal consumption	Calorific value	Percent of O_2	Theoretical exhaust velocity
Li_2O	4.98	1.85	2	4710	53.5	6270
LiF	5.82	2.33	7	4450	44.7	6100
B_2O_3	5.85	1.53	1	3900	68.5	5700
H_2O upper	5.92	–	–	3240	–	5200
$Mg(OH)_2$	6.06	2.09	4	3750	55.1	5600
Al_2O_3	6.07	2.68	8	3730	47.0	5590
$Na_2B_4O_7$	6.14	2.24	6	3700	55.5	5550
MgO	6.34	2.69	9	3560	39.7	5450
MgF_2	6.64	2.18	5	3400	61.4	5320
Al_2F_6	6.78	1.87	3	3320	68.0	5260
H_2O lower	6.92	–	–	3830	–	5660
CaF_2	8.04	3.61	10	2800	48.7	4830
NaF	8.59	4.50	11	2640	45.3	4690
Gasoline	9.71	–	–	2350	77.5	4430

O_2 constitutes, however, 68.5% of the weight of B_2O_3; this is a high percentage and may cause difficulties. The compound most favorable from this point of view is MgO, which contains only 39.7% O_2. For air-breathing jet engines, Li or B turn out to be better due to their low atomic weight and consequent low total consumption. In view of the comparatively high price required for processing of the metal, the price of the raw materials is, therefore, less important. The price of Li is, nevertheless, rather high, and it may therefore be assumed that in the meantime either MgO or Al_2O_3 or Al_2F_6 will be used; in this case the combustible metal's weight is only 1.87 times the weight of the empty rocket.

Among liquid fuels, we may point to liquid methane, which gives about 13,000 cal/kg and is much cheaper than liquid hydrogen. In view of the considerable increase in rocket performance by the use of powerful cooling means, low freezing fuels as, e.g., toluene (−100° C) will be generally advantageous.

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Pressurized kerosene will be useful in engines, specially those using oxygen, because of its lubrication properties.

When using a metal as a fuel it should not be forgotten that if atmospheric air is not used, one must always burn together with the metal a material which yields gaseous combustion products. This may be celluloid or naphthalene or some other solid material, so that the flight can be carried out without any liquid fuel; this may increase considerably the strength of individual sections of the flying apparatus. In air-breathing jet engines, nitrogen constitutes the gaseous material and therefore the presence of a liquid fuel is not necessary. The following experiment has been conducted with aviation engines: 500 g of gasoline per hour per hp were supplied to the engine. Only after half an hour's operation did its valves become strongly obstructed. It may therefore be assumed that in particularly simply designed engines for flight outside the terrestrial atmosphere it will be possible to utilize metallic fuel. Towards the end of the first world war, the French already built engines of particularly simple design intended for one flight. If this is done in the conditions of super-aviation, such an engine may be built of a suitable metal which will be used as metallic fuel, leaving in the airplane only a small engine for landing.

When using metallic fuel, a ceiling for airplanes or rockets will not exist, since there will be no limit to the amount of fuel.

In designing a rocket or an airplane, using exclusively liquid fuel, the problem of the designer is to design them so that they will carry along the greatest amount of fuel for a given calorific capacity. For that purpose it is necessary to use the strength of the material up to the admissible limit, and to simplify the rocket as much as possible, since a large number of small, sufficiently strong parts increases the weight reducing the amount of liquid fuel which may be stored in the rocket.

The problem of how to cope with the two requirements of material quality and minimum rocket complexity is reduced, in a device using metallic fuel, to another problem – namely to design it so that some parts fall into others and are melted in them until, finally, almost nothing is left.

The solution of the latter problem is, however, considerably simpler [sic!].

11. ADVANTAGES OF VARIOUS TYPES OF ROCKETS. ACCESSORIES FOR ROCKETS

Detailed designs of stage rockets, consisting of two or several rockets one within the other have been made in Germany and in America. These rockets may also give rather high flight altitudes, but their initial weight is many times larger than that of rockets using solid propellants. This is due to the fact that after the ascent of a large rocket together with a small one to a high altitude, the large rocket comes down gliding whereas in my design all its weight is used for increasing the flight altitude further. Since the large rocket weighs 10-20 times more than the liquid propellant contained in the small rocket, its use as a propellant may increase the flight altitude by a huge factor. The use of the large rocket as a propellant might be discarded with time, however, if it will become possible by gradual improvement of air-breathing jet engines and of combinations of the propeller-

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motor group with rockets, to fly away from the Earth without consuming solid propellant or with a very small consumption of it.

The Russian scientist K. E. Tsiolkovskii proposed the use of rocket trains for flights; in these the rockets are uncoupled from the train and descend one after the other; the last, small rocket can achieve a high flight velocity and even fly to another planet.

For rockets intended for research of the higher layers of the atmosphere, the following accessories are required: 1) a meteorograph; 2) a device for regulating the fuel and oxygen consumption, as well as the air inflow in air-breathing jet engines; 3) gyroscopes for controlling the flight direction; 4) a device for ejecting the parachute in the return landing.

The advantages of those or other types of rockets can be decided in relation to the purpose for which a given rocket is intended. Thus, multi-stage rockets will be better suited to attain the highest altitudes, or the highest flight velocities, than single-stage ones; rockets using a certain proportion of atmospheric air and liquid oxygen will be more advantageous than those using only atmospheric air or only liquid oxygen from the beginning of the flight to its end. This is so since near the surface of the Earth there is much atmospheric air and it will be disadvantageous, therefore, to carry along liquid oxygen. At high altitudes over the Earth's surface the opposite is true.

As indicated already above, rockets which throw away certain parts of their structure during the flight will be less advantageous than those which use them as a propellant.

As to the comparison of solid propellant rockets with those working with liquid or liquefied propellants (for example, metallic), it should be noted that liquid propellant rockets are extremely simple but, on the other hand, the gas pressure in them is rather high, if we do not add to the propellant a considerable amount of carbon or of some other suitable inert material which, however, lowers the calorific value of the propellant. Propellant powder belongs to materials containing all the oxygen for combustion. The calorific value of smokeless powder is only 1240 cal/kg, and from Table 1 we see, that, for example, gasoline and to an even larger extent, hydrogen and light metals possess a considerably larger calorific capacity per kg of combustion products.

Further, the feeding of a large amount of solid fuel in a comparatively small combustion chamber of one rocket may present difficulties. But such feeding may be required in a rocket airplane, where much fuel has to be taken for a small rocket. However, the total efficiency of solid propellant rockets may be improved by introducing atmospheric air.

A comparison of solid-propellant rockets with those using other types of propellant, leads therefore to the conclusion that rockets, working on different propellants, have a certain superiority. However, for certain purposes, for example, for rather short duration jumps or for fast take-off rockets not carrying persons where, consequently, safety does not play an important role, solid propellant rockets will also be used.

Air-breathing jet engines working on metallic fuel will, probably, be the safest, since in this case one not only avoids the explosive liquid oxygen, but the metal itself does not yield gaseous products when heated.

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12. AIRPLANES EQUIPPED WITH A ROCKET AND ENGINES. PART OF THE STRUCTURE USED AS FUEL

Figure 28 shows my schematic design of an airplane, whose external sections may be pulled in. The wings consist of separate sections within a special frame, and by means of cables mounted on conical drums of suitable shape, the wing sections and all the rest of the parts are pulled into vessel A for melting and subsequently used as fuel. Since the path of the individual sections is not longer than 5-8 m on the average, small drums are required. I studied to some extent the sections of the airplanes which can be used, and calculated their strength; it turns out that such an airplane could carry only approximately 10% less liquid fuel than an ordinary airplane due to the weight of the dismantable joints. The airplane wings occupy the greatest area of all parts which have to be moved; however, in certain airplane constructions, the wing area may be reduced during the flight to 1/3 of its normal value in order to increase flight velocity, so that in our case the displacement will be only one step forward. The rest of the sections, e. g., the rudders of a large airplane and the elevator can, according to my calculations, be easily pulled in. At the end of the flight only the body of the airplane might remain and on it the small wings and rudders shown in Figure 28. If necessary, certain parts of the body may also be used as fuel, after the weight of the ship has decreased considerably. The use of the engine as fuel was mentioned above. The schemes for folding and retracting the sections, and the order of performing these operations, may be very different and a wide field is open for inventions. The combustion should be started with the least necessary and cheapest sections. In many cases, only the combustion of a small number of sections may be required. One has to aim at the greatest simplicity and cheapness of the combustible sections. Upon development, the amount of combustible sections will be reduced; however, for the "conquest" of interplanetary space, the price of one airplane will play only a rather insignificant role.

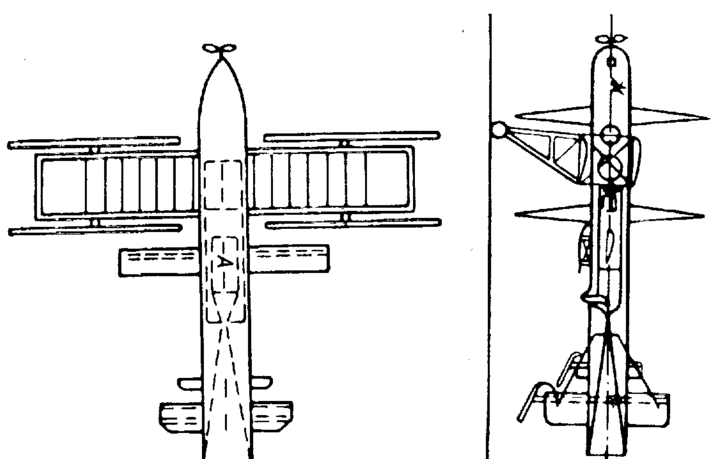

FIGURE 28. Schematic diagram of an airplane with retractable parts driven by an engine and a rocket

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With other methods it is still impossible to fly away from the Earth, and by the method proposed here a final weight of the empty flying device equal to one hundredth of the initial weight can be easily imagined, i.e., the flying device will receive thermal energy from its structure whose weight is 99 times larger than its final weight. This together with the designs of jet engines examined above guarantees completely the attainment of interplanetary velocities.

13. A CENTRAL ROCKET SURROUNDED BY A CLUSTER OF LATERAL ROCKETS AND FUEL AND OXYGEN TANKS

The diagram in Figure 29 shows a central rocket with a cluster of lateral rockets, arranged along branches of diverging spirals. Two lateral tanks placed already inside the central rocket for melting are shown. By increasing the number of lateral rockets and tanks on the branches of the spirals, the flight altitude increases. The spiral branches may consist of pipes in which fuel and oxygen for combustion can be passed by a special valve system. The diagram of a lateral rocket is denoted by the letter b in Figure 29 and of that a lateral tank by c. Fuel and liquid oxygen tanks are seen in the

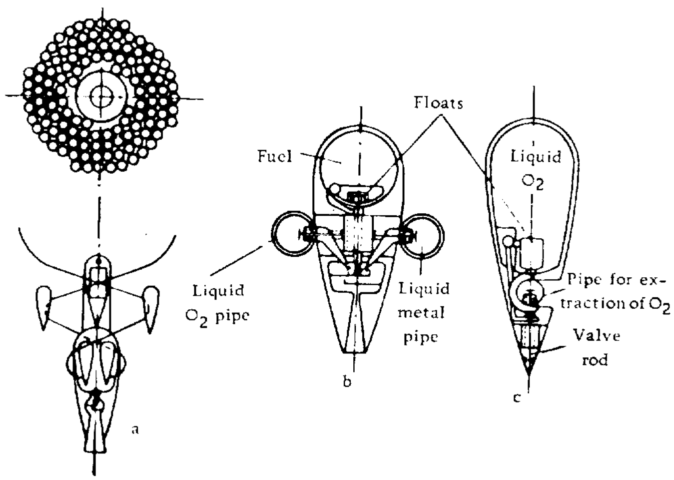

FIGURE 29. Diagram of a central rocket with a cluster of lateral rockets and liquid fuel and oxygen tanks.

nose section. They contain floats which release a spring by means of a lever. The spring closes and opens valves as necessary and allows the tank to glide in to the central rocket for melting after the liquid is consumed. In this case too one can conceive a great number of variants, including one in which a series of central rockets fly together and fall subsequently into the rocket placed at the center of the whole device, i.e., the process described above is repeated. In view of the fact that the individual tanks and lateral rockets may fold like an umbrella, they may initially weigh considerably more than the central rocket and nevertheless melt in it, so that it can be imagined that the weight at the end of the flight will be equal to only one

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thousandth of the initial weight, i.e., one section receives the energy of 999 consumed sections; such a high fuel consumption is not required even for a flight to another planet. As was shown above, the flight can be also performed without any liquid fuel. Then, individual sections of the structure can be made especially strong and all the thick sections are used later on as fuel. Thus, notwithstanding the more complex structure, the final weight is not increased for a given initial weight.

The total air resistance for the above-examined arrangement of rockets will be larger at the beginning of the flight and at high flight velocities at the end of the flight it will be considerably smaller than in a single rocket without lateral rockets.

According to the studies of K.E. Tsiolkovskii and H. Obert, the air resistance to vertically ascending rockets does not play an important role if the acceleration is sufficiently large. The work required to overcome the rocket's weight is small for rather gently sloping flights if the acceleration is not too high. By inclining slightly the rocket axis with respect to the flight direction, a sufficient air resistance may be obtained for overcoming the weight, in particular for a considerable flight velocity.

The rockets discussed above can be constructed as completely or partially air-breathing jet engines. In this case, due to the low weight of the liquid oxygen, they will be even more advantageous.

14. ROCKET AIRPLANE TAKE-OFF

Notwithstanding the tremendous experience accumulated in aviation, it may be assumed that man will have to fly first not in a rocket but in an airplane on which a rocket will be mounted along with the engine which drives the propellers. The action of the rocket on the plane can be stopped at any moment and the pilot can pass to a safe flight by means of the propeller assembly, or to gliding.

I have calculated (in 1921) flight curves for an airplane with a rocket for a total weight of $G_0 = 5000$ kg and a rocket with a reaction force of $P = 1500$ kg (examined above in detail), for an altitude starting from 35 km and a constant rate of fuel consumption. In the calculation I assumed a flat terrestrial surface. The ratio of the acceleration to the gravitational acceleration may be expressed by

$$\frac{1}{g_0}\frac{dv}{dt} = \frac{P}{G} - \frac{2R\cos\alpha}{A} - \sin\alpha,$$

where $\frac{dv}{dt}$ is the flight acceleration;

G, the weight of the airplane;

$\frac{2R}{A}\cos\alpha$, the magnitude of air friction force under constant attack angles;

α, flight trajectory inclination with respect to horizon;

$\sin\alpha$ is proportional to lifting force of the airplane;

$\frac{P}{G}$, the apparent weight in airplane.

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The curves are given in Figure 30, where the following notation has been used:

t, the flight time;
h, the flight altitude;
v, the flight velocity attained;
A_B, the work done to overcome air resistance, in ton km;
A, the entire work done by rocket to attain altitude h;
N, the useful power of rocket;
N_B, the power necessary to overcome atmospheric resistance;
s, the path traversed by rocket.

We see from the figure that a velocity of 8 km/sec is attained for a weight of ~500 kg and a flight time of $t\sim$ 20 minutes. The altitude is then h=80 km, the apparent weight at the end of the flight is $P/G=3$, the flight range s=3300 km, i.e., ~1/12 of the Earth's circumference. The work for overcoming the air resistance decreases strongly towards the end of the flight: $A_B/A=0.15$ or 15% of the total work. The work for overcoming the weight is even lower, as can be seen from the curves of $\sin\alpha$ and $\frac{2R}{A}\cos\alpha$.

If we take into account the curvature of the Earth, then the results are considerably better, since the centrifugal force, appearing in flight, very much helps the airplane to rise.

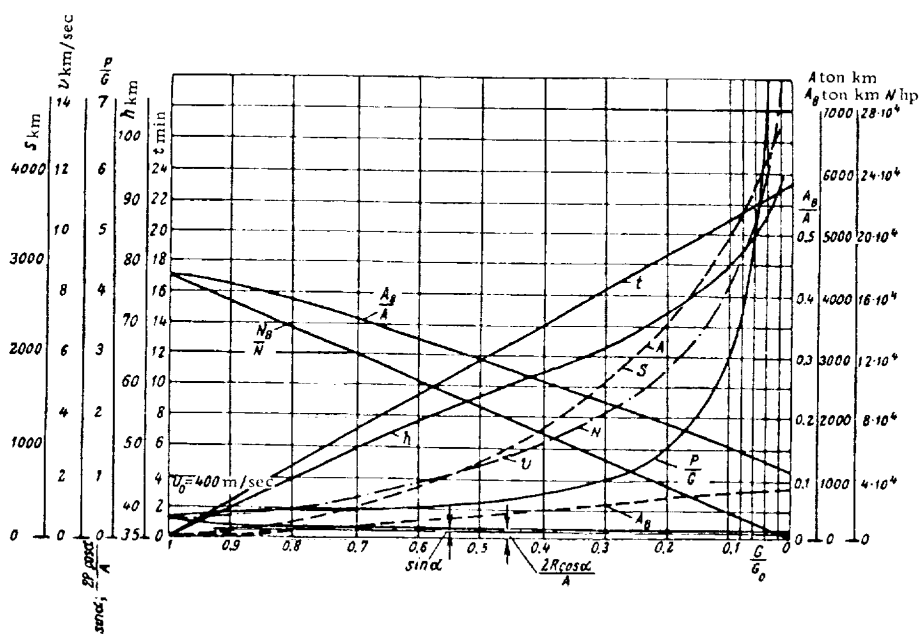

FIGURE 30. Flight curves of a rocket airplane

Up to an altitude of 35 km the flight could be conducted by means of a combination of an aviation engine and a rocket, and at lower altitudes with

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the engine only. My calculations show that when flying with the above-mentioned external combustion engine which I started develcping, the power is limited not by the engine, but by the propellers, which must have a rather large diameter and which would revolve so rapidly at high flight velocities that they turn out to be too heavy.

If we have reached the interplanetary empty space, which is most suitable for inertial flight, and we wish to descend again, then a glide landing is a natural choice, since no fuel is needed for it; otherwise we must build a heavier and more expensive structure. In studying the possibility of varying the flight path at these high velocities, I have obtained the following result (Figure 31): If we fly even with a velocity of 11.3 km/sec, attained when

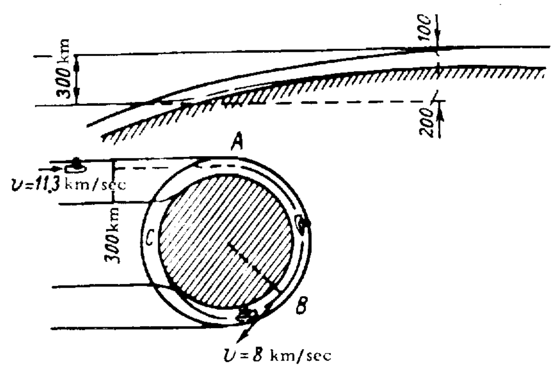

FIGURE 31. Glide landing from interplanetary space on the Earth

falling on the Earth from infinity, it is possible to land safely (apparent weight less than five times the gravitational weight) in a space of annular cross section with a total height of 300 km. The atmosphere takes up 100 km of this space and 200 km are left for skirting the Earth. Car-drivers on Earth skirt a vehicle or a lamppost with a velocity of approximately 25 m/sec (90 km/hour). Our velocity is larger by a factor of $\frac{11,300}{25} = 450$. Arranging the wings and rudders of the airplane in such a way that under high air pressures they would open to protect against breakage, we have to increase the width of the road (10 m) available to the driver proportionally to the velocity of our flight; this yields a path width for the airplane flying at 11:3 km/sec of $\frac{10 \cdot 450}{1000} = 4.5$ km, whereas we have at our disposal a width of 300 km. Therefore, such a landing can be regarded as possible. The airplane can be protected against spinning by the same wing structure and by automatically regulated rudders in too fast a descent. At lower flight velocities, the zone, which is not too dangerous for flight, widens considerably.

15. FLIGHT OF ROCKETS REACHING FAR BEYOND THE ATMOSPHERE

I have calculated the range outside the atmosphere of rockets with a given initial velocity for a static Earth (Figure 32). The highest

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flight altitude is attained when traversing a quarter of the terrestrial globe (1330 km); the initial rocket velocity is then already equal to 7.2 km/sec. One sees that it is comparatively easy to achieve greater distances, since in order to traverse the entire terrestrial globe an initial velocity of 7.9 km/sec is required, i. e. only 0.7 km/sec larger. To traverse half of the terrestrial globe if we do not choose the shortest path a velocity of 7.9 km/sec is also theoretically required, and the flight will take place practically along a somewhat extended ellipse. The propellant consumption for a gasoline rocket is shown in Figure 33 and that for a hydrogen rocket in Figure 34; the curves without a prime denote propellant consumption for the theoretical exhaust velocity in the following cases:

a) only fuel is taken along, air is taken from the atmosphere, i. e., we have an air-breathing jet engine;

b) fuel and liquid oxygen are carried along and the air sucked in is used to increase the efficiency at low flight velocities;

c) fuel and liquid oxygen are carried along again, but external air is not used to increase the efficiency, i. e., this is the ordinary rocket case.

The curves marked with a prime denote the same as above, assuming that in the third case the exhaust velocity is 0.75 of the theoretical velocity, i. e., $(1-0.75^2) \cdot 100 = 44\%$ of the entire heat energy is lost, and that in the first and second cases the same fraction of the total energy is lost. We see that to traverse distances, making an arc of 20° of the entire circumference of the Earth, the fuel consumption (neglecting atmospheric resistance and the work spent on overcoming the weight) requires in the first case, 27% in the second case, 63% and in the third case, 73% of the total weight of the ship; these results were for gasoline. For hydrogen the corresponding values are 12, 56 and 67%.

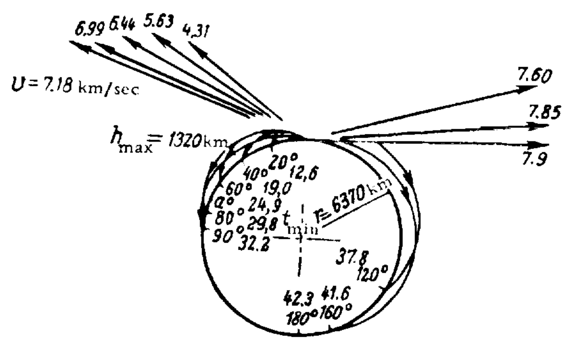

FIGURE 32. Ranges of rockets flying far outside the atmosphere

Figure 35 and 36 give the following trajectory elements: large and small semiaxes, a and b of the elliptical flight trajectory, the largest and smallest distances r_2 and r_3 from the center of the Earth, the greatest height h over the surface of the Earth, the angle β of the trajectory with respect to the horizon at the initial and final points of the flight, the central angle α corresponding to the flight range, the angle $\vartheta°$ from the take-off point to the perigee, the relative eccentricity of the trajectory Σ and its absolute

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eccentricity e, the average flight velocity c, the velocity v at the moment of take-off and the flight duration t.

The curves show that initially, for short ranges, the ellipses are rather narrow, and they get more circular with increasing range, until they become a circle when the range is half the Earth's circumference. The average flight velocity increases rapidly at the beginning, attaining 10,500 km/hour already for a range of 20° (2200 km). The time it takes to cover half the globe is only 42.3 min.

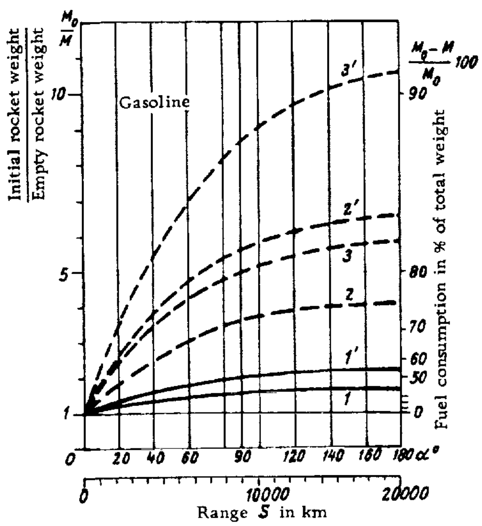

FIGURE 33. Fuel consumption of a gasoline rocket

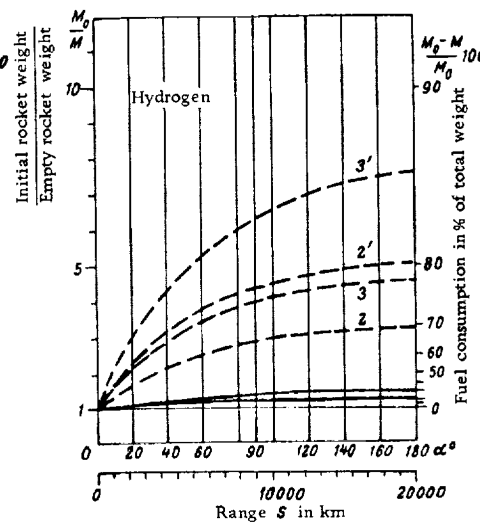

FIGURE 34. Fuel consumption of a hydrogen rocket

A rather considerable economy may be obtained by taking into account the rotation of the Earth about its axis and about the Sun.

Flight of rockets in the atmosphere and vertical ascent above the atmosphere have already been dealt with by a series of authors such as Tsiolkovskii, Obert, Rynin, and others. Little has been written about the flight of long range rockets outside the atmosphere although this region will play a tremendous role in the near future in the transportation of express loads and persons and also in the launching of missiles from one point on the Earth to another through interplanetary space.

For those interested in more detailed calculations of the flight of long-range rockets we present below formulas for the determination of all the quantities, characterizing the flight; we also examined the conditions most suitable for achieving maximum range with greatest fuel economy for a given initial velocity, i.e., we calculated the cheapest flight per km of distance traversed.

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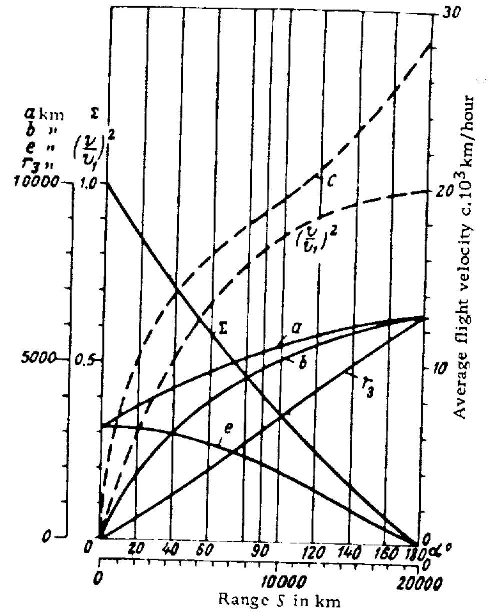

FIGURE 35. Elements of elliptical trajectories of long-range rockets

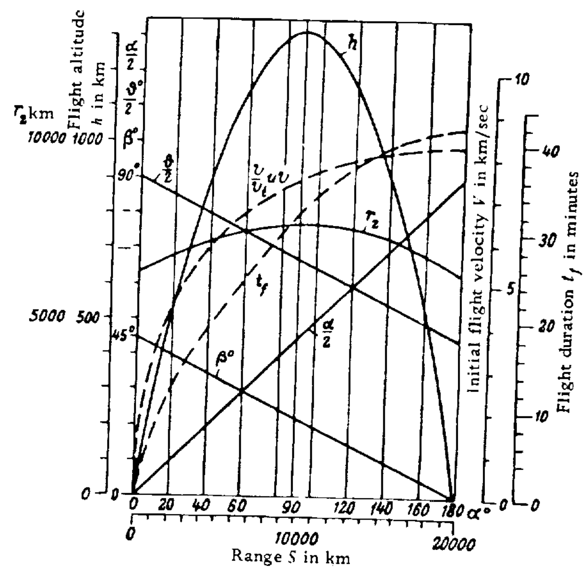

FIGURE 36. Elements of elliptical trajectories of long-range rockets

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Optimum trajectories. Trajectory elements. Flight duration

Consider an ellipse having one of the foci at the center of the Earth.

A flight can be characterized by the following quantities: the flight range or the corresponding angle as seen from the center of the Earth; the highest flight altitude; the initial velocity and the initial inclination of the trajectory with respect to the horizon; the flight duration and the propellant consumption required for obtaining a given initial velocity.

We shall use the large and small semiaxes and the eccentricity of the ellipse to describe the trajectory; for convenience, we shall use polar coordinates (Figure 37).

For the trajectory section under consideration we introduce the notation:

r_1, radius vector of ellipse at initial and final flight points;
$h=r_2-r_1$, the highest altitude reached by the rocket;
ϑ, the true anomaly of initial flight point;
v_3, orbital velocity (v_3 = 7.9 km/sec)
S, flight range, measured at a distance r_1 from Earth's center;
$\alpha°$, central angle with vertex at Earth's center, corresponding to flight range S;
T, half period in an elliptic orbit, assuming the entire mass of the Earth concentrated in its center;
T_1, half circling period at a distance of r_1 from Earth's center;
t, the time required to fly from take-off point to perigee of ellipse;
t_f, flight duration from initial to final point;
E, eccentric anomaly of initial flight point.

We then obtain from Kepler's second law, according to which the radius vector draws equal areas in equal time intervals,

$$r_1 \cdot v \cos\beta = \frac{\pi ab}{T}. \tag{1}$$

Next, we have the equation of the ellipse in polar coordinates

$$r_1 = \frac{a - \sum e}{1 + \sum \cos\vartheta}. \tag{2}$$

We have

$$e = \sum a; \tag{3}$$

$$r_2 = a + e = a\left(1 + \sum\right), \tag{4}$$

hence

$$\sum = \frac{r_2 - a}{a} \tag{4a}$$

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and

$$b = \sqrt{a^2 - e^2} = \\ = \sqrt{r_2(2a - r_2)} = \sqrt{r_2 r_3}. \quad (5)$$

Next, from Kepler's third law

$$\left(\frac{T}{T_1}\right)^2 = \left(\frac{a}{r_1}\right)^3. \quad (6)$$

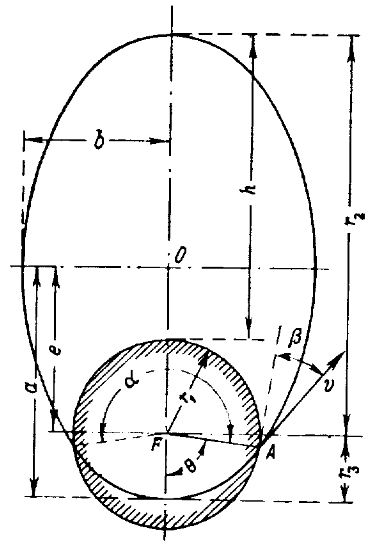

FIGURE 37. Diagram for the calculation of the trajectories of long-range rockets

Substituting e from (3) in (2) and Σ from (4a) in the result obtained, we get

$$\cos\vartheta = \frac{a(2r_2 - r_1) - r_2^2}{r_1(r_2 - a)}. \quad (7)$$

The rocket traverses a path corresponding to a central angle

$$\alpha = 360° - 2\vartheta°, \quad (8)$$

so that the range, measured along a sphere of radius r_1, is equal to

$$s = r_1 \alpha° \frac{\pi}{180°} = \frac{\pi r_1}{90}(180° - \vartheta°). \quad (9)$$

Let us determine next the angle β between the horizon and the initial velocity v of the rocket at the initial point of the trajectory.

From the properties of the ellipse it is known that the normal AB (Figure 38) to its contour decreases by a half the angle between the radius vectors

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AF_1 and AF; the required angle β is equal to the angle BAF, so that the angle between the normals to the ellipse and to a circle of radius r_1 is equal to the angle between the corresponding tangents. It is also known that the sum of the radii-vectors AF and AF_1 is equal to $2a$

$$AF+AF_1=2a.$$

By introducing the notation $AF_1=r_1'$ we obtain

$$r_1'=2a-r_1. \qquad (10)$$

Next, from Figure 38, we find

$$F_1F=2e=2(a-r_3). \qquad (11)$$

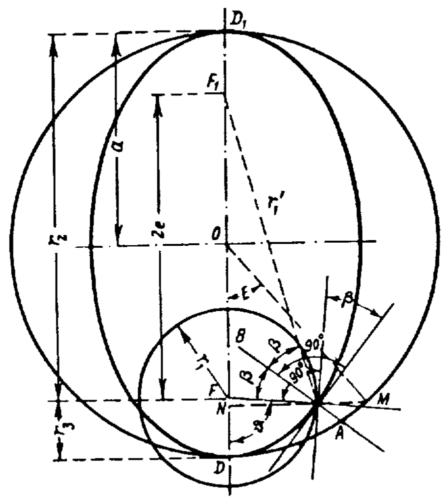

FIGURE 38. Diagram for the calculation of the trajectories of long-range rockets

From the triangle F_1AF in which all the three sides are known, we obtain for half its perimeter

$$s=\frac{1}{2}(2a+2a-2r_3)=2a-r_3=r_2,$$

and for $\cos\beta$

$$\cos\beta=\sqrt{\frac{s(s-2e)}{r_1'r_1}}=\sqrt{\frac{r_2(r_2-2a+2r_3)}{(2a-r_1)r_1}}=\sqrt{\frac{r_2\cdot r_3}{(2a-r_1)r_1}}. \qquad (12)$$

The initial rocket velocity is now found from formula (1) by substituting in it b from (5), T from (6), $\cos\beta$ from (12), and taking also into consideration that

$$T_1=\frac{\pi r_1}{v_3}, \qquad (13)$$

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we obtain

$$v=\frac{\pi ab}{r_1\cos\beta T}=\frac{\pi a\sqrt{r_2r_3}}{r_1\sqrt{\frac{r_2r_3}{(2a-r_1)r_1}}\left(\frac{a}{r_1}\right)^{\frac{3}{2}}\frac{\pi r_1}{v_3}}=v_3\sqrt{2-\frac{r_1}{a}}. \tag{14}$$

We have now five independent equations:

$$e=r_2-a, \tag{4}$$

$$\sum=\frac{r_2-a}{a}, \tag{4a}$$

$$b=\sqrt{r_2(2a-r_2)}, \tag{5}$$

$$\cos\vartheta=\frac{a(2r_2-r_1)-r_2^2}{r_1(r_2-a)}, \tag{7}$$

$$v=v_3\sqrt{2-\frac{r_1}{a}}. \tag{14a}$$

From these equations we determine the seven quantities, r_2, a, b, e, v, Σ and ϑ, having in the general case two independent variables.

When we consider the most advantageous case of lowest initial velocity v for a given flight range, or of a given ϑ, we obtain one more equation and all the quantities will be functions of one independent quantity only.

In the general case, any two quantities may be taken as the independent variables, depending on the conditions of the problem, and we calculate the other quantities from them.

In the case of the minimum initial velocity trajectory, i.e., minimum energy spent for a given flight range, we shall use in the meantime ϑ as the independent variable and determine r_2 so that we get the smallest velocity v for a given ϑ. We can write

$$\frac{dv}{dr_2}=\frac{dv}{da}\,\frac{da}{dr_2}, \tag{15}$$

and v has a minimum, if $\frac{dv}{dr_2}=0$.

To find the derivative of a with respect to r_2 we transform equation (7)

$$r_1\cos\vartheta\,(r_2-a)=a(2r_2-r_1)-r_2^2;\quad a=\frac{r_1r_2\cos\vartheta+r_2^2}{2r_2-r_1+r_1\cos\vartheta}. \tag{7a}$$

Then for ϑ = const, we obtain

$$\frac{da}{dr_2}=\frac{(r_1\cos\vartheta+2r_2)(2r_2-r_1+r_1\cos\vartheta)-2(r_1r_2\cos\vartheta+r_2^2)}{(2r_2-r_1+r_1\cos\vartheta)^2}=$$

$$=\frac{2r_2^2-(r_1\cos\vartheta+2r_2)r_1(1-\cos\vartheta)}{(2r_2-r_1+r_1\cos\vartheta)^2}. \tag{16}$$

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Differentiating equation (14) with respect to a, we obtain

$$\frac{dv}{da} = -\frac{v_3 r_1 a^{-2}}{2\sqrt{2-\frac{r_1}{a}}}.$$

Substituting (16) and (17) in (15) and replacing a by its expression from (7a) we obtain

$$\frac{dv}{dr_2} = \frac{v_3 r_1}{2} \frac{2r_2^2 - (r_1 \cos\vartheta + 2r_2)\cdot r_1(1-\cos\vartheta)}{(r_1 r_2 \cos\vartheta + r_2^2)^{\frac{3}{2}} \left[2r_1 r_2 \cos\vartheta + 2r_2^2 - 2r_1 r_2 + r_1^2(1-\cos\vartheta)\right]^{\frac{1}{2}}}. \quad (15a)$$

Equating this expression to zero we obtain values of r_2 for which v is either a maximum or a minimum. This condition is satisfied either by taking $r_2 = \infty$, which corresponds to the greatest value of v, or by making the numerator of (15a) equal to zero, which gives

$$r_2^2 - r_2 r_1 (1-\cos\vartheta) - \frac{r_1^2}{2}\cos\vartheta(1-\cos\vartheta) = 0 \quad (17)$$

or

$$r_2 = \frac{r_1}{2}(1-\cos\vartheta) \pm \sqrt{\frac{r_1^2}{4}(1-\cos\vartheta)^2 + \frac{r_1^2}{2}\cos\vartheta(1-\cos\vartheta)} =$$

$$= \frac{r_2'}{r_2''} = \frac{r_1}{2}(1-\cos\vartheta \pm \sin\vartheta) = \pm r_1\sqrt{2}\sin\frac{\vartheta}{2}\cos\left(\frac{\vartheta}{2} \mp 45^\circ\right). \quad (18)$$

From (8): $\alpha = 360^\circ - 2\vartheta^\circ$, we find $\alpha = 0$, for $\vartheta = \frac{360^\circ}{2} = 180^\circ$, $\frac{\vartheta}{2} = 90^\circ$ and $\alpha = 180^\circ$ for $\vartheta = \frac{180^\circ}{2} = 90^\circ$, $\frac{\vartheta}{2} = 45^\circ$; therefore in our case $90^\circ \geqslant \frac{\vartheta}{2} \geqslant 45^\circ$.

In Table 2 the ratios $r_2' : r_1$ and $r_2'' : r_1$ for the range $0^\circ \leqslant \alpha^\circ \leqslant 360^\circ$ are given. It follows from it that in the range $0^\circ < \alpha^\circ < 180^\circ$ only $r_2' : r_1$ is larger than 1 and the flight corresponds to a minimum initial velocity. The rest of the cases give $r_2 < r_1$; they are of no value for flight from the Earth. The situation is different if it is required later on in interplanetary space to transfer the rocket from one place to another with the smallest initial velocity. For $180^\circ < \alpha^\circ < 360^\circ$ flight in a circle around the Earth is theoretically most advantageous; practically in this case, either a circle, lying outside the atmosphere, or an ellipse, rising somewhat above the atmosphere, may be chosen.

Let us examine more exactly the case $r_2 : r_1 > 1$, $0 \leqslant \alpha^\circ \leqslant 180^\circ$ (see Table 3). In this case we have to choose the upper sign in (18); we then obtain

$$r_1' = r_2 = r_1\sqrt{2}\sin\frac{\vartheta}{2}\sin\left(45^\circ - \frac{\vartheta}{2}\right). \quad (18a)$$

The following quantities are given in Table 3 for various values of the central angle α, i.e., the flight distances:

the true anomaly ϑ° at the moment of take-off;
the take-off angle β°;
the initial flight velocity v;

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the rocket's greatest distance r_2 from the Earth's center and the highest altitude h over the take-off point, for which we have

$$h=r_2-r_1=\frac{r_1}{2}(\sin\vartheta-\cos\vartheta-1)=r_1\sqrt{2}\cos\frac{\vartheta}{2}\sin\left(\frac{\vartheta}{2}-45^\circ\right). \qquad (19)$$

Figure 32 shows the optimum flight trajectories corresponding to the data of Table 3.

TABLE 2

α°	0	40	80	120	160	180
$\vartheta^\circ:2$	90	80	70	60	50	45
$r_2':r_1$	1.000	1.140	1.206	1.184	1.080	1.000
$r_2'':r_1$	1.000	0.800	0.562	0.318	0.094	0
α°	200	240	280	320	360	
$\vartheta^\circ:2$	40	30	20	10	0	
$r_2':r_1$	0.907	0.683	0.439	0.201	0	
$r_2'':r_1$	-0.079	-0.183	-0.204	-0.141	0	

TABLE 3

α°	0	20	40	60	80	90
$\vartheta^\circ/2$	90	85	80	75	70	67.5
β°	45	40	35	30	25	22.5
v/v_3	0	0.545	0.713	0.815	0.885	0.910
v, km/sec	0	4.305	5.63	6.14	6.99	7.18
r_2, km	6370	6875	7266	7535	7671	7690
$h=(r_2-r_1)$, km	0	505	896	1165	1301	1320
α°	100	120	140	160	180	
$\vartheta^\circ/2$	65	60	55	50	45	
β°	20	15	10	5	0	
v/v_3	0.930	0.962	0.982	0.995	1	
v, km/sec	7.35	7.60	7.75	7.85	7.90	
r_2, km	7671	7535	7266	6875	6370	
h, km	1301	1165	895	505	0	

In order to find the maximum height h_{max} we determine $\frac{dr_2}{d\vartheta/2}$ and equate it to zero. From (18a) we obtain

$$\frac{dr_2}{d\frac{\vartheta}{2}}=r_1\sqrt{2}\left[\cos\frac{\vartheta}{2}\cos\left(\frac{\vartheta}{2}-45^\circ\right)-\sin\frac{\vartheta}{2}\sin\left(\frac{\vartheta}{2}-45^\circ\right)\right]=0,$$

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or

$$\operatorname{ctg}\frac{\vartheta}{2}=\operatorname{tg}\left(\frac{\vartheta}{2}-45^\circ\right);\quad 90^\circ-\frac{\vartheta}{2}=\frac{\vartheta}{2}-45^\circ;$$

$$\frac{\vartheta}{2}=67^\circ.5;$$

$$\alpha=360^\circ-2\vartheta=360^\circ-270^\circ=90^\circ.$$

We see therefore that the maximum flight altitude for maximum range at a given initial velocity is obtained for a range of one quarter of the Earth's circumference. Substituting $\vartheta/2=67.5^\circ$ in equation (19) we obtain for the altitude in this case

$$h_{max}=r_1\sqrt{2}\cos 67.5^\circ\sin(67^\circ.5-45^\circ)=0.208r_1.$$

Assuming $r_1=6370$ km for the radius of the Earth, we obtain

$$h_{max}=0.208\cdot 6370=1320\text{ km}.$$

From equation (7a) we determine for an arbitrary angle ϑ the major semi-axis a of the ellipse. By substituting r_2 obtained from equation (18), and taking the upper sign, we obtain

$$a=\frac{r_1(1-\cos\vartheta+\sin\vartheta)(2\cos\vartheta+1-\cos\vartheta+\sin\vartheta)}{4\sin\vartheta},$$

or

$$a=\frac{r_1}{4\sin\vartheta}[(1+\sin\vartheta)^2-\cos^2\vartheta]=r_1\frac{2\sin\vartheta+2\sin^2\vartheta}{4\sin\vartheta},$$

or

$$a=\frac{r_1}{2}(1+\sin\vartheta)=r_1\cos^2\left(\frac{\vartheta}{2}-45^\circ\right). \qquad (7b)$$

Let us also express the remaining quantities as functions of ϑ. From equation (14) we obtain, substituting a from (7b), the initial flight velocity v

$$v=v_3\sqrt{2-\frac{2}{1+\sin\vartheta}}=v_3\sqrt{2}\sqrt{\frac{\sin\vartheta}{1+\sin\vartheta}}=v_3\frac{\sqrt{\sin\vartheta}}{\cos\left(\frac{\vartheta}{2}-45^\circ\right)}. \qquad (14a)$$

The eccentricity Σ is obtained from equation (4a) by substituting r_2 and a from (18) and (7b), taking the upper sign in (18)

$$\Sigma=\frac{r_2}{a}-1=\frac{1-\cos\vartheta+\sin\vartheta}{1+\sin\vartheta}-1=-\frac{\cos\vartheta}{1+\sin\vartheta};$$

$$\Sigma=\operatorname{tg}\left(\frac{\vartheta}{2}-45^\circ\right). \qquad (4b)$$

The linear eccentricity is obtained from equations (4), (18), and (7b)

$$e=r_2-a=-\frac{r_1\cos\vartheta}{2}. \qquad (4c)$$

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The minor semiaxis of the ellipse is found from equations (5), (18), and (7b); we determine initially the minimum value of the radius vector of the ellipse r_3

$$r_3 = 2a - r_2 = r_1(1 + \sin\vartheta) - \frac{r_1}{2}(1 - \cos\vartheta + \sin\vartheta);$$

$$r_3 = \frac{r_1}{2}(1 + \sin\vartheta + \cos\vartheta) = r_1\sqrt{2}\cos\left(\frac{\vartheta}{2} - 45°\right)\sin\frac{\vartheta}{2}; \qquad (20)$$

and find then

$$b = \sqrt{r_2 r_3} = \frac{r_1}{2}\sqrt{(1 + \sin\vartheta)^2 - \cos^2\vartheta} = r_1\sqrt{\frac{\sin\vartheta(1 + \sin\vartheta)}{2}} =$$

$$= r_1\cos\left(\frac{\vartheta}{2} - 45°\right)\sqrt{\sin\vartheta}. \qquad (5a)$$

The inclination angle β of the rocket's trajectory to the horizon at the initial point, is obtained from equations (12), (18), (7b) and (20), using the relation

$$2a - r_1 = r_1\sin\vartheta; \qquad (21)$$

$$\cos\beta = \sqrt{\frac{(1 - \cos\vartheta + \sin\vartheta)(1 + \sin\vartheta + \cos\vartheta)}{2}} = \cos\left[\pm\left(45° - \frac{\vartheta}{2}\right)\right]; \qquad (12a)$$

$$\beta = \pm\left(45° - \frac{\vartheta}{2}\right);$$

where $\frac{\vartheta}{2} \geqslant 45°$ and $\beta > 0$, so that the lower sign has to be taken, i.e., we have

$$\beta = \frac{\vartheta}{2} - 45°. \qquad (12b)$$

From this formula we obtain for a flight around half the circumference of the Earth, i.e., for $\vartheta = 90°$, $\beta = 0$, and for $\vartheta = 180°$, i.e., for short ranges

$$\beta = \frac{180°}{2} - 45° = 45°.$$

Taking into consideration equation (8), we also obtain

$$\beta = \frac{360° - \alpha}{4} - 45° = 45° - \frac{\alpha°}{4}, \qquad (12c)$$

i.e., for $\vartheta = 180°$, $\beta = 45°$, we indeed obtain $\alpha = 0$, i.e., short ranges; for $\vartheta = 90°$, $\beta = 0$, we obtain $\alpha = 4.45° = 180°$, i.e., a flight around the Earth.

From (12c) we see that increasing the central angle α by 20° corresponds to a reduction of the inclination β by $20°/4 = 5°$ for the cases of maximum flight range for a given initial velocity examined above.

According to the type of problem which has to be solved, it is possible to choose also any other of the quantities r_2; r_3; h; a; b; Σ; e; α; β; ϑ; ν as the independent variable, instead of ϑ, for example, β or α.

Let us collect all the formulas derived for the case of maximum flight range:

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$$r_2=\frac{r_1}{2}(1-\cos\vartheta+\sin\vartheta)=r_1\sqrt{2}\sin\frac{\vartheta}{2}\cos\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (18b)$$

$$h=\frac{r_1}{2}(-1-\cos\vartheta+\sin\vartheta)=r_1\sqrt{2}\cos\frac{\vartheta}{2}\sin\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (19)$$

$$r_3=\frac{r_1}{2}(1+\sin\vartheta+\cos\vartheta)=r_1\sqrt{2}\cos\frac{\vartheta}{2}\cos\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (20)$$

$$a=\frac{r_1}{2}(1+\sin\vartheta)=r_1\cos^2\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (7b)$$

$$b=r_1\sqrt{\frac{\sin\vartheta(1+\sin\vartheta)}{2}}=r_1\sqrt{\sin\vartheta}\cos\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (5a)$$

$$e=-\frac{r_1}{2}\cos\vartheta; \qquad (4c)$$

$$\sum=\operatorname{tg}\left(\frac{\vartheta}{2}-45^\circ\right); \qquad (4b)$$

$$\alpha=360^\circ-2\vartheta; \qquad (8)$$

$$\beta=\frac{\vartheta^\circ}{2}-45^\circ; \qquad (12b)$$

$$v=v_3\sqrt{\frac{2\sin\vartheta}{1+\sin\vartheta}}=v_3\frac{\sqrt{\sin\vartheta}}{\cos\left(\frac{\vartheta}{2}-45^\circ\right)}; \qquad (14a)$$

$$s=r_1\frac{\pi}{180^\circ}\alpha^\circ=r_1\frac{\pi}{90}(180^\circ-\vartheta^\circ). \qquad (9)$$

These formulas allow us to determine the angle ϑ for any given quantity and then to calculate all remaining parameters.

Particularly simple formulas are obtained, if all the quantities are expressed as functions of β. From (12b) we obtain

$$\frac{\vartheta}{2}=\beta+45^\circ$$

or

$$\vartheta=2\beta+90^\circ. \qquad (12d)$$

And so

$$r_2=r_1\sqrt{2}\cos(45^\circ-\beta)\cos\beta; \qquad (18c)$$

$$h=r_1\sqrt{2}\sin(45^\circ-\beta)\sin\beta; \qquad (19a)$$

$$r_3=r_1\sqrt{2}\sin(45^\circ-\beta)\cos\beta; \qquad (20a)$$

$$a=r_1\cos^2\beta; \qquad (7c)$$

$$b=r_1\sqrt{\cos 2\beta}\cos\beta; \qquad (5b)$$

$$e=\frac{r_1}{2}\sin 2\beta; \qquad (4d)$$

$$\sum=\operatorname{tg}\beta; \qquad (4e)$$

$$\alpha=180^\circ-4\beta; \qquad (8a)$$

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$$v = v_3 \frac{\sqrt{\cos 2\beta}}{\cos \beta};\qquad (14b)$$

$$s = \frac{\pi r_1}{180}(180^\circ - 4\beta) = \pi r_1 - \frac{\pi r_1 \beta}{45}.\qquad (9a)$$

Usually, the range s will be given. We then obtain from (9a)

$$\beta = \frac{45^\circ}{\pi r_1}(\pi r_1 - s) = 45^\circ - \frac{45 s}{\pi r_1}.\qquad (9b)$$

The determination of all the important quantities characterizing the flight is now quite simple.

The flight duration is determined by Kepler's equation; we have

$$t_f = 2(T - t)\qquad (22)$$

and Kepler's equation

$$t = \frac{T}{\pi}\left(E - \sum \sin E\right).\qquad (23)$$

where the semirotation period T around the Earth is determined by equation (6)

$$T = T_1 \left(\frac{a}{r_1}\right)^{\frac{3}{2}}.\qquad (6a)$$

The relation between the eccentric anomaly, appearing in Kepler's equation, the true anomaly ϑ, and the relative eccentricity Σ is given by

$$\operatorname{tg}\frac{E}{2} = \sqrt{\frac{1 - \sum}{1 + \sum}}\operatorname{tg}\frac{\vartheta}{2},\qquad (24)$$

geometrically this is shown in Figure 38, if we draw a normal to the major axis of the ellipse DD_1 from the take-off point A and continue it up to its intersection M with the circle of radius $OD = a$ and center O. Connecting points M and O by a straight line we have: $E = \measuredangle MOD$.

Substituting a from equations (7b) and (7c) in (6a), we obtain in the case of maximum flight range for a given initial velocity

$$T = T_1 \cos^3\left(\frac{\vartheta}{2} - 45^\circ\right) = T_1 \cos^3 \beta.\qquad (6b)$$

Next, by substituting Σ from (4e) and ϑ from (12d) in (24) we obtain

$$\operatorname{tg}\frac{E}{2} = \sqrt{\frac{1 - \operatorname{tg}\beta}{1 + \operatorname{tg}\beta}}\operatorname{tg}(\beta + 45^\circ) = \sqrt{\operatorname{tg}(45^\circ - \beta)} \cdot \operatorname{tg}(\beta + 45^\circ)$$

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or

$$\operatorname{tg}\frac{E}{2}=\sqrt{\operatorname{tg}(\beta+45^\circ)}=\sqrt{\operatorname{tg}\frac{\vartheta}{2}}. \tag{24a}$$

or

$$\sin E=\frac{2\sqrt{\operatorname{tg}(\beta+45^\circ)}}{1+\operatorname{tg}(\beta+45^\circ)}=\frac{2\sqrt{\operatorname{tg}\frac{\vartheta}{2}}}{1+\operatorname{tg}\frac{\vartheta}{2}}, \tag{24b}$$

and from equations (22) and (23) we find

$$t_f=\frac{2T}{\pi}\left(\pi-E+\sum\sin E\right). \tag{22a}$$

Using formulas (24b),(6a), and (22a), we easily find the flight duration as a function of the central angle α of the path traversed (Table 4).

The velocity v_3 required to circle the Earth at a radius of r_1=6370 km, is obtained from the condition of equality of the gravitational attraction to the centrifugal force developed in the circular motion

$$g=\frac{v_3^2}{r_1};\quad v_3=\sqrt{gr_1}=\sqrt{\frac{9.81}{1000}6370}=7.90\ \text{km/sec}.$$

We can now find the half circling period around the Earth at a radius of r_1 = 6370 km

$$T_1=\frac{\pi r_1}{v_3};\quad T_1=\frac{6370}{7.90\cdot 60}=42.3\ \text{min}.$$

The path s traversed, corresponding to the central angle α, is equal by (9) to

$$s=r_1\frac{\pi}{180^\circ}\alpha=6370\frac{\pi}{180^\circ}\alpha=111.4\alpha.$$

The corresponding average velocity on the Earth is equal to $c=s/t_f$; s and c are also given in Table 4.

Propellant consumption

The minimum amount of propellant required for given flight range and exhaust velocity, is obtained for rather high rocket accelerations, if air resistance can then be neglected. This case gives the lower limit for propellant consumption. It may serve for comparison with the real performance of rockets.

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For this limiting case we have the equation

$$\frac{M_0}{M} = e^{v/w}, \tag{26}$$

where w is the exhaust velocity of the gases with respect to the rocket;
M_0 and M are the masses of the full and empty rocket;
e is the base of natural logarithms (it should not be confused with the eccentricity of the elliptic orbit!);
v is the burnout velocity of the rocket.

In addition to rockets with constant exhaust velocity, we have examined above rockets capable of using the external atmospheric air. In these rockets, the exhaust velocity may be variable.

TABLE 4

α	0°	20°	40°	60°	80°	90°
$\vartheta:2$	90°	85°	80°	75°	70°	67.5°
$\sin E$	0	0.544	0.712	0.816	0.882	0.910
$180°-E$	0	32°53′	45°28′	54°40′	61°50′	65°30′
Σ	1	0.8391	0.7002	0.5774	0.4663	0.414
$T:T_1$	0.355	0.451	0.550	0.650	0.745	0.790
T_{min}	15.0	19.1	23.2	27.5	31.5	33.4
$t_f:T$	0	0.660	0.821	0.907	0.948	0.966
t_f min	0	12.6	19.0	24.9	29.8	32.2
s, km	0	2230	4450	6690	8910	10 000
$c\ \frac{1000\text{ km}}{\text{hour}}$	—	10.63	14.2	16.1	17.94	18.65

α	100°	120°	140°	160°	180°	
$\vartheta:2$	65°	60°	55°	50°	45°	
$\sin E$	0.930	0.964	0.984	0.996	1	
$180°-E$	68°30′	74°35′	79°40′	84°50′	90°	
Σ	0.3640	0.2680	0.1763	0.0875	0	
$T:T_1$	0.831	0.901	0.955	0.990	1	
T	35.2	38.1	40.4	41.8	42.3	
$t_f:T$	0.976	0.993	0.996	0.998	1	
t_f	34.4	37.8	40.2	41.6	42.3	
s	11 140	13 370	15 600	17 800	20 000	
c	19.46	21.2	23.3	25.6	28.4	

My recent studies in this field clearly show the possibility of building such rockets with a rather high efficiency. For these rockets an even lower fuel consumption minimum is obtained, since in this case, in the limit, all the energy contained in the fuel can be transformed into kinetic energy of the rocket. Since in the rocket's nozzle the air may be expanded to rather low temperatures and pressures with secondary compression to atmospheric pressure, inflow of heat from the external air to the low temperature place is possible. When cooling the secondary compressed jet by liquid air the

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heat of the external atmospheric air may be used in a rocket in addition to the heat, obtained from the fuel.

Not considering here this last possibility, we obtain for a rocket suitable for flight in air a lower limit for fuel consumption. Equating the kinetic energy of the rocket to the work equivalent to the heat contained in the consumed fuel, we obtain

$$\frac{H}{A}(M_0 - M)g = M\frac{v^2}{2}, \tag{27}$$

here H is the calorific capacity of the fuel per kg of combustion products, if the oxygen for combustion is carried along, and per kg of the fuel itself, if atmospheric air is compressed by an injector or by another method into the combustion chamber and the fuel only is carried in the rocket; $A = 1/427$ is the mechanical equivalent of heat and g is the gravitational acceleration at the Earth's surface.

We can introduce in formula (27) the maximum exhaust velocity

$$\frac{w^2}{2g} = \frac{H}{A}, \tag{28}$$

where w is the real value if oxygen is taken along and a purely calculated one if fuel only is taken in the rocket.

From formulas (27) and (28) we obtain

$$\frac{M_0 - M}{M} = \frac{v^2}{w^2} \quad \text{or} \quad \frac{M_0}{M} = 1 + \frac{v^2}{w^2}, \tag{27a}$$

and from (28) we find

$$w = \sqrt{2gH/A} = \sqrt{2 \cdot 981 \cdot 427H} = 91.5\sqrt{H}. \tag{28a}$$

The amount of fuel consumed is equal to $M_0 - M$, so that its ratio to the final mass of the rocket is

$$\frac{M_0 - M}{M}.$$

Numerical values for the ratios of the initial to final mass (M_0/M) are given in Table 5 for two different fuels: for gasoline and for liquid hydrogen.

For gasoline the following values were used: heat capacity per kg $H_b = 10{,}500$ cal/kg; amount of air, required for the combustion of 1 kg of gasoline – 14.9 kg; fraction of oxygen in air – 23.1%.

Then, the heat capacity of gasoline per kg of combustion products (when burning gasoline in pure oxygen) is equal to

$$H = \frac{10\,500}{14.9 \cdot 0.231 + 1} = 2370 \text{ cal/kg}.$$

The maximum exhaust velocities, corresponding to H_b and H are

$$w = 91.5\sqrt{10\,500} = 9380 \text{ m/sec},$$

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and

$$w = 91.5\sqrt{2370} = 4450 \text{ m/sec.}$$

Similarly, we obtain for hydrogen

$$H_{H_2} = 28\,900 \text{ cal/kg of } H_2 \text{ and } H = \frac{28\,900}{9} = 3210 \text{ cal/kg of } H_2O,$$

hence,

$$w = 91.5\sqrt{28\,900} = 15\,520 \text{ m/sec}$$

$$w = 91.5\sqrt{3210} = 5180 \text{ m/sec.}$$

TABLE 5

	α	0°	20°	40°	60°	80°	90°
	v, km/sec	0	4.395	5.63	6.44	6.99	7.18
Gasoline	3. $\frac{M_0}{M} = e^{\frac{v}{4.45}}$	1	2.63	3.54	4.25	4.80	5.01
	2. $\frac{M_0}{M} = \left(\frac{v}{4.45}\right)^2 + 1$	1	1.939	2.60	3.10	3.46	3.61
	1. $\frac{M_0}{M} = \left(\frac{v}{9.38}\right)^2 + 1$	1	1.212	1.360	1.470	1.557	1.587
Hydrogen	3. $\frac{M_0}{M} = e^{\frac{v}{5.18}}$	1	2.29	2.96	3.46	3.85	3.96
	2. $\frac{M_0}{M} = \left(\frac{v}{5.18}\right)^2 + 1$	1	1.833	2.18	2.55	2.82	2.92
	1. $\frac{M_0}{M} = \left(\frac{v}{15.52}\right)^2 + 1$	1	1.077	1.132	1.172	1.203	1.214
	α		100°	120°	140°	160°	180°
	v, km/sec		7.35	7.60	7.75	7.85	7.90
Gasoline	3. $\frac{M_0}{M} = e^{\frac{v}{4.45}}$		5.20	5.51	5.69	5.83	5.89
	2. $\frac{M_0}{M} = \left(\frac{v}{4.45}\right)^2 + 1$		3.73	3.94	4.04	4.13	4.16
	1. $\frac{M_0}{M} = \left(\frac{v}{9.38}\right)^2 + 1$		1.616	1.658	1.683	1.700	1.710
Hydrogen	3. $\frac{M_0}{M} = e^{\frac{v}{5.18}}$		4.13	4.33	4.45	4.54	4.59
	2. $\frac{M_0}{M} = \left(\frac{v}{5.18}\right)^2 + 1$		3.02	3.16	3.25	3.30	3.33
	1. $\frac{M_0}{M} = \left(\frac{v}{15.52}\right)^2 + 1$		1.225	1.240	1.250	1.256	1.260

Table 5 gives the lower limits for fuel consumption in the cases indicated above. We see that the necessary amounts of fuel are astonishingly small: to orbit the Earth, i.e., to attain the first cosmic velocity, the fuel consumption required (if the entire energy contained in the fuel is used)

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constitutes for benzene $(1-1/1.71)\cdot 100 = 41.5\%$, and for hydrogen $(1-1/1.26)\cdot 100 = 20.6\%$ of the total initial weight of the rocket.

Rockets may take much more fuel, in particular if we include in it a part of the rocket's structure itself*.

For the closest approach to optimal fuel consumption it is possible, as was shown above, to solve the following practical problem. Let us assume that we take part of the air for the combustion from the terrestrial atmosphere and that we supplement the insufficiency in the high layers of the atmosphere by oxygen carried along. We have then to find the trajectory, the velocity, and the consumption of oxygen carried along, which give the closest approach to the ideal case. Taking also the price into consideration, it is possible to indicate those quantities which correspond to the cheapest flight for a given final rocket weight and for a given altitude or flight range.

The exhaust velocities, obtained practically by Professor Goddard in America in 1919 (see Obert, "Die Rakete zu den Planetenräumen, pp. 90-91. 1925) for powder are the following: for the smokeless powder "infallible", an exhaust velocity w up to 2434 m/sec and a combustion heat $H = 1238.5$ cal/kg; for pistol powder No. 3 an exhaust velocity $w = 2290$ m/sec and a combustion heat $H = 972.5$ cal/kg.

Using these data, we can determine the theoretical exhaust velocities from (18a)

$$w_{max} = 91.5\sqrt{H}.$$

Introducing next the following notation for the ratio of the practical exhaust velocity, to the theoretical one

$$\xi = \frac{w}{w_{max}}, \tag{29}$$

and writing for the thermal efficiency

$$\eta_t = \left(\frac{w}{w_{max}}\right)^2, \tag{30}$$

we obtain for smokeless powder

$$w_{max} = 91.5\sqrt{1238.5} = 3210 \text{ m/sec}.$$

$$\xi = \frac{2434}{3210} = 0.76;$$

$$\eta_t = 0.76^2 = 0.577;$$

* I first proposed in public to use the solid structural material of a rocket as fuel in December 1923 in a lecture given before the theoretical section of the Moscow Society of Amateurs of Astronomy. I have published a paper on the subject in July 1924, in No. 13 of the Journal "Tekhnika i Zhizn'" and on 8 June 1924 I described this idea in my statement to the Committee for Inventions. The idea of the advantages obtainable by using the entire mass of a rocket as a fuel is already found, however, in my manuscript of 11 March 1909. I have always expressed this view in discussions with relatives and friends on the possibility of interplanetary travels. In 1917 a factory prepared for me a crucible for performing experiments on the burning of molten metal. Numerical data on the calorific value of magnesium oxide and other materials may be found in my manuscript for the first time on 11 January 1918. I therefore consider that to my best knowledge I was the first to have expressed this idea and I was the first to publish it. Kondratyuk's manuscripts start in 1916 and his book – Zavoevanie mezhplanetnykh prostranstv (The Conquest of Interplanetary Space) appeared only in 1929. In a series of lectures, which I gave in 1924 and 1925 in various towns, I also explained the principle of the method of burning solid structural material and gave the example of my own project for an interplanetary ship.

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and for pistol powder No. 3

$$w_{max} = 91.5\sqrt{972.5} = 2850\ \text{m/sec};$$
$$\xi = \frac{2290}{2850} = 0.803; \quad \eta_t = 0.803^2 = 0.645.$$

If we take $\xi = 0.75$ for the combustion of gasoline or hydrogen with oxygen, then we obtain for the practical exhaust velocity of the combustion products of gasoline with oxygen: $w = 0.75\sqrt{4450} = 3340$ m/sec; for hydrogen and oxygen: $w = 0.75\sqrt{5180} = 3380$ m/sec. The calculated velocities in the case of sucking in the external air for combustion are: for benzene $w = 0.75\sqrt{9380} = 7040$ m/sec; for hydrogen $w = 0.75\sqrt{15,520} = 11,650$ m/sec.

Table 6 was calculated using these data. Used together with Table 5, it gives an idea of the influence of the thermal efficiency on fuel consumption.

TABLE 6

	α	0°	20°	40°	60°	80°	90°
Gasoline	3'. $\frac{M_0}{M} = e^{\frac{v}{3.34}}$	1	3.64	5.39	6.88	8.11	8.60
	2'. $\frac{M_0}{M} = \left(\frac{v}{3.34}\right)^2 + 1$	1	2.67	3.85	4.73	5.38	5.63
	1'. $\frac{M_0}{M} = \left(\frac{v}{7.04}\right)^2 + 1$	1	1.376	1.640	1.839	1.990	2.04
Hydrogen	3'. $\frac{M_0}{M} = e^{\frac{v}{3.88}}$	1	3.04	4.27	5.25	6.05	6.35
	2'. $\frac{M_0}{M} = \left(\frac{v}{3.88}\right)^2 + 1$	1	2.24	3.10	3.75	4.25	4.43
	1'. $\frac{M_0}{M} = \left(\frac{v}{11.65}\right)^2 + 1$	1	1.138	1.235	1.306	1.360	1.381

	α		100°	120°	140°	160°	180°
Gasoline	3'. $\frac{M_0}{M} = e^{\frac{v}{3.34}}$		9.02	9.75	10.18	10.48	10.66
	2'. $\frac{M_0}{M} = \left(\frac{v}{3.34}\right)^2 + 1$		5.85	6.20	6.40	6.54	6.61
	1'. $\frac{M_0}{M} = \left(\frac{v}{7.04}\right)^2 + 1$		2.09	2.165	2.22	2.24	2.26
Hydrogen	3'. $\frac{M_0}{M} = e^{\frac{v}{3.88}}$		6.65	7.11	7.35	7.57	7.68
	2'. $\frac{M_0}{M} = \left(\frac{v}{3.88}\right)^2 + 1$		4.60	4.85	5.00	5.09	5.15
	1'. $\frac{M_0}{M} = \left(\frac{v}{11.65}\right)^2 + 1$		1.400	1.426	1.442	1.455	1.460

In order to obtain a more complete picture of the dependence of all parameters on the distance traversed, the following quantities are also calculated in Table 7:

$$a = r_1\cos^2\beta = 6370\cos^2\beta; \quad b = r_1\sqrt{\cos^2\beta}\cos\beta = 6370\sqrt{\cos^2\beta}\cos\beta;$$
$$e = r_2 - a; \quad r_3 = 2a - r$$

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as well as the quantity $(v/v_3)^2$, which is proportional to the kinetic energy given to the rocket at burnout.

The results of Tables 3 and 7 for the values of the parameters characterizing the optimum range for a given fuel consumption as functions of this range are given in Figures 35 and 36.

In Figure 36 we can see a linear dependence between the flight range s and the quantities α, ϑ and β; next we see a parabolic dependence for h, and also a rapid increase of the initial velocity v for small ranges s while it grows slowly for distances larger than 6000 km.

TABLE 7

α	0°	20°	40°	60°	80°	90°
a, km	3185	3738	4274	4777	5232	5436
b	0	2036	3050	3900	4635	4950
e	3185	3137	2992	2758	2439	2254
r_3	0	601	1282	2019	2793	3182
$(v/v_3)^2$	0	0.298	0.510	0.665	0.785	0.830
μ	–	1.580	1.540	1.576	1.628	1.640
α		100°	120°	140°	160°	180°
a, km		5620	5943	6177	6321	6370
b		5245	5735	6090	6300	6370
e		2051	1592	1089	554	0
r_3		3569	4351	5088	5767	6370
$(v/v_3)^2$		0.867	0.929	0.967	0.990	1.000
μ		1.655	1.683	1.700	1.710	1.720

The quantity $(v/v_3)^2$, which is proportional to the kinetic energy of the rocket at burnout, increases strongly up to distances of 7000-8000 km and varies little for larger distances. It follows therefore that longer flights will be cheaper per km of distance traversed than short-range flights. From Table 7 we see that the energy required to fly around one quarter of the Earth, $\alpha = 90°$, $s = 10{,}000$ km, is 83% of the energy required to go around half the Earth.

Figures 33 and 34 show the curves of the required fuel consumption as a function of the flight range. In this case we also see the same picture, since the fuel consumption of a rocket with constant efficiency is proportional to v and for a rocket with constant exhaust velocity it is approximately proportional to v. The advantages of using rockets which take air from the atmosphere for combustion are obvious if this turns out to be possible after a more detailed examination of a rocket's flight in the atmosphere.

Further, we see from Figures 33 and 34 that within the given velocity limits the exponential and quadratic velocity laws give very similar consumption curves and only the absolute value of the consumption is different. Table 7 shows the ratio of fuel consumption in a rocket with constant exhaust velocity to that of a rocket utilizing a specific fraction of the fuel's energy

$$\mu = \frac{M_0' - M}{M_0 - M} = \frac{\frac{M_0'}{M} - 1}{\frac{M_0}{M} - 1} = \frac{e^{\frac{v}{3.34}} - 1}{\frac{v^2}{3.34^2}},$$

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where M_0' is the initial mass of the rocket in the exponential law, and
M_0 is the initial mass of the rocket in the quadratic law for the same mass M of empty rocket.

It can be seen from Table 7 that this ratio varies from 1.54 to 1.72 for central angles varying from $\alpha = 20°$ to $\alpha = 180°$. We have then $v_{max} : w = 7.9:3.34 = 2.37.$

Expanding $e^{\frac{v}{w}}$ in a series, we may write for any w

$$\mu = \frac{M_0' - M}{M_0 - M} = \frac{w}{v} + \frac{1}{2} + \frac{v}{6w} + \frac{1}{4}\left(\frac{v}{w}\right)^2 + \frac{1}{5}\left(\frac{v}{w}\right)^2 + \ldots \tag{31}$$

For $v = 0$ and for $v = \infty$, we obtain $\mu = \infty$.

The curve has a minimum, which is found by equating to zero the first derivative of the expression $(M_0' - M) : (M_0 - M)$ with respect to $v : w$.

We obtain

$$\frac{d\mu}{d\frac{v}{w}} = \frac{e^{\frac{v}{w}}\left(\frac{v}{w}\right)^2 - \left(e^{\frac{v}{w}} - 1\right)2\frac{v}{w}}{\left(\frac{v}{w}\right)^4} = 0$$

or

$$f\left(\frac{v}{w}\right) = e^{\frac{v}{w}}\frac{v}{w} - 2e^{\frac{v}{w}} + 2 = 0. \tag{32}$$

This is a transcendental equation, whose solution gives $\frac{v}{w} = 1.590$ and $\mu_{min} = 1.542$.

We see therefore that in a gravity free medium a rocket with constant exhaust velocity comes closest to a rocket utilizing completely the energy corresponding to the velocity w if the flight velocity is $v = 1.590\,w$; it has utilized by then 64.8 % i. e. ≈65 % of the entire energy, corresponding to the velocity v.

For particularly low and particularly high values of $v : w$, a constant exhaust velocity rocket is especially unfavorable.

In our case, the advantages of a constant efficiency rocket strongly increase only for flight ranges shorter than 2200 km ($\alpha = 20°$).

All formulas derived above refer directly only to a stationary coordinate system with the origin of coordinates at the center of the Earth. Since the Earth rotates, we must take into account the rotation velocity at the take-off and arrival points. We find then the value of the trajectory inclination known from the ballistic theory: in the northern hemisphere the rocket deviates to the right and in the southern hemisphere – to the left. The velocities, which have to be given to the rocket, are obtained from the velocity triangle: absolute velocities with respect to a stationary coordinate system; relative velocities with respect to the surface of the Earth and to the velocity of the take-off point.

Flights to the east, for which the relative velocity and the rotation velocity of the Earth are added, give ranges longer than calculated above, those to the west, on the contrary, give shorter ranges. The take-off angle and the absolute initial velocity should also be modified somewhat in order to obtain the longest range for a given fuel consumption.

Coming still closer to practice, it is possible to take into account the air resistance and the gravitational attraction. However, from Table 6 we

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already see that when atmospheric air is used for combustion and for increasing the efficiency of a rocket, it is possible to carry in the rocket the entire amount of fuel in liquid form, i. e., metals must not be used as fuel. When using metals for part of the flight lying inside the atmosphere, leaving the liquid fuel for the flight outside the atmosphere, flight to great distances and even escape from the Earth and flight into interplanetary space will be considerably simplified.

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JET ENGINES*

Our usual aviation engines have certain features and insufficiencies which compel us to look for new types of engines radically different from those employed at present.

Such new types of engines are the jet engines and the gas turbines. Between these two types of machines there exists still an entire series of combinations. These are: reactive propellers, engines in which the heat of the outgoing gases or their pressure is used to obtain an additional reaction force, and gas turbines operated by the outgoing gases of the engine and

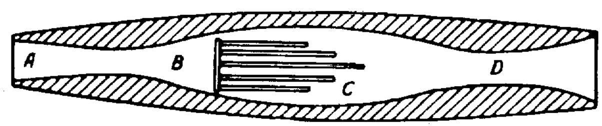

FIGURE 1. Schematic diagram of Loren's air-breathing jet engine

coupled to a centrifugal compressor supplying the engine with compressed air or driving a special propeller. Recently, the so-called air-breathing jet engines began to be distinguished from the series of jet engines. This name refers to two types of engines: in one type instead of carrying along oxygen or some other combustion oxidizer, external atmospheric air is introduced or compressed into the engine and is used subsequently in the combustion; in another type of engine, atmospheric air is mixed with the combustion products in various ways in order to decrease losses by the outgoing gases which carry with them much heat and kinetic energy.

The engines of the famous French designer René Loren belong to the first type. Three diagrams of these engines are shown in Figures 1-3.

In Figure 1 we see the diagram of a simple air-breathing continuous combustion jet engine [ramjet]. Air enters into the engine through the frontal section *A*, it is compressed in the cavity *B*, and then mixed with fuel coming from injectors in the chamber *C* where it is ignited. Coming out of the engine, the combustion products expand to atmospheric pressure and acquire a considerable velocity, attaining in section *D* the critical velocity, which is equal to the sound velocity in the hot gases. The reaction force of the outgoing combustion products pushes the engine forward.

Another type of air-breathing continuous combustion jet engine is shown in Figure 2. In this engine, the compartment *E* represents a compressor of centrifugal or piston type. In the first type of engine the kinetic energy

* This article was published in the Journal "Samolet", No. 1. 1932.

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of the incident air is transformed in a special duct into potential energy contained in the compressed air, and only part of it is lost due to friction, thus reducing the useful axial thrust obtained, while in the second type of engine all, or almost all the kinetic energy of the air entering the engine is destroyed; therefore this type of engine is applicable only at low flight velocities, where the kinetic energy of the incident air does not yet play an important role. Part of the kinetic energy of the incident air can be used only by building an air-compressor which is half centrifugal and half axial.

Nevertheless, jet engines designed according to Figure 2, will have a low efficiency even at low flight velocities since they use only a small fraction of the kinetic energy corresponding to the high exhaust velocity.

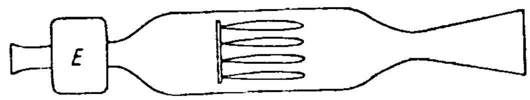

FIGURE 2. Schematic diagram of an air-breathing jet engine with compressor

Figure 3 shows the diagram of a jet engine, in which the compressor *E* feeds compressed air to the combustion chamber *H* periodically and not continuously. The mixture is ignited by a spark plug *G*, and the combustion products escape periodically through the nozzle *J*. The valve *F* admits a new portion of mixture into the combustion chamber only when the pressure in the combustion chamber has dropped to a value below the pressure in the space behind the compressor.

If a piston-type compressor is used instead of a centrifugal one, the valve is operated by a distributor mechanism.

A combustion chamber, in which the pressure and temperature fluctuate, is more easily cooled than a continuous combustion chamber; for the same average jet thrust, however, the stresses in the walls of the engine will be larger with fluctuating chamber pressure than in the case of constant pressure. It is easy to show that the amount of energy spent is smaller in the latter case.

The kinetic energy of the jet is proportional to the square of the relative velocity, and the momentum of the force, which is proportional to the useful work, grows only with the first power of this velocity.

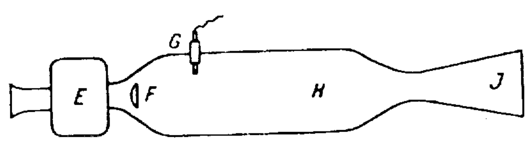

FIGURE 3. Schematic diagram of an air-breathing discontinuous combustion jet engine

Let us introduce the following notation:

w, the relative velocity acquired by combustion products particles inside jet engine's nozzle;

dm, an infinitely small mass which at a given moment acquired velocity w;

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P, jet thrust at a given moment of time;
dt, the time element;
C, the flight velocity;
L, useful work performed during time interval t;
E, work spent on acceleration of combustion products;
M, ejected mass of combustion products.

For a series of consecutive explosions (Figure 3) taking place under high pressure in the combustion chamber, the gas particles will come out of the nozzle with high velocity. With decreasing pressure the exhaust velocity will also decrease, diminishing, at the end of the cycle, almost to zero. The whole work spent on acceleration is equal to the kinetic energy

$$E=\int_0^{m_1}\frac{w^2\,dm}{2}. \qquad (1)$$

The useful work spent per cycle of duration t in moving the jet engine forward is equal to

$$L=\int_0^t P\cdot C\,dt=C^*\int_0^t P\,dt. \qquad (2)$$

From the law of momentum we have

$$\int_0^t P\,dt=\int_0^{m_1} w\,dm. \qquad (3)$$

Substituting $\int_0^t P\,dt$ from (3) in (2) we obtain

$$L=C\int_0^{m_1} w\,dm. \qquad (2a)$$

Denoting by subscript m all quantities for constant exhaust velocity w, we obtain from (1)

$$E_m=\frac{m_1 w_m^2}{2} \qquad (4)$$

and from (2a)

$$L_m=Cw_m m_1. \qquad (5)$$

Let us now consider two cases: one with constant pressure (see Figure 2) and the other with varying pressure in the combustion chamber (see Figure 3). Let the useful work performed at a given flight velocity be equal in both cases

$$L=L_m$$

we then obtain from formulas (5) and (2a)

$$w_m m_1=\int_0^{m_1} w\,dm. \qquad (6)$$

Let us prove that the kinetic energy E_m for constant exhaust velocity is smaller than the kinetic energy E in the case of variable w. We denote by

* [Here the author assumes implicitly that the flight velocity C is constant.]

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w_1 the average exhaust velocity which would give the same kinetic energy E for the same amount of combustion products. Then

$$E = \frac{m_1 w_1^2}{2} \tag{7}$$

and from (1) and (7) we obtain

$$w_1^2 m_1 = \int_0^{m_1} w^2\, dm. \tag{8}$$

Let us compare formulas (6) and (8). Since the integral $\int w^2 dm$ is changed only slightly by small values of w, while large values of w increase it considerably, w_1 is always larger than w_m.

Let us demonstrate this by an example. Let the exhaust velocity be proportional to the mass already ejected since the beginning of the cycle, i. e.,

$$w = km, \tag{9}$$

where k is a proportionality factor. Upon substitution in (6) and (8) we find

$$w_m m_1 = \frac{k_1 m_1^2}{2}; \quad w_m = \frac{k_1 m_1}{2}; \tag{10}$$

$$w_1^2 m_1 = \frac{k_1^2 m_1^3}{3}; \quad w_1 = \frac{k_1 m_1}{\sqrt{3}}. \tag{11}$$

Dividing (4) by (7) and substituting w_m and w_1 from (10) and (11), we obtain

$$\frac{E_m}{E} = \frac{w_m^2}{w_1^2} = \frac{3}{4}. \tag{12}$$

We see that the amounts of energy which have to be given to the combustion products for constant and variable velocities are in the ratio of 3 to 4.

If half of the mass is ejected at a velocity n times greater than the ejection velocity of the second half, we obtain

$$\frac{E_m}{E} = \frac{n+1}{n^2+1}. \tag{13}$$

If $n = 10$, we obtain $\frac{E_m}{E} = \frac{11}{101} = 0.109$.

We see that constant exhaust velocity engines will be in general more advantageous than variable exhaust velocity engines. The engines proposed by Loren may be very much improved by introducing a closed working cycle; this will be shown later on.

By careful study of jet engine designs, many deficiencies, characteristic of our present aviation engines, can be overcome.

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THERMAL CALCULATION OF A LIQUID PROPELLANT ROCKET ENGINE

Paper One*

INTRODUCTION

In the present article we consider the calculation of the thermal processes taking place in a direct jet rocket engine. The combustion of fuel and oxidizer takes place under a certain pressure in the combustion chamber, and the combustion products expanding in the nozzle are ejected directly to the atmosphere with a velocity which is constant or almost constant with respect to the nozzle.

The parameters which characterize the thermal phenomena in a rocket engine, are: the pressure in the combustion chamber, the combustion temperature of the working mixture, the flow temperature, pressure and velocity in the nozzle critical section and in its exit section.

These parameters depend on the physicochemical properties of the components of the working mixture and on the pressure in the combustion chamber.

In the present case we take as fuel high grade aviation gasoline, and as oxidizer – liquid air enriched by oxygen. To be more general, the calculation has been performed for liquid air with oxygen content varying from 23 to 100%. Various pressures in the combustion chamber were also chosen – from 3.5 to 11 atm.

DETERMINATION OF THE COMBUSTION TEMPERATURE

Approximate determination of the combustion temperature for variable heat capacities making no allowance for dissociation

We shall use the following notation:

x_0, percentage of oxygen (by weight) in the air used;
$(1-x_0)$, percentage of nitrogen (by weight) in the air;
L_0, weight consumption of air per kg of fuel;
h, percentage of hydrogen in fuel;
c, percentage of carbon in fuel;
H_2O_{mol}, amount of water vapor molecules obtained from 1 kg of fuel,

* This article was published in the Journal "Raketnaya Tekhnika", No. 1. 1936.

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CO_{2mol}, same for carbon dioxide;
N_{2mol}, same for nitrogen left from air after combustion;
M_{1mol}, amount of mixture molecules per kg of fuel, in g;
R_O, μ_O, gas constant and molecular weight of oxygen;
R_N, μ_N, gas constant and molecular weight of nitrogen;
R_2, μ_2, gas constant and molecular weight of combustion products;
μ_{gaso}, molecular weight of gasoline;
T_0, temperature of air supplied for combustion;
T_i, combustion temperature;
H_u, calorific value of fuel;
$R_1 \mu_1$, gas constant and molecular weight of evaporated air.

Let us take as fuel gasoline of the following composition: $h = 14.5\%$ by weight, $c = 85.5\%$ by weight.

The amount of oxygen required for the combustion of 1 kg of fuel is determined from

$$x_0 L_0 = 32\left(\frac{c}{12}+\frac{h}{4}\right) = 32\left(\frac{0.855}{12}+\frac{0.145}{4}\right) = 3.44 \text{ kg}.$$

The amount of liquid air required for the combustion of 1 kg of fuel will be

$$L_0 = \frac{3.44}{x_0} \text{ kg}.$$

The gas constant of evaporated air, taking $R_O = 26.5$ and $R_N = 30.2$, is equal to

$$R_1 = x_0 R_0 + (1-x_0) R_N = x_0 26.5 + (1-x_0) 30.2.$$

The average molecular weight of evaporated air is equal to

$$\mu = \frac{848}{R_1}.$$

The amount of mixture in kg-mol per kg of fuel, taking for gasoline a molecular weight μ_{gaso} of 100, will be

$$M_{1mol} = \frac{L_0}{\mu_1} + \frac{1}{100}.$$

The amount of combustion products in kg-mol, obtained from 1 kg of fuel, is equal to:

for H_2O_{mol}

$$\frac{h}{2} = \frac{0.145}{2} = 0.0725 \; mol;$$

for $CO_{2\,mol}$

$$\frac{c}{12} = \frac{0.855}{12} = 0.0712 \; mol;$$

for $N_{2\,mol}$

$$\frac{(1-x_0) L_0}{\mu_N} = \left[\frac{(1-x_0) L_0}{28.08}\right] mol.$$

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Denoting the amount of molecules of combustion products per kg of fuel by M_{2mol}, we obtain

$$M_{2mol} = H_2O_{mol} + CO_{2mol} + N_{2mol} = 0.1437 + \frac{(1 - x_0) L_0}{28.08}.$$

The average molecular weight of the combustion products is

$$\mu_2 = \frac{1 + L_0}{M_{2mol}}.$$

The gas constant of the combustion products is

$$R_2 = \frac{848}{\mu_2}.$$

The average heat capacity of the combustion products is:
for H_2O

$$0.0725(6.67 + 15.2 \cdot 10^{-4}T) = 0.584 + 1.102 \cdot 10^{-4}T;$$

for CO_2

$$0.0712(9.55 + 10.2 \cdot 10^{-4}T) = 0.679 + 0.726 \cdot 10^{-4}T;$$

for the sum of $H_2O + CO_2$

$$1.263 + 1.828 \cdot 10^{-4}T;$$

for N_2

$$N_{2\,mol}(6.59 + 5.167 \cdot 10^{-4}T).$$

The total heat capacity of M_{2mol} of combustion products will be

$$A_2 + B_2T = 1.263 + 1.828\ 10^{-4}T + N_{2mol}(6.59 + 5.167 \cdot 10^{-4}T) =$$
$$= (2.163 + N_{2mol}6.59) + (1.828 + N_{2mol}5.167)\ 10^{-4}T.$$

The heat contained in the combustion products is equal to

$$(A_2 + B_2T_i)\,T_i.$$

Finally, we determine the heat contained in the mixture before combustion at a temperature T_0 of 288°K, taking an evaporation heat r of about 90 cal/kg for gasoline, and obtain, similarly to the previous case, the expression

$$(A_1 + B_1T_0)\,T_0 - r = M_{1mol}(6.59 + 5.167 \cdot 10^{-4}\ 288)\ 288 - 90 =$$
$$= (1940 M_{1mol} - 90)\ \text{cal/kg}.$$

To determine the combustion temperature we have the equation

$$H_u + (A_1 + B_1T_0)\,T_0 - r = (A_2 + B_2T_i)\,T_i. \qquad (1)$$

Transforming it as follows:

$$T_i^2 + \frac{A_2}{B_2}T_i - \frac{1}{B_2}[H_u + (A_1 + B_1T_0)\,T_0 - r] = 0, \qquad (1a)$$

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and solving it with respect to the combustion temperature T_i, we obtain

$$T_i = -\frac{A_2}{2B_2} \pm \sqrt{\left(\frac{A_2}{2B_2}\right)^2 + \frac{1}{B_2}[H_u + (A_1 + B_1 T_0) T_0 - r]}. \qquad (2)$$

Figure 1 presents the values of T_i for the combustion of gasoline in air containing different amounts of oxygen.

It can be seen from the figure that the combustion temperature for high oxygen content in the enriched air is considerably higher than the temperature actually observed. This is due to the fact that we did not take into account the dissociation of the gases. Gas dissociation is accompanied by heat absorption and, consequently, by a temperature decrease which is particularly noticeable at high temperatures.

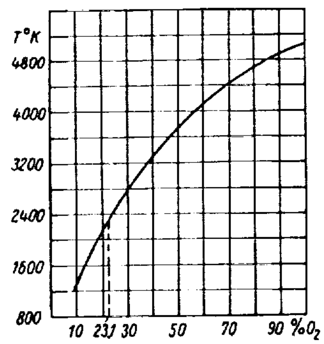

FIGURE 1. Combustion temperature of gasoline (neglecting dissociation) as a function of oxygen content in the air

The temperature reduction due to dissociation affects, however, the exhaust velocity only partially, since the temperature of the gases drops during their expansion in the nozzle. We thus have association, a process opposite to dissociation, which is accompanied by heat release and increases the exhaust velocity.

Determination of the combustion temperature accounting for gas dissociation

In the first approximation, we assume that instead of gasoline the initial materials are carbon and hydrogen in proportions corresponding to the chemical composition of gasoline. Furthermore, we shall neglect the decomposition heat of gasoline. Then we may write

$$Q_T = Q_{CO_2}\frac{c}{12} + Q_{H_2O}\frac{h}{2}, \qquad (3)$$

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where Q_{CO_2} is the heat of formation of CO_2; and Q_{H_2O}, the heat of formation of H_2O.

In the general case, the calorific value Q at any temperature is given by the formula

$$Q=Q_0+T(c_1-c_2), \tag{4}$$

where Q_0 is the calorific value at the absolute zero temperature; c_1 and c_2 are the average heat capacities of the reacting materials before and after combustion.

The heat of formation of carbon dioxide is equal to*

$$Q_{CO_2}=(67\,300+27\,600+3.4T-0.0036T^2)\,\text{cal/kg mol of } CO_2$$
$$=(94\,900+3.4T-0.0036T^2)\,\text{cal/kg mol of } CO_2. \tag{5}$$

The heat of formation of water vapor is**

$$2Q_{H_2O}=113\,900+4.067T+3.5\cdot 10^{-4}T^2-4\cdot 10^{-10}T^4. \tag{5a}$$

We obtain an equation for the combustion temperature, by comparing the heat of reaction to the difference in the enthalpies of the combustion products and the combustion mixture, taking into account the latent heat of evaporation of gasoline r and its enthalpy at a temperature T_0, i. e.,

$$Q_u=Q_{CO_2}\frac{c}{12}+Q_{H_2O}\frac{h}{2}=(A_2+B_2T_i)T_i-(A_1+B_1T_0)T_0+$$
$$+r-c_{gaso}T_0. \tag{6}$$

To be precise, we should account for the change in composition of the combustion products due to dissociation. To simplify matters, it may be assumed that the enthalpy hardly varies due to this. Furthermore, the degree of dissociation depends on the pressure in the combustion chamber, a fact which we will neglect***. The previous equation for the combustion temperature is solved graphically. For this purpose we determine initially the calorific value of gasoline as a function of temperature. Then, we determine the enthalpy of the combustion products as a function of temperature. The intersection point of the curves gives the combustion temperature.

As mentioned before, the calorific value is a function of temperature, as given by (4)

$$Q=Q_0+T(c_1-c_2).$$

* See Izgaryshev. – Khimicheskaya termodinamika (Chemical Thermodynamics), pp. 90-94. 1927. This book gives the heat of combustion of CO to CO_2. We add to this the quantity 23·12 = 27,600 cal/kg mol°C, which we assume constant since the dissociation of CO may be neglected.

** According to Pir's formula, the heat capacity of water vapor is $2(c_1-c_2)=4.155+3.5\cdot 10^{-4}\cdot T-4\cdot 10^{-10}T^3$.

*** By neglecting the influence of the pressure on the degree of dissociation reduces the accuracy of the calculation. Since higher pressures reduce dissociation, the errors will be smaller at high pressures than at low pressures. – Editor's note.

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From equation (6) we obtain for gasoline

$$Q = 0.0712\,(94\,900 + 3.47T + 0.00367T^2) + \frac{0.0725}{2}\,(113\,900 + 4.07T + $$
$$+ 3.5 \cdot 10^{-4}T^2 - 4 \cdot 10^{-10}T^4) = 10\,875 + 0.3897T - 2.433 \cdot 10^{-4}T^2 = $$
$$= 2.433 \cdot 10^{-4}T^2 - 0.145 \cdot 10^{-10}T^4. \qquad (7)$$

Figure 2 shows the calorific value of gasoline as a function of its absolute temperature T.

The enthalpy of the mixture, taking for the heat capacity of gasoline $c_{gaso} = 0.5$ cal/kg °C and assuming an initial temperature T_0 of 288°K, is given by

$$(A_1 + B_1T_0)\,T_0 - r + c_{gaso}T_0 =$$
$$= 1940M_{1mol} - 90 + 0.50 \cdot 288 = 1940M_{1mol} + 54. \qquad (8)$$

The enthalpy of the combustion products is equal to

$$A_2 + B_2T_i)\,T_i = [1.163 + 1.828 \cdot 10^{-4}T_i + N_{2mol}(6.59 +$$
$$+ 5.167 \cdot 10^{-4}T_i)]\,T_i. \qquad (9)$$

Plotting the difference in the enthalpies calculated by these formulas as a function of the temperature for given percentages of oxygen in the air, and plotting also Q as in Figure 1, we obtain the required combustion temperature at the intersection points (Figure 3). Figure 4 gives the values of T_i obtained, accounting for dissociation.

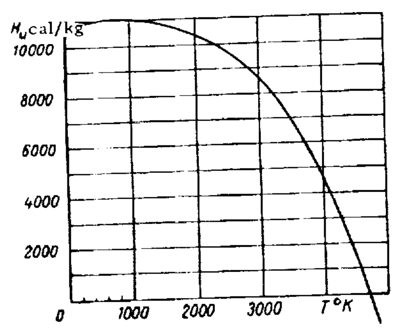

FIGURE 2. Calorific value of gasoline as a function of its absolute temperature, T

The highest temperature for the combustion of gasoline in pure oxygen is 3650° abs, or approximately 3400° C. This temperature is by 1440° lower than the temperature obtained neglecting dissociation. Thus, for an oxygen content larger than 40-50%, dissociation of the combustion products becomes so noticeable that it cannot be neglected.

For an oxygen content of 23.1, 30 and 40%, the combustion temperature can be calculated with adequate accuracy by the formula which does not account for dissociation.

FORMULAS FOR DETERMINING THE EXPANSION LAW OF THE COMBUSTION PRODUCTS, THE NOZZLE CROSS SECTION AREA AND THE EXHAUST VELOCITY

It has been shown above that part of the heat released in the combustion chamber is spent on dissociating the combustion products. The dissociation phenomenon is more noticeable, the higher the combustion temperature.

In the expansion of the combustion products in the nozzle, however, their temperature decreases and part of the heat spent on dissociation is released back and increases the expansion work of the gases.

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One obtains the expansion law in the nozzle, taking into account the heat release due to the decrease in dissociation degree, from the energy equation; we shall write it in the form

$$\frac{Aw\,dw}{2g} = dQ' - dQ_s - di = -AV\,dp - A\,dR_s, \tag{10}$$

with A, mechanical equivalent of heat;
w, gas velocity at given section of nozzle;
dQ', amount of heat per kg released by changes in state of gas;
dQ_s, amount of heat escaping through walls;
di, increase in gas enthalpy;
V, specific volume;
dp, increment in gas pressure;
dR_s, friction work arising in expansion of gas per kg.

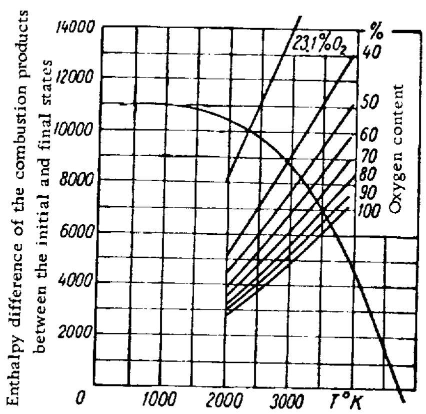

FIGURE 3. Determination of the combustion temperatures of gasoline in air enriched by oxygen (accounting for dissociation)

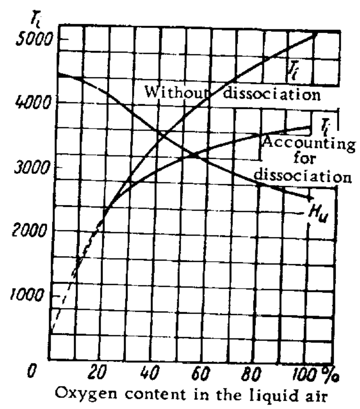

FIGURE 4. Heat release in the combustion of gasoline. Combustion temperature of gasoline versus oxygen content in the air (neglecting dissociation as well as allowing for it)

The amount of heat dQ', spent in dissociating 1 kg of gasoline or (L_0+1) kg of combustion products, will be as before equal to

$$dQ' = -\frac{dQ}{L_0+1} - (c_1 - c_2)\,dT. \tag{11}$$

By substituting dQ' in equation (10) and integrating, we obtain

$$\frac{A}{2g}(w_0^2 - w_i^2) = -\frac{1}{L_0+1}(Q_2 - Q_1) - \\ -(Q_{s_2} - Q_{s_1}) - (i_2 - i_1). \tag{12}$$

Replacing $Q_2 - Q_1$ in (12) by its value which is obtained by integrating (11),

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we obtain

$$\frac{A}{2g}(w_0^2-w_i^2)=-\frac{1}{L_0+1}\int_{T_1}^{T_2}(c_1-c_2)\,dT-(Q_{s_2}-Q_{s_1})-(i_2-i_1). \qquad (13)$$

where the index 1 refers to the initial state, and the index 2 – to the final state.

If in the first approximation we neglect the heat loss through the walls, i.e., we assume $Q_{s2}-Q_{s1}=0$, then the exhaust velocity w will depend only on the temperature.

Assuming that the enthalpy of the combustion products is related to the true heat capacity c_2' of the reacting materials after combustion by the expression

$$di=\frac{c_2'\,dT}{L_0+1}, \qquad (14)$$

which upon integration yields

$$i_2-i_1=\frac{1}{L_0+1}\int_{T_1}^{T_2}c_2'\,dT, \qquad (15)$$

then, after substituting in equation (13) the expression of (i_2-i_1) from (15) and transforming it, this equation will have the form

$$\frac{A}{2g}(w_0^2-w_i^2)=\frac{1}{L_0+1}\int_{T_1}^{T_2}c_1\,dT-(Q_{s_2}-Q_{s_1}).\text{*} \qquad (16)$$

It follows therefore that if we have dissociation, i.e., when heat is released in the process of gas expansion, as well as in the ordinary case considered, the increment of the kinetic energy of the combustion products is equal to the difference of two quantities: the enthalpy decrease of the original gases of the working mixture (and not of the combustion products, as is usually taken), taken at the initial and final temperatures, and the amount of heat escaping through the walls.

Since the heat capacity of gasoline vapors at high temperatures is not well known and since at high temperatures the dissociation of gasoline will be associated with heat release or its absorption, which complicates the derivation of the formulas, we may, as was done in the determination of the combustion temperature, take carbon and hydrogen instead of gasoline as the initial materials in a first approximation, thus neglecting the heat of formation or of decomposition of gasoline.

Formulas for the enthalpy at high temperatures are also not well known and may not be completely exact.

We may say that in formulas (10), (12) and (13), the quantity di, corresponding to the difference (i_2-i_1), should refer to the true heat capacities of the combustion products, assuming that they are not dissociated. More

* [Here the author assumes implicitly $c_2'=c_2$.]

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exactly, the enthalpy variation, when accounting for the heat release due to a decrease in dissociation, is equal to

$$i_1' - i_2' = i_1 - i_2 + \frac{H_{u_2} - H_{u_1}}{L_0 + 1} = i_1 - i_2 +$$
$$+ \frac{1}{L_0 + 1} \int_{T_1}^{T_2} (c_1' - c_2')\, dT. \quad (17)$$

c_2' refers to the dissociated combustion products and, therefore, calculations cannot be done directly by formulas (14) and (15).

Not insisting on a special investigation of this problem, in view of its complexity and the lack of experimental data, we shall establish the law of expansion of the gases in the nozzle. We shall assume that this process takes place so that the heat which was spent on dissociation of the combustion products is not released back during the expansion and the cooling of the combustion products, since it is difficult to establish what fraction of this heat returns back in the expansion. Direct experiments with rocket engines will allow improvement of the results. In the meantime we can find a lower limit for the exhaust velocity by assuming that the formulas for the heat capacity are applicable in all cases, and that the combustion temperature corresponds to the value determined by the methods examined above.

Thus, let us derive the formulas for the velocity of the combustion products in various sections of the nozzle. Instead of equation (10), we shall start from the equation

$$\frac{Aw\, dw}{2g} = -di - dQ_s = -AV\, dp - A\, dR_s. \quad (18)$$

Upon integration we obtain

$$\frac{A}{2g}(w_2^2 - w_1^2) = i_1 - i_2 + (Q_{s_1} - Q_{s_2}). \quad (19)$$

From equation (18), i.e.,

$$di + dQ_s = AV\, dp + A\, dR_s \quad (20)$$

after substituting V from Clapeyron's equation

$$V = \frac{RT}{p} \quad (21)$$

and neglecting the heat loss through the walls ($dQ_s = 0$) and the friction work ($dR_s = 0$), we obtain a second equation

$$\frac{AR\, dp}{p} = \frac{di}{T}. \quad (22)$$

We shall use equations (18) and (22) for the determination of the quantities w, T, p.

Neglecting in equation (18) the friction forces dR_s, we obtain

$$\frac{w\, dw}{2g} = -V dp. \quad (23)$$

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Replacing the specific volume V by $\frac{1}{\gamma}$ we have

$$\frac{w\,dw}{2g} = -\frac{dp}{\gamma}. \tag{23a}$$

The whole work of expansion of the gases from p_1 to p_2 is hence equal to

$$\int_{w_1}^{w_2} \frac{w\,dw}{2g} = -\int_{p_1}^{p_2} \frac{dp}{\gamma} = -\int_{p_1}^{p_2} V\,dp. \tag{24}$$

$$\frac{w_2^2 - w_1^2}{2g} = -\int_{p_1}^{p_2} V\,dp = -\int_{p_1}^{p_2} \frac{dp}{\gamma}. \tag{25}$$

The relation between p and γ depends on the thermal process through which the state of the particles varies in their motion. In the present case we assume the process to be adiabatic.

For an adiabatic process we have

$$p = c\gamma^k, \tag{26}$$

and, consequently,

$$dp = ck\gamma^{k-1}d\gamma, \tag{26a}$$

hence

$$-\int_{p_1}^{p_2} \frac{dp}{\gamma} = -\int_{\gamma_1}^{\gamma_2} ck\gamma^{k-2}d\gamma = c\,\frac{k}{k-1}\,\gamma^{k-1}. \tag{27}$$

Substituting for c its expression from equation (26), we find

$$-\int_{p_2}^{p_1} \frac{dp}{\gamma} = -\left|\frac{k}{k-1}\,\frac{p}{\gamma}\right|_{p_2}^{p_1} = \frac{k}{k-1}\left[\frac{p}{\gamma_1} - \frac{p_2}{\gamma_2}\right]. \tag{28}$$

Assuming in equation (25), $w_1 = 0$, we have from (28)

$$w_2 = \sqrt{2g\,\frac{k}{k-1}\,p_1V_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}. \tag{29}$$

From thermodynamics we have for an adiabatic process

$$\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1} = \frac{T_2}{T_1}. \tag{30}$$

Equation (29) can be written as

$$w_2 = \sqrt{2g\,\frac{k}{k-1}\,R_1T_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}. \tag{29a}$$

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or

$$w_2=\sqrt{2g\frac{k}{k-1}R_1(T_1-T_2)}. \tag{29b}$$

The derivation of the formulas for the pressures, densities and temperatures in the nozzle critical section and in its exit section is not difficult and may be found in any course of thermodynamics for an adiabatic process.

The cross sections of the nozzle are obtained from the equation of continuity of the jet

$$G=\gamma_x f_x w_x, \tag{31}$$

where G, is the rate of flow of the combustion products;
γ_x, the specific weight of the gases;
f_x, the nozzle cross section area;
w_x, the velocity of the combustion products.

TEMPERATURE, PRESSURE AND VELOCITY OF THE COMBUSTION PRODUCTS IN THE CRITICAL SECTION OF THE NOZZLE

Let us first determine the value of the adiabatic exponent k, neglecting the variation of the gas constant due to dissociation. It is known from thermodynamics that

$$k=\left(\frac{c_p}{c_v}\right)_2=1+\frac{AR_2\mu_2}{\mu_2 c_{v_2}}, \tag{32}$$

where c_{p_2} and c_{v_2} are the specific heats of the combustion products under constant pressure and constant volume and $AR_2\mu_2 = 1.985$. Substituting this value we obtain

$$k=1+\frac{1.985}{\mu_2 c_{v_2}}. \tag{32a}$$

To calculate $\mu_2 c_{v_2}$, we have a formula which was used previously for the determination of the combustion temperature

$$\mu_2 c_{v_2}=\frac{1}{\mu_2}(A_2+2R_2T_1)-1.985.$$

The critical temperature, i. e., the temperature in the critical section of the nozzle, is determined by a formula known from thermodynamics

$$T_c=T_1\frac{2}{1+k}. \tag{33}$$

k is a variable, depending on T_c. We shall find an approximate value of k, which we need for the determination of T_c. For this, let us take a critical temperature approximately equal to

$$T_c=0.90T_1. \tag{33a}$$

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The average temperature between the combustion temperature and the critical one is

$$T_{c\ av}=0.95T_1. \tag{33b}$$

We calculate $\mu_2 c_{v_2}$ and k, for this temperature, using the formulas given above. Then we determine the critical temperature T_c from (33).

A more exact value of the average temperature $T_{c\ av}$ or the expansion from T_i to T_c, for which we take a constant value of k, can be determined by the formula

$$T_{c\ av}=\frac{T_i+T_c}{2}. \tag{33c}$$

Next, we determine more exact values of $\mu_2 c_{v_2}$ and k and also of T_c and of the actual ratio $T_c:T_i$. Calculations show that $T_c:T_i$ is equal to 0.90 for air containing approximately 23-40% of oxygen. By increasing the oxygen content, $T_c:T_i$ increases too and for pure oxygen it attains the value of 0.94.

In Figure 5, values of k for various temperatures and pressures are given; Figure 6 shows T_c versus the oxygen percentage in air.

The critical pressure p_c, i.e., the pressure in the critical section of the nozzle, and the ratio p_c/p_i are given by the formulas

$$\frac{p_c}{p_i}=\left(\frac{T_c}{T_i}\right)^{\frac{k}{k-1}} \tag{34}$$

and

$$p_c=p_i\left(\frac{T_c}{T_i}\right)^{\frac{k}{k-1}}. \tag{34a}$$

Figure 7 shows the critical pressure as a function of the oxygen content in liquid air with the combustion chamber pressure as parameter. The ratio p_c/p_i is approximately 0.56 and varies insignificantly with the temperature: from 0.556 for $T_i = 2000°$ (for 23% O_2) to 0.579 for $T_i = 3500°$ (for 100% O_2), i.e., an increase of 2.3%.

The critical velocity, i.e., the velocity in the critical section of the nozzle, is determined by a formula, also known from thermodynamics

$$w_c=\varphi\sqrt{2g\frac{k}{k+1}RT_c}. \tag{35}$$

Figure 8 gives values of w_c, calculated by the above formula for $p_i = 6$ atm, as a function of oxygen content in the liquid air.

TEMPERATURE AND EXHAUST VELOCITY OF THE COMBUSTION PRODUCTS AT THE OUTLET FROM THE NOZZLE

The temperature of the combustion products at the outlet from the nozzle

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is determined, assuming an adiabatic process, by the formula

$$T_a = T_i \left(\frac{p_a}{p_i}\right)^{\frac{k-1}{k}}. \quad (36)$$

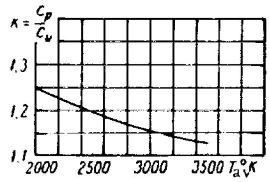

FIGURE 5. Adiabatic exponent versus temperature

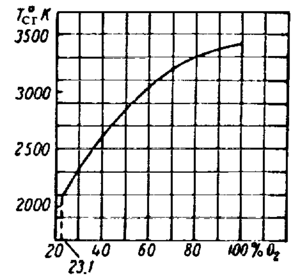

FIGURE 6. Critical temperature versus oxygen content in air

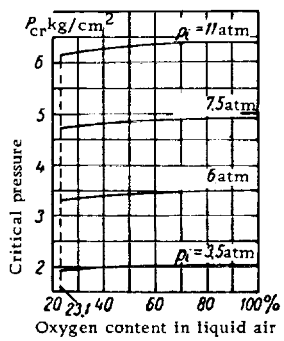

FIGURE 7. Critical pressure versus oxygen content in air with combustion chamber pressure as parameter

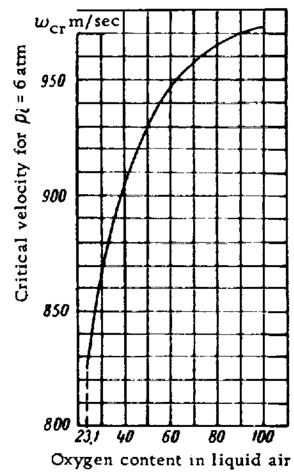

FIGURE 8. Critical velocity of combustion products versus oxygen content in air

We take the pressure at the nozzle exit section $p_a = 1$ atm. The calculation is similar to that for the nozzle critical section. First, T_a is determined approximately for a value of k corresponding to T_c, i.e.,

$$T'_a = T_i \left(\frac{p_a}{p_i}\right)^{\frac{k_{T_c}-1}{k_{T_c}}}; \quad (36a)$$

and then the adiabatic exponent k is determined for a more exact value of the average temperature, equal to

$$T'_{a\,av} = \frac{T_i + T'_a}{2}. \quad (36b)$$

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Finally, the temperature of the combustion products at the exit from the nozzle will be equal to

$$T_a = T_i \left(\frac{p_a}{p_i}\right)^{\frac{k'_{a\,\mathrm{av}} - 1}{k'_{a\,\mathrm{av}}}}. \qquad (36c)$$

Figure 9 shows T_a versus the oxygen content in the air for different pressures p_i in the combustion chamber, namely 3.5, 6, 8.5 and 11 atm.

The exhaust velocity of the combustion products can be determined by formula (29b)

$$w_a = \varphi \sqrt{2g \frac{k}{k-1} R_i (T_i - T_a)}.$$

The factor φ, taken as 0.95, represents velocity losses due to friction.

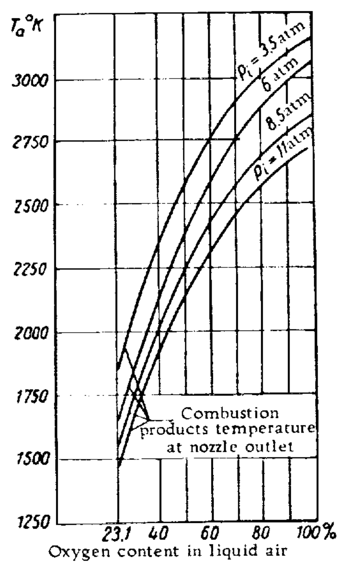

FIGURE 9. Combustion products temperature at nozzle outlet versus oxygen content in liquid air for various values of combustion chamber pressure

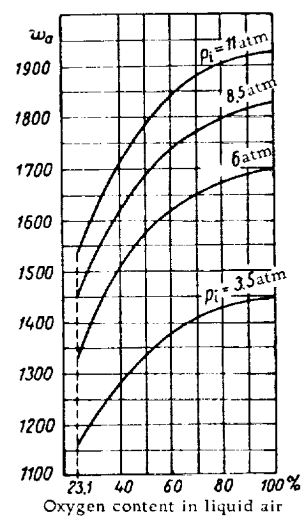

FIGURE 10. Combustion products exhaust velocity versus oxygen content in liquid air for various values of combustion chamber pressure

The numerical values of the exhaust velocity of the combustion products from the nozzle, calculated by formula (29b), are given in Figure 10.

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CONSUMPTION RATE OF WORKING MIXTURE

The required fuel mixture consumption per second G is easily determined from the given reaction force P and the known exhaust velocity w_a of the gases, using the expression for the reaction force

$$P = \frac{G}{g} w_a; \tag{37}$$

the total consumption per second is therefore equal to

$$G = \frac{Pg}{w_a}. \tag{38}$$

The gasoline consumption per second is

$$G_1 = \frac{G}{L_0 + 1} \tag{38a}$$

and the air consumption per second is

$$G_2 = G - G_1 = \frac{GL_0}{L_0 + 1}. \tag{38b}$$

DETERMINATION OF THE DIMENSIONS OF THE NOZZLE CRITICAL AND EXIT SECTIONS

The dimensions of the nozzle critical and exit sections are determined from the equation of continuity of the jet (31)

$$G = \gamma_x f_x w_x$$

and from the equation of state of the gases

$$\gamma_x = \frac{1}{V_x} = \frac{p_x}{RT_x}.$$

From these equations we find

$$f_x = \frac{GRT_x}{p_x w_x}. \tag{39}$$

For the critical section of the nozzle we have

$$f_c = \frac{GRT_c}{p_c w_c}. \tag{40}$$

and for the exit section of the nozzle we have

$$f_a = \frac{GRT_a}{p_a w_a}. \tag{41}$$

The diameters of the corresponding sections of the nozzle are given by

$$d_c = \sqrt{\frac{4}{\pi} f_c} \tag{40a}$$

and

$$d_a = \sqrt{\frac{4}{\pi} f_a}. \tag{41a}$$

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EFFECTIVE EFFICIENCY OF A ROCKET ENGINE

The effective efficiency of a rocket engine is equal to the ratio of the kinetic energy of 1 kg of combustion products, i. e., $\frac{w_a^2}{2g}$ to the thermal energy of 1 kg of fuel, which is equal to

$$\frac{H_u}{A(L_0+1)}, \text{ where } A=\frac{1}{427}.$$

Thus, we have

$$\eta_e=\frac{w_a^2(L_0+1)A}{2gH_u}. \tag{42}$$

Figure 11 presents curves of the effective efficiency of a rocket engine as function of the oxygen content in the air for various combustion chamber pressures.

CALCULATION OF THE COMBUSTION CHAMBER WALL TEMPERATURE

Let us assume that in a preliminary calculation we have already determined by the above method the temperature, density and pressure of the gases, the mixture consumption and the velocity of the gases inside the engine. Let us also assume that the temperature decrease due to heat losses through the walls is insignificant. In this case, knowing the initial temperature of the cooling liquid or gas and its velocity, we can determine the temperature of the walls and the heat exchange between the walls and the cooling medium.

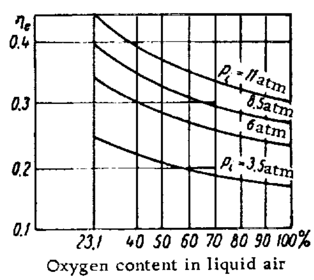

FIGURE 11. Effective efficiency of a rocket engine versus oxygen content in air for various values of combustion chamber pressure

Assume an engine in the shape of a cylindrical combustion chamber with a nozzle surrounded by an external envelope of the same shape. The envelope is so mounted that between it and the walls of the chamber there is a gap for the passage of the cooling fluid.

The calculation has been carried out assuming that the walls of the external envelope do not participate in the heat transfer.

The oxidizer, i. e. air enriched by oxygen, is used for cooling the engine. The air temperature before entering the chamber is taken as $t_0 = 15\,°C$ or $T_0 = 288\,°K$.

The coefficient of heat transfer through the walls is given by

$$k=\frac{1}{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{\delta}{\lambda}}, \tag{43}$$

with α_2 and α_1, heat transfer coefficients from gases to wall and from wall to cool air; λ, wall heat conductivity; δ, wall thickness.

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The coefficients α_2 and α_1 are determined by Nusselt-Nernst's formula

$$\alpha_1 = \frac{15.9\lambda_{1W}}{d_1^{0.214}}\left(\frac{w_1 b_1}{\lambda_1}\right)^{0.786} \tag{44}$$

and

$$\alpha_2 = \frac{15.9\lambda_{2W}}{d_2^{0.214}}\left(\frac{w_2 b_2}{\lambda_2}\right)^{0.786}, \tag{45}$$

with w_1, air velocity in annular space between envelope and combustion chamber;
w_2, velocity of combustion products in combustion space;
b_1, heat capacity of cool air in cal/m³;
b_2, heat capacity of combustion products in cal/m³;
λ_1, heat conductivity of air at cool air temperature;
λ_{1W}, heat conductivity of air at chamber wall temperature;
λ_2, heat conductivity of combustion products at combustion temperature;
λ_{2W}, heat conductivity of combustion products at chamber wall temperature;
d_1 and d_2, average calculated diameter of annular space (approximately equal to the hydraulic radius) and diameter of combustion chamber.

For a wall thickness of $\delta = 2\text{ mm} = 0.002\text{ m}$ and for

$$\lambda = 33\,\frac{\text{cal}}{\text{m}^2\,\text{hour}\,^\circ\text{C}}$$

(the mean value for steel) we obtain

$$\frac{\delta}{\lambda} = \frac{0.002}{33} = \frac{1}{16\,500}.$$

This is such a small quantity as compared with $1/\alpha_1$ and $1/\alpha_2$, that it can be neglected in the present calculation. The temperature difference between the external and internal surfaces of the combustion chamber walls is small.

Therefore, we can neglect the term δ/λ in equation (43) and use the formula

$$k = \frac{1}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)}, \tag{43a}$$

for the determination of the mean temperature of the combustion chamber wall.

The amount Q/Fz of heat, passing through 1 m² of surface F of the walls during 1 hour, can be expressed by

$$\frac{Q}{Fz} \approx \alpha_2(t_i - \vartheta_{av}) \approx \alpha_1(\vartheta_{av} - t) = k(t_i - t_0) = \frac{\lambda}{\delta}(\vartheta_2 - \vartheta_1), \tag{46}$$

where ϑ_1, ϑ_2 and ϑ_{av} are respectively the external, internal and average temperatures of the combustion chamber wall.

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From (46) we obtain

$$\vartheta_{av}=t_i-\frac{k}{\alpha_2}(t_i-t_0). \tag{47}$$

In (44) and (45), only λ_{1w} and λ_{2w} are variable quantities for the sections considered. They grow linearly with the temperature ϑ_{av}. We may therefore write

$$\alpha_1=\alpha_{1_0}(1+\beta_1\vartheta_{av}) \tag{44a}$$

and

$$\alpha_2=\alpha_{2_0}(1+\beta_2\vartheta_{av})^*, \tag{45a}$$

where the index 0 denotes the initial values of α_1 and α_2.

Substituting the expression of k from (43a) in formula (47), we obtain

$$(t_i-\vartheta_{av})=\frac{1}{\alpha_2\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)}(t_i-t_0)=\frac{\alpha_1(t_i-t_0)}{\alpha_1+\alpha_2}.$$

Substituting in this formula the expressions of α_1 and α_2 from (44a) and (45a), we obtain after some transformations

$$\vartheta_{av}^2-\frac{t_i\alpha_{2_0}\beta_2+t_0\alpha_{1_0}\beta_1-\alpha_{1_0}-\alpha_{2_0}}{\alpha_{1_0}\beta_1+\alpha_{2_0}\beta_2}\vartheta_{av}-\frac{t_0\alpha_{1_0}+t_i\alpha_{2_0}}{\alpha_{1_0}\beta_1+\alpha_{2_0}\beta_2}=0. \tag{48}$$

Denoting the coefficient of ϑ_{av} by a, and the free term by b we obtain

$$\vartheta_{av}^2-a\vartheta_{av}-b=0,$$

hence

$$\vartheta_{av}=\frac{a}{2}\pm\sqrt{\frac{a^2}{4}+b}, \tag{49}$$

with

$$a=\frac{\alpha_{1_0}(1-\beta_1t_0)+\alpha_{2_0}(1+\beta_2t_i)}{\alpha_{1_0}\beta_1+\alpha_{2_0}\beta_2} \tag{50}$$

and

$$b=\frac{t_0\alpha_{1_0}+t_i\alpha_2}{\alpha_{i_0}\beta_1+\alpha_{2_0}\beta_2}. \tag{51}$$

Using formula (48), the temperature ϑ_{av} of the combustion chamber walls can be easily determined, if the values of t_i, t_0, β_1, β_2 are given, and those of α_{1_0} and α_{2_0} are calculated.

* β_1 and β_2 are the heat transfer temperature coefficients. – Editor's note.

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Determination of the heat transfer coefficient α_{2} and its temperature coefficient β_2 for the heat transfer from the combustion products to the combustion chamber wall

The heat conductivity coefficients of the individual gases N_2, CO_2 and H_2O are

$$\left.\begin{aligned} \lambda_{N_2} &= 0.0190(1+0.0035t), \\ \lambda_{CO_2} &= 0.0119(1+0.00542t), \\ \lambda_{H_2O} &= 0.01405(1+0.00369t). \end{aligned}\right\} \qquad (52)$$

In the present calculation we have used, as a first approximation, a formula for the mixture which gives too high values for λ. More exact values may be obtained by experiments.

The amounts of CO_2 and H_2O, which are obtained by the complete combustion of 1 kg of gasoline, were calculated previously and are respectively equal to 3.145 and 1.305 kg.

The amount of N_2 obtained by the combustion of 1 kg of gasoline is

$$N_2 = \frac{3.44(1-x_0)}{x_0}\,\text{kg}.$$

According to the mixture rule the heat conductivity of a mixture of combustion products is equal to

$$(N_2+CO_2+H_2O)\lambda = N_2\lambda_{N_2} + CO_2\lambda_{CO_2} + H_2O\lambda_{H_2O}. \qquad (53)$$

We may also write for the total weight of the combustion products obtained from the combustion of 1 kg of fuel

$$N_2+CO_2+H_2O = L_0+1. \qquad (54)$$

Thus, we have

$$\lambda = \frac{N_2}{L_0+1}0.0190(1+0.0035t) +$$
$$+\frac{1}{L_0+1}[3.145\cdot 0.0119(1+0.00542t+1,305\cdot 0.01405)(1+0.00369t)] =$$
$$= \frac{N_2 0.0190 + 0.0557}{L_0+1} + \frac{N_2 66.5 + 271}{(L_0+1)10^6}t. \qquad (54a)$$

Using this formula we determine λ_2 and λ_{2w} in (45) for the respective temperatures t_i and ϑ_{av}.

Next, (45) contains the heat capacity b_2 of the combustion products at the temperature t_i. It is obtained from

$$b_{2t_i} = \frac{\gamma_i}{\mu_2 M_2}(A_2+2B_2t_i), \qquad (55)$$

where, as before

$$\mu_2 M_2 = L_0+1 \text{ and } \gamma_1 = \frac{p_i}{R_2 T_i}.$$

Then we have for the combustion chamber (taking an internal diameter d_2 of 60 mm = 0.06 m)

$$\lg d_2^{0.214} = 0.2141 \lg 0.06 = -0.214 \cdot 1.2218 = -0.262 = \lg 0.548,$$

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and consequently

$$d_2^{0.214}=0.548.$$

The combustion chamber cross section area is

$$f_i=\frac{\pi}{4}\,0.06^2=28.27\,\mathrm{cm}^2.$$

The velocity of the combustion products in the chamber is obtained from the continuity equation of the jet

$$G=\gamma_i f_i w_i,$$

and hence using the equation of state of the gases, we obtain

$$w_i=\frac{G}{\gamma_i f_i}=\frac{GR_i T_i}{p_i f_i}. \qquad (56)$$

Since, λ_{2f_i} and b_{2f_i}, as well as w_i are important independent quantities, they have to be calculated and then introduced in (45).

α_{2_0} is obtained by substituting in (45) λ_{2_0}, obtained for $t = 0$, and the temperature coefficient β_2 is obtained simply from (54a), by taking outside the brackets the term free of t; we then obtain for the coefficient of t

$$\beta_2=\frac{N_2 66.5+271}{10^6(N_2 0.0190+0.0557)}. \qquad (57)$$

We may also write

$$\beta_2=\frac{N_2 0.665+2.71}{N_2 190+557}. \qquad (57a)$$

Determination of the heat transfer coefficient α_{1_0} and its temperature coefficient β_1 for the heat transfer from the combustion chamber walls to the cold air moving in the annular space between the external envelope and the combustion chamber

The heat conductivity coefficient of the air produced from liquid air was also calculated by the mixture rule. We have

$$\lambda_1=x_0\lambda_{O_2}+(1-x_0)\lambda_{N_2}. \qquad (58)$$

The heat conductivity coefficient of nitrogen is given in (52); for oxygen it may be calculated by

$$\lambda_{O_2}=0.0200(1+0.0035t), \qquad (52a)$$

where t corresponds to ϑ_{av}.

Substituting in (58) we obtain

$$\lambda_1=x_0 0.0200(1+0.0035t)+(1-x_0)0.0190(1+0.0035t)=$$
$$=(1+0.0035t)(0.0190+0.001x_0). \qquad (58a)$$

With the aid of this formula we determine the values of λ_1 for the temperature at which the air is assumed to enter the envelope, and also for the temperature ϑ_1 of the walls themselves.

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For $t_0 = 15°C$ we have

$$\lambda_1 = (1+0.0035 \cdot 15)(0.0190+0.001 x_0. \quad (58b)$$

Taking as the heat capacities of oxygen and nitrogen

$$c_{pO_2} = 0.217 \text{ cal/kg °C and } c_{pN_2} = 0.247 \text{ cal/kg °C}$$

the heat capacity for air (1 kg) at a temperature of $t = 15°$ will be

$$c_{p1} = x_0 0.217 + (1-x_0)0.247. \quad (59)$$

The heat capacity of 1 m³ of air at a temperature t_0 is

$$b_1 = \frac{\gamma_0 c}{p_0} p_i = \frac{1.188 \cdot 29.26 p_i [x_0 \cdot 0.217 + (1-x_0) 0.247]}{R_1 p_0}, \quad (60)$$

where we have taken for the density that of atmospheric air which is inversely proportional to the gas constant, i. e.,

$$\gamma_0 = \frac{1.188 \cdot 29.26}{R_1}.$$

For pipes of annular section there is no straightforward formula for the heat transfer coefficient. Let us assume that the diameters of the section are equal to $d_a \approx 88$ mm and $d_i = 64$ mm.

If we assume that in formula (44) the value of d_1 is equal to twice the hydraulic radius, we obtain

$$d_1 = d_a - d_i.$$

If we assume that d_i is equal to the distance between the external and internal walls, then, introducing the symbol "prime", we obtain

$$d_1' = \frac{d_a - d_i}{2},$$

so that

$$\frac{d_1}{d_1'} = 2.$$

However, for formula (44), only the quantity $d_i^{0.214}$ is important and, therefore, we have for the ratio of the corresponding heat transfer coefficients

$$\frac{d_1'}{d_1} = 2^{0.214} = 1.160.$$

i. e., considering the first case, we obtain d_1 smaller by 16% only which means a somewhat higher calculated temperature of the combustion chamber walls.

Thus, taking

$$d_1 = d_a - d_i. \quad (61)$$

we obtain

$$d_1 = 88 - 64 = 24 \text{ mm},$$

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and

$$d_1^{0.214} = 0.450.$$

The air velocity is determined by a formula analogous to (56), i. e.,

$$w_1 = \frac{G_{air} R_1 T_0}{p_i f_1}, \quad (62)$$

with G_{air} air consumption in kg/sec; R_1, gas constant of air for corresponding oxygen content; T_0, air temperature at given section.

When determining the temperature of the combustion chamber walls, it is necessary to give first the dimensions of the combustion chamber and the consumption per second. After calculating all the auxilliary quantities, i. e., α_1, α_{2_0}, β_1, β_2, etc., they have to be substituted in (48a), (50), and (51).

The value of λ_1 is calculated by formula (58a), in which we omit the temperature-dependent terms, i. e., we assume that

$$\lambda_1 = (\lambda_{av})_0.$$

Furthermore, we substitute in (44) $d_1^{0.214} = 0.450$. The temperature coefficient β_1 is taken equal to 0.0035 according to (58a).

APPROXIMATE DETERMINATION OF THE DIMENSIONS OF THE REQUIRED COMBUSTION SPACE

The flame length is in general a function of many variables. It depends on the following factors: the flow velocity of the gases; the pressure, the temperature and the air surplus; the composition of the air of the kind of oxidizer; the kind of fuel; the degree of mixing; the temperature* of the air to be mixed; the temperature of the fuel; the degree of its spraying and evaporation; the combustion chamber wall temperature; the presence of extraneous bodies (fuel sprayers) and of incandescent parts in the chamber; the direction, in which the fuel is injected into the combustion chamber; eventual vortical motions or artificial mixing of the gases; and finally, the fuel's preliminary partial carburation. The shape of the combustion chamber itself plays also an important role.

Of particular importance at high temperatures is the dissociation of gases especially that of carbon dioxide. The dissociation lengthens the flame appearing inside the combustion chamber so much that its end comes out of the nozzle where it is mixed with atmospheric air. This lowers the combustion temperature and may even end the combustion. Inside the chamber, the combustion process will approach asymptotically the theoretical one, i. e., for a smaller combustion chamber, the fraction of unburned fuel will be larger.

We may approach the determination of the dimensions of the combustion chamber in different ways. We may, as was done abroad (see the Journal "Die Rakete", No. 3. 1929) take into account only the evaporation rate of the drops determining the length of the path traversed by the drop until its

* By increasing the pressure in the chamber the temperature of the chamber walls increases too due to the increased velocity of the gases. This increase is so small that the curves for the considered pressure interval practically coincide.

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complete evaporation. However, it will be incorrect to assume that the evaporated fuel burns out. This would take place only under the most perfect mixing.

It is therefore worthwhile to use experimental data and derive formulas enabling one to pass from a successfully operating chamber to another one of similar form.

The ratio of the flame length L to the internal diameter of the combustion chamber d_i gives the first relation for determining the dimensions of the combustion space

$$\frac{L}{d_i}=x. \tag{63}$$

The second relation is obtained from the continuity equation of the jet

$$G=\frac{\pi d_i^2 w_i p_i}{4RT_i}. \tag{64}$$

To obtain a third equation we assume that the length of the flame for a given temperature is approximately proportional to the gas flow velocity independently of the combustion space cross section. This is plausible, since for a given reaction rate, the individual particles of the burning mixture will be carried during combustion to distances proportional to the gas velocity in the combustion chamber.

Next, it is natural to assume that the flame length is inversely proportional to the gas density, i. e. that it is inversely proportional to its pressure for a given temperature. By increasing the pressure, the gas particles are situated closer one to another and the paths of individual molecules are shorter. It may therefore be assumed that the combustion intensity, i. e., the number of molecules combining per unit time in unit volume, is proportional to the gas pressure, whereas the combustion rate does not increase noticeably, since the combustion temperature does not increase. This means that for a given gas velocity, the volume in which the combustion takes place will be inversely proportional to the pressure and directly proportional to the working mixture consumption per unit time, i. e.,

$$\left.\frac{V}{V_0}\right|_{w=\mathrm{const}}=\frac{p_0 G_x}{p_x G_0}. \tag{65}$$

Since the total volume of the combustion chamber will also be proportional (for a constant pressure) to the gas velocity multiplied by the square of the diameter, we obtain a new equation in which the index "0" refers to the initial case and the volume of the combustion chamber is denoted by V.

Then

$$V=\frac{\pi d_x^2}{4}L_x \tag{66}$$

and

$$V_0=\frac{\pi d_0^2}{4}L_0. \tag{67}$$

We shall not consider here variations of the combustion temperature since it is almost constant, assuming that the kind of fuel and oxidizer and the degree of mixing do not vary.

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From expression (64) we find

$$\frac{G_x}{G_0}=\frac{p_x d_x^2 w_x}{p_0 d_0^2 w_0}. \tag{68}$$

Substituting the expression of $\frac{G_x}{G_0}$ from (68) in (65), we obtain for identical chamber pressures

$$\frac{V}{V_0}=\frac{d_x^2 L}{d_0^2 L_0}\bigg|_{p=\text{const}}=\frac{d_x^2 w_x p_x p_0}{d_0^2 w_0 p_x p_0}=\frac{d_x^2 w_x}{d_0^2 w_0}. \tag{69}$$

For different experimental and designed chamber pressures we have

$$\frac{V}{V_0}=\frac{w_x p_0 G_x}{w_0 p_x G_0}, \tag{70}$$

and, making the substitution

$$V_0=\frac{\pi d_0^2}{4}L_0,$$

we obtain the final expression for the volume

$$V=\frac{w_x p_0 G_x}{w_0 p_x G_0}\cdot\frac{\pi d_0^2}{4}L_0. \tag{71}$$

All the formulas derived above have an approximate character and require improving on the basis of theoretical studies and of results of tests conducted directly on rocket engines.

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THERMAL CALCULATION OF A LIQUID PROPELLANT ROCKET ENGINE

Paper Two*

THE THRUST OF A ROCKET

According to Newton's second law the total axial thrust on a rocket is equal to the product of the mass of gases leaving it per unit time by their exhaust velocity relative to the rocket.

Let us use the notation:

P, thrust on rocket (reaction force);
dM, mass of gases leaving rocket during time dt;
v, velocity of rocket;
c_1, absolute velocity of propellant before ejection;
c'_1, absolute velocity of gases at nozzle inlet (positive in flight direction);
c_2, absolute exit velocity of gases from rocket's nozzle (positive in flight direction);
w_1, relative velocity of propellant before ejection;
w'_1, velocity of gases relative to rocket at nozzle inlet (positive opposite to flight direction);
w_2, the same as w'_1 but at nozzle exit.

When the velocity of the rocket increases, the propellant in the tanks is also accelerated. Thus it acquires a certain amount of kinetic energy, which is later partially used in the combustion and plays, therefore, a tremendous role at high flight velocities. Before ejection the propellant absolute velocity, c_1, is always equal to the flight velocity, v, and the corresponding relative velocity, w_1, is always zero for rockets which do not use atmospheric air but carry along all the required fuel and oxygen. When entering the rocket nozzle, the propellant has in general already produced a certain small propulsive force corresponding to the change in relative velocity from $w_1 = 0$ to w'_1 or to the equivalent change in absolute velocity from c_1 to c'_1.

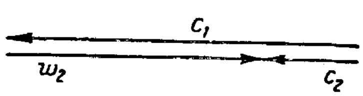

FIGURE 1

In calculating the axial thrust P one must, therefore, take into account the change in absolute gas velocity from $c_1 = v$ to c_2 or the equal change in relative velocity from $w_1 = 0$ to w_2. According to Figure 1

$$w_2 = c_1 - c_2; \tag{1}$$

* This article published in the Journal "Raketnaya Tekhnika", No. 5. 1937.

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the change of momentum of the mass dM will, therefore, be

$$w_2 dM = (c_1 - c_2) dM,$$

and since it is equal to the impulse of the force Pdt, we have

$$Pdt = w_2 dM \tag{2}$$

hence

$$P = w_2 \frac{dM}{dt}. \tag{3}$$

Denoting by G_s the propellant consumption per unit time (in 1 sec) in kg and by g_0 – the gravitational acceleration at the Earth's surface, we have

$$\frac{dM}{dt} = \frac{G_s}{g_0},$$

hence

$$P = w_2 \frac{G_s}{g_0}. \tag{4}$$

The velocity w_2 can be determined if we know the propellant's calorific value, the gases' initial and final states, and the heat transfer coefficient to the nozzle and the combustion chamber's walls.

Let us write the general energy equation

$$\frac{i_1 - i_2}{m} + Ab = AL + Q_s + A\left(\frac{c_2^2}{2g_0} - \frac{c_1^2}{2g_0}\right). \tag{5}$$

with m, molecular weight of combustion products;

i_1, propellant enthalpy per kg-mol of combustion products before combustion in rocket;

i_2, enthalpy of 1 kg-mol of combustion products after their combustion;

$i_1 - i_2$, so-called thermal head (a term analogous to hydraulic head);

L, work, in kg, usefully delivered by rocket engine per kg of propellant mixture;

A, 1:427 cal/kg – thermal equivalent of work;

Q_s, heat transferred to rocket walls per kg of propellant mixture;

b, kinetic energy acquired by 1 kg of propellant when fed into combustion chamber.

Equation (5) can be expressed in words as follows: enthalpy change plus work performed by the external forces is equal to the sum of three quantities:

1) the thermal equivalent of the useful work AL;
2) the heat Q_s given to the rocket walls;
3) the kinetic energy of the combustion products per kg of propellant mixture,

$$A\left(\frac{c_2^2}{2g_0} - \frac{c_1^2}{2g_0}\right).$$

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Friction losses at the walls were neglected in (5). The work $G_s L$, performed by the rocket in unit time, is equal to the work of the force P, acting on the rocket at the velocity v, i. e.,

$$G_s L = Pv.$$

By substituting P from equation (3) we obtain

$$G_s L = \frac{w_2 G_s}{g_0} v,$$

hence

$$L = \frac{w_2 c_1}{g_0}. \tag{6}$$

Substituting L from (6) and w_2 from (1) into (5), we obtain

$$-Q_s + \frac{i_1 - i_2}{m} + Ab = A\,\frac{w_2 c_1}{g_0} + \frac{A}{2g_0}(c_2 - c_1)(c_2 + c_1) =$$

$$= \frac{A}{2g_0}[2w_2 c_1 - w_2(c_2 + c_1)] = \frac{Aw_2}{2g_0}(c_1 - c_2) = \frac{Aw_2^2}{2g_0};$$

hence

$$w_2 = \sqrt{\frac{2g_0}{A}\left(\frac{i_1 - i_2}{m} - Q_s + Ab\right)}. \tag{7}$$

Introducing the exhaust coefficient φ, i. e., the ratio of the actual to the theoretical exhaust velocity, we obtain

$$w_2 = \varphi\sqrt{\frac{2g_0}{A}\left(\frac{i_1 - i_2}{m} - Q_s + Ab\right)}. \tag{8}$$

Inserting the numerical values of $g_0 = 9.81$ m/sec² and of $A = \frac{1}{427}$, we have

$$w_2 = 91.5\varphi\sqrt{\frac{i_1 - i_2}{m} - Q_s + Ab}\ \text{m/sec} \tag{9}$$

where i_2 is taken from the iS diagram, plotted without allowance for friction.

The entire thermal head, divided by the molecular weight of the combustion products $\frac{i_1 - i_2}{m}$, is equal to the calorific value $H_0 = H/(1+x)$ of 1 kg of propellant mixture. H is the calorific value of 1 kg of fuel and x – the weight of oxygen or air required for the combustion of 1 kg of fuel.

In particular, using the entire thermal head and taking $\varphi = 1$, $Q_s = 0$ and $b = 0$, we obtain for the maximum exhaust velocity

$$w_{2max} = \sqrt{\frac{2g_0}{A}\,\frac{i_1 - i_2}{m}} = 91.5\sqrt{H_0}\ \text{m/sec}. \tag{10}$$

Equation (9) shows that the relative exhaust velocity w_2 depends only on the calorific value of the propellant $\frac{i_1 - i_2}{m}$, on the amount of heat Q_s escaping

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through the rocket walls, and on a certain coefficient φ which is the closer to 1 the smaller the friction in the rocket.

The rocket's thrust P is equal, according to (4), to the product of the velocity w_2 by the mass of gas ejected from the rocket per unit time. For a constant velocity w_2, constant pressure in the combustion chamber and constant area of the nozzle exit section this rate of mass flow is also constant. The rocket is an apparatus, possessing for a given constant propellant consumption rate a constant propulsive (pushing) force. Since the power is equal to Pv, it increases proportionally to flight velocity and may be raised without limit. The efficiency, however, tends to zero at high velocities (much higher than 11 km/sec) as we shall see in the next section.

EFFICIENCIES OF A ROCKET*

Let us denote by η_m the ratio of the work G_sLdt received by the rocket during time dt, to the heat transformed during that time interval into kinetic energy of the combustion products, i. e., to the quantity

$$dt\,\frac{G_s w_2^2}{2g_0}=\frac{G_s}{A}\left(\frac{i_1-i_2}{m}-Q_s\right)\varphi^2\,dt.$$

We then obtain**

$$\eta_m=\frac{G_sL\,dt}{\frac{G_s}{A}\,\frac{i_1-i_2-mQ_s}{m}\,\varphi^2\,dt}=\frac{LmA}{(i_1-i_2-mQ_s)\,\varphi^2}. \tag{11}$$

Substituting in (11) the value of L from (6) and of w_2 from (8), we have

$$\eta_m=\frac{w_2c_1}{g_0}\cdot\frac{2g_0}{w_2^2}$$

or, finally

$$\eta_m=\frac{2c_1}{w_2}. \tag{12}$$

Equation (11) shows that at the speed of our ordinary airplanes η_m is a very small quantity; a hydrogen-oxygen propellant, e. g., with $H_u=$ $=29000$ cal/kg and $x=8$, gives $H_0=29000:9=3220$ cal/kg and w_{2max} $=91.5\sqrt{3220}=5200$ m/sec for $\varphi=1$. Taking for w_2 a realistic value of 3000 m/sec, we obtain, according to (12), for a flight velocity of $v=c_1=$ $=45$ m/sec (162 km/hour)

$$\eta_m=\frac{2\cdot 45}{3000}=0.03.$$

It follows that at low altitudes over the Earth's surface and at the velocities of our ordinary airplanes, it is impossible to use a rocket which is unable to use additional air or does not have a very high acceleration.

* This section is a more complete exposition of Chapter 7 of the book by Tsander F. A. "Problema poleta pri pomoshchi reaktivnykh apparatov" (Problem of Flight with the Aid of Jet Propulsion Machines). In that chapter, Tsander gives only final formulas for the efficiencies. Here, their detailed derivation is presented. – Editor's note.

** Here Tsander neglects the quantity Ab, i. e., the kinetic energy given to the propellant when it is fed to the combustion chamber. This can be done since this energy is usually small. – Editor's note.

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The thermal efficiency η_t of a rocket may be defined as the ratio of the thermal head (i_1-i_2) theoretically transformed by adiabatic expansion into energy corresponding to the velocity w to the full thermal head (i_1-i_0), i.e.,

$$\eta_t=\frac{i_1-i_2}{i_1-i_0}. \tag{13}$$

If by full thermal head we mean the upper calorific value, then $i_0=0$. However, in general one cannot hope for liquefaction of the water vapor contained in the combustion products during expansion in the nozzle and therefore $i_0\neq 0$ is frequently assumed. The denominator (i_1-i_0) in (13) is the lower calorific value, and i_0 – the thermal head just before liquefaction.

Denoting by η_i the ratio of the energy transmitted to the rocket to the work corresponding to the full thermal head at a given moment, we have

$$\eta_i=\eta_m\varphi^2\eta_t. \tag{14}$$

The total efficiency η_{i+E} is defined as the ratio of the energy given to the device during the time dt to the sum of two quantities: 1) the energy, corresponding to the entire heat contained in the propellant consumed during dt, and 2) the absolute kinetic energy contained in the fuel before combustion. The work capacity of 1 kg of propellant mixture is

$$L_0=\frac{i_1-i_0}{Am}+\frac{c_1^2}{2g_0} \tag{15}$$

and the work given by the rocket engine is

$$L=\frac{i_1-i_2-Q_sm}{Am}\varphi^2+\frac{c_1^2-c_2^2}{2g_0}; \tag{16}$$

therefore

$$\eta_{i+E}=\frac{L}{L_0}=\frac{(i_1-i_2-mQ_s)\varphi^2 2g_0+(c_1^2-c_2^2)Am}{(i_1-i_0)2g_0+c_1^2Am}. \tag{17}$$

In (15) the first term is the full thermal head, the second term the kinetic energy of the propellant immediately before combustion, when still in the tanks. η_{i+E} cannot be larger than 1, whereas the other above-examined efficiencies η_i and η_m have values larger than 1 for certain high velocities and are not, therefore, true efficiencies.

Since

$$G_sL=Pv=Pc_1$$

and

$$L=\frac{Pc_1}{G_s}=\frac{w_rc_1}{g_0},$$

and also

$$\frac{i_1-i_0}{Am}=\frac{w_{2\,max}^2}{2g_0},$$

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we obtain

$$\eta_{i+E}=\frac{L}{L_0}=\frac{L}{\frac{i_1-i_0}{Am}+\frac{c_1^2}{2g_0}}=\frac{2g_0Pc_1}{G_s(w_{2\,max}^2+c_1^2)}=$$

$$=\frac{2g_0L}{w_{2\,max}^2+c_1^2}=\frac{2w_2c_1}{w_{2\,max}^2+c_1^2}. \quad (18)$$

Next, from (14), we have

$$\eta_i=\eta_m\eta_t\varphi^2=\frac{LAm}{(i_1-i_2-mQ_s)\varphi^2}\frac{(i_1-i_0-mQ_s)}{i_1-i_0}\varphi^2=\frac{LAm}{i_1-i_0}=\frac{2g_0L}{w_{2\,max}^2}=\frac{2w_2c_1}{w_{2\,max}^2}=\frac{2g_0Pc_1}{G_sw_{2\,max}^2}. \quad (19)$$

We therefore obtain

$$\eta_{i+E}=\frac{\eta_i}{1+\frac{c_1^2}{w_{2\,max}^2}}=\frac{\eta_m\eta_t\varphi^2}{1+\frac{c_1^2}{w_{2\,max}^2}}=\frac{\eta_m\eta_t\varphi^2}{1+\frac{\eta_m^2}{4}\eta_t\varphi^2}=\frac{\eta_i}{1+\frac{\eta_m\eta_i}{4}}. \quad (20)$$

The maximum value $\eta_{i+E}=1$ is obtained, as seen from (18) for $w_2=c_1$, with $\eta_t=1$ and $\varphi=1$. From formula (19) we see that then $\eta_i=2$, i. e., the useful work is equal to twice the enthalpy of the fuel and, consequently, the useful kinetic energy is equal to the fuel's enthalpy. The absolute exhaust velocity of the gases is zero in this case; $c_2=c_1-w_2$.

The coefficients η_m, η_t and η_{i+E} give a clear picture of the efficiency of flight at a given velocity v; the coefficient η_t enables us to determine: 1) how usefully the thermal energy is exploited and 2) the effect of the counter-pressure (the pressure at the end of expansion) on heat exploitation. Assuming atmospheric pressure as the final pressure, we may construct the curves η_t as function of the altitude h over the Earth's surface for a given combustion chamber pressure.

From formula (4), together with formulas (8) and (13) we can easily determine the axial thrust P as a function of η_t

$$P=\frac{G_s}{g_0}\varphi\sqrt{\frac{2g_0}{A}\frac{i_1-i_2}{m}}=\varphi G_s\sqrt{\frac{2(i_1-i_2)\eta_t}{Amg_0}}=\varphi G_s\sqrt{\frac{2H_0}{Ag_0}\eta_t}. \quad (21)$$

For example, for the combustion of hydrogen with oxygen we have

$$P=\varphi G_s\sqrt{\eta_t}\sqrt{\frac{2\cdot3220\cdot427}{9.81}}=530\varphi G_s\sqrt{\eta_t}. \quad (22)$$

(Curves, giving η_t as function of h for the combustion of hydrogen with oxygen, were given by the author in his book "Problema poletov pri pomoshchi reaktivnykh apparatov" (Problems of Flight with the Aid of Jet Propulsion Machines))*. In determining the coefficient η_t it was assumed that the gases actually expanded to external pressure in the rocket by decreasing the nozzle minimum cross section or by increasing its maximum cross section.

The four coefficients η_m, η_t, η_i and η_{i+E} together characterize the performance of a rocket at a given moment of flight.

* The sentence in brackets was added in the editing. – Editor's note.

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Let us now look at the work performed on the part of the rocket remaining at the end of engine operation. Denoting its mass by $M_1=\frac{G_1}{g_0}$ where G_1 is the weight which the mass M_1 would have at the Earth's surface, we find that the work usefully spent during the time dt on lifting and accelerating the mass M_1 only is smaller than G_sLdt. Let M denote the mass of the entire rocket at a given moment and dA, the air resistance work during the time dt. Then $G_sLdt-dA$ is the work required to lift and accelerate the entire rocket, and

$$G_sL\,dt-dA)\frac{M_1}{M}$$

is the same, for the mass M_1.

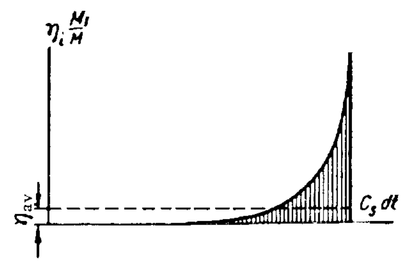

FIGURE 2

Consequently, for that part of the trajectory in which air resistance may be neglected, the efficiencies introduced earlier must be multiplied by M_1/M, i.e.,

$$\eta_m\frac{M_1}{M};\quad \eta_t\frac{M_1}{M};\quad \eta_i\frac{M_1}{M}\text{ and }\eta_{i+E}\frac{M_1}{M},$$

since, as before, these coefficients characterize the performance of the rocket at a given moment of flight.

If we plot η_iM_1/M as the ordinate and the weight G_sdt of the propellant consumed as the abscissa, then the ordinate η_{av}, i.e., the height of a rectangle whose area is equal to that bound by the curve η_iM_1/M, represents the average efficiency for the entire flight (Figure 2). We have

$$\eta_{av}=\frac{\int\eta_i\frac{M_1}{M}G_s\,dt}{\int G_s\,dt}.$$

THERMAL CALCULATION OF A ROCKET

The nozzles of steam turbines are little rockets; therefore, all calculations of pressures, specific weights of the gases, velocities, temperatures, friction with the walls, and enthalpy decrease for various sections

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perpendicular to the nozzle's longitudinal axis can be carried out, as for turbine nozzles with the difference, however, that in rockets using very hot gases, their cooling by the walls must also be taken into account.

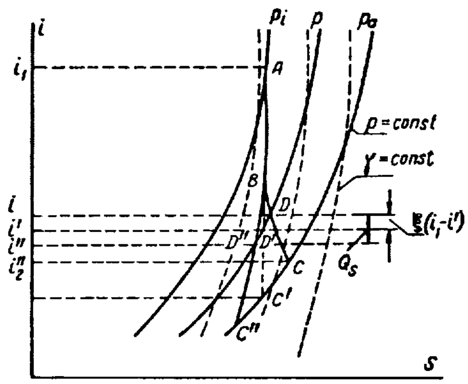

FIGURE 3

We shall now make a somewhat generalized calculation of the processes taking place inside the rocket, analogous to the calculation of steam turbine nozzles. Since the motion of the gases in the engine is steady state, the amount G_s of gases passing through all sections of the nozzle perpendicular to its axis in unit time will be constant. Denoting by f the area of a cross section of the nozzle, by p, the absolute pressure of the gases, and by γ, their specific weight, we have

$$G_s = \gamma f w \quad \text{or} \quad f = \frac{G_s}{\gamma w}. \tag{23}$$

Next, we have for the enthalpy decrease

$$\frac{i_1 - i_2}{m} = \frac{A w^2}{2 g_0} + Q_s. \tag{24}$$

Given the initial pressure p_i and enthalpy i_1, we find point A (Figure 3), corresponding to the pressure p_i in the iS diagram (where S is the entropy). A line parallel to the ordinate axis i corresponds then to adiabatic expansion. If the final pressure in the nozzle p_a is given, and the corresponding enthalpy for adiabatic motion is i_2', we can determine the actual final enthalpy i_2 for known friction losses with the walls (according to data of Professor A. Stodola* their value ranges from 0.05 to 0.15 of the thermal head $i_1 - i_2'$). To do this we move upwards along a line p = const up to

$$i_2 = i_2' + \xi (i_1 - i_2'), \tag{25}$$

where ξ is a coefficient taken from Stodola.

We shall first neglect the heat lost through the rocket walls. Down to the critical pressure at point B we can take an adiabatic line and further,

* Stodola, A. Die Dampf- und Gasturbinen. 6th Edition.

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in a first calculation, a smooth curve connecting B to C. Thus, for each point of the curve, p, γ, and i are known. It is assumed that in the iS diagram, p = const and γ = const curves are drawn. Determining w from equation (24) as function of the pressure p, for a given consumption G_s per unit time, we may also obtain from (23) f as a function of p. This function has a minimum. The pressure at f_{min} is called the critical pressure.

The final pressure p_a determines the maximum final cross section of the rocket nozzle, and the critical pressure determines its minimum cross section. These two pressures determine the rocket nozzle. In designing it, one should make a smooth transition near the narrow section of the nozzle and not expand it too steeply, in order to avoid separation of the jet from the walls (this phenomenon occurs at large divergence angles). If the divergence angle is too small, then the length becomes large increasing friction losses considerably; generally a divergence angle of about 10° is used. The temperature depends on i and may also be determined.

By means of the computed values of T, w and γ one can determine the amount of heat which must be removed through the nozzle walls in order to avoid overheating. If the coefficient of gas friction with the walls is known, one can also determine the total friction work with the walls.

Next, in a second, more exact calculation, we can draw on the iS diagram a new curve $ABD''C''$ (see Figure 3), which is required for the determination of the friction coefficient ξ not only at the final point C but for any point of the curve. For this purpose we measure for any pressure, p, i.e., for point D', the quantity $\xi(i_1-i')$ vertically upwards. We obtain point D. Measuring then the thermal head, corresponding to the amount of heat removed through the walls up to the moment considered, downwards along the ordinate axis and moving horizontally till the intersection with the line p= const, we obtain point D'', i.e., the point of the required expansion curve. Since

$$-dQ_s+AR_s=di-AVdp,$$

and $dp = 0$, then

$$di=-dQ_s+AR_s,$$

where R_s is the friction work.

For each point of the new curve, p, γ and i will be known. In determining the cross sections f of the nozzle, we should use the values of γ obtained from the curve $ABD''C''$, and determine w from the difference i_1-i of the thermal heads at points A and D of the curve $ABDC$, since the heat escaping through the walls does not accelerate the gases. The calculation proceeds then as before; the temperature T'' should be determined from i'' which corresponds to point D'' lying on the curve $ABD''C''$.

If friction with the walls and heat removal through the walls are already noticeable before point B, then curves ADC and $AD''C''$ diverge, of course, already near point A. However, in experiments with nozzles of Laval turbines it has been found*, that friction losses down to the critical cross section are practically insignificant. The same is also obtained theoretically from the $w:d$ diagram (see Section 5 on friction). It also follows from experiments that if the counter-pressure at the nozzle exit is large, we have

* Zerkowitz, Thermodynamik der Turbomaschinen, p. 39. 1913.

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inside the nozzle first an expansion of the gases and further on a compression, associated with a shock, while the total amount of gases passing in unit time through the section remains constant. If the counter-pressure approaches the initial pressure, reduction of the propellant flow rate is observed. Guthermuht observed in this case a sucking action of the nozzles*.

PERFECT GASES

If we deal with perfect gases, then we obtain for Laval type nozzles, as before,

$$G_s = \frac{fw}{V_x}, \tag{26}$$

where $V_x = \frac{1}{\gamma}$ is the specific volume of the gases at section f.

Let us further use the notation:

- n, polytropic exponent;
- k, adiabatic exponent;
- p_i, V_i, T_i, f_i, pressure, specific volume and temperature of gases, and area of chamber's cross section before nozzle;
- p_x, V_x, T_x, f_x, same for section f of nozzle;
- p_a, V_a, T_a, f_a, same after expansion in final nozzle cross section;
- p_k, w_k, f_k, pressure, gas velocity and nozzle area in critical cross section;
- L', work corresponding to energy increment by expansion from p_i to p_x.

We have

$$w_a = \varphi w_{\text{theor}} = \varphi \sqrt{2g_0 \frac{k}{k-1} p_i V_i \left[1 - \left(\frac{p_a}{p_i}\right)^{\frac{k-1}{k}}\right]}; \tag{27}$$

$$G_s = \varphi f_x \sqrt{\frac{2g_0 k}{k-1} \frac{p_i}{V_i} \left[\left(\frac{p_x}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right]}; \tag{28}$$

$$p_k = p_i \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}; \tag{29}$$

$$w_k = \varphi \sqrt{2g_0 \frac{k}{k+1} p_i V_i}; \tag{30}$$

$$G_{s\,\max} = \varphi f_k \sqrt{2g_0 \frac{k}{k+1} \left(\frac{p_k}{p_i}\right)^{\frac{2}{k}} \frac{p_i}{V_i}}; \tag{31}$$

$$L'_{\text{theor}} = -\int_{p_i}^{p_x} V_x\,dp = \frac{ck}{k-1}\left(p_i^{\frac{k-1}{k}} - p_x^{\frac{k-1}{k}}\right), \tag{32}$$

where

$$c = p_i^{\frac{1}{k}} V_i;$$

* Guthermuht, VDI. 1904.

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$$T_x = T_i - \frac{A}{c_p}\frac{w^2}{2g_0}; \qquad (33)$$

$$V_x = V_i\left(\frac{p_i}{p_x}\right)^{\frac{1}{k}} + \frac{k-1}{kp_x}\left(\frac{1}{\varphi^2} - 1\right)\frac{w^2}{2g_0}. \qquad (34)$$

Nozzles calculated by these formulas have an exhaust velocity coefficient of

$$\varphi = \frac{w}{w_{\text{theor}}} = 0.975 - 0.92.$$

The above-mentioned formulas were given by Stodola* for adiabatic expansion and friction only was included in the calculation since in the nozzles he neglected heat losses through the walls. However, these formulas are also applicable for the calculation of rocket nozzles if we take a polytropic expansion.

The calculation can be done by the following method. We give, for example, the initial and final pressures p_i and p_a and determine the combustion temperature by the general rules**, after which the specific volume of the gases before expansion may be found from the equation

$$V_i = \frac{RT_i}{p_i}. \qquad (35)$$

Next, we choose the polytropic exponent**, of which one can say that for rockets it will be closer to the adiabatic exponent and for small rockets probably further from it, than in the case of internal combustion engines.

* Stodola, A. Die Dampf- und Gasturbinen. 6th Edition.

** To determine n two methods may be used. In the first it is determined from the thermal balance of the nozzle flow:

$$Q = q_1 + q_2 + q_3,$$

where Q is the algebraic sum of the heats participating in the process;
q_1, the combustion heat;
q_2, the association heat;
q_3, the heat removed through the walls.

$$Q = c_v \frac{n-k}{n-1}(T - T_1).$$

In the second method n is determined from the ratio of the theoretical to the real velocity. If in equation (27) we replace k by n, we have

$$\varphi\sqrt{2g_0\frac{k}{k-1}p_iV_i\left[1 - \left(\frac{p_a}{p_i}\right)^{\frac{k-1}{k}}\right]} = \sqrt{2g_0\frac{n}{n-1}p_iV_i\left[1 - \left(\frac{p_a}{p_i}\right)^{\frac{n-1}{n}}\right]},$$

whereby n is determined. (See the paper "Teplovoi raschet raketnogo dvigatelya" (Thermal Calculation of a Rocket Engine), first paper). – Editor's note.

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From equations (4), (27) and (28), the thrust of the rocket will be equal to

$$P=2g_0\varphi^2 f_a \frac{n}{n-1} p_i \left[1-\left(\frac{p_a}{p_i}\right)^{\frac{n-1}{n}}\right]\left(\frac{p_a}{p_i}\right)^{\frac{1}{n}}. \tag{36}$$

To obtain this formula, the quantities f_x and p_x were replaced in formula (28) by their values f_a and p_a at the final nozzle cross section.

Prescribing the rocket's thrust P it is possible to determine f_a from (36); next we determine w_a and V_a by (27) and (34) and then the propellant flow rate G_s from (26). Equations (29), (30) and (31) give p_k, w_k and f_k for the narrowest section of the nozzle. From (27), (28), (33) and (34) we can also determine w_x, f_x, T_x and V_x for any pressure. Designing, as in the previous case, the expanding section of the nozzle in the form of a cone, we can determine the friction and heat loss through the nozzle walls for any section, as will be shown later, and then the total friction loss and the total amount of heat escaping through the walls up to a given section.

Between the coefficient φ and the relative kinetic energy loss ξ due to friction we have the relation

$$\varphi=\sqrt{1-\xi}, \tag{37}$$

which may be derived from equations (25) and (8). Thus, for a second, more exact calculation, we may determine φ for any section; furthermore, we can determine for any section the polytropic exponent*.

Dividing the expansion curve into sections, for which φ and n are almost constant, we can determine more exactly the expansion curve. If necessary, we may obtain similarly a third approximation.

FRICTION WITH THE WALLS

The heat, corresponding to the friction work in a rocket engine, is expressed by

$$z=A\int \frac{U\xi}{4F}\frac{w^2}{2g_0}\,dl \text{ cal/kg} \tag{38}$$

with ξ, a coefficient varying from 0.02 to 0.03 for water vapor on smooth walls;
- U, perimeter of nozzle cross section;
- F, cross section area;
- dl, length differential measured along nozzle generator;
- A = 1:427, thermal equivalent of work.

For a circular cross section we have

$$\frac{U}{4F}=\frac{1}{d},$$

where d is the cross section diameter.

* See previous footnote.

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In this case we obtain

$$z = A\int \xi \frac{w^2}{2g_0 d}\, dl = \frac{A\xi}{2g_0}\int \frac{w^2}{d}\, dl. \tag{39}$$

If the walls are very smooth, we obtain, taking $\xi = 0.02$,

$$z = \frac{0.02}{2\cdot 9.81\cdot 427}\int \frac{w^2}{d}\, dl = 2.39\cdot 10^{-6}\int \frac{w^2}{d}\, dl. \tag{40}$$

If we compare two similar rockets of different dimensions, having identical state curves on the iS diagram, but different consumption rates G_s, the nozzle cross sections areas f will be proportional to G_s. The diameters of the nozzle cross sections of two such rockets will be in ratio $\sqrt{f}:\sqrt{f_1}$ or $\sqrt{G_s}:\sqrt{G_{s1}}$. Equation (39) shows that the friction work for the same length dl will be in ratio $d_1 : d = \sqrt{G_{s1} : G_s}$. For identical cone angles α for both rockets, the total length of the nozzles of the two rockets will be in the ratio of their cross section diameters, and, therefore, we have in this case

$$\frac{dl_1}{dl} = \frac{d_1}{d},$$

and, consequently, for such nozzles the friction work per kg of propellant will be the same.

We may therefore say that for large rockets the relative friction loss will be in general the same as for small rockets if the initial and final states of the gases are identical.

(A rough calculation of an example for a hydrogen-oxygen rocket follows in the manuscript of Tsander. It has been carried out by the methods of the present article, taking into account friction losses. The results of this calculation were given by Tsander in his book "Problema poleta pri pomoshchi reaktivnykh apparatov" (Problems of Flight with the Aid of Jet-Propulsion Machines), Chapter 5.)

RESULTS OF CALCULATIONS FOR HEAT FLOW RATE THROUGH THE COMBUSTION CHAMBER'S WALLS

Let us assume that we have already calculated the temperature, density, pressure, consumption rate and velocity of the gases for the case of completely heat-proof walls.

We also assume that the gas temperature decrease in the chamber due to heat losses through the walls is not very significant. In this case we can determine the temperature of the walls and the heat exchange with them, if we know the initial temperature and velocity of the cooling liquid or, when cooling by gases, their initial temperature, density, consumption and velocity. If the calculated temperature decrease of the hot gases is considerable we may say that due to the too large temperature difference between the gases and the cooling medium, the actual temperature decrease will be smaller than calculated. Starting from the results just obtained we must perform a second calculation, which will give us new, more exact results; these may be improved still further.

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When choosing the design of the cooling system, the cooling method is of particular importance; the walls can be cooled by letting the coolant flow along the walls in the same direction as the combustion products or opposite to them. It can be passed in a spiral around the engine walls, and it is also possible to cool the most dangerous places by one medium (e. g., a liquid) and other places by another medium (e. g., gases).

In order that the cooling liquid (or gas) should come into contact with the walls near the critical section of the nozzle, where cooling is most required, we take the direction of coolant flow coincident with that of gas flow in the nozzle; this also simplifies the calculation.

Next, we assume that the velocity of the cooling medium is constant. This may be achieved by a suitable gap between the jacket and the combustion chamber.

At the beginning of the thermal calculation we neglected the amount Q_s of heat given to the rocket walls per kg of propellant mixture (see e. g., (24)). Let us now calculate it.

If the cooling medium is also a gas, it is possible to write for it a formula, analogous to (24), where, however, $-Q_s$ has to replace Q_s and, furthermore, the velocity w_1 of the cooling medium with respect to the rocket cannot be neglected. Using subscript a for quantities referring to the gaseous coolant, we have

$$\frac{i_{1a}-i_{2a}}{m_a}=\frac{A_1\left(w_{2a}^2-w_{1a}^2\right)}{2g_0}-Q_s. \tag{41}$$

In case we take liquid oxygen as the cooling medium, we could first evaporate it holding it under a pressure of 20 atm (temperature of 141 °C), then heat it in the gaseous state and finally, use it in the engine for combustion. We can also cool by water, using its latent heat of evaporation, and then recondense the water vapor by cooling it with liquid oxygen, liquid hydrogen or any other propellant with low boiling temperature. To evaluate the cooling effect attainable with liquid oxygen, we determine the number of calories which can be taken away by 1 kg of liquid oxygen. It is equal to the latent evaporation heat, i. e., 51 cal/kg, if the oxygen is at atmospheric pressure and is transformed into vapor by heating.

For water vapor, we easily obtain the number of calories from the *iS* diagram; if the heating takes place under constant pressure, we follow the curve p = const from the initial water temperature t_1 to the final temperature t_2 chosen by us, and calculate from the diagram the enthalpy difference i_2-i_1*. Water vapor can, of course, be liquefied again, whereas oxygen and hydrogen (or other materials) will best be used in the combustion process after heating.

We determine the heat, passing through the walls of the rocket engine, using Nusselt's formula

$$\alpha_1=19.23\frac{\lambda_W}{d^{1.786}}\left(\frac{G_s c_p}{\lambda}\right)^{0.786}=15.9\frac{\lambda_W}{d^{0.214}}\left(\frac{w c_p}{\lambda}\right)^{0.786}\text{cal kg/m}^2\text{ hour °C}$$

* Schüle, W. Technische Thermodynamik, 4th edition, Vols. 1 and 2. 1923. Tables III and IVa for water vapor, Table IIa for atmospheric air and diatomic gases.

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with λ_w, heat conductivity of gas at temperature of pipe walls;
λ, heat conductivity of gas at average temperature in pipe;
w, average velocity of gases, m/sec;
c_p, heat capacity of gas per m^3 at constant pressure and for given state of gas in pipe;
d, pipe diameter, m;
G_s, weight of gas passing through pipe per sec.

Let us determine the heat loss for the combustion chamber assuming duraluminum walls. We shall take fast flowing water as coolant, an area of $F = 1.965\ m^2$, a consumption per second of $G_s = 3.33$ kg/sec and hydrogen and oxygen as propellants.

It should be noted that if the temperature of the walls' inner side is lower than that of saturated steam (for $p = 20$ atm, $t_{sat} = 211.33°C$), partial liquefaction of the gas may occur. In this case, α_i will probably be considerably larger than the value obtained from Nusselt's formula.

Omitting all calculations and denoting by ξ the percent of heat lost through the walls, i.e.,

$$\xi = \frac{100q}{q_0},$$

where q_0 is the enthalpy of the amount of propellant consumed per sec, and q, the amount of heat lost by the gases per sec, we obtain for duraluminum walls $\xi = 0.39\%$, and the wall temperature is found to be about +100°C.

Since strong mixing takes place in the combustion chamber, it is better to assume that α_i increases towards the beginning of the pipe. For this case $\xi = 0.63\%$.

Let us now determine the heat transfer through the walls of the same combustion chamber if they are made of steel and have an internal refractory lining of 2 mm thick asbestos*. The coolant will again be water. The value of α_i will be inconsiderable in this case. Omitting as before all calculations, we obtain the percentage of heat loss, $\xi = 0.97\%$, and for the increased value of α_i at the beginning of the pipe, $\xi = 1.207\%$. The asbestos will then reach a temperature of 2500°C, whereas it melts at 1500°C. One must, therefore, use an insulation material with a large coefficient λ (according to Hütte for asbestos $\lambda = 0.186$ for 300°C and 0.180 for 200°C), or decrease still further the thickness of the lining. Nonuniform thickness resulting from the production process creates a danger of local overheating and, eventually of melting the lining.

Thus, it is better to build the combustion chamber without a refractory lining. However, in this case the temperature of the walls should be such as to eliminate the possibility of liquefying the super-heated steam in the combustion products, since otherwise it is very likely that the amount of heat lost will increase considerably. This is particularly important for a hydrogen-oxygen rocket. The layer of cooling water should be of such a thickness that it becomes heated, as it traverses the pipe, from 0 to 100°C. The higher the water velocity, the smaller the amount required. However,

* Asbestos cannot in general be considered a refractory for combustion chambers of rocket engines, since it has a relatively low melting point; what is most important – it crumbles at 800°C. The results of Tsander's calculations are of relative interest; they show how low heat conductivity of the chamber's material affects the walls' temperature. – Editor's note.

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at high water velocities, friction losses may attain such a magnitude that it becomes necessary to feed it by means of a pump. For low velocities gravity feeding may be sufficient. Calculations I performed show that quite probably it will be possible to use gravity feeding to start the water motion.

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CALCULATION OF THE EXPERIMENTAL ROCKET ENGINE ER-1*

Tsander dealt not only with theoretical investigations of rocket engines and their working processes. He also spent a good deal of time on pure engineering work, including calculations of experimental engine models, organization of tests with them and evaluation of the experimental data.

In order to give an idea of this aspect of Tsander's activity, we give below the most characteristic extracts from his calculation of rocket engine model ER-1 (15 of the 145 pages of his manuscript), which he built in 1929. The calculations were begun personally by Tsander in October 1929**.

A range for testing jet engines was just started in these years and the material possibilities were very poor. Therefore, according to present-day scale, the dimensions of the engine chosen were very small (the rate of consumption of fuel and oxidizer was 1.69 g/sec). For simplicity, atmospheric air was chosen as the oxidizer. Nevertheless, the diagram, the method of calculation, and the testing of the engine are characteristic of liquid rocket engines.

Subsequently, developing the idea of the use of atmospheric air for improving the thermodynamic cycle of liquid rocket engines, Tsander arrived at an engine scheme with direct and inverted cones (see his article, Problema poleta pri pomoshchi reaktivnykh apparatov (Problems of Flight with the Aid of Reactive Devices)).

The ER-1 engine was constructed and more than 50 tests were carried out with it. The following modifications of liquid rocket engines were completed by Tsander's pupils after his death.

Editor

BASIC DATA OF THE ER-1 ENGINE

The engine was based on a gasoline-air burner with the following parameters:

Capacity of gasoline tank	~ 1 liter;
Fuel consumption	350–400 g/hour
Air pump:	
Piston diameter	16 mm;
Stroke	107 mm;
Outlet nozzle bore diameter	22 mm.

* This article was prepared for print by E. S. Shchetinkov, Dr. of Technical Sciences. – Editor's note.

** This date was taken from the manuscript. Formulas connected with the calculation of the engine ER-1 are met, however, in the stenographical records of Tsander of 1922-1929 (not yet deciphered). A more exact date of the performance of this calculation will be established after deciphering his works.

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The gasoline was heated in a copper tube with an external diameter of 8 mm.

The nozzle was changed: it was surrounded by an envelope into which air was injected under pressure. Using a special pipe, a combustion chamber was constructed within the envelope. An interchangeable nozzle, giving flow velocities larger than the sound velocity, was mounted at the end of this pipe. The copper tube for heating the gasoline was replaced by a longer one, screwed into a new conical nozzle.

The tank was equipped with a manometer and a nipple to let out the air. A thermometer for measuring the temperature of the tank cover was attached to it by a special holder, and a safety wire was soldered to the tank cover by its upper end. A valve for regulating the fuel consumption was available.

For further calculations, we take, on basis of preliminary tests with the burner:

gasoline consumption $Q = 400$ g/hour for an overpressure of

$$p_{over} = 2 \text{ atm};$$

flame length, measured from the outlet orifice of the nozzle

$$L = 14 \text{ cm};$$

theoretical air consumption per kg of gasoline

$$G_{air} = 14.2 \text{ kg};$$

capacity of gasoline tank 1 liter; specific weight of gasoline 0.720 kg/liter.

We then obtain:

gasoline consumption per second: $\frac{Q}{3600} = \frac{0.400 \cdot 10^3}{3600} = \frac{1}{9}$ g/sec;

air consumption per second: $G_{air} = 14.2 \cdot \frac{Q}{3600} = \frac{14.2}{9} = 1.58$ g/sec;

air consumption per hour: $14.2 \cdot Q = 14.2 \cdot 0.4 = 5.69$ kg/hour;

duration of combustion:

$$\frac{1 \cdot 720 \cdot 9}{60} = 108 \text{ min} = 1 \text{ hour } 48 \text{ min};$$

amount of combustion products, per second and per hour:

$$0.111 + 1.58 = 1.691 \text{ g/sec or } 0.4 + 5.69 = 6.09 \text{ kg/hour}.$$

The power which an internal combustion engine would give for this rate Q of gasoline consumption and for a specific consumption of 240 g/eff. hp-hr, is

$$\frac{400}{240} = 1.67 \text{ eff. hp}.$$

The power of the propeller of an internal combustion engine for a propeller efficiency of $\eta_{prop} = 0.75$ is

$$N_{prop} = \eta_{prop} N_{eng} = 0.75 \cdot 1.67 = 1.25 \text{ eff. hp}.$$

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The duration of operation of the compressed air vessel with a capacity of 20 liter at a pressure of 150 atm, i. e., of $\frac{20 \cdot 150}{1000} = 3\,m^3$, for an air pressure of 760 mm of mercury, a temperature of $t = 15°$ C and a specific air weight of $\gamma_0 \approx 1.23\ kg/m^3$, is

$$\frac{1.23 \cdot 3000}{1.58 \cdot 60} = 38.9\ \text{min}.$$

The number of compressed air vessels required for the consumption of the gasoline in the tank of the rocket engine, is

$$\frac{108}{38.9} = 2.78,\ \text{i. e. } 3$$

and when oxygen is used: $2.78 \cdot 0.209 = 0.58$.

DETERMINATION OF THE COMBUSTION TEMPERATURE AT CONSTANT PRESSURE

The lower calorific value of gasoline is:

$$H_u = 10\,400\ \text{cal/kg}.$$

The composition of gasoline is:

hydrogen $H = 14.5\%$;

carbon $C = 85.5\%$.

For complete combustion of 1 kg of gasoline the combustion products are:

$$\text{Water } (H_2O)\ \frac{H}{2} = 0.0725\ \text{mole} = 0.0725 \cdot 18 = 1.306\ \text{kg};$$

$$\text{Nitrogen } (N_2)\ 0.79\,L_0 = 0.79\,\frac{14.2}{28.95} = 0.79 \cdot 0.490 = 0.388\ \text{mole};$$

$$\text{Carbon dioxide } (CO_2)\ \frac{C}{12} = \frac{0.855}{12} = 0.0712\ \text{mole}.$$

Altogether $M_2 = 0.5317$ mole $= 15.2$ kg.

The number of kilogram-moles per kg of gasoline before combustion is

$$M_1 = L_0 + b_V = \frac{14.2}{28.95} + \frac{1}{100} = 0.500\ \text{mole},$$

taking the molecular weight of gasoline equal to 100.

The coefficient of molecular variation is:

$$\beta_m = \frac{M_2}{M_1} = \frac{0.5317}{0.500} = 1.0634.$$

The difference between the molecular specific heats at constant pressure and constant volume is equal to

$$AR = \frac{848}{427} = 1.985 = \mu\,(c_p - c_v).$$

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The average molecular specific heats at constant pressure for gases (see Professor Brilling, Teplovoi raschet aviatsionnogo dvigatelya (Thermal Calculation of an Aviation Engine), pp. 39-40, 1927), are, for a temperature T between 300° C and 750° abs:

for diatomic gases $(\mu c_p)_m = 1.985 + 4.605 + 5.167 \cdot 10^{-4}T = 6.590 + 5.167 \cdot 10^{-4}T$;
for CO_2 $(\mu c_p)_m = 1.985 + 5.035 + 32.5 \cdot 10^{-4}T = 7.020 + 32.5 \cdot 10^{-4}T$;
for H_2O $(\mu c_p)_m = 1.985 + 6.08 + 4.5 \cdot 10^{-4}T = 8.065 + 4.5 \cdot 10^{-4}T$.

For temperatures T between 1600° and 2100° abs:

for diatomic gases $(\mu c_p)_m = 1.985 + 4.605 + 5.167 \cdot 10^{-4}T = 6.590 + 5.167 \cdot 10^{-4}T$;
for CO_2 $(\mu c_p)_m = 1.985 + 7.565 + 10.2 \cdot 10^{-4}T = 9.550 + 10.2 \cdot 10^{-4}T$;
for H_2O $(\mu c_p)_m = 1.985 + 4.685 + 15.2 \cdot 10^{-4}T = 6.670 + 15.2 \cdot 10^{-4}T$.

The average heat capacity of the M_2 mole after combustion is:

for N_2 $0.388(6.59 + 5.167 \cdot 10^{-4}T) = 2.555 + 2.006 \cdot 10^{-4}T$;
for CO_2 $0.0712 \cdot (9.55 + 10.2 \cdot 10^{-4}T) = 0.679 + 0.726 \cdot 10^{-4}T$;
for H_2O $0.0725 \cdot (6.67 + 15.2 \cdot 10^{-4}T) = 0.484 + 1.102 \cdot 10^{-4}T$.
Altogether $A_2 + B_2T = 3.718 + 3.834 \cdot 10^{-4}T$.

The average heat capacity of the M_1 mole before combustion is:

for L_0 $0.490 \cdot (6.59 + 5.167 \cdot 10^{-4}T) = 3.23 + 2.535 \cdot 10^{-4}T = A_1 + B_1T$.

For $T = T_0 = 288°$ we obtain

$$A_1 + B_1T_0 = 3.23 + 2.535 \cdot 10^{-4}288 = 3.303.$$

For comparison, for $c_p = 0.238$ we obtain

$$A_1 + B_1T_0 = L_0 \cdot c_p = 28.95 \cdot 0.490 \cdot 0.238 = 3.38.$$

Since $T_0 < 300°$, we shall adopt the latter value.

For the theoretical combustion temperature we have

$$H_u + (A_1 + B_1T_0)T_0 = (A_2 + B_2T_z)T_z.$$

$$T_z^2 + \frac{3.718 \cdot 10^4}{3.834}T_z - \frac{10\,400 + 3.38 \cdot 288}{3.834 \cdot 10^{-4}} = T_z^2 + 9698T_z - 2966 \cdot 10^4 = 0.$$

$$T_z = -4849 + \sqrt{4849^2 + 29.66 \cdot 10^6} = -4819 + 7290 = 2441°.$$

Since, for complete combustion $T_z > 2100°$, let us repeat the calculation for the specific heats in the range $T = 2000$-$3000°$ abs.

For diatomic gases $(\mu c_p)_m = 6.59 + 5.167 \cdot 10^{-4}T$;
for CO_2 $(\mu c_p)_m = 10.16 + 7.333 \cdot 10^{-4}T$;
for H_2O $(\mu c_p)_m = 5.28 + 21.65 \cdot 10^{-4}T$.

The average heat capacity of the M_2 mole after combustion is:

for N_2 $0.388 \cdot (6.59 + 5.167 \cdot 10^{-4}T) = 2.555 + 2.006 \cdot 10^{-4}T$;
for CO_2 $0.0712 \cdot (10.16 + 7.333 \cdot 10^{-4}T) = 0.723 + 0.522 \cdot 10^{-4}T$;
for H_2O $0.0725 \cdot (5.28 + 27.65 \cdot 10^{-4}T) = 0.3825 + 1.57 \cdot 10^{-4}T$

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Altogether $A_2+B_2T=3.661+4.098\cdot 10^{-4}T$.
For the combustion temperature we have

$$T_z^2+\frac{3.661}{4.098}\cdot 10^4\cdot T_z-\frac{11\,375\cdot 10^4}{4.098}=T_z^2+8936\cdot T_z-2775\cdot 10^4=0.$$

$$T_z=-4468+\sqrt{4468^2+27.75\cdot 10^6}=-4468+6910=2442°.$$

As we see, the difference is negligible (2442°-2441° = 1°). Since we want to obtain a velocity close to the critical one at the outlet section of nozzle No. 1, we obtain for the approximate value of the initial pressure p_z, assuming an adiabatic expansion with $k=1.3$,

$$\frac{p_0}{p_z}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}=\left(\frac{2}{2.3}\right)^{\frac{1.3}{0.3}}=\beta.$$

$$\lg\beta=\frac{1.3}{0.3}\cdot\lg\frac{2}{2.3}=\frac{1.3}{0.3}\cdot\lg 0.8696=-\frac{1.3}{0.3}\cdot 0.06068=$$

$$=-0.2629=\bar{1}.7371=\lg 0.5459;$$

$$p_z\cong p_0:0.546.$$

For an adiabatic expansion, the temperature T_0 at the exit section is

$$T_0=T_z\beta^{\frac{k-1}{k}}=T_z\frac{2}{k+1}=2442\cdot\frac{2}{2.3}=2123°.$$

The average temperature of the gases in the nozzle is

$$T_{av}=\frac{T_0+T_z}{2}=\frac{2442+2123}{2}=2283°.$$

The adiabatic exponent is

$$k=\frac{c_p}{c_v}=1+\frac{AR}{\mu c_v}.$$

$$\mu c_p=\frac{1}{M_2}(A_2+2B_2T)=\frac{3.661}{0.5317}+\frac{2\cdot 4.098}{0.5317}\cdot 10^{-4}T=$$

$$=6.888+15.42\cdot 10^{-4}T.$$

$$k=1+\frac{1.985}{4.903+15.42\cdot 10^{-4}T}=1+\frac{1.985}{4.903+15.42\cdot 0.2283}=1.236.$$

By a second substitution we obtain

$$T_0=2442\cdot\frac{2}{2.236}=2184°;\quad T_{av}=\frac{2442+2184}{2}=2313°;$$

$$k=1+\frac{1.985}{4.903+15.42\cdot 0.2313}=1.235.$$

Therefore, finally

$$T_0=2442\cdot 0.895=2185°;\quad \beta^{\frac{k-1}{k}}=\frac{2}{2.235}=0.895;$$

$$\lg\beta=\frac{1.235}{0.235}\cdot\lg 0.895=-\frac{1.235}{0.235}\cdot 0.04818=\bar{1}.7469=\lg 0.5583;\ \beta=0.5583.$$

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The initial pressure for an adiabatic expansion is exactly

$$p_z = \frac{p_0}{\beta} = \frac{1.033}{0.5583} = 1.855 \backsim 1.86 \text{ kg/cm2}.$$

The thermal efficiency for constant heat capacities is

$$\eta_t = 1 - \frac{T_0}{T_z} = 1 - 0.895 = 0.105.$$

The thermal efficiency, assuming variable heat capacities is

$$\eta_t = \frac{(\mu c_{pz})_m T_z - (\mu c_{p0})_m T_0}{(\mu c_{pz})_m T_z} = 1 - \frac{(\mu c_{p0})_m T_0}{(\mu c_{pz})_m T_z};$$

$$(\mu c_p)_m = 6.888 + 7.71 \cdot 10^{-4} T;$$

$$(\mu c_{pz})_m = 6.888 + 7.71 \cdot 0.2442 = 8.771;$$

$$(\mu c_{p0})_m = 6.888 + 7.71 \cdot 0.2185 = 8.575;$$

$$\eta_t = 1 - \frac{8.575}{8.771} \cdot 0.895 = 1 - 0.979 \cdot 0.895 = 1 - 0.875 = 0.125.$$

The theoretical exhaust velocity is

$$w_1 = \sqrt{2g \frac{k}{k+1} p_z V_z};$$

the average molecular weight of the combustion products being

$$\mu_2 = \frac{14.2 + 1}{M_2} = \frac{15.2}{0.5317} = 28.59.$$

The gas constant of the combustion products is

$$R_2 = \frac{848}{28.59} = 2967 \text{ kgm}.$$

The specific volume of the combustion products at the pressure $p_z =$ $= 1.86 \text{ kg/m}^3$ and the temperature $T_z = 2442°$ abs is

$$V_z = \frac{R_z T_z}{p_z} = \frac{29.67 \cdot 2442}{1.86 \cdot 10^4} = 3.90 \text{ m}^3/\text{kg}.$$

Then, the theoretical exhaust velocity is equal to

$$w_1 = \sqrt{2 \cdot 9.81 \cdot \frac{1.235}{2.235} \cdot 29.67 \cdot 2442} = \sqrt{78.7 \cdot 10^4} = 887 \text{ m/sec}.$$

Assuming that 5 % of the velocity or 10 % of the kinetic energy is lost by friction in the nozzle, the practical exhaust velocity is

$$w = \varphi w_1, \ \varphi = 0.95, \ w = 0.95 \cdot 887 = 843 \text{ m/sec}.$$

The axial thrust of the rocket engine is

$$P_a = \frac{G}{g}; \quad w = \frac{1.691}{9.81} 843 = 145.2 \text{ gram}.$$

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The power N at a velocity «c» is

$$N=\frac{P_a c}{75}=\frac{0.1452c}{75}=\frac{1.935c}{1000}\ \text{hp}.$$

The apparent mechanical efficiency referred to the kinetic energy of the relative motion of the gases is

$$\eta_\kappa=\frac{N}{N_{rel}};\quad N_{rel}=\frac{sw^2}{75\cdot 2g};$$

$$\eta_\kappa=\frac{Gws}{75g};\quad \frac{Gw^2}{75\cdot 2g}=\frac{2c}{w}.$$

The total efficiency is

$$\eta_{tot}=\eta_t\cdot\eta_\kappa\cdot\varphi^2.$$

Only at low velocities, the coefficient η_κ is the true efficiency, since at high velocities a part of the kinetic energy that the fuel possesses by virtue of its velocity c is transformed into useful work.

Velocity c m/sec	3.89	13.9	27.8	55.6	843
Velocity c cm/hour	14	50	100	200	3030
Power N, hp	0.0075	0.027	0.054	0.107	1.63
Apparent mechanical efficiency η_κ	0.0094	0.033	0.066	0.132	2.0
Total efficiency η_{tot}	0.00104	0.0037	0.0074	0.0149	0.226

The equipment is suitable for experiments for determining the combustion temperature chamber and that of the nozzle. For practical purposes it is required:

1) to increase the initial air pressure;

2) to construct a device for sucking in external air to reduce the average outlet velocity;

3) to increase the fuel consumption per unit time. For fast rockets condition 1 has to be fulfilled; for motion on the Earth, on water, on snow or ice and for airplane models – conditions 1 and 2; for attaining high power – condition 3 too.

The present experimental equipment is also suitable, without modifications, for atomizing and burning molten metal alloys (for example, of Elektron) in a jet of combustion products with a surplus of air, since the temperature of the outgoing gases is very high.

THE NOZZLE OF THE EXPERIMENTAL ROCKET ENGINE ER-1

The nozzle is cooled by the pipe which serves for heating the gasoline and heat escapes from it also by radiation and conduction by the air.

Let us calculate the amount of heat needed to heat the gasoline. We take the specific heat of gasoline, as for kerosene, equal to $c = 0.50$ cal/kg degree; the gasoline consumption $Q = 400$ g/hr; we calculate the temperature of the

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heated gasoline, approximately equal to the temperature of the saturated vapors under the pressure prevailing in the gasoline tank, by the formula

$$\ln p_1 = \ln p_2 - \frac{\mu r}{2} \frac{(T_2 - T_1)^*}{T_2 T_1}$$

where p_1 and T_1 are the required pressure and temperature;
p_2 and T_2, the initial pressure and temperature;
r, the latent heat of evaporation;
μ, the apparent molecular weight of the fuel.

$$\mu = 107; \quad r = 74; \quad p_2 = \frac{100}{735.5} = 0.136\,\text{atm}; \quad T_2 = 273 + 51 = 324°$$

For a pressure p_z of 1.86 atm, in the combustion chamber and for p_{over} = 2 atm we have:

$$p_1 = p_z + p_{over} = 3.86\,\text{atm}.$$

We then obtain

$$\frac{2}{\mu r} \ln \frac{p_2}{p_1} = \frac{T_2 - T_1}{T_2 T_1}; \quad T_2 - T_1 = \left(\frac{2T_2}{\mu r} \ln \frac{p_2}{p_1}\right) T_1;$$

$$T_1 = \frac{T_2}{\frac{2T_2}{\mu r} \ln \frac{p_2}{p_1} + 1} = \frac{T_2}{\frac{4.605 T_2}{\mu r} \lg \frac{p_2}{p_1} + 1},$$

and in our case

$$T_1 = \frac{324}{1 - \frac{4.605 \cdot 324}{107 \cdot 74} \lg \frac{3.86}{0,136}} = \frac{324}{1 - 0.1885 \cdot 1.454} = \frac{324}{0.726} = 446°\,\text{abs};$$

$$t_1 = 446° - 273° = 173°\,\text{C}.$$

The rate of heat inflow, required to heat the gasoline from t_0 = 25° C to t_1 = 173° C during one hour is

$$q_t = cQ(t_1 - t_0) = 0.50 \cdot 0.400 \cdot (173 - 25) = 29.6\,\text{cal/hour}.$$

The rate of heat inflow required to evaporate the gasoline during one hour is

$$q_r = rQ = 74 \cdot 0.4 = 29.6\,\text{cal/hour}.$$

Up to complete evaporation the total rate of heat inflow required is

$$q = q_t + q_r = 29.6 + 29.6 = 59.2\,\text{cal/hour}.$$

The dimensions of the nozzle are:
Initial diameter d_4 = 2.2 cm;
Final diameter d_5 = 0.40 cm;
Inclination angle of internal surface α = 10°31';
Nozzle length

$$l = \frac{d_4 - d_5}{2} \operatorname{ctg} \alpha = \frac{2.2 - 0.4}{2} \operatorname{ctg} 10°31' = 4.85\,\text{cm}.$$

We shall now calculate the coefficient of heat transfer from the gases to the walls. For this purpose we shall use everywhere approximate average values.

* See Brilling. Issledovanie aviamotornykh topliv (Investigation of Aviation Fuels). 1922.

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The average temperature of the gases in the nozzle is

$$T_{av}=\frac{2442+2185}{2}=2314° C;$$

$$t=2314°-273°=2041° C.$$

The average velocity of the gases in the nozzle is

$$W_{av}=\frac{17.3+843}{2}=430 \text{ m/sec}.$$

The average pressure of the gases in the nozzle is

$$p_{av}=\frac{1.86+1.0}{2}=1.43 \text{ atm}.$$

The average internal diameter of the nozzle (along the span) is

$$d_{av}=\frac{2.2+0.4}{2}=1.3 \text{ cm}.$$

The nozzle area is

$$F=\frac{\pi}{2}(2.2+0.4)\,4.85=19.81 \text{ cm}^2.$$

We therefore obtain the heat transfer coefficient at the temperature of the gases

$$\lambda=0.01894\cdot(1+0.00228\cdot 2041)=0.1070 \text{ kcal/m hour°C},$$

the specific weight of the gases

$$\gamma=\frac{p}{RT}=\frac{1.43\cdot 10^4}{29.67\cdot 2314}=0.208 \text{ kg/m}^3,$$

the heat content of the gases at constant pressure

$$b=\frac{\gamma}{\mu}\cdot(6.888+15.42\cdot 10^{-4}T)=\frac{0.208}{28.59}\cdot(6.888+$$
$$+15.42\cdot 0.2314)=0.0761 \text{ cal/m}^3,$$

and

$$\lg d^{0.214}=0.214\lg 0.013=-0.214\cdot\bar{1}.886=-0.404=\lg 0.394;$$
$$d^{0.214}=0.394.$$

The coefficient of heat transfer from the gases to the nozzle walls is

$$\alpha_1=15.9\frac{0.01894\cdot(1+0.00228\cdot\vartheta_1)}{0.394}\cdot\left(\frac{430\cdot 0.0761}{0.107}\right)^{0.786} 0.786\cdot\lg 306=$$
$$=0.786\cdot 2.486=1.955=\lg 90;$$
$$\alpha_1=68.8(1+0.00228\cdot\vartheta_1).$$

The amount of heat, passing through the nozzle walls per hour, taking

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a wall temperature equal to $t_1 = 173° = \vartheta_1$, is

$$q_1 = \alpha_1 F(t - t_1) =$$
$$= 68.8 \cdot (1 + 0.00228 \cdot 173) \cdot 19.81 \cdot 10^{-4} \cdot (2041 - 173) = 355 \text{ cal/hour.}$$

We see that the amount of heat required for heating and evaporating the gasoline is equal to $\frac{59.2 \cdot 100}{355} = 16.7\%$ only of the total amount of heat passing through the walls.

At the average nozzle wall temperature, the rate of radiation q_1 cal/hour is

$$q_1 = \frac{F\left[\left(\frac{\theta_1}{100}\right)^4 - \left(\frac{\theta_0}{100}\right)^4\right]}{\frac{1}{b_1} + \frac{1}{b_2} - \frac{1}{b}},$$

where $b = 4.61$ is the radiative coefficient for an absolutely black body;
$b_1 = 4.40$, the radiative coefficient for iron with oxidized, dull surface;
$b_2 \approx 4.61$, the radiative coefficient of the surrounding medium, assuming it is empty space.

Taking $\theta_0 = 288°$, we obtain

$$\left(\frac{\theta}{100}\right)^4 = \frac{q_1}{b_1 F} + \left(\frac{\theta_0}{100}\right)^4 = \frac{355 \cdot 10^4}{4.40 \cdot 19.81} + 2.88^4 = 40\,700 + 68 = 40\,770;$$
$$\theta = 1420°; \quad \vartheta = 1420 - 273 = 1147°.$$

Repeating this calculation for $\vartheta_1 = 1147°$, we obtain

$$\frac{q_1}{F} = \alpha_1 (t - t_1) = 68.8 \cdot (1 + 0.00228 \cdot 1147) \cdot (2041 - 1147) =$$
$$= 68.8 \cdot 3.62 \cdot 894 = 222\,000 \text{ cal/hour m}^2$$
$$q_1 = 222\,000 \cdot \frac{19.81}{10^4} = 440 \text{ cal/hour}$$

hence

$$\left(\frac{\theta}{100}\right)^4 = \frac{222 \cdot 10^3}{4.40} + 70 = 50570; \quad \theta = 1500°;$$
$$\vartheta = 1500° - 273° = 1227°.$$

In the presence of radiation only the temperature would rise higher than $\vartheta = 1227°C$.

Taking into account also heat transfer by the air, we have, according to the experiments of Valiler and Ginlein on the transfer from a horizontal pipe to quiet air

$$\alpha_2 = 1.02 \sqrt[4]{\frac{(\vartheta_2 - t_0)}{d}},$$

where d is the pipe diameter in meters,

$$\sqrt[4]{0.013} = \sqrt{0.114} = 0.338;$$
$$\alpha_2 = \frac{1.02}{0.338} \sqrt[4]{\vartheta_2 - t_0} = 3.02 \sqrt[4]{\vartheta_2 - t_0}.$$

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For

$$\vartheta_2 = \vartheta = 1227° \text{ we obtain:}$$

$$\alpha_2 = 3.02 \sqrt[4]{1227-15} = 3.02 \cdot 5.90 = 17.8,$$

and the heat, carried away per m^2 per hour, is equal to

$$\alpha_2(\vartheta_2 - t_0) = 17.8 \cdot (1227-15) = 21\,600 \text{ cal/hour m}^2,$$

which constitutes approximately 0.1 of the radiated heat. The external surface of the nozzle is somewhat larger than the internal surface and is equal to

$$F_a = \frac{\pi}{2}(2.71 + 0\ 63) \cdot 4.85 = 25.4 \text{ cm}^2.$$

Taking into account all three kinds of cooling, we obtain

$$q = 68.8(1 + 0.00228\vartheta) F(t - \vartheta) =$$

$$= q_s + F_0 b_1 \left[\left(\frac{\vartheta + 273}{100}\right)^4 - \left(\frac{\vartheta_0}{100}\right)^4\right] + 3.02 \sqrt[4]{\vartheta - t_0} F_a (\vartheta - t_0).$$

$$q_1 = 68.8 \cdot 19.81 \cdot 10^{-4} \cdot (2041 - \vartheta) \cdot (1 + 0.00228\vartheta) =$$

$$= 59.2 + 25.4 \cdot 10^{-4} \cdot 4.40 \cdot \left[\left(\frac{\vartheta + 273}{100}\right)^4 - 70\right] +$$

$$+ 25.4 \cdot 10^{-4} \cdot 3.02 \cdot \sqrt[4]{\vartheta - 15} \cdot (\vartheta - 15);$$

$$q_1 = 0.1362(2041 - \vartheta)(1 + 0.00228\vartheta) =$$

$$= 59.2 + 0.01118 \cdot \left[\left(\frac{\vartheta + 273}{100}\right)^4 - 70\right] +$$

$$+ 0.00766 \cdot \sqrt[4]{\vartheta - 15}(\vartheta - 15) = q_s + q_{con} + q_{rad}.$$

TABLE 1

Determination of the nozzle wall temperature

Temperature of the walls ϑ° C	900	1000	1100	1200	800	700
$2041 - \vartheta$	1141	1041	941	841	1241	1341
$1 + 0.00228\vartheta$	3.05	3.28	3.51	3.74	2.825	2.596
q_1 cal/hour	474	465	451	429	479	475
$\vartheta + 273$	1173	1273	1373	1473	1073	973
$\left(\frac{\vartheta+273}{100}\right)^4 - 70$	19 800	26 100	35 200	46 800	13 130	8900
q_{rad}	221	292	394	523	146.9	99.5
$\vartheta - 15$	885	985	1085	1185	785	685
$\sqrt[4]{\vartheta - 15}$	5.46	5.60	5.75	5.88	5.30	5.13
q_{con}	37	42	48	53	31.9	26.9
$59.2 + q_{rad} + q_{con}$	317	393	501	635	238	185.6
$3(q_{rad} + q_{con})$	774	1602	1326	1728	536.4	379.2

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The required temperature is equal to $\vartheta_x = 1065°C$.

The amount of heat, passing then through the walls per hour, is equal to $q_1 = 456$ cal/hour or $\frac{156 \cdot 100}{4160} \approx 11\%$ of the total amount of heat (Figure 1). If the gasoline is heated by radiation and not by direct contact with the nozzle, the wall temperature is increased to $\vartheta_x = 1085°$.

The temperature may be reduced in various ways.

1) Cooling ribs may be added, increasing the radiation surface and the heat transfer to the air.

2) Cooling by compressed air coming out of a high pressure pipe can be arranged, the air current accelerating as the burning gases. For that purpose it is possible to eliminate the external conical envelope over the nozzle and to equip the internal nozzle with slots for balancing the pressures.

3) It is possible to arrange cooling by compressed air which then goes into the combustion tube for burning the fuel.

4) It is possible to mount an external nozzle, in which atmospheric air is sucked in for cooling the main nozzle.

5) It is possible to improve the performance of the rocket by sucking external air into the low pressure space, up to the point where the velocities of the air and the combustion products become equal, subsequently increasing the pressure again ("Tsander's direct and inverted cone").

6) It is possible to design cooling by liquid metal or by fused salt and to equip the metal tank with cooling ribs (using atmospheric air).

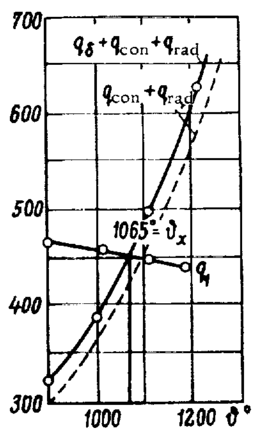

FIGURE 1. Rate of heat transfer through the walls (without cooling ribs)

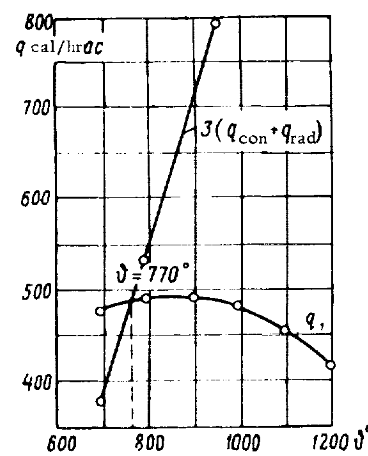

FIGURE 2. Rate of heat transfer with cooling ribs which enlarge threefold the heat exchange surface

7) It is possible to design for water cooling with formation of water vapor, which can then serve for the burning of magnesium.

8) It is possible to reduce in the tests:

a) the initial pressure;

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b) the amount of fuel;

c) the ratio of the amount of fuel to the amount of air or inversely.

9. It is possible to build the nozzle with very thick walls in order to increase the cooling surface and to have a uniform temperature along the whole nozzle, reducing in this way the stresses from the internal pressure.

Figure 2 shows curves of q_1, which has to be removed, and of $3(q_{rad}+q_{con})$, removed by radiation and convection when cooling ribs increasing threefold the cooling surface are added.

We see from the figure that the wall temperature is then equal to $\vartheta = 770°$. The strength of wrought iron K_z at different temperatures in relation to its strength K_{z20} at $t = 20°C$ (K_z/K_{z20}, %) is shown in Table 2 and Figure 3* (according to Hütte, ed. XI, page 537**):

TABLE 2

t	100	200	300	400	500	600	700	800
$K_z : K_{z20}$	104	112	116	96	76	42	25	15

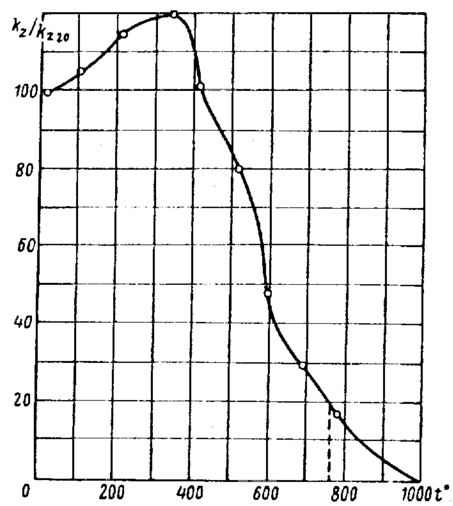

FIGURE 3. Ratio of strength of wrought iron to its strength at 20°C for various temperatures

By taking $K_{z20} = 3300$ kg/cm², we obtain $K_{z770} = 0.18 \cdot 3300 = 596$ kg/cm². At the inlet, for $d_t = 2.2$ cm and $\sigma_z = 60$ kg/cm², the required nozzle wall thickness is equal to

$$\delta = \frac{pd}{2\sigma_z} = \frac{(1.86 \quad 1)\cdot 2.2}{2\cdot 60} = 0.016 \text{ cm}.$$

We take

$$\delta = \frac{27.1 - 22.0}{2} = 2.55 \text{ mm},$$

* [The tabulated and the graphic data are not in complete agreement.]

** [Probably, Russian edition is meant.]

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so that

$$\sigma_z = 60 \cdot \frac{0.0158}{0.255} = 3.7\,\mathrm{kg/cm^2}.$$

The stress at the exit diameter is equal to zero, since the overpressure is equal to zero.

If we assume the overpressure to be $p = 0.86$ atm, then the stress at the exit diameter, equal to $d_5 = 4$ mm, will be

$$\delta = \frac{0.63 - 0.4}{2} = 0.115\,\mathrm{cm};$$

$$\sigma_z = \frac{0.86 \cdot 0.4}{2 \cdot 0.115} = 1.5\,\mathrm{kg/cm^2}.$$

To these stresses we have to add temperature stresses.

The difference between the external and internal temperatures is obtained from

$$\frac{q_1}{F} = \frac{\lambda}{\delta}(\vartheta_1 - \vartheta_2) \quad \text{or} \quad \vartheta_1 - \vartheta_2 = \frac{q_1 \delta}{F\lambda}.$$

In our case

$$q_1 = 478 \ \mathrm{cal/hr}, \quad F \approx \frac{25.4 + 19.8}{2} = 22.6\,\mathrm{cm^2}$$

$$\lambda \approx 33 \text{ for steel.}$$

Then, the temperature difference at the entry to the nozzle is

$$\vartheta_1 - \vartheta_2 = \frac{478 \cdot 10^4 \cdot 0.255}{22.6 \cdot 33 \cdot 100} = 16.3^\circ\,\mathrm{C},$$

and at the outlet from the nozzle

$$\vartheta_1 - \vartheta_2 = \frac{478 \cdot 10^4 \cdot 0.115}{22.6 \cdot 33 \cdot 100} = 7.35^\circ\,\mathrm{C}.$$

The stress due to the nonuniform expansion is equal to

$$\sigma_t = E\varepsilon = E\frac{\Delta d}{d} = E\beta\left(\frac{\vartheta_1 - \vartheta_2}{2}\right) = \frac{E\beta}{2}\frac{q_1\delta}{F\lambda}.$$

Taking for steel: $E = 2.2 \cdot 10^6\,\mathrm{kg/cm^2}$ and $\beta = 11 \cdot 10^{-6}$ we obtain at the nozzle inlet

$$\sigma_t = 2.2 \cdot 10^6 \cdot 11 \cdot 10^{-6} \cdot \frac{16.3}{2} = 197\,\mathrm{kg/cm^2}$$

$$\frac{1}{s} = \frac{\sigma_t + \sigma_z}{k_z} = \frac{197 + 3.7}{596} \approx \frac{1}{3};$$

and at the nozzle outlet

$$\sigma_t = \frac{2.2 \cdot 11}{2} \cdot 7.35 = 89\ \mathrm{kg/cm^2}$$

$$\frac{1}{s} = \frac{\sigma_t + \sigma_z}{k_z} = \frac{89 + 1.5}{596} = \frac{1}{6.6}.$$

We therefore see that at the nozzle inlet a decrease in the thickness δ results in a further decrease in the stress, but the ribs, whose external section is colder, modify the stresses. A more exact calculation for different places in the nozzle and for different wall temperatures gives more exact results.

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USE OF METALLIC PROPELLANT IN ROCKET ENGINES*

In the investigations of Tsander on rocket engines and their application to cosmic flights, the idea of the use of metal as a propellant, which he proposed for the first time already in 1909, is stressed throughout the many years of his activity in this field.

Tsander's idea of the use of metallic propellant for rocket engines is based on the assumption that it can be exploited in two ways: a) it increases the exhaust velocity of the combustion products of liquid propellants by burning them together with metals having a high calorific value (e. g., aluminum, magnesium, lithium, berilium, and others), in the form of a colloidal solution of the metal in the liquid propellant; b) it increases the mass fraction of fuel in the rocket since the metallic sections of the structure are used as propellants when they are no longer required for flight or landing.

The idea of using sections of the rocket's structure as propellants is extremely important with modern propellants; these have a relatively low exhaust velocity, due to their comparatively low calorific value, and therefore the mass fraction of propellant required in order to attain high altitudes, even within the limits of the atmosphere, is so high that the practical realization of such a rocket becomes impossible.

This idea of Tsander's constitutes an original solution to the problem of constructing rockets, in which the total amount of propellant may reach up to 90% of the initial weight of the rocket; thus flights to altitudes inaccessible at the present time would be feasible.

By using stage rockets, in which the rocket sections which have finished their job are discarded, the ratio of the initial to the final mass can also be increased. However, stage rockets lag far behind rockets which burn part of their structures, since in these the energy stored for the same initial weight is considerably larger than in stage rockets due to the additional burning of the metal.

Tsander attributed tremendous importance to rockets which use part of their structure as fuel, and assumed that they would be the first devices to succeed in overcoming the Earth's attraction and in flying in cosmic space.

However, the problem of metallic propellant should not be considered only from the point of view of cosmic rockets. The need to look for the most efficient propellant for high altitude rockets is clear and was stressed many times in the technical literature on rocket technology.

* This article of Tsander was prepared for print by L. S. Dushkin in the Journal "Raketnaya Tekhnika", No. 1. 1936. In the present edition, Dushkin's foreword and the article are published without modification. – Editor's note.

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The use of metals with high calorific values as one of the fuel's constituents is a general problem of rocket technology. It is not astonishing, therefore, that notwithstanding the tremendous difficulties connected with the technical use of metallic propellant, Tsander was occupied with this problem from the first days in which the possibility of practical work on liquid propellant rockets appeared.

The untimely death of Tsander on 28 March 1933 prevented him from fully developing his great talent and enriching rocket technology with new ideas.

The present article by him is called in his manuscript "Rocket Engines, Using Materials Which Give Not Only Volatile But Also Solid Combustion Products" and is one of his many works in the field of rocket propulsion. In this article, he examines the influence of solid combustion products mixed with gaseous products on the reactive force.

In his considerations, Tsander aims at a rough evaluation of the possible results and, therefore, the derivations of the formulas have an approximate character and do not take into account a series of factors which, however, cannot change the qualitative side of his results. In other manuscripts by him there are references to more exact investigations he carried out, which, unfortunately, have not yet been found.

No essential modifications have been introduced in the article; some stylistic corrections have been introduced and rarely used notation has been replaced.

Editor

INTRODUCTION

Let us examine rocket engines, in which, under normal operation conditions, part of the combustion products is released as solid particles or liquid drops. All solid propellant* and fireworks rockets can be included in this type. Among them are those rocket engines which, in the author's opinion, will probably be the first to enable a complete escape from the Earth and flight into cosmic space, due to the fact that part of their structure can be used as additional fuel for the rocket.

As material for building the rocket engine and its associated parts one may use light metals and their alloys: aluminum, magnesium, berilium, lithium and also a series of other elements, which may be used subsequently as fuel.

When burning metals with oxygen (which is carried along) a tremendous amount of heat is released per kg of combustion products, exceeding considerably the amount obtained in the combustion of hydrogen with oxygen (see table).

If we burn in a rocket some material giving solid combustion products together with materials giving gaseous combustion products**, then the "smoke" produced will become more and more nonuniform structurally as

* By this he means ordinary black powder rockets.

** The need to burn, together with materials which give solid combustion products, others giving gaseous combustion products is due to the fact that solid combustion products do not have molecular velocities like gases and, therefore, they are incapable of delivering a reaction force. Coming out of the rocket's nozzle, the gaseous combustion products will carry with them the solid particles. – L. S. Dushkin's note.

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it is accelerated while approaching the rocket exit section. This is due to the fact that the solid combustion products will be entrained by the gaseous products only to a certain extent even for very small dimensions of the solid particles as a result of the enormous accelerations in the nozzle. However, since the difference between the velocities of the solid and gaseous combustion products is large, the heat from the solid particles will be rapidly transferred to the gases and the "smoke" will have an almost uniform temperature.

Calorific value of certain metals, hydrogen and gasoline

Fuel	Calorific value of the mixture, cal/kg	Theoretical exhaust velocity, m/sec
Lithium	4780	6330
Aluminum	3700	5560
Beryllium	5430	6750
Magnesium	3600	5500
Hydrogen	3240	5170
Gasoline	2350	4430

Note: The calorific values of the metals were taken from Landolt.

In general the molecular weight of the solid particles will be considerably larger than that of the gases and since the molecular heat capacity of compounds of identical structure is approximately constant, the solid particles' enthalpy will constitute, as a rule, only a small fraction of the gases' enthalpy. Therefore, almost the entire heat passes from the solid particles to the gases still in the combustion chamber and the temperature of the gases is raised considerably. Thus, the use of materials which give solid combustion products gives favorable results, in particular, if we take into account that our rockets are not yet capable of carrying with them an amount of liquid fuel sufficient for the rocket's complete escape from the Earth.

APPROXIMATE DETERMINATION OF THE REACTION FORCE FOR AN ENGINE IN WHICH PARTICLES WITH TWO CONSIDERABLY DIFFERENT VELOCITIES ARE SIMULTANEOUSLY EJECTED FROM THE NOZZLE

At different moments the gas molecules in the combustion chamber and in the nozzle move with quite different velocities. The exhaust velocity is an average of the molecules' velocities. Only when the final pressure is very low with respect to the initial pressure, the temperature drop makes the relative motions of the individual molecules, so small relative to the flow velocity that one can speak about more or less equal velocities of all the molecules.

If the flying particles are of different kinds, complete equalization of the velocities is impossible, and in nonelastic collisions between the gas

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molecules and the particles of the solid or liquid combustion products, part of the energy is always transformed into heat.

This will also happen, although to a lesser extent, if the exhaust products consist of gases with very different atomic weights. As we shall see below, the different velocities of the particles cause a reduction of the reaction force for a given kinetic energy of all the particles. A certain decrease in the reaction force, in particular for low excess pressures in the combustion chamber, is observed also in the case of a homogeneous gas, since the impulse per unit time is equal to the product of the mass of one molecule by the arithmetic sum of all molecules' velocities, whereas the kinetic energy of all the molecules taken together, which serves for the calculation of the average exhaust velocity, is equal to the product of half the mass of a molecule by the sum of the squares of all molecules' velocities.

Let us consider the case of two material systems, whose particles fly with different mean velocities.

Let us use the following notation:

g, gravitational acceleration on Earth's surface;
m_1 and m_2, mass of particles of first and second mediums (gases and solid particles) leaving nozzle in unit time;
G_1 and G_2, corresponding weights;
w_1 and w_2, particle velocities at nozzle outlet;
E_1 and E_2, kinetic energies of masses m_1 and m_2 at nozzle outlet;
$E_0=E_1+E_2$, sum of energies E_1 and E_2;
w_3, common velocity of masses m_1 and m_2 after velocity equalization, assuming conservation of momentum and heat release in impact;
E', kinetic energy of the whole mass (m_1 plus m_2) coming out of nozzle with velocity w_3;
Δt, approximate temperature increase of both systems due to heat release by equalization velocity;
P, thrust (taking m_1 and m_2 as consumptions per second);
$A=\frac{1}{427}$, mechanical equivalent of heat;
w_4, common velocity of masses m_1 and m_2 after velocity equalization when all heat, released by gradual velocity equalization at nozzle outlet, is transformed into kinetic energy;
H_1 and H_2, calorific values of material systems in calories per kg of combustion products;
P_{max}, maximum thrust, attainable only by full use of thermal effect and for $w_1=w_2$;
P_{min}, minimum thrust, obtainable for one particular value $q=w_2/w_1$ (we shall indicate below the conditions in which the reaction force has a minimum);
c_1 and c_2, specific heats of systems in process of velocity equalization.

For the kinetic energy with respect to the rocket engine we may write:

$$E_1=\frac{m_1 w_1^2}{2} \tag{1}$$

and

$$E_2=\frac{m_2 w_2^2}{2}. \tag{2}$$

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Consequently*,

$$E_0 = E_1 + E_2 = \frac{m_1 w_1^2 + m_2 w_2^2}{2}. \qquad (3)$$

Let us assume that the faster particles entrain the slower particles in such a way that only collisions between the particles take place but the pressure on the nozzle walls does not vary. This phenomenon is associated with energy losses of particle motion directed towards the nozzle outlet, but causes at the same time particle motion in all directions, thus producing a certain amount of heat.

In this case we have a certain analogy with the nonelastic collision of two spheres. Therefore, the velocities which the particles acquire will be the minimum which may be expected.

The total momentum before velocity equalization is equal to $m_1 w_1 + m_2 w_2$, and after equalization $(m_1 + m_2) w_3$.

We obtain therefore

$$w_3 = \frac{m_1 w_1 + m_2 w_2}{m_1 + m_2}. \qquad (4)$$

The kinetic energy of the entire mass after velocity equalization is equal to

$$E' = (m_1 + m_2)\frac{w_3^2}{2}. \qquad (5)$$

Substituting the expression of w_3 from (4) in (5) we obtain

$$E' = \frac{(m_1 w_1 + m_2 w_2)^2}{2(m_1 + m_2)}. \qquad (6)$$

The amount of kinetic energy transformed into heat is given by

$$E_0 - E' = \frac{1}{2(m_1 + m_2)}\left[(m_1 w_1^2 + m_2 w_2^2)(m_1 + m_2) - \right.$$
$$\left. - (m_1 w_1 + m_2 w_2)^2\right] = \frac{m_1 m_2 (w_2 - w_1)^2}{2(m_1 + m_2)}. \qquad (7)$$

The problem of what amount of heat is transferred to each of the material systems remains open. In the case of two gases, it could be assumed that both gases are equally heated but in the present case, i.e., when we have in addition to the gas molecules the relatively large and very hot particles of the metal oxide, it is closer to the truth to assume that almost the entire kinetic energy, released as heat, will be concentrated in the gas molecules.

For the sake of generality we write

$$m_1 c_1 \Delta t_1 + m_2 c_2 \Delta t_2 = (E_0 - E') A =$$
$$= \frac{A m_1 m_2}{2(m_1 + m_2)}(w_2 - w_1)^2,$$

* Equation (2) must be interpreted as a formal expression determining the amount of thermal energy contained in the metallic fuel since in reality the metals' oxides do not possess kinetic energy like the gases. – L. S. Dushkin's note.

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hence

$$\Delta t_1 = \frac{Am_1 m_2 (w_2 - w_1)^2}{2(m_1 + m_2)\left(m_1 c_1 + m_2 c_2 \frac{\Delta t_2}{\Delta t_1}\right)}. \tag{8}$$

In each individual case it is necessary to investigate what change in the state of the gases was effected by the velocity equalization.

The calculated velocity w_3 represents that average velocity of the combustion products which determines the thrust of the jet. Indeed, from the momentum law for a unit time interval we obtain

$$P = m_1 w_1 + m_2 w_2 = (m_1 + m_2) w_3. \tag{4a}$$

We could obtain a higher average velocity if we succeeded in transforming the heat, corresponding to $E_0 - E'$, into kinetic energy in the velocity equalization process. However, this is possible only if the velocities w_1 and w_3 are attained not at the nozzle outlet but inside it. In this case, the entire kinetic energy E would be transformed into kinetic energy* of the homogeneous jet. We would then have

$$\frac{(m_1 + m_2) w_4^2}{2} = E_0 = \frac{m_1 w_1^2 + m_2 w_2^2}{2}$$

or

$$w_4 = \sqrt{\frac{m_1 w_1^2 + m_2 w_2^2}{m_1 + m_2}}. \tag{9}$$

The maximum thrust which is then obtained is equal to

$$P_{max} = (m_1 + m_2) w_4. \tag{10}$$

Substituting in (10) the expression of w_4 from (9), we obtain

$$P_{max} = \sqrt{(m_1 w_1^2 + m_2 w_2^2)(m_1 + m_2)}. \tag{10a}$$

For convenience, let us introduce the following notation:

$$q = \frac{w_2}{w_1}$$

and

$$p = \frac{m_2}{m_1}.$$

Furthermore, let us denote the thrust, obtained from a mass m_1, ejected with a velocity w_1 by P_1; then

$$P_1 = m_1 w_1. \tag{11}$$

* In reality such a velocity (formula (9)) cannot be achieved, since the oxides of the metals, not possessing motion energy, will give their energy in the form of heat to the gaseous combustion products and at the same time will absorb part of the kinetic energy of the gaseous particles when the latter collide with the solid particles. This law was not taken into consideration here. – L. S. Dushkin's note.

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Taking $m_1 w_1$ in formula (10a) out of the square root, we obtain, after substituting q, p and P_1

$$P_{max} = P_1 \sqrt{(1+pq^2)(1+p)}. \tag{10b}$$

In a similar way we obtain from formula (4a) for the actual thrust

$$P = P_1 (1+pq). \tag{4b}$$

Dividing (4b) by (10b), we obtain an important formula which gives the ratio of the actual thrust, obtained when the particles of one system lag somewhat behind those of the other, to the theoretical thrust, which would be available if the particles' velocities could be equalized and all the heat transformed into exhaust energy. This formula has the form

$$\frac{P}{P_{max}} = \frac{1+pq}{\sqrt{(1+pq^2)(1+p)}}. \tag{12}$$

Let us examine this formula in more detail. For

$$q = \frac{w_2}{w_1} = 1$$

we find

$$\frac{P}{P_{max}} = 1,$$

i.e., if the velocities are equal, then the actual thrust is equal to the maximum thrust, as it should be.

Next, for

$$q = \frac{w_2}{w_1} = 0$$

we obtain

$$\frac{P}{P_{max}} = \frac{1}{\sqrt{1+p}}. \tag{13}$$

This formula gives the magnitude of the actual thrust for the case when the velocity of the particles of one system may be neglected ($w_2 = 0$) or when one system serves only for heating the particles of the other system. From equation (13) we see (remembering that $p = m_2/m_1$) the rather fast decrease of the axial thrust P relative to the maximum thrust P_{max} for a relatively small amount of solid particles. For small p we have approximately, by series expansion:

$$\frac{P}{P_{max}} = 1 - 0.5p.$$

For larger values of p we have a parabolic dependence of P_{max}/P, as can be seen from equation (13). The thrust P, being small, decreases extremely slowly. For example, if the mass of the heating material is three times larger than the mass of the outflowing particles ($p = 3$), then

$$\frac{P}{P_{max}} = \frac{1}{\sqrt{1+3}} = \frac{1}{2},$$

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i.e., the actual thrust P is half the maximum thrust P_{max}, which would be obtained if both materials would come out of the nozzle with the same velocity. If $p = 100$, then

$$\frac{P}{P_{max}} \approx 0.10.$$

The curves in Figure 1 were drawn for various values of $q = w_2/w_1$. The abscissa gives the mass ratio and the ordinate – the thrust ratio. We see that the curves

$$p = f\left(\frac{P}{P_{max}}\right)$$

have certain minima, and also that for a small lag of particles of one material behind those of the other one, the actual thrust P is almost equal to the maximum force which would be obtained if the velocities were equalized without energy loss. If the lag attains even 50% ($q = 50\%$), the loss in force amounts, in this most unfavorable case, to 5.5%.

The position of the minimum of P/P_{max} for a given $q = w_2/w_1$ is easily determined by equating to zero the first derivative of P/P_{max} with respect to p. From (12) we obtain

$$\frac{d}{dp}\left(\frac{P}{P_{max}}\right) = \frac{q\sqrt{(1+pq^2)(1+p)}}{(1+pq^2)(1+p)} - \\ - \frac{(1+pq)\frac{1}{2}\left[(1+pq^2)(1+p)\right]^{-1/2}\left[q^2(1+p)+(1+pq^2)\right]}{(1+pq^2)(1+p)} = \\ = \frac{2q(1+pq^2)(1+p)-(1+pq)(1+q^2+2pq^2)}{2\left[(1+pq^2)(1+p)\right]^{\frac{3}{2}}} = 0. \qquad (14)$$

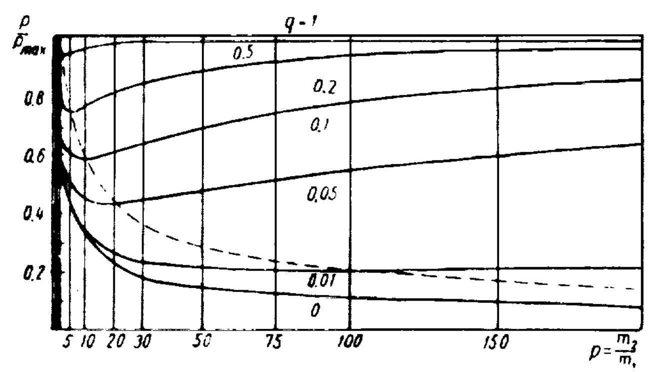

FIGURE 1. Ratio of the actual thrust of a jet to the theoretical maximum thrust achieved for velocity equalization, vs the mass ratio

We have p^2 in the numerator and p^3 in the denominator. Therefore for $p = \infty$

$$\frac{d}{dp}\left(\frac{P}{P_{max}}\right) = 0,$$

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i. e., as seen from the figure, we have here a maximum. This result may be formulated as follows.

If the mass of the solid particles which come out of the nozzle is many times larger than that of the gas, i. e., if $p=\frac{m_2}{m_1}\to\infty$ then the thrust of the jet approaches the maximum possible thrust, occurring for velocity equalization.

In this connection one should, however, remark that, in a real case, as $p\to\infty$ a considerable amount of heat remains in the solid particles and is not used, while in this formula we assumed that the sum of the kinetic energies of both materials is constant.

If the numerator of formula (14) is made equal to zero, we obtain

$$2q(1+pq^2)(1+p)-(1+pq)(1+q^2+2pq^2)=0,$$

or, after opening the brackets:

$$p(q-2q^2+q^3)=1+q^2-2q,$$

hence

$$pq=1. \tag{15}$$

Introducing in this formula the values of p and q, we obtain

$$m_1w_1=m_2w_2. \tag{15a}$$

For these values of q and p we obtain the minimum of P/P_{max}, shown in Figure 1.

This result may be formulated as follows. The thrust of a jet made up of two kinds of particles, possessing different flow velocities, becomes a minimum if the momenta of both kinds of particles are equal.

Substituting (15) in (12) we obtain the minimum value of the thrust ratio:

$$\frac{P_{min}}{P_{max}}=\frac{2\sqrt{q}}{1+q}=\frac{2\sqrt{p}}{1+p}. \tag{16}$$

Further, it is interesting to find for the general case the ratio of the thrust P, obtained from the whole jet, to the thrust P_1 of the mass m_1. We see from (4b) that for given q, P/P_1 is a linear function of p, and for given p it is a linear function of q. We may then prescribe constant values either for p or for q.

If we look for the maximum theoretical thrust obtainable, we must use the maximum exhaust velocities which would be obtained by transforming all the heat, corresponding to the calorific value, into exhaust kinetic energy. For solid particles, which cannot acquire a velocity independently, this value is arbitrary.

Denoting these velocities by w_{1max} and w_{2max}, we can write using expressions (1) and (2):

$$E_1=\frac{m_1w_{1max}^2}{2}=gm_1\frac{H_1}{A}, \tag{17}$$

hence

$$w_{1max}=\sqrt{\frac{2gH_1}{A}}=91.5\sqrt{H_1}. \tag{17a}$$

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In a similar way we have for the velocity w_{2max}

$$E_{2max} = \frac{m_2 w_{2\,max}^2}{2} = g m_2 \frac{H_2}{A}, \tag{18}$$

hence

$$w_{2max} = \sqrt{\frac{2gH_2}{A}} = 91.5\sqrt{H_2}. \tag{18a}$$

Dividing (18a) by (17a), we obtain

$$\frac{w_{2max}}{w_{1max}} = \frac{\sqrt{H_2}}{\sqrt{H_1}}$$

or, denoting

$$x = \frac{H_2}{H_1}$$

and remembering that,

$$\frac{w_2}{w_1} = q,$$

we have

$$q_{max}^2 = x. \tag{19}$$

Substituting this value of q_{max} in (4b) we obtain for the actual thrust:

$$P = P_1\left(1 + p x^{\frac{1}{2}}\right), \tag{20}$$

and in the case of velocity equalization with full use of the heat corresponding to the calorific value, we obtain by substituting q_{max} in (10b):

$$P_{max} = P_1\sqrt{(1 + px)(1 + p)}, \tag{21}$$

where

$$P_1 = m_1 w_{1max} = 91.5 m_1\sqrt{H_1}. \tag{11a}$$

Substituting in (20) and (21) for x and p their expressions

$$x = \frac{H_2}{H_1},$$

$$p = \frac{m_2}{m_1},$$

and for P_1 – its expression from (11a), we find also

$$P = 91.5\left(m_1\sqrt{H_1} + m_2\sqrt{H_2}\right) \tag{20a}$$

and

$$P_{max} = 91.5\sqrt{(m_1 H_1 + m_2 H_2)(m_1 + m_2)}. \tag{21a}$$

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Since

$$m_1 = \frac{G_1}{g} \tag{22a}$$

and

$$m_2 = \frac{G_2}{g}, \tag{22b}$$

and also

$$p = \frac{G_2}{G_1},$$

we obtain for the thrust

$$P = 91.5 \frac{1}{g} (G_1 \sqrt{H_1} + G_2 \sqrt{H_2}) \tag{20b}$$

and correspondingly

$$P_{max} = 91.5 \frac{1}{g} \sqrt{(G_1 H_1 + G_2 H_2)(G_1 + G_2)}. \tag{21b}$$

APPROXIMATE DETERMINATION OF THE DIFFERENCE IN THE VELOCITIES OF GASEOUS AND SOLID COMBUSTION PRODUCTS AT NOZZLE OUTLET

Let us use the following notation:

$w' = w_1 - w_2$, difference in velocity of gaseous and solid combustion products at nozzle outlet, m/sec;

γ_1, average specific weight of gaseous products at nozzle outlet, kg/m3;

γ_2, specific weight of solid products during their stay in the nozzle, kg/m3;

F_2, maximum (mid-particle) cross section of solid particles, mm^2;

d_2, average diameter of solid particles, mm;

t, time, sec;

M_2, average mass of a solid particle;

W, average aerodynamic resistance of a solid particle is given by the formula*

$$W = \psi \gamma_1 \frac{(w')^2}{2g} F_2, \tag{23}$$

where ψ is the coefficient of air resistance of solid particles.

In order to determine the shape and dimensions of the solid particles an experiment was performed with magnesium fuel, which gives magnesium oxide particles. A film of magnesium was burnt and the smoke produced was deposited on a microscope glass. The microscopic examination showed

* This is a commonly used experimental formula for resistance to motion in air of bodies with sonic velocity. It was not verified for the case considered, i. e., for small particles of a diameter of the order of 0.001 mm. – L. S. Dushkin's note.

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that the particles have oval and spherical shapes with a diameter from 0.001 to 0.0005 mm.

As the solid particles get further from the flame, their temperature decreases rapidly and their light dies out. This shows that the heat from the oxide is indeed transferred rapidly to the surrounding air. The same result is obtained when burning magnesium powder, e. g., for photographic purposes: the spark is short-lived and the heat is rapidly given away to the surrounding medium. Experience justifies us in considering the particles as spherical in first approximation.

Let us investigate the case when the acceleration of the oxide particles is constant.

The acceleration i_2 is equal to $\frac{dw_2}{dt}$ and, if the velocity of the particles is increased in the nozzle along its length l from zero to w_2, then

$$i_2 = \frac{w_2^2}{2l}. \tag{24}$$

Assuming that the oxide particles are accelerated by aerodynamic drag, we find

$$W = M_2 \frac{dw_2}{dt}. \tag{25}$$

The spherical oxide particles' mass and cross section are given by

$$M_2 = \frac{\gamma_2}{g} \frac{1}{6} \pi d_2^3 \tag{26}$$

and

$$F_2 = \frac{\pi d_2^2}{4}. \tag{27}$$

Eliminating W from (25) and (23) and substituting in the equation obtained the expressions of M_2, F_2 and $\frac{dw_2}{dt}$ from (26), (27) and (24) we find

$$\frac{\gamma_2}{g} \frac{1}{6} \pi d_2^3 \frac{w_2^2}{2l} = \psi \gamma_1 \frac{(w')^2}{2g} \frac{\pi}{4} d_2^2$$

or

$$\frac{(w')^2}{w_2^2} = \frac{2\gamma_2 d_2}{3\gamma_1 \psi l}. \tag{28}$$

Remembering that $w' = w_1 - w_2$, we finally obtain for the ratio of the oxide particles' velocity to the velocity of the gaseous particles:

$$\frac{w_2}{w_1} = \frac{1}{1 + \sqrt{\frac{2\gamma_2 d_2}{3\gamma_1 \psi l}}}. \tag{29}$$

As an example we take the case of magnesium oxide particles entrained by the combustion products of detonating gas. We take as the average diameter of the oxide particles $d_2 = 0.00075\ \text{mm} = 0.75 \cdot 10^{-6}\ \text{m}$.

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The specific weight of the particles, is, for magnesium, $\gamma_2 = 3.2 \cdot 10^3\,kg/m^3$. For a sphere we have $\psi = 0.60$. As a first approximation we can use for an oxygen-hydrogen rocket the theoretical curves for gas expansion that I gave in 1932*. These curves were calculated for a rocket with an initial gas pressure of 20 atm, a mixture consumption of 3.33 kg/sec and an angle of 10° between the axis and the generator of the cone.

Let us assume that the amount of oxide is so small that the temperature and, therefore, also the specific weight of the gases do not vary noticeably for a given pressure. Then we find from the expansion diagram that for a pressure ranging from 1 to 0.01 atm at the end of the nozzle, the average product $\gamma_1 l$, appearing in the denominator of formula (29), takes values from 0.014 to 0.0073 kg/m throughout the entire process of gas expansion. Substituting all the above-indicated numerical values in formula (29), we find that the value of w_2/w_1 fluctuates between 0.70 and 0.62.

For these values of $q = w_2/w_1$ we find from formula (12) a very important result, namely: the actual thrust in these conditions constitutes about 97% of the maximum thrust, which would be obtained if the velocity of the oxide particles was equal to the velocity of the gaseous particles. For a small amount of oxide ($p < 1.5$) the loss will be smaller than 3%.

From formula (29) we see that the velocity ratio of the oxide particles to the gas particles will be the closer to 1, the longer the nozzle and the higher the specific weight of the gas, and also the lower the specific weight and average diameter of the oxide particles. Consequently, in rockets with high fuel consumption and small cone angle the lag of the oxide will be small.

If the solid particles are of streamlined shape their relative lag will, of course, be larger than that of spherical particles. If the oxide particles have a crystalline structure, e.g. like snow-flakes, their relative lag will be even smaller than that calculated above.

For oxide particles of nonspherical shape we find easily from formulas (23), (24) and (25):

$$W = \psi \gamma_1 \frac{(w')^2}{2g} F_2 = M_2 \frac{w_2^2}{2l},$$

hence

$$\frac{(w')^2}{w_2^2} = \frac{M_2 g}{F_2 \gamma_1 \psi l} = \frac{G_2}{F_2 \gamma_1 \psi l}, \tag{30}$$

where $G_2 = M_2 g$ is the average weight of a solid particle.

We may introduce here also the ratio

$$\delta = \frac{G_2}{F_2}, \tag{31}$$

representing the specific load, due to the proper weight of the particle, per unit area of the maximum cross section of the particle. Introducing again the relative velocity $w' = w_1 - w_2$ between the solid and gaseous combustion

* See the article, Problema poleta pri pomoshchi reaktivnykh apparatov (Problems of Flight with the Aid of Jet Propulsion Machines). – Editor's note.

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products, we obtain for w_2/w_1:

$$\frac{w_2}{w_1}=\frac{1}{\sqrt{\frac{\delta}{\gamma_1\psi l}+1}}. \tag{32}$$

By comparing (32) with (29), we see that for a sphere

$$\delta=\frac{2}{3}\gamma_2 d_2. \tag{33}$$

The accelerations which the solid particles undergo in the nozzle are very large since the velocities increase very rapidly in a comparatively short distance.

The values of all the quantities, which may be obtained on the basis of the formulas derived above, represent only a first approximation, since we neglected, on one hand, the influence of dissociation which lowers the gas particles' velocity, and on the other hand – the increase of the coefficient ψ in formula (23) which takes place at supersonic velocities. For a large percentage of solid or liquid particles and a large counter-pressure at the nozzle outlet, the relative velocity is not very high and, therefore, it is better in first approximation not to start from the gas particles' velocity, but from the velocity which would be obtained in the case of velocity equalization.

In addition to the acceleration of the solid particles, we must also calculate the slowing down of the gas particles. Using the approximate results which are obtained in a first calculation, it is possible to carry out a second calculation more exactly.

The stay of the gaseous and solid particles in the rocket nozzle is determined in the general case by the formulas

$$t_1=\int_0^l \frac{dl}{w_1} \tag{34}$$

and

$$t_2=\int_0^l \frac{dl}{w_2}, \tag{35}$$

where w_1 and w_2 vary in these formulas from zero to their maximum values. In the case of constant acceleration, we obtain:

$$t_1=\frac{2l}{w_1} \tag{34a}$$

and

$$t_2=\frac{2l}{w_2}, \tag{35a}$$

where, in this case w_1 and w_2 are the magnitudes of the velocities at the outlet from the nozzle. Calculations show that the stay of particles in the

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nozzle is extremely short. For the large nozzle considered above its range will be between 1/5000 and 1/570 sec for magnesium oxide particles.

TEMPERATURE DROP OF THE OXIDE PARTICLES DURING THEIR STAY IN THE NOZZLE*

Due to the high temperature in the nozzle, radiation will predominate over heat transfer by convective current. Therefore, to simplify matters, let us first examine heat transfer by radiation.

Let us use the following notation:

T_1 and T_2, the absolute temperature of gases and oxide at a given moment;
dQ, amount of heat released by one oxide particle during time dt;
c_2, specific heat of solid (or liquid) particles;
C_s, Stefan-Boltzmann radiation constant.

Let us consider a spherical oxide particle. In this case, the ratio of its surface to volume is a minimum, and therefore, the particle will be cooled less than particles with different forms.

The amount of heat lost by the gas when the temperature is lowered by dT_2, is equal for one particle to

$$dQ = -c_2\gamma_2 \frac{\pi d_2^3}{6} dT_2. \quad (36)$$

On the other hand, the spherical surface πd_2^2 radiates during the time dt the amount of heat

$$dQ = C_s \pi d_2^2 (T_2^4 - T_1^4)\, dt. \quad (37)$$

From (36) and (37) we obtain

$$-c_2\gamma_2 \frac{\pi d_2^3}{6} dT_2 = C_s \pi d_2^2 (T_2^4 - T_1^4)\, dt$$

or

$$dt = -\frac{c_2\gamma_2 d_2}{6C_s} \frac{dT_2}{T_2^4 - T_1^4}. \quad (38)$$

When T_1 is small relative to T_2, we may take for it an average value T_{1m} and integrate. Then we obtain

$$t = -\frac{c_2\gamma_2 d_2}{6C_s} \int_{T_2'}^{T_2''} \frac{dT_2}{T_2^4 - T_{1m}^4}. \quad (39)$$

where t is the time during which the oxide temperature decreases from T_2' to T_2''.

* The writing of this section has not been completed, since Tsander intended to make a more exact and final calculation after completing a series of experiments which he had planned. – L. S. Dushkin's note.

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The integral gives

$$t=\frac{c_2\gamma_2 d_2}{24C_s T_{1m}^3}\left[2\,\mathrm{arc\,tg}\,\frac{T_2''}{T_{1m}}-2\,\mathrm{arc\,tg}\,\frac{T_2'}{T_{1m}}+\right.$$

$$\left.+\ln\frac{(T_2''+T_{1m})(T_2'-T_{1m})}{(T_2''T_{1m})(T_2'+T_{1m})}\right]. \tag{39a}$$

Knowing the specific heat c_2 of the solid particles, their density γ_2, the radiation coefficient C_s, the diameter d_2, the average temperature of the gas and the initial temperature of the solid particles T_2' before their heat is transferred to the gases, we can calculate the time t as function of T_2'' and then also T_2'' as function of the time t.

CONCLUSION

The previous discussion points to definite advantages obtained by using metals as an additional fuel for a rocket engine. It is possible to burn a metal in such proportions to the liquid fuel that no decrease in the thrust will be observed. We do not take into account the fact that metal oxides (as, for example, Al_2O_3, and MgO) will probably be in a gaseous state in the chamber and partly in the nozzle, since the combustion temperature exceeds the vaporization temperature of certain oxides. This problem requires an additional investigation.

We did not touch at all on the problems of the dynamics of a rocket in which part of the structure built of high calorific value metals is used as fuel. If we take into consideration these possibilities, too, then the advantage of using a metallic fuel will be even greater than shown in the present article.

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DESIGN PROBLEMS OF A ROCKET USING METALLIC FUEL*

METAL ALLOYS, USED AS CONSTRUCTION MATERIALS AND AS FUEL

When the combustible metal constitutes part of the rocket's structure, before being melted, it must have the following qualities:

1) sufficiently low melting temperature;

2) sufficient strength at the temperatures to which it is subjected before falling into the melting chamber;

3) the heat required for heating and melting the metal should not be too large, so that the time needed for melting will be as short as possible;

4) the calorific value of the metal should be as high as possible;

5) the ignition temperature of the metal should be as low as possible.

To condition 1) we may add that if the design contains lateral chambers for melting the metals, then these chambers should also be melted in turn in one or several central chambers**, and it is consequently expedient to use metals with different melting temperatures:

a) for the liquid oxygen tanks, we may use alloys with melting temperatures between 50° and 200°C;

b) for the sections which are subjected to an approximate average temperature of 28-50°, alloys with melting temperatures between 200° and 500°;

c) for the chambers serving for melting sections indicated under a), alloys with melting temperatures between 500° and 700°;

d) for the chambers intended for melting the sections mentioned in b), alloys with melting temperatures between 500° and 900°; these alloys may also be used for intensely cooled rocket nozzle walls;

e) for chambers intended for melting the sections mentioned in c) and d), alloys with melting temperatures between 900° and 2000°; these alloys may be used for rocket combustion chamber walls, and for weakly cooled parts of rocket nozzles and also for the nose sections of rockets***.

For intensely heated sections of rockets which are not to be melted it is possible to use refractory materials and very high-melting alloys. Structures which are fusible at low temperature may also contain high-melting alloys if they do not disturb the melting. Subsequently these sections may be melted together with the chambers into which they fell, or they may be removed from the chamber by special automatic grids attached to levers.

* This article was published in the Journal "Raketnaya Teknika", No. 5. 1937.

** For an example of such a design see the article, Problema poleta pri pomoshchi reaktivnykh apparatov (Problems of Flight with the Aid of Jet Propulsion Machines). – Editor's note.

*** These sections may get intensely heated when flying at high velocities in the atmosphere. – Editor's note.

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To condition 4) the following should be added:

a) if the air for combustion is taken from the atmosphere, the calorific value per kg of the fuel itself should be a maximum;

b) if pure liquid oxygen is taken in the rocket for combustion, the calorific value per kg of metal oxide should be a maximum;

c) if a material containing oxygen only partially (for example, liquid air which possesses 85% of O_2 or H_2O_2 or $Al(NO_3)_3$), the calorific value should be determined for all the combustion products taken together.

To condition 5) we should add that if it is intended to burn simultaneously also gasoline or oil or, for example, celluloid, then the molten metal may be sprayed by means of a burner, in which gasoline or oil burns with a large surplus of oxygen.

DETERMINATION OF THE RELATION BETWEEN THE WEIGHTS OF METAL, HYDROGEN, SOLID AND GASEOUS COMBUSTION PRODUCTS, SOLID AND LIQUID PROPELLANTS (INCLUDING OXYGEN)

(Since we must burn together with a metal some other material, giving gaseous combustion products, a rocket capable of burning its metallic sections must carry also some other fuel, either liquid (for example, liquid hydrogen, gasoline, oil, etc.), or solid (for example, celluloid and others). It is interesting and important to determine the weight ratios between the individual components of the propellant in such a case).

As an example we take a rocket which is propelled by liquid-oxygen, liquid hydrogen and aluminum.

Let us use the following notation:

A_O, A_H, A_{Al}, atomic weight of oxygen, hydrogen and aluminum ($A_O = 16$, $A_H = 1.008$, $A_{Al} = 27.1$);

a_H, a_{Al}, number of hydrogen and aluminum atoms combining with one atom of oxygen ($a_H = 2$; $a_{Al} = \frac{2}{3}$), according to the formulas H_2O and Al_2O_3;

G_O, G_{OH}, $G_{O\,Al}$, total and partial weight of oxygen, required for combustion in kg;

G_H, G_{Al}, weight of hydrogen and aluminum, required for combustion in kg;

G, G_f, initial and final weight of a long-range rocket (before and after obtaining the entire kinetic energy) in kg;

G_{H_2O}, $G_{Al_2O_3}$, weight of water and aluminum oxide produced in kg;

G_p, total weight of propellant, including oxygen.

We may then write

$$G_O = G_{OH} + G_{O\,Al}, \tag{1}$$

$$G = G_O + G_H + G_{Al} + G_f = G_{H_2O} + G_{Al_2O_3} + G_K, \tag{2}$$

$$G_p = G - G_f = G_O + G_H + G_{Al}, \tag{3}$$

$$G_{H_2O} = G_{OH} + G_H, \tag{4}$$

$$G_{Al_2O_3} = G_{O\,Al} + G_{Al}, \tag{5}$$

$$G_{OH} = G_H \frac{A_O}{A_H a_H}, \tag{6}$$

$$G_{O\,Al} = G_{Al} \frac{A_O}{A_{Al} a_{Al}}. \tag{7}$$

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The calorific value of detonating gas per kg of H_2O is 3230 cal/kg, and of aluminum oxide, Al_2O_3 is 3690 cal/kg (see "Chimiker Kalender"). Assuming a somewhat smaller utilization factor for Al_2O_3 than for hydrogen, since the heat has to pass from the Al_2O_3 particles to the gases, we may take in a first approximation Al_2O_3 and H_2O as equivalent fuels. Equivalent fuels will be those which give the same value for the velocity acquired by the rocket, according to the formula

$$v = w \ln \frac{G}{G_f}.$$

In the present case we may assume this, since the variation of the exhaust velocity w of the gases and the solid combustion products with the percentage of Al_2O_3 is small. If G/G_f is given, then for a given w a definite v is obtained.

Thus, when the amount of aluminum used varies, it is possible to assume, for a given final weight G_f, that the weight of all the propellant $(G-G_f)$ does not vary, so that the initial weight G is also constant.

From equations (1), (2), (3), (6), and (7) we obtain

$$G - G_f = G_O + G_H + G_{Al} = G_H\left(1 + \frac{A_O}{A_H a_H}\right) + G_{Al}\left(1 + \frac{A_O}{A_{Al} a_{Al}}\right), \qquad (8)$$

hence

$$G_{Al} = \frac{(G - G_f) - G_H\left(1 + \frac{A_O}{A_H a_H}\right)}{1 + \frac{A_O}{A_{Al} a_{Al}}}. \qquad (9)$$

(Further in the manuscript of Tsander we find a derivation of formulas giving the cost of a rocket as a function of the amount of hydrogen, oxygen and aluminum in it. Tsander writes:

"The prices and the amounts of liquid oxygen, liquid hydrogen or gasoline, aluminum or magnesium and other fuels required, are very different. If we use as part of the fuel in a rocket molten metal or some other material which before melting was in the form of bracings, longerons, airplane ribs, engine parts, propellers and so on, the cost will be even higher and a very serious problem arises: is the expenditure on such use of rocket parts justified or not. Let us find therefore under what conditions it will be economical to use metals as fuel in a rocket and how much the cost will rise relative to the case in which only liquid fuel is used."

The following calculation has no value today but its results are curious. If the cost of liquid oxygen is 0.15 rubles per kg and that of machined aluminum parts for the rocket 10 rubles per kg, one finds that if 1 kg of liquid hydrogen costs more than 46.8 rubles, then a rocket which uses as fuel aluminum sections of the structure will be cheaper than a pure hydrogen-oxygen rocket. Otherwise the contrary is true. For a rocket with oxygen and aluminum with

$$\frac{G_p}{G} = \frac{19}{20}$$

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we obtain a cost of 5.6 rubles for 1 kg of rocket at the start. This includes only the cost of the material, propellant and manufacturing.)

Let us denote by

$$V_{Al}=\frac{G_{Al}}{G_p};\quad V_H=\frac{G_H}{G_p};\quad V_O=\frac{G_O}{G_p}$$

the ratios of the corresponding weights of aluminum, hydrogen and oxygen to the total weight of propellant (including oxygen).

Denoting by

$$\xi=\frac{G_{Al}+G_{O\,Al}}{G_H+G_{OH}}=\frac{G_{Al_2O_3}}{G_{H_2O}}$$

the weight ratio of aluminum oxide to water vapor (the ratio of solid to gaseous combustion products) and, finally, by

$$\beta=\frac{G_O+G_H}{G_{Al}}=\frac{V_O+V_H}{V_{Al}}$$

the ratio of the sum of weights of oxygen and hydrogen to that of combustible aluminum (the weight ratio of liquid to solid propellants).

From (3) we obtain

$$\frac{G_O}{G_p}+\frac{G_H}{G_p}+\frac{G_{Al}}{G_p}=V_O+V_H+V_{Al}=1. \qquad (10)$$

Dividing both sides of equation (1) by G_p and introducing relations (6) and (7), we obtain

$$V_O=A_O\left(\frac{V_H}{A_H a_H}+\frac{V_{Al}}{A_{Al}a_{Al}}\right). \qquad (11)$$

Substituting in (10) we get

$$V_O=1-V_H-V_{Al}=\frac{A_O V_H}{A_H a_H}+\frac{A_O V_{Al}}{A_{Al}a_{Al}}$$

or

$$V_H\left(\frac{A_O}{A_H a_H}+1\right)+V_{Al}\left(\frac{A_O}{A_{Al}a_{Al}}+1\right)=1, \qquad (12)$$

hence

$$V_{Al}=\frac{1-V_H\left(\frac{A_O}{A_H a_H}+1\right)}{\frac{A_O}{A_{Al}a_{Al}}+1}. \qquad (13)$$

Next, dividing (6) and (7) by G_p, we have

$$\frac{G_{OH}}{G_p}=V_H\frac{A_O}{A_H a_H} \qquad (14)$$

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and

$$\frac{G_{O\,A1}}{G_p} = V_{A1}\frac{A_O}{A_{A1}a_{A1}}. \tag{15}$$

Substituting in the expression for ξ we obtain

$$\xi = \frac{V_{A1} + \frac{G_{O\,A1}}{G_p}}{V_H + \frac{G_{OH}}{G_p}} = \frac{V_{A1}\left(1 + \frac{A_O}{A_{A1}a_{A1}}\right)}{V_H\left(1 + \frac{A_O}{A_H a_H}\right)}. \tag{16}$$

Substituting V_{A1} from (13) in (16), we obtain

$$\xi = \frac{1}{V_H\left(1 + \frac{A_O}{A_H a_H}\right)} - 1. \tag{17}$$

Using this equation, we can determine V_H, V_{A1}, and V_0 as functions of ξ. We find immediately

$$V_H = \frac{1}{(\xi+1)\left(1 + \frac{A_O}{A_H a_H}\right)}. \tag{18}$$

Substituting V_H in equation (13) we obtain

$$V_{A1} = \frac{\xi}{(\xi+1)\left(1 + \frac{A_O}{A_{A1}a_{A1}}\right)}. \tag{19}$$

Finally, we obtain V_O by substituting the expressions found for V_H and V_{A1} in equation (11):

$$V_O = \frac{\xi\left(1 + \frac{A_O}{A_H a_H}\right)\frac{A_O}{A_{A1}a_{A1}} + \left(1 + \frac{A_O}{A_{A1}a_{A1}}\right)\frac{A_O}{A_H a_H}}{(\xi+1)\left(1 + \frac{A_O}{A_{A1}a_{A1}}\right)\left(1 + \frac{A_O}{A_H a_H}\right)}. \tag{20}$$

Next, we have from (10):

$$V_O + V_H = 1 - V_{A1};$$

substituting this equation in the expression for β we obtain

$$\beta = \frac{1 - V_{A1}}{V_{A1}}.$$

Substituting V_{A1} from (19) we have

$$\beta = \frac{(\xi+1)\left(1 + \frac{A_O}{A_{A1}a_{A1}}\right) - \xi}{\xi} = \frac{(\xi+1)\frac{A_O}{A_{A1}a_{A1}} + 1}{\xi}. \tag{21}$$

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Now we can determine V_H by (18), V_{Al} by (19), V_O by (20) and β as function of ξ from (21). The weight ratio of aluminum oxide to water vapor, which we denote by ξ, plays a decisive role in the calculation of the rocket performance: the greater the amount of aluminum oxide mixed with water vapor, the larger the amount of heat leaving the rocket with the oxide unused.

The weight ratio of the liquid to the solid propellant β, plays a role in choosing the stresses in the rocket's material: the more liquid propellant we have, the greater the stresses in the material.

We shall denote by δ the ratio of the weight of liquid propellants, including oxygen, to the initial weight of the rocket. It is equal to

$$\delta=\frac{G_H+G_O}{G}=(V_H+V_O)\frac{G_P}{G}=(1-V_{Al})\left(1-\frac{G_f}{G}\right)=$$
$$=\frac{1+\frac{A_O}{A_{Al}a_{Al}}(\xi+1)}{(\xi+1)\left(1+\frac{A_O}{A_{Al}a_{Al}}\right)}\left(1-\frac{G_f}{G}\right). \tag{22}$$

Rearranging this equation we obtain for ξ:

$$\xi=\frac{1-\frac{G_f}{G}}{\left(1+\frac{A_O}{A_{Al}a_{Al}}\right)\delta-\frac{A_O}{A_{Al}a_{Al}}\left(1-\frac{G_f}{G}\right)}-1. \tag{23}$$

Knowing the mechanical properties of the material of which the rocket has to be built, we find δ from structural considerations and determine ξ by equation (23). Then, we find V_H, V_{Al}, V_O, and β as shown above.

If we start from the rocket's design, it is better to express all the quantities through β. Then we have from equation (21)

$$\xi=\frac{1+\frac{A_O}{A_{Al}a_{Al}}}{\beta-\frac{A_O}{A_{Al}a_{Al}}}. \tag{24}$$

Therefore

$$\xi+1=\frac{1+\beta}{\beta-\frac{A_O}{A_{Al}a_{Al}}}. \tag{25}$$

Substituting the expressions of $(\xi+1)$ and of ξ in equations (18), (19) and (20) we obtain

$$V_H=\frac{\beta-\frac{A_O}{A_{Al}a_{Al}}}{(1+\beta)\left(1+\frac{A_O}{A_Ha_H}\right)}, \tag{26}$$

$$V_{Al}=\frac{1}{1+\beta}, \tag{27}$$

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$$V_O = \frac{\beta \frac{A_O}{A_H a_H} + \frac{A_O}{A_{Al} a_{Al}}}{(1+\beta)\left(1 + \frac{A_O}{A_H a_H}\right)}. \tag{28}$$

Using (25) and (23), we can express β as function of δ:

$$\xi + 1 = \frac{1+\beta}{\beta - \frac{A_O}{A_{Al} a_{Al}}} = \frac{1 - \frac{G_f}{G}}{\left(1 + \frac{A_O}{A_{Al} a_{Al}}\right)\delta - \frac{A_O}{A_{Al} a_{Al}}\left(1 - \frac{G_f}{G}\right)}, \tag{29}$$

hence

$$\beta = \frac{\delta}{1 - \frac{G_f}{G} - \delta}. \tag{30}$$

On the other hand,

$$\delta = \frac{\beta\left(1 - \frac{G_f}{G}\right)}{1+\beta}. \tag{31}$$

Knowing the admissible limits for δ, we find the limits for β from equation (30) and limits for V_H, V_{Al}, V_O and ξ respectively from (26), (27), (28) and (24).

Substituting numerical values, we may now determine all the quantities indicated above and draw a diagram for all cases, since there is only one independent variable.

In our example we used aluminum. If we take another metal, giving also an oxide as a result of combustion, it is possible to replace A_{Al} and a_{Al} by the corresponding quantities for this metal, leaving the formulas unchanged.

If hydrogen is replaced by gasoline or another fuel which gives only gaseous combustion products, we must transform the formulas correspondingly*.

Let us assume from the beginning that we did not take any hydrogen in the rocket. All the oxygen was used for the combustion of aluminum only.

It is simple to determine the values taken in this case by ξ, β and δ.

To do this we take $V_H = 0$ in the formulas derived above and get $\xi = \infty$. From (26) we have:

$$\beta = \frac{A_O}{A_{Al} a_{Al}};$$

from (31) we obtain

$$\delta = \frac{A_O}{A_{Al} a_{Al} + A_O}\left(1 - \frac{G_f}{G}\right). \tag{32}$$

* It is necessary to take into consideration the other combustion products too. In the case of gasoline CO_2 is obtained in complete combustion in addition to H_2O. – Editor's note.

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Next, from (19) we have

$$V_{Al} = \frac{1}{1 + \frac{A_O}{A_{Al} a_{Al}}} \tag{33}$$

and from (10):

$$V_O = 1 - V_{Al}.$$

In a similar way, when we have only oxygen and hydrogen, $V_{Al} = 0$; then, $\xi = 0$.

In this case, we obtain from (18):

$$V_H = \frac{1}{1 + \frac{A_O}{A_H a_H}} \tag{34}$$

and from (10):

$$V_O = 1 - V_H = \frac{1}{\frac{A_H a_H}{A_O} + 1}. \tag{35}$$

Furthermore, for this case $\beta = \infty$. From (30) we find

$$\delta = 1 - \frac{G_f}{G}. \tag{36}$$

All possible cases are situated between the limits $V_H = 0$ and $V_{Al} = 0$.

If $V_H > 0$ and $V_{Al} > 0$, then we obtain from (23), (18), (19), (20) and (21) ξ, V_H, V_{Al}, V_O and β as functions of δ; taking $\frac{G_f}{G} = \frac{1}{20}$ these expressions become:

$$\xi = \frac{1}{1.985\delta - 0.885} - 1; \quad V_{Al} = \frac{\xi}{(\xi + 1)\, 1.885};$$

$$V_H = \frac{1}{(\xi + 1)\, 8.95}; \quad V_O = 1 - V_H - V_{Al}; \quad \beta = \frac{1 - V_{Al}}{V_{Al}}.$$

The numerical values of all these quantities are given in Table I.

TABLE 1

$\delta = \frac{G_O + G_H}{G}$	0.446	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
$\xi = \frac{G_{Al_2O_3}}{G_{H_2O}}$	∞	8.18	3.84	2.26	1.47	0.98	0.656	0.421	0.246	0.110	0
$100 V_H$ %	0	1.22	2.31	3.42	4.52	5.64	6.74	7.85	8.95	10.05	11.16
$100 V_{Al}$ %	53.0	47.4	42.0	36.8	31.6	26.2	21.05	15.70	10.38	5.25	0
$100 V_O$ %	47.0	51.78	55.7	59.8	63.9	68.2	72.2	76.5	80.6	84.7	88.8
$\beta = \frac{G_O + G_H}{G_{Al}}$	0.885	1.128	1.380	1.717	2.16	2.82	3.75	5.37	8.55	18.05	∞

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If we wish to know the amount of hydrogen, aluminum or oxygen as a percent of the initial weight G, then the numbers $100V_H$, $100V_{Al}$ and $100V_O$ of the table have to be multiplied in our example by $G_p/G = 0.95$.

We should be interested here in the values of δ and V_0. We see that for the combustion of aluminum an amount of oxygen, equal to 47% of the weight of the aluminum oxide produced is required.

If the final weight of the rocket G_f is 1/20 of the initial weight, then the liquid material must amount to 44.6% of the initial weight; we see further that for $\delta > 0.446$, the rocket should contain more than 44.6% of liquid propellant, since in addition to the aluminum, we must burn hydrogen to get gaseous combustion products.

In order to reduce the amount of liquid fuel, oxygen could of course be taken in an amount insufficient for complete combustion, or we could make partial use of other metals, for example steel, which do not require much oxygen for combustion; furthermore, a light incombustible gas could be used in part instead of hydrogen. All these reduce the performance of the rocket; however, in view of the high calorific value of aluminum (or magnesium, lithium and certain other metals) this method may find applications.

(The values of δ and ξ are of extreme importance. The first, showing the weight fraction of the liquid propellants, determines the technical feasibility of the rocket.)

Airplanes may carry from 40 to 60% of liquid fuel, i. e., their full weight may be by 1.67-2.5 times larger than the weight of the unfueled airplane. Assuming that a rocket may carry up to 75% of liquid propellant, we obtain that its full weight may be 4 times the weight of the empty rocket.

(Therefore, we should consider as technically feasible in our example, rockets using metallic fuel with $\delta \leqslant 0.75$. On the other hand, the value of ξ, which gives the ratio between solid and gaseous combustion products, also determines the choice of δ. The value of ξ may be determined only experimentally. In case it is smaller than 0.656, the rocket on metallic fuel with $G_f/G = 1/20$, taken by us as an example, will be practically unaccomplishable at the present state of aviation technology.)

EXPERIMENTS ON THE PRODUCTION AND COMBUSTION OF LIGHT ALLOYS FOR HIGH ALTITUDE ROCKETS

Together with co-workers, I have carried out in the autumn of 1928 experiments on the production of light alloys containing magnesium and tested their combustibility in air. The motive for the experiments was my idea to use metals as fuel for rockets, high altitude semi-rocket airplanes, and spaceships in addition to liquid fuel, since liquid fuel only is not sufficient for the attainment of extreme altitudes and flight velocities. The most radical method – burning, as additional fuel, parts of the rocket (or airplane), made e. g. of Magnallium or Elektron, specially for this purpose - may give a mass ratio sufficiently high to achieve practically, with the appropriate design, the launching of payloads and man into interplanetary space and even to other planets.

It is therefore of paramount importance to demonstrate that alloys which are suitable for rocket construction, burn well and give high efficiency in a rocket. I have proved the last point theoretically.

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The thrust of a rocket is

$$P=\frac{dM_g}{dt}w_g+\frac{dM_0}{dt}w_0,$$

where $\frac{dM_g}{dt}$ and $\frac{dM_0}{dt}$ are the rates of ejection of the gas and solid particles (metal oxides); w_g and w_0, the corresponding ejection velocities.

Since in collisions of gas particles with solid particles the total momentum does not vary, i.e.,

$$dM_g w_g+dM_0 w_0=\text{const},$$

then for a given consumption

$$P=\text{const},$$

i.e., the rocket's thrust does not vary even if the solid particles have some lag. It is then assumed that the particles of the gas and of the metal oxide will behave like perfectly elastic bodies and that the rotation energy of the solid particles about their axis can be neglected.

We give below a list of alloys which may be of interest for constructing combustible parts of rockets (Table 2).

TABLE 2

Serial No.	Alloy	Melting point, °C	Remarks
1	65% Sn, 35% Pb	182	Eutectic alloy
2	95% Zn, 5% Mg	335	Eutectic alloy $\gamma=6.15$
3	60% Zn, 40% Mg	338	Eutectic alloy $\gamma=2.85$
4	75% Mg, 20% Al, 1.7% Cu, 3.3% Zn	–	Complete combustion of aluminum
5	88% Mg, 8% Al, 2% Cu, 2% Zn		Complete combustion of Elektron
6	59% Sn, 32% Pb, 5% Zn, 4% Mg	–	Ignites but does not burn completely
7	63% Sn, 34% Pb, 3% Mg	–	Burns only in powder form
8	80% Al, 20% Cu	–	
9	50% Al, 50% Cu	–	Alloy
10	92% Al, 8% Cu	638	Brittle alloy; incomplete combustion of aluminum
11	59% Al, 41% Mg	–	
12	87% Zn, 10% Mg, 3% Al	–	Brittle alloy; burns well
13	90.5% Zn, 4.75% Mg, 4.75% Al	–	Burns only in thin sheets
14	93.5% Zn, 3.2% Al, 3.3% Mg	–	Good mechanical qualities, slightly brittle, burns poorly

Difficulties in preparing the alloys

1. Since magnesium oxidizes readily, its mixtures must be prepared either under a protective film of NaCl and $MgCl_2$ ($\gamma=2.17$ and 2.32, melting

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temperatures 800° and 708°) or in an inert gas atmosphere. We succeeded in preparing Elektron by removing the flame from the crucible each time we introduced magnesium and subsequently added $MgCl_2$ to form a protective film of MgO.

F. Regelsberger* gives the following indications: "For the melting of magnesium and of alloys with high magnesium content, iron crucibles are used with tightly closing iron covers (it is recommended to use U-shaped vessels for melting light alloys). Clay or graphite crucibles should be avoided, since the magnesium may combine with the silicon contained in them, forming magnesium silicide which the metallurgists consider undesirable. Furthermore, due to the separation of other silicon salts, a variation of the metal's structure may occur and a danger of spoiling the cast object appears".

However, in order to prepare the casting mold, very fine grain sand, free from clay particles, and permanent molds consisting, for example, of aluminum brass (62% Cu, 31% Zn, 5% Al, 2-3.25% Fe) are frequently used in order to achieve rapid cooling or slow hardening if kept warm.

2. When magnesium, zinc and aluminum are melted, the oxides (e.g., MgO, ZnO) are always formed. It is interesting that the carbides of magnesium, zinc and aluminum give acetylene, C_2H_2, when water is poured over them; when HCl ($\gamma = 1.12$) is poured over them more acetylene and hydrogen are evolved. Aluminum carbide gives methane, CH_4.

3. By passing over molten zinc and also over an alloy of 60% Zn with 40% Mg gasoline vapors, which are ignited in a crucible, we succeeded in preventing the metal from burning, but at high temperatures carbide was formed. It is better to use hydrogen as protection against burning.

4. In order to prevent overheating of the metal, we prepared crucibles, enclosed one within the other, and filled the gap between them with molten zinc; in this way we succeeded in keeping the slightly oxidized metal in a crucible without cover.

5. By placing a cover over the crucible it was always possible to stop the spontaneous combustion of the metal, even of magnesium.

Evaporation of metals

1. Experiments on the evaporation of magnesium, zinc and cadmium have been contemplated but were not yet carried out. According to the indications of Kerl and Stohman good results may be obtained with magnesium**.

2. The following information*** is interesting. In order to prepare zinc oxide, it is heated in retorts to white heat; coming out of the retort the zinc vapors meet with air heated to 300°C, and burn ZnO.

3. By burning the magnesium contained in an alloy, the other metals are heated and after evaporation they burn too; this method leads to results more easily than the previous one.

* Regelsberger. Chemische Technologie der Leichtmetalle, S. 307. 1926.
** Kerl u. Stohman. Enziklopäd. – Handbush d. Technischen Chemie, B. II, S. 1047-1057.
*** Partheil. – Lehrbuch d. Chemie, S. 403. 1903.

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Combustion of alloys in the solid state

1. It is well known that a film of magnesium burns readily. If it is lubricated e. g. , by Gargoyle, it can be fed through a stuffing box into a high pressure chamber and used in a rocket.

There exist lamps for burning magnesium films for signaling at sea and for fog lanterns. Thus, the film feed can be mechanized.

2. Many other alloys burn well in air; such are, for example, the alloys Nos. 2, 3, 4 and 5 in Table 2.

It should be noted that by adding to alloys 5% magnesium or more they become combustible in many cases. The addition of zinc also increases the combustibility, although with the addition of copper, tin, and lead, it hardly changes. On the contrary, the addition of large amounts of aluminum hinders combustion due to the formation of a protective film of Al_2O_3.

3. The combustibility in pure oxygen or in heated air will be better than in cold air due to the considerable heating and evaporation of the metal.

4. By melting together aluminum and a well burning alloy, it is possible to achieve the burning of aluminum too. This method is applicable also to other metals.

5. I have tested a perforated cone (a rocket nozzle), in which a film of magnesium of some alloy was burned, by feeding gases from a bunsen burner through the perforations from an external steel cone.*

A deposit of 23% was obtained in the first model and one of 13% in the second model. Probably a design of a nozzle without a perforated cone is possible since the evaporation temperature of magnesium is 2800° C and its combustion temperature in air is 4500° C.

Spraying of liquid metal

(The spraying of liquid metal may be performed with an atomizer designed so that the liquid metal, flowing in a pipe, meets on its way a high velocity gas current which brings about the atomization. The corresponding diameters of the outlet orifices of the sprayer pipes and the flow velocities of the liquid metal and the gas can be easily calculated.)

Let us use the following notation:

d_1 and d_2, diameters of outlet orifices of pipes feeding oxygen (or air) and molten metal to place of metal spraying;

h_1 and h_2, excess pressures in mm of water at outlet orifices;

γ_1 and γ_2, specific weights of oxygen (or air) and molten metal;

ξ_1 and ξ_2, coefficients of head loss due to resistance at outlet orifices of pipes;

v_1 and v_2, outflow velocity in orifices of diameters d_1 and d_2 respectively;

V_1 and V_2, volume of oxygen (or air) and liquid metal consumed per unit time;

G_1 and G_2, corresponding weights, consumed per unit time;

$a = G_1/G_2$, weight ratio of oxygen (or air) to liquid metal, sprayed by it.

* For more detailed data on the experiments with this cone see the article: Problema poleta pri pomoshchi reaktivnykh apparatov (Problems of Flight with the Aid of Jet Propulsion Machines). –Editor's note.

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The velocities of the gas and the metal may be expressed by

$$v_1 = \sqrt{\frac{2gh_1}{\gamma_1(1+\xi_1)}}; \tag{37}$$

$$v_2 = \sqrt{\frac{gh_2}{\gamma_2(1-\xi_2)}}; \tag{38}$$

and since

$$V_1 = \frac{\pi}{4} d_1^2 v_1;$$

$$V_2 = \frac{\pi}{4} d_2^2 v_2;$$

$$G_1 = \gamma_1 V_1 = \frac{\pi}{4} d_1^2 v_1 \gamma_1, \tag{39}$$

$$G_2 = \gamma_2 V_2 = \frac{\pi}{4} d_2^2 v_2 \gamma_2, \tag{40}$$

then, dividing (39) by (40), we obtain

$$a = \frac{d_1^2 v_1 \gamma_1}{d_2^2 v_2 \gamma_2}.$$

Substituting here v_1 and v_2 from equations (37) and (38), we obtain

$$\frac{d_1^2}{d_2^2} = \frac{a\gamma_2}{\gamma_1} \sqrt{\frac{h_2}{\gamma_2(1+\xi_2)} \frac{\gamma_1(1+\xi_1)}{h_1}} = a \sqrt{\frac{h_2\gamma_2(1+\xi_1)}{h_1\gamma_1(1+\xi_2)}}$$

or

$$\frac{d_1}{d_2} = \sqrt{a} \cdot \sqrt[4]{\frac{h_2\gamma_2(1+\xi_1)}{h_1\gamma_1(1+\xi_2)}}. \tag{41}$$

Next, by substituting the expressions of v_1 and v_2 in (39) and (40) we find:

$$G_1 = \frac{\pi}{4} d_1^2 \sqrt{\frac{2gh_1\gamma_1}{(1+\xi_1)}}, \tag{42}$$

$$G_2 = \frac{\pi}{4} d_2^2 \sqrt{\frac{2gh_2\gamma_2}{(1+\xi_1)}}. \tag{43}$$

d_1 and d_2 are therefore immediately found:

$$d_1 = \sqrt{\frac{4G_1}{\pi} \sqrt{\frac{1+\xi}{2gh_1\gamma_1}}}. \tag{44}$$

$$d_2 = \sqrt{\frac{4G_2}{\pi} \sqrt{\frac{1+\xi_2}{2gh_2\gamma_2}}}. \tag{45}$$

(Air (for initial experiments) or oxygen should be taken as the atomizing gas and the metal should be burned immediately as it comes out from the atomizer.)

1. For the first experiments an alloy consisting of 65 % Sn and 35 % Pb may be used. In this case we should be able to obtain homogeneous melting, since this alloy constitutes approximately an eutectic alloy and is completely melted at 182° C. A different ratio of the metals would give only

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partial melting at a given temperature, which might either lead to obstruction of atomizers smaller than the outlet orifice of the tube, or cause the metal with lower melting point to be consumed first, with possible unfavorable consequences in the combustion of the atomized metal.

In Table 3 the physicochemical quantities, characterizing the constituent elements of the alloy mentioned are listed.

Tin is oxidized on the surface when melted; in heating to white heat it burns to tin oxide, emitting strong white light.

When lead is melted in open air it gets covered by lead "ash", consisting of a mixture of lead monoxide and dioxide. By continuous melting in open air, lead may be completely transformed into an oxide.

Tin vessels, covered by thick felt may be used successfully for storing liquid oxygen. Since at the temperature of liquid oxygen, lead becomes elastic and acquires the mechanical properties of silver, it could also be used for this purpose. This refers also to the above-mentioned alloy (65 % Sn+35 % Pb). By theoretical calculations we find that the amount of oxygen required for the combustion of this alloy is 20.19 %. If it is burned in air containing 23.1 % of oxygen, 87.3 % of the air is required for complete combustion.

TABLE 3

Material	Melting point, °C	Boiling point, °C	Atomic weight	Calorific value cal/mole	Combustion reaction
Tin	231.8	2200	118.7	141.3	$Sn+O_2$
Lead	327.4	1600	207.2	50.8	$Pb+O$

In the experiments, we succeeded in atomizing 65 % Sn+35 % Pb alloy by cold air and oxygen. Combustion was not achieved, but grains of diameter 0.1-0.3 mm shaped like threads and drops were formed. These results were obtained by microphotography. We did not obtain combustion due to the cooling of the alloy in the cold air*.

(2. Further, Tsander mentions his experiments on the atomizing of the 60 % Zn+40 % Mg alloy, and then of pure zinc and of anti-friction metal. All these experiments had the purpose of establishing the conditions for a satisfactory atomizer design. They showed the need of heating the air and reducing the path length of the molten metal from the crucible (or from another vessel) to the atomizer**.

Tsander constructed a special crucible with direct feeding of the liquid metal to the atomizer and with heating of the air in a copper tube, surrounded by a tin casing. With this instrument he conducted only preliminary experiments.)

* In Tsander's manuscript there is a detailed comparison of a rocket using the 65 % Sn + 35 % Pb alloy with one using gasoline. Since the final conclusions are in complete agreement with those of section 3 of the present article, this comparison has been omitted. – Editor's note.

** Since modern technology already has at its disposal satisfactory equipment for atomizing metals (for example, the well-known Shoop apparatus), Tsander's ideas on the possibility of atomization are now realizable.

The subsequent burning of the metal is also perfectly feasible. Here one may refer to the aluminum-oxygen welding burner (see: Novosti Tekhniki, No. 232, item 5555, 20 November 1932; or Tikhonravov. –Raketnaya Tekhnika, p. 21, ONTI. 1935). This burner worked on aluminum dust. – Editor's note.

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3. The atomizing of a metal with air and with ammonium nitrate, sulfur or some other material developing sufficient heat is also of interest.

4. Furthermore, metals may be used as powders or as powders distributed in some other material: for example, magnesium filings in resin or in celluloid.

(5. Tsander always paid much attention to the economical aspect of the problem and therefore further in his manuscript he gives a summary of the prices of the metals, which may be used as rocket fuels. At the present time this summary is not interesting, since it was composed on the basis of sources from 1900-1924 from the book of Regelsberger, Technologie d. Leichtmetalle. S. 347.)

On the design of high-altitude rockets

(Here Tsander mentions rockets with lateral liquid propellant tanks. First the liquid propellant is used and then the tanks burn in the main rocket which is equipped with a meteorograph and a gyroscope*. Further on, he presents two tables of the calorific values of certain metals and other compounds for combustion in oxygen or fluorine. A selection from these tables appears in the present book (see p. 97).

ROCKET ENGINES OPERATING ON METALLIC OR SOLID FUEL IN GENERAL

The design of such an engine should have the purpose of investigating the application of a solid fuel, mainly metallic and plastic. Due to the impossibility of reaching high altitudes and long ranges by means of liquid fuels only, it is proposed to construct a rocket of special design, enabling the use of a larger or smaller part of the rocket itself as fuel. This part should be made either of a metal or of plastic. As was shown in more detail in my book, Problem poleta pri pomoshchi reaktivnykh apparatov (Problems of Flight with the Aid of Jet Propulsion Machines) pp. 43-51 and 71-72, designs in which almost the entire mass of the jet propulsion machine serves as fuel are quite possible. The strength of materials does not restrict us any longer in these devices, – all depends on the skill of the designer. In my opinion, rockets which use a great part of their structure as fuel will be the first to succeed practically in sending vehicles to enormous distances, over the oceans and up to a complete escape from the Earth.

The importance of the experimental construction of rocket engines of the above-mentioned type is therefore obvious. The first experimental engine, should be adapted for mounting on a rocket. Possible fuels are, on one hand, powders of magnesium, berillium, aluminum, coal, etc. On the other hand, it is contemplated to melt their alloys, plastics, and, later, part of the structure in order to increase the rocket's range. In order to prevent clogging up of the combustion chamber and of the inlet section of the nozzle

* This section is, obviously, a conclusion of Tsander's lecture and referred to the immediate perspectives of using metallic fuel. As can be seen from the list of points given above which Tsander considered, he had in mind a stratospheric rocket with a gyroscope for maintaining stability. – Editor's note.

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by the solid fuel, a certain amount of liquid fuel: gasoline, ethyl alcohol, kerosene, etc., should be used.

As an oxidizer it is proposed to use liquid oxygen or nitrogen tetroxide. It is proposed to use the powdered fuel partly as thermal insulation for the hydrogen tanks.

According to the above description, three tanks are required: one – for liquid oxygen and its surrounding powdered fuel, the second – for the liquid fuel and the third – for the lumped solid fuel. The latter tank should be provided with fire-tubes for melting fuel lumps. The burning of the liquid fuel also takes place in the fire-tubes; a combustion chamber, surrounded by a jacket, inside which the melting of the solid fuel takes place, may be indicated as an alternative; in this case special heating of the solid fuel tank is not required.

It will probably be possible to feed the liquid fuel and the oxidizer into the combustion chamber by centrifugal pumps, and the molten metal – by an injector operated by the vaporized oxidizer. The centrifugal pumps may be driven either by a wind-turbine or by a special gas turbine or, finally, by using the nozzle jet to drive the turbine.

For better evaporation of the oxidizer it can be passed around the combustion chamber. It should be possible to replace the nozzle water cooling system and the oxidizer water-system heating by a similar one, in which the liquid fuel circulates instead of water.

The pipes should be provided with valves which will serve to control the flow or to shut off parts of the engine.

The rear part of the combustion chamber and of the nozzle may have a jacket designed to suck in external air which increases the reaction of the jet, acting as an air-breathing rocket engine.

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COMPARISON OF FUEL CONSUMPTION BETWEEN A VEHICLE USING ATMOSPHERIC OXYGEN AND ONE USING OXYGEN STORED IN THE ROCKET*

It is possible to conceive a flying apparatus in the form of an airplane, which is accelerated to an escape velocity in the Earth's atmosphere. At a flight velocity of 8 km/sec, the centrifugal force which appears in orbital flight is equal to the Earth's attraction force. Therefore, when flying at velocities exceeding 8 km/sec, the airplane should fly on its back so that the force on its wings should be directed downward and not upward; otherwise the apparatus will fly away from the terrestrial atmosphere and it will be impossible to accelerate it further if its engines use atmospheric oxygen.

These engines may be piston engines, turbines or rockets, and also combinations of rockets with other engines in which the first work as injectors in steam-boilers. Furthermore, the engines can be used to feed the rocket with liquid fuel and a certain amount of atmospheric air under pressure, while the rest of the air can be drawn into the rocket from all sides. If a sufficient velocity head is then formed in the rocket, it is possible to burn in the rocket itself the fuel mixture and obtain in addition to the work of the engines a large amount of work from the pressure and expansion of the gases burned in the rocket.

An advantage of this method of flight is the low fuel consumption, compared with the case in which the oxygen is stored in the rocket. However, the weight of the engines and of the propellers, which at high flight velocities turn out to be large and rather heavy, are a great difficulty.

In combinations of engines with a rocket, in particular if the latter burns also solid structural material, we have a flight method which may have great importance.

Let us use the following notation:

- g_0, gravitational acceleration at Earth's surface;
- G_0, total weight of device before take-off;
- G_1, weight of device at end of engine rocket operation, if weight is determined for acceleration g_0;
- G_f, weight of fuel required for flight (excluding oxygen);
- G_{ox}, weight of oxygen required for flight;
- $x = G_{ox}/G_f$, weight ratio of oxygen to fuel;
- H, calorific value per kg of fuel, in cal;
- A, work performed in lifting weight G to given height;
- R, Earth's radius.

In this notation, the work required to remove a load G_1 from the surface

* This article was written by Tsander in 1925. – Editor's note.

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of the Earth to infinity will be

$$A_0 = RG_1,$$

and the work required in order to bring a load G_1 to orbital velocity will be

$$A_{\text{crit}} = \frac{1}{2} RG_1.$$

If A is expressed in kgm, then

$$A = \eta H G_f \cdot 427, \tag{1}$$

where η is the average efficiency of the device, i. e., the ratio of the useful work to the total energy contained in the fuel.

The required weight of fuel is

$$G_f = \frac{A}{\eta H \cdot 427}, \tag{1a}$$

and the weight of oxygen

$$G_{ox} = xG_f = \frac{xA}{\eta H \cdot 427}. \tag{2}$$

Therefore, the total weight of fuel and oxygen is

$$G_f + G_{ox} = (1+x)\,G_f = \frac{(1+x)\,A}{\eta H \cdot 427}. \tag{3}$$

If the oxygen is not carried along, we obtain for the total weight of the device

$$G_0 = G_1 + G_f = G_1\left(1 + \frac{A}{G_1}\frac{1}{427\,H\eta}\right), \tag{4}$$

if it is carried along,

$$G_0' = G_1 + G_f + G_{ox} = G_1\left(1 + \frac{A}{G_1}\frac{1+x}{427\,H\eta}\right). \tag{5}$$

Let us determine the ratios of the total weight of the device to its final weight, i. e., G_0/G_1 and G'_0/G_1 for the case in which the propellants are hydrogen and oxygen. Let us assume that we wish to overcome the attraction of the Earth.

Then

$$R = 6370 \cdot 10^3\ \text{m}$$

$$H = 34\,000\ \text{cal/kg}$$

and

$$x = \frac{G_{ox}}{G_f} = \frac{16}{2} = 8.$$

Substituting these values of R, H and x in (4) and (5) we obtain

$$\frac{G_0}{G_1} = 1 + \frac{6370 \cdot 10^3}{427 \cdot 34\,000\eta} = 1 + \frac{0.439}{\eta} \tag{4a}$$

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and

$$\frac{G_0'}{G_1} = 1 + \frac{(1+8)\,0.439}{\eta} = 1 + \frac{3.95}{\eta}. \qquad (5a)$$

If we wish only to orbit the Earth, then, as was shown above, the required useful work will be

$$A = \frac{1}{2} R G_1.$$

Consequently

$$\frac{G_0}{G_1} = 1 + \frac{0.439}{2\eta} \qquad (4b)$$

and

$$\frac{G_0'}{G_1} = 1 + \frac{3.95}{2\eta}. \qquad (5b)$$

The ratio of the fuel's weight to the total weight of the device will be in the first case:

$$\frac{G_f}{G_0} = 1 - \frac{G_1}{G_0} = \frac{\frac{A}{G_1}}{\frac{A}{G_1} + 427H\eta} \qquad (6)$$

and in the second case

$$\frac{G_f + G_{ox}}{G_0'} = 1 - \frac{G_1}{G_0'} = \frac{\frac{A}{G_1}}{\frac{A}{G_1} + \frac{427H\eta}{1+x}}. \qquad (7)$$

According to formulas (4)-(7) calculations were done of the ratios of the initial to the final weight of the device and of the fuel weight to the initial weight of the device in the cases when oxygen is carried along and when it is taken from the atmosphere. In the two cases two calculations were made, one for escape velocity, the other for orbital velocity.

All calculations were performed for various efficiencies of the apparatus. The results are presented in Table 1.

This table gives a clear idea of the amount of fuel that must be carried and of the ratio of the initial to the final weight of the device, if the average efficiency of all the engines, including the rocket, is known. As is seen from the table, it is possible to orbit the Earth for $\eta = 0.20$, with $G_f : G_0 = 0.523$ in case I; this is very difficult to accomplish. Case II requires, as can be seen from the table, an even larger consumption of fuel, since the exclusive use of hydrogen and oxygen requires an amount of fuel plus oxidizer constituting 90.8 % of the initial weight of the vehicle for orbiting the Earth if η is 0.2.

It is seen from formulas (4) and (6) that case I requires a fuel whose calorific value H is as great as possible. This may correspond to hydrogen. From formulas (5) and (7) it is seen that case II requires a fuel for which $H : (1+x)$, i.e., the calorific value per kg of mixture, is a maximum.

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TABLE 1

		Overcoming the entire gravitation of the Earth $A/G_1 = R$				Orbiting the Earth $A/G_1 = R/2$			
Efficiency		1.0	0.5	0.2	0.1	1.0	0.5	0.2	0.1
Case I Only fuel (not oxygen) is carried	Ratio of initial to final weight G_0/G_1	1.439	1.878	3.195	5.39	1.220	1.439	2.098	3.195
	Ratio of fuel weight to initial weight of device G_f/G_0	0.305	0.467	0.687	0.814	0.180	0.305	0.523	0.687
Case II Fuel and oxygen are carried	Ratio of initial to final weight of device G_0'/G_1	4.95	8.90	20.75	40.5	2.98	4.95	10.89	20.75
	Ratio of fuel + oxygen weight to initial weight of device $(G_f + G_{ox})/G_0'$	0.798	0.8875	0.9518	0.9753	0.666	0.798	0.908	0.9518

TABLE 2

Name	Components	Compound	Molecular weight	Calorific value per kg of mixture
Lithium oxide	Li_2+O	Li_2O	30	4 710
Boron anhydride (amorphous)	B_2+O_3	B_2O_3	70	3 900
Aluminum oxide	Al_2+O_3	Al_2O_3	102	3 730
Magnesium oxide	Mg+O	MgO	40	3 690
Water (gaseous products)	H_2+O	H_2O	18	3 240
Water (liquid products)	H_2+O	H_2O	18	3 830

There are several mixtures of metals with oxygen, for which $H:(1+x)$ is larger than for hydrogen with oxygen. Using solid material, i.e., metallic sections of the apparatus which are not required for further flight, we will be in a position to overcome the gravitation of the Earth by consuming, for $\eta = 0.2$ (as is seen from Table 1), 95.2 % of the initial weight of the apparatus as fuel.

The final weight of the device will then be approximately 20.8 times smaller than its initial weight.

Below we give a table of the calorific values of various metals and other fuels (Table 2), which are distinguished either by a high calorific value per unit weight of their mixture with oxygen or by the simplicity of their use.

If it will be possible to use ozone instead of oxygen, then the calorific value is increased by 0.640 cal per each gram of it in 1 kg of the compound.

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FLIGHTS TO OTHER PLANETS

Paper One*

As a result of my long standing interest in the mathematical and design aspects of the investigation of inter-planetary flights, I have performed detailed calculations regarding this problem. My conclusion is that at the level of present-day technology flights to other planets will become possible, during the next years.

I have worked out the following principal points:

1. For flight in the upper layers of the atmosphere, and also for landing on planets which possess an atmosphere, it will be advantageous to use an airplane as the structure supporting the spaceship in the atmosphere. Airplanes which are able to perform a glide landing in the case of engine failure are much superior to a parachute which was proposed for landing back on the Earth by Oberth in his book The Rocket to the Planets.

When using a parachute one is not free to choose the landing place and to fly further in the case of temporary stoppage of the engine so that it should be used only for unmanned flights. That part of the rocket which is controlled by a man should be equipped with an airplane. For landing on a planet which possesses a sufficient atmosphere the use of a rocket, as proposed by K. E. Tsiolkovskii, will also be less advantageous than a glider or a powered airplane, since a rocket consumes a large amount of propellant on landing. Even a one-man rocket will cost tens of thousands of rubles, whereas an airplane landing costs only several tens of rubles, and glide landing costs nothing. The calculations carried out show clearly the perfect possibility of a slow safe glide landing on the Earth.

2. In the lower layers of the atmosphere the flight velocities have to be low and they can be increased continuously with increasing height over the Earth's surface as the air density decreases.

3. The driving power in the lower layers of the atmosphere should be supplied by a special high-pressure engine which operates on fuel and liquid oxygen. The engine should drive propellers which may have either adjustable or ordinary blades. In the last case the engine should be so designed that on the Earth it has a low number of rotations and as the altitude increases the number of rotations increases too. Instead of the propeller group we may use a rocket which is capable of flying in the air and supplying its thrust in the flight direction.

By "rocket" we mean a nozzle analogous to turbine nozzles: through its narrow throat the combustion products proceed into the nozzle under high pressure; the gases, repelled from the nozzle walls, expand rapidly and

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acquire a high velocity, equal to 4000-5000 m/sec in the direction of the nozzle's axis.

A rocket capable of flying in air is one which sucks into the nozzle external atmospheric air. Upon mixing with the gases, a lower ejection velocity is obtained but the ejected mass and the efficiency are larger than for an ordinary rocket, whose efficiency at flight velocities up to 400 m/sec is very small.

4. At flight velocities exceeding 400 m/sec, either a rocket capable of flying in the air or an ordinary pure reactive rocket should be used to supply the driving power.

5. During the flight of the rocket parts of the supporting surfaces (propellers, engine and other sections of the airplane) should be drawn into the rocket and melted in a special chamber; the molten metal should then be ejected to improve the rocket's performance. For this purpose the airplane should be of suitable design, equipped with cables and devices to carry out all the necessary displacements. According to the calculations performed, the weight of the dismountable airplane will be only slightly larger than that of an ordinary airplane.

6. At velocities close to 8 km/sec, it is advantageous to fly away from the atmosphere under a small inclination to the horizon, since at this velocity the centrifugal force, appearing as a result of orbiting the Earth, is equal to the Earth's attractive force, i.e., the vehicle, left to itself, will not fall back to the Earth and if it is situated already outside the atmosphere it will orbit the Earth forever like the moon. The air, which served as a support for the airplane would now only slow down the vehicle; in interplanetary space, the airplane is no longer necessary and serves again only for landing in the atmosphere.

7. When we wish to fly to other planets, velocities of 11-18 km/sec must be attained. In this case it is possible to use a rocket but it will be probably more advantageous to fly with the aid of mirrors or screens made of thin sheets. The screens should rotate about their central axis in order to give them rigidity. The mirrors do not require fuel and in case of need they may be used as fuel in the rocket. These are the two advantages of the mirrors; in addition they do not produce large stresses in the material of the ship and have a smaller weight than that of a rocket together with the propellant. However, the mirrors can be damaged by meteors more easily than a rocket.

8. Instead of screens it will very likely be possible to use rings in which an electric current will flow. Inside the ring there will be iron dust, held close to the ring plane by the forces of the electric field. The dust specks should be charged with static electricity so as to maintain a certain distance from one another.

If sunlight falls on a mirror, screen or dust, it exerts a certain pressure on them. At the immense distances of interplanetary spaces weak forces give relatively high flight velocities.

9. If we built in interplanetary space huge concave mirrors which will rotate together with astronomical sensors around the planets, then solar light, concentrated by the mirrors and directed on the spaceship flying to another planet, would give velocities exceeding by many times the spaceships.

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10. On the basis of all these considerations it is possible to construct a series of spaceships.

According to my calculations the following results may be achieved.

The huge dimensions of the carrier rocket can be eliminated. Tsiolkovskii proposed to use a rocket for interplanetary trips, but not in combination with an airplane. In his proposal the reaction force of the rocket supports the entire weight of the spaceship and accelerates it too. Such rockets are called carrier or lifting rockets. According to my design the rocket is located in the airplane and its reaction force should be between 1/3 to 1/7 of the ship's weight. It is much simpler to build such a rocket than Tsiolkovskii's huge rocket. In my design the stresses in the material will be much smaller than in the carrier rocket.

Further, using the structural material of the airplane as fuel reduces the stresses in the spaceship, since it enables partial replacement of liquid fuel by solid structural material and this increase in the amount of structural material makes it possible to distribute the acting forces over larger beam cross sections.

The possibility of consuming 9500 kg of a ship's weight of 10,000 kg, so that 500 kg remain (the weight of small terrestrial airplanes) appears practical. This ensures completely the attainment of the immense velocities required for overcoming the attraction of the Earth. This large fuel consumption eliminates the need of using very high-energy propellants.

The large acceleration produced by the lifting rocket is the reason for the appearance of a large apparent weight which forces the pilot to lie in a bath filled with liquid during the acceleration period. In my rocket design this is eliminated, since its accelerations are much smaller and the period of accelerated motion can be longer than in the case of a lifting rocket.

Since the engine and the rocket can be stopped and restarted during the flight, it is very simple to conduct experiments with a spaceship of my design, gradually increasing the altitude and the flight velocity.

The combination of rocket and airplane, and the use of the airplane's structural material as fuel in the rocket eliminates the obstacle to interplanetary flights, consisting of the lack of a propellant with sufficiently high energy, mentioned by Ya. I. Perel'man.

In my design no very high-energy propellants are required. The very heavy carrier rocket is replaced by a rocket which is lighter by a factor of 10 to 30 than the lifting rocket, mentioned by Perel'man, and thus, the obstacles to interplanetary flights, also mentioned by him, are eliminated.

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FLIGHTS TO OTHER PLANETS

Paper Two

Is there anyone who, looking at the sky on a clear spring night and seeing the twinkling stars, has not thought of the possibility that the distant planets may be inhabited by intelligent creatures a thousand years more culturally advanced than ourselves? What incalculable cultural wealth may be brought back to Earth through science should man be capable of travelling there, and how cheap is the cost of such a tremendously important achievement in comparison to the vast amounts that are wasted by humanity? A million airplanes weighing 10,000 kg each could be built with the money spent on the last world war. It is very likely that the first spaceships, which will carry only one person, will not be heavier than these airplanes.

In what follows, the author of this article will attempt to acquaint a wide circle of readers with the mathematical and engineering studies he has conducted for many years in order to clarify all problems related to spaceships and space travel.

Till now only the booster rocket, with a thrust exceeding by a factor of 4 to 10 the weight of the entire spaceship, has a scientific basis for space travel. Such a booster rocket can be compared to the helicopter, i. e., a device which is driven by an engine with lifting propellers.

A booster rocket requires a tremendous thrust as compared with a rocket mounted on an airplane; the dimensions of the latter are therefore smaller. As a new design this is simpler to build than the booster rocket and the combination of a rocket with an airplane would enable us to exploit, for space travel, the immense experimental material accumulated in aviation.

The author proposes, to the best of his knowledge for the first time, to use for space travel a rocket mounted in an airplane instead of an ordinary rocket supporting the spaceship. The airplane would be accelerated, still within the limits of the atmosphere, to the velocity required for orbiting the Earth like a satellite (approximately 8 km/sec). With the aid of such a rocket it is easy to leave the terrestrial atmosphere and then fly to other planets.

These rockets should supply a thrust not larger than the propeller thrust of an ordinary airplane. They are therefore 10-30 times smaller than booster rockets. As to the amount of fuel which should be carried along, the calculations show that it should constitute from 90-98 % of the entire weight of the vehicle in order to overcome completely the Earth's attraction, i. e., in order to attain a velocity of 11.18 km/sec. The farther a body having this velocity goes from Earth, the more it slows down, and will reach

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zero velocity at infinity, never returning to Earth. This 90-98% is required if hydrogen and oxygen are used as propellants. For gasoline and liquid oxygen, the above-mentioned fraction is sufficient only to achieve orbital velocity.

The use of gasoline and oxygen for the experiments would be desirable. This, however, gives rise to the following problem. Our present airplanes may carry only 40-60% of liquid fuel, and 95% propellant is needed if we assume a low efficiency of the devices at the beginning. Otherwise we will run short of fuel and be forced to return to the Earth, not having attained that minimum velocity of 8 km/sec necessary to equilibrate gravity and orbit the Earth out of the atmosphere. How are we, therefore, to prevent this shortage of propellant? In designing the spaceship, I introduced a radical step, which, to the best of my knowledge, has not yet been proposed by anyone.

The cost of an airplane is in general insignificant. It is of even less consequence when we are dealing with interplanetary travel and the conquest of whole planets by the inhabitants of the Earth. Therefore, I propose the following method. As the fuel is consumed and the weight is reduced, the airplane wings should be pulled in or folded back, the frame should be pulled into the fuselage where we set up a melting chamber which can be either left open or closed and there we melt the retracted sections of the airplane as well as its engine*. We assume that all parts are, as far as possible, made of duraluminum or some similar alloy. Certain parts of the engine should have steel, cast iron or bronze bushes, which, like other high-melting metals, will remain in a special net in the melting chamber. After melting, the liquid metal is fed by an injector or by a centrifugal pump into the rocket and burns there with oxygen together with gasoline or hydrogen. According to my calculations, the immense heat which is obtained by the burning of aluminum in oxygen is transferred in great part to the volatile combustion products of gasoline or hydrogen and only a small fraction of the heat developed remains in the aluminum oxide. This is a result of the fact that the specific heat of gases increases strongly with temperature while the specific heat of solid or liquid bodies (of aluminum oxide) is small. Instead of aluminum it is also possible to use some alloy of magnesium, e.g., magnalium. At atmospheric pressure, aluminum boils at a temperature of 1800°, and magnesium at only 1200°C. It is possible that even lower-melting alloys of sufficient strength will be found. Aluminum and magnesium release more heat than hydrogen per kg of fuel-oxygen mixture: 3730 cals against 3240 cals per kg for the lower calorific value of hydrogen. Lithium releases 4710 cals, i.e., 1.45 times more heat than hydrogen with oxygen. My calculations in which I assumed that individual particles are small diameter spheres (having thus minimum surface for a given volume), have also established: a) the aluminum oxide particles have enough time to transfer a large part of the heat while still in the rocket and b) the aluminum oxide particles flying out of the exhaust hardly lag behind the particles of the gaseous combustion products consumed simultaneously in the rocket. It follows therefore that the efficiency of a rocket using simultaneously materials with solid and gaseous combustion products is only slightly lower than if we used materials with gaseous products only.

* The internal combustion engine with which the airplane is equipped in order to overcome the lower, denser layers of the atmosphere is meant here. – Editor's note.

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The only objection, therefore, to the use of solid combustion products is the possibility of the rocket's contamination by the metal oxide. On one hand, our industrial pipes through which a huge amount of fuel flows are not cleaned every day and, on the other hand, vibration of the rocket, the very high particle velocities and the spaceship's acceleration during flight, force the metal oxide particles to fall from the rocket's internal walls. Furthermore, it is possible to let the gases flow near the walls and the aluminum oxide nearer the rocket's axis.

In this way, i.e., by using sections of the airplane's structural material, it will be possible, even with low efficiency, to achieve velocities and altitudes sufficient to get beyond the terrestrial atmosphere and even to fly to other planets already now, with present day technology. There is nothing extravagant in this if we remember that larger airplanes of poor manufacture, intended for a single flight only, were built already during the First World War. Furthermore, in our case we should take into account the large amount of heat evolved by aluminum-oxygen combustion, so that the amount of fuel required will be small.

Taking an initial weight of 10,000 kg for the spaceship and 95% propellant (gasoline, oxygen and aluminum), the final weight would be 500 kg, i.e., the weight of our small terrestrial airplanes. Suppose we take off from Earth either in an airplane driven by a rocket capable of operating in the air, e.g., of the Melo design, or, as in the case I considered, in an airplane with a high pressure engine working on liquid oxygen instead of atmospheric air. Suppose also that we rise at a certain angle, adjusting the flight velocity as a function of the altitude so that the air resistance remains a minimum for a given lift force; the total air resistance will then not be large. At a certain altitude over the Earth we can stop the engine and start the rocket instead, dismantling gradually sections of the apparatus so that finally only the spaceship, weighing 500 kg, will remain.

At the comparatively low velocities at which one has to fly in order to reduce the resistance up to an altitude of approximately 28 km, a rocket which does not use external air has a low efficiency. Therefore, up to the above-mentioned height, it is necessary to fly either by means of a rocket drawing additional air, or by a propeller engine of special design.

If such an airplane will have a velocity of 30 m/sec at the Earth's surface, then at a height of 28 km, according to my calculations, it should fly, as a result of the decrease in atmospheric density, with a velocity of approximately 400 m/sec. At such a velocity it is already possible to use a pure rocket, and when flying higher, to regulate the thrust of the rocket so that its velocity will increase in accordance with the height and with the corresponding decrease in atmospheric density. Calculations have shown that a velocity of 11.18 km/sec will thus be attained at a height of approximately 85-90 km above the Earth's surface, whereas a velocity of 8 km/sec is already sufficient to fly away from the terrestrial atmosphere and start circling in the vacuum around the Earth.

In order to return to Earth, it is advantageous to give the remaining small spaceship the shape of an airplane (second airplane) with lifting surfaces.

This combination of rocket and airplane and the use of the airplane's structural material as fuel, eliminates, by the way, also the obstacle to interplanetary flights mentioned by Ya.I. Perel'man in his book

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"Mezhplanetnye puteshestviya" (Interplanetary Travel). On page 77 (edition 4-5, 1923) he says: "The main, perhaps even the only, obstacle to the immediate accomplishment of a jet propelled spaceship is the lack of a sufficiently powerful propellant. At the present state of technology we are not aware of a source which could develop a force sufficient for driving a huge rocket".

In my proposed spaceship, however, no powerful propellant is necessary since the 95% of conventional propellant required is obtained by pulling into the chamber parts of the airplane and using them as fuel; furthermore, I replaced Perel'mans very heavy rocket by one 10-30 times smaller.

It would be desirable to conduct suitable experiments with separate sections of the mechanism of such a spaceship. In particular, institutes manufacturing liquid hydrogen could conduct experiments with small rockets working on hydrogen and oxygen and also on light metals, i.e., aluminum or magnesium, as described above. Such experiments could lead to much progress.

In addition, experiments on the use of a light-weight melting chamber for light metals are needed. The problem of dismantling the airplane and the rudders has been already solved in principle by aviation technology and there exist nowadays airplanes which pull in the frame and the wings. Due to the use of hinged connections, gears and mechanisms for pulling in the parts, the increase in the structure's weight, according to my calculations, is small since the required forces and movements are small. Furthermore, it should be noted that by increasing the amount of solid combustion products, the amount of liquid fuel is reduced and at the same time the proportion in which magnesium or some other metal alloy is added to the gaseous combustion products is also reduced.

Nowadays, airplanes weighing much more than 10,000 kg are already built. They can be controlled by starting and shutting off auxiliary engines of simple construction which can subsequently serve as fuel. It follows therefore, that if we use a spaceship of 30,000 kg initial weight, which reduces, after leaving the Earth's atmosphere, to 500 kg, then the efficiency of all the engines could be much lower than in the case of an initial weight of 10,000 kg; the required final velocity would nevertheless have been attained.

If we mount on the airplane lower power engines and smaller rockets, the flight will be more gently sloping and the work needed to overcome the air resistance will increase as compared with the climbing and acceleration work. In this case, the rocket should be started earlier, when the velocity of the vehicle, and consequently, the efficiency of the rocket, are still low. In return we have the big advantage that the stresses in the material of the engines will be smaller than in large engines and rockets. The thrust of the engines can be reduced to that of our ordinary airplane propellers. In these conditions flights to other planets will be safe and will be easily accomplished but the initial weight will be very likely somewhat higher than 10,000 kg.

Flight by means of a rocket is only advantageous up to a velocity of 8 km/sec, i.e., up to that velocity at which it is possible to orbit the Earth, stop the operation of the rocket and rest as on a natural station. There is no danger of falling back to the Earth, since the centrifugal force appearing when orbiting is equal to the attraction force of the Earth. If we want to fly to other planets, however, a velocity larger than 11.18 km/sec is required.

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To reach Mars, we need according to calculations, at least 2.7 km/sec more, so that altogether $\sqrt{11.18^2+2 \cdot 7^2} = 11.5$ km/sec is required. For the closest planet – Venus – an additional velocity of 2.3 km/sec is required or altogether 11.4 km/sec. To reach other planets a still greater velocity is required.

Under these conditions, the rocket consumes unnecessarily huge amounts of propellant in each flight. Even if it operates on hydrogen and oxygen, we obtain the following figures: if the weight of the vehicle on Earth was 10,000 kg, it will, due to propellant consumption, weigh about 1000 kg when it reaches a velocity of 8 km/sec, and only 500 kg at 11.18 km/sec. For a further increase in the velocity the weight is reduced in geometrical progression.

If, on the other hand, we calculate the weight of an aluminum mirror of 100,000 m^2 area and 0.001 mm thickness which could supply a spaceship of 500 kg weight with a sufficient light force (even if the direction of this force is unfavorable), we find that for flight, for example, to Mars, the weight of this mirror will be approximately 300 kg. In the mirror's design it is assumed that it has a central axis and thin wires which maintain its form and transfer the slow acceleration of the axis' rotation to the umbrella itself. The above-mentioned thickness of the aluminum sheet has a linear tearing strength of 1.66 kg per cm and can withstand a rather large centrifugal force. Since the light pressure on such a mirror is not larger than 46 g/m^2, the entire mirror will maintain its form. There exist, for measuring light pressure, aluminum sheets whose thickness is only 0.0004 mm (see the article edited by Professor Lazarev in the journal "Uspekhi Fizicheskikh Nauk", Vol. I, No. 2, p. 144. 1918). The thickness at which silver sheets start passing light is still smaller (see the article in the journal "Annalen der Physik", No. 14, pp. 763-790. 1915, about experiments on the refraction index of thin metallic layers). Edison already prepared nickel sheets of 0.001 mm thickness and 1600 m × 2 m area (see Tsiolkovskii, Gondola metallicheskogo dirizhablya, (The Gondola of the Metallic Dirigible) p. 24). Had Edison been asked to prepare from these sheets the above-mentioned type of mirror for a spaceship, it is very likely that he would have solved this problem easily*.

If the mirrors are made larger, the duration of the flight is shortened. In addition, they may be used for many flights, whereas the expensive propellant of a rocket is consumed in one flight. Further, by means of still larger mirrors in permanent orbits around the planets and directed by telescopes, it should be possible to concentrate the solar light and direct it to a mirror which drives a spaceship to another planet. Flight time will then be much shorter than it is possible to achieve with rockets since their weight increases in geometrical progression when the velocity increases in arithmetic progression, while the mirror's weight increases only proportionally to the square of the vehicle's velocity. To attain a velocity of 100 km/sec one needs a mirror 100 times larger than that required to attain 10 km/sec (the same path length in both cases), and the initial weight of a rocket has to be 37,000 million times larger than that required to attain 10 km/sec.

We arrive at the conclusion that a rocket with its huge propellant consumption and large thrust should be employed only to get out of the

* Goldsmiths hammer gold for plating to a thickness of 0.0001 mm. – Editor's note.

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terrestrial atmosphere and to accelerate up to 8 km/sec, and eventually only for rapid variations of its trajectory to avoid meteoric currents. In combination with an airplane, the rocket produces acceleration already in the lower layers of the Earth's atmosphere starting from small velocities, and, as though sliding slantwise upwards through the air it reaches the required velocity already in its upper layers. In interplanetary space with its huge distances and the possibility of applying small forces, it is much better to use light pressure or transmission of light energy to distances by means of very thin mirrors; these should rotate in order to gain rigidity as do, for example, the flexible propellers of dirigibles of the Parseval system.

As regards the use of thin mirrors, Perel'man is mistaken when, in his book "Mezhplanetnye puteshestviya" (Interplanetary Travel), he denies the possibility of using them for interplanetary travel. It should be stressed that mirrors cannot be used to escape from Earth (in the case considered above, a light force of 500 kg would be needed for this, corresponding to a mirror area of about 1000 km^2, where 1 m^2 weighs 1 mg). Mirrors are perfectly applicable for flights in interplanetary space itself, however, if the trajectory of the spaceship does not intersect a planet or its atmosphere (1 m^2 should then weigh 10 g, which is perfectly attainable in practice).

It is also worthwhile considering an airplane, with all its parts subject to compression or bending, with hollow beams manufactured from pipes containing gaseous or liquid propellant under high pressure. If these pipes are fitted with valves for discharging the propellant at their ends, then they can be compression loaded to half the stress admissible for wires, and therefore, they will be much lighter than legs and longerons of ordinary structures. If the beams are filled with liquid methane, ethane, methyl* or some similar fuel, which does not have such a low boiling temperature as liquid hydrogen, there is another possible advantage: by lowering the temperature the strength of the materials increases considerably (the brittleness, however, is also increased). If the temperature is suitably lowered so that the pipes become considerably stronger but not too brittle, it will be possible to use very light ones. This would allow the amount of liquid fuel to increase, and would make for economy in solid structural material.

The use of a rocket drawing in air for combustion, should also be investigated. If it is mounted on an airplane, the flight up to a speed of 8 km/sec will take place in the Earth's atmosphere, and therefore at least part of the oxygen for combustion will be taken from the atmosphere. In this way a rocket containing only 60%, or slightly more, fuel, might reach a velocity of 8 km/sec. Then it will be possible to use only a small amount of metal for combustion, leaving the remaining parts of the airplane for emergencies.

Some of our proposals have already been realized: airplanes with folding frames and wings are already being manufactured; airplanes are being built of duraluminum; there are airships which are much larger than those required for a spaceship; a rocket capable of flying in air has been produced; liquid oxygen is taken along for high-altitude flights and was used

* [Methyl-alcohol is probably meant.]

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in an automobile engine in the Paris Exhibition already in 1900. We have approached so very close to the possibility of flight beyond the terrestrial atmosphere, that for the type of spaceship proposed, only some experiments and a comparatively small expenditure are required to realize flight to other planets.

The main experiments should be conducted in the use of molten metals as rocket fuel, on the construction of the chamber for melting the metal, and also on finding the most convenient design for dismantling the airplane and feeding its sections to the melting chamber. Further, it is possible to construct, for studying air resistance and the heating of airplane sections at low pressures and very high velocities, a wind tunnel using two cones joined at their broad ends, and a ventilator. It is essential that all scientists and engineers interested in this problem should study the design problems and conduct as many experiments as possible in this direction; this could push forward an extremely interesting field of science and technology, most promising for the future.

In conclusion, the author considers it his duty to say that in his subsequent works he will present calculations confirming the assumptions made in the present paper.

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DESCRIPTION OF TSANDER'S SPACESHIP*

High-altitude rocket designs proposed till now have the following drawbacks:

1. The rocket's huge size increases the explosion danger since fuel and oxidizer constitute a great fraction of the entire rocket's weight.

2. The huge amount of fuel needed for landing on Earth with the aid of a reaction force increases the weight many times and makes the flight considerably more expensive.

3. Dynamical support of the rocket by the gases ejected may lead, even when using a parachute, to a catastrophe in the case of accidental rocket failure.

4. When landing in the atmosphere by means of a parachute, or by operating the rocket, or by both methods together, one cannot choose the landing place freely and the consumption of an additional amount of fuel is required.

5. To attain cosmic velocities one must use tremendous accelerations which have a negative physiological effect on man's organism.

6. Finally, the need to have a low G_f/G_0 ratio, where G_f is the structure's weight and G_0 is the initial weight, makes all the proposals impractical from structural considerations.

In my design, the rocket is coupled structurally to two airplanes: a large one for take-off, and a second, much smaller one, for landing. At low altitudes the rocket is driven by a special kind of engine, since normally a rocket has low efficiency at low velocities.

Sections of the large airplane's structure are used as additional fuel together with the liquid fuel, since the latter is not sufficient for reaching cosmic velocities. It is proposed, therefore, to build the airplane of duraluminum, Elektron or some other similar alloy.

The proposed rocket (Figure 1) consists of a body, externally similar to a projectile. The wings of a large folding airplane with retractable frame and special kinds of engine driven propellers are attached to the body. The large airplane's rudders are attached at the back of the body and they, as well as the wings and the propellers, can be pulled into the body. Inside the body there is a small airplane with rudders, frame, propeller and engine. In addition it contains propellant tanks, a chamber for melting the sections of the large airplane after pulling them in, and a rocket engine operating on liquid propellant and metal. The combustion chamber is located under the melting chamber. The liquid propellant coming out of the

* First published in the Journal "Raketnaya Tekhnika", No. 5. 1937. In the collection of Tsander's articles (Oborongiz, pp. 24-29, 1947) this paper was published under the name "Konstruktsiya daleko letayushchei rakety" (Design of a Long-Range Rocket).

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tanks first cools the combustion chamber, is then injected into it and burns to gases which melt the metal and then proceed to the rocket's throat. The molten metal proceeds through a special pipe to the combustion chamber either by an injector or by a special pump, being heated in that part of the pipe entering into the combustion chamber. There the liquid metal is sprayed, partly evaporated and mixed with oxygen, and burned in the chamber. The combustion of a metal with high calorific value increases the temperature of the gaseous products and therefore increases considerably the rocket's efficiency. Sections of the airplane (propellers, frame rudders, etc.,) are pulled into the body through openings in the vertical lateral protrusions of the body.

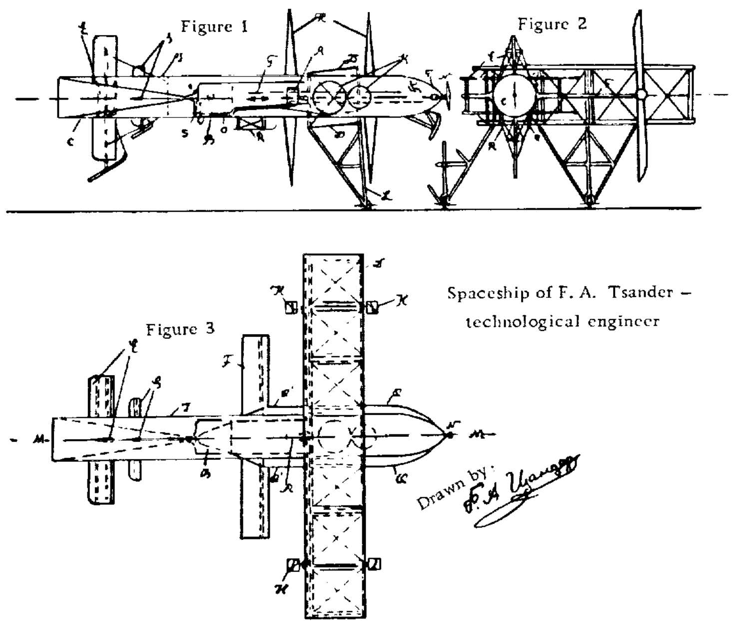

FIGURE 1. Tsander's spaceship (project)

If the quantity of fuel required for attaining cosmic velocities is very large, the fuel tanks and the engine* can also be melted. The melting chamber must then be provided with a special grid to prevent high-melting materials from getting into the discharge pipe. Airplanes of various types can be used and it is also possible to use the rocket's body as a supporting surface.

The proposed rocket climbs as an ordinary airplane. A special high-pressure engine using liquid oxygen, enables the airplane to climb to an altitude of about 25-30 km over the Earth's surface.

With decreasing air density, the flight velocity should be increased in such a way that the wings' angle of attack, required to support the airplane

* The author refers here to the large airplane's engine which drives its propellers. – Editor's note.

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at a given altitude, does not vary strongly. At an altitude of about 28 km over the surface of the Earth, a velocity of 350-450 m/sec is attained. Up to this altitude, where the rocket should be started, the engine's exhaust gases may be let through it to increase thrust. The airplane engine should be stopped when starting the rocket (or slightly later), since the propellers cannot stand too high peripheral velocities; at lower peripheral velocities they will have a low efficiency.

The frame is pulled into the melting chamber, melted, and then used as fuel for the rocket. The latter ascends higher and higher accelerating during the flight, while with decrease of the rocket's weight, the wings are pulled in. The two extreme external sections of the wings are symmetrically pulled in first, followed after some time by the second section, then the third one, and so on.

The sections may be retracted in various ways, e.g. as in shortening a telescope; this is performed easily in monoplanes. The external sections may be pulled into the melting chamber through all the remaining sections of the wings and there is no particular need to strip off the wings. The large airplane's rudders are next pulled in and melted. The engine and its tanks are also introduced into the melting chamber and the length of the ship's body is shortened. Finally we melt the pulling in mechanisms themselves.

Towards the end of the acceleration period, when the ship reaches the velocities required to fly beyond the limits of the terrestrial atmosphere, to orbit the Earth or even to fly to other planets, it consists of a small airplane having a shortened body with wings, rudders, a melting chamber, a propeller, an engine, a frame and the necessary internal equipment.

To move in space, starting with a velocity of 8 km/sec (required for orbiting the Earth), it is possible to use instead of a rocket special low-weight, large area mirrors which change the flight velocity and its direction by the pressure of light falling on them*.

In order to land on the Earth or on a planet possessing an atmosphere, the flight in the atmosphere has to be horizontal. In order to maintain a constant altitude in the atmosphere at velocities larger than 8 km/sec, the airplane should fly on its back so that a negative lift towards the Earth is produced. At velocities lower than 8 km/sec it is possible to fly in the usual position, describing a curve for landing in the desired place; a small engine with propellers may help.

The main advantages of the design proposed here, as compared with previous proposals, are the following factors of technical and economical nature (not taking into consideration the suggestion to use metallic fuel):

1. The use of a small winged airplane enables glide landing on the Earth and on planets possessing an atmosphere. This reduces the spaceship's total weight by a factor of 15 to 20 in comparison to one using rocket thrust for landing (assuming the same landing weight in both cases).

2. The use of an airplane for take-off and landing on the Earth or on a planet with an atmosphere guarantees the safety of the passengers to a larger extent than the use of parachutes and lifting rockets. In case of accidental engine failure during take-off, it is possible to pass to safe glide landing and to restart the engine. With a parachuted rocket it will be rather

* See Tsander's paper "The Use of Light-Pressure for Flight in Interplanetary Space", on pp. 303–321 of this book. – Editor's note.

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difficult to pull in the supporting parachute and restart the engine. In previously proposed designs, a failure of the rocket's engine would force a landing on Earth, and failure immediately after take-off or even at high velocities slightly lower than 8 km/sec, is very likely to be lethal to the passengers. If the failure occurs immediately after take-off, the parachute has no time to open, and in a high velocity descent the parachute may easily tear off. This danger is eliminated in my proposed combination of airplane and rocket. Instead of being supported by the exhaust gases, the rocket is supported in my scheme by the airplane's wings. At any moment, the rocket rests safely on the airplane; this is not the case in the booster rocket with parachute. The high development of aviation technology and the advanced stage of airplane design, will enable us to overcome difficulties which might be encountered in flight.

3. For returning to Earth we can use the huge amount of kinetic energy acquired by the rocket in the ascent. With this store of energy for a glide landing with a small airplane, we may circle half the globe and land at any place.

4. The required rocket thrust in my design is equal to only one sixth to one third of the whole ship's weight. The thrust required with booster rockets is equal to 4 to 10 times the ship's initial weight.

5. Due to the smaller apparent gravity, the pilot may control the rocket freely, while in booster rockets he will be subjected to the harmful effect of large accelerations during the initial period. Furthermore, these accelerations hamper the observation of the machine's operation.

6. Since the rocket proposed here is 10 to 60 times smaller than the corresponding booster rocket, the servicing is much simpler. The danger to the passengers in the event of damage or explosion of a small rocket is smaller than in the explosion of a large rocket.

7. It is rather simple to conduct flight experiments using a ship of the proposed design since one can stop and start the rocket's engine every minute.

I have also solved the problems of take-off and landing of long-range rockets of my system.

This rocket design may serve, in addition to interplanetary travel, also for express flights transporting loads and passengers on Earth, in the atmosphere's upper layers, and above.

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FLIGHTS TO OTHER PLANETS

(The theory of interplanetary travel)

The present paper, written by Tsander in 1924-1925*, is indeed a basis for the calculation of any cosmic flight. The paper consists of nine more or less connected sections devoted to a series of problems which cosmonauts will encounter when choosing flight trajectories to planets.

Tsander investigates in detail the motion of a spaceship in the gravitational field of the Sun, and he gives a method for determining the magnitude and direction of the additional velocity which should be given to a spaceship moving together with the departure planet along its orbit in order that the ship reaches the destination planet with minimum propellant consumption. He investigates how the planets' gravitational fields modify the flight trajectory and change thereby the spaceship's kinetic energy, producing acceleration or deceleration. At the same time the author investigates the problem of correcting the flight trajectory when approaching the destination planet. In this work Tsander shows the influence of the moment of departure to another planet on the magnitude of the additional velocity and on the flight duration and gives also a very detailed calculation of a series of possible flight trajectories to Mars.

Editor

1. DETERMINATION OF AN INTERPLANETARY FLIGHT TRAJECTORY AND OF THE MAGNITUDE OF THE ADDITIONAL VELOCITIES REQUIRED FOR ITS REALIZATION

The paths which are traversed by a spaceship in flying to other planets may be divided into a number of domains, which are not too sharply outlined.

The first domain extends from the Earth's surface to the terrestrial atmosphere's limits.

When the airplane ascends in the atmosphere, two forces act on it: lift and drag. In the atmosphere's lower layers, when the velocity is still small, it is possible to use either propeller engine of special design or a rocket with small exhaust velocity for propulsion. In the higher layers of

* These dates (1924-1925) were given in Tsander's typewritten article. In his stenographical records of 1922 there are also formulas referring to this paper. The exact time at which the present paper was compiled will be established after deciphering the shorthand notes.
The paper was prepared for print by V. I. Sevast'yanov. – Editor's note.

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the atmosphere a rocket with a high exhaust velocity may be used since the low air resistance allows high flight velocities, at which the efficiency of such a rocket is already sufficient, to develop.

For a direct take-off from Earth by means of a rocket it is possible to use a rocket with high exhaust velocity. This is advantageous, as shown by Tsiolkovskii, if the acceleration is several times higher than the gravitational acceleration.

The second domain extends from the atmosphere's upper layers at about 70-100 km over the Earth's surface, to the point where the Earth's sphere of attraction ends and that of the Sun and the Moon begins. The boundary of this sphere can be indicated only approximately.

The aerodynamic forces can be neglected in this domain but a centrifugal force appears when flying in a trajectory following the Earth's curvature. At velocities of 5-6 km/sec and more this force plays a decisive role; it gives an apparent relative lifting force, helping flight in interplanetary space. Up to velocities of 8 km/sec a rocket is needed for acceleration since there is still danger of falling back on the Earth. For a velocity of 8 km/sec in horizontal flight, the Earth's attraction is equal to the centrifugal force and the spaceship describes a circle around the Earth's center.

For a further velocity increase, the spaceship's trajectory may be chosen so that it will not intersect the Earth's atmosphere. Here one can use methods of accelerating the interplanetary flight essentially different from those used for take-off from Earth. First, it is possible to use the pressure of light or of other rays*, which give only negligible accelerations; however, acting for a long time, these may accelerate the spaceship to considerable velocities. Second, it is possible to unfold in interplanetary space huge concave rotating mirrors which collect the solar rays and send out almost parallel beams to the mirrors connected to the spaceships. After the action of the rocket engine or of the rays is stopped, the trajectory will be an ellipse, a parabola, or a hyperbola, one of whose foci is situated at the Earth's center. If the velocity is increased above a certain limit, the ship, getting further from the Earth, falls into the next domain (if it is not upheld by the rocket engine's thrust, which in this case should be directed towards the Earth).

The third domain differs essentially from the second one since here the attraction of three celestial bodies: the Earth, the Sun and the Moon acts on the spaceship. The trajectory can be modified by the same methods as in the second domain. The ship, left to itself, describes a complicated curve**.

The fourth domain is the one in which the attraction of the Sun predominates. In its extension in space this domain exceeds all the others and extends over the entire solar system with the exclusion of the zones of influence of the planets, of their satellites, and also of the places occupied by comets or meteor streams. Modification of the flight trajectory can be introduced by the same methods as in the second and third domains, but in this domain the trajectory of a ship left to itself (i.e., if the rocket engine is not operating or the ship is not subjected to radiation pressure) will be

* A separate article of Tsander's is devoted to this problem (see pp. 303–321). – Editor's note.

** These curves have been partly studied during the past 12 years by Strömgren in Copenhagen using mechanical integration.

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an ellipse, a parabola or a hyperbola with a focus at the center of the Sun. The first three domains (if there is no atmosphere – the second and third only) surround each celestial body.

Motion of spaceship in the zone of action of the Sun's gravitational field

To determine the time, path, and the flight velocity, we shall concentrate on a flight to another planet and examine the phenomena which take place in the fourth domain since the flight through this domain takes the longest time. In the first three domains it will be simple to obtain the direction and velocity of flight required so that at the moment of leaving the third domain and entering the fourth, we are sure that the destination planet will be reached by the desired path. If the trajectories are determined assuming the fourth domain occupies the whole solar system, the sections lying in the first three domains should be replaced by curves; for these a special calculation is made.

For simplicity, we shall assume that the Earth and the destination planet move in circles having the center of the Sun as common center. It is thus possible in many cases to obtain average values for the flight range, the additional velocity and the flight duration.

Various considerations will be important when choosing a flight trajectory. Let us first consider a flight by means of a rocket which gives the necessary initial velocity in the immediate neighborhood of the Earth. We require the following:

a) To minimize the amount of propellant consumed, i.e., the additional velocity imparted by the rocket*.

b) To shorten the flight time in order to reduce the danger of meeting meteors, and generally to shorten the flight duration.

c) To shorten the trajectory's length, thus also reducing the danger of meeting meteors.

d) In addition to individual sporadic meteors, the spaceship may meet on its way whole swarms of meteors which are as yet unknown to us since they do not cross the Earth's orbit. In such a case it would be necessary to avoid as far as possible the whole swarm of meteors. Another possibility is to irradiate the meteors by low velocity cathode rays. If the spaceship itself is placed in an electrically charged sphere, the meteors charged by the rays (electrons) will be deflected by the electric forces of the ship without reaching it. This problem is examined in more detail below (see p. 365). It will also be possible to protect the spaceship against sporadic meteors. This can be done by using sufficiently tough material for the external walls, by building separate compartments with automatically operating devices which close the passages between them in case of air pressure drop, and also by building moving internal walls. In each individual case it is necessary to determine the flight trajectory in such a way that the danger of meeting meteors will be as small as possible. Using data on the number of "falling stars" observed, I have found that a spaceship encounters on the

* The carrier rocket is meant here. – Editor's note.

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average (unless it meets a meteor stream) approximately one meteor every 10 years of an average weight of 6 grams.

e) Inaccurate injection of a spaceship into orbit may lead either to escape from the solar system or to falling into a zone near the Sun due to trajectory changes caused by the destination planet. If the spaceship will get to the intersection point of the flight path with the planet's orbit later than the latter, its trajectory will be deflected in a counter-clockwise direction. If it arrives at this point earlier than the planet, the deflection will be in the opposite direction. I investigated the magnitude of the deflection by various planets and will discuss this below.

In choosing a flight trajectory, it is always necessary to find whether a given planet is in a position to deflect the flight trajectory so much that one of the above-mentioned cases will take place. Such trajectories should be avoided as far as possible as it will be difficult to catch up with a vehicle having already flown far from the Sun and one may not succeed in rescuing a vehicle flying towards the Sun.

A natural trajectory seems one in which the aphelion (the point of the trajectory furthest from the Sun) is not situated further than required, i. e., making the spaceship's orbit tangent to the orbit of the destination planet. Calculations show, however, that this is advantageous only close to the minimum possible additional velocity.

f) The trajectory in the form of an ellipse, a parabola, or a hyperbola is convenient since all the velocity required for attaining a given planet is given at the beginning of the flight. However, such a flight trajectory will not be the shortest for a given fuel consumption: it requires also a longer flight time than the optimum trajectory. The optimum trajectory can be determined by variational calculus. It is then necessary, however, to assume that the engine works during the entire flight time. In this case damage to the rocket engine (caused for example by meteor piercing) may prevent the spaceship from reaching the destination planet.

If we use as driving force light pressure, which produces only extremely small accelerations, the choice of the optimum trajectory (from the point of view of flight duration) is of paramount importance.

Turning to the calculation of flight trajectories of spaceships from one planet to another or in interplanetary space in general, let us first consider trajectories having the destination planet at the aphelion of the elliptical flight trajectory. The formulas will have then a more general value. In the general case, the velocities for reaching a given aphelion, and their directions, are not changed, but the planet will be reached before or after passing through the aphelion.

Let us introduce the following notation (Figure 1):

r_0 and r_2, perihelion and aphelion radii of flight trajectory;
r_1, radius of departure planet's (Earth) orbit;
a, flight trajectory semimajor axis;
b, flight trajectory semiminor axis;
e, flight trajectory linear eccentricity;
Σ, flight trajectory relative eccentricity;
V, absolute flight velocity at take-off moment;
V_1, V_2, orbital velocities of departure and destination planets;
V_z, relative velocity of spaceship with respect to Earth at take-off moment; the spaceship should possess this additional velocity after leaving the Earth's sphere of attraction;

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β, angle between V and V_1 at moment of take-off from Earth's orbit;

ξ, angle between V_2 and V_1 at moment of take-off from the Earth's orbit;

ϑ, angle between ellipse major axis, measured from perihelion, and radius-vector at moment of take-off from Earth (spaceship's true anomaly);

α, angle between major axis and normal to ellipse at moment of take-off from Earth;

τ, spaceship's semirotation period around Sun;

τ_1, τ_2, semirotation periods of departure and destination planets in circles of radii r_1 and r_2;

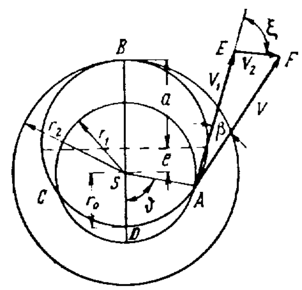

FIGURE 1

m, spaceship's mass;

M, mass of Sun;

k, Newton's gravitational constant;

K, force of attraction between Sun and spaceship;

$$p = r_1/a;\ \gamma = r_2/r_1;$$

$$\gamma_0 = r_0/r_1;\ u = V_2^2/V_1^2;$$

V_p, V_a, spaceship's velocities at perihelion and aphelion of flight trajectory;

t, time required for spaceship's flight from trajectory's perihelion to point of intersection with Earth's orbit;

t_f, time required to fly to another planet;

E, spaceship's eccentric anomaly.

For uniform motion of the planets in circular orbits (using the above notation), we have:

$$\tau_1 = \frac{\pi r_1}{V_1}; \tag{1}$$

$$\tau_2 = \frac{\pi r_2}{V_2}, \tag{2}$$

and, according to Kepler's third law:

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3. \tag{3}$$

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Hence

$$\frac{V_2^2}{V_1^2}=\frac{r_1}{r_2}. \tag{4}$$

We may write, in analogy to expression (3),

$$\left(\frac{\tau}{\tau_1}\right)^2=\left(\frac{a}{r_1}\right)^3. \tag{5}$$

For an ellipse we have

$$a+e=r_2; \tag{6}$$

$$\sum=\frac{e}{a}=\frac{r_2-a}{a}; \tag{7}$$

$$b=\sqrt{r_2(2a-r_2)}. \tag{8}$$

From the law of areas, according to which the radius-vector describes equal areas in equal time intervals, we find

$$r_1 V\cos\beta=\frac{\pi ab}{\tau}, \tag{9}$$

and from ΔAEF (see Figure 1) we obtain

$$V\cos\beta=V_1+V_z\cos\xi, \tag{10}$$

and

$$\frac{V_z}{\sin\beta}=\frac{V}{\sin\xi}. \tag{11}$$

Eliminating V from these two equations we find

$$V_z\sin\xi\operatorname{ctg}\beta=V_1+V_z\cos\xi,$$

or

$$V_z=\frac{V_1}{\sin\xi\operatorname{ctg}\beta-\cos\xi}=\frac{V_1\sin\beta}{\sin(\xi-\beta)}. \tag{12}$$

Substituting the expression of $V\cos\beta$ from (10) in (9) and introducing for b and τ in (9) their expressions found from (8), (5) and (1), we find:

$$r_1(V_1+V_z\cos\xi)=\frac{\pi ab}{\tau}=$$

$$=\frac{\pi a\sqrt{r_2(2a-r_2)}r_1^{3/2}}{\tau_1 a^{3/2}}=\frac{V_1\sqrt{r_1r_2(2a-r_2)}}{\sqrt{a}};$$

or

$$V_z=\frac{V_1}{\cos\xi}\left(\sqrt{\frac{r_2(2a-r_2)}{r_1a}}-1\right). \tag{13}$$

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By eliminating V_r from (12) and (13), we obtain a relation between angles ξ and β:

$$\operatorname{tg}\xi=\operatorname{tg}\beta\left(1+\frac{\sqrt{r_1a}}{\sqrt{r_2(2a-r_2)}-\sqrt{r_1a}}\right). \tag{14}$$

It is then possible to derive the following formula from the basic properties of the ellipse:

$$\operatorname{tg}\beta=\sqrt{\frac{\sum^2a^2-(a-r_1)^2}{(1-\sum^2)a^2}}, \tag{15}$$

or, using (7),

$$\operatorname{tg}\beta=\sqrt{\frac{(r_2-r_1)(r_2+r_1-2a)}{r_2(2a-r_2)}} \tag{16}$$

and

$$\cos\beta=\sqrt{\frac{r_2(2a-r_2)}{r_1(2a-r_1)}}. \tag{17}$$

From equations (14) and (16) we have

$$\operatorname{tg}\xi=\frac{\sqrt{(r_2-r_1)(r_2+r_1-2a)}}{\sqrt{r_2(2a-r_2)}-\sqrt{r_1a}}, \tag{18}$$

and, therefore

$$\cos\xi=\frac{\sqrt{r_2(2a-r_2)}-\sqrt{r_1a}}{\sqrt{3ar_1-r_1^2-2\sqrt{r_2r_1a(2a-r_2)}}}. \tag{19}$$

Upon substituting (19) in equation (13) we finally obtain

$$V_z=V_1\sqrt{3-\frac{r_1}{a}-2\sqrt{\frac{r_2}{r_1}\left(2-\frac{r_2}{a}\right)}}. \tag{20}$$

By substituting $\cos\beta$ from (17) and $V_z\cos\xi$ from (13) in expression (10) we obtain

$$V=V_1\sqrt{2-\frac{r_1}{a}}. \tag{21}$$

Next, we are interested in the angle ϑ (see Figure 1), i.e., in the spaceship's true anomaly at the start.

From the equation of the ellipse in polar coordinates

$$r_1=\frac{a-\sum e}{1+\sum\cos\vartheta}, \tag{22}$$

and using expression (7) we obtain

$$\cos\vartheta=\frac{a(2r_2-r_1)-r_2^2}{r_1(r_2-a)}. \tag{23}$$

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Let us return to the additional velocity V_z which is determined by (20). For given radii r_1 and r_2, we have here only one independent variable – a. Let us find the condition for the minimum additional velocity V_z by finding the minimum of

$$u=\frac{V_z^2}{V_1^2}=3-\frac{r_1}{a}-2\sqrt{\frac{r_2}{r_1}\left(2-\frac{r_2}{a}\right)}. \qquad (24)$$

u becomes a minimum, if $\frac{du}{da}=0$;

$$\frac{du}{da}=\frac{1}{V_1^2}\,2V_z\frac{dV_z}{da},$$

or

$$\frac{dV_z}{da}=\frac{V_1^2}{2V_z}\frac{du}{da}.$$

Consequently, if $\frac{du}{da}=0$, we have also $\frac{dV_z}{da}=0$.

A special investigation is required only in the case when $V_z=0$. In this case the spaceship will, obviously, remain in Earth's orbit, i.e., $a=r_1$ and $r_2=r_1$, as can be easily shown by substituting these values in equation (20). We can, therefore, omit the case $V_z=0$.

If $V_z\neq 0$, we obtain by differentiating equation (24) with respect to a:

$$\frac{du}{da}=\frac{r_1}{a^2}-\sqrt{\frac{r_2}{r_1}}\frac{r_2}{a^2\sqrt{2-\frac{r_2}{a}}}=$$

$$=\frac{1}{a^2}\left(r_1-\sqrt{\frac{r_2}{r_1}}\frac{r_2}{\sqrt{2-\frac{r_2}{a}}}\right). \qquad (25)$$

From this equation we find that when $a\to\infty$, $du/da=0$ and, consequently, $dV_z/da=0$.

From equation (b), however, it follows that $a<r_2$ since $e>0$, i.e., the case $a\to\infty$ drops out for finite r_2.

We are left therefore with the equation

$$r_1-\sqrt{\frac{r_2}{r_1}}\frac{r_2}{\sqrt{2-\frac{r_2}{a}}}=0$$

or

$$a=\frac{r_2}{2-\left(\frac{r_2}{r_1}\right)^3}. \qquad (26)$$

By substituting in equation (20) we find:

$$V_z=V_1\sqrt{-\frac{r_2+2r_1}{r_2}\left(\frac{r_2}{r_1}-1\right)}.$$

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which is always an imaginary quantity.

It follows therefore that in the entire domain considered, du/da, and therefore also dV_z/da, do not change sign, i.e., when a increases V_z either increases or decreases monotonically. Let us show that the latter happens.

Since the radius-vector r_0 at the perihelion is a minimum, r_0 is smaller than r_1. In the limit for $\vartheta = 0$, we obtain $r_{0\,max} = r_1$, and in the general case

$$r_0 = a - e, \tag{27}$$

and the smallest value of r_0 is $r_{0\,min} = 0$.

From equations (27) and (6) we have

$$r_0 + r_2 = 2a; \quad a = \frac{r_0 + r_2}{2}, \tag{28}$$

which gives for $r_{0\,max} = r_1$

$$a_{max} = \frac{r_1 + r_2}{2}; \tag{29}$$

and for $r_{0\,min} = 0$

$$a_{min} = \frac{r_2}{2}. \tag{30}$$

Then for the first case ($r_{0\,max} = r_1$) we obtain, substituting a_{max} from expression (29) in equation (25) and using the notation

$$\gamma = \frac{r_2}{r_1}, \tag{31}$$

$$\left.\frac{du}{da}\right|_{a=a_{max}} = \frac{4}{(1+\gamma)^2 r_1}\left[1 - \gamma\sqrt{\frac{\gamma(1+\gamma)}{2}}\right]; \tag{32}$$

and for the second case ($r_{0\,min} = 0$)

$$\left.\frac{du}{da}\right|_{a=a_{min}} \to -\infty.$$

For $\gamma > 1$ we obtain $1 < \gamma\sqrt{\frac{\gamma(1+\gamma)}{2}}$, which means that $\left.\frac{du}{da}\right|_{a=a_{max}} < 0$, hence $\frac{du}{da} < 0$, i.e. if a decreases, the values of u and V_z increase monotonically; the minimum of V_z is obtained for

$$a_{max} = \frac{1}{2}(r_1 + r_2).$$

For this case we obtain from equations (16) and (18)

$$\operatorname{tg}\beta = 0;\ \beta = 0;\ \operatorname{tg}\xi = 0;\ \xi = 0.$$

From equation (23) we obtain:

$$\cos\vartheta = 1;\ \vartheta = 0;$$

from expression (8)

$$b = \sqrt{r_1 r_2};$$

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from expression (7)

$$\sum=\frac{r_2-r_1}{r_2+r_1};$$

and from equations (20) or (13):

$$V_{z\,\min}=V_1\left(\sqrt{\frac{2r_2}{r_1+r_2}}-1\right). \quad (33)$$

Thus we have the following law:

In order to reach another planet with the minimum additional velocity (V_z) or with the minimum expenditure of energy, it is necessary to take-off from the orbit of the departure planet parallel to its direction of motion around the Sun ($\beta=0, \xi=0$) with the additional velocity:

$$V_{z\,\min}=V_1\left(\sqrt{\frac{2r_2}{r_1+r_2}}-1\right),$$

the trajectory will then be tangent to the orbit of the departure planet at the perihelion, and to the orbit of the destination planet at the aphelion: it will be half an ellipse.

This law refers only to circular planetary orbits. For elliptic orbits it is possible to find mutual positions of the planets for which the trajectory's length and the flight duration will be somewhat shortened. The inclination of the planets' orbital planes with respect to one another requires that the rocket be given some additional velocity perpendicular to the orbital plane of the departure planet.

The formulas have been derived for a flight to an external planet but they can be easily adapted for a flight to an internal planet. The additional velocity V_z will then be in a direction opposite to the motion of the departure planet.

From the condition $du/da \neq 0$ it follows that the corresponding variations of u and a are of the same order. Let us now determine the quantities $d\vartheta/da$ and $d\beta/da$. Differentiating (23) with respect to a, we obtain

$$-r_1\sin\vartheta\, d\vartheta=\frac{(2r_2-r_1)(r_2-a)+a(2r_2-r_1)-r_2^2}{(r_2-a)^2}\,da,$$

or

$$r_1\frac{d\vartheta}{da}=\frac{r_2(r_2-r_1)}{(r_2-a)^2\sin\vartheta}. \quad (34)$$

In the same way we obtain from (17):

$$-2\cos\beta\sin\beta\frac{d\beta}{da}=\frac{r_2[2(2a-r_1)-2(2a-r_2)]}{r_1(2a-r_1)^2},$$

or

$$r_1\frac{d\beta}{da}=-\frac{2r_2(r_2-r_1)}{\sin 2\beta\,(2a-r_1)^2}. \quad (35)$$

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For $\vartheta = 0$ and $\beta = 0$, we obtain $da/d\vartheta = 0$ and $da/d\beta = 0$, i.e., near $V_{z\,min}$, ϑ and β vary much more rapidly than a.

We also have

$$\frac{du}{d\vartheta}=\frac{du}{da}\frac{da}{d\vartheta}; \quad \left.\frac{du}{d\vartheta}\right|_{\vartheta=0}=0.$$

The angle ϑ is proportional to the times it takes the Earth to move in its orbit. We see that u, and consequently also V_z, vary rather little near $V_{z\,min}$. The angle β, on the contrary, varies quite rapidly.

Mathematically V_z will be a true minimum with respect to the variable ϑ, but only a smallest value with respect to the variable a.

Let us calculate the spaceship's velocity at its perihelion and at its aphelion. Denoting them by V_p and V_a we obtain from the law of areas:

$$\frac{1}{2} r_1 V\cos\beta=\frac{1}{2} r_2 V_a=\frac{1}{2} r_0 V_p.$$

Hence, we obtain easily, using (17) for $\cos\beta$, (21) for V and (28) for r_0:

$$V_a=V_1\sqrt{\frac{r_1}{r_2}\left(2-\frac{r_2}{a}\right)} \tag{36}$$

and

$$V_p=V_1\sqrt{\frac{r_1 r_2}{a(2a-r_2)}}. \tag{37}$$

Let us find the interesting quantities for two cases:

1) For $a_{min} = r_2/2$ we obtain:

$$\operatorname{tg}\beta=\infty;\quad \beta=90^\circ;\quad \operatorname{tg}\xi=-\sqrt{\frac{2(r_2-r_1)}{r_2}}; \tag{38}$$

$$\cos\vartheta=-1;\quad \vartheta=180^\circ;\quad V_z=V_1\sqrt{3-\frac{2r_1}{r_2}}; \tag{39}$$

$$b=0;\quad \sum=1;\quad V=V_1\sqrt{2\left(1-\frac{r_1}{r_2}\right)}; \tag{40}$$

$$V_a=0;\quad V_p=\infty.$$

This case corresponds to the shortest flight path: the spaceship flies along a radius-vector from one planet to another, getting further from the Sun. The ellipse is transformed into a segment of a straight line, with the destination planet at one end and the Sun at the other. The angle ξ, between the direction of the velocity V_z and the velocity of the Earth V_1 is larger than a right angle. The velocity V_z is considerably larger than $V_{z\,min}$. The resultant velocity V is pointing radially from the Sun.

2) For some intermediate case, when the velocity V_z is in a direction pointing away from the Sun, we have $\xi = 90^\circ$, i.e., $\operatorname{tg}\xi = \infty$, and from equation (18) we find:

$$a=\frac{r_2^2}{2r_2-r_1}.$$

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From equation (23) we have

$$\cos\vartheta = 0; \quad \vartheta = 90^\circ;$$

from equation (16)

$$\operatorname{tg}\beta = 1 - \frac{r_1}{r_2};$$

from equation (20)

$$V_z = V_1\left(1 - \frac{r_1}{r_2}\right);$$

from equation (8)

$$b = r_2\sqrt{\frac{r_1}{2(r_2 - r_1)}};$$

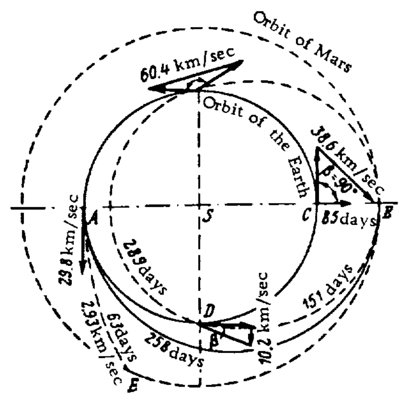

FIGURE 2

from equation (7)

$$\sum = 1 - \frac{r_1}{r_2};$$

from equation (21)

$$V = V_1\sqrt{1 + \left(1 - \frac{r_1}{r_2}\right)^2};$$

from equation (36)

$$V_a = V_1\frac{r_1}{r_2};$$

from equation (37)

$$V_p = V_1\left(2 - \frac{r_1}{r_2}\right);$$

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from equation (28)

$$r_0 = \frac{r_1 r_2}{2(r_2 - r_1)}.$$

Figure 2 shows the flight paths and also the triangles of the velocities V_1, V_2 and V at the take-off moment from Earth orbit for a flight from the Earth to Mars, assuming that Mars is at an average distance from the Sun corresponding to $\gamma = 1.52$.

Determination of the flight duration

Let us consider right away the general case, assuming that the destination planet is situated at an arbitrary point of the ellipse.

The flight duration is determined in this case by Kepler's equation which is easily derived. Let us express the radius-vector r and the angle ϑ in the equation of the ellipse

$$r = \frac{a - e\Sigma}{1 + \Sigma\cos\vartheta} \tag{22}$$

by the angle E, shown in Figure 3, which is customarily called the eccentric anomaly of the trajectory.

From point A on the ellipse we draw a perpendicular to the major axis OD and continue it through point A till its intersection M with a circle of radius $OD = a$ and center O. Connecting points M and O by a straight line we have $\angle MOD = E$.

For the ellipse we have

$$e = \Sigma a.$$

and expressing the segment SN by the angles ϑ and E, we find from Figure 3

$$SN = r\cos\vartheta = a\cos E - a\Sigma.$$

Together with equation (22) this expression gives:

$$a - a\Sigma^2 = r + \Sigma r\cos\vartheta = r + \Sigma a\cos E - a\Sigma^2,$$

or

$$r = a(1 - \Sigma\cos E). \tag{41}$$

In order to obtain E as a function of ϑ we find from equations (41) and (22):

$$1 - \Sigma\cos E = \frac{1 - \Sigma^2}{1 + \Sigma\cos\vartheta}, \tag{42}$$

or

$$\cos E = \frac{\Sigma + \cos\vartheta}{1 + \Sigma\cos\vartheta}. \tag{43}$$

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Using the trigonometric identities:

$$\operatorname{tg}\frac{E}{2}=\sqrt{\frac{1-\cos E}{1+\cos E}}$$

and

$$\operatorname{tg}\frac{\vartheta}{2}=\sqrt{\frac{1-\cos\vartheta}{1+\cos\vartheta}},$$

we get:

$$\operatorname{tg}\frac{E}{2}=\sqrt{\frac{1+\sum\cos\vartheta-\sum-\cos\vartheta}{1+\sum\cos\vartheta+\sum+\cos\vartheta}}=\sqrt{\frac{1-\sum}{1+\sum}}\operatorname{tg}\frac{\vartheta}{2}. \qquad (44)$$

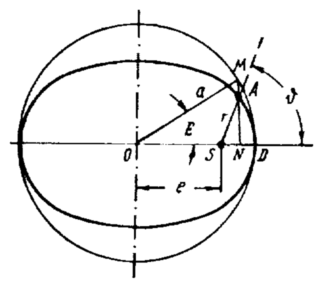

FIGURE 3

If we now denote by t the time required to fly from the perihelion of the elliptical path to a point on the ellipse corresponding to a given angle ϑ, and by τ the semirotation period of the Sun along the ellipse, we have from the law of areas:

$$\frac{t}{\tau}=\frac{2}{\pi ab}\int_0^{\vartheta}\frac{r^2}{2}d\vartheta, \qquad (45)$$

where πab is the area of the ellipse, and $\int_0^{\vartheta}\frac{r^2}{2}d\vartheta$ is the area swept by the radius-vector during time t.

To calculate the integral we express $d\vartheta$ by dE. Differentiating equation (42), we obtain

$$\sum\sin E\,dE=\frac{(1-\sum^2)(-1)(-\sum\sin\vartheta\,d\vartheta)}{(1+\sum\cos\vartheta)^2},$$

or

$$\frac{d\vartheta}{dE}=\frac{\sin E}{\sin\vartheta}\,\frac{(1+\sum\cos\vartheta)^2}{1-\sum^2}. \qquad (46)$$

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Expressing ϑ in equation (42) in terms of E, we find:

$$\cos\vartheta = \frac{\cos E - \sum}{1 - \sum\cos E}, \tag{47}$$

hence

$$\sin\vartheta = \frac{\sqrt{(1-\sum\cos E)^2 - (\cos E - \sum)^2}}{1-\sum\cos E} = \frac{\sqrt{1-\sum^2}\sin E}{1-\sum\cos E}, \tag{48}$$

and consequently:

$$\frac{\sin E}{\sin\vartheta} = \frac{1-\sum\cos E}{\sqrt{1-\sum^2}}.$$

Substituting this expression and the one for $\cos\vartheta$ found in (47), in equation (46), we have:

$$\frac{d\vartheta}{dE} = \frac{\sqrt{1-\sum^2}}{1-\sum\cos E}. \tag{49}$$

Substituting now r from (41) and $d\vartheta$ from (49) in $\int_0^\vartheta r^2 d\vartheta$, we obtain:

$$\int_0^\vartheta r^2 d\vartheta = a^2 \int_0^\vartheta \frac{(1-\sum\cos E)^2\sqrt{1-\sum^2}}{1-\sum\cos E} dE =$$
$$= a^2\sqrt{1-\sum^2}\,[(E-\sum\sin E) - (E_0 - \sum\sin E_0)].$$

For $t_0 = 0$ we have $\vartheta_0 = 0$. In this case we see from (44) that $E_0 = 0$, so that we can write:

$$\int_0^\vartheta r^2 d\vartheta = a^2\sqrt{1-\sum^2}\,(E-\sum\sin E).$$

Substituting this expression in (45), we obtain:

$$t = \frac{\tau a^2\sqrt{1-\sum^2}}{\pi a b}\,(E-\sum\sin E).$$

Remembering that $b = a\sqrt{1-\sum^2}$, we find finally that

$$t = \frac{\tau}{\pi}\left(E - \sum\sin E\right). \tag{50}$$

For the case considered above, when the destination planet is at the aphelion of the elliptical path of the spaceship at the moment of arrival there, it is convenient to express all quantities in terms of the constant $\gamma = r_2/r_1$ and the variable

$$\rho = \frac{r_1}{a} = \frac{r_1}{r_2}\frac{r_2}{a} = \frac{r_2}{\gamma a}. \tag{51}$$

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For ϑ, β, ξ, V_z, V, V_a, V_p, r_0, a, b, Σ, E, τ, t, t_f and t_f' we obtain the following expressions:

$$\cos\vartheta = \frac{2\gamma - 1 - \gamma^2\rho}{\gamma\rho - 1}; \tag{52}$$

$$\cos\beta = \sqrt{\frac{\gamma(2-\gamma\rho)}{2-\rho}}; \tag{53}$$

$$\operatorname{tg}\beta = \sqrt{\frac{(\gamma-1)(\gamma\rho+\rho-2)}{\gamma(2-\gamma\rho)}}; \tag{54}$$

$$\operatorname{tg}\xi = \frac{\sqrt{(\gamma-1)(\gamma\rho+\rho-2)}}{\sqrt{\gamma(2-\gamma\rho)}-1}; \tag{55}$$

$$V_z = V_1\sqrt{3-\rho-2\sqrt{\gamma(2-\gamma\rho)}}; \tag{56}$$

$$V = V_1\sqrt{2-\rho}; \tag{57}$$

$$V_a = V_1\sqrt{\frac{2-\gamma\rho}{\gamma}}; \tag{58}$$

$$V_p = V_1\rho\sqrt{\frac{\gamma}{2-\gamma\rho}}; \tag{59}$$

$$V_2 = \frac{V_1}{\sqrt{\gamma}}; \tag{60}$$

$$\frac{r_0}{r_1} = \frac{2}{\rho} - \gamma; \tag{61}$$

$$a = \frac{r_1}{\rho}; \tag{62}$$

$$b = r_1\sqrt{\frac{\gamma(2-\gamma\rho)}{\rho}}; \tag{63}$$

$$\Sigma = \gamma\rho - 1; \tag{64}$$

$$\frac{1-\Sigma}{1+\Sigma} = \frac{2-\gamma\rho}{\gamma\rho}; \tag{65}$$

$$\operatorname{tg}\frac{E}{2} = \sqrt{\frac{2-\gamma\rho}{\gamma\rho}}\operatorname{tg}\frac{\vartheta}{2}; \tag{66}$$

$$\tau = \frac{\tau_1}{\rho^{3/2}}; \tag{67}$$

$$t = \frac{\tau_1}{\pi\rho^{3/2}}[E - (\gamma\rho - 1)\sin E]; \tag{68}$$

$$t_f = \tau - t; \tag{69}$$

$$t_f' = \tau + t. \tag{70}$$

The formulas for V_z and ξ are somewhat modified if angles ϑ, lying between 180° and 360°, are considered. In the range $0 < \vartheta < 180°$, the additional velocity, V_z, which the rocket should give the spaceship, is relatively small, in particular near $\vartheta = 0$. In the range $180° < \vartheta < 360°$ it increases, since in this case the flight around the Sun has to be done more or less against the Earth's direction of motion.

From Figure 4 it follows that the velocity V does not change if we start, instead of from A, from point C on the Earth's orbit, which is symmetrical to A with respect to the major axis for a given form of elliptical trajectory.

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The ellipse elements r_0, a, b, Σ, E, remain the same. This is also true for the magnitudes of the velocities V_a and V_p which change only their direction, and the times τ and t are unchanged too. Only the quantities ϑ, β, ξ and V_z take new values.

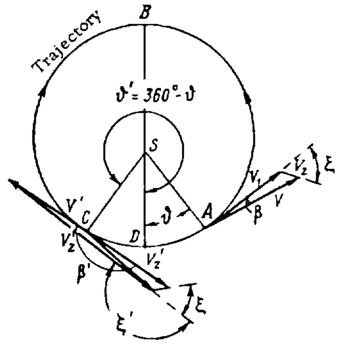

FIGURE 4

If we denote by a "prime" the values of the quantities for point C, we see from Figure 4 that:

$$\vartheta' = 360° - \vartheta; \ \beta' = 180° - \beta.$$

Because the square root may have a positive as well as a negative value, we obtain from equations (20) and (18) expressions for ξ and V_z

$$V_z = V_1 \sqrt{3 - \frac{r_1}{a} \mp 2 \sqrt{\frac{r_2}{r_1}\left(2 - \frac{r_2}{a}\right)}} \qquad (71)$$

$$\operatorname{tg} \xi = \frac{\pm \sqrt{(r_2 - r_1)(r_2 + r_1 - 2a)}}{\sqrt{r_2(2a - r_2)} \mp \sqrt{r_1 a}}, \qquad (72)$$

the upper sign referring to the case $0 < \vartheta < 180°$, and the lower sign – to the case $180° < \vartheta < 360°$.

Instead of flying from point A or C straight to point B (as assumed till now), it is possible to fly from these points to point B through the perihelion D. By symmetry we then obtain velocity triangles analogous to those considered above (see Figure 4), except that the angles β and ξ will be directed towards the Sun (measuring from the velocity vector V_1), while in the case considered before they were directed away from it. Along the shorter path CB, the flight time will be

$$t_f = \tau - t = \frac{\tau}{\pi}[\pi - E + (\gamma\rho - 1)\sin E]; \qquad (73)$$

and along the path CDB:

$$t_f' = \tau + t = \frac{\tau}{\pi}[\pi + E - (\gamma\rho - 1)\sin E]. \qquad (74)$$

Instead of (68) we can use the formula:

$$t = \frac{\tau_1}{\pi \rho^{3/2}}\left[E - (\gamma - 1)\rho \operatorname{tg}\frac{E}{2}\right], \qquad (75)$$

which is obtained after a simple transformation. Instead of (52) we have:

$$\operatorname{tg}\frac{\vartheta}{2} = \sqrt{\frac{1 - \cos\vartheta}{1 + \cos\vartheta}} = \sqrt{\frac{\gamma[\rho(1 + \gamma) - 2]}{(\gamma - 1)(2 - \gamma\rho)}}, \qquad (76)$$

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and instead of expression (66)

$$\operatorname{tg}\frac{E}{2}=\sqrt{\frac{\rho(1+\gamma)-2}{(\gamma-1)\rho}}. \qquad (77)$$

If we take for the flight from the Earth to Mars, $\gamma = 1.52$, we obtain:

$$V_z=V_1\sqrt{3-\rho\mp 3.04\sqrt{\frac{2}{1.52}-\rho}}. \qquad (78)$$

For small values of the angle ϑ, (78) gives inaccurate values of V_z; we shall give two more accurate expressions. From equations (11) and (12) we find:

$$V_z=\frac{V_1\sin\beta}{\sin(\xi-\beta)}=\frac{V\sin\beta}{\sin\xi}. \qquad (79)$$

By calculating β and ξ initially as functions of ϑ, or of ρ, V_z can be easily determined. Formula (53) for β can then be transformed by replacing ρ by ϑ. For $\vartheta = 0$, we have from (76)

$$\rho_0=|\rho|_{\vartheta=0}=\frac{2}{(1+\gamma)}. \qquad (80)$$

We had

$$\operatorname{tg}\beta=\sqrt{\frac{(\gamma-1)[(\gamma+1)\rho-2]}{\gamma(2-\gamma\rho)}}. \qquad (54)$$

In this formula

$$(\gamma+1)\rho-2=(\gamma+1)[\rho-2/(\gamma+1)]=(\gamma+1)(\rho-\rho_0);$$

and by substituting ρ from (76), we obtain:

$$(\gamma+1)(\rho-\rho_0)=\frac{(\gamma-1)(1-\cos\vartheta)}{\gamma(\cos\vartheta+\gamma)}, \qquad (81)$$

and

$$2-\gamma\rho=\frac{1+\cos\vartheta}{\cos\vartheta+\gamma}.$$

After reduction, equation (54) takes the form

$$\operatorname{tg}\beta=\frac{\gamma-1}{\gamma}\operatorname{tg}\frac{\vartheta}{2}. \qquad (82)$$

This formula is accurate for small angles ϑ, and it can be used successfully for the calculation of V_z by (79).

In the second method we can determine V_z by the formula (see Figure 1)

$$V_z^2=V_1^2+V^2-2VV_1\cos\beta=(V-V_1)^2+$$
$$+2VV_1(1-\cos\beta)=(V-V_1)^2+4VV_1\sin^2\frac{\beta}{2};$$

or

$$V_z=(V-V_1)\sqrt{1+\frac{4VV_1\sin^2\frac{\beta}{2}}{(V-V_1)^2}}. \qquad (83)$$

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The velocity $V - V_1$ is calculated by a series expansion. From (57) we have:

$$\frac{V-V_1}{V_1} = \sqrt{2-\rho} - 1 = \frac{1}{2}(1-\rho) - \frac{1}{8}(1-\rho)^2 + \frac{1}{16}(1-\rho)^3 + \ldots$$

$$\ldots + (-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-3)}{2^n\cdot n!}(1-\rho)^n + \ldots$$

For the case of flight from Earth to Mars ($\gamma = 1.52$), we have

$$\rho = \rho_0 = \frac{2}{2.52} = 0.7938;$$

$$1 - \rho_0 = 0.2062;$$

and the series gives

$$\frac{V-V_1}{V_1} = 0.1031 - 0.005315 + 0.000548 - 0.000071 + 0.000010 =$$

$$= 0.09827.$$

Therefore, since $V_{z\,\min} = V - V_1$, we find

$$V_{z\,\min} = 0.09827 V_1 = 0.09827 \cdot 29.77 = 2.927 \text{ km/sec}.$$

The value of V_z' is found in a similar way by the formula (see Figure 4)

$$V_z' = V_1 \sin\beta / \sin(\beta + \xi' - 180°) = V \sin\beta / \sin\xi'. \tag{84}$$

In equation (55), tg ξ and tg $\frac{E}{2}$ in (77) contain the quantity

$$\rho - \frac{2}{1+\gamma} = \rho - \rho_0.$$

Since for small ϑ this is a very small quantity which cannot be determined accurately from the calculated values of ρ and ρ_0, we find it directly. From equation (81) we have

$$\rho - \rho_0 = \frac{(\gamma - 1)\, 2 \sin^2\frac{\vartheta}{2}}{(\gamma+1)\,\gamma\,(\cos\vartheta + \gamma)}. \tag{85}$$

The denominator of expression (55) for tg ξ contains the quantity

$$\gamma\sqrt{\frac{2}{\gamma} - \rho} - 1 = \sqrt{\gamma(2-\gamma\rho)} - 1,$$

which is rather difficult to determine since the first term is only slightly larger than 1. Denoting this quantity by y,

$$y = \sqrt{\gamma(2-\gamma\rho)} - 1,$$

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and introducing another variable χ, defined by the equation

$$\sin\chi=\gamma(2-\gamma\rho)-1,$$

we obtain

$$y=\sqrt{1+\cos(90^\circ-\chi)}-1=\sqrt{2}\cos\left(45^\circ-\frac{\chi}{2}\right)-1=2\sqrt{2}\sin\left(45^\circ-\frac{\chi}{4}\right)\sin\frac{\chi}{4}. \quad (86)$$

Substitution of the value of ρ from equation (76) in the expression for $\sin\chi$ yields:

$$\sin\chi=2\gamma-1-\gamma\frac{2\gamma-1+\cos\vartheta}{\cos\vartheta+\gamma}=\frac{\cos\vartheta(\gamma-1)}{\cos\vartheta+\gamma}. \quad (87)$$

Therefore,

$$\operatorname{tg}\xi=\frac{\sqrt{(\gamma-1)(\rho-\rho_0)}}{y}. \quad (88)$$

Flight to Mars

We now investigate the flight to the planet Mars, assuming it to be situated at an average distance from the Sun corresponding to $\gamma=1.52$.

The results of the calculations are given in Table 1 and the basic relations of the kinematic parameters of the flight trajectory are given in Figure 5.

We note that since we required the aphelions of the flight trajectories to be on the orbit of Mars, the value of ρ had to lie between the two limits determined by formulas (29) and (30):

$$a_{max}=\frac{(r_1+r_2)}{2}, \quad (29)$$

$$a_{min}=\frac{r_2}{2}. \quad (30)$$

From these two equations we obtain

$$\rho_{min}=\frac{r_1}{a_{max}}=\frac{2r_1}{(r_1+r_2)}=\frac{2}{(1+\gamma)} \quad (89)$$

and

$$\rho_{max}=\frac{r_1}{a_{min}}=\frac{2}{\gamma}, \quad (90)$$

as the limiting values of ρ.

In our case we have:

$$\rho_{min}=0.7938;\ \rho_{max}=1.316.$$

The angle ϑ was chosen as the independent variable for the interval $0<\vartheta<90^\circ$ and the quantity ρ for the range $90^\circ<\vartheta<180^\circ$. This choice was required since in the interval $0^\circ<\vartheta<90^\circ$ the value of ρ hardly varies so that determining ϑ from a given ρ would give inaccurate results.

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From (52) we obtain:

$$\rho = \frac{2\gamma - 1 + \cos\vartheta}{\gamma(\cos\vartheta + \gamma)}. \qquad (91)$$

The values of ρ obtained by this formula, served for the calculation of the corresponding part of the table.

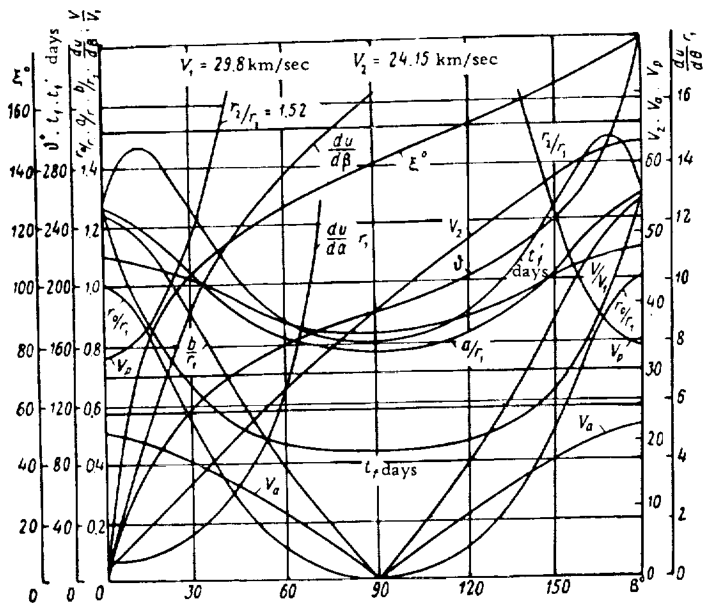

Figure 5

The curve of r_0/r_1 shows how close the spaceship approaches the Sun at the perihelion (if it flies through it).

The velocity of Mars in its orbit (assumed circular) is determined by (60) and is equal to V_2 = 24.15 km/sec (the Earth's average velocity is V_1 = 29.77 km/sec).

The curve of $(V_2 - V_a)$ gives the difference between the velocity of Mars and that of the spaceship in its neighborhood. It can be seen from the table that in the case considered we always have $V_2 - V_a > 0$, i.e., Mars moves faster than the spaceship which is about land. This means that the spaceship approaches Mars from that part of space into which the latter is moving. The relative velocity is equal, in the most favorable case, to 2.63 km/sec (see Table 1). The resultant of this velocity and the velocity due to the attraction of Mars itself, must be cancelled by gliding descent on the planet or by using a parachute or a rocket for braking.

Table 1 gives values of the following quantities, calculated by formulas (52) - (70) for V_1 = 29.77 km/sec and γ = 1.52:

$\rho = r_1/a$, ratio of Earth's orbit radius to semimajor axis of flight trajectory;

V_z, required additional velocity, km/sec;

β, angle between resulting velocity V and Earth velocity V_1, degrees;

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ξ, angle between additional velocity V_z and Earth velocity V_1, degrees;
ϑ, spaceship's true anomaly at take-off moment, degrees;
a/r_1, ratio of semimajor axis of flight trajectory to Earth orbit radius;
b/r_1, ratio of semiminor axis of flight trajectory to Earth orbit radius;
r_0/r_1, ratio of radius-vector of flight trajectory perihelion to Earth orbit radius;
Σ, eccentricity of elliptical flight trajectory;
V_z', additional velocity required when flying against the Earth's direction of motion, km/sec;
$180-\xi'$, angle between additional velocity V_z' and the Earth's direction of motion, degrees;
V, absolute velocity at moment of take-off from Earth, km/sec;
V_a, flight velocity at elliptical trajectory aphelion, km/sec;
V_p, flight velocity at perihelion, km/sec;
V_2-V_a, difference between Mars velocity and spaceship velocity in the planet's neighborhood (not accounting for attraction of Mars);
E, eccentric anomaly of spaceship at moment of take-off from Earth, degrees;
τ, semirotation period of spaceship around the Sun, days;
t, time required by spaceship to fly from perihelion to flight trajectory's point of intersection with Earth orbit, days;
$\tau-t$, flight time of spaceship from Earth to Mars (take-off from nearest point), days;
$\tau+t$, flight time of spaceship from Earth to Mars (take-off from more distant point), days.

If the spaceship's rocket engine is switched on once more shortly before arriving at Mars, it is then possible to reach it somewhat faster. Similarly it is possible to switch on the rocket engine again sometime after take-off from the Earth in order to shorten the time and path of the flight. This will be true, of course, for similar flight to other planets.

Turning to Figure 5, we see that when angle β increases, the flight time t_f first decreases rapidly and then remains almost constant for β between 55° and 125°, while the additional velocity V_z increases continuously and with it the propellant mass ratio required for the flight.

The maximum absolute flight velocity V is attained for $\beta = 0$, where $V = 1.10V_1$; for $\beta = 90°$ the velocity decreases to $V = 0.83V_1$ and for $\beta = 180°$ it increases again to $V = 1.10V_1$. This shows the advantage of flying with a low additional velocity if the required propellant mass ratio plays an important role.

It can also be seen from Figure 5 that when flying from A to B through C, the maximum flight time is equal to $t_f' \approx 293$ days only, while for $\beta = 0$ it is equal to 258.4 days, and for $\beta = 90°$, i.e., when flying directly opposite to the Sun, the flight time attains a minimum equal to approximately 85 days, i.e., about 3 months.

The angle ξ of the relative velocity V_z increases rapidly with β.

For $\beta = 0$, we have: $V_{z\,min} \approx 2.93$ km/sec; $\xi = 0$; $\vartheta = 0$; $V \approx V_1 + 2.93 \approx$ ≈ 32.7 km/sec; $V_a \approx 21.5$ km/sec; $V_p \approx 32.7$ km/sec; $a \approx 1.26\,r_1$; $b \approx 1.23\,r_1$ and $\Sigma \approx 0.21$.

Comparing a and b, we see that in the present case the ellipse is almost a circle.

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TABLE 1

Quantities characterizing the flight from Earth to Mars for the case when the latter is at the aphelion of the spaceship's elliptical trajectory
(Earth velocity $V_1 = 29.77$ km/sec; $\gamma = r_2/r_1 = 1.52$)

$\rho = r_1/a$	0.7937	0.7970	0.8073	0.8271	0.8600	0.8829	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.316
V_z	2.927	3.42	4.73	6.53	8.81	10.18	11.92	13.71	16.09	18.41	20.8	22.3	25.0	29.3	34.3	38.6
β	0°	3°27'	7°6'	11°11'	16°2'	18°58'	20°55'	26°14'	31°20'	36°28'	41°50'	47°50'	54°38'	63°20'	77°23'	90°
ε	0°	35°2'	58°13'	73°25'	84°54'	90°	93°4'	100°15'	105°37'	110°30'	114°50'	119°10'	123°30'	128°18'	134°53'	140°28'
ϑ	0°	20°	40°	60°	80°	90°	101°42'	110°30'	121°20'	130°10'	138°10'	146°3'	152°7'	161°	173°	180°
a/r_1	1.260	1.255	1.239	1.209	1.163	1.133	1.110	1.054	1.000	0.951	0.909	0.870	0.834	0.800	0.769	0.760
b/r_1	1.233	1.226	1.207	1.170	1.107	1.064	1.032	0.943	0.855	0.765	0.674	0.576	0.474	0.348	0.160	0
r_0/r_1	1.000	0.989	0.957	0.897	0.807	0.746	0.702	0.585	0.480	0.385	0.299	0.219	0.1474	0.080	0.0169	0
Σ	0.206	0.212	0.227	0.257	0.308	0.342	0.369	0.444	0.520	0.596	0.671	0.748	0.823	0.900	0.978	1.000
V_z'	62.47	62.37	62.18	61.83	61.04	60.44	59.9	58.6	57.3	55.9	54.0	52.3	50.0	47.3	42.8	38.6
$180° - \xi'$	0°	1°48'	3°42'	5°48'	8°16'	9°42'	10°42'	13°18'	15°40'	18°1'	20°23'	22°55'	26°8'	29°10'	34°30'	39°40'
V	32.69	32.65	32.52	32.25	31.78	31.46	31.2	30.5	29.77	29.0	28.3	27.4	26.6	25.8	24.9	24.6
V_a	21.52	21.45	21.23	20.81	20.10	19.59	19.19	18.00	16.73	15.35	13.84	12.12	10.19	7.65	3.50	0
V_p	32.69	32.94	33.71	35.23	37.93	39.95	41.5	46.6	52.9	60.5	70.3	83.8	104.7	144.8	321	–
$V_2 - V_a$	2.63	2.70	2.92	3.34	4.05	4.56	4.96	6.15	7.42	8.80	10.31	12.03	13.96	16.50	20.57	24.15
E	0°	16°12'	33°12'	47°50'	62°48'	70°1'	74°14'	83°20'	90°	94°40'	98°30'	101°38'	104°	106°8'	107°54'	108°20'
τ	258.4	256.7	251.7	242.8	229	220.1	214.5	197	182.6	169.5	158.7	148.3	138.9	130.3	123.2	121.7
t	0	18.28	35.3	49.8	60.0	63.1	64.55	63.8	61.25	57.5	53.5	49.4	45.1	41.9	37.65	36.9
$\tau - t$	258.4	238.4	216.4	193.0	169.0	157.0	149.9	133.2	121.4	112	105.2	98.9	93.8	88.4	85.6	84.8
$\tau + t$	258.4	275.0	287.0	292.6	289.0	283.2	279.1	260.8	243.9	227	212.2	197.7	184.0	172.2	160.9	158.6

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For large β, such high velocities V_z are required that the rocket will hardly be able to develop them. In particular, for $180°<\vartheta<360°$ it will be more advantageous to take the longer path from C to B through A, if in this case the proximity of the Sun at the perihelion does not disturb the flight and if the date of arrival permits an increased flight duration. In general, however, already for $\vartheta<180°$, it will be more advantageous, from the point of view of propellant consumption for a given flight time, to choose for the trajectory an ellipse whose aphelion lies farther than the given planet. This refers in particular to the case when we want to reach a given planet in the shortest time for a given propellant consumption, i.e., for a given V_z.

For comparison with the case considered we will determine the additional velocity V_z and the flight duration t_f for the case when the aphelion of the trajectory is at infinity and the flight begins in the direction of the Earth's motion around the Sun. From formula (33) we obtain:

$$V_z = V_1\left(\sqrt{\frac{2}{r_1/r_2+1}} - 1\right)\Bigg|_{r_2\to\infty} = $$
$$= V_1(\sqrt{2}-1) = 0.416\cdot 29.77 = 12.3 \text{ km/sec}.$$

The time required to fly from the Earth to Mars is obtained from a formula given in astronomy books. Using the equation of the parabola in polar coordinates

$$r = \frac{2r_0}{1+\cos\vartheta} \tag{92}$$

and Kepler's second law, we obtain the expression

$$t = \frac{\sqrt{2}\tau_1}{\pi}\left(\operatorname{tg}\frac{\vartheta}{2} + \frac{1}{3}\operatorname{tg}^3\frac{\vartheta}{2}\right). \tag{93}$$

In our case $r = 1.52\, r_0$ and $r_0 = r_1$ (the flight starts at the trajectory perihelion). From (92) we obtain:

$$\cos\vartheta = \frac{2r_1}{r} - 1 = \frac{2}{1.52} - 1 = 0.318; \quad \vartheta = 71°28'.$$

From (93) we find:

$$t = \frac{365.26\sqrt{2}}{2\pi}\left(\operatorname{tg} 35°44' + \frac{1}{3}\operatorname{tg}^3 35°44'\right) = 69.5 \text{ days}.$$

We see that an additional velocity of 12.30 km/sec gives us a flight time of 69.5 days for this trajectory, while the same additional velocity gave a flight time of approximately 146 days in the case considered before. If, however, the rocket engine is damaged, or if the flight trajectory is not chosen accurately, it may happen that a vehicle flying along a parabolic trajectory will escape forever from the solar system, as explained before. Attention should also be drawn to the fact that in the cases considered, each flight trajectory corresponds to a perfectly determined moment of arrival at the other planet.

The relations given in Figure 5, refer also to a flight from Mars to Earth. In this case $(V_2 - V_a)$ is the additional velocity which has to be

TABLE 2

Magnitude of the initial velocity c which must be given to a spaceship at the Earth's surface for flight to Mars and vehicle's initial weight G_0 (assuming final weight of $G = 1$ ton) for various values of the spaceship's true anomaly ϑ at the moment of take-off (see Table 1)

ϑ	0	20°	40°	60°	80°	90°	101°42'	110°30'	121°20'	130°10'	138°10'	146°03'	152°07'	161°	175°	180°
V_z, km/sec	2.927	3.42	4.73	6.53	8.81	10.18	11.92	13.71	16.09	18.41	20.8	22.3	25.0	29.3	34.3	38.6
V_z^2	8.00	11.72	22.4	42.5	78.0	104	142	189	259	340	435	499	625	860	1180	1495
$V_z^2 + 11.18^2$	131.9	137.02	147.7	167.8	203.3	229.3	267.3	314.3	384.3	465.3	560	624	750	985	1305	1620
c	11.57	11.70	12.11	12.91	14.21	15.1	16.32	17.7	195	21.55	23.6	25.0	27.4	31.4	36.0	40.2
$\lg \frac{M_0}{M} = \frac{c}{w}\frac{n}{n-1}\lg e$; $n = 3$, $w = 4000$	1.88	1.201	1.970	2.10	2.31	2.455	2.658	2.88	3.17	3.50	3.84	4.07	445	5.105	5.85	6.54
$n = 10$, $w = 3000$	1.853	1.879	1.948	2.075	2.28	2.423	2.62	2.84	3.13	3.46	3.78	4.01	4.40	4.045	5.78	6.45
$n = 3$, $w = 3000$	2.505	2.54	2.625	2.80	3.08	3.275	3.54	3.84	4.23	4.67	5.12	5.42	5.95	6.81	7.80	8.72
G_0 for $G = 1$ ton, tons; $n = 10$, $w = 4000$	24.6	25.7	28.8	35.8	51.5	66.0	92.9	137	224	394	690	1 023	2 000	6 050	22 000	70 000
$n = 3$, $w = 4000$	76.0	79.5	93.4	126	204	286	454	760	1 480	3 160	6 900	11 700	28 200	127 000	710 000	$3.47 \cdot 10^6$
$n = 10$, $w = 3000$	71.3	75.1	88.8	119	191	265	417	690	1 350	2 880	6 030	10 300	25 000	110 000	600 000	$2.821 \cdot 10^6$
$n = 3$, $w = 3000$	320	347	422	630	1210	1880	3480	6930	17 000	47 000	130 000	260 000	8.105	$6.45 \cdot 10^6$	$6.30 \cdot 10^7$	$5.31 \cdot 10^8$
$t_a = \vartheta/360° \cdot 783$ (time of take-off from nearest point, days)	0	43.5	87	130.5	174	196.7	221.5	240.5	264	284	301	318	332	350	376	392
$t_e = t + t_f$ (time of arrival for take-off from nearest point, days)	258.4	281.9	303.4	323.5	343	353.7	371.4	373.7	385.4	396	406.2	416.9	425.8	438.4	461.6	176.8
$t_a' = 318 - t_a$ (time of take-off from far point, days)	318	274.5	231	187.5	144	121.3	96.5	77.5	54	34	17	0	-	-	-	-
$t_e' = t_a' + t_f'$ (time of arrival for take-off from far point, days)	576.4	549.5	518	480.1	433	404.5	375.6	338.3	297.9	261	229.2	197.7	-	-	-	-

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imparted to the spaceship near Mars in a direction opposite to the direction of motion of Mars around the Sun. The velocity V_t and the velocity due to the Earth's attraction must be cancelled for landing on the Earth. The flight trajectories themselves represent a mirror reflection of the trajectories along which the spaceship flew to Mars, and the velocities at the Earth point in the opposite direction.

The total velocity c which must be given to the rocket when it takes-off from the Earth (or cancelled when it returns), is found from conservation of energy:

$$\frac{mc^2}{2}=\frac{mc_0^2}{2}+\frac{mV_z^2}{2};$$

$$c=\sqrt{c_0^2+V_z^2}, \tag{94}$$

where c_0 = 11.16 km/sec is the initial velocity which must be given to a body at the Earth's surface if its velocity at infinity is zero.

If the velocity c is given to the vehicle in the immediate vicinity of the Earth and the rocket acceleration is n times the gravitational acceleration, then the ratio of the initial mass M_0 to the final empty mass M of the rocket, for a vertical take-off, should be according to Tsiolkovskii:

$$\frac{M_0}{M}=e^{\frac{c}{w}\frac{n}{n-1}},$$

where w is the exhaust velocity of the combustion products.

Taking the values n = 10 and n = 3, for which w = 4000 m/sec, w = = 3000 m/sec, we obtain 4 cases for which Table 2 was calculated.

We can see from this table that it is very difficult to attain velocities $c \geqslant 22$ km/sec by means of a rocket due to the huge initial weight required. If we limit the initial weight to 1000 times the final weight, we obtain for the maximum values of ϑ:

1. For w = 4 km/sec and n = 10, we have $\vartheta_{max} \approx 146°$.
2. For w = 4 km/sec and n = 3, we have $\vartheta_{max} \approx 114°$.
3. A slightly larger angle is obtained for w = 3 km/sec and n = 10.
4. For w = 3 km/sec and n = 3, we have ϑ_{max} = 73°.

If we do not fly through the perihelion then ϑ_{min} = 0; the minimum initial weights will then be 24.6, 76 and 320 ton. In the case of trajectories which do pass through the perihelion, the time interval during which it is possible to take-off for Mars is doubled; in this case $\vartheta_{min} \approx -146°$, $-114°$ and $-73°$.

The period between two oppositions of Mars, in which it is possible to cross by these trajectories can be determined as follows. Let us denote by t_1 and t_2 the rotation times of the planets about the Sun and by t_x the time between two oppositions of the external planet or, the so-called synodical rotation time.

The angular velocity of the internal planet is $360°/t_1$, and that of the external planet $360°/t_2$. Their relative angular velocity is $\frac{360°}{t_1}-\frac{360°}{t_2}$, and the angle between the radius-vectors varies by 360° during time t_x, where

$$t_x\left(\frac{360°}{t_1}-\frac{360°}{t_2}\right)=360°,$$

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or

$$t_x = \frac{t_1 t_2}{t_2 - t_1}.$$

For Mars and the Earth, $t_1 \approx 365.3$ days, $t_2 \approx 684.5$ days, and therefore, $t_x \approx 783$ days*. Thus, in the conditions indicated, the communication with Mars would always be interrupted for 783-209 = 574 days and only in the course of 209/783 = 0.268 = 26.8% of the whole time would it be possible to despatch the spaceship. It would be possible to arrive at Mars only in the course of 110 days or during 110/783 = 0.141 = 14.1% of the whole time.

2. DETERMINATION OF TAKE-OFF MOMENT TO ANOTHER PLANET ENSURING ADDITIONAL VELOCITY CLOSE TO MINIMUM

We will now determine the mutual position of the planets in their orbits at the time of take-off and arrival of the spaceship, the time interval between two take-offs and between the corresponding moments of arrival for flights in various trajectories, and the time interval during which flights to other planets are possible.

We will use the notation (Figure 6):

Δt_c, time between two take-offs considered;
Δt_e, time between two arrivals considered;
t_1, period of revolution of internal planet around Sun;
t_2, period of revolution of external planet around Sun;
$A'AE'E$, orbit of destination planet;
$CC'G'G$, orbit of departure planet;
AE, direction of semimajor axis of flight trajectory ellipse for minimum additional velocity (in this case CtE is the flight trajectory and C and E are the perihelion and aphelion respectively);
$A'E'$, direction of semimajor axis of flight trajectory ellipse for flight from internal planet along $C't'E'$ (in this case E' is the aphelion of the flight trajectory);

$$\delta = \angle A'OA;$$
$$\eta = \angle AOC';$$
$$\vartheta = \angle A'OC = \eta + \delta.$$

We will assume for simplicity that the planetary orbits are circles with the Sun at the center, that the external planet is at point B (B') at the moment of take-off from the internal planet, that the internal planet is at point G (G') at the moment of arrival of the spaceship, and we will write

$$\angle AOB = \alpha;\ \angle AOB' = \alpha';\ \angle COG = i;\ \angle COG' = i'.$$

Assuming the orbital motion of the planets to be uniform, we obtain

$$\Delta t_c = \frac{t_1(\vartheta - \delta)}{360} = \frac{t_2(\alpha' - \alpha)}{360}, \tag{95}$$

$$\Delta t_e = \frac{t_1(i - i')}{360} = \frac{t_2\delta}{360}, \tag{96}$$

* A more exact figure for Mars is 779 days on the average.

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since the planets describe the following angles during the time between two take-offs: the internal planet angle $COC' = \eta - \delta$; the external one, angle $BOB' = \alpha' - \alpha$; similarly between two arrivals: the internal planet describes angle $G'OG = i - i'$; and the external one, angle $E'OE = \delta$. If we start measuring time from the moment the internal planet is at point C, then it will be at point C' at the moment Δt_c, the spaceship flying along the trajectory $C'E'$ arrives at the other planet at the moment $\Delta t_c + t'$, and the external planet will be at point E at the moment $\Delta t_c + t' + \Delta t_e$. However, this last time interval should be equal to the flight time along the path CE, i. e., to t. Thus, we have the equation:

$$t = \Delta t_c + t' + \Delta t_e; \qquad (97)$$

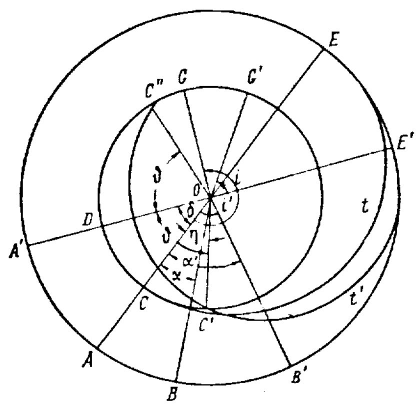

FIGURE 6

or substituting the expressions for Δt_c and Δt_e from equations (95) and (96), we obtain:

$$t = \frac{t_1(\vartheta - \delta)}{360} + t' + \frac{t_2 \delta}{360}.$$

Hence, $\angle E'OE$ is determined:

$$\delta = \frac{(t - t')\,360 - t_1 \vartheta}{t_2 - t_1}. \qquad (98)$$

It is then simple to find the angles described by the planets during the time interval Δt_c:

$$\angle COC' = \vartheta - \delta = \eta; \quad \angle BOB' = \alpha' - \alpha = \frac{\Delta t_c \cdot 360}{t_2}; \qquad (99)$$

$$\angle G'OG = i - i' = \frac{t_2}{t_1}\delta; \qquad (100)$$

$$\angle E'OE = \delta_1,$$

and from equations (95), (96) and (98) the values of Δt_c and Δt_e, if the flight times t and t', the angle ϑ and the revolution times of the planets t_2 and t_1 are given.

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We see that the external planet describes the angle $BOE = \frac{t}{t_2} 360$ during the flight time t, and the internal planet – the angle

$$i = \angle COG = \frac{t}{t_1} 360. \quad (101)$$

Similarly during the flight time t'

$$\angle B'OE' = \frac{t'}{t_2} \cdot 360, \quad \angle C'OG = \frac{t'}{t_1} \cdot 360,$$

therefore,

$$\alpha = \angle AOB = 180 - \angle BOE;$$

$$\alpha = 180 - \frac{t}{t_2} 360; \quad (102)$$

$$\alpha' = \angle AOB' = 180° - \angle BOE - \delta;$$

or

$$\alpha' = 180 - \frac{t'}{t^2} 360 - \delta; \quad (103)$$

and

$$i' = \angle COG' = \eta + \angle C'OG';$$

or

$$i' = \eta + \frac{t'}{t_2} 360. \quad (104)$$

The quantity Δt_c is particularly important since it determines the time interval during which it is possible to take-off with an additional velocity close to the minimum, i. e., to fly with minimum propellant consumption. The time Δt_e, which shows how many days earlier it is possible to arrive at the other planet if the additional velocity is increased, is also important.

For a flight from point C'' to point E' through point C' the formulas are somewhat modified: in this case $(+\vartheta)$ should be replaced everywhere by $(-\vartheta)$.

Thus, we have in the general case:

$$\delta = \frac{(t - t') \cdot 360 \mp t_1 \vartheta}{t_2 - t_1}; \quad (105)$$

$$\Delta t_c = \frac{t_1 (\pm \vartheta - \delta)}{360}; \quad (106)$$

$$\Delta t_e = \frac{t_2 \cdot \delta}{360}; \quad (107)$$

$$\eta = \pm \vartheta - \delta; \quad i = \frac{t}{t_1} 360; \quad (108)$$

$$i' = \eta + \frac{t'}{t_1} 360; \quad (109)$$

$$\alpha = 180 - \frac{t}{t_2} 360; \quad (102)$$

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$$\alpha' = 180 - \frac{t'}{t_2} 360 - \delta; \qquad (103)$$

where the upper sign refers to flight from point C', and the lower sign to flight from point C''.

The curves for flight to Mars (see Figure 5) based on the formulas derived above, show interesting results. It should be noted that in view of the elliptical orbit of Mars, these results represent only average values. For each new revolution of Mars around the Sun, the angles, times, and other quantities will vary, but their average values will coincide with those calculated. The inclination of the orbital planes with respect to one another hardly changes the results.

It can be seen from Figure 5 that for flights from point C', the flight time decreases rapidly for an increase of the additional velocities; for an additional velocity of 8 km/sec it becomes only 172 days, whereas for the minimum additional velocity of $V_2 = 2.73$ km/sec it is equal to 257 days. In the first case it is possible to take-off $\Delta t_c = 74$ days later than for $V_2 = 2.73$ km/sec, although the arrival on Mars is then only 19 days earlier than for the additional velocity $V_2 = 2.73$ km/sec.

For an additional velocity $V_2 = 8$ km/sec and take-off from point C'', the flight time is 290 days. If it is required to arrive as early as possible on Mars, then people will very probably fly from point C'' notwithstanding the long flight time. If the time of stay on the way has to be shortened, then it is better to start the flight from point C. For flight from point C'', the perihelion lies at a distance of 0.81 times the Earth's orbital radius from the Sun and the ship's velocity at the perihelion is 35 km/sec which exceeds only slightly the Earth's orbital velocity (29.8 km/sec).

The first flights will very likely be accomplished with an additional velocity close to the minimum of $V_2 \approx 2.7$ km/sec. To reduce the flight duration it can be started from some point C' which leads, however, to increased propellant consumption.

If the aphelion of the trajectory lies on the orbit of Mars, and if we assume a maximum V_2 of 8 km/sec, then take-off to Mars is possible only during 74+135 = 209 days. Then we have a period during which communication with Mars must be interrupted. This requirement is important. For example, if correction of the trajectory becomes impossible due to damage to the rocket by meteors, or if the control system fails, then the deviation caused by the attraction of Mars (if the spaceship flies near it) may modify the flight trajectory in such a way that the vehicle will escape from the Sun forever. It will be very hard to overtake such a vehicle with another spaceship.

However, some shifting of the aphelion of the spaceship's trajectory farther behind the orbit of the external planet lengthens somewhat the take-off period and shortens the flight time.

3. TRAJECTORY CORRECTION ON APPROACH TO PLANETS TO ACHIEVE SAFE LANDING IN DESIRED PLACE

When a spaceship approaches a planet, the flight velocity is in general hyperbolic (if it is not reduced by means of the rocket engine near the

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planet); we shall therefore assume that the trajectory of the spaceship is a hyperbola, with the given planet as one of its foci (Figure 7). We shall assume for this purpose that the distance from the vehicle to the given planet is so small that the attraction of the Sun and of the planet's satellites can be neglected. If the relative velocity V_1 near the destination planet is found by calculation, then the velocity at the planet's surface can be easily obtained by using conservation of energy:

$$\frac{mV_1^2}{2} + mg_0R = \frac{mV_0^2}{2}, \qquad (110)$$

where m is the spaceship's mass;
R, the planet's radius;
g_0, the gravitational acceleration at the planet's surface;
g_0R, the surface potential of the planet per unit mass;
V_0, the velocity at a distance R from the planet's center.

From equation (110) we obtain

$$V_0 = \sqrt{V_1^2 + 2g_0R}. \qquad (111)$$

Since the sum of the kinetic and potential energies of a body is constant, it is also possible to determine the velocity at any distance from the planet.

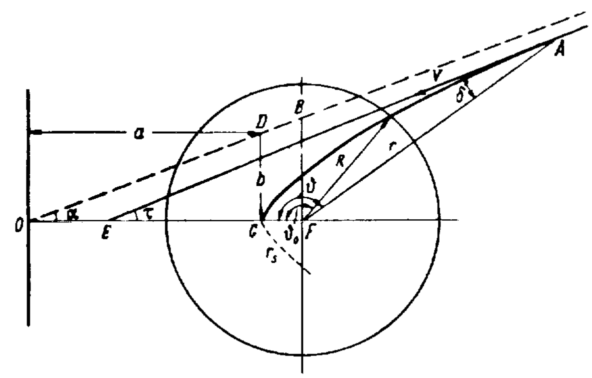

FIGURE 7

The potential energy of a body with mass m, situated at infinity, with respect to a sphere of radius r, is equal to:

$$A = mgr, \qquad (112)$$

where g is the planet's gravitational acceleration at a distance r from its center:

$$g = \frac{R^2}{r^2} g_0. \qquad (113)$$

Substituting this expression in formula (112), we obtain:

$$A = mg_0 \frac{R^2}{r}.$$

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If the body has reached the sphere of radius r, its kinetic energy $mV^2/2$ is equal to the sum of its initial kinetic energy $mV_1^2/2$ and to the variation A in its potential energy, i. e.,

$$\frac{mV^2}{2}=\frac{mV_1^2}{2}+mg_0\frac{R^2}{r};$$

or

$$V=\sqrt{V_1^2+\frac{2g_0R^2}{r}}. \tag{114}$$

For correction of the flight we are interested in three curves: two hyperbolas (before and after correction) and the flight trajectory during the correction.

Let us determine the elements of the hyperbolic trajectory, the velocity and the flight time.

Let the spaceship be at point A at a given moment, let r be the distance between the point A and the center F of the planet, V, the spaceship's velocity with respect to the planet and δ, the angle between AF and V (see Figure 7).

Let us introduce the following notation:

Σ, hyperbola's relative eccentricity;
a, hyperbola's real semi-axis;
b, hyperbola's imaginary semi-axis;
$2p$, hyperbola's parameter;
ϑ, ϑ_0, angles between radius-vectors r, R and line OF (O is the hyperbola's center);
τ, angle between tangent and real axis;
α, angle between asymptote and real axis;
R, planet's radius;
$r_s=FC$, radius-vector to the hyperbola's vertex (point C).

In polar coordinates the hyperbola's equation has the form:

$$r=\frac{p}{1+\Sigma\cos\vartheta}, \tag{115}$$

where

$$p=-a\left(1-\Sigma^2\right)=\frac{b^2}{a}. \tag{116}$$

Therefore,

$$r=\frac{a\left(\Sigma^2-1\right)}{1+\Sigma\cos\vartheta}. \tag{117}$$

For the vertex of the hyperbola, $\vartheta=0$, and, consequently,

$$r=r_s=\frac{a\left(\Sigma^2-1\right)}{1+\Sigma},$$

or

$$r_s=a(\Sigma-1). \tag{118}$$

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For $r=R$ we find, for the angle ϑ_0:

$$\cos\vartheta_0=\frac{a\left(\sum^2-1\right)}{R\sum}-\frac{1}{\sum}. \qquad (119)$$

If we take the hyperbola's equation in Cartesian coordinates

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$$

and differentiate it, we obtain

$$\frac{2x\,dx}{a^2}-\frac{2y\,dy}{b^2}=0,$$

or, since

$$\operatorname{tg}\tau=\frac{dy}{dx},$$

then

$$\frac{dy}{dx}=\frac{x}{y}\,\frac{b^2}{a^2}=\operatorname{tg}\tau.$$

However, it follows from Figure 7 that

$$x=\sqrt{a^2+b^2}-r\cos\vartheta;$$
$$y=r\cdot\sin\vartheta;$$
$$OF=\sqrt{a^2+b^2}.$$

so that

$$\operatorname{tg}\tau=\frac{\left(\sqrt{a^2+b^2}-r\cos\vartheta\right)}{r\sin\vartheta}\,\frac{b^2}{a^2}. \qquad (120)$$

Eliminating b in this expression with the aid of the formula

$$\sum=\frac{\sqrt{a^2+b^2}}{a}, \qquad (121)$$

or

$$\sum^2-1=\frac{b^2}{a^2}, \qquad (122)$$

i. e.,

$$b=a\sqrt{\sum^2-1}, \qquad (123)$$

we obtain

$$\operatorname{tg}\tau=\frac{\sum a-r\cos\vartheta}{r\sin\vartheta}\left(\sum^2-1\right). \qquad (124)$$

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Eliminating r from equations (117) and (124), we have

$$\operatorname{tg}\tau=\frac{\cos\vartheta+\sum}{\sin\vartheta}. \tag{125}$$

From Figure 7 we have

$$\operatorname{tg}\alpha=\frac{b}{a}=\sqrt{\sum^2-1}. \tag{126}$$

The velocity at the vertex of the hyperbola is obtained from equation (114) for $r=r_s$:

$$V_s=\sqrt{V_1^2+\frac{2g_0R^2}{r_s}}. \tag{127}$$

From the law of areas we obtain the value $rV\sin\delta\,dt/2$ for the area swept during the time dt at point A and $r_sV_s\,dt/2$ at the vertex of the hyperbola. Therefore:

$$r_sV_s=rV\sin\delta; \tag{128}$$

$$r_sV_s=RV_0\sin\delta_0. \tag{129}$$

From the triangle FAE (see Figure 7) we obtain

$$180°=\delta+\tau+\vartheta, \tag{130}$$

hence

$$\operatorname{tg}\delta=\operatorname{tg}(180°-\tau-\vartheta)=-\operatorname{tg}(\tau+\vartheta)=-\frac{\operatorname{tg}\tau+\operatorname{tg}\vartheta}{1-\operatorname{tg}\tau\operatorname{tg}\vartheta}. \tag{131}$$

Substituting in (131) $\operatorname{tg}\tau$ from equation (125) and ϑ from (117), we obtain:

$$\operatorname{tg}\delta=\frac{\cos\vartheta+\frac{1}{\sum}}{\sin\vartheta}, \tag{132}$$

or

$$\operatorname{tg}\delta=\frac{a(\sum^2-1)}{r\sum\sin\vartheta}. \tag{133}$$

From the condition that the centripetal acceleration at the vertex of the hyperbola is due to the planet's attraction, we obtain one more equation.

The radius of curvature of the hyperbola at its vertex is $\frac{b^2}{a}=p$, therefore, the centripetal acceleration is equal to

$$\frac{V_s^2}{p}=\frac{V_s^2a}{b^2}.$$

However, the acceleration due to the planet's attraction is equal to

$$g_0\frac{R^2}{r_s^2},$$

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and, therefore

$$V_s^2 \frac{a}{b^2} = g_0 \frac{R^2}{r_s^2}.$$

Hence introducing Σ according to equation (121) and substituting for a its expression from (118) we obtain

$$V_s^2 = \frac{g_0 R^2}{r_s}\left(\sum + 1\right). \tag{134}$$

In this problem we are interested in the quantities:

$$V_1,\ V_0,\ V_s,\ a,\ b,\ \Sigma,\ \alpha,\ r,\ V, r_s,\ \vartheta,\ \vartheta_0,\ \delta,\ \delta_0,\ \tau.$$

To determine these 15 quantities we have 12 equations: (111), (114), (117), (118), (119), (123), (125), (126), (127), (128), (129), (134), and, therefore, given any three of the quantities, we can determine all the others. This is obvious. Since the general equation of the hyperbola contains five arbitrary constants and our quantities refer to the focus of the hyperbola, two of the constants are determined and only three arbitrary constants, which we are free to choose, remain. The interplanetary travellers can determine the distance r from the planet by measuring its angular diameter, the velocity V – by measuring the rate of variation of the planet's angular diameter, and the angle δ – by using some measuring instrument.

Assuming that r, δ, and V are given, let us determine the remaining quantities. From equation (114) we find

$$V_1 = \sqrt{V^2 - \frac{2 g_0 R^2}{r}}. \tag{135}$$

Then from equation (111) we obtain:

$$V_0 = \sqrt{V_1^2 + 2 g_0 R} = \sqrt{V^2 + 2 g_0 R^2 \left(\frac{1}{R} - \frac{1}{r}\right)}. \tag{136}$$

Eliminating the product $r_s V_s$ from equations (128) and (129) and introducing V_0 from equation (136), we obtain:

$$\sin\delta_0 = \frac{r V \sin\delta}{R V_0} = \frac{r V \sin\delta}{R \sqrt{V^2 + 2 g_0 R^2 \left(\frac{1}{R} - \frac{1}{r}\right)}}. \tag{137}$$

Eliminating then V_s, from equations (127) and (128), we obtain an equation for r_s:

$$V_s^2 = \frac{(r V \sin\delta)^2}{r_s^2} = V_1^2 + \frac{2 g_0 R^2}{r_s} \tag{138}$$

which gives

$$r_s = -\frac{g_0 R^2}{V_1^2} + \sqrt{\frac{g_0^2 R^4}{V_1^4} + \frac{(r V \sin\delta)^2}{V_1^2}}, \tag{139}$$

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where V_1 is given by (135).

From equation (128) we obtain

$$V_s = \frac{rV\sin\delta}{r_s}. \tag{140}$$

Instead of equation (138) we can also write

$$\frac{2g_0R^2}{(rV\sin\delta)^2}\frac{1}{r_s} - \frac{V_1^2}{(rV\sin\delta)^2} = 0;$$

$$\frac{1}{r_s} = \frac{g_0R^2}{(rV\sin\delta)^2} + \sqrt{\frac{g_0^2R^4}{(rV\sin\delta)^4} + \frac{V_1^2}{(rV\sin\delta)^2}}. \tag{141}$$

From equation (134) we find the relative eccentricity

$$\Sigma = \frac{V_s^2r_s}{g_0R^2} - 1, \tag{142}$$

and from equation (127)

$$V_s^2r_s = V_1^2r_s + 2R^2g_0.$$

Substituting this expression in equation (142), we have

$$\Sigma = \frac{V_1^2r_s}{g_0R^2} + 1 = \sqrt{1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2}, \tag{143}$$

where we have used the value of r_s found from equation (138).

From equation (118) we obtain

$$a = \frac{r_s}{\Sigma - 1}, \tag{144}$$

then, using equation (143) we have

$$a = \frac{g_0R^2}{V_1^2} = \frac{g_0R^2}{V^2 - \frac{2g_0R^2}{r}}. \tag{145}$$

By substituting in equation (119) a from equation (145) and Σ from equation (143), we obtain:

$$\cos\vartheta_0 = \left[\frac{g_0R^2}{RV_1^2}\left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2 - 1\right]\frac{1}{\sqrt{1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2}} =$$

$$= \left[\frac{(rV\sin\delta)^2}{g_0R^3} - 1\right]\frac{1}{\sqrt{1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2}}. \tag{146}$$

From equation (117) we obtain for $\cos\vartheta$ the same formula as for $\cos\vartheta_0$, with the only difference that now rR^2 replaces R^3 in the denominator of the first term:

$$\cos\vartheta = \left[\frac{(rV\sin\delta)^2}{g_0R^2r} - 1\right]\frac{1}{\sqrt{1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2}}. \tag{147}$$

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τ is found from equation (130):

$$\tau = 180^\circ - \delta - \vartheta. \tag{148}$$

or, if we want to express τ through the basic quantities, we substitute ϑ from equation (147) and Σ from equation (143) in equation (125), obtaining:

$$\operatorname{tg}\tau = \frac{\left[\frac{(rV\sin\delta)^2}{g_0R^2r} - 1\right] + 1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2}{\sqrt{1 + \left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2 - \left[\frac{(rV\sin\delta)^2}{g_0R^2r} - 1\right]^2}} =$$

$$= \frac{\left(\frac{rV\sin\delta}{g_0R^2}\right)^2\left(V_1^2 + \frac{g_0R^2}{r}\right)}{\sqrt{\left(\frac{rV\sin\delta V_1}{g_0R^2}\right)^2 + \frac{2(rV\sin\delta)^2}{g_0R^2r} - \frac{(rV\sin\delta)^4}{g_0^2R^4r^2}}}.$$

Substituting in this equation the expression of V_1 from (135), we get

$$\operatorname{tg}\tau = \frac{\left(\frac{rV\sin\delta}{g_0R^2}\right)^2\left(V^2 - \frac{g_0R^2}{r}\right)}{\frac{V\sin\delta}{g_0R^2}\sqrt{r^2\left(V^2 - \frac{2g_0R^2}{r}\right) + 2rg_0R^2 - r^2(V\sin\delta)^2}} =$$

$$= \frac{(rV\sin\delta)^2\left(V^2 - \frac{g_0R^2}{r}\right)}{V\sin\delta g_0R^2\sqrt{r^2V^2 - r^2V^2\sin^2\delta}};$$

or

$$\operatorname{tg}\tau = \frac{(rV\sin\delta)^2\left(V^2 - \frac{g_0R^2}{r}\right)}{rV^2\sin\delta\cos\delta g_0R^2} = \operatorname{tg}\delta\left(\frac{rV^2}{g_0R^2} - 1\right). \tag{149}$$

Finally, by substituting a from (145) and Σ from (143) we obtain the value of the semi-axis b from equation (123);

$$b = a\sqrt{\Sigma^2 - 1} = \frac{g_0R^2}{V_1^2}\,\frac{rV\sin\delta V_1}{g_0R^2};$$

or

$$b = \frac{rV\sin\delta}{V_1} = \frac{rV\sin\delta}{\sqrt{V^2 - \frac{2g_0R^2}{r}}} \tag{150}$$

and

$$\operatorname{tg}\alpha = \frac{b}{a} = \sqrt{\Sigma^2 - 1} = \frac{rV\sin\delta V_1}{g_0R^2} =$$

$$= \frac{rV\sqrt{V^2 - \frac{2g_0R^2}{r}}\sin\delta}{g_0R^2}. \tag{151}$$

If we introduce in this formula the velocity V_∞ required to escape from the surface of the planet to infinity, then the formulas are simplified.

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Putting $V_1=0$, we obtain from equation (111) for V_∞

$$V_\infty^2=2g_0R. \tag{152}$$

The case $r_s=R$, when the vertex of the hyperbola touches the planet, is particularly important.

Then, $V_s=V_0$ and we obtain from equation (128):

$$r_sV_s=RV_0=rV\sin\delta. \tag{153}$$

In this case equations (135) and (136) remain unchanged.

From equation (137) we have $\delta_0=90°$, $\sin\delta_0=1$, and from equations (143) and (152)

$$\sum=\sqrt{1+\left(\frac{RV_0V_1}{g_0R^2}\right)^2}=\sqrt{1+\left(2\frac{V_0V_1}{V_\infty^2}\right)^2}=$$
$$=\sqrt{1+\frac{2V_1^2}{V_\infty^4}(V_1^2+V_\infty^2)}=\sqrt{1+\frac{2V_1^4}{V_\infty^4}+\frac{2V_1^2}{V_\infty^2}}. \tag{154}$$

The value of the semi-axis a is calculated by (145). For ϑ_0 we obtain from equation (146):

$$\cos\vartheta_0=\left(\frac{R^2V_0^2}{g_0R^3}-1\right)\frac{1}{\sqrt{1+\left(\frac{RV_0V_1}{g_0R^2}\right)^2}}=\left(\frac{V_0^2}{g_0R}-1\right)\frac{1}{\sqrt{1+\left(\frac{V_0V_1}{g_0R}\right)^2}}=$$
$$=\frac{V_0^2-g_0R}{\sqrt{g_0^2R^2+V_0^4-2g_0RV_0^2}}=1;$$

i.e., $\vartheta_0=0$.

From equation (147), we have

$$\cos\vartheta=\left(\frac{R^2V_0^2}{g_0R^2r}-1\right)\frac{g_0R}{V_0^2-g_0R}=\frac{R}{r}\,\frac{V_0^2-g_0r}{V_0^2-g_0R},$$

or

$$\cos\vartheta=\frac{-\frac{V_0^2}{r}+g_0}{-\frac{V_0^2}{R}+g_0}. \tag{155}$$

Next, we obtain τ from equation (148), or directly from equation (149):

$$\operatorname{tg}\tau=\operatorname{tg}\delta\left(\frac{rV^2}{g_0R^2}-1\right)=\frac{RV_0V}{g_0R^2\cos\delta}-\operatorname{tg}\delta=\frac{V_0V}{g_0R\cos\delta}-\operatorname{tg}\delta \tag{156}$$

From equation (150) we find

$$b=\frac{RV_0}{V_1}=\frac{R\sqrt{V_1^2+2g_0R}}{V_1}=R\sqrt{1+\left(\frac{V_\infty}{V_1}\right)^2}; \tag{157}$$

and from equation (151)

$$\operatorname{tg}\alpha=\frac{RV_0V_1}{g_0R^2}=\frac{2V_1}{V_\infty}\sqrt{1+\left(\frac{V_1}{V_\infty}\right)^2}. \tag{158}$$

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For $r=\infty$ we should have

$$\tau_\infty=\alpha,\ \vartheta_\infty=180°-\alpha,\ \delta_\infty=0.$$

From equation (155) we have for this point of the hyperbola

$$\cos\vartheta_\infty=\frac{g_0}{g_0-\frac{V_0^2}{R}}=\frac{g_0R}{g_0R-V_0^2}=-\frac{V_\infty^2}{2\left(V_0^2-\frac{V_\infty^2}{2}\right)},$$

but, from equation (111)

$$V_0^2=V_1^2+V_\infty^2, \tag{159}$$

so that

$$\cos\vartheta_\infty=-\frac{V_\infty^2}{2\left(V_1^2+\frac{V_\infty^2}{2}\right)}=-\frac{V_\infty^2}{2V_1^2+V_\infty^2};$$

$$\operatorname{tg}\vartheta_\infty=\pm\sqrt{\frac{1}{\cos^2\vartheta_\infty}-1}=\pm\sqrt{\frac{(2V_1^2+V_\infty^2)^2}{V_\infty^4}-1}=$$

$$=\pm\frac{1}{V_\infty^2}\sqrt{4V_1^4+4V_1^2V_\infty^2}=$$

$$=\pm 2\frac{V_1}{V_\infty}\sqrt{1+\frac{V_1^2}{V_\infty^2}}=\pm\operatorname{tg}\alpha_\infty \text{ (see formula (158)).}$$

4. DETERMINATION OF ADDITIONAL VELOCITY REQUIRED FOR CORRECTING FLIGHT TRAJECTORY

Let us examine the two hyperbolas before and after the trajectory modification, assuming initially for simplicity that the modification takes place instantly. It is then simple to determine the required additional velocity.

Since a hyperbola which touches the surface of the planet with its vertex (Figure 8) corresponds to the normal landing case, we shall first assume that our hyperbola, after the trajectory modification, is precisely such a one. Let us denote the quantities which refer to the modified trajectory by a "prime". If the hyperbolas intersect at the radius $r_2=r_2'$, if r_2, V_2 and δ_2 are given and V_2' and δ_2' have to be determined, we have the equations:

$$r_s'=R;$$
$$V_z^2=V_2^2+V_2'^2-2V_2V_2'\cos(\delta_2-\delta_2'). \tag{160}$$

Furthermore, we have still the 12 equations indicated in the previous section. Since 3 of the 15 quantities which interest us must be given, and only two, r_s' and $r=r_2'$ are determined, we introduce the condition for minimum V_z, i.e.,:

$$\frac{dV_z}{dV_2}=0. \tag{161}$$

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We have introduced a new variable V_z and obtained two new equations (160) and (161), so that we can solve the problem completely. From the remaining equations we have to deduce initially a relation between δ_2' and V_2'.

For $r_s = R$ we obtain from equation (128)

$$RV_s = r_2' V_2' \sin \delta_2', \tag{162}$$

where $V_s = V_0$.

From equation (136) we have

$$V_0 = \sqrt{V_2'^2 - 2g_0 R + \frac{2g_0 R^2}{r_2'}}, \tag{163}$$

and from equations (162) and (163) we find

$$\sin \delta_2' = \frac{RV_0}{r_2' V_2'} = \frac{R}{r_2' V_2'} \sqrt{V_2'^2 - 2g_0 R + \frac{2g_0 R^2}{r_2'}},$$

or

$$\sin \delta_2' = \frac{R}{r_2'} \sqrt{1 - \frac{2g_0 R}{V_2'^2}\left(1 - \frac{R}{r_2'}\right)}. \tag{164}$$

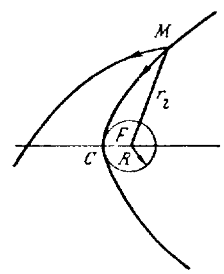

FIGURE 8

Differentiating equations (160) and (164) with respect to V_2', we obtain

$$V_z \frac{dV_z}{dV_2'} = V_2' - V_2 \cos(\delta_2 - \delta_2') + V_2 V_2' \sin(\delta_2 - \delta_2') \frac{d\delta_2}{dV_2'}; \tag{165}$$

$$\cos \delta_2' \frac{d\delta_2'}{dV_2'} = \frac{R}{r_2'} \frac{1}{2} \frac{4g_0 R\left(1 - \frac{R}{r_2'}\right)}{V_2'^3 \sqrt{1 - \frac{2g_0 R}{V_2'^2}\left(1 - \frac{R}{r_2'}\right)}}. \tag{166}$$

For $V_{z\,\min}$ we have $\frac{dV_z}{dV_2'} = 0$ and from equations (165) and (166) we obtain

$$V_2' - V_2 \cos(\delta_2 - \delta_2') = -V_2 V_2' \sin(\delta - \delta_2') \frac{d\delta_2'}{dV_2'} = -V_2 \frac{\sin(\delta_2 - \delta_2')}{\cos \delta_2'} \times$$
$$\times \frac{2g_0 R^2 \left(1 - \frac{R}{r_2'}\right)}{r_2' (V_2')^2 \sqrt{1 - \frac{2g_0 R}{V_2'^2}\left(1 - \frac{R}{r_2'}\right)}}. \tag{167}$$

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Let us introduce the notation:

$$E = 2g_0R\left(1 - \frac{R}{r_2'}\right); \tag{168}$$

$$x = \sqrt{1 - \frac{E}{V_2'^2}}, \tag{169}$$

then

$$\frac{E}{V_2'^2} = 1 - x^2 \tag{170}$$

and

$$V_2' = \sqrt{\frac{E}{1 - x^2}}. \tag{171}$$

Equation (167) takes the form

$$\sqrt{\frac{E}{1-x^2}} - V_2\cos(\delta_2 - \delta_2') = -V_2\frac{R}{r_2'}\frac{1-x^2}{x}\frac{\sin(\delta_2 - \delta_2')}{\cos\delta_2'}, \tag{172}$$

and equation (164) can be written in the form

$$\sin\delta_2' = \frac{R}{r_2'}x. \tag{173}$$

δ_2' and x can be determined from equations (172) and (173), and then V_2' from equation (171).

These formulas are, however, inconvenient for calculations. It is much simpler to plot a series of curves which would enable the interplanetary travellers to determine graphically the required additional velocity V_2 and the angle δ_2' between the direction of the velocity V_2 and the radius-vector. It is then possible to give the velocity V_0 for the radius R, to determine V_2' for the radius r_2' by formula (163), to find δ_2' from equation (164) and to determine then V_2 as a function of δ_2 and r_2' from equation (167). For this purpose we find from equation (164)

$$\frac{2g_0R}{V_2'^2}\left(1 - \frac{R}{r_2'}\right) = 1 - \sin^2\delta_2'\frac{r_2'^2}{R^2}. \tag{174}$$

Substituting (174) and (164) in equation (167) we obtain

$$V_2' - V_2\cos(\delta_2 - \delta_2') = -V_2\frac{\sin(\delta_2 - \delta_2')}{\cos\delta_2'\sin\delta_2'}\left(\frac{R}{r_2'}\right)^2\left(1 - \sin^2\delta_2'\frac{r_2'^2}{R^2}\right), \tag{175}$$

or

$$V_2 = \frac{V_2'}{\cos(\delta_2 - \delta_2') - \frac{\sin(\delta_2 - \delta_2')}{\cos\delta_2'\sin\delta_2'}\left[\left(\frac{R}{r_2'}\right)^2 - \sin^2\delta_2'\right]}. \tag{176}$$

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Given the value of δ_2, we determine V_2 and then V_z, from equation (160). For the angle ξ between V_z and r_2' (Figure 9) we have

$$\xi = 180° - \zeta - \delta_2, \tag{177}$$

where

$$\frac{\sin \zeta}{V_2'} = \frac{\sin (\delta_2 - \delta_2')}{V_z}. \tag{178}$$

We will first determine the angle ζ from (178) and then the angle ξ from (177).

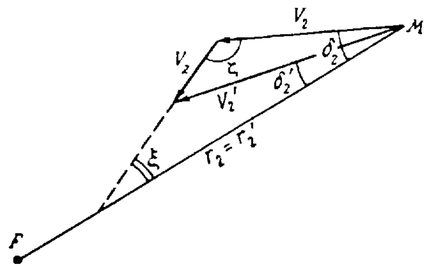

FIGURE 9

Thus, it is simple to obtain the additional energy required for correcting the flight trajectory at a given distance from the planet. Similarly the most favorable way of modifying a trajectory can be calculated when the correction is done far from the planet, when only the attraction of the Sun has to be taken into consideration. R is then the radius of the orbit of the destination planet and g, the acceleration due to the attraction of the Sun.

The problem is, however, more complicated due to the fact that the orbital motion of the destination planet has to be taken into consideration.

5. MODIFICATION OF FLIGHT TRAJECTORY AROUND SUN BY PLANETS' GRAVITATIONAL FIELDS

Let V_1 be the absolute velocity of the spaceship near some planet, calculated taking into account the attraction of the Sun only, S being the center of the Sun and A, the center of the planet (Figure 10). Assuming in a first approximation that the radius of the sphere of influence of the planet's gravitational field is so small that the trajectory modification can be assumed to take place at one point, we obtain for the moment before the trajectory modification a triangle of the velocities V_1, V_z, V_p where V_p is the velocity of the planet, and V_z, the relative velocity with respect to this planet.

If V_1, V_p and the angle α_1 between them are given we have from this triangle (Figure 11)

$$V_z^2 = V_1^2 + V_p^2 - 2V_1 V_p \cos \alpha_1. \tag{179}$$

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Similarly, after passing the planet we obtain

$$V_z^2 = V_2^2 + V_p^2 - 2V_2V_p\cos\alpha_2, \qquad (180)$$

where V_2 is the absolute velocity and α_2, the angle between the absolute velocity and the planet's velocity.

The velocity of the planet and the relative velocity V_z can be considered as unchanged, although the angle α_{z1} is changed by a given angle β, so that

$$\alpha_{z2} = \alpha_{z1} \pm \beta; \qquad (181)$$

the upper sign referring to the case when the ship passed the planet on its

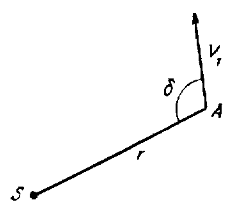

FIGURE 10

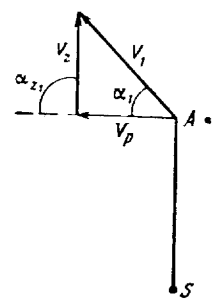

FIGURE 11

left, and the lower sign, when it did so on its right. We also have from the velocity triangles

$$\frac{\sin\alpha_{z1}}{V_1} = \frac{\sin\alpha_1}{V_z}; \qquad (182)$$

and

$$\frac{\sin\alpha_{z2}}{V_2} = \frac{\sin\alpha_2}{V_z}. \qquad (183)$$

If V_1, V_p, α_1 and the relation between the deflection angle β and V_z*,

$$\beta = f(V_z), \qquad (184)$$

are given, then the values of V_z, α_{z1}, α_{z2}, β, and V_2 must be calculated.

From equation (179) we find V_{z1} directly and then we determine from equation (182):

$$\sin\alpha_{z1} = \frac{V_1}{V_z}\sin\alpha_1. \qquad (185)$$

From equation (184) we determine $\beta = f(V_z)$, and from equation (181), α_{z2}. Instead of equation (180) we can use:

$$V_2^2 = V_z^2 + V_p^2 - 2V_zV_p\cos(180° - \alpha_{z2}). \qquad (186)$$

* The method for finding the function $\beta = f(V_z)$ is described in the following section. – Editor's note.

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From equation (183) we obtain

$$\sin\alpha_2 = \frac{V_z}{V_2}\sin\alpha_{z2}. \tag{187}$$

6. KINETIC ENERGY INCREMENT OF SPACESHIP FLYING AROUND PLANET

It is simple to determine all the quantities required from the velocity triangles before and after the trajectory modification, and from the hyperbola representing the flight trajectory near the planet (Figure 12, a).

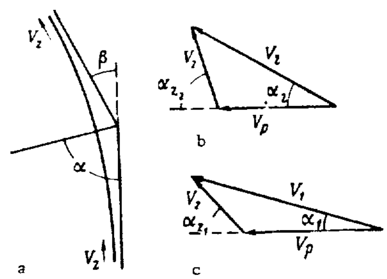

FIGURE 12

Formula (186) can be written in the form (12, b)

$$V_2^2 = V_z^2 + V_p^2 + 2V_zV_p\cos\alpha_{z2}, \tag{188}$$

while from the triangle for V_1 (Figure 12, c) we obtain

$$V_1^2 = V_z^2 + V_p^2 + 2V_zV_p\cos\alpha_{z1}. \tag{189}$$

The energy increment per unit mass is

$$\Delta E = \frac{V_2^2 - V_1^2}{2} = V_zV_p(\cos\alpha_{z2} - \cos\alpha_{z1}). \tag{190}$$

Taking into consideration that

$$\alpha_{z2} - \alpha_{z1} = \pm\beta; \quad \alpha_{z2} + \alpha_{z1} = 2\alpha_{z1} \pm \beta, \tag{191}$$

we obtain

$$\Delta E = -2V_zV_p\sin\left(\frac{\alpha_{z2}+\alpha_{z1}}{2}\right)\sin\frac{(\alpha_{z2}-\alpha_{z1})}{2} =$$
$$= \pm 2V_zV_p\sin\left(\alpha_{z1} \pm \frac{\beta}{2}\right)\sin\frac{\beta}{2}. \tag{192}$$

For given values of β and V_z, ΔE has a maximum for $\sin\left(\alpha_{z1} \pm \frac{\beta}{2}\right) = \pm 1$, or

$$\alpha_{z1} = \pm 90° \pm \frac{\beta}{2}. \tag{193}$$

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We obtain then from equation (181)

$$\alpha_{z2} = \pm 90^\circ \pm \beta \mp \frac{\beta}{2} = \mp 90^\circ \pm \frac{\beta}{2}. \qquad (194)$$

Drawing the velocity triangles with V_p as base (Figure 13, a), and determining β from the general equation

$$\beta = 180^\circ - 2\alpha, \qquad (195)$$

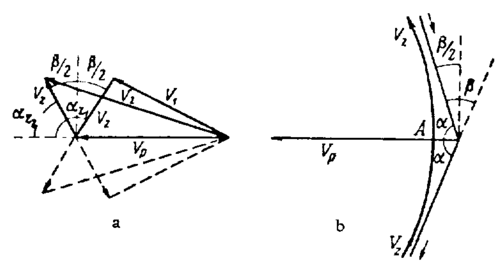

FIGURE 13

we find

$$\alpha_{z1} = \mp 90^\circ \mp (90^\circ - \alpha) = \mp (180^\circ - \alpha);$$
$$\alpha_{z2} = \mp 90^\circ \pm (90^\circ - \alpha) = \mp \alpha;$$

or, adding in the first case 360°,

$$\alpha_{z1} = 180^\circ \pm \alpha,$$

we obtain for the lower sign (the solid lines) the same path as for the upper sign (the dotted lines) only in the opposite direction. In this case the principal axis of the hyperbola (Figure 13, b) has the direction of the planet's orbit. Formula (192) takes the form

$$\Delta E_{max} = 2 V_z V_p \sin \frac{\beta}{2}. \qquad (196)$$

It is easy to determine the value of β which gives the absolute maximum of ΔE.

From formula (195) we have

$$\sin \frac{\beta}{2} = \sin (90^\circ - \alpha) = \cos \alpha. \qquad (197)$$

Noting that $\cos \alpha = \frac{1}{\Sigma}$ and using (154) we obtain

$$\cos \alpha = \frac{1}{\sqrt{1 + \frac{2V_z^4}{V_\infty^4} + \frac{2V_z^2}{V_\infty^2}}}, \qquad (198)$$

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where in this case V_∞ is the velocity which a body acquires at a distance r_s from the planet if its velocity at infinity is zero and r_s is the distance of the hyperbola's vertex from the planet's center.

From (196), (197) and (198) we obtain

$$\Delta E_{max}=\frac{2V_zV_p}{\sqrt{1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}}. \tag{199}$$

Let us determine the value of V_z which gives the maximum of ΔE_{max}:

$$\frac{d\Delta E_{max}}{dV_z}=\frac{2V_p}{\left(1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}\right)}\left(\sqrt{1+\frac{2V^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}-\right.$$
$$\left.-\frac{1}{2}V_z\frac{\frac{8V_z^3}{V_\infty^4}+\frac{4V_z}{V_\infty^2}}{\sqrt{1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}}\right)=0. \tag{200}$$

Hence

$$\frac{2V_z^4}{V_\infty^4}=1,$$

or

$$V_z=\frac{V_\infty}{\sqrt[4]{2}}=0.841V_\infty. \tag{201}$$

For this value of V_z we have

$$(\Delta E_{max})_{max}=\frac{2V_\infty V_p}{\sqrt[4]{2}\sqrt{1+1+\frac{2}{\sqrt{2}}}}=\frac{2V_\infty V_p}{\sqrt{2(\sqrt{2}+1)}}=0.91V_\infty V_p, \tag{202}$$

and

$$\Sigma\big|_{(\Delta E_{max})_{max}}=\sqrt{2+\sqrt{2}}; \tag{203}$$
$$\cos\alpha\big|_{(\Delta E_{max})_{max}}=\frac{1}{\sqrt{2+\sqrt{2}}}=\frac{1}{\sqrt{3.414}}=0.541;$$
$$\alpha\big|_{(\Delta E_{max})_{max}}=57°18'.$$

Therefore: $\alpha_{z1}=180°\pm57°18'$; $\alpha_{z2}=\pm57°18'$; $\beta=65°42'$.

These values refer to the absolute maximum of ΔE, since ΔE may be regarded as a function of $\left(\alpha_{z1}\pm\frac{\beta}{2}\right)$ and of V_z (see (192)).

We have

$$\frac{\partial\Delta E}{\partial\left(\alpha_{z1}\pm\frac{\beta}{2}\right)}=0 \quad \text{for} \quad \alpha_{z_1}=\mp 90°\mp\frac{\beta}{2}$$

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(see (193)) and $\frac{\partial \Delta E}{\partial V_z} = 0$ for $V_z = \frac{V_\infty}{\sqrt[4]{2}}$ (see (201)), i.e., we obtain our formulas (196) and (202).

If we only substitute the condition $\frac{\partial \Delta E}{\partial V_z} = 0$, and $\alpha_{z1} \pm \frac{\beta}{2} = \frac{\alpha_{z1} + \alpha_{z2}}{2}$ is left unchanged, then by multiplying (202) by $\sin \frac{\alpha_{z1} + \alpha_{z2}}{2}$ (see (192)) we obtain

$$\Delta E \Big|_{\frac{\partial \Delta E}{\partial V_z} = 0} = (\Delta E_{max})_{max} \sin \frac{\alpha_{z1} + \alpha_{z2}}{2} = = \frac{2 V_\infty V_p \sin \frac{\alpha_{z1} + \alpha_{z2}}{2}}{\sqrt{2(\sqrt{2}+1)}} \qquad (204)$$

This formula shows that for a given average direction $\left(\frac{\alpha_{z1} + \alpha_{z2}}{2}\right)$ of the relative velocities there exists a certain velocity $V_z = \frac{V_\infty}{\sqrt[4]{2}}$, for which the energy increase is a maximum.

By interchanging the indices 2 and 1 in all formulas, we obtain the maximum decrease in the energy; this follows from the symmetry of the corresponding formulas and drawings with respect to quantities with indices 2 and 1.

In order to determine the value of $(\Delta E_{max})_{max}$ for individual planets we calculate first

$$V_\infty = \sqrt{2 g_0 R} = V_{\infty 1} \sqrt{\frac{g_0 R}{g_{01} R_1}}, \qquad (205)$$

where the index 1 refers to the Earth. Taking $V_{\infty 1} = 11.18$ km/sec and knowing $\frac{g_0}{g_{01}}$ and $\frac{R}{R_1}$, we determine V_∞ by formula (205) (see Table 3).

The velocity corresponding to the energy variation per unit mass $(\Delta F_{max})_{max}$, is determined by

$$V = \sqrt{2 (\Delta E_{max})_{max}}.$$

In reality, for $\alpha_{z1} = 180° \pm 57°18'$, we shall have according to (189):

$$V_1^2 = V_z^2 + V_p^2 + 2 V_z V_p \cos(180° \pm 57°18') = = V_z^2 + V_p^2 - 2 V_z V_p \cos 57°18' = V_z^2 + V_p^2 - - 2 \cdot 0.541 V_z V_p = V_z^2 + V_p^2 - (\Delta E_{max})_{max}. \qquad (206)$$

Similarly we have from (188):

$$V_2^2 = V_z^2 + V_p^2 + 2 V_z V_p \cos 57°18' = V_z^2 + V_p^2 + (\Delta E_{max})_{max}, \qquad (207)$$

or from (190):

$$V_2^2 = V_1^2 + 2 (\Delta E_{max})_{max}. \qquad (208)$$

We can see from Table 3 that the energy increase per unit mass $(\Delta E_{max})_{max}$ is equal, for the Earth, to 283 km²/sec², which is smaller only than the value for Venus (315 km²/sec²) and for Jupiter (661 km²/sec²). It is interesting to make a comparison of the maximum distances from the Sun

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TABLE 3

Table of the additional energies and the corresponding velocities acquired by a spaceship flying around a planet

Names of planets	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Average radius r/r_0 of orbit	0.3871	0.72333	1.000	1.52369	5.2028	9.53886	19.18329	30.05508
Acceleration g_{01}/g_{e0} at surface	0.44	0.80	1.00	0.38	2.26	0.89	0.75	1.14
Radius r_{01}/r_{e0} of planet	0.373	0.999	1.000	0.528	11.061	9.299	4.234	3.798
$\sqrt{\frac{2r_2r_1}{1+r_2/r_1}}-1$	−0.253	−0.082	0	10.10	0.294	0.345	0.378	0.391
$V_{z\,min}$ km/sec	−7.05	−2.27	0	2.78	8.18	9.60	10.50	10.85
Average orbital velocity, km/sec	47.0	34.7	27.8	24.0	13.0	9.5	6.5	5.4
Velocity of free fall on planet from infinity, km/sec	4.52	9.98	11.18	5.00	55.9	32.2	19.9	23.3
Additional energy acquired by flying around planet $(\Delta E_{max})_{max}$, km^2/sec^2	193.2	315	283	109.1	661	278	117.7	114.4
Corresponding velocity $\sqrt{2(\Delta E_{max})_{max}}$, km/sec	19.63	25.1	23.8	14.75	36.4	23.6	16.6	16.2

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which may be attained with velocities V_1 and V_2 for various planets. The velocity difference V_2-V_1 is equal for the Earth to 9.7 km/sec, for Saturn 9.8 km/sec and for Jupiter 13.65 km/sec; for the rest of the planets it is smaller.

If it is required to determine the conditions under which the difference V_2-V_1 attained by flying around a planet is a maximum, it is possible to proceed as follows. We join the ends of the vectors V_z (Figure 14) by the line CB. Then $CB=V'_z$ is the velocity which has to be added geometrically to the velocity V_1 in order to obtain the velocity V_2. Since for a given velocity V_z the deflection angle $\beta=180°-2\alpha$ and the velocity V'_z will have maximum values if the vertex of the hyperbola is situated near the planet's surface, and since for a given V'_z the difference V_2-V_1 will be a maximum for $\alpha_1=\alpha_2$, we obtain:

$$V_2-V_1=V_z. \tag{209}$$

Let us find the velocity V_z which gives the maximum of V'_z. From Figure 14 we have:

$$V'_z=2V_z\sin\frac{\beta}{2}=2V_z\sin(90°-\alpha)=2V_z\cos\alpha. \tag{210}$$

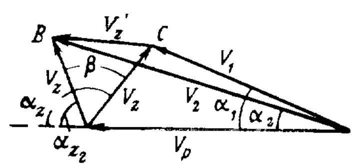

FIGURE 14

Substituting $\cos\alpha$ from equation (198) we obtain

$$V'_z=\frac{2V_z}{\sqrt{1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}}. \tag{211}$$

We find the maximum value of V'_z for a given V_∞ from the equation

$$\frac{dV'_z}{dV_z}=0. \tag{212}$$

Differentiating (211), we obtain

$$\frac{dV'_z}{dV_z}=\frac{2}{\sqrt{1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}}-\frac{V_z}{\left(1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}\right)^{3/2}}\times$$

$$\times\left(\frac{8V_z^3}{V_\infty^4}+\frac{4V_z}{V_\infty^2}\right)=\frac{2\left[1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}-\frac{4V_z^4}{V_\infty^4}-\frac{2V_z^2}{V_\infty^2}\right]}{\sqrt{\left(1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}\right)^3}}=0.$$

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Hence

$$\frac{2V_z^4}{V_\infty^4}=1.$$

or

$$V_z=\frac{V_\infty}{\sqrt[4]{2}}=0.841\,V_\infty. \qquad (213)$$

This value of V_z is, therefore, the same as the value obtained for $(\Delta E_{max})_{max}$ (see (201)).

From (198) we have again $\alpha = 57°18'$, and

$$\beta=180°-2\alpha=65°42'.$$

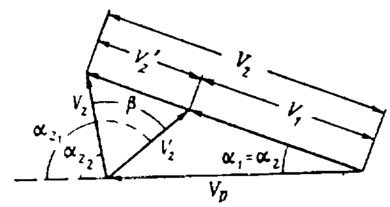

FIGURE 15

From (211) we obtain

$$V_z=\frac{2V_\infty}{\sqrt[4]{2}\sqrt{1+\frac{2}{2}+\frac{2}{\sqrt{2}}}}=V_\infty\sqrt{\frac{2}{\sqrt{2}+1}}0.910V_\infty. \qquad (214)$$

Let us now determine the velocities V_1 and V_2 and the angles $\alpha_1=\alpha_2$, α_{z1} and α_{z2}. First of all we see from Figure 15 that for $V_p<V_z$ the velocities V_1 and V_2 cannot now lie on the same line.

This case requires a special investigation. Assuming for the meantime that $V_p>V_z$, we obtain from Figure 15:

$$V_1=\sqrt{V_p^2-V_z^2\cos^2\frac{\beta}{2}}-\frac{V_z'}{2}, \qquad (215)$$

$$V_2=\sqrt{V_p^2-V_z^2\cos^2\frac{\beta}{2}}+\frac{V_z'}{2}, \qquad (216)$$

and

$$\sin\alpha_1=\frac{V_z}{V_p}\cos\frac{\beta}{2}. \qquad (217)$$

For $V_p=V_z$, we have $V_1=0$, $V_2=V_z'$, and $\alpha_1=\alpha_2=90°-\frac{\beta}{2}$. If $V_p<V_z$, then we obtain from triangle ADB (see Figure 16):

$$V_1=\sqrt{V_p^2+V_z^2-2V_pV_z\cos\gamma}. \qquad (218)$$

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and from triangle *ACB*:

$$V_2=\sqrt{V_p^2+V_z^2-2V_pV_z\cos(\beta+\gamma)}. \qquad (219)$$

We have therefore:

$$V_2-V_1=\sqrt{V_p^2+V_z^2-2V_pV_z\cos(\beta+\gamma)}-$$
$$-\sqrt{V_p^2+V_z^2-2V_pV_z\cos\gamma}. \qquad (220)$$

Let us determine now the angle γ for which V_2-V_1 attains a maximum for $V_z=\text{const}$, and consequently, also for $\beta=\text{const}$.

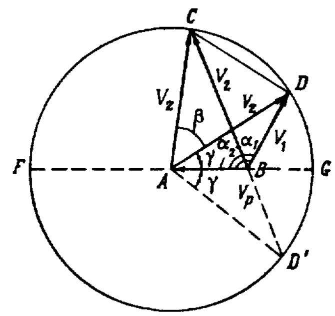

FIGURE 16

The condition that has to be satisfied is:

$$\frac{d(V_2-V_1)}{d\gamma}=0. \qquad (221)$$

Differentiating equation (220) with respect to γ we obtain

$$\frac{1}{2V_2}2V_pV_z\sin(\beta+\gamma)-\frac{1}{2V_1}2V_pV_z\sin\gamma=0,$$

or

$$\frac{V_2}{V_1}=\frac{\sin(\beta+\gamma)}{\sin\gamma}. \qquad (222)$$

From Figure 16 we see that

$$V_z\sin\gamma=V_1\sin\alpha_1; \qquad (223)$$

$$V_z\sin(\beta+\gamma)=V_2\sin\alpha_2; \qquad (224)$$

therefore

$$\frac{\sin(\beta+\gamma)}{\sin\gamma}=\frac{V_2\sin\alpha_2}{V_1\sin\alpha_1},$$

which, together with equation (222), gives $\sin\alpha_2=\sin\alpha_1$.

We obtain therefore,

$$\alpha_1=\alpha_2 \text{ or } \alpha_1=180°-\alpha_2. \qquad (225)$$

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The condition $\alpha_1 = \alpha_2$ can be satisfied only for $V_z < V_p$ when the point B lies outside a circle of radius V_z and center A; this is the first case which we have already investigated. For $V_z > V_p$ we have $\alpha_1 = 180° - \alpha_2$. Producing AB and CB until they intersect with the circle of radius V_z and center A, we obtain two intersecting lines FG and CD'. The angle α_2 between them is of course equal to half the sum of the central angles corresponding to the arcs FC and $D'G$, i. e.,

$$\alpha_2 = \frac{1}{2}(\angle CAF + \angle GAD), \text{but } \angle CAF = 180° - (\beta + \gamma),$$

since $\angle DBG = 180° - \alpha_1 = \alpha_2$ and $\angle GBD' = \alpha_2$. The point D' is symmetrical to D with respect to the line FG, i. e., $\angle GAD' = \angle DAG = \gamma$. Hence we have

$$\alpha_2 = \frac{1}{2}[180° - (\beta + \gamma) + \gamma] = 90° - \frac{\beta}{2}, \tag{226}$$

which gives

$$\alpha_1 = 180° - 90° + \frac{\beta}{2} = 90° + \frac{\beta}{2}. \tag{227}$$

Therefore

$$\angle CBD = \alpha_1 - \alpha_2 = \beta. \tag{228}$$

From triangle ADB we obtain

$$V_z^2 = V_1^2 + V_p^2 - 2V_1 V_p \cos\alpha_1, \tag{229}$$

and from triangle ACB

$$V_z^2 = V_2^2 + V_p^2 - 2V_2 V_p \cos\alpha_2. \tag{230}$$

Solving these equations with respect to V_1 and V_2, we get

$$V_1 = V_p \cos\alpha_1 \pm \sqrt{V_p^2 \cos^2\alpha_1 - V_p^2 + V_z^2} = $$

$$= V_p \cos\alpha_1 \pm \sqrt{V_z^2 - V_p^2 \sin^2\alpha_1}; \tag{231}$$

$$V_2 = V_p \cos\alpha_2 \pm \sqrt{V_z^2 - V_p^2 \sin^2\alpha_2}. \tag{232}$$

We have therefore

$$V_2 - V_1 = V_p(\cos\alpha_2 - \cos\alpha_1) =$$

$$= -2V_p \sin\frac{\alpha_2 - \alpha_1}{2} \sin\frac{\alpha_2 + \alpha_1}{2}, \tag{233}$$

and since

$$\frac{\alpha_2 + \alpha_1}{2} = \frac{180°}{2} = 90° \text{ and } \frac{\alpha_2 - \alpha_1}{2} = -\frac{\beta}{2};$$

then

$$V_2 - V_1 = 2V_p \sin\frac{\beta}{2}. \tag{234}$$

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Obviously, when V_z varies, we obtain the maximum of $V_2 - V_1$ at the maximum of $\sin\frac{\beta}{2}$. From equations (195) and (198) we have:

$$\frac{\beta}{2} = 90^\circ - \alpha \text{ and } \sin\frac{\beta}{2} = \frac{1}{\sqrt{1 + \frac{2V_z^4}{V_\infty^4} + \frac{2V_z^2}{V_\infty^2}}}. \tag{235}$$

The maximum value of $\sin\frac{\beta}{2}$ is obtained for the minimum value of V_z which is equal for the case considered to V_p, i.e.,

$$\sin\frac{\beta_{max}}{2} = \frac{1}{\sqrt{1 + \frac{2V_p^4}{V_\infty^4} + \frac{2V_p^2}{V_\infty^2}}}, \tag{236}$$

and therefore

$$V_2 - V_1 = 2V_p \sin\frac{\beta_{max}}{2} = \frac{2V_p}{\sqrt{1 + \frac{2V_p^4}{V_\infty^4} + \frac{2V_p^2}{V_\infty^2}}}. \tag{237}$$

The velocity $V_z = \frac{V_\infty}{\sqrt[4]{2}}$ is larger than V_p for large external planets, as can be seen from Table 3. In this case, formulas (209) and (214) are inapplicable since they refer to the case $V_z < V_p$, and formulas (236) and (237) should be used.

To determine V_1 and V_2 for this case, we turn to formulas (231) and (232). For $\alpha_1 = 90^\circ + \frac{\beta}{2}$ and $\alpha_2 = 90^\circ - \frac{\beta}{2}$, we obtain

$$V_1 = -V_p \sin\frac{\beta}{2} \pm \sqrt{V_z^2 - V_p^2 \cos^2\frac{\beta}{2}}; \tag{238}$$

$$V_2 = +V_p \sin\frac{\beta}{2} \pm \sqrt{V_z^2 - V_p^2 \cos^2\frac{\beta}{2}}. \tag{239}$$

Omitting the lower sign which corresponds to the maximum decrease of the velocity, we obtain for $V_z = V_p$:

$$V_1 = 0;$$

$$V_2 = 2V_p \sin\frac{\beta_{max}}{2}.$$

If we wish to acquire an additional velocity when flying around large planets, we must approach them with zero initial absolute velocity ($V_1 = 0$).

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7. FLIGHT AROUND PLANET'S SATELLITE FOR ACCELERATING OR DECELERATING SPACESHIP

Let T be the planet's center, L, the center of its satellite, V_T, the velocity of the planet around the Sun and V_L, the velocity of the satellite around the planet (Figure 17).

The satellite's absolute velocity around the Sun is obtained by geometrical addition of the velocities V_T and V_L (Figure 18). Let us denote the resultant velocity by V. To find the maximum of the difference $(V_2-V_1)_{max}$ we can proceed as in the case $V_p>V_t$. From (209) and (214) we obtain:

$$V_2-V_1=V_z'=0.910V_\infty \qquad (240)$$

where V_∞ is the velocity a body acquires by falling on the satellite from infinity.

Denoting by V_3 the velocity of the spaceship relative to the planet, we see that one must distinguish between the maximum increase of the absolute velocity, i.e., $(V_2-V_1)_{max}$ and the maximum difference $V_2-(V_T+V_3)$.

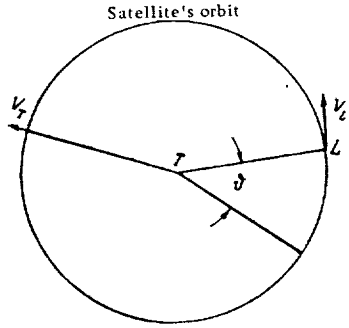

FIGURE 17

Taking-off from the planet and consuming a certain amount of propellant, we reach a certain velocity V_3 after overcoming approximately the entire pull of the planet. The question arises as to which course is the best.

Either we could direct V_3 along V_T without flying around the satellite (in this case the final velocity is V_3+V_T) or fly around the satellite and direct V_3 to obtain a maximum absolute velocity V_2 after going around the satellite.

In order to solve this problem and to determine the maximum advantage which can be obtained by flying around a satellite, we determine the maximum of the velocity: $W=V_2-(V_3+V_T)$. This calculation, as well as the calculation made above of the variation of the spaceship's kinetic energy in flying around planets, is not quite exact. Firstly, the action of the Sun on the planet, on the satellite and on the spaceship is not quite the same. Secondly, due to the orbital motion of the planet, the velocity V_T after flying around the satellite is inclined by some small angle with respect to the velocity V_T before. Thirdly, the velocities V_1, V_2, V_3, V_z and V_z' are completely attained in magnitude and direction only at an infinitely large distance from the planet and from the satellite. The error in the direction of V_T will be rather small due to the shortness of the time taken to fly around the satellite. Furthermore, the distance to the Sun is large compared to the distance between the spaceship and the planet or the satellite, so that the action of the Sun on all the three bodies is almost identical. The greatest inaccuracy in the case of flying around a satellite is caused by the third of the above-mentioned reasons. The approximate value of this error may be estimated by determining the "residual potentials" of the planet and of the satellite at the distances at which the beginning and end of the fly-by is assumed in the calculation.

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In Figure 19, BAC is the velocity triangle after the fly-by, BAD, the one before the fly-by, triangle BEA gives the geometrical sum of the velocities V_T and V_L and finally, in triangle BED, the side DE represents the spaceship's velocity relative to the planet before flying around the satellite.

From triangles CDB and DEB we see that $BC-BD \leqslant DC$ or $V_2-V_1 \leqslant V'_z$; and that for $V_1>V_T, BE+ED \geqslant BD$ or $V_T+V_3 \geqslant V_1$. Adding both inequalities, we obtain $V_2 \leqslant V'_z+V_T+V_3$ or $V'_z \geqslant V_2-V_T-V_3$ or $V'_z \geqslant W$. For a given V_z the maximum value of W, $W=V'_z$, is obtained if the points D and E lie on the line CB, since then $V_2-V_1=V'_z$ and $V_1=V_T+V_3$.

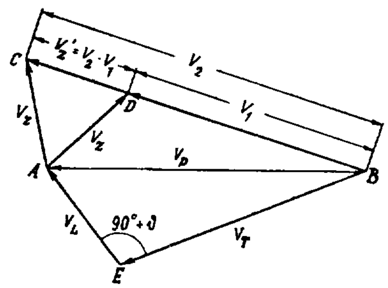

FIGURE 18

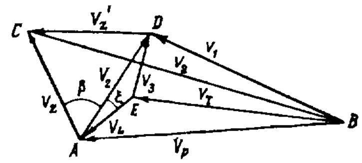

FIGURE 19

If $V_1<V_T$, then $V_2-V_1 \leqslant V'_z$. Replacing in this inequality V_1 by V_T, we obtain $V_2-V_T<V'_z$ and, subtracting V_3 from both sides, we obtain $V_2-V_T-V_3<V'_z-V_3$ or $W<V'_z-V_3$.

If the points E and D lie on the line CB, then $V_2-V_1=V'_z$ and $V_1=V_T-V_3$. Therefore,

$$W=V_2-V_T-V_3=(V_2-V_1)+(V_1-V_T)-V_3=V'_z-2V_3.$$

Therefore, for a given V'_z, i. e., for given V_z and β (see equations (210) and (235)) we obtain the maximum value of W if the points D and E lie on CB, and E lies between D and B (Figure 20).

In the case of maximum V'_z we also obtain the maximum of W, $W=V'_z$.

But

$$V'_z=2V_z \sin\frac{\beta}{2}; \tag{210}$$

$$\sin\frac{\beta}{2}=\frac{1}{\sqrt{1+\frac{2V_z^4}{V_\infty^4}+\frac{2V_z^2}{V_\infty^2}}}. \tag{235}$$

Since

$$V_{z\,max}=0.841\,V_\infty; \quad V'_{z\,max}=0.910\,V_\infty$$

and V'_z first increases with V_z and then decreases, for $V_L>V_{z\,max}$, i. e., for $V_L>0.841\,V_\infty$, we obtain

$$W_{max}=V_{z\,max}=0.910V_\infty,$$

where V_∞ is the velocity which a body acquires by falling on the satellite if it had zero velocity relative to it at infinity.

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If $V_L > 0.841\ V_\infty$, then we obtain $V'_{z\,max}$ for the maximum value of V_z, equal to V_L, i.e., when the points D and E coincide.

In this case

$$V_z = V_L \quad W = V'_z = 2V_L \sin\frac{\beta}{2}. \tag{241}$$

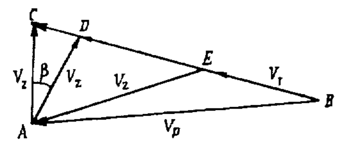

FIGURE 20

8. ADVANTAGES OF ACCELERATING SPACESHIP BY ROCKET ENGINE AT HIGH FLIGHT VELOCITY

If we examine the flight of a rocket relative to a body whose attraction it must overcome, it turns out that in order to obtain a high final flight velocity it is rather advantageous to accelerate the vehicle at high flight velocity. In flying around celestial bodies, the maximum flight velocity is obtained at the minimum distance from them. It can be easily shown that the best result for flight around the Sun is obtained if we accelerate the vehicle near the perihelion; for flight around planets or their moons, we should accelerate at locations nearest to their surface.

Let us examine flight around the Sun.

We shall use the following notation:

r_1, radius-vector to point of elliptical, parabolic or hyperbolic trajectory at which rocket was accelerated for comparison with effect of acceleration given at perihelion;

r_0, radius-vector of perihelion of initial trajectory;

V_1, flight velocity at distance r_1 from Sun before acceleration;

V_0, flight velocity at perihelion at distance r_0 from Sun before acceleration;

V_z, additional velocity imparted to vehicle for consumption of given amount of propellant (this velocity increment does not depend on vehicle velocity);

P_1, potential of Sun per unit mass at distance r_1 from Sun's center;

P_0, same at distance r_0 from Sun's center;

V', flight velocity at distance r_1 from Sun after accelerating vehicle at perihelion (at distance r_0 from Sun);

V'', same when vehicle was accelerated at distance r_1 from Sun;

V''', flight velocity at distance r_0 from Sun when vehicle was accelerated at this point.

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The maximum flight velocity after acceleration is obtained if the direction of the additional velocity V_z coincides with the direction of the flight velocity before the acceleration, since in this case the velocities are added algebraically.

For acceleration at a distance r_1 from the Sun we have

$$V'' = V_z + V_1. \tag{242}$$

and for acceleration at a distance r_0 from the Sun

$$V''' = V_z + V_0. \tag{243}$$

The increment of kinetic energy per unit mass is equal to the variation of the solar potential, so that

$$\frac{V_0^2 - V_1^2}{2} = P_1 - P_0, \tag{244}$$

$$\frac{(V_0 + V_z)^2 - (V')^2}{2} = P_1 - P_0. \tag{245}$$

Equation (244) refers to flight from A to B before acceleration (Figure 21), and equation (245) – to flight from B to C' after acceleration at r_0.

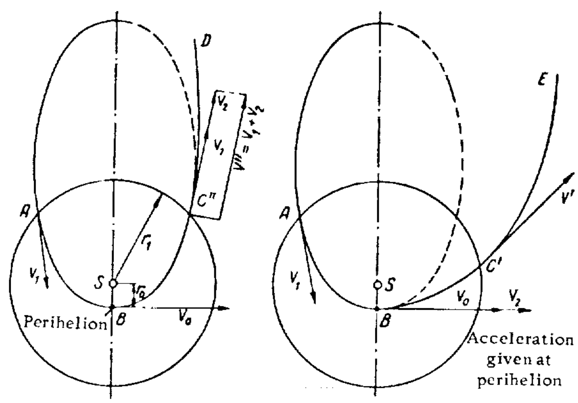

FIGURE 21

From equations (244) and (245) we obtain

$$V_0^2 - V_1^2 = (V_0 + V_z)^2 - (V')^2.$$

Hence

$$V' = \sqrt{V_1^2 + 2V_zV_0 + V_z^2}. \tag{246}$$

This formula may also be written in the form

$$(V')^2 = (V_1^2 + V_z^2 + 2V_1V_z) - 2V_1V_z + 2V_0V_z =$$
$$= (V_z + V_1)^2 + 2V_z(V_0 - V_1) = (V'')^2 + 2V_z(V_0 - V_1).$$

Then we have

$$(V')^2 - (V'')^2 = 2V_z(V_0 - V_1). \tag{247}$$

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We see that the velocity V' obtained after acceleration at the perihelion, will be larger than the velocity V'' obtained after acceleration at a distance r_1 from the Sun, both velocities V' and V'' referring to the same distance r_1 from the Sun. In formula (247), we always have $V_0 > V_1$.

Assuming, for example, that

$$V_1 = 30 \text{ km/sec}, V_0 = 70 \text{ km/sec and } V_z = 10 \text{ km/sec},$$

we obtain from (242):

$$V'' = 10 + 30 = 40 \text{ km/sec},$$

and from (246)

$$V' = \sqrt{30^2 + 2 \cdot 10 \cdot 70 + 10^2} = 49 \text{ km/sec}.$$

The additional velocity V'_z, obtained at a distance r_1 from the Sun and resulting from acceleration given at the perihelion, is equal to

$$V'_z = V' - V_1. \tag{248}$$

In our example

$$V'_z = 49 - 30 = 19 \text{ km/sec},$$

whereas direct acceleration at a distance r_1 from the Sun would give only

$$V_z = 10 \text{ km/sec}.$$

If we denote by x the relative gain obtained from acceleration at the perihelion:

$$x = \frac{V'_z - V_z}{V_z}, \tag{249}$$

then

$$x = \frac{V'_z}{V_z} - 1 = \frac{\sqrt{V_1^2 + 2V_zV_0 + V_z^2} - V_1}{V_z} - 1 = \\ = \sqrt{\left(\frac{V_1}{V_z}\right)^2 + \frac{2V_0}{V_z} + 1} - \frac{V_1}{V_z} - 1. \tag{250}$$

Let us determine the values of the additional velocity V_z for which the value of x will be a minimum or a maximum if the velocities V and V_0 are given.

From (250) we obtain

$$[(x+1)V_z + V_1]^2 = V_1^2 + 2V_zV_0 + V_z^2 = (x+1)^2 V_z^2 + \\ + 2(x+1)V_zV_1 + V_1^2,$$

or, after simplification:

$$(x+1)^2 V_z - V_z + 2(x+1)V_1 - 2V_0 = 0.$$

Further

$$x^2 V_z + 2xV_z + 2xV_1 + 2V_1 - 2V_0 = 0.$$

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Hence

$$V_z = \frac{2(V_0 - V_1 - xV_1)}{x(x+2)} \tag{251}$$

or

$$\frac{V_z}{2V_1} = \frac{\frac{V_0 - V_1}{V_1} - x}{x(x+2)}. \tag{252}$$

Using the notation:

$$\xi = \frac{V_z}{2V_1} \tag{253}$$

and

$$\eta = \frac{V_0 - V_1}{V_1}, \tag{254}$$

(252) takes the form:

$$\xi = \frac{\eta - x}{x(x+2)}. \tag{255}$$

From (249) we obtain

$$V_z' = (x+1)V_z \tag{256}$$

and from (253)

$$V_z = 2V_1\xi. \tag{257}$$

Thus, we have

$$V_z' = (x+1)2V_1\frac{(\eta - x)}{x(x+2)}, \tag{258}$$

$$V_z = \frac{2V_1(\eta - x)}{x(x+2)}. \tag{259}$$

These formulas may be used if the velocities V_0 and V_1 and the relative gain x are given. From equation (259) we see that $x \geqslant \eta$, if $V_z > 0$.

In other words, for $V_z = 0$

$$x_{max} = \eta = \frac{V_0 - V_1}{V_1}, \tag{260}$$

and for $x = 0$ we obtain $V_z = \infty$.

Thus, the maximum relative gain in the case of acceleration at the perihelion is obtained for an infinitely small additional velocity V_z, if the velocity V_1 at the distance r_1 from the Sun and the velocity V_0 at the perihelion are given before the acceleration. This maximum relative gain is equal to

$$x_{max} = \frac{V_0 - V_1}{V_1}.$$

There is no relative gain $(x = 0)$ if the additional velocity is equal to infinity $(V_z = \infty)$.

The maximum relative gain increases with the ratio $V_0 : V_1$. If the Sun was concentrated at one point, then the maximum value of V_0, as well as the

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corresponding x_{max} would be equal to infinity; the limit is the possibility to approach the Sun.

As an example, let $V_0 = 70$ km/sec and $V_1 = 7$ km/sec. Then we obtain

$$\eta = x_{max} = \frac{70-7}{7} = 9.$$

Calculating ξ, V'_z and V_z by (255), (256) and (257), we obtain for the present example the following table of additional velocities V_z and V'_z. They are obtained at a distance r_1 from the Sun's center if the acceleration was given at point C'' (velocity V_z) or at the perihelion (point B, velocity V'_z) for the same propellant mass ratio in both cases.

x	9	8	7	6	5	4	3	2	1	0
ξ	0	0.0125	0.0318	0.0625	0.114	0.208	0.40	0.875	2.67	∞
V_z, km/sec	0	0.175	0.455	0.875	1.60	2.92	5.60	12.25	37.3	∞
V'_z, km/sec	0	1.577	3.56	6.12	9.60	14.60	22.4	36.7	74.6	∞
$V'_z - V_z$	0	1.402	3.115	5.245	8.00	11.68	16.8	24.45	37.3	63

For the absolute gain $(V'_z - V_z)$ from acceleration at the perihelion, we obtain from (258) and (259) the equation

$$V'_z - V_z = 2V_1 \frac{\eta - x}{x+2}. \tag{261}$$

The maximum absolute gain is obtained for $x = 0$ and $V_z = \infty$. In this case we have

$$(V'_z - V_z)_{max} = V_1\eta = V_0 - V_1. \tag{262}$$

In our example

$$(V'_z - V_z)_{max} = 63 \text{ km/sec}.$$

This gain increases with the additional velocity V_z. In the best case it is possible, as we see from equation (262), to gain the whole difference between the velocity V_0 at the perihelion and the velocity V_1 at a distance r_1 from the Sun.

For planets the case when they are first at practically infinite distance $(r_1 \approx \infty)$ and then at the nearest distance from the spaceship (r_1 denotes the distance between the spaceship and the planet considered) is of interest. All the formulas derived above are applicable to this case, only the velocities should be taken relative to the given planet. How the velocities relative to the Sun then vary, will be investigated in a special article.

It is always better to fly in trajectories which pass near the Sun or the planets when travelling in a spaceship from one planet to another and desiring to increase the flight velocity, since they may give a considerable decrease to flight duration. However, while the maximum increase in

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kinetic energy is obtained for acceleration at the perihelion, the minimum flight duration is obtained if the vehicle is accelerated somewhat earlier, on section AB of the flight trajectory.

9. DETERMINATION OF FLIGHT TRAJECTORIES IN COSMIC SPACE WITH RETURN TO EARTH AFTER INTEGRAL NUMBER OF YEARS

Let us introduce the ratios of an arbitrary radius-vector to the radius-vectors at the perihelion and at the aphelion as variables.

Denoting (see Figure 22) the radius-vector to the perihelion by r_0, and that to the aphelion by r_2, and putting $r_0/r = \gamma_0$; $\frac{r_2}{r} = \gamma_2$, we can give the formulas derived above a particularly symmetric form.

We have

$$a = \frac{r_0 + r_2}{2},$$

or

$$\frac{r_1}{a} = \rho = \frac{2}{\gamma_0 + \gamma_2}. \qquad (263)$$

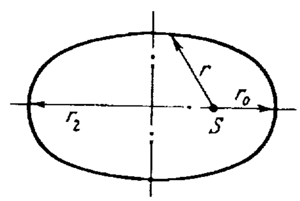

FIGURE 22

Above, we have introduced the quantity $\gamma = r_2/r_1$, where r_2 was the radius-vector at the aphelion (in the previous calculation it coincided with the radius of the orbit of the destination planet) and r_1 was an arbitrary radius-vector (in the previous calculation it coincided with the radius of the Earth's orbit). Therefore, we have now to replace γ by γ_2 and ρ by $\frac{2}{\gamma_0 + \gamma_2}$. Substituting these expressions in the corresponding equations we obtain:

$$V_z = V_1 \sqrt{3 - \frac{2}{\gamma_0 + \gamma_2} \mp 2\sqrt{\frac{2\gamma_0\gamma_2}{\gamma_0 + \gamma_2}}}; \qquad (264)$$

$$\cos\beta = \sqrt{\frac{\gamma_0\gamma_2}{\gamma_0 + \gamma - 1}}; \qquad (265)$$

$$\operatorname{tg}\beta = \sqrt{\frac{(1 - \gamma_0)(\gamma_2 - 1)}{\gamma_0\gamma_2}}; \qquad (266)$$

$$\operatorname{tg}\xi = \frac{\sqrt{(1 - \gamma_0)(\gamma_2 - 1)}}{\sqrt{\gamma_0\gamma_2} \mp \sqrt{\frac{\gamma_0 + \gamma_2}{2}}}; \qquad (267)$$

$$\cos\vartheta = \frac{\gamma_0(2\gamma_2 - 1) - \gamma_2}{\gamma_2 - \gamma_0} = \frac{2\gamma_0\gamma_2 - \gamma_0 - \gamma_2}{\gamma_2 - \gamma_0}, \qquad (268)$$

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$$\frac{a}{r}=\frac{\gamma_0+\gamma_2}{2}; \tag{269}$$

$$\frac{b}{r}=\sqrt{\gamma_0\gamma_2}; \tag{270}$$

$$\frac{r_0}{r}=\gamma_0; \tag{271}$$

$$\sum=\frac{\gamma_2-\gamma_0}{\gamma_2+\gamma_0}; \tag{272}$$

$$V=V_1\sqrt{2\left(1-\frac{1}{\gamma_0+\gamma_2}\right)}; \tag{273}$$

$$V_a=V_1\sqrt{\frac{2\gamma_0}{\gamma_2(\gamma_0+\gamma_2)}}; \tag{274}$$

$$V_p=V_1\sqrt{\frac{2\gamma_2}{\gamma_0(\gamma_0+\gamma_2)}}; \tag{275}$$

$$V_2=\frac{V_1}{\sqrt{\gamma_2}}; \tag{276}$$

$$\operatorname{tg}\frac{E}{2}=\sqrt{\frac{1-\gamma_0}{\gamma_2-1}}; \tag{277}$$

$$\tau=\tau_1\left(\frac{\gamma_0+\gamma_2}{2}\right)^{3/2}; \tag{278}$$

$$t\Big|_0^{\vartheta}=\frac{\tau_1}{\pi}\left(\frac{\gamma_0+\gamma_2}{2}\right)^{3/2}\left(\varepsilon-\frac{\gamma_2-\gamma_0}{\gamma_2+\gamma_0}\sin\xi\right). \tag{279}$$

In addition, we also have

$$t\Big|_0^{\vartheta}=\frac{\tau}{\pi}\left(\frac{\gamma_0+\gamma_2}{2}\right)^{3/2}\left(\varepsilon-\frac{2(\gamma_2-1)}{\gamma_0+\gamma_2}\operatorname{tg}\frac{\xi}{2}\right). \tag{280}$$

and using (272) we have from equation (44)

$$\operatorname{tg}\frac{\vartheta}{2}=\sqrt{\frac{\gamma_2}{\gamma_0}}\operatorname{tg}\frac{\xi}{2}=\sqrt{\frac{\gamma_2(1-\gamma_0)}{\gamma_0(\gamma_2-1)}}. \tag{281}$$

From (272) we obtain:

$$\frac{\gamma_2}{\gamma_0}=\frac{1+\sum}{1-\sum}, \tag{282}$$

and from equations (274) and (275)

$$V_a=V_1\sqrt{\frac{r_1r_0}{r_2a}}; \tag{283}$$

$$V_p=V_1\sqrt{\frac{r_1r_2}{r_0a}}. \tag{284}$$

Let us determine the trajectories from which it is possible to return to Earth after m years (m being an integer) and n complete revolutions around the Sun if the Earth served as the departure planet.

The formulas will have a general form.

Taking

$$\tau=\frac{m}{n}\tau_1, \tag{285}$$

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we obtain from (278)

$$\frac{m}{n}=\left(\frac{\gamma_0+\gamma_2}{2}\right)^{3/2},$$

or

$$\gamma_0+\gamma_2=2\left(\frac{m}{n}\right)^{2/3}. \qquad (286)$$

In order that the Earth's orbit of radius r should intersect the ellipse, we must have $0<\gamma_0<1$.

We therefore obtain for γ_2 the condition:

$$2\left(\frac{m}{n}\right)^{2/3}\geqslant \gamma_2\geqslant 2\left(\frac{m}{n}\right)^{2/3}-1. \qquad (287)$$

Since $\gamma_2\geqslant 1$, then

$$\frac{m}{n}\geqslant\frac{1}{2^{3/2}} \text{ or } \frac{m}{n}\geqslant\frac{1}{2.8284}.$$

Therefore, $n_{max}\leqslant 2.8284\ m$.

For a given value of m we obtain from Table 4 the maximum number of revolutions of the spaceship around the Sun.

TABLE 4

m	1	2	3	4	5	6	8	10	15	20	30
n_{max}	2	5	8	11	14	16	22	28	42	56	85

Let us first consider the case when only one revolution of the spaceship around the Sun during m years is desired. Then $n=1$, and we obtain from equation (287):

$$2m^{2/3}\geqslant\gamma_2\geqslant 2m^{2/3}-1. \qquad (288)$$

In this case the radius-vector at the aphelion will lie between the limits

$$\gamma_{2\,min}=2m^{2/3}-1; \qquad (289)$$

$$\gamma_{2max}=2m^{2/3}, \qquad (290)$$

and the minimum distances from the Sun will be $\gamma_0=1$ and $\gamma_0=0$. In the second case the ellipse is transformed into a straight line segment with the Sun at one end. This follows also from formula (270) for the semiminor axis of the ellipse.

For $\gamma_0=1$ we have $b/r=\sqrt{\gamma_2}$, and for $\gamma_0=0$ we have $b/r=0$.

From equation (268) it follows that for $\gamma_0=1$, we have $\cos\vartheta=1$, i. e., $\vartheta=0$, and for $\gamma_0=0$ we have $\cos\vartheta=-1$, i. e., $\vartheta=180°$. For $\gamma_0=1$ we obtain the minimum required additional velocity V_2, and for $\gamma_0=0$ we obtain the maximum additional velocity. Correspondingly, we obtain in both cases the minimum and maximum propellant consumption for the flight.

TABLE 5

Table for determining the limiting flight trajectories and the additional velocities which give 1 flight around the Sun in m years

1	m		1	2	3	4	5	6	8	10	15	20	30	60
2	V/V_1		1.000	1.171	1.232	1.268	1.289	1.307	1.322	1.337	1.356	1.363	1.378	1.392
3	V km/sec		29.97	34.86	36.67	37.74	38.37	38.9	39.35	39.80	40.37	40.58	41.03	41.42
4	γ_2	For $\gamma_0 = 1$	1.000	2.175	3.160	4.040	4.848	5.604	7.000	8.283	11.164	13.74	18.31	29.62
5	b/r		1.000	1.475	1.778	2.010	2.202	2.367	2.645	2.878	3.341	3.707	4.280	5.443
6	Σ		0	0.370	0.519	0.004	0.658	0.697	0.750	0.785	0.836	0.865	0.896	0.935
7	V_z/V_1		0	0.171	0.232	0.268	0.289	0.307	0.322	0.337	0.356	0.363	0.378	0.392
8	V_∞/V_1		1.000	0.539	0.390	0.314	0.265	0.233	0.189	0.1615	0.1216	0.0992	0.0752	0.0470
9	V_z km/sec		0	5.09	6.91	7.98	8.60	9.14	9.60	10.02	10.6	10.8	11.25	11.67
10	V_a		29.8	16.02	11.60	9.35	7.89	6.94	5.63	4.81	3.62	2.95	2.24	1.399
11	V_z/V_1	For $\gamma_0 = 0$	1.414	1.540	1.586	1.611	1.630	1.642	1.659	1.670	1.184	1.691	1.705	1.712
12	V_z		42.1	45.8	47.2	48.0	48.5	48.9	49.4	49.7	50.1	50.4	50.7	51.0
13	$\xi°$		135°	130°30'	129°4'	128°16'	127°48'	127°25'	127°6'	126°48'	126°24'	126°16'	125°58'	125°42'
14	γ_2	For $\gamma_0 = 0.3$	1.700	2.875	3.860	4.740	5.548	6.304	7.700	8.983	11.864	14.44	19.01	30.32
15	a/r		1.000	1.5874	2.0801	2.5198	2.9240	3.3019	4.0000	4.6416	6.0822	7.3681	9.1549	15.32
16	b/r		0.714	0.928	1.076	1.190	1.290	1.376	1.520	1.640	1.888	2.080	2.387	3.016
17	$\beta°$		44°26'	50°57'	52°44'	53°38'	54°8'	54°25'	54°54'	55°11'	55°32'	55°48'	56°1'	56°21'
18	$\xi°$		112°12'	106°10'	104°31'	103°46'	103°16'	102°53'	102°30'	102°17'	101°50'	101°42'	101°29'	101°13'
19	Σ		0.700	0.811	0.856	0.881	0.899	0.909	0.925	0.935	0.951	0.960	0.970	0.981
20	$180° - \vartheta$		45°34'	55°48'	58°52'	60°20'	61°19'	61°58'	62°48'	63°15'	64°1'	64°36'	64°59'	65°32'
21	V_z/V_1		0.755	0.948	1.013	1.050	1.073	1.090	1.107	1.123	1.141	1.153	1.166	1.1805
22	V_a/V_1		0.420	0.256	0.193	0.159	0.136	0.120	0.0988	0.0850	0.0643	0.0532	0.0405	0.0254
23	V_p/V_1		2.38	2.455	2.49	2.50	2.51	2.52	2.53	2.54	2.55	2.555	2.56	2.569
24	V_z (km/sec)		22.5	28.2	30.2	31.2	31.9	32.4	32.9	33.4	34.0	34.3	34.6	35.15
25	V_a (km/sec)		12.5	7.61	5.74	4.73	4.04	3.57	2.94	2.53	1.912	1.582	1.205	0.757
26	V_p (km/sec)		70.8	73.0	74.0	74.4	74.6	75.0	75.2	75.5	75.8	75.9	76.1	76.5
27	t_0^ϑ days		50.5	43.5	41.5	41.0	39.5	39.5	39.0	39.0	38.5	38.5	38.0	37.0

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The form of the flight trajectory and the required velocities are characterized by the quantities, given in Table 5, where for a given m the velocity

$$V = V_1 \sqrt{2 - \left(1 - \frac{1}{\gamma_0 + \gamma_2}\right)}$$

is constant, since $\gamma_0 + \gamma_2 = 2a$ is a constant.

We have

$$V = V_1 \sqrt{\frac{2(\gamma_{2max} - 1)}{\gamma_{2max}}} = V_1 \sqrt{\frac{2\gamma_{2min}}{\gamma_{2min} + 1}}.$$

For $\gamma_0 = 1$ we have:

$$\sum = \frac{\gamma_{2min} - 1}{\gamma_{2min} + 1}; \quad \beta = 0; \quad \xi = 0;$$

$$V_z = V - V_1;$$

$$V_a = V_1 \sqrt{\frac{2}{\gamma_{2min}(\gamma_{2min} + 1)}} = \frac{V}{\gamma_{2min}}.$$

For $\gamma_0 = 0$ we have:

$$\beta = 90°; \quad V_a = 0; \quad V_p = \infty;$$

$$V_z = V_1 \sqrt{3 - \frac{2}{\gamma_{2max}}};$$

$$\operatorname{tg} \xi = -\sqrt{\frac{2(\gamma_{2max} - 1)}{\gamma_{2max}}} = -\frac{V}{V_1}.$$

$\gamma_0 = 0$ is interesting only as a limit case; in reality it is impossible to approach the Sun closer than, say, $\gamma_0 = 0.3$, i. e., to a distance somewhat shorter than that of Mercury.

For $\gamma_0 = 0.3$ we have:

$$\gamma_2 = 2m^{2/3} - 0.3;$$

$$a/r = m^{2/3}; \quad b/r = \sqrt{0.3\gamma_2};$$

$$r_0/r = 0.3;$$

$$\cos \beta = \sqrt{\frac{0.3\gamma_2}{2m^{2/3} - 1}};$$

$$\operatorname{tg} \xi = \frac{\sqrt{0.7(\gamma_2 - 1)}}{\sqrt{0.3\gamma_2 - m^{2/3}}}; \quad \cos \vartheta = \frac{0.3\gamma_2 - m^{2/3}}{m^{2/3} - 0.3};$$

$$\sum = \frac{m^{2/3} - 0.3}{m^{2/3}}; \quad \operatorname{tg} \frac{\xi}{2} = \sqrt{\frac{0.7}{\gamma_2 - 1}}; \quad t\Big|_0^\beta = \frac{\tau}{\pi}\left(\xi - \sum \sin \xi\right);$$

$$V_z = \frac{V \sin \beta}{\sin \xi}; \quad V_a - V_1 \sqrt{\frac{0.3}{\gamma_2 m^{2/3}}};$$

$$V_p = V_1 \sqrt{\frac{\gamma_2}{0.3 \cdot m^{2/3}}}.$$

The quantities calculated by these formulas are given in Table 5.

Let us now consider the radius-vectors at the aphelion for the case when the number, n, of revolutions around the Sun till return to Earth is larger than 1.

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For $\gamma_0 = 1$, (287) gives

$$\gamma_{2min} = 2\left(\frac{m}{n}\right)^{2/3} - 1, \tag{291}$$

and for $\gamma_0 = 0$

$$\gamma_{2max} = 2\left(\frac{m}{n}\right)^{2/3}. \tag{292}$$

It should be noted that if m and n have a common factor, cancellation is possible so that the flight around the Sun with return to Earth is made in a shorter period. This means that for $m > 1$, we need not perform the calculations for all integral values $0 < n < 2^{3/2} m$. The results of the calculations are also given in Table 5.

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THE USE OF LIGHT PRESSURE FOR FLIGHT IN INTERPLANETARY SPACE

FROM THE EDITORIAL STAFF

The present article consists of two of Tsander's manuscripts: "The Use of Very Thin Sheets for Flight in Interplanetary Space" and "Light Pressure on Multiple Mirrors". The first comprises sections 1-4, the second section 5. Both manuscripts are unfinished. The first manuscript was prepared for print personally by Tsander, as can be seen by his notes; the date "18 July 1924" appears near Table 1. The second manuscript bears the date 1925. The article was prepared for print by M. K. Tikhonravov, Doctor of Technical Sciences. Figure 4 was redrawn.

In his book "Mezhplanetnye puteshestviya" (Interplanetary Travel) (1923 edition) Ya. E. Perel'man investigates the possibility of flight to other planets with the aid of very thin mirrors. Unfortunately, he does not distinguish between take-off from Earth until orbital velocity is attained and flight in interplanetary space itself, where it is possible to change the velocity by means of mirrors.

It is, however, necessary to establish the radical difference between the requirements which these two sections of the flight path impose on the driving force. The force required to attain orbital velocity of 8 km/sec varies between 1/6 of the spaceship's weight on Earth (if wings are used as on airplanes) and 10 times its weight (for a vehicle without wings). It is then desirable that the acceleration period should be as short as possible. On the other hand, rather small forces are sufficient to accelerate the flight after attaining orbital velocity, as will be shown presently. If the accelerating force P is perpendicular to the direction of the Earth's attraction, the smallest force will produce a velocity increment, since in the absence of P the counteracting force of the solar attraction together with the Earth's attraction would return the ship to the point "A" with its previous velocity (Figure 1). One can also say that the work of an arbitrary small force P, which is equal to $P\int \cos\alpha\, ds$, increases the energy level of the orbit of the spaceship, where α is the angle between the flight direction and the force P, and ds is a path element. At the same time, this small force changes the flight trajectory. The same is true for flight around the Sun in interplanetary space far from any planet.

There is only one case in which it is impossible to use small forces. This happens when the spaceship enters a zone near a planet or near the Sun along its path, notwithstanding the action of the small force. It is then necessary to use a large thrust rocket near the dangerous places. These cases are rare, however, since the planets and the Sun occupy only a small part of the solar system's huge volume, so that for almost all motions there

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it is possible to use rather small forces, carrying along a high thrust rocket only for emergencies and for correcting the path near a planet.

The possibility of using small forces over prolonged periods also speaks in their favor, since flights to other planets will take a long time. This will be shown in a special article. Furthermore, the use of light pressure does not require propellant consumption and therefore this kind of flight will be particularly cheap compared with rocket flight. It will be shown below that very high velocities can be attained with the aid of mirrors for considerably lower total weights of the spaceship's entire structure than is possible by means of rockets.

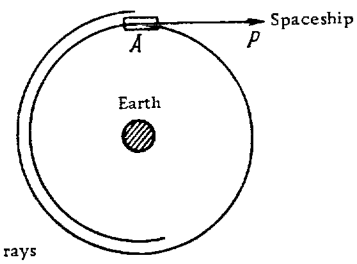

FIGURE 1

The light pressure on sheets is extremely small, but the length of the paths along which this pressure may act, i.e., the distances between the planets, is immense. Therefore, it is interesting to examine what size a mirror should have in order to develop along these immense distances velocities similar to those achieved by rockets. The weight of the sheets, their manufacture, the stresses arising in them and the economic aspects of such a flight are also important topics.

1. LIGHT PRESSURE INTENSITY

Let us use the following notation:

p, specific light pressure, g/m^2;
f, sheet area, m^2;
P, total light force on surface f, g;
E, amount of radiant energy falling on entire sheet per unit time if direction of rays is perpendicular to sheet surface, g·m/sec;
E′, amount of reflected radiant energy;
E, amount of radiant energy contained in unit volume before falling on sheet, kg·m/m^3;
E', same after reflection;
V, light velocity;
v, mirror velocity (positive in direction of light beam), m/sec.

Let us deal first with the case of a beam perpendicular to the surface. We have then for the incident beam:

$$\mathrm{E}=fE(V-v), \tag{1}$$

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and for the reflected beam (for perfect reflection)

$$E' = fE'(V+v). \tag{2}$$

It can be shown that:

$$\frac{E'}{E} = \left(\frac{V-v}{V+v}\right)^2. \tag{3}$$

On the other hand, the useful work per unit time is equal to fpv and this is equal to the energy lost in the reflection

$$fpv = E - E' = f[E(V-v) - E'(V+v) = fE[(V-v) - \frac{(V-v)^2}{V+v}] = \frac{2fEv(V-v)}{V+v}.$$

Hence

$$p = \frac{2E(V-v)}{V+v}. \tag{4}$$

For nonreflecting sheets the light pressure p is equal to half this amount, i.e.,

$$p' = E\frac{V-v}{V+v} = \frac{p}{2}. \tag{5}$$

If the rays fall on the surface at an angle $\alpha < 90°$ the perpendicular component of the pressure is

$$p_{\alpha\,max} = p_{max}\sin^2\alpha; \tag{6}$$

$$p'_{\alpha} = p'\sin^2\alpha. \tag{7}$$

If the reflection is not perfect, the light pressure will be between p_α and p'_α.

Since the flight velocities will very probably be lower than 100 km/sec, and therefore,

$$\frac{v}{V} = \frac{100}{300\,000} = \frac{1}{3000},$$

we can neglect v relative to V and we then obtain with sufficient accuracy*

$$p_{max} = 2E; \tag{8}$$

$$p' = E, \tag{9}$$

i.e., for nonreflecting surfaces, the specific light pressure on a surface perpendicular to the light rays is practically equal to the radiant energy contained in unit volume, and for totally reflecting surfaces it is twice as large.

The solar constant, i.e., the amount of radiant energy falling in 1 minute on 1 cm^2 of surface perpendicular to the rays** is equal to

* Tsander based this derivation on Newtonian mechanics. The same result is obtained from the law of momentum in the special theory of relativity. – Editor's note.

** According to Abbot the average value is $I = 1.938$ cal/min·cm^2. For this value of I we find $p' = 0.46$ and $p_{max} = 0.92$ mg/m^2. (Russell et. al. "Stars and Their Spectra", 1927; "Fundamental Problems of Cosmic Physics", 1934.)– Editor's note.

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$I \approx 1.91$ cal/min·cm². The amount of radiant energy falling on 1 m² per second is

$$L = 427 \frac{10^4}{60} I = 136\,000 \text{ g·m/sec·m}^2$$

but since

$$L = \frac{E}{f} = EV,$$

then

$$p' = E = \frac{L}{V} = \frac{136\,000}{3 \cdot 10^8} = 0.453 \text{ mg/m}^2$$

$$p_{max} = 0.906 \text{ mg/m}^2.$$

2. SHEET THICKNESS

Edison produced very thin sheets* of 0.001 mm thickness, 2 m width and 1000 m length by electrolysis of nickel on a slowly rotating shaft.

"Metod izmereniya davleniya sveta pri pomoshchi tonkogo metallicheskogo listka" (A Method for Measuring Light Pressure with Thin Metallic Sheets), which appeared in "Uspekhi fizicheskikh nauk" Vol. I, No. 2, p. 144. 1918, described the following experiment. The light of an electric bulb fell on an aluminum sheet, of length $l = 7.43$ cm and surface weight $q = 1.16 \cdot 10^{-4}$ g/cm², placed at various distances. The energy of the bulb at a distance of 10.5 cm was $E = 1.95 \cdot 10^{-5}$ erg/cm³.

The following deflections were obtained:

Distance of the sheet from the lamp	Deflections of the sheet	
	measured	calculated
10.5 cm	$a_0 = 2.5 \cdot 10^{-3}$ cm	$a = 2.6 \cdot 10^{-3}$ cm
11.5 cm	2.0	2.3
12.5 cm	1.5	2.0

Taking $\gamma = 2.8$ g/cm³, for the specific weight of aluminum we found that the thickness of the sheet was

$$\delta = \frac{1.16}{2.8} 10^{-4} = 4.14 \cdot 10^{-4} \text{mm}.$$

To check, let us determine the sheet's thickness from the moment it is deflected. For a perfectly reflecting surface, the light pressure on the

* Tsiolkovskii, K. E. Gondola metallicheskogo dirizhablya (The Gondola of the Metallic Dirigible). Kaluga. 1918.

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sheet is theoretically equal to

$$p_{max} = 2E = \frac{2 \cdot 1.95 \cdot 10^{-5}}{9.81} = 3.98 \cdot 10^{-6}\ \mathrm{g/cm^2}.$$

The moment of the light force with respect to the sheet's rotation axis $P_{max}\frac{l^2}{2}$ is equal to the moment of the gravitational force: $ql\frac{a}{2}$, where l is the sheet's length and a its deflection.

Since

$$\frac{qla}{2} = P_{max}\frac{l^2}{2},$$

and since

$$q = \gamma\delta,$$

we obtain

$$\delta = \frac{P_{max}\,l}{\gamma a} = \frac{3.98 \cdot 10^{-6} \cdot 7.43}{2.8 \cdot 2.6 \cdot 10^{-3}} = \frac{4.06}{10^5}\ \mathrm{cm} = \frac{1}{2460}\ \mathrm{mm}.$$

Both values of the sheet thickness are approximately the same. It follows therefore that the calculations of the sheet deflection (see table) were done for a perfectly reflecting surface. Let us calculate the ratio a_0/a, i.e., the ratio of the actual force to the theoretical one. We obtain:

$$\left.\begin{array}{l} 2.5 : 2.6 = 0.962 \\ 2.0 : 2.3 = 0.87 \\ 1.5 : 2.0 = 0.75 \end{array}\right\} \text{average} - 0.86.$$

This calculation shows that aluminum sheets of $\sim 4 \cdot 10^{-4}$ mm thickness give light pressure values which are closer to the pressure p for a perfectly reflecting surface than to the pressure $p' = 0.5p$ of a nonreflecting surface.

The same very important result is obtained in experiments with metal films illuminated through* .....

It can be concluded that sheets of 0.001 mm thickness can be manufactured** and that their opacity will be admissible.

A new problem, however, appears: is their strength sufficient?

Assuming for pure aluminum a yield point k_t of 1500 kg/cm² and an admissible stress of 350 kg/cm², we find that for a thickness of 0.001 mm a force of 35 g on 1 cm width is allowed; for a thickness of 0.01 mm a force of 350 g per centimeter is allowed. Assuming that the widths of sheets used for interplanetary flights will be rather large, and the light force rather small, we may conclude that the stresses in the mirrors cannot be considerable; they will be determined later after calculating the surface required.

* In Tsander's manuscript there is a blank space here. – Editor's note.

** Goldsmiths prepare even thinner sheets. They brought the thickness of gold sheets down to $\delta = 0.0001$ mm.

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3. SHEET SURFACE AREA REQUIRED FOR FLIGHT IN INTERPLANETARY SPACE

In this section we shall determine the forces acting on the sheets, the surface required, and the magnitude of the possible velocities.

Let us consider first the simplest case, i.e., the action of a force in the absence of gravity. We shall use the following notation:

t, acceleration time;
s, path length;
c, acceleration magnitude;
g_0, gravitational acceleration at Earth's surface;
M, mass of spaceship including mirror;
G, weight of spaceship including mirror.
g_s, gravitational acceleration of Sun measured at distance of Earth from Sun;
R, spaceship's distance from Sun;
R_e, Earth's distance from Sun;
V_e, Earth's velocity around Sun;
T_e, Earth's period of revolution around Sun;
G_m, mirror weight measured at Earth's surface.

Then, at an average distance equal to the distance of the Earth from the Sun, if the direction of the light rays coincides with the flight direction, the work spent on accelerating the ship will be equal to

$$Ps = pfs.$$

On the other hand, this work is equal to

$$\frac{M}{2}(V^2 - V_0^2),$$

where V_0 and V are the initial and final flight velocities.
Consequently

$$\frac{M}{2}(V^2 - V_0^2) = pfs. \tag{10}$$

In order to obtain an increase in kinetic energy due to light pressure, when orbiting the Earth (Figure 2), we should use the mirror in each revolution from point B_n through C_n to B_n', while along the path $B_n'D_nB_{n+1}$ the mirror should be inclined so that the rays slide by it without performing work. In this way the attraction force of the Sun may be much larger than the force P. Since point B_n will lie higher than point B_1, relative to the Earth's center, a part of the useful work will be transformed into potential energy

$$M\int_{h_1}^{h_n} g\,dh,$$

where Mg is the Earth's force of attraction, and h_1 and h_2 are the vehicle's initial and final heights, i.e., the distance of B_1 and B_n from the Earth's surface.

The exact formula for the energy acquired by the vehicle is

$$A = M\int_{h_1}^{h_2} g\,dh + \frac{M}{2}(V_k^2 - V_1^2) = pf(s_n - s_1), \tag{11}$$

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where indexes 1 and n refer to points B_1 and B_n, and the difference $s_n - s_1$ is the sum of the differences of the distances of points B'_n and B_n from the Sun*.

The total work which has to be done to overcome completely the Earth's attraction is equal to Mg_0r, where $r = 6370$ km is the Earth's radius.

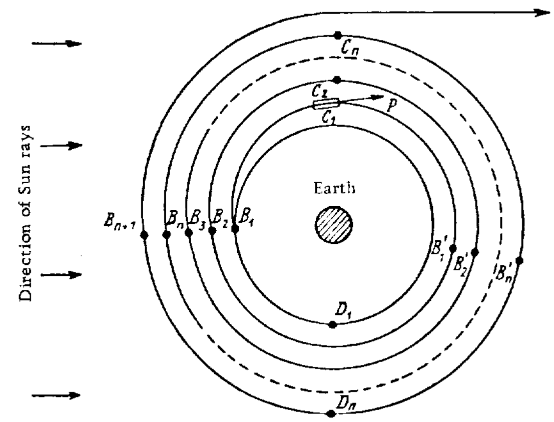

FIGURE 2

In order to orbit the Earth it is necessary to expend half this amount of work. In order to escape to infinity from a circular orbit, it is therefore required to expend the work $Mg_0r/2$. Let us consider this case. We have

$$A = \frac{Mg_0r}{2} = pj(s_n - s_1).$$

As an example we take $G = Mg_0 = 1000$ kg, $p = 0.75\,p_{max} = 0.75 \cdot 0.906 = 0.68$ mg/m^2.

Then

$$j(s_n - s_1) = 4.69 \cdot 10^{15} \text{ m}3.$$

If the mirror area is equal to $100{,}000 \text{ m}^2 = 0.1 \text{ km}^2$, we obtain $s_n - s_1 = 4.69 \cdot 10^{10}$ m $= 46.9 \cdot 10^6$ km.

If the thickness of the mirror is 0.001 mm it weighs $G_m = 280$ kg. We shall approximately determine the time during which the work A is performed. For an average velocity of $V_1 = 8$ km/sec, a path of $46.9 \cdot 10^6$ km will be traversed during

$$\frac{s_n - s_1}{V_1} = \frac{46.9 \cdot 10^6}{8} = 5.85 \cdot 10^6 \text{sec.} = 67.7 \text{ days.}$$

Since the length of time during which the mirror will be in action will constitute approximately 40% of the entire orbiting time, the total time will be equal to 169.0 days**.

* $(s_n - s_1)$ is numerically equal to the sum of the distances between the points B_i and B'_i for all loops of the orbit, where $i = 1, 2, \ldots, n$. – Editor's note.

** Notwithstanding the fact that the average value of V_1 is overestimated, the total time will be larger, since the plane of the mirror is assumed to be perpendicular to the direction of the solar rays during the entire orbiting time.

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We note that if larger mirrors with an area 100 times greater, say, will be in orbit around the Earth, and if their light will be directed to the spaceship's mirror with the aid of guiding astronomical tubes*, then the time may be shortened to approximately 1.69 days, depending on the degree of reflection from the mirror.

Let us consider the case when the plane of the mirror attached to the spaceship is perpendicular to the direction of the solar rays incident on it. For a constant area the light pressure is inversely proportional to the square of the distance. The Sun's attraction follows the same law so that their ratio is constant.

Let use use the following notation:

K, the force of attraction between the Sun and the spaceship;

K_1, the same, minus the light pressure on the mirror;

M_s, the Sun's mass;

k, the Newtonian gravitational constant.

Then, we may write

$$K=\frac{kMM_s}{R^2}. \tag{12}$$

and

$$K_1=K-P. \tag{13}$$

The force P can be expressed by

$$P=\frac{C}{R^2}, \tag{14}$$

where C is a constant.

We then have

$$K_1=\frac{kMM_s-C}{R^2}. \tag{15}$$

Introducing a new constant

$$k_1=k-\frac{C}{MM_s}, \tag{16}$$

we obtain

$$K_1=\frac{k_1MM_s}{R^2}. \tag{17}$$

Comparing (12) and (17) we see that the action of the mirror reduces the resultant attraction force K_1 relative to the Sun's attraction in the ratio:

$$\frac{K_1}{K}=1-\frac{C}{kMM_s}=\frac{k_1}{k}. \tag{18}$$

Since (17) has the same form as the attraction law (12), a spaceship with a mirror will move around the Sun along the same curves as those of celestial bodies, i.e., along a circle, an ellipse, a parabola, or a hyperbola. We must, however, take the reduced value k_1 in place of the gravitational constant. Thus, an apparent weakening of the solar attraction is obtained.

* In other words, the solar radiation density will be increased.

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After overcoming the Earth's attraction, the spaceship still possesses the velocity of the Earth around the Sun, but it will not remain in the Earth's orbit, as it would without a mirror. Due to the weakening of the solar attraction it will move instead in an orbit whose aphelion lies farther from the Sun than the Earth's orbit. This allows it to reach other planets.

In order to determine the extent of weakening of the solar attraction we calculate the spaceship's attraction by the Sun at the Earth's distance and compare it with the light pressure on the mirror.

The Earth's mean orbital velocity is $V_e = 29.2$ km/sec, and the rotation period around the Sun is $T_e = 365.25$ days. The average distance of the Earth from the Sun is $R_e = 147 \cdot 10^6$ km. The spaceship's centrifugal force, equal to the Sun's attraction is

$$K_e = \frac{MV_e^2}{R} = 5.9 \cdot 10^{-4} G. \qquad (19)$$

Therefore, the force of attraction of any mass by the Sun at the distance of the Earth's orbit is in the ratio

$$\frac{K_e}{G} = 0.00059.$$

to the weight of this mass on the Earth's surface.

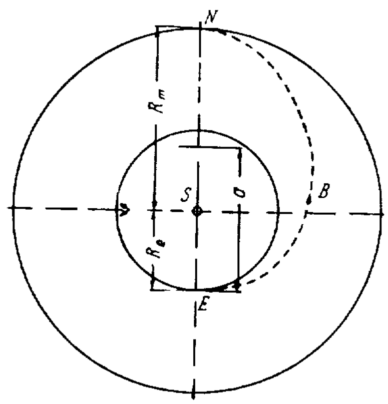

FIGURE 3

We see that the attraction of the Sun is rather weak. A spaceship weighing 1000 kg is attracted to the Sun by a force of

$$K_e = 0.59 \text{ kg}.$$

The total light force on a mirror with an area of 10^5m^2, assuming as above that the pressure is only 0.75 of the value for a perfectly reflecting mirror, is equal to $P = 68$ g. Therefore

$$\frac{P}{K_e} = 0.115.$$

Thus, the mirror reduces the Sun's attraction force by 11.5 percent.

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Furthermore, we have

$$\frac{K_1}{K}=\frac{k_1}{k}=1-\frac{P}{K_e}=1-0.115=0.885.$$

Let us turn now to the determination of flight orbits and flight times. Let use use the notation (Figure 3):

S, the Sun;

E, a point on the Earth's orbit serving as the departure point for the elliptic trajectory EBN;

N, the aphelion of the trajectory which determines the radius R_m of the orbit of the planet farthest from the Sun which can still be reached by a given mirror if the mirror's surface is perpendicular to the solar rays. For simplicity the orbits of the Earth and of the planet are assumed circular.

Also the notation:

V'_e, the velocity with which the spaceship with a mirror would revolve in the Earth's orbit (under a reduced solar attraction);

U_1, the velocity at point E for motion along the ellipse EBN using the coefficient k of solar attraction (without a mirror);

U'_1, the same, using the coefficinet k' of solar attraction (with a mirror);

U_{av}, the average velocity for motion along the ellipse EBN (without a mirror);

U'_{av}, the average velocity for motion along the ellipse EBN (with a mirror);

T_{el}, the time of flight from E to N along the semiellipse (without a mirror);

T'_{el}, the time of flight from E to N along the semiellipse (with a mirror);

Σ the eccentricity of the ellipse EBN;

R_m, the radius of the orbit of the farthest planet that may be reached;

R_e, the radius of the Earth's orbit;

a, the semimajor axis of the flight trajectory, i.e., of the ellipse EBN.

Then, from equations (12), (15) and (19) we have:

$$K_e=\frac{MV_e^2}{R_e}=\frac{kMM_s}{R_e^2},$$

hence

$$kM_s=V_e^2R_e. \tag{20}$$

Similarly

$$K'_e=\frac{MV_e'^2}{R_e}=\frac{kMM_s-C}{R_e^2},$$

hence

$$kM_s-\frac{C}{M}=V_e'^2R_e. \tag{21}$$

From (20) and (21) we obtain

$$V_e^2-V_e'^2=\frac{C}{MR_e}.$$

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For elliptical motion of celestial bodies we have

$$U_1 = U_{av}\sqrt{\frac{1+\Sigma}{1-\Sigma}}; \quad (22)$$

$$U_{av} = \frac{2\pi a}{T_{el}}, \quad (23)$$

where

$$a = \frac{R_e + R_m}{2};$$

$$\sum = \frac{R_m - R_e}{R_m + R_e}. \quad (24)$$

Therefore

$$U_1 = \frac{2\pi a}{T_{el}}\sqrt{\frac{1+\Sigma}{1-\Sigma}}; \quad (25)$$

$$V_e = \frac{2\pi R_e}{T_e}. \quad (26)$$

Also, from Kepler's third law,

$$\frac{T_e^2}{T_{el}^2} = \frac{R_e^3}{a^3}. \quad (27)$$

Dividing equation (25) by equation (26), and using equations (27) and (24) we obtain

$$\frac{U_1}{V_e} = \frac{a}{R_e}\frac{T_e}{T_{el}}\sqrt{\frac{1+\Sigma}{1-\Sigma}} = \frac{a}{R_e}\left(\frac{R_e}{a}\right)^{3/2}\sqrt{\frac{1+\Sigma}{1-\Sigma}} =$$
$$= \left(\frac{2R_e}{R_m + R_e}\right)^{1/2}\sqrt{\frac{1+\Sigma}{1-\Sigma}} = \sqrt{1+\Sigma}.$$

hence

$$U_1 = V_e\sqrt{1+\Sigma}. \quad (28)$$

We assumed that the mirror starts to act when the Earth's attraction has been overcome completely. In this case we shall have $U'_1 = V_e$, and, therefore, we obtain $U_1 = U'_1\sqrt{1+\Sigma}$. If the attraction is reduced by the light pressure, we obtain, instead of (28), a similar formula

$$U'_1 = V'_e\sqrt{1+\Sigma}. \quad (29)$$

and also

$$V_e = V'_e\sqrt{1+\Sigma}. \quad (30)$$

The elliptical rotation periods are inversely proportional to the corresponding velocities

$$\frac{T'_{el}}{T_{el}} = \frac{U_1}{U'_1} = \sqrt{1+\Sigma}.$$

hence

$$T'_{el} = T_{el}\sqrt{1+\Sigma} = T_{el}\sqrt{\frac{2R_m}{R_m + R_e}}. \quad (31)$$

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The time T_{el} is determined from (27):

$$T_{el} = T_e\left(\frac{a}{R_e}\right)^{3/2} = T_e\left(\frac{R_m + R_e}{2R_e}\right)^{3/2} = \frac{T_e}{(1-\Sigma)^{3/2}}. \tag{32}$$

Determining V_e from equation (30) and C from equation (21) we obtain:

$$C = MR_e(V_e'^2 - V_e^2) = MR_eV_e^2\left(1 - \frac{1}{1+\Sigma}\right) = MR_eV_e^2\frac{\Sigma}{1+\Sigma}. \tag{33}$$

From (14), however, we have, putting $R = R_e$

$$P = \frac{C}{R_e^2} = \frac{MV_e^2}{R_e}\frac{\Sigma}{1+\Sigma}. \tag{34}$$

If Σ and M are given it is easy to determine the required mirror area. It is equal to

$$f = \frac{GV_e^2}{gpR_e}\frac{\Sigma}{1+\Sigma}. \tag{35}$$

For $G = 1000$ kg and $p = 0.68$ mg/m² we have

$$f = 8.7\cdot 10^5\frac{\Sigma}{1+\Sigma}\ \text{m}^2.$$

TABLE 1

	Mars	Jupiter	Saturn	Uranus	Neptune
R_m/R_e	1.5237	5.203	9.539	19.183	30.055
Σ	0.2075	0.677	0.810	0.901	0.935
$1-\Sigma$	0.7925	0.823	0.190	0.099	0.065
$(1-\Sigma)^{-3/2}$	1.419	5.45	12.10	32.2	60.4
T_{el} $\frac{\text{days}}{\text{years}}$	$\frac{259}{0.710}$	$\frac{995}{2.72}$	$\frac{2210}{6.35}$	$\frac{5880}{16.1}$	$\frac{11020}{30.2}$
$\sqrt{1+\Sigma}$	1.097	1.291	1.342	1.379	1.390
T'_{el} $\frac{\text{days}}{\text{years}}$	$\frac{284}{0.780}$	$\frac{1284}{3.52}$	$\frac{2970}{8.13}$	$\frac{8100}{22.2}$	$\frac{15310}{42.0}$
$\frac{\Sigma}{1+\Sigma}$	0.172	0.404	0.447	0.474	0.483
f, km²	0.1496	0.351	0.389	0.412	0.420
G_m, kg ($\delta = 10^{-3}$ mm)	389	912	1013	1075	1090
G_m, kg ($\delta = 4\cdot 10^{-4}$ mm)	156	365	405	430	437
$r_2 = \sqrt{f/\pi}$	0.218	0.334	0.351	0.362	0.366
n min⁻¹ ($k_z = 350$ kg/cm²)	7.90	5.12	4.86	4.72	4.67

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The mirror's weight is given by

$$G_m = \gamma f \delta.$$

For chemically pure aluminum $\gamma = 2.6\ g/cm^3$.
Taking $\delta = 10^{-3}$ mm we obtain

$$G_m = 2.6 \cdot 10^{-3} f \text{ kg}.$$

The foregoing table was prepared for the external planets. For comparison, times of flight from the Earth to a planet are given both with a mirror and without it.

4. STRESSES IN REVOLVING MIRRORS

According to Professor Stodola's book "Dampf- und Gasturbinen" (6th Edition) one can determine the radial and tangential stresses in turbine wheels of constant thickness. By these formulas one can also calculate the stresses in revolving thin mirrors since in this case we have only expansion forces. We have

$$\sigma_r = \frac{E}{1-\nu^2}\left[-(3+\nu)\frac{Ax^2}{8} + (1-\nu)b_1 - (1-\nu)\frac{b_2}{x^2}\right]; \qquad (36)$$

$$\sigma_t = \frac{E}{1-\nu^2}\left[-(1+3\nu)\frac{Ax^2}{8} + (1+\nu)b_1 + (1-\nu)\frac{b_2}{x^2}\right]; \qquad (37)$$

$$\xi = -\frac{A}{8}x^3 + b_1 x + \frac{b_2}{x}; \qquad (38)$$

where

$$A = \frac{(1-\nu^2)\mu\omega^2}{E}. \qquad (39)$$

We have used the following notation:

σ_r, the radial stress;
σ_t, the tangential stress;
E, the material's modulus of elasticity;
ν, the transverse compression coefficient;
x, the distance of the section considered from the rotation axis;
b_1 and b_2, constants which should be determined from the mirror's boundary conditions;
γ, the specific weight of the mirror's material;
ω, the angular rotation velocity;
$\mu = \frac{\gamma}{g}$, the mass density of the mirror's material;
ξ, the elongation.

If the mirror does not have a central hole, then for $x = 0$ we obtain $\xi = 0$ since the linear elongation at the sheet's center is zero.

This condition gives $b_2 = 0$, $b_2/x^2 = 0$ since near the sheet's center $\sigma_r = \sigma_t$ should hold. If the external edge of the sheet is not loaded, as it may be, e. g., by an enclosed wire, then here too $\sigma_r = 0$. From equation (36) we find

$b_1 = \frac{3+\nu}{1+\nu}\frac{A}{8}r_2^2$, where r_2 is the radius of the mirror.

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Substituting the values of b_1, b_2 and A in equations (37) and (38) we obtain

$$\text{for } x=0 \quad \sigma_r=\sigma_t=\frac{3+\nu}{8}\sigma_n;$$

$$\text{for } x=r_2 \quad \sigma_r=0, \quad \sigma_t=\frac{1-\nu}{4}\sigma_n,$$

$$\xi=\frac{1-\nu}{E}x\sigma_n;$$

where $\sigma_n=\mu\omega^2r_2^2$ is the stress which would appear in a freely rotating ring of radius r_2.

Taking $k_t = 350\,\mathrm{kg/cm^2}$, $\gamma = 0.0026\,\mathrm{kg/cm^3}$, $g = 981\,\mathrm{cm/sec^2}$ and $\nu = 0.3$ we can calculate ω and from it the number n of rotations per minute which is given in Table 1.

5. LIGHT PRESSURE ON MULTIPLE MIRRORS

Light pressure in the direction of the light source

If a beam of parallel light rays (the same holds for any kind of waves) falls on a surface A, is reflected under a certain angle, falls on a second surface B and is reflected again, then the light force will have a component in the initial direction of the rays (surface A) and a component in the opposite direction (surface B). The question arises: is it possible to find such an inclination of the mirrors that the difference of the above mentioned components will be directed towards the light source?

Let us use the following notation:

P_r and P_t, the components of the resultant of the light forces on both mirrors in the direction of, and perpendicular to, the source;

P, the light force on mirror A when it is perpendicular to the rays;

α, the light's incidence angle on mirror A;

β, the light's incidence angle on mirror B;

δ, the angle between the normal to the plane of mirror B and a plane normal to the initial direction of the light rays (Figure 4).

Let us consider the case when both mirrors are perfectly reflecting. The normal forces on the mirrors A and B are equal to

$$P_A=P\sin^2\alpha;$$

$$P_B=P\sin^2\beta.$$

The components in the direction of the light source and perpendicular to it are equal, for the two mirrors together, to

$$P_r=P_A\sin\alpha-P_B\sin\delta; \tag{40}$$

$$P_t=P_A\cos\alpha-P_B\cos\delta, \tag{41}$$

It follows from the diagram that $\angle SAB=180°-2\alpha$. Let us draw a perpendicular from B to AS, intersecting AS at F; then

$$\angle FBA=90°-(180°-2\alpha)=2\alpha-90°.$$

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By definition $\angle GBA = \beta$, and, therefore, $\angle GBF = \beta - (2\alpha - 90°)$

As can be easily seen, $\delta = 90° - \angle GBF = 90° - \beta + (2\alpha - 90°) = 2\alpha - \beta$. Substituting this value of and those of P_A and P_B in equations (40) and (41), we obtain

$$P_r = P[\sin^3\alpha - \sin^2\beta \sin(2\alpha - \beta)]; \tag{42}$$

$$P_t = P[\sin^2\alpha \cos\alpha - \sin^2\beta \cos(2\alpha - \beta)]. \tag{43}$$

The direction of P_r coincides with the initial direction of the light rays, i. e., from the light source to the mirror. As long as P_r is positive, the light pressure will tend to remove the mirror farther from the light source. The question arises: is it possible to choose such angles α and β that P_r will become negative? In this case the light pressure will tend to bring the mirror nearer to the light source.

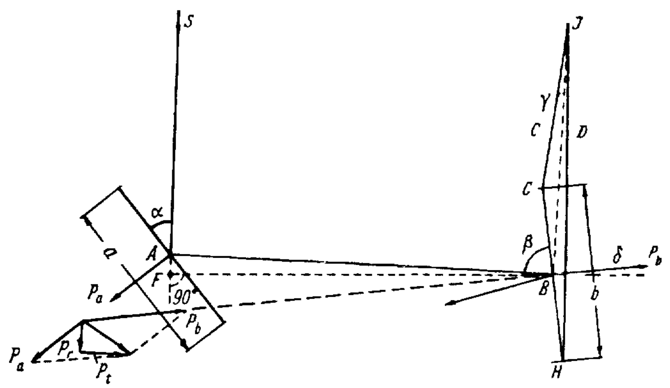

FIGURE 4

To find the minimum of the force P_r we equate the first order derivatives of P_r with respect to the angles α and β to zero:

$$\frac{\partial P_r}{\partial \alpha} = P[3\sin^2\alpha\cos\alpha - 2\sin^2\beta\cos(2\alpha - \beta)] = 0; \tag{44}$$

$$\frac{\partial P_r}{\partial \beta} = P[-2\sin\beta\cos\beta\sin(2\alpha - \beta) + \cos(2\alpha - \beta)\sin^2\beta] = 0. \tag{45}$$

From equation (45) we obtain

$$\operatorname{tg}\beta = 2\operatorname{tg}(2\alpha - \beta); \tag{46}$$

and from equation (44)

$$x = 3\sin^2\alpha\cos\alpha - 2\sin^2\beta\cos(2\alpha - \beta) = 0. \tag{47}$$

Since the solution of these equations is cumbersome, we proceed as follows: we give the angle β and determine the angle α by (46); with these values of α and β we calculate the value of x by (47). Substituting in equations (42) and (43) the values α_0 and β_0 which give $x = 0$, we obtain $(P_r)_{min}$ and the corresponding value of P_t. Calculations are given in Table 2.

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The table shows that P_r has a minimum for $\beta_0 = 60°$ and $\alpha_0 = 50°27'$. Substitution of these angles in equations (42) and (43) yields:

$$(P_r)_{min} = -0.0334P;$$
$$P_t = -0.189P.$$

Thus, 3.34% of the maximum light force P on the mirror A can be obtained as a force directed toward the light source, e. g., towards the Sun. The force P_t constitutes 18.9% of P. If we place another pair of mirrors symmetrically with respect to the Sun's direction we can balance the force P_t and we have only a force $2(P_r)_{min}$ pointing straight to the light source. If we place two mirrors C and D at a very small angle to the direction of the rays from the light source so that the component of the force acting on them, in the direction of the light source, is smaller than $(P_r)_{min}$, then, of course, the force resulting from the mirror system will also be pointing in the light source's direction.

TABLE 2

Determination of $(P_r)_{min}$

β	0	20°	40°	60°	80°	90°
$(\mathrm{tg}\,\beta)/2$	0	0.1828	0.4195	0.8660	2.836	
$2\alpha-\beta$	0	10°18'	22°45'	40°54'	70°32'	90°
α	0	15°09'	31°23'	50°27'	75°16'	90°
$\sin^2\alpha$	0	0.0680	0.270	0.596	0.940	1
$\cos\alpha$	1	0.964	0.853	0.638	0.255	0
$\sin^2\beta$	0	0.1175	0.415	0.752	0.972	1
$\cos(2\alpha-\beta)$	1	0.986	0.924	0.757	0.333	0
$3\sin^2\alpha\cos\alpha$	0	0.1965	0.691	1.14	0.719	0
$2\sin^2\beta\cos(2\alpha-\beta)$	0	0.232	0.766	1.14	0.646	0
x	0	−0.0357	−0.075	0	+0.073	0

If we denote the width of mirror A by a, that of mirror B by b and those of mirrors C and D, placed at equal angles γ to the light rays, by c and d, then the width of the light beam falling on A and reflected from it is equal to $a\sin\alpha$. Since this width should be equal to $b\sin\beta$ we have:

$$b = a\frac{\sin\alpha}{\sin\beta}. \tag{48}$$

Let us determine c and d. In the triangle IGH we have

$$\angle IGH = 180° - \gamma - \delta;$$

$$\angle IHG = 180° - \gamma - (180° - \delta) = \delta - \gamma.$$

We obtain therefore

$$\frac{c}{\sin(\delta-\gamma)} = \frac{b}{\sin 2\gamma};$$

$$c = b\frac{\sin(\delta-\gamma)}{\sin 2\gamma}; \tag{49}$$

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and similarly,

$$d = \frac{b \sin(\delta + \gamma)}{\sin 2\gamma}. \tag{50}$$

The maximum value of γ is obtained from the condition that the component of the light force on both mirrors C and D in the direction of IB is equal to $(P_r)_{min}$. Denoting this component by P_{CD}, we have

$$P_{CD} = P\frac{c}{a}\sin^3\gamma + P\frac{d}{a}\sin^3\gamma,$$

hence

$$\sin^3\gamma = \frac{P_{CD}a}{P(c+d)}. \tag{51}$$

Substituting the values of a, c, and d from (48), (49) and (50) and performing some trigonometric transformations we obtain

$$\sin^3\gamma = \frac{P_{CD}}{P}\frac{\sin\gamma\sin\beta}{\sin(2\alpha-\beta)\sin\alpha},$$

or

$$\sin^2\gamma = \frac{P_{CD}}{P}\frac{\sin\beta}{\sin(2\alpha-\beta)\sin\alpha}. \tag{52}$$

In our case, for $(P_{CD})_{max} = (P_r)_{min} = 0.0334P$, we have $\sin^2\gamma_{max} = 0.0575$; $\gamma_{max} = 13°51'$, and since $\delta = 2\alpha - \beta = 40°54'$ then

$$d = 1.758\text{ b}, \quad c = 0.977\text{ b}.$$

Light force on two inclined mirrors

Let AB and AC be two mirrors (Figure 5) on which light rays fall from the direction SD. The angle of incidence of the rays and the angle of reflection, EDA, are equal to α.

From the triangle DEA we see that the angle of incidence on the second mirror is

$$\angle DEC = \angle EDA + \angle DAE = \alpha + 2\alpha = 3\alpha.$$

The angle of reflection from this mirror is also equal to 3α. From the triangle EFA we see that $\angle AFE$, i.e., the new angle of incidence on the first mirror, is again larger by 2α than the previous angle of incidence, i.e., is equal to

$$3\alpha + 2\alpha = 5\alpha.$$

In the general case, the incidence and reflection angles at the m-th contact of the rays with the mirrors are equal to

$$\alpha_m = \alpha_{m-1} + 2\alpha = \alpha + (m-1)2\alpha = (2m-1)\alpha. \tag{53}$$

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The pressure component, normal to the surface, obtained by m reflections is

$$P\sin^2\alpha_m = P\sin^2(2m-1)\alpha,$$

and the force component in a direction parallel to SD is

$$P\sin^2\alpha_m\ \sin\alpha = P\sin^2(2m-1)\alpha\sin\alpha.$$

The sum of these forces is

$$Q = P[\sin^2\alpha+\sin^2 3\alpha+\ldots+\sin^2(2m-1)\alpha+\sin^2(2n-1)\alpha]\sin\alpha. \qquad (54)$$

If we choose the angle α so that after the last reflection the rays have

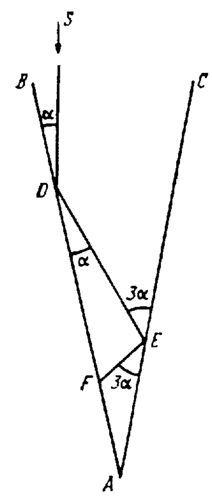

FIGURE 5

a direction exactly opposite to the direction of the source rays, then

$$(2n-1)\alpha = 180^\circ-\alpha,$$

or

$$n = \frac{90^\circ}{\alpha}, \qquad (55)$$

i. e., the angle α should be contained in 90° an integral number of times. We also see that the sum of the first and last terms in the expression for Q is equal to $\sin^2\alpha+\sin^2(180^\circ-\alpha) = 2\sin^2\alpha$. The same can be said of the following pair of terms equally distant from the ends of the expression, i. e.,

$$\sin^2 3\alpha+\sin^2(180^\circ-3\alpha) = 2\sin^2 3\alpha \text{ and so on.}$$

Thus

$$Q = 2P(\sin^2\alpha+\sin^2 3\alpha+\ldots)\sin\alpha. \qquad (56)$$

Equation (54) contains n terms. If n is an even number we shall have $n/2$ terms in equation (56). Remembering that each term in equation (56)

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is equal to twice the corresponding term in equation (54) and making use of formula (55) we may represent (54) in the form:

$$Q=2P[\sin^2\alpha+\sin^2 3\alpha+\ldots+\sin^2(90-3\alpha)+\sin^2(90-\alpha)]\sin\alpha. \quad (57)$$

The sum of the first and last terms in the square brackets of this expression is equal to

$$\sin^2\alpha+\sin^2(90^\circ-\alpha)=1,$$

and the sum of each pair of terms equally distant from the ends is also equal to 1, so that

$$Q=\frac{1}{2}Pn\sin\alpha,$$

and since

$$n=\frac{90^\circ}{\alpha^\circ}=\frac{\pi}{2\alpha}, \quad (58)$$

then

$$Q=\frac{\pi\sin\alpha}{4\alpha}P.$$

If the angle α is very small, $\sin\alpha\approx\alpha$, and we obtain

$$Q=\frac{\pi}{4}P=0.786\,P.$$

If the number of terms n in equation (54) is odd, then one term is left in the middle; it is obviously equal to $\sin^2 90^\circ = 1$, since the equally distant lateral terms, which differ by 2α from the middle one, are equal ;to one another only if they have the values $\sin^2(90^\circ-2\alpha)$ and $\sin^2(90^\circ+2\alpha)$.

(At this point Tsander's manuscript is interrupted. Yet, already from the investigation of the case with even n, the conclusion may be drawn that in certain cases, for small angles α a corner arrangement of the two mirrors (see Figure 5) is better than putting them separately under the same angle to the direction of the rays from the light source. This should be taken into account in designing the equipment.)

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CALCULATIONS OF SPACESHIP FLIGHT IN THE EARTH'S ATMOSPHERE (DESCENT)

Tsander paid much attention to the problem of the return of a spaceship to Earth. He stated clearly that the problem of the descent of a spaceship from interplanetary space to Earth is, figuratively speaking, the threshold through which man will directly enter the cosmos.

Listing in his autobiography what he considers his principal proposals, Tsander writes: "Equipping a rocket with wings ... for a gliding descent from interplanetary space to Earth and to other planets possessing an atmosphere ...".

The published article is dated 28 October 1927* and is devoted to ballistic and aerodynamic problems connected with the landing of a spaceship from interplanetary space to Earth.

In the first part of the article a glide landing of a spaceship on Earth is considered. Tsander was the first to propose the use of the lift of the spaceship's wings in the return to Earth. The spaceship reduces its velocity when flying in the less dense layers of the atmosphere, and thus the surface temperature due to aerodynamic heating is lower. The author gives an estimate of the maximum flight velocities for which the centrifugal force can be neglected and gives an approximate method for calculating the trajectory of a glide landing for an isothermal atmosphere in the cases: 1) flight at constant altitude; 2) constant drag of the vehicle; 3) constant lift to drag ratio of the vehicle. The second part of the article deals with some problems of ballistic landing – zero lift flight of a spaceship in the Earth's atmosphere.

Now, when the conquest of the cosmos has made great steps forward, the requirements for choosing the optimum landing trajectory of a spaceship on Earth, which Tsander indicated but did not examine in detail in his article, come to the forefront. These are the limitations of the maximum overloads acting on the vehicle, the limitation of the maximum surface temperatures and also the maximum reduction of the total heat flux received by the vehicle during the landing which determined the weight of its thermal insulation.

Notwithstanding this, the article is of unquestionable interest even at the present time.

Editor

* It will be possible to establish the exact date after deciphering Tsander's stenographic records. The calculation was undoubtedly performed earlier than this date since the calculation of the ascent is dated 1924.

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1. GLIDE LANDING OF A SPACESHIP FROM INTERPLANETARY SPACE ON EARTH

From the general equation of motion of a rocket

$$P = Q - G\sin\theta + \frac{G}{g}\frac{dV}{dt}, \qquad (1)$$

(where g is the gravitational acceleration at an altitude h over the Earth's surface), we obtain for $P = 0$, i.e., in the case of glide landing

$$\frac{G}{g}\frac{dV}{dt} = G\sin\theta - Q;$$

or

$$\frac{dV}{dt} = g\left(\sin\theta - \frac{Q}{G}\right). \qquad (1a)$$

The components of the forces in a direction perpendicular to the motion

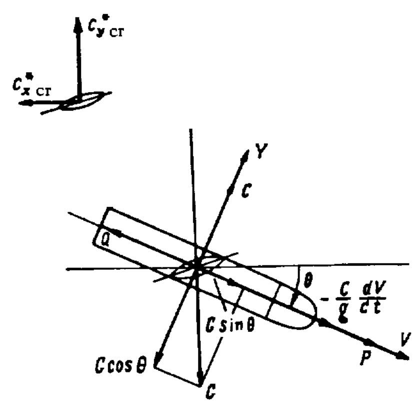

FIGURE 1

(Figure 1) give:

$$C + Y = G\cos\theta, \qquad (2)$$

where

$$C = \frac{V^2}{r}\frac{G}{g}, \qquad (3)$$

is the centrifugal force which appears as a result of the curvature of the trajectory (r is the radius of curvature).

Determination of the maximum limit of flight velocities for which the centrifugal force can be neglected

If we take into consideration that in a gently sloping glide landing the radius of curvature r of the flight trajectory is relatively large, it becomes

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clear that at flight velocities much lower than 8 km/sec (a velocity at which the centrifugal force is equal to the Earth's gravitational attraction and a gliding descent is transformed into a flight at constant altitude over the Earth's surface) C can be neglected as compared with the vehicle's weight G.

Then instead of (2) we obtain the expression

$$Y = G\cos\theta. \tag{2a}$$

Values of r and V, obtained from (3) for $n = G/C = 9.81$ and 98.1* are given in Table 1. From (3) we obtain

$$V^2 = \frac{rg}{n}, \tag{3a}$$

or, in our example

$$V\,[\mathrm{m/sec}] = \sqrt{r\,[\mathrm{m}]}$$

and

$$V\,[\mathrm{m/sec}] = \sqrt{\frac{r\,[\mathrm{m}]}{10}}.$$

The values of r and V are given for various values of the flight altitude h which are obtained when the inclination angle of the trajectory varies from $\theta = 0°$ to $\theta = 7°08'$ along a circular arc of radius r.

TABLE 1

h, km	0.1	0.495	1	4.195	10	49.5	100
V [m/sec] for $n=9.81$	113.2	252	358	796	1132	2520	3580
V [m/sec] for $n=98.1$	35.8	79.6	113.2	252	358	796	1132
r [km]	12.85	63.7	128.7	637	1287	6370	12 870

In order to estimate the trajectory curvature's influence on the resulting drop in altitude (if we take initially the Earth's radius as infinitely large) we calculate the drop in height h.

In lowering the velocity, the kinetic energy of the flying rocket plane acts like an engine, reducing the inclination angle of the gliding trajectory. For steady-state motion, when $\frac{dV}{dt} = 0$, we obtain from (1a)

$$Q = G\sin\theta. \tag{1b}$$

From (1b) and (2a) we obtain

$$\sin\theta_{max} = \frac{Q}{G} = \frac{Q}{Y}\cos\theta_{max}.$$

* The value of n characterizes the so-called "centrifugal load", i.e., it shows what fraction of the vehicle's weight is balanced by the centrifugal force. – Editor's note.

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or

$$\operatorname{tg}\theta_{max}=\frac{Q}{Y}.$$

Introducing the notation*

$$\frac{Q}{Y}=\operatorname{tg}\beta,$$

we obtain

$$\operatorname{tg}\theta_{max}=\operatorname{tg}\beta;\ \theta_{max}=\beta;$$

which for $\operatorname{tg}\beta = 0.125$ gives

$$\beta=\theta_{max}=7°08'$$

Therefore, in the flight $\theta<7°08'$.

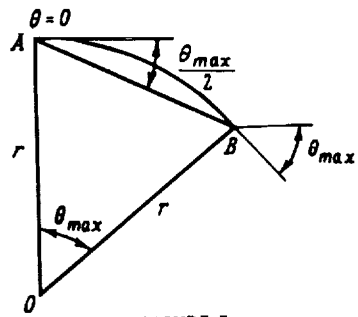

FIGURE 2

Assuming that in continuous landing** we have always $\theta>0$, it is easy to determine the drop in height for a variation of the angle from $\theta = 0°$ to $\theta = 7°08'$, if the flight is along a gently sloping arc of a circle. Since (see Figure 2),

$$AB=s=2r\sin\frac{\theta_{max}}{2},$$

and

$$h=s\sin\frac{\theta_{max}}{2},$$

we obtain

$$h=2r\sin^2\frac{\theta_{max}}{2}, \tag{4}$$

and in our example

$$h=2r\sin^2 3°34'=0.00778\,r. \tag{4a}$$

* Q/Y is the reciprocal of the lift to drag ratio of the rocket plane. Taking $\operatorname{tg}\beta = 0.125$, the author considers a rocket plane with a lift to drag ratio of 8. – Editor's note.

** The author had in mind a so-called "smooth" landing trajectory which differs from a wavy trajectory by the absence of waves, i.e., the angle of inclination of the landing trajectory does not change sign. Angles of inclination of the trajectory are considered here positive if measured downwards from the local horizon at the initial point of the landing trajectory. It is now usual to regard $\theta>0$ upwards and $\theta<0$ downwards, from the local horizon.

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From (3a) and (4) we also obtain

$$V=\sqrt{\frac{rg}{n}}\sqrt{\frac{hg}{2n\sin^2\frac{\theta_{max}}{2}}}=\frac{1}{\sin\frac{\theta_{max}}{2}}\sqrt{\frac{gh}{2n}}; \quad (5)$$

or in the example with $n = 9.81$

$$V[\mathrm{m/sec}]=11.32\sqrt{\frac{h}{10}}=3.58\sqrt{h\,[\mathrm{m}]}. \quad (5a)$$

for $n = 98.1$

$$V[\mathrm{m/sec}]=\sqrt{\frac{h}{2}}\frac{1}{\sin 3°34'}=11.32\sqrt{h\,[\mathrm{m}]}; \quad (5a')$$

From (4a) we have:

$$r=\frac{10^3 h}{7.78}=128.7\,h. \quad (4b)$$

V and r can be calculated as functions of h by using (5a), (5a') and (4b) (see Table 1). If we neglect C, accept an error not larger than $\frac{C}{G}=\frac{1}{n}=$ $=\frac{1}{9.81}\approx 10\%$, take the radius of curvature not smaller than $r=R/10 = 637$ km ($R = 6370$ km being the Earth's radius), and neglect the centrifugal force (in the present case $<G/9.81$), then it is possible to attain a velocity $V \ll 796$ m/sec. If we increase n and decrease r by a factor of ten, then $V=$ $= 79.6$ m/sec.

In the first case $h = 4950$ m and in the second case $h = 495$ m. We see, therefore, that at flight velocities up to 800 m/sec the centrifugal force can be neglected in lift to within an error of 10%.

If we determine by a more exact calculation the angle of inclination θ of the gliding trajectory for velocities from 800 to 1500 m/sec ($\theta > 0$), the centrifugal force can be neglected for these flight velocities, due to the still larger radius of curvature. In this case we must take into account the Earth's radius of curvature which we took equal to infinity when calculating the drop in height.

If we neglect the centrifugal force and assume $\cos\theta\approx 1$ (actually $\cos\theta_{max}=$ $=\cos 7°08' = 0.9923$, so that the error is $\approx 0.8\%$), we have one more equation expressing the lift of the incident stream on the rocket plane's wings

$$Y=G\cos\theta\approx G=\frac{c^{*}_{y\,cr}}{\gamma_0}\gamma F V^2, \quad (6)$$

where G, $c^{*}_{y\,cr}$, γ_0, and F are constants. Hence

$$\gamma V^2=\frac{G\gamma_0}{Fc^{*}_{y\,cr}} \quad (6a)$$

is also a constant*.

Therefore, knowing h as a function of γ, it is easy to determine by (5) the average value of n for landing on Earth from a given height h. We then

* The author does not take into account the lift of the rocket plane's body. At hypersonic velocities the vehicle's body creates a significant part of the lift.
$c^{*}_{y\,cr}=\frac{c_{y\,cr}\gamma_0}{2}$ (kg·sec^2/m^4) is a constant coefficient proportional to the aerodynamic wing lift coefficient, where $c_{y\,cr}$ is the aerodynamic wing-section lift coefficient, and γ_0 is the atmospheric density at sea level. – Editor's note.

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calculate V from (6a) for a given value of h, and determine from (5) the quantity

$$n = \frac{gh}{2} \frac{1}{V^2 \sin^2 \frac{\theta_{max}}{2}},$$

assuming in first approximation a range of variation of the angle of inclination of the trajectory from $\theta = 0$ to θ_{max}.

After calculating the angle θ for the maximum velocity we obtain a more exact value of n.

Approximate calculation of gliding descent neglecting the centrifugal force

If the velocities in gliding descent are so high that we cannot neglect the Earth's curvature since it produces a centrifugal force reducing the gliding angle, we can still obtain a lower curve for the flight trajectory by neglecting the Earth's curvature and the centrifugal force perpendicular to flight direction. The angle θ will then be too high since the centrifugal force increases the lift of the rocket plane. After determining the trajectory curvature it is easy to determine the centrifugal force and with the new results to calculate a new trajectory which will lie higher than the real one. Determining the curvature of the new trajectory we can finally find a landing trajectory close to the one required.

For velocities higher than 8 km/sec, even for constant altitude flight, the centrifugal force is larger than the weight of the rocket plane and the gliding descent is transformed into gliding ascent for which the centrifugal force cannot be neglected in a first calculation.

Equation (1a) shows that the maximum deceleration is obtained when the gravitational force is not accelerating the vehicle and the air drag Q decelerates it. Therefore, neglecting initially the quantity $G \sin \theta$, we obtain too fast a landing and also too curved a landing trajectory.

$G \sin \theta \approx 0$ can be assumed only at relatively high flight velocities when the rocket plane possesses a large amount of kinetic energy. If we then assume a constant angle of attack of the rocket plane wings we have $Q = \text{const}$*, and from (1a)

$$\frac{dV}{dt} = -\frac{Q}{G} = -\operatorname{tg} \beta^{**}.$$

Hence we find by integration

$$V - V_0 = (t_0 - t) \operatorname{tg} \beta. \tag{7}$$

We also have equation (6a) and

$$\frac{dh'}{dt} = -V \sin \theta. \tag{8}$$

* This is true for fixed trajectories of equilibrium gliding if we neglect the dependence of the lift coefficient on the flight Mach number M.

** In this case the author neglects the centrifugal force C in equation (2) and assumes $\cos \theta \approx 1$, since the trajectory's angle of inclination is small, and then $Y = G$ and $\operatorname{tg} \beta \approx \frac{Q}{Y} = \frac{Q}{G}$. – Editor's note.

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Substituting in (6a) the expression for V from (7), and knowing h' as function of γ, we obtain h' as function of t. This means that we can determine dh'/dt, and from (8) also $\sin\theta$, as functions of t.

Taking into account the term $G\cos\theta$ in (1b), allowing for the Earth's curvature, and neglecting only C in (2), the calculation may be carried out as follows.

Let h' be the height over the Earth's surface at the given point (the Earth's surface is assumed plane);
h, the same for a spherical surface of the Earth;
r', the flight trajectory's radius of curvature;
r, the distance to the Earth's center.

Formula (8) can be written in the form

$$V = -\frac{1}{\sin\theta}\frac{dh'}{dt}, \tag{8a}$$

or, differentiating V with respect to t,

$$\frac{dV}{dt} = -\frac{d^2h'}{dt^2}\frac{1}{\sin\theta} + \frac{dh'}{dt}\frac{d\theta}{dt}\frac{\cos\theta}{\sin^2\theta}. \tag{9}$$

We see from Figure 3 that $dh' = r'\sin\theta\, d\theta$, and hence the trajectory's radius of curvature is

$$r' = -\frac{dh'}{dt}\frac{dt}{d\theta}\frac{1}{\sin\theta}. \tag{10}$$

The centrifugal acceleration is found from (8a) and (10)

$$\frac{V^2}{r'} = \frac{1}{\sin^2\theta}\left(\frac{dh'}{dt}\right)^2\frac{d\theta}{dt}\frac{dt}{dh'}\sin\theta = -\frac{dh'}{dt}\frac{d\theta}{dt}\frac{1}{\sin\theta}.$$

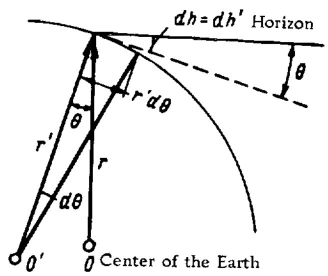

FIGURE 3

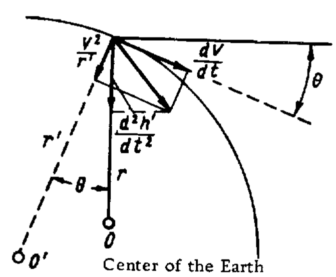

FIGURE 4

We obtain the same result from

$$\frac{V^2}{r'} = V\omega = -\frac{dh'}{dt}\frac{d\theta}{dt}\frac{1}{\sin\theta}. \tag{11'}$$

We find, therefore, from (9)

$$\frac{dV}{dt} = -\left(\frac{d^2h'}{dt^2} + \frac{V^2}{r'}\cos\theta\right)\frac{1}{\sin\theta}. \tag{11}$$

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From Figure 4 it follows that

$$\frac{d^2h'}{dt^2} = -\frac{dV}{dt}\sin\theta - \frac{V^2}{r'}\cos\theta; \tag{11a}$$

hence we obtain the same formula as (11).

From the formulas of mechanics, however, we have

$$\frac{d^2h'}{dt^2} = \frac{d^2h}{dt^2} - \frac{V^2}{r}. \tag{11b}$$

Substituting $\frac{d^2h'}{dt^2}$ from this formula in (11) we get

$$\frac{dV}{dt} = -\frac{d^2h}{dt^2}\frac{1}{\sin\theta} + \frac{V^2}{\sin\theta}\left(\frac{1}{r} - \frac{\cos\theta}{r'}\right), \tag{11c}$$

and since $\cos\theta \approx 1$ and $r \approx r'$, we obtain in first approximation

$$\frac{dV}{dt} \approx -\frac{d^2h}{dt^2}\frac{1}{\sin\theta}. \tag{11d}$$

From (1a) and (11d) we obtain for $Q/G = \mathrm{tg}\,\beta$

$$\frac{dV}{dt} \approx \frac{d^2h}{dt^2}\frac{1}{\sin\theta} = g(\sin\theta - \mathrm{tg}\,\beta);$$

or

$$\frac{d^2h}{dt^2} + g(\sin^2\theta - \sin\theta\,\mathrm{tg}\,\beta) = 0. \tag{12}$$

Furthermore, from (6) and (8a) we have

$$\frac{c^{*}_{y\,\mathrm{cr}}}{\gamma_0}\gamma F\left(\frac{dh}{dt}\right)^2 - G\sin^2\theta = 0, \tag{6'}$$

since $\frac{dh}{dt} = \frac{dh'}{dt}$, or, if we introduce the constant*

$$k = \frac{c^{*}_{y\,\mathrm{cr}}}{\gamma_0}\frac{F}{G}, \tag{13}$$

then

$$\left(\frac{dh}{dt}\right)^2 = \frac{\sin^2\theta}{k\gamma}. \tag{14}$$

Eliminating the angle θ from (12) and (14) we obtain

$$\frac{d^2h}{dt^2} + k\gamma g\left(\frac{dh}{dt}\right)^2 - \mathrm{tg}\,\beta\sqrt{k\gamma}\,g\frac{dh}{dt} = 0. \tag{15}$$

Denoting

$$\frac{dh}{dt} = V_y, \tag{16}$$

* k is the reciprocal of the so-called effective specific wing load. – Editor's note.

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which gives

$$\frac{d^2h}{dt^2}=V_y\frac{dV_y}{dh}, \tag{17}$$

equation (15) assumes the form

$$V_y\frac{dV_y}{dh}+k\gamma g V_y^2-g\,\mathrm{tg}\,\beta\sqrt{k\gamma}\,V_y=0,$$

or

$$\frac{dV_y}{dh}+k\gamma g V_y-g\,\mathrm{tg}\,\beta\sqrt{k\gamma}=0.$$

Introducing in the last equation new dependent variables u and q, so that

$$V_y=uq, \tag{18''}$$

we obtain*

$$\frac{dh}{dt}=V_y=e^{-kg\int\gamma\,dh}\left[g\sqrt{k}\,\mathrm{tg}\,\beta\int\sqrt{\gamma}\,e^{kg\int\gamma\,dh}dh+C_1\right]. \tag{19}$$

Knowing γ as a function of h, we find $\frac{dh}{dt}$ and

$$t=\int\frac{dh}{V_y}+C_2, \tag{20}$$

and then the gliding angle from (14)

$$\sin\theta=\frac{dh}{dt}\sqrt{k\gamma}. \tag{14a}$$

The velocities dh/dt and dh'/dt are equal, as follows from Figure 5, at the point A at which the heights h and h' are equal (the vertical variations of the heights dh and dh' are identical).

Consequently, (8a) can be written in the form

$$V=-\frac{1}{\sin\theta}\frac{dh}{dt}, \tag{8b}$$

which, using (14), gives

$$V=\frac{1}{\sqrt{k\gamma}}. \tag{8c}$$

The same is obtained from (6) by substituting expression (13), and from (8c) we determine the flight velocity V.

If we take $\cos\theta\approx 1$, the horizontal component of the path is approximately equal to

$$s=\int V\,dt. \tag{21}$$

From (19) we have

$$dt=\frac{dh}{V_y},$$

* C_1 and C_2 are integration constants which are determined from the initial conditions. – Editor's note.

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so that

$$s=\int\frac{V\,dh}{V_y}; \tag{21a}$$

where V and V_y may be regarded as given functions of h. By plotting V/V_y as the ordinate and h as the abscissa, we obtain a diagram whose integration between the limits h_1 and h_2 gives s.

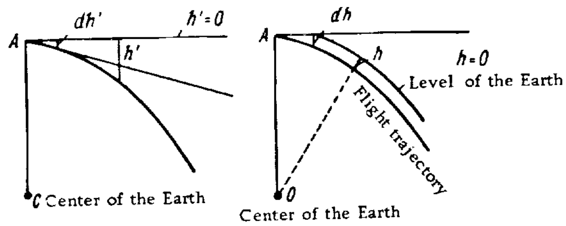

FIGURE 5

The trajectory's radius of curvature r' is obtained from (10) where $d\theta/dt$ can be determined from (14a):

$$\cos\theta\frac{d\theta}{dt}=\frac{d^2h}{dt^2}\sqrt{k\gamma}+\frac{dh}{dt}\frac{\sqrt{k}}{2\sqrt{\gamma}}\frac{d\gamma}{dh}\frac{dh}{dt},$$

or

$$\frac{d\theta}{dt}=\frac{d^2h}{dt^2}\frac{\sqrt{k\gamma}}{\cos\theta}+\frac{\sqrt{k}}{2\sqrt{\gamma}}\left(\frac{dh}{dt}\right)^2\frac{d\gamma}{dh}. \tag{22}$$

Differentiating the equation $\gamma=f(h)$ we obtain $d\gamma/dh$, and if γ is given in the form of a curve, we obtain a curve $d\gamma/dh=f(h)$.

Substituting expression (22) for $\frac{d\theta}{dt}$ into equation (10), the expression for the trajectory's radius of curvature takes the form

$$r'=\frac{-\frac{dh}{dt}}{\sin\theta}\,\frac{1}{\frac{d^2h}{dt^2}\frac{\sqrt{k\gamma}}{\cos\theta}+\sqrt{\frac{k}{\gamma}}\,\frac{1}{2}\left(\frac{dh}{dt}\right)^2\frac{d\gamma}{dh}}. \tag{10a}$$

Having calculated r', it is possible to calculate the term $V^2\left(\frac{1}{r}-\frac{\cos\theta}{r'}\right)$ omitted in (11d). Then, replacing (11d) by (11c) in (1a), we obtain

$$g(\sin\theta-\mathrm{tg}\,\beta)=\frac{1}{-\sin\theta}\left[\frac{d^2h}{dt^2}-\left(\frac{1}{r}-\frac{\cos\theta}{r'}\right)V^2\right].$$

or

$$\frac{d^2h}{dt^2}+g\sin\theta(\sin\theta-\mathrm{tg}\,\beta)-\left(\frac{1}{r}-\frac{\cos\theta}{r'}\right)V^2=0. \tag{12a}$$

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Using equation (22) and assuming $dh/dt = dh'/dt$, we obtain from (11') an expression for the centrifugal acceleration

$$\frac{V^2}{r'} = \frac{1}{\sin\theta}\left[\frac{dh}{dt}\frac{d^2h}{dt^2}\frac{\sqrt{k\gamma}}{\cos\theta} + \frac{1}{2}\sqrt{\frac{k}{\gamma}}\left(\frac{dh}{dt}\right)^3\frac{d\gamma}{dh}\right]. \qquad (11a)$$

Instead of equation (6') we use now the corresponding equation obtained by using equations (2), (3), (8a), and (6) and assuming $\cos\theta \approx 1$:

$$\sin^2\theta\frac{V^2}{r'}\frac{G}{g} + \frac{c^{*}_{y\,\mathrm{cr}}}{\gamma_0}\gamma F\left(\frac{dh}{dt}\right)^2 - G\sin^2\theta = 0. \qquad (6'')$$

Calculating the correction terms

$$\varphi_1(V_y) = \sin^2\theta\frac{V^2}{r'}\frac{1}{g}$$

in equation (6'') and

$$\varphi_2(V_y) = \left(\frac{1}{r} - \frac{\cos\theta}{r'}\right)V^2$$

in equation (12a) as functions of V_y or h, we are able to solve a more general equation instead of equation (15).

Using k from (13), and the remarks made above, (6'') can be written in the form

$$\left(\frac{dh}{dt}\right)^2 = \frac{\sin^2\theta - \varphi_1(V_y)}{k\gamma}, \qquad (14')$$

and (12a) – in the form

$$\frac{d^2h}{dt^2} + g\sin\theta(\sin\theta - \mathrm{tg}\,\beta) - \varphi_2(V_y) = 0. \qquad (12a')$$

Eliminating the angle θ from (14') and (12a'), we obtain:

$$\frac{d^2h}{dt^2} + gk\gamma\left(\frac{dh}{dt}\right)^2 + g\varphi_1(V_y) - g\,\mathrm{tg}\,\beta\sqrt{k\gamma\left(\frac{dh}{dt}\right)^2 + \varphi_1(V_y)} - \varphi_2(V_y) = 0,$$

or, using (16) and (17)

$$V_y\frac{dV_y}{dh} + k\gamma gV_y^2 + g\varphi_1(V_y) - V_y g\,\mathrm{tg}\,\beta\sqrt{k\gamma}\sqrt{1 + \frac{\varphi_1(V_y)}{k\gamma V_y^2}} - \varphi_2(V_y) = 0.$$

We have finally

$$\frac{dV_y}{dh} + k\gamma gV_y + g\frac{\varphi_1(V_y)}{V_y} - g\,\mathrm{tg}\,\beta\sqrt{k\gamma}\sqrt{1 + \frac{\varphi_1(V_y)}{k\gamma V_y^2}} - \frac{\varphi_2(V_y)}{V_y} = 0. \qquad (18')$$

Assuming that V found from (19) for a given altitude h or a given density γ differs only slightly from the more exact value found from (18'), and

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replacing V_y in (18') by V'_y, i. e., taking $V_y \approx V'_y$ and subtracting (18') from (18), we can calculate $V'_y - V_y$:

$$\frac{d(V'_y - V_y)}{dh} + \frac{g}{V_y}[\varphi_1(V_y) - \varphi_2(V_y)] - \\ - g \operatorname{tg} \beta \sqrt{k\gamma}\left[\sqrt{1 + \frac{\varphi_1(V_y)}{k\gamma(V_y)^2}} - 1\right] = 0. \quad (23)$$

Assuming V_y to be a given function of γ or h, we have

$$\frac{d(V'_y - V_y)}{dh} = \varphi_3(h), \quad (23')$$

hence

$$V'_y - V_y = \int \varphi_3(h)\, dh. \quad (23a)$$

With the aid of (20) and replacing V_y by V'_y, we now determine the landing time t. We then obtain from (14'):

$$\sin\theta' = \sqrt{k\gamma(V'_y)^2 + \varphi_1(V'_y)}. \quad (14')$$

Instead of (8b) we have

$$V' = \frac{V'_y}{\sin\theta}; \quad (8b')$$

and instead of (21a)

$$s' = \int \frac{V'\,dh}{V'_y}. \quad (21a')$$

Taking the results of the second approximation for a third, even more accurate calculation, we obtain a more accurate landing trajectory. By successive approximation any degree of accuracy can be obtained.

It is now also simple to determine at what velocities and altitude the centrifugal force can be neglected in formula (2).

Calculation of gliding descent for an isothermal atmosphere

For an isothermal atmosphere, which exists at high altitudes ($h > 11$ km), we have

$$\frac{\gamma}{\gamma_0} = \frac{p}{p_0} = e^{-\frac{h}{H}}, \quad (24)$$

where H = const is the altitude of the uniform atmosphere.

In this case

$$\int \gamma\, dh = \gamma_0 \int e^{-\frac{h}{H}}\, dh = -\gamma_0 H e^{-\frac{h}{H}}, \quad (25)$$

$$e^{-kg\int \gamma\, dh} = e^{kg\gamma_0 He^{-\frac{h}{H}}} = e^{z^2}, \quad (25a)$$

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where we have introduced a new variable z, which, using (24), can be written in the form

$$z^2 = -kg\int \gamma\, dh = kg\gamma H. \tag{26}$$

If we put

$$\psi = \int \sqrt{\gamma}\, e^{kg\int \gamma\, dh}\, dh, \tag{27}$$

we can introduce the variable z in equation (19) too. From (26) we obtain

$$dz = -\frac{kg\gamma}{2z}\, dh = -\frac{z}{2H}\, dh. \tag{26a}$$

Substituting this expression in (27) we obtain

$$\psi = \int \frac{z}{\sqrt{kgH}}\, e^{-z^2} \frac{(-2H)}{z}\, dz = -2\sqrt{\frac{H}{kg}} \int e^{-z^2} dz. \tag{27a}$$

The value of $\int e^{-z^2} dz$ can be found by a series expansion.

Substituting z from (26) and ψ from (27a) in (19) we obtain

$$\frac{dh}{dt} = V_y = e^{z^2}\left(-2\sqrt{gH}\,\mathrm{tg}\,\beta \int e^{-z^2} dz + C_1\right). \tag{19a}$$

The integration constant C_1 is determined from the initial conditions.
The velocity V is given by formula (8c). It can also be expressed by z. From (8c) and (26) we obtain

$$V = \frac{\sqrt{gH}}{z}, \tag{8d}$$

and then V_y can be represented as a function of V.
Differentiating (8d) we obtain

$$dz = -\frac{\sqrt{gH}}{V^2}\, dV. \tag{8d'}$$

Substituting (8d) and (8d') in (19a), we find

$$V_y = e^{\frac{gH}{V^2}}\left(+2gH\,\mathrm{tg}\,\beta \int e^{-\frac{gH}{V^2}} \frac{dV}{V^2} + C_1\right). \tag{19b}$$

For convenience it is possible to introduce a new variable

$$p = \frac{V^2}{gH} = \frac{1}{z^2}; \tag{28}$$

$$dz = -\frac{dp}{2p^{3/2}}. \tag{28a}$$

Finally we have

$$V_y = e^{\frac{1}{p}}\left(\sqrt{gH}\,\mathrm{tg}\,\beta \int e^{-\frac{1}{p}} \frac{dp}{p^{3/2}} + C_1\right). \tag{19c}$$

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To determine the constant of integration C_1, the vertical velocity of descent V_{y0} is given for a given altitude h.

Then we find from (19a)

$$V_{y0}=e^{z_0^2}\left(-2\sqrt{gH}\,\mathrm{tg}\,\beta\int_{z_1}^{z_0}e^{-z^2}dz+C_1\right), \tag{29}$$

or from (19c)

$$V_{y_0}=e^{\frac{1}{p_0}}\left(\sqrt{gH}\,\mathrm{tg}\,\beta\int_{p_1}^{p_0}\frac{e^{-\frac{1}{p}}}{p^{3/2}}dp+C_1\right); \tag{29a}$$

hence

$$C_1=V_{y0}e^{-z_0^2}+2\sqrt{gH}\,\mathrm{tg}\,\beta\int_{z_1}^{z_0}e^{-z^2}dz, \tag{29'}$$

or

$$C_1=V_{y_0}e^{-\frac{1}{p}}-\sqrt{gH}\,\mathrm{tg}\,\beta\int_{p_1}^{p_0}\frac{e^{-\frac{1}{p}}}{p^{3/2}}dp. \tag{29a}$$

Substituting C_1 in (19a) and (19c) we get

$$V_y=e^{z^2}\left[-2\sqrt{gH}\,\mathrm{tg}\,\beta\left(\int_{z_1}^{z}e^{-z^2}dz-\int_{z_1}^{z_0}e^{-z^2}dz\right)+V_{y0}e^{-z_0^2}\right]=$$
$$=-2\sqrt{gH}\,\mathrm{tg}\,\beta e^{z^2}\int_{z_0}^{z}e^{-z^2}dz+V_{y0}e^{z^2-z_0^2}, \tag{19a'}$$

or

$$V_y=e^{\frac{1}{p}}\left[\sqrt{gH}\,\mathrm{tg}\,\beta\left(\int_{p_1}^{p}\frac{e^{-\frac{1}{p}}dp}{p^{3/2}}-\int_{p_1}^{p_0}\frac{e^{-\frac{1}{p}}dp}{p^{3/2}}\right)+V_{y0}e^{-\frac{1}{p_0}}\right]=$$
$$=\sqrt{gH}\,\mathrm{tg}\,\beta e^{\frac{1}{p}}\int_{p_0}^{p}\frac{e^{-\frac{1}{p}}dp}{p^{3/2}}+V_{y0}e^{\frac{1}{p}-\frac{1}{p_0}}. \tag{19c'}$$

We see that, as expected, the arbitrary quantities z_1 and p_1 indeed disappear.

Spaceship flight at constant altitude and high flight velocity

For constant altitude ($\theta=0$), throttled engine ($P=0$) flight, the general equation of motion (1) takes the form

$$-\frac{G}{g}\frac{dV}{dt}=Q, \tag{1c}$$

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or

$$Q = Y\frac{c^{\bullet}_{x\,cr}}{c^{\bullet}_{y\,cr}} + c_{x\,pa}F_M\gamma V^2, \tag{1d}$$

where the total drag Q is decomposed into the drag of the lifting surfaces $Y\frac{c^{\bullet}_{x\,cr}}{c^{\bullet}_{y\,cr}}$ and the parasitic drag of the remaining sections of the rocket plane* $c_{x\,pa}F_M\gamma V^2$. Here, F_M is the cross section area of a plate perpendicular to the incident stream and having the same drag as the whole vehicle. From (2) and (3) we obtain:

$$C + Y = G = \frac{V^2}{r}\frac{G}{g} + Y, \tag{2b}$$

where r is the distance from the center of the Earth to the point considered on the flight trajectory.

Furthermore, we have also

$$Y = \frac{c^{\bullet}_{y\,cr}}{\gamma_0}\gamma F V^2. \tag{6}$$

In these formulas γ, γ_0, F_M, F, g, r, $c_{x\,pa1}$, G are constant quantities at a fixed moment of time. The lift and drag coefficients $c^{\bullet}_{y\,cr}$ and $c^{\bullet}_{x\,cr}$ of the carrying surfaces are functions of the angle of attack α, so that one can write

$$c^{\bullet}_{y\,cr} = f_1(\alpha) \text{ and } c^{\bullet}_{x\,cr} = f_2(\alpha).$$

It is also possible to regard $c^{\bullet}_{x\,cr}$ as a function of $c^{\bullet}_{y\,cr}$, i.e., $c^{\bullet}_{x\,cr} = \psi(c^{\bullet}_{y\,cr})$. If the airfoil profile is given, these three functions may also be regarded as known*.

α, Y and V are still unknown as functions of time. Eliminating the lift Y of the carrying surfaces from (6) and (2b) we obtain:

$$\frac{V^2}{r}\frac{G}{g} + \frac{c^{\bullet}_{y\,cr}}{\gamma_0}\gamma F V^2 = G$$

or

$$V^2 = \frac{G}{\frac{G}{rg} + c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F}. \tag{2c}$$

Eliminating Q, Y and V from (1c), (1d), (6) and (2c) we have

$$-\frac{G}{g}\frac{dV}{dt} = c^{\bullet}_{x\,cr}\frac{\gamma}{\gamma_0}F V^2 + c_{x\,pa1}F_M\gamma V^2;$$

* $c^{\bullet}_{x\,cr} = \frac{c_{x\,cr}\gamma_0}{2}$ (kg sec^2/m) is a constant coefficient proportional to the aerodynamic wing section drag coefficient;

$c_{x\,cr}$ is the aerodynamic wing section drag coefficient;

$c_{x\,pa1} = \frac{c_{x\,pa}}{2}$ is half the aerodynamic parasitic drag coefficient of the apparatus. – Editor's note.

** The author does not take into consideration the dependence of the aerodynamic coefficients on the flight Mach number. For very large Mach numbers this is admissible in first approximation.

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or

$$-\frac{G}{g}\frac{dV}{dt}=\frac{\left(c^{\bullet}_{x\,cr}\frac{\gamma}{\gamma_0}F+c_{xpa}F_M\gamma\right)G}{\frac{G}{rg}+c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F}. \quad (1e)$$

Differentiating (2c) with respect to t we obtain

$$2V\frac{dV}{dt}=-G\left(\frac{G}{rg}+\frac{\gamma}{\gamma_0}c^{\bullet}_{y\,cr}F\right)^{-2}\frac{\gamma}{\gamma_0}F\frac{dc^{\bullet}_{y\,cr}}{dt},$$

or using (2c)

$$\frac{dV}{dt}=-\frac{\sqrt{G}}{2}\left(\frac{G}{rg}+c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{-3/2}\frac{\gamma}{\gamma_0}F\frac{dc^{\bullet}_{y\,cr}}{dt}. \quad (2d)$$

Substituting (2d) in (1e) we have:

$$\frac{dc^{\bullet}_{y\,cr}}{dt}=\frac{1}{F}\frac{2g}{\sqrt{G}}\left(c^{\bullet}_{x\,cr}F+c_{xpa1}F_M\gamma_0\right)\left(\frac{G}{rg}+c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{\frac{1}{2}}=\varphi(c^{\bullet}_{y\,cr}). \quad (2e)$$

If we substitute this equation $c^{\bullet}_{x\,cr}=\psi(c^{\bullet}_{y\,cr})$, the right-hand side becomes a function of $c^{\bullet}_{y\,cr}$ and we get:

$$t=\int_{(c^{\bullet}_{y\,cr})_0}^{c^{\bullet}_{y\,cr}}\frac{dc^{\bullet}_{y\,cr}}{\varphi(c^{\bullet}_{y\,cr})}. \quad (20a)$$

From (20a) we determine $c^{\bullet}_{y\,cr}$ as function of the flight time t and from $c^{\bullet}_{y\,cr}=f_1(\alpha)$ we obtain the angle of inclination of the carrying surfaces of the rocket plane to its direction of motion, i.e., the angle of attack α of the wing for constant altitude flight of the rocket plane.

From (2c) we obtain now the velocity V, and from (2d) – the deceleration dV/dt.

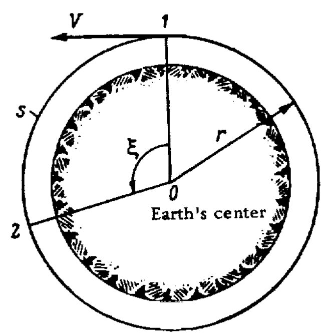

FIGURE 6

The path traversed by the rocket plane in time t is

$$s=\int_0^t V\,dt, \quad (21)$$

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and the central angle ξ (measured at the Earth's center), defined by the arc of the flight path (Figure 6) is

$$\xi^{\circ}=\frac{s}{r}\frac{180^{\circ}}{\pi}. \qquad (21')$$

If $c^{*}_{x\,cr}=\psi(c^{*}_{y\,cr})$ is given analytically for small angles of attack α, t can also be expressed in final form.

In general, the time t can be found graphically by measuring the area under the calculated function $1/\varphi(c^{*}_{y\,cr})$ between the limits $(c^{*}_{y\,cr})_0$ and $c^{*}_{y\,cr}$.

In order to determine the velocities at the points of the trajectory between which flight at a given altitude is possible, it is best to use expression (2c).

If we denote by $(c^{*}_{y\,cr})_1$ the initial* value of $c^{*}_{y\,cr}$ corresponding to the maximum angle of attack of the wings in flight "on the back", and by $(c^{*}_{y\,cr})_2$ the final value corresponding to flight in the usual position, we obtain

$$|(c^{*}_{y\,cr})_1|=|(c^{*}_{y\,cr})_2|.$$

Flight "on the back" is required at velocities larger than 8 km/sec in order that the rocket plane should not fly back into interplanetary space.

The resultant force of the incident stream on the wings of the rocket plane should be directed towards the Earth's center. Then $(c^{*}_{y\,cr})_1<0$, $(c^{*}_{y\,cr})_1$ corresponds to V_{max}, and $(c^{*}_{y\,cr})_2$, to V_{min}. From (2c) we obtain

$$V^2_{max}=\frac{G}{\frac{G}{rg}-(c^{*}_{y\,cr})_1\frac{\gamma}{\gamma_0}F}, \qquad (2c')$$

and

$$V^2_{min}=\frac{G}{\frac{G}{rg}+(c^{*}_{y\,cr})_2\frac{\gamma}{\gamma_0}F}. \qquad (2c'')$$

If here $(c^{*}_{y\,cr})_2$ is the maximum attainable value of the lift coefficient lying near the maximum of the curve $c^{*}_{y\,cr}=f(c^{*}_{x\,cr})$, it is possible to construct curves of V_{max} and V_{min} as functions of the density γ. Obviously, the greater γ is, i.e., the nearer the flight is to the Earth's surface, the higher V_{max} and the lower V_{min}. For high flight velocities we have, however, still another limitation – apparent gravity.

If we take $(c^{*}_{y\,cr})_2=0$ in (2c'), we obtain $V=\sqrt{rg}$. This is the velocity at which the centrifugal force is equal to the gravitational attraction. At this instantaneous velocity the vehicle flies freely as a satellite of the Earth, remaining at constant distance from the Earth's center. The apparent gravity is equal to zero in this case.

Let us denote by a the ratio of the apparent gravity to the Earth's attraction**. If the horizontal component of the apparent gravity due to flight deceleration $\frac{dV}{dt}$ by drag can be neglected, then

$$a=\frac{Y}{G}=c^{*}_{y\,cr}V^2_{max}\frac{\gamma}{\gamma_0}\frac{F}{G}. \qquad (6b)$$

* During constant-altitude flight. – Editor's note.

** I. e., the "g"-loading. – Editor's note.

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If, on the other hand, the rocket plane flies "on the back" at a constant distance from the Earth's center, the centrifugal acceleration is equal to the sum of the gravitational acceleration g and the apparent gravitational acceleration, ag, i.e.,

$$g + ag = \frac{V^2_{max}}{r};$$

hence

$$V^2_{max} = rg(1+a). \tag{6c}$$

We see, that if a is constant, V_{max} is also constant. In this case, $c^{\bullet}_{y\,cr}$ is inversely proportional to γ (see (6b)).

To determine the value γ_x we obtain from (6b) $c^{\bullet}_{ycr} = (c^{\bullet}_{ycr})_2$; in this case

$$\gamma_x = \frac{a\gamma_0 G}{(c^{\bullet}_{y\,cr})_2 Frg(1+a)}. \tag{6b'}$$

If $\gamma > \gamma_x$, then $c^{\bullet}_{ycr} < (c^{\bullet}_{ycr})_2$, i.e., the density is larger than γ_x, and we have from (6c) $V_{max} = \text{const}$; for air density lower than γ_x, formula (2c') will hold, and V_{max} will decrease with γ.

At high densities γ there are still the following three limitations to high velocity flight:

1) apparent gravity in the horizontal direction due to the flight deceleration*;
2) vehicle heating in the atmosphere's dense layers;
3) high drag which may cause vehicle failure. The deceleration is obtained from (2d) and the two other limitations will be considered separately.

Gliding descent for $c_{x\,cr} = \text{const}$

If we take the wing section drag coefficient $c^{\bullet}_{x\,cr}$ as constant for a certain range of small angles of attack α of the carrying surfaces, then introducing the quantities

$$\varkappa = \frac{G}{rg} + c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F; \tag{a}$$

$$A = \frac{(c^{\bullet}_{x\,cr}F + c_{x\,pai}F_M\gamma_0)\,g\gamma}{\gamma_0\sqrt{G}}; \tag{b}$$

we obtain from (2e)

$$\frac{dc^{\bullet}_{y\,cr}}{dt} = \frac{2\gamma_0}{\gamma F}A\varkappa^{1/2}.$$

Differentiating expression (a) with respect to t, we obtain

$$\frac{d\varkappa}{dt} = \frac{\gamma}{\gamma_0}F\frac{dc^{\bullet}_{y\,cr}}{dt},$$

* I.e., the maximum admissible value of the longitudinal "g" – loading. – Editor's note.

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consequently,

$$\frac{d\varkappa}{dt}=2A\varkappa^{1/2}. \qquad \text{(a')}$$

Hence

$$t=\frac{1}{2A}\int\limits_{\varkappa_0}^{\varkappa}\frac{d\varkappa}{\sqrt{\varkappa}}=\frac{1}{A}\left(\sqrt{\varkappa}-\sqrt{\varkappa_0}\right), \qquad \text{(c)}$$

or

$$\sqrt{\varkappa}=At+\sqrt{\varkappa_0}. \qquad \text{(c')}$$

Substituting $\varkappa$ from (c') in (a) we obtain:

$$c^{*}_{y\,\mathrm{cr}}=\frac{\left[(At+\sqrt{\varkappa_0})^2-\frac{G}{rg}\right]\gamma_0}{\gamma F}.$$

Substituting (a)' in (2c) we obtain

$$V^2=\frac{G}{\varkappa},$$

or

$$V=\frac{\sqrt{G}}{At+\sqrt{\varkappa_0}}. \qquad \text{(2c')}$$

The path is

$$s=\int\limits_0^t V\,dt=\sqrt{G}\int\limits_0^t\frac{dt}{At+\sqrt{\varkappa_0}},$$

or

$$s=\frac{\sqrt{G}}{A}\ln\left(\frac{A}{\sqrt{\varkappa_0}}+1\right). \qquad \text{(21a)}$$

Finally from (1e) and (b) we obtain for the deceleration

$$-\frac{dV}{dt}=\frac{g}{G}\left(\frac{c^{*}_{x\,\mathrm{cr}}F}{\gamma_0}+c_{x\,\mathrm{pat}}F_M\right)\gamma V^2,$$

or

$$-\frac{dV}{dt}=\frac{A\sqrt{G}}{(At+\sqrt{\varkappa_0})^2}. \qquad \text{(1e)}$$

We can also determine the resultant apparent gravity in the vehicle

$$j=\sqrt{\left(\frac{dV}{dt}\right)^2+\left(\frac{V^2}{r}-g\right)^2}. \qquad \text{(d)}$$

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Gliding descent for constant lift to drag ratio of the wing

If we consider the domain of the curve $c^{\bullet}_{x\,cr}/c^{\bullet}_{y\,cr}$, in which the quantity*

$$u=\frac{c^{\bullet}_{x\,cr}}{c^{\bullet}_{y\,cr}},$$

can be considered constant (instead of $c^{\bullet}_{x\,cr}$ as before), then we obtain from (2e):

$$\frac{dc^{\bullet}_{y\,cr}}{dt}=\frac{2g\gamma_0}{F\gamma\sqrt{G}}\left(uc^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F+c_{x\,\mathrm{pal}}F_M\gamma\right)\times$$
$$\times\left(\frac{G}{rg}+c^{\bullet}_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{\frac{1}{2}} \tag{2e'}$$

or using (a)

$$\frac{dc^{\bullet}_{y\,cr}}{dt}=\frac{2g\gamma_0}{F\gamma\sqrt{G}}\left(u\varkappa-\frac{nG}{rg}+c_{x\,\mathrm{pal}}F_M\gamma\right)\varkappa^{\frac{1}{2}}. \tag{2e''}$$

Differentiating (a) with respect to t, we have

$$\frac{d\varkappa}{dt}=\frac{\gamma}{\gamma_0}F\frac{dc^{\bullet}_{y\,cr}}{dt}. \tag{a''}$$

Using the notation:

$$C_1=\frac{2g}{\sqrt{G}}u \tag{f}$$

and

$$C_2=\frac{c_{x\,\mathrm{pal}}F_M\gamma}{u}-\frac{G}{rg}, \tag{g}$$

and then substituting $dc^{\bullet}_{y\,cr}/dt$ from (2e'') in (a''), we obtain

$$\frac{d\varkappa}{dt}=C_1(\varkappa+C_2)\varkappa^{\frac{1}{2}}, \tag{a'''}$$

or

$$C_1t=\int_{\varkappa_0}^{\varkappa}\frac{d\varkappa}{(\varkappa+C_2)\varkappa^{\frac{1}{2}}}. \tag{aIV}$$

Introducing

$$y=\varkappa^{1/2} \tag{h}$$

and, therefore

$$dy=\frac{1}{2}\frac{d\varkappa}{\varkappa^{1/2}}, \tag{h'}$$

* u – the reciprocal of the lift to drag ratio of the wing; $k_{\mathrm{aer}}=\frac{1}{u}=c^{\bullet}_{y\,cr}/c^{\bullet}_{x\,cr}$. – Editor's note.

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we obtain, on substituting in (a^{IV}),

$$C_1 t = 2\int \frac{dy}{y^2 + C_2} = \frac{2}{\sqrt{C_2}} \operatorname{arc\,tg}\left(\frac{y}{\sqrt{C_2}}\right) + C_3 =$$

$$= \frac{2}{\sqrt{C_2}} \operatorname{arc\,tg} \sqrt{\frac{x}{C_2}} + C_3.$$

or

$$C_1 t = \frac{2}{\sqrt{C_2}} \left(\operatorname{arc\,tg} \sqrt{\frac{x}{C_2}} - \operatorname{arc\,tg} \sqrt{\frac{x_0}{C_2}}\right). \qquad (a^V)$$

Using the notation

$$\xi = \operatorname{arc\,tg} \sqrt{\frac{x}{C_2}}, \qquad (i)$$

or

$$\operatorname{tg} \xi = \sqrt{\frac{x}{C_2}}; \qquad (i')$$

$$\operatorname{tg} \xi_0 = \sqrt{\frac{x_0}{C_2}}, \qquad (i'')$$

then

$$\frac{C_1 \sqrt{C_2}}{2} t = \xi - \xi_0 \qquad (a^{VI})$$

and

$$x = C_2 \operatorname{tg}^2 \xi = C_2 \operatorname{tg}^2\left(\xi_0 + \frac{C_1 \sqrt{C_2}}{2} t\right) = \frac{G}{rg} + c^{*}_{y\,cr} \frac{\gamma}{\gamma_0} F,$$

or

$$c^{*}_{y\,cr} = \frac{\gamma_0}{\gamma F}\left[C_2 \operatorname{tg}^2\left(\xi_0 + \frac{C_1 \sqrt{C_2}}{2} t\right) - \frac{G}{rg}\right]. \qquad (k)$$

Determining initially the values of C_1, C_2 and ξ_0 by formulas (f), (g), (i''), and (a), it is possible to find the dependence of $c^{*}_{y\,cr}$ on the time t by formula (k). Then we can determine from $c^{*}_{y\,cr}$ the angle of attack α of the wings, and the drag coefficient $c^{*}_{x\,cr}$.

From (2c) we find

$$V = \sqrt{\frac{G}{x}} = \frac{\sqrt{G}}{\sqrt{C_2} \operatorname{tg}\left(\xi_0 + \frac{C_1 \sqrt{C_2}}{2} t\right)}, \qquad (2c''')$$

and from (21)

$$s = \int_0^t V\,dt = \sqrt{\frac{G}{C_2}} \int_0^t \frac{dt}{\operatorname{tg}\left(\psi_0 + \frac{C_1 \sqrt{C_2}}{2} t\right)} =$$

$$= \sqrt{\frac{G}{C_2}} \frac{2}{C_1 \sqrt{C_2}} \int_{\xi_0}^{\xi} \frac{d\xi}{\operatorname{tg} \xi}. \qquad (21b)$$

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The value of the integral in (21b) is

$$\int_{\xi_0}^{\xi} \frac{d\xi}{\operatorname{tg}\xi} = \int_{\xi_0}^{\xi} \frac{d(\sin\xi)}{\sin\xi} = \ln\frac{\sin\xi}{\sin\xi_0},$$

so that

$$s = \frac{2\sqrt{G}}{C_1 C_2} \ln \frac{\sin\left(\xi_0 + \frac{C_1\sqrt{C_2}t}{2}\right)}{\sin\xi_0}. \qquad (21c)$$

From formula (21') the central angle is equal to

$$\xi = \frac{s}{r}\frac{180°}{\pi}.$$

Finally, differentiating (2c''') with respect to t, we obtain an expression for the deceleration:

$$-\frac{dV}{dt} = \frac{\sqrt{\frac{G}{C_2}}\,\frac{C_1\sqrt{C_2}}{2}}{\sin^2\left(\xi_0 + \frac{C_1\sqrt{C_2}}{2}t\right)},$$

or

$$-\frac{dV}{dt} = \frac{C_1\sqrt{G}}{2\sin^2\left(\xi_0 + \frac{C_1\sqrt{C_2}}{2}t\right)}. \qquad (2c^{IV})$$

To calculate V, s and $\frac{dV}{dt}$ it is more convenient to calculate initially ξ with the aid of formula (a^{VI}).

Calculation of gliding descent with allowance for the centrifugal force

As in the calculation of constant altitude flight we take into account the flight trajectory's radius of curvature not only in formula (1a) (and correspondingly (11c) or (11d)) but also in formula (2). In a first approximation we take this radius equal to the distance r to the Earth's center for some average flight altitude, and we calculate the flight trajectory.

Taking (as above) the trajectory found as a first approximation, we may say that due to the increase in the angle θ during the descent, the radius of curvature will actually be smaller than r. This means that the centrifugal force for the same velocity will be larger than assumed, i.e., that the actual trajectory will lie higher than the one found.

Making an approximate determination of the real radius of curvature from the first calculation, we can determine the flight trajectory more accurately in a second calculation. The second trajectory found will lie higher than the real one but will be already very close to it. Similarly we can find a third approximation.

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The corresponding trajectory will lie between the two first and will be even closer to the true one. The true one will be situated between the flight trajectories found in the second and third approximations.

We obtained above the relations

$$\frac{d^2h}{dt^2} = -\frac{dV}{dt}\sin\theta; \tag{11d}$$

$$\frac{dV}{dt} = g\left(\sin\theta - \frac{Q}{G}\right); \tag{1a}$$

$$Q = Y\frac{c^{\bullet}_{x\,\text{cr}}}{c^{\bullet}_{y\,\text{cr}}} + c_{x\,\text{pal}}F_M\gamma V^2. \tag{1d}$$

Furthermore, from (2) and (3) we have

$$\frac{G}{g}\frac{V^2}{r} + Y = G\cos\theta; \tag{3a}$$

$$Y = c_{y\,\text{cr}}\frac{\gamma}{\gamma_0}FV^2. \tag{6}$$

Since the trajectory is very gently sloping we can take $\cos\theta \approx 1$. Then substituting (6) in (3a) we obtain

$$V^2 = \frac{G}{\frac{G}{rg} + c^{\bullet}_{y\,\text{cr}}\frac{\gamma}{\gamma_0}F} \tag{2c}$$

and from (1a) and (1d) we obtain a formula analogous to formula (1e)

$$-\frac{G}{g}\frac{dV}{dt} = c^{\bullet}_{x\,\text{cr}}\frac{\gamma}{\gamma_0}FV^2 + c_{x\,\text{pa}}F_M\gamma V^2 - G\sin\theta. \tag{22}$$

From (22) we find

$$\sin\theta = \frac{1}{g}\frac{dV}{dt} + \gamma\left(c^{\bullet}_{x\,\text{cr}}\frac{F}{\gamma_0} + c_{x\,\text{pal}}F_M\right)\frac{V^2}{G}, \tag{22a}$$

and (11d) takes the form

$$\frac{d^2h}{dt^2} = -\frac{1}{g}\left(\frac{dV}{dt}\right)^2 - \gamma\left(c^{\bullet}_{x\,\text{cr}}\frac{F}{\gamma_0} + c_{\ \text{pal}}F_M\right)\frac{V^2}{G}\frac{dV}{dt}. \tag{23}$$

From (2c) we obtain

$$\frac{dV}{dt} = -\frac{\sqrt{G}}{2}\left(\frac{G}{rg} + c^{\bullet}_{y\,\text{cr}}\frac{\gamma}{\gamma_0}F\right)^{-\frac{3}{2}}\frac{c^{\bullet}_{g\,\text{cr}}F}{\gamma_0}\frac{d\gamma}{dt}. \tag{2cV}$$

Next, knowing how the air density varies with altitude, i.e., $h = f(\gamma)$, we obtain

$$\frac{dh}{dt} = \frac{df}{d\gamma}\frac{d\gamma}{dt} = f'\frac{d\gamma}{dt}; \tag{24a}$$

$$\frac{d^2h}{dt^2} = \frac{df'}{d\gamma}\left(\frac{d\gamma}{dt}\right)^2 + f'\frac{d^2\gamma}{dt^2} = f''\left(\frac{d\gamma}{dt}\right)^2 + f'\frac{d^2\gamma}{dt^2}. \tag{24b}$$

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Substituting expressions (24b) and (2c) in (23) we obtain the differential equation

$$f''\left(\frac{d\gamma}{dt}\right)^2+f'\left(\frac{d^2\gamma}{dt^2}\right)+\frac{1}{g}\frac{G}{4}\left(\frac{G}{rg}+c^*_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{-3}\left(\frac{c^*_{y\,cr}}{\gamma_0}\right)^2F^2\left(\frac{d\gamma}{dt}\right)^2-$$
$$-\frac{\sqrt{G}}{2}\frac{\left(c^*_{x\,cr}\frac{F}{\gamma_0}+c_{x\,\mathrm{pal}}F_M\right)}{\left(\frac{G}{rg}+c^*_{y\,cr}\frac{\gamma F}{\gamma_0}\right)^{5/2}}\frac{d\gamma}{dt}\frac{c^*_{y\,cr}\gamma F}{\gamma_0}=0,$$

or

$$\frac{d^2\gamma}{dt^2}+\left(\frac{d\gamma}{dt}\right)^2\left[f''+\frac{G}{4g}\left(\frac{G}{rg}+c^*_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{-3}\left(\frac{c^*_{y\,cr}}{\gamma_0}\right)^2F^2\right]\frac{1}{f'}-$$
$$-\frac{d\gamma}{dt}\frac{\sqrt{G}\,c^*_{y\,cr}\gamma F\left(\frac{c^*_{x\,cr}F}{\gamma_0}+c_{x\,\mathrm{pal}}F_M\right)}{2f'\gamma_0\left(\frac{G}{rg}+\frac{c^*_{y\,cr}\gamma F}{\gamma_0}\right)^{5/2}}. \qquad (25)$$

This equation has the form

$$\frac{d^2\gamma}{dt^2}+\varphi_1(\gamma)\left(\frac{d\gamma}{dt}\right)^2+\varphi_2(\gamma)\frac{d\gamma}{dt}=0. \qquad (25a)$$

By introducing the new variable

$$z_1=\frac{d\gamma}{dt}, \qquad (26)$$

so that

$$\frac{dz_1}{d\gamma}=\frac{d^2\gamma}{dt^2}\frac{1}{z_1}, \qquad (26a)$$

equation (25a) assumes the form

$$z_1\frac{dz_1}{d\gamma}+\varphi_1 z_1^2+\varphi_2 z_1=0, \qquad (25b)$$

where φ_1 and φ_2 are functions of γ:

$$\varphi_1(\gamma)=\frac{1}{f'}\left[f''+\frac{G}{4g}\left(\frac{G}{rg}+c^*_{y\,cr}\frac{\gamma}{\gamma_0}F\right)^{-3}\left(\frac{c^*_{y\,cr}}{\gamma_0}\right)^2F^2\right]; \qquad (25c)$$

$$\varphi_2(\gamma)=\frac{\sqrt{G}\,c^*_{y\,cr}F\left(\frac{c^*_{x\,cr}F}{\gamma_0}+c_{x\,\mathrm{pal}}F\right)\gamma}{2f\gamma_0\left(\frac{G}{rg}+\frac{c^*_{y\,cr}\gamma F}{\gamma_0}\right)^{5/2}}. \qquad (25d)$$

If equation (25b) is divided by z_1, it can be solved like equation (2b) (here we introduced the independent variable h instead of γ).

We have

$$\frac{dz_1}{d\gamma}+\varphi_1 z_1+\varphi_2=0. \qquad (25e)$$

Putting

$$z_1=u\cdot q \qquad (26)$$

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we obtain

$$z_1 = \frac{d\gamma}{dt} = e^{-\int \varphi_1 d\gamma}\left[\int \varphi_2 e^{\int \varphi_1 d\gamma} d\gamma + C_3\right] \tag{27}$$

and

$$t = \int \frac{d\gamma}{z_1} + C_4. \tag{28}$$

Thus, as in the previous case, we find γ as a function of t. From equation (24) we find h, from (2c) the velocity V then, from ($2c^{IV}$) the deceleration $\frac{dV}{dt}$, and from (22a) $\sin\theta$ as function of t.

It is possible to plot all the quantities on the same diagram and also to determine by integration the path traversed

$$s = \int V\,dt + C, \tag{28a}$$

as well as its projection on the Earth's surface and the central angle defined by it (Figure 7):

$$L = \int V\cos\theta \frac{R}{R+h} dt; \tag{29}$$

$$\xi = \frac{L}{R}. \tag{30}$$

In this calculation the angle of attack of the wings is taken constant, i. e., a constant position of the rudder is assumed*. The trajectory's inclination angle can then be such that the flight will be sloping most gently when the wing drag and the parasitic drag are approximately equal. Such a flight is possible at velocities lower than 8 km/sec, since at this velocity the centrifugal force is equal to the airplane's weight and a positive angle of attack would lead to ascent and not to descent of the rocket plane.

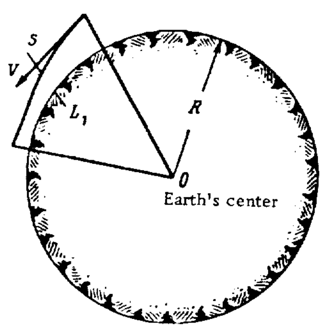

FIGURE 7

At high velocities flight "on the back" is possible, descending to a certain altitude. In this case the wing lift will be negative, i. e., $Y<0$. In order not to get buried in the atmosphere in this case and not to fall into too low layers, a mechanical lift limiter must be used. For example, the wings should automatically let through air when the lift becomes larger than a certain value Y_1, a multiple of the ship's weight. This is required also because of possible wing failure.

Gliding descent "on the back" may be required if in approaching the Earth the spaceship goes too far from the Earth's center and must drop from the highest layers of the terrestrial atmosphere to the lower ones, where it can fly "on the back" at a constant altitude, decelerating slowly for the landing.

* The assumption that the angle of inclination of the altitude rudder has to be constant in order to keep the vehicle balanced at a constant angle of attack is true only if the dependence of the aerodynamic coefficients on the flight Mach number is disregarded. – Editor's note.

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For flight velocities lower than 8 km/sec such lift limitations are also required but in this case only to prevent wing failure. As can be easily seen, the rocket plane in its usual attitude remains at almost constant altitude. If we give it too large an angle of attack α, it rises slightly higher, but because it reaches more rarefied air, further ascent is stopped. If the angle of attack is chosen too small, the rocket plane falls into denser layers of the atmosphere and a balance is again rapidly achieved.

When flying "on the back" the opposite situation prevails. If the angle of attack α is too large, the rocket plane plunges into the lower, denser layers of the atmosphere where the force Y increases even more and, without the limitation due to wing lift, the rocket plane may crash to the Earth. The centrifugal force increases indeed due to the larger curvature of the trajectory, but a twofold reduction of the trajectory's radius of curvature corresponds at a velocity of 11 km/sec to a doubling of Y. Since the atmospheric density varies sharply, Y grows faster; this means that the descent will be unstable. If variable surface wings are used, e. g., laminated wings, or if the rudders are reset at large values of Y, the force Y can be kept small and the vehicle is stabilized to some extent. The radius of curvature r' of the trajectory, corresponding to a given Y_{max}, can be calculated by the formula

$$Y_{max}=\frac{G}{g}\frac{V^2}{r'}-G\cos\theta,$$

hence

$$r'=\frac{GV^2}{g\,(Y_{max}-G\cos\theta)}. \quad (31)$$

Instead of achieving stability in descent by the action of the wings, it is also possible to place a special surface, on which the air pressure acts from one side and a spring* from the other side, perpendicular to the incident stream. When the air pressure varies, such a surface moves and may easily set the rudders so that the machine will be kept steady at a suitable height. If the vehicle enters the Earth's atmosphere from interplanetary space, these control surfaces can be operated when an atmospheric density normal for the given velocity is reached.

As long as the atmospheric density is smaller than the normal one for the given flight velocity, the angle of attack and also flight "on the back" or in the usual attitude should be chosen, depending on the shortest distance from the Earth's surface which the vehicle would pass without the action of wing lift.

We have not yet investigated the section of the descent from interplanetary space to that height at which, due to the action of the aerodynamic forces, the flight trajectory's radius of curvature is approximately equal to the Earth's radius. This section extends into the high layers of the atmosphere. Although in this section of the landing trajectory we can fly in a great number of trajectories, controlling the flight by inclination of the rudders so that a convenient landing will be achieved, we have first to study free landing. Here an additional lift would reduce the danger of falling to

* Due to the strong aerodynamic heating all surfaces of the landing device will have a high temperature, and therefore a metallic spring will not function. In this case a gaseous cushion or any other similar device can be used as a spring. – Editor's note.

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the Earth and an additional negative lift would give a closer approach to the Earth if we fly "on the back" because of too high an altitude.

2. BALLISTIC LANDING OF A SPACESHIP ON EARTH

Ballistic landing taking into account the curvature of the Earth's surface

The drag of the wing sections (with zero lift), as well as that of all other external surfaces of the rocket airplane, decelerate it, and the gravitational force $G \sin \theta$, accelerates it, so that we have

$$-\frac{G}{g}\frac{dV}{dt}=c_{x\,\text{pal}}F_M\gamma V^2-G\sin\alpha, \qquad (32)$$

where $c_{x\,\text{pal}}F_M\gamma V^2$ is the parasitic drag of the wing sections and all other parts taken together. Furthermore, for a small angle of inclination of the trajectory to the horizon, the centrifugal force is approximately equal to the vehicle's weight. In the general case we have

$$C=\frac{G}{g}\frac{V^2}{r'}=G\cos\theta,$$

or

$$V^2=gr'\cos\theta. \qquad (33)$$

There are great theoretical difficulties in making an exact calculation. A graphical solution, however, can give us the flight trajectory approximately and also all the interesting quantities. Given the initial velocity V_0 and the initial inclination angle θ_0 of the trajectory (Figure 8), we determine the initial radius of curvature r_0' by (33).

Knowing the dependence of the atmospheric density on the altitude over the Earth's surface, we determine the deceleration dV_0/dt from (32).

For motion along an arc of a circle, the velocity of a spaceship after a time interval Δt, and also the path traversed, are given by the formulas

$$V_1= V_0+\frac{dV_0}{dt}\,\Delta t;$$

$$\Delta s=V_0\,\Delta t=r'\Delta\tau.$$

For the point A_1 reached we determine from Figure 8 the height h_1 over the Earth's surface, we calculate the corresponding density γ_1, and the new angle of inclination θ_1 of the trajectory. From (33) we calculate the new radius of curvature

$$r_1'\frac{V_1^2}{g\cos\theta_1},$$

and also $\Delta V_1/\Delta t$ from (32). We repeat this calculation until we reach the desired final height h_k.

We proceed similarly in other cases. If, for example, the height at which the landing should pass into a constant altitude flight is given, then

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h_k and $\theta_k=0$ are given. If in this case the velocity V_0 is given instead of V_k, we must construct a series of curves. Choosing an arbitrary V_k, we determine r'_k by (33), $-dV_k/dt$ by (32), and then construct the flight trajectory in the reverse order.

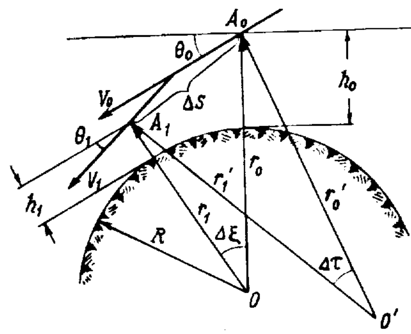

FIGURE 8

Ballistic landing neglecting the curvature of the Earth's surface

Considering the accelerations in the vertical and horizontal directions (Figure 9), we obtain the following equations of motion*:

$$\frac{G}{g}\frac{d^2x}{dt^2}=-Q\cos\theta; \tag{34}$$

$$\frac{G}{g}\frac{d^2h}{dt^2}=Q\sin\theta-G. \tag{35}$$

Since

$$Q=c_{x\,\text{pa}1}F_M\gamma V^2, \tag{36}$$

then

$$-\frac{d^2x}{dt^2}=\frac{gc_{x\,\text{pa}1}F_M\gamma V^2}{G}\cos\theta,$$

or

$$-\frac{d^2x}{dt^2}=k_1\gamma V^2\cos\theta, \tag{34a}$$

where

$$k_1=\frac{gc_{x\text{pa}}F_M}{G}. \tag{37}$$

* The author disregards the centrifugal forces. – Editor's note.

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For the horizontal and vertical components of the accelerations dV/dt and V^2/r' we obtain

$$\frac{dV}{dt}\cos\theta-\frac{V^2}{r'}\sin\theta=\frac{d^2x}{dt^2}; \qquad (38)$$

$$\frac{dV}{dt}\sin\theta+\frac{V^2}{r'}\cos\theta=-\frac{d^2h}{dt^2}. \qquad (39)$$

If the velocity V is not too large or the angle θ is not too small, we can

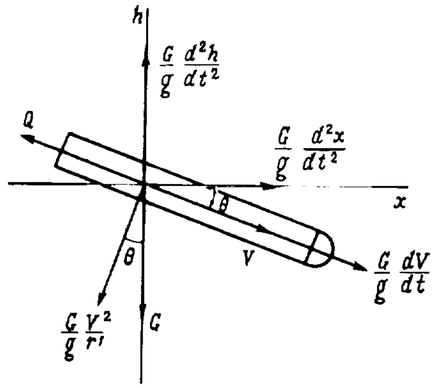

FIGURE 9

assume that $\frac{V^2}{r'}\sin\theta\approx 0$. In this case equation (38) assumes the form

$$\frac{dV}{dt}\cos\theta=\frac{d^2x}{dt^2}. \qquad (38a)$$

If next we neglect the quantity $Q\sin\theta$ in equation (35), we obtain

$$\frac{d^2h}{dt^2}=-g;$$

and hence

$$h=-\frac{gt^2}{2}+C_1t+C_2.$$

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CALCULATIONS OF SPACESHIP FLIGHT IN THE EARTH'S ATMOSPHERE (ASCENT)*

Let us use the following notation:

P, force produced by propellers or rocket engine in the direction of vehicle axis which coincides with flight direction;

α, angle of inclination of vehicle axis with respect to horizon;

$G=Mg_0$, spaceship's weight at Earth's surface;

M, spaceship's mass;

g_0, gravitational acceleration at Earth's surface;

V, flight velocity;

R/A, ratio of drag of airplane's lifting surfaces in flight direction to their lift perpendicular to flight direction;

F_w, area of plate perpendicular to flight direction, having a drag equivalent to that of all parts of the vehicle exclusive of lifting surfaces;

k, drag coefficient per unit area of a plate perpendicular to stream, for unit air density and unit velocity;

γ, air density at altitude h above Earth's surface;

W, vehicle's total drag;

Q, lift of lifting surfaces perpendicular to flight direction.

Further, let ξ be a coefficient defined by the formula:

$$\xi=\frac{\gamma V^2}{Q}. \tag{1}$$

Then the total drag will be equal to

$$W=Q\frac{R}{A}+kF_W\gamma V^2=\left(\frac{R}{A}+kF_W\xi\right)Q, \tag{2}$$

where $Q\frac{R}{A}$ is the drag of the wing sections and $kF_W\gamma V^2$ is the drag of the plate F_W.

Neglecting the centrifugal force due to the flight trajectory's curvature we can also write

$$Q=G\cos\alpha. \tag{3}$$

The propellers thrust P should overcome the drag W, and the component of the ship's weight $G\sin\alpha$, and should supply in addition an accelerating force

* A manuscript dealing with the problem of ascent, "Calculations of Space Vehicle Flight in the Atmosphere", was discovered in Tsander's archives. The calculation is dated 1924. The end of this manuscript has not been found, and the editors, therefore, undertook its completion, enclosing the end in brackets. – Editor's note.

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$\frac{G}{g_0}\frac{dV}{dt}$, i.e.:

$$P = W + G\sin\alpha + \frac{G}{g_0}\frac{dV}{dt} =$$

$$= \left(\frac{R}{A} + kF_w\xi\right) G\cos\alpha + G\sin\alpha + \frac{G}{g_0}\frac{dV}{dt}. \tag{4}$$

Hence, we find the useful power of the engines or of the rocket

$$N = PV = \left(\frac{R}{A} + kF_w\xi\right) GV\cos\alpha + GV\sin\alpha + \frac{G}{g_0}\frac{V\,dV}{dt}. \tag{5}$$

For simplicity it has been assumed in this calculation that the gravitational acceleration is constant within the atmosphere. In reality it varies with the altitude h as

$$g = g_0\frac{R^2}{(R+h)^2},$$

where $R = 6370$ km is the Earth's radius.

Therefore, for $h = 100$ km we have

$$g = g_0\frac{1}{\left(1 + \frac{100}{6370}\right)^2} = 0.97 g_0,$$

and the ship's acceleration will end less than 100 km above the Earth's surface.

For the rocket thrust we have

$$P = V_2\frac{B}{g_0} = -\frac{V_2}{g_0}\frac{dG}{dt}, \tag{6}$$

where V_2 is the relative exhaust velocity of the gases from the rocket, and B is the propellant consumption per unit time.

The rate of propellant consumption B is equal to the rate of decrease in the ship's weight $-\frac{dG}{dt}$.

Substituting (6) in (4) we obtain

$$\frac{dV}{dt} = \frac{-g_0}{G}\frac{1}{g_0}\frac{dG}{dt}V_2 - \left(\frac{R}{A} + kF_w\xi\right)\frac{G\cos\alpha}{G}g_0 - G\sin\alpha\frac{g_0}{G},$$

or

$$\frac{dV}{dt} = -\frac{dG}{G}\frac{V_2}{dt} - \left(\frac{R}{A} + kF_w\xi\right)g_0\cos\alpha - g_0\sin\alpha. \tag{7}$$

It is known that when flying in conditions of maximum lift to drag ratio, the wing sections' drag and the parasitic drag of the vehicle are approximately equal. This takes place if

$$\frac{R}{A} = kF_w\xi. \tag{8}$$

Introducing the notation

$$\operatorname{tg}\beta = \frac{R}{A} + kF_w\xi, \tag{9}$$

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and assuming the most favorable conditions:

$$\operatorname{tg}\beta = 2\frac{R}{A}. \tag{10}$$

we have

$$\frac{dV}{dt} = -\frac{d\ln\frac{G}{G_0}}{dt}V_2 - g_0(\operatorname{tg}\beta\cos\alpha + \sin\alpha),$$

or

$$\frac{dV}{dt} = -\frac{d\ln\frac{G}{G_0}}{dt}V_2 - g_0\frac{\sin(\alpha+\beta)}{\cos\beta}. \tag{11}$$

This equation can be integrated for constant V_2 if the dependence of the angle α on time is known.

In the general case we have

$$V - V_0 = V_2\ln\frac{G_0}{G} - g_0\int_{t_0}^{t}\frac{\sin(\alpha+\beta)}{\cos\beta}\,dt, \tag{12}$$

and if $\alpha = \text{const}$, and $\beta = \text{const}$, then

$$V - V_0 = V_2\ln\frac{G_0}{G} - g_0(t - t_0)\frac{\sin(\alpha+\beta)}{\cos\beta}. \tag{13}$$

It follows therefore that the shorter the acceleration time, the higher the velocity V attained for a given weight ratio G_0/G (i.e., propellant consumption) and for given V_2, α and β.

A constant rate of propellant consumption B gives also a constant thrust $P = V_2\frac{B}{g_0}$. We then have from the relation $\frac{dG}{dt} = -B$

$$G_0 - G = B(t - t_0). \tag{14}$$

Substituting this expression in (13) we obtain

$$V - V_0 = V_2\ln\frac{G_0}{G} - g_0\frac{G_0 - G}{B}\frac{\sin(\alpha+\beta)}{\cos\beta} = V_2\left[\ln\frac{G_0}{G} - \frac{G_0 - G}{P}\frac{\sin(\alpha+\beta)}{\cos\beta}\right]. \tag{15}$$

It follows from (12) that if the angle of ascent α is increased, the rocket's thrust must also increase, because in equation (4) the term $G\sin\alpha$ increases, and because the acceleration time should be made much shorter in order that the second term in equation (12) should remain small. At the same time it can be seen from (1) and (2) that the drag also varies.

It is possible to find the maximum angle α which would give $V - V_0 = 0$. From equation (15) we obtain

$$\sin(\alpha_{max} + \beta) = \frac{\ln\frac{G_0}{G}P\cos\beta}{G_0 - G}. \tag{16}$$

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In order that at a given moment of the flight the velocity should not decrease, we determine α_{max} from (7), (11) and (6) for $dV/dt = 0$, i.e., for the case of constant flight velocity. We have

$$-\frac{dG}{G}\frac{V_2}{dt} - \operatorname{tg}\beta g_0 \cos\alpha_{max} - g_0 \sin\alpha_{max} = 0,$$

or

$$g_0 \frac{\sin(\alpha_{max}+\beta)}{\cos\beta} = -\frac{V_2}{G}\frac{dG}{dt} = \frac{Pg_0}{G},$$

or

$$\sin(\alpha_{max}+\beta) = \frac{P}{G}\cos\beta. \qquad (17)$$

If we fly at an angle α for which

$$\sin(\alpha+\beta)/\sin(\alpha_{max}+\beta) = \varepsilon, \qquad (18)$$

then for $\varepsilon<1$, the ship will be accelerating all the time.

As an example, let us take the vehicle's total weight at the moment the rocket starts as $G_0 = 500$ kg (it is then assumed that 19 % of the vehicle's total weight has already been consumed by the engines), the rocket thrust $P = 1500$ kg, $R/A = 0.0624$. We obtain from (10):

$$\operatorname{tg}\beta = 0.1248;\ \beta = 7°08';\ \cos\beta = \cos 7°08' = 0.992$$

Equation (12) can now be easily integrated for constant P and B. Taking $t_0 = 0$ at the start of the rocket flight we obtain

$$G = G_0 - Bt. \qquad (19)$$

For the integral on the right-hand side of (12) we have

$$g_0\int_0^t \frac{\sin(\alpha+\beta)}{\cos\beta}dt = g_0\varepsilon\int_0^t \frac{\sin(\alpha_{max}+\beta)}{\cos\beta}dt = \varepsilon g_0\int_0^t \frac{P}{G}dt =$$

$$= g_0\varepsilon P\int_0^t \frac{dt}{G_0-Bt} = \varepsilon Pg_0\int_0^t -\frac{1}{B}\frac{d(G_0-Bt)}{G_0-Bt} = -\frac{\varepsilon Pg_0}{B}\ln\frac{G_0-Bt}{G_0}$$

and the equation takes the form

$$V - V_0 = V_2\ln\frac{G_0}{G} + \varepsilon\frac{P}{B}g_0\ln\frac{G_0-Bt}{G_0} = V_2\left(\ln\frac{G_0}{G} + \varepsilon\ln\frac{G_0-Bt}{G_0}\right) =$$

$$= V_2\ln\left(\frac{G_0}{G}\frac{(G_0-Bt)^{\varepsilon}}{G_0^{\varepsilon}}\right) = V_2\ln\frac{G_0^{1-\varepsilon}}{G^{1-\varepsilon}} = V_2(1-\varepsilon)\ln\frac{G_0}{G},$$

or

$$V - V_0 = V_2(1-\varepsilon)\ln\frac{G_0}{G}. \qquad (20)$$

In order to make $\varepsilon = \text{const}$ we could choose a flight angle α for which

$$\frac{\sin\alpha}{\sin\alpha_{max}} = \frac{1}{n},$$

with $n>1$.

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It is also possible to obtain a solution by finding α_{max} as a function of t from equations (17) and (19), and then integrating graphically the second term on the right-hand side of equation (12). However, formula (20) gives the result quicker. In order to have $\alpha>0$ always, ε must be positive at the beginning of the rocket's flight; then G has a maximum and the angle α therefore a minimum.

From equations (17) and (18) we have

$$\sin(\alpha+\beta)=\varepsilon\frac{P\cos\beta}{G}, \tag{21}$$

and taking $\alpha=0$ at the beginning of the flight we obtain

$$\varepsilon_{min}=\frac{G_0 \operatorname{tg}\beta}{P}, \tag{22}$$

which is the minimum admissible value of ε. In our example

$$\varepsilon_{min}=\frac{5000\cdot 0.1248}{1500}=0.416.$$

Let us take $V_2 = 4130$ m/sec for hydrogen plus oxygen and assume that the rocket starts at a velocity $V_0 = 0.400$ km/sec which increases to $V = 0.8$ km/sec (see equation 20). We then have

$$\lg\frac{G_0}{G}=\frac{V-V_0}{2.303V_2(1-\varepsilon)}=\frac{8.0-0.4}{2.303\cdot 4.13(1-\varepsilon)}=\frac{0.799}{1-\varepsilon}.$$

For $\varepsilon = 0$ we find $G_0/G = 6.30$ and for $\varepsilon = 0.5$ we have $G_0/G = 6.30^2 = 39.5$. This unfavorable ratio was obtained because we took $\varepsilon = 0.5$ for the entire flight, whereas ε can in fact be reduced with the decrease in weight G.

If we take $G_0\varepsilon/G=\overline{V}=\text{const}$ (and not $\varepsilon=\text{const}$) then we obtain for the integral on the right-hand side of equation (12)

$$g_0\int_{t_0}^{t}\frac{\sin(\alpha+\beta)}{\cos\beta}\,dt=g_0\int_{t_0}^{t}\varepsilon\,\frac{\sin(\alpha_{max}+\beta)}{\cos\beta}\,dt=$$

$$=\frac{g_0\overline{V}}{G_0}\int_{t_0}^{t}G\,\frac{P}{G}\,dt=\frac{g_0\overline{V}P}{G_0}(t-t_0),$$

and if $t_0 = 0$, then

$$V-V_0=V_2\ln\frac{G_0}{G}-\frac{g_0\overline{V}P}{G_0}t=V_2\ln\frac{G_0}{G}-$$

$$-\frac{g_0\overline{V}P}{G_0}\,\frac{G_0-G}{B}=V_2\left[\ln\frac{G_0}{G}-\overline{V}\frac{(G_0-G)}{G_0}\right],$$

or

$$V-V_0=V_2\left[\ln\frac{G_0}{G}-\overline{V}\left(1-\frac{G}{G_0}\right)\right]. \tag{23}$$

In our case $\overline{V}_{min}=\varepsilon_{min} = 0.416$. Taking $\overline{V}_{min} = 0.5$, and omitting in an approximate calculation, the last term of equation (12), i.e., the small

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quantity $\frac{G}{G_0}\overline{V}$, we obtain

$$\frac{V - V_0 + \overline{V}V_2}{V_2} \approx \ln\frac{G_0}{G}, \tag{24}$$

or

$$\lg\frac{G_0}{G} = \frac{1}{2.303\cdot 4.13}(8.0 - 0.4 + 0.5\cdot 4.13) = 1.018,$$

i. e.,

$$\frac{G_0}{G} = 10.43. \tag{25}$$

For a velocity $V = 11.3$ km/sec we find

$$\lg\frac{G_0}{G} = \frac{1}{2.303\cdot 4.13}(11.3 - 0.4 + 0.5\cdot 4.13) = 1.365;$$

$$\frac{G_0}{G} = 23.2.$$

In the limit, for $\overline{V} = \overline{V}_{min} = 0.416$, we would have $\frac{G_0}{G} = 21.4$, and without any losses

$$\lg\frac{G_0}{G} = \frac{10.9}{2.303\cdot 4.13} = 1.146; \quad \frac{G_0}{G} = 14.0.$$

(*This formula is valid as long as $\sin(\alpha+\beta) \leqslant 1$, i. e., as long as (see (17) and (18)) $\frac{P}{G}\cos\beta \leqslant 1$, or $G \geqslant \varepsilon P\cos\beta$. In our example this would take place for

$$\frac{G}{G_0} = \frac{\varepsilon P\cos\beta}{G_0} = \frac{0.5\cdot 1500\cdot 0.992}{5000} = \frac{1}{6.72}.)$$

If we write equation (15) in the form:

$$\frac{V - V_0}{V_2} = \ln\frac{G_0}{G} - \frac{G_0 - G}{P}(\mathrm{tg}\,\beta\cos\alpha + \sin\alpha), \tag{26}$$

then we can calculate the flight time average of the angle α_{max} for which $V - V_0 = 0$. If we assume $\cos\alpha \approx 1$, then for large α, corresponding to small G, we overestimate somewhat the air friction. Then

$$\sin\alpha'_{max} = \frac{\ln\frac{G_0}{G} - \frac{G_0 - G}{P}\,\mathrm{tg}\,\beta}{\frac{G_0 - G}{P}}. \tag{27}$$

If we fly at an angle α_1 for which $\sin\alpha/\sin\alpha'_{max} = 1/n$, then we obtain from equation (26)

$$\frac{V - V_0}{V_2} \approx \ln\frac{G_0}{G} - \frac{G_0 - G}{P}\,\mathrm{tg}\,\beta - \frac{G_0 - G}{P}\sin\alpha,$$

(since $\cos\alpha \approx 1$), hence

$$\frac{V - V_0}{V_2} = \frac{G_0 - G}{P}\left(\sin\alpha'_{max} - \frac{\sin\alpha'_{max}}{n}\right).$$

* If we neglect the altitude and velocity already attained with the aid of the engine ($h = 28$ km, $V_0 = 400$ m/sec).

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or

$$\frac{V - V_0}{V_2} = \frac{G_0 - G}{P} \frac{n-1}{n} \sin \alpha'_{max}. \tag{28}$$

If G_0, β, P, V_2, V_0 and n are given, it is simple to calculate from equations (27) and (28) the velocity V and the angle α as functions of G. The larger the decrease in weight G_0-G, the larger is the angle α.

It is then possible to determine by (14) the time that has elapsed since the beginning of the rocket's operation:

$$t = \frac{G_0 - G}{B}. \tag{29}$$

We can now determine the altitude suitable for flight with velocity V. To do this we use formula (1):

$$\xi = \frac{\gamma V^2}{Q} = \frac{\gamma V^2}{G \cos \alpha} \approx \frac{\gamma V^2}{G}.$$

If we denote by F the area of the wings' supporting surface, we can write for the lift

$$Q = G_0 \cos \alpha = \frac{A}{\gamma_0} \gamma F V^2. \tag{30}$$

where A is the lift of unit wing surface for unit flight velocity; $Q_0 = AFV_0^2$, $\xi_0 = \frac{\gamma_0 V_0^2}{Q_0} = \frac{\gamma_0}{AF}$, are the values of Q and ξ at the Earth's surface.

The parasitic drag of all the sections of the vehicle is equal to

$$kF_W \gamma V^2 = kF_W \xi Q = kQ \frac{\gamma_0}{A} \frac{F_W}{F},$$

and from formula (2) we obtain the total drag

$$W = Q\left(\frac{R}{A} + k \frac{\gamma_0}{A} \frac{F_W}{F}\right). \tag{31}$$

(Here Tsander's manuscript is interrupted.)

(The altitude suitable for flight with velocity V can be calculated if we remember that

$$\xi = \frac{\gamma_0}{AF} \approx \frac{\gamma V^2}{G}.$$

The quantity γ/γ_0, which characterizes the flight altitude, can be expressed as:

$$\frac{\gamma}{\gamma_0} = \frac{G}{AFV^2}.$$

The flight velocity is determined from (28)

$$V = V_2 \frac{G_0 - G}{P} \frac{n-1}{n} \sin \alpha'_{max} + V_0$$

and then

$$\frac{\gamma}{\gamma_0} = \frac{G}{AF\left[V^2 \frac{G_0 - G}{P} \frac{n-1}{n} \sin \alpha'_{max} + V_0\right]^2}.$$

Thus, the altitude is a function of the rocket's weight.)

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SPACESHIP'S TEMPERATURE IN GLIDE LANDING ON EARTH

Tsander devoted a series of theoretical works to problems of the return of spaceships to Earth. He showed that in order to realize interplanetary flight with subsequent return to Earth one must solve a series of complex scientific-technical problems. One of them is to protect the vehicle's body against aerodynamic heating when it moves through the atmosphere at high supersonic velocities. This is dealt with in the present article.

Some of his estimates of the body temperatures and of the weight of the vehicle's thermal protection in gliding descent, which were obtained on a concrete example, are close to the values obtained by modern heat exchange calculations. It should be noted that problems of spaceship thermal protection during landing are mostly unsolved even now and require many theoretical and experimental investigations of heat exchange at supersonic velocities. The studies conducted by Tsander already in 1925, on the thermal conditions of spaceships, are therefore still of great interest. The article was prepared for print by B. P. Plotnikov, Candidate of Technical Sciences.

Editor

A vehicle entering the Earth's atmosphere from interplanetary space possesses a large amount of kinetic energy which it must lose by deceleration in the atmosphere. If the re-entry velocity is 11.2 km/sec we must, in order to make a gliding descent possible, create a negative lift by an appropriate angle of attack. This is because at this velocity the centrifugal force C_1 which appears in orbiting the Earth is twice the vehicle's weight. At $V = 8$ km/sec, $C_8 = G$, and since $11.2/8 = \sqrt{2}$ and $C = V^2/R$ (R being the Earth's radius), then at $V = 11.2$ km/sec we obtain

$$C_{11.2} = C_8 (\sqrt{2})^2 = 2C_8 = 2G.$$

The airplane has to be subjected to a force $C_{11.2} - G = G$ in the direction of the Earth. Only because of the large drag coefficient at high velocities can one use a somewhat smaller angle of attack, thus reducing the total drag.

Let us assume a total drag W at a velocity of 11.2 km/sec equal to $1/n$ of the vehicle's weight, i.e., $W = G/n$ (with $n \approx 6$)*.

The work performed by the drag in unit time is equal to

$$L = WV = \frac{GV}{n}.$$

When the velocity of the spaceship decreases to 8 km/sec the drag drops since at this velocity the ship can orbit freely and the wings' angle of attack

* The coefficient $n = 6$ was probably chosen by the author on the basis of data on airplane drag existing at the time.

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can be reduced to zero. Therefore at V_{max} the power L and the rate of temperature increase will be maxima.

Let us find the temperature which the spaceship would reach for constant total power. The actual heating will be smaller due to the gradual decrease in power brought about by vehicle deceleration. The power L corresponds to a thermal energy AL, where $A = 1/427$ kcal/kg is the thermal equivalent of work. At high altitudes the air is quite rarefied and the heat transferred from the air to the vehicle can be neglected. The amount of heat Q which is radiated in unit time (in 1 hour) is equal to

$$Q = cF(T^4 - T_0^4),$$

where F is the area of the spaceship's heat radiating surface in m^2;
T, the temperature of the spaceship's heated surface;
T_0, the temperature of the surrounding medium;
c, the radiation coefficient for a perfect black body, where

$$c = 4.40 \cdot 10^{-8} \text{ kcal/m}^2 \text{ hour}$$

(we can take $c \approx 4 \cdot 10^{-8}$).

Assuming that the vehicle's surface is in thermal equilibrium, we can write the equation of heat balance

$$Q = \varphi AL = \varphi \frac{AGV}{n} = cF(T^4 - T_0^4),$$

where φ is some coefficient.

$\varphi < 1$ since a part of the energy L is spent in creating vortices in the air*.

G should be expressed in kg, and V in m/hr.

Let us take as an example $G = 1000$ kg, a wing surface (sum of the upper and lower sides) equal to $2 \times 40 = 80$ m^2, and a body surface of 94.3 m^2 (assuming a length of 15 m and an average diameter of 2 m). The total surface is equal to

$$F_1 = 80 + 94.3 \approx 170 \text{ m}^2.$$

If on the average $\frac{1}{m}$ of this area will be heated, then

$$F = \frac{F_1}{m}.$$

We then obtain

$$T^4 = T_0^4 + \varphi \frac{AGVm}{ncF_1},$$

and taking for T_0 its probable upper limit of $T_0 = 273$ °K, we obtain

$$T^4 = 273^4 + \frac{1000 \cdot 11.2 \cdot 10^3 m\varphi 10^8 \cdot 3.6 \cdot 10^3}{6 \cdot 427 \cdot 4.0 \cdot 170} = 55.8 \cdot 10^8 + 23\,100 \cdot 10^8 m\varphi.$$

If we take $\varphi = \frac{1}{70}$ as the most probable value (this value was found for meteors which enter the terrestrial atmosphere at high velocity)**, and

* The coefficient φ gives the part of the kinetic energy which is transformed into heat and reaches the vehicle's surface.

** At the time of Tsander's work on this article there existed no fundamental research on the heat exchange of bodies moving with high supersonic velocities in the Earth's atmosphere, on which the true value of the coefficient φ could be based. Research conducted recently on heat exchange in supersonic flows has shown that the value $\varphi = 1/70$ adopted by Tsander is close to the actual value.

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$m \approx 20$, i.e., that only the leading sections of the body and of the wings are heated, then

$$\frac{T^4}{10^8} = 55.8 + 23\,100 \frac{20}{70} \approx 6656; \quad T = 902^\circ \text{ K};$$

$$t = 902 - 273 = 629^\circ \text{ C}.$$

Even if all the braking energy was given to 1/20 of the ship's surface, we would have $\varphi = 1$, $m = 20$ and

$$\frac{T^4}{10^8} = 55.8 + 23\,100 \cdot 20 = 462\,056;$$

$$T = 2610^\circ \text{ K}; \; t = 2610 - 273 = 2337^\circ \text{ C}.$$

If we build the moving surface in such a way that all its sections are uniformly heated, we can take $\varphi = \frac{1}{70}$ and $m = 1$, i.e.,

$$\frac{T^4}{10^8} = 55.8 + \frac{23\,100}{70} = 386;$$

$$T = 444^\circ \text{ K}; \; t = 444 - 273 = 171^\circ \text{ C}.$$

We can also remove all the heat absorbed by the spaceship by water which is converted into steam. If we take the water in the form of ice, then 1 kg of water under a pressure of 1 atm can remove $q = 80 + 637 = 717$ kcal/kg.

The entire energy which the spaceship uses is equal to $E = GV^2/2g$, and the heat which has to be removed is equal to

$$R = \varphi EA = \varphi \frac{V^2 AG}{2g}.$$

Therefore, the weight of water G_w which is required to remove all the heat is equal to

$$G_w = \frac{R}{q} = \frac{\varphi V^2 AG}{2gq},$$

and, therefore,

$$\frac{G_w}{G} = \varphi \frac{V^2 A}{2gq}.$$

Substituting the numerical values $V = 11.2$ km/sec, $\varphi = \frac{1}{70}$, $q = 717$ kcal/kg, and $g = 9.81$ m/sec , we obtain

$$\frac{G_w}{G} = \frac{11.2^2 \cdot 10^6}{70 \cdot 427 \cdot 2 \cdot 7.81 \cdot 717} = 0.299,$$

i.e., the water constitutes 30% of the vehicle's weight.

We now have the question of what should be the area of the heated surface needed to evaporate the amount of water required to cool the ship. Reference books give the amount of coal burned per m² of boiler heating surface and the plant efficiency; from these it is simple to determine the number of calories transferred in 1 hour from 1 m² of surface to the water. It turns out that for passenger locomotives, the latter figure is approximately 37,000 cal/hour m² or $\frac{37{,}000}{3600} = 10.3$ cal/sec m².

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The maximum rate of heat removal is

$$Q = \frac{\varphi A G V}{n} \text{ cal/sec or in our example:}$$

$$Q = -\frac{1000 \cdot 11.2 \cdot 10^3}{70 \cdot 427 \cdot 6} = 62.5 \text{ cal/sec;}$$

which means that only $625/10.3 = 6.06 \sim 6$ m^2 of surface are required to remove the amount of heat which arrives at the vehicle's surface. It can be shown that the temperature of the vehicle's external wall will not be too high.

Taking, for example, the wall thickness of the copper water cooling jacket equal to 2 mm (copper's coefficient of thermal conductivity λ is 300 cal/m hour°C), and assuming that the water is boiling all the time, we can take its heat transfer coefficient as

$$\alpha = 10\,000 \text{ cal/m}^2 \text{hour °C}.$$

We then obtain for the total heat transfer coefficient

$$k = \frac{1}{\frac{1}{\alpha} + \frac{\delta}{\lambda}} = \frac{1}{\frac{1}{10\,000} + \frac{2}{300 \cdot 1000}} = 9390 \text{ cal/m}^2 \text{hour °C}.$$

Taking the cooling water's temperature t as 100°C, the formula $Q = kF(t_k - t)$ gives for the cooling jacket's external wall a temperature

$$t_k = t + \frac{Q}{kF} = 100 + \frac{62.5 \cdot 3600}{9390F} = 100 + \frac{24}{F}.$$

If $F = 6.06$ m^2, we obtain $t = 100 + \frac{24}{6.06} = 104$°C, and for $F = 1$ m^2, we obtain $t = 124$°C.

We see that it is very easy to cool a spaceship during landing, even if only the leading edges, i.e., only a small surface receives the heat. The temperature of the cooled walls will not be high.

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DEFLECTION AND REPULSION OF METEORS BY ELECTROSTATIC CHARGES EMITTED BY THE SPACESHIP*

In the present article we shall investigate the problem of protecting a spaceship from collisions with meteors during its travel in interplanetary space. The following method is proposed. The spaceship is connected to a negatively charged sphere built of very thin metallic sheets. The spaceship itself can be placed either inside the sphere or behind it, so that the meteors cannot reach it. A beam of electrons, for example, in the form of cathode rays, is emitted towards the meteors. The electrons charge the meteors which are repelled from the charged sphere by electrostatic forces. As a result the number of meteor hits on the spaceship may become zero. The smaller the meteors, the easier it will be to hold them back. Protection against meteor dust is the easiest task.

Let us use the following notation:

- m, meteor mass;
- V_∞, meteor velocity at an infinitely large distance from spaceship;
- R, meteor distance from spaceship at a given moment;
- R_0', radius of sphere attached to spaceship;
- K and K_0, forces with which meteor is repelled from spaceship at distances R and R_0;
- μ, μ_1, amounts of charge situated at a given moment on meteor and on spaceship;
- μ_0, amount of charge on meteor when distance between it and spaceship is R_0;
- U, U_1, U_0, potentials of meteor and of spaceship corresponding to charges μ, μ_0 and μ_1;
- γ/g, meteor mass density;
- r, meteor radius;
- R_0, minimum distance between meteor and center of sphere attached to spaceship;
- ν, amount of charge emitted by spaceship in unit solid angle per sec;
- σ, solid angle into which charge is emitted.

We can write that the meteor's energy at infinity is equal to $E=\frac{mV_\infty^2}{2}$. If we assume that already at ∞ the meteor has a potential U_0, then the interaction force between the meteor and the spaceship at a distance R is equal to $K=K_0\frac{R_0^2}{R^2}$. If we also assume that the spaceship moves along the meteor's line of motion and is not noticeably slowed down by it, either due to its large

* The article was written in the period from 26 June to 6 July 1925. It was prepared for print by L. V. Letuchikh. – Editor's note.

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mass or due to the fact that it emits rays in other directions too and so is repelled also by other meteors, we can write for the potential work of the force K

$$E_1 = \int_{R_0}^{\infty} K\,dR = k_0 R_0^2 \int_{R_0}^{\infty} \frac{dR}{R^2} = K_0 R_0. \tag{1}$$

If the meteor's velocity relative to the sphere is zero and, in particular, when they touch each other, we have

$$E = E_1, \text{ i. e. } \frac{mV_\infty^2}{2} = K_0 R_0 \quad \text{or} \quad K_0 = \frac{mV_\infty^2}{2R_0}, \tag{2}$$

where V_∞ is the velocity of the meteor with respect to the spaceship.

By Coulomb's law we have

$$K_0 = \frac{\mu_1 \mu_0}{R_0^2}, \tag{3}$$

where $\mu_1 = R_0 U_1$ and $\mu_0 = rU_0$, R_0 and r being the capacities of the two spheres. We have assumed here that the meteor has a spherical shape. For other shapes these quantities are larger, but the amount of charge falling on a meteor of given mass per unit time will not differ considerably from the figure given above, since the projection of an object on a plane perpendicular to the cathode rays will have on the average approximately the same cross section as a sphere. For bodies of different shapes this will be examined later*.

If at its nearest position from the center of the spaceship's sphere the meteor touches the spaceship, and if their potentials are equal, i.e., $U_1 = U_0$, then

$$R_0' = R_0 - r;\ \mu_1 = (R_0 - r)U_1;\quad \mu_0 = 2V_1.$$

Substituting this in (3) we obtain

$$K_0 = \frac{(R_0 - r)\,rU_1^2}{R_0^2}.$$

or in (2)

$$K_0 = \frac{mV_\infty^2}{2R_0} = \frac{\left(1 - \frac{r}{R_0}\right) rU_1^2}{R_0}.$$

Hence

$$U_1^2 = V_\infty^2 \frac{m}{2r\left(1 - \frac{r}{R_0}\right)}.$$

The mass of a sphere is equal to $m = \frac{4}{3}\pi r^3 \frac{\gamma}{g}$ so that

$$U_1^2 = V_\infty^2 r^2 \frac{2\pi\gamma}{3g\left(1 - \frac{r}{R_0}\right)}. \tag{4}$$

* All Tsander's studies were for spherical meteors. Studies of different shapes of meteors are not included in the present article and have not yet been found in Tsander's archive.

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We see, therefore, that the potentials required for stopping the meteors at the surface of the spaceship's sphere are proportional to the meteor's velocity, to its radius, and to the square root of its density. If we take a meteor of radius $r = 1$ cm consisting of iron with specific weight $\gamma = 7.86$ kg/dm^3 and having a velocity V of 25 km/sec = $25\cdot 10^5$ cm/sec, relative to the spaceship, we obtain in the CGS system of units:

$$U_1 = U_0 = 300\cdot 25\cdot 10^5\cdot 1.0\sqrt{\frac{2\pi\cdot 7.86}{3\left(1-\frac{r}{R_0}\right)}},$$

or for $r/R_0 \approx 0$, we have $U_1 = U_0 = 3.04\cdot 10^9$ volts.

According to the theory of Maxwell and Exner, the potential of the Earth is equal to $1.9\cdot 10^9$ volts. This potential can stop meteors whose radius is equal to $r = 1.0\cdot 19/30.4 = 0.625$ cm, which corresponds to a weight of

$$mg = \frac{4}{3}\pi r^3\gamma = \frac{4}{3}\pi\cdot 0.625^3\cdot 7.86 \cong 8 \text{ g}.$$

Meteors whose trajectory does not pass through the center of the spaceship's sphere, will be deflected with greater ease.

If we take $r \ll R$, i.e., $\frac{r}{R} \approx 0$, then it follows from (4) that the potential required to stop meteors does not depend on the radius of the spaceship's sphere. If $R_0' \approx R_0$, then $K_0 \approx \frac{\mu_0 U_1}{R_0}$, i.e., the interaction force between the meteor and the sphere under contact is inversely proportional to the radius of the spaceship's sphere.

Meteors will not in general have the Earth's potential. This is demonstrated by the fall of meteoric interplanetary dust on the Earth. It can also be said that meteors possessing sharp edges and corners will lose their charge easily. Therefore, the spaceship should in general emit negative charges.

We can collect solar energy by large convex mirrors attached to the spaceship, using this heat to evaporate water in a steam boiler. With the steam we drive a steam engine and by it a dynamo feeding an instrument which emits cathode rays. In this way we can continuously throw large amounts of negative electricity into space. The principal energy carriers are cathode rays which contain high velocity electrons. Let us, therefore, examine in detail the electron ejection, and the process of charging and discharging of meteors.

An amount of electricity of $\nu d\sigma\, dt$ coulombs is ejected into a solid angle $d\sigma$ during time dt. At a distance R from the spaceship, a spherical meteor of radius r will be seen at an angle of $\frac{\pi r^2}{R^2} = \Delta\sigma$. During time dt an amount of electricity equal to $\nu\frac{\pi r^2}{R^2}dt$ falls on the meteor. At the moment considered the meteor contains a charge of $\mu = rU$ coulombs and the interaction force is equal to $K = \frac{\mu\mu_1}{R^2}$, where $\mu_1 = R_0' U_1$. From the laws of mechanics we have also $K = -m\frac{dV}{dt}$ where $V = -\frac{dR}{dt}$. We take the velocity of the meteor at a given moment as negative since we are considering an approach of the

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meteor to the spaceship. We obtain, therefore

$$K = \frac{\mu\mu_1}{R^2} = -m\frac{dV}{dt},$$

or

$$-\frac{dV}{dt} = \frac{\mu\mu_1}{mR^2} = \frac{\mu_1 r}{m}\frac{U}{R^2}. \tag{5}$$

On the other hand the charge increase on the meteor is equal to

$$d\mu = \nu\frac{\pi r^2}{R^2}dt,$$

hence

$$\frac{d\mu}{dt} = r\frac{dU}{dt} = \frac{\pi\nu r^2}{R^2}. \tag{6}$$

Introducing the constants

$$K_1 = \frac{\mu_1 r}{m} \text{ and } K_2 = \pi\nu r,$$

we obtain two differential equations

$$-\frac{dV}{dt} = K_1\frac{U}{R^2}; \tag{5a}$$

$$\frac{dU}{dt} = \frac{K_2}{R^2}. \tag{6a}$$

Dividing them by one another we obtain

$$\frac{dV}{dU} = -\frac{K_1}{K_2}U,$$

or

$$dV = -\frac{K_1}{K_2}U\,dU,$$

hence

$$V - V_\infty = \frac{K_1}{2K_2}(U_\infty^2 - U^2),$$

or also

$$V = -\frac{K_1}{2K_2}U^2 + b_1, \tag{7}$$

where U_∞ is the potential which the meteor possessed at an infinitely large distance from the spaceship and $b_1 = \frac{K_1}{2K_2}U_\infty^2 + V$ is a constant.

If we substitute $V = -\frac{dR}{dt}$, equations (5a) and (7) become

$$\frac{d^2R}{dt^2} = K_1\frac{U}{R^2}, \quad \frac{dR}{dt} = \frac{K_1}{2K_2}U^2 - b_1.$$

Eliminating U from these equations we obtain

$$\frac{dR}{dt} - \frac{K_1}{2K_2}\left(\frac{d^2R}{dt^2}\right)^2\frac{R^4}{K_1^2} + b_1 = 0,$$

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or

$$R^2 \frac{d^2R}{dt^2} = \pm \sqrt{2K_1K_2} \sqrt{b_1 + \frac{dR'}{dt}}.$$

We introduce the constant

$$b_2 = \mp \sqrt{2K_1K_2} = \sqrt{\frac{\mu\pi\nu \cdot r}{m}}.$$

Since $m = \frac{4}{3}\pi r^3 \frac{\gamma}{g}$, we have

$$b_2 = \mp \sqrt{\frac{3g\mu\nu}{2r\gamma}}.$$

We now obtain

$$\frac{d^2R}{dt^2} = -\frac{b_2}{R^2}\sqrt{b_1 + \frac{dR}{dt}},$$

or by substituting

$$V = -\frac{dR}{dt} \text{and} \frac{dV}{dt} = -\frac{d^2R}{dt^2}$$

we obtain

$$\frac{dV}{dt} = \frac{+b_2}{R^2}\sqrt{b_1 - V}.$$

Since

$$\frac{dV}{dt} = \frac{dV}{dR}\frac{dR}{dt} = -\frac{V\,dV}{dR},$$

then

$$-\frac{V\,dV}{\sqrt{b_1 - V}} = \frac{b_2\,dR}{R^2};$$

or after integration

$$+b_2\left(\frac{1}{R} - \frac{1}{R_\infty}\right) = +\int\limits_{V_\infty}^{V} \frac{V\,dV}{\sqrt{b_1 - V}}. \tag{8b}$$

To calculate the integral on the right-hand side we introduce a new variable

$$z = b_1 - V. \tag{9}$$

Then $dz = -dV$, and the right-hand side of equation (8) takes the form

$$\int \frac{(b_1 - z)(-dz)}{\sqrt{z}} = -b_1 \int \frac{dz}{\sqrt{z}} +$$
$$+\int \sqrt{z}\,dz = (\sqrt{z_\infty} - \sqrt{z}) + \frac{2}{3}(z^{3/2} - z_\infty^{3/2}).$$

Substituting this in equation (8) and introducing the notation

$$b_3 = +\frac{b_2}{R_\infty} + 2b_1\sqrt{z_\infty} - \frac{2}{3}\sqrt{z_\infty^3},$$

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we obtain

$$\frac{b_2}{R} = -b_3 - 2b_1\sqrt{z} + \frac{2}{3}\sqrt{z^3},$$

or

$$R = \frac{-b_2}{-b_3 + 2b_1\sqrt{z} - \frac{2}{3}\sqrt{z^3}}.$$

Returning to the variable V (using (9)) we have finally

$$R = \frac{-b_2}{-b_3 + 2b_1(b_1 - V)^{1/2} - \frac{2}{3}(b - V)^{3/2}}.$$

This expression enables us to examine the behavior of a meteor in the spaceship's electrostatic field. In particular it enables us to determine the specific velocities V for each distance R between the meteor and the sphere in the electrostatic field characterized by the parameters μ_1 and ν.

If the meteors are stopped at the surface of the sphere, which means that the meteor's relative velocity V is 0 for $R = R_0$, the equation takes the form

$$R_0 = \frac{-b_2}{-b_3 + \frac{4}{3}b_1^{3/2}}.$$

By means of this expression, which represents a complex functional relation between the parameters R_0, μ_1, ν, r, U_∞, V_∞, we can determine the amount of electricity ν which should be ejected by the sphere in unit time in order to stop the meteors at the spacehip's surface.

To determine the actual electric parameters of the sphere which will provide a reliable protection of the spaceship against bombardment by meteors, it is necessary in future calculations to choose the most probable values for r, U_∞, V_∞.

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PROBLEMS OF SUPER-AVIATION AND IMMEDIATE OBJECTIVES OF SPACE RESEARCH*

The paper considers first the influence of pressure, temperature, density, and composition of the atmosphere on spaceflight. Methods for increasing the power of aeroengines at high altitudes are then described. The author's innovations consist in replacing compressor pumps by special thermal injectors, and in introducing high-pressure motors operating on liquid oxygen and fuel, as well as special motor-rocket combinations. Results of calculations of a jet engine having an axial thrust of 1500 kg are presented. Various jet engines, new working cycles and their conditions of operation, combustion of metal fuels and their advantages are discussed. Designs of rockets and airplanes utilizing solid fuel which constituted part of the structure before combustion, are presented. Results of calculations of the ascent of an airplane with a rocket, and of range curvature of rockets flying over the greater part of the globe are also given. The principal problems of super-aviation are listed and the projects which should be undertaken immediately in preparation for space travel are reviewed.

Let us summarize briefly the problems of super-aviation. The goal of super-aviation, i.e., the ascent to the upper atmospheric layers with temporary exit from the atmosphere, requires that a flight velocity of 7.9 km/sec be attained, since this is the value of the orbital velocity necessary to eliminate completely the possibility of falling back to Earth. Making allowance for the Earth's rotation, this velocity can be lowered to $7.9 - 0.46 = 7.44$ km/sec if the rocket is launched on the equator in the west-east direction. Flight velocities of 3 – 3.5 km/sec at an altitude of about 60 km are satisfactory for super-aviation provided that high-altitude atmospheric hydrogen can be used.

To achieve super-aviation the following points should be investigated.

1. In the field of aircraft power-plants: centrifugal compressors with enhanced cooling and more than two stages, with the carburetor between the stages; large-size engines utilizing oxygen-enriched fuel mixtures and operating on an increased compression ratio; high-pressure engines utilizing liquid oxygen and operating on the principle of internal and external combustion, and their components; jet compressors (thermal injectors) replacing compressor pumps; ordinary engines being combined with jet engines, either placed at the propeller's tips or fixed rigidly inside the airplane; engines made of combustible metals and designed for a single run.

2. In the field of jet engines: engines operating on the principle of direct action with constant exhaust velocity and utilizing various kinds of fuel; engines with an inverted cone to increase the exhaust velocity by cooling (with

* This article is the summary of a paper written by the author on this subject. It includes the last pages of the manuscript which give a general résumé of the problems of super-aviation. – Editor's note.

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air, liquid fuel, or water); engines utilizing the atmospheric heat by cooling the ambient air to extremely low temperatures; jet engines with preliminary compression and direct exhaust or inverted cone to increase the exhaust velocity; engines with air intake either accompanied by a shock wave or shockless; engines in which the combustible air is compressed by a jet compressor; combustion of the $2H_2 + O_2$ mixture which supposedly exists at an altitude of 60 km, at pressures from about 10^{-4} to 1 atm; rockets operating in combination with airplane models.

3. In the field of fuels and materials: solid fuels, including metals, and their use in jet engines and aeroengines; the applicability of gases with low liquefaction point as fuels and coolants; solid fuels of high hardness and viscosity, their melting and solubility; containers for melting metals and compounds; low temperature properties of materials; methods for heating cooled parts of the spaceship.

4. Heating of frontal components at high flight velocities; blowing of cold air on these components from the low-pressure pocket in the inverted cone.

5. In the field of construction of rockets and airplanes: one-stage rockets, rockets equipped with lateral rockets and vessels, compound rockets, rocket trains; construction of airplanes with combustible parts and devices for moving parts of the airplane.

6. In the field of investigation of high-altitude atmospheric layers: launching of rockets and airplanes; radio reception of rocket reports.

7. In the field of pilot hygiene: development of apparatus for generation of oxygen and absorption of carbon dioxide; cabin design; methods for cleaning the observation ports.

8. Theoretical studies of the flight trajectory, development of apparatus for indicating stresses in materials.

Research projects following this outline will prepare us not only for a temporary exit from the terrestrial atmosphere, but also for full-scale interplanetary travel. For the last objective, the problem should be formulated in a wider sense. We shall therefore consider those points not related with super-aviation, and refer exclusively to preparatory studies for interplanetary travel.

The main attention should be given to experiments on the purification of air in closed quarters where human beings will be confined for several months rather than for a few hours. To a certain extent this problem can be solved on Earth. A human being, or some experimental animals at first, should be placed in sealed quarters equipped with apparatus for air regeneration. It is possible to design chemical cyclic processes in which carbon dioxide is absorbed by some reagent and is then released elsewhere; oxygen can be supplied in high-pressure containers. These problems are obviously the concern of our chemists and doctors. The low temperatures of interplanetary space may be possibly used for the purification of air from all undesirable gases. This can be tested on Earth under laboratory conditions. Apparatus for the control of fresh oxygen supply is also necessary.

Experiments on protection from meteors could also be devised. The meteoric velocities are very high: 40 to 50 km/sec on the average. These velocities are obviously unattainable under experimental conditions, so that the actual research should be done at low velocities and the results extended by analogy. The best way would be to design an apparatus which sweeps the

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fast-moving meteors away from the spaceship. My proposal is to deflect the meteors by an electron beam of so-called low-velocity cathode rays emitted by a sphere charged with negative electricity. Inside this sphere the spaceship will be safe from meteors. The lower the rays' velocity, the higher the pressure they produce for a given energy. These rays will charge the meteors which approach the sphere, and having acquired a charge of the same sign the meteors will be repelled. Calculations have shown that fine dust and meteors up to 1 cm in diameter can be deflected by this method provided that the ejection of electrons from their sharp points is not excessive. It is highly desirable to carry out some experiments in a very high vacuum. Large meteors are encountered very seldom, and they may puncture spaceships with a frequency not higher than once in 10-60 years.

As regards the food supply, it can be brought along for the initial part of the journey: but since unexpected delays may occur, it is absolutely necessary to make some experiments with greenhouses of suitably light space-borne design. I carried out the first experiments in 1915-1917, in which I succeeded in growing peas, cabbage, and some other vegetables in charcoal, which is 3-4 times as light as ordinary soil. The experiments showed that charcoal fertilized by human refuse is perfectly suitable. Charcoal is highly absorbent, and can therefore be used to keep the air in the greenhouse fairly clean. In 1926 I grew beans in a glass of water fertilized with refuse in a ratio of 1:200.

From this elementary type of hydroponics we may pass to direct sprinkling of nutrient liquids on the plants' roots. Here the plants will grow without any soil. The method of aeration is suitable for converting all refuses into useful fertilizers within 24 hours. At the high temperatures attainable in interplanetary space, very rich crops can be expected in these greenhouses in an atmosphere of pure oxygen mixed with carbon dioxide. Tsiolkovskii calculated that a greenhouse of bananas, the most prolific of plants, can feed one person from an overall area of $1\,m^2$. Even if $100\,m^2$. are necessary, the area is still less than the craft's surface*. Plants can be grown in empty quarters where liquid is sprinkled, the quarters being enlarged as the plants grow so that maximum economy in the greenhouse area is attained. The weight of the device is very low. The greenhouse will also supply pure oxygen and utilize the carbon dioxide exhaled by man. The energy is derived from the Sun, and man can live together with the plants without requiring either additional air or additional food.

The provision of interplanetary stations near the Earth and other planets is very important. Crafts and rockets launched from the Earth may stay here so that the pilots may rest after the hardship of the climb. Interplanetary travel will become much cheaper if these stations are available, since they can store all the articles necessary for further flight to another planet. These stations may also transmit signals over large distances. The erection of a suitable building on the Moon or some other planet is highly desirable, since soon after the first conquest of interplanetary space, i.e., after leaving the terrestrial atmosphere, it will become more expedient to build spaceships not on Earth, but on some lesser celestial body, where the energy required for launching a vehicle is considerably smaller. Moreover, any work in the widest sense of this word is more easily carried out on small

* Here, and in what follows, craft is taken to indicate a spaceship designed after an aircraft. – Editor's note.

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celestial bodies. It can, therefore, be maintained that life will be cheaper on small celestial bodies than on Earth.

The most difficult problem in interplanetary travel is how to attain the first interplanetary velocity equal to about 8 km/sec. After reaching this velocity we shall be able to go on to other projects. Further acceleration of the flight velocity is possible by means of forces 100 to 1000 times smaller than the forces required for launching. Moreover, in interplanetary space there are some sources of energy which can be tapped quite free of charge.

In conclusion I still wish to enumerate briefly those methods which will be suitable for traversing interplanetary space.

Departing from an interplanetary station on a long-range vehicle, we may use the following means of propulsion:

1. Rockets of very small size which occupy only a limited space in the vehicle.
2. Light rays falling on large rotating disks or mirrors. Over the enormous interplanetary distances, the radiant energy may increase the flight velocity substantially, since the Sun's attraction at a distance equal to the distance of the Earth from the Sun is 1/1700 of the terrestrial attraction on the Earth's surface.
3. My investigations show that it is possible to design compound mirrors which would give an attraction, rather than repulsion, towards the Sun.
4. It is possible to design a system of mirrors with one prism from which light, once having entered, will not emerge. This system can generate very large forces and can also be used in solar power engines.
5. Rockets may be designed whose action is boosted by concentrated solar light.
6. It is possible to find ways to transmit the energy of concentrated solar rays over large distances and combine this energy with a rocket boosted by the action of concentrated solar rays. At present this is the only possible way to reach high flight velocities; we hope that these methods may be suitable for journeys to other solar systems.
7. My studies show that it we build a current-carrying solenoid and put inside it iron dust charged with static electricity, the pressure of Sun rays on this dust propels the interplanetary structure.
8. The flight trajectories may sometimes by chosen so that the vehicle travels round planets or outside their atmosphere. In this case the flight velocity can be increased. Revolution about the Moon may raise the flight velocity by about 2 km/sec. A ship traveling around the Earth outside the atmosphere will gain about 10 km/sec, and inside the atmosphere, 50-55 km/sec. A ship traveling round Jupiter outside its atmosphere may increase its flight velocity by 24 km/sec.
9. It is generally advisable to accelerate the flight near the points at which the flight velocity is high, i.e., near planets.
10. Journeys in interplanetary space proper, and also directly over the surface of minor planets, can be carried out in a charged, very light, hollow sphere, provided the planets or the asteroids are electrically charged. In this case the sphere will be repelled by the planet if it is charged by electricity of the same sign (obtained, for instance, by bringing the sphere into contact with the surface of the celestial body), and it will be attracted if it is charged with electricity of opposite sign.

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11. There are certain indications that the fairly powerful magnetic fields of the planets and of the Sun can be utilized for the same purpose. Traversing a magnetic field with a very high velocity and putting an electric current through a conductor closed in the space outside the vehicle, we may obtain a force acting on the conductor in a definite direction. This can be used for changing the course of the vehicle and for lifting it from the surface of a minor planet, particularly if the low temperatures are suitable for using the superconducting properties of the metals.

In the very near future, intensive research on problems of interplanetary travel, with the participation of workers from other related fields, should enable humanity to enter interplanetary space and travel to other planets.

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Appendix 1

SUMMARY OF THE LECTURE

ON MY SPACESHIP, DELIVERED AT THE THEORETICAL SECTION OF THE MOSCOW SOCIETY OF AMATEUR ASTRONOMERS, 20 JANUARY 1924

A. Introduction: outline of lecture: the fundamental propositions, operation and advantages of my design, and the relevant calculations.

B. Lecture about the spaceship.
   I) Read the article from "Samolet" and show the drawings.
   II) Calculations for the rocket of a spaceship:
      a) rocket force;
      b) velocity of rocket gases;
      c) efficiency and performance;
      d) rocket cross sections, velocities, pressures, specific weights, and temperatures;
      e) friction at the rocket walls;
      f) heat transmitted by the walls and the wall thickness;
      g) actual example of calculations of the magnitudes quoted in a)–f) and of the thermal efficiency as a function of altitude.
   III) Calculation of a voyage in a rocket and the overall consumption of fuel.
   IV) Calculation of a voyage in terms of the engine and the engine power.
   V) Velocities required to reach other planets. Calculation of trajectory, duration of journey, least-velocity trajectories for given duration of voyage.
   VI) Use of mirrors and screens instead of rockets in interplanetary space. Calculation and advantages.
   VII) List of danger spots and list of possible and interesting calculations.

C. Conclusion: a detailed study of the design is required; it is desirable to organize a society of amateur investigators of interplanetary travel.

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Appendix 2

REPORT OF F.A. TSANDER

ON THE PROPOSED PROJECTS OF THE SCIENTIFIC RESEARCH SECTION OF THE SOCIETY OF INTERPLANETARY COMMUNICATION

Delivered 15 July 1924

My Friends!

We all are united by a common idea: The necessity to investigate the field of Interplanetary Travel!

This is a vast project and many fervently wish to advance it. However, there are problems: how are we to conduct the research, where are we to start, and how are we to unfold our activities?

Our Scientific Research Section is obviously now in a position to penetrate the heart of the matter. On the one hand we shall study the phenomena theoretically, sketch their trend, and develop, partly in joint projects, the necessary designs, and on the other hand we shall test, under actual conditions, models and small rockets for investigating upper atmospheric layers; we shall strive for immediate success. We shall thus verify by experiment the validity of the theoretical conclusions, the quality of fuels, etc.

Our problem thus consists in:

1) laboratory work;
2) draftsmanship;
3) theoretical scientific work,

taking care that all three projects are intimately linked with one another.

It appears that in the more distant future we shall also add by concerted work:

4) the construction of large machines for carrying people to the upper atmospheric layers and to interplanetary space; we hope that we shall be able to shake hands on other planets.

Laboratory and practical work

1. The most intensive work in the laboratory is obviously conducted in the field of:

A) Investigation of small rockets operated by various fuels. After the first successful experiments we should already be able to launch such rockets and thus test our gyroscopic devices. The competition for inventing small rockets to be used in investigations of the upper-atmospheric layers will be highly conducive to this end. It will be necessary to test: 1) the influence of the initial and final pressure of the gases; 2) the influence of wall smothness; 3) the transmission of heat through the walls; 4) the determination of the propelling force for all cases; 5) various fuels including metals; 6) materials for rockets with reference to the wall temperature and the internal pressure; 7) various gyroscopic devices;

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8) the injector rocket operating on atmospheric air; 9) compound multistage rockets inserted one into another; 10) the determination of designs capable of carrying the highest percentage of liquid fuel.

Then we have:

B) Construction and testing of collapsible and noncollapsible airplane models of different designs propelled by rockets and engines or by rockets only.
C) Testing of the influence of high accelerations in specially designed centrifugal machines.
D) Construction and design of engines operating on liquid oxygen or on solar energy.
E) Testing of a diving suit for high-altitude and space flight and of suitable protective devices.
F) Testing of apparatus for regenerating the exhaled air, etc.
G) Investigations towards a greenhouse light enough to be carried by a rocket.
H) Testing of television for rockets.
I) Testing of components of spaceships at low pressures and high accelerations in a wind tunnel made of two rockets brought into contact at their widest parts; determination of resistance, lift, and heating.
K) Investigation of the upper atmospheric layers with rockets, balloon sondes, and photometric observations of twilight and instruments for these.
L) Testing very thin sheets for screens.
M) Testing of coils carrying an electric current with an iron dust core.

Appendix 3

17 October 1926

TABLE OF CONTENTS (summary)

of the book by F. A. Tsander proposed for publication under the title

"FLIGHTS TO OTHER PLANETS: THE FIRST STEP INTO THE VAST UNIVERSE"

(Theory of interplanetary flight)

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3) Efficiency of a rocket:
   a) ratio of work done by ship in time dt to the heat converted into kinetic energy (η_m);
   b) thermal efficiency (η_t);
   c) total thermal efficiency ($\eta_i = \eta_m \cdot \varphi \cdot \eta_t$). Graph of its dependence on flight velocity;
   d) total efficiency of a rocket (η_{i+E}), equal to the ratio of work transmitted to the spaceship in time dt to the sum of the kinetic and the thermal energy of the fuel consumed. Graph of its dependence on flight velocity;
   e) average efficiency for a given period of time. Graph of its dependence on fuel consumption.
4) Thermal calculation of a rocket following Professor Stodol's method for exhaust nozzles generalized by the introduction of heat transmitted through the rocket walls. Sample calculations and drawings. Calculations based on the iS diagram, where i is the heat content, and S the entropy of the combustion products.
5) Calculations of exhaust velocity, fuel consumption, and axial thrust of a rocket using perfect gas equations.
6) Determination of friction losses on rocket walls. The effect of friction in rockets of various sizes. Plot of gas friction on rocket walls.
7) Heat transmitted through rocket walls. Quantity of cooling liquid required. Temperature of rocket walls at various points. Difficulties involved in cooling small rockets in comparison to large ones.
8) Determination of thickness of rocket walls*.
9) Numerical examples:
   a) hydrogen-oxygen rocket for an axial thrust P of 1500 kg. Diagrams of axial thrust and thermal efficiency for various final diameters of the rocket. Pressure, temperature, density, and gas velocity for various cross sections of the rocket. Limiting (maximum) and actual values of these quantities, taking into consideration gas friction and heat transmission;
   b) gasoline-oxygen rocket.
10) Calculation of rockets where the fuel and the oxygen for combustion are heated by some means, e. g.:
   a) by another component of the fuel, giving either gaseous or solid combustion products;
   b) by cooling the rocket walls with the fuel;
   c) by the economizer mounted in the exhaust nozzle of the rocket near the inlet;
   d) by concave mirrors which collect solar rays inside the spaceship, heating the fuel to combustion or heating the gases in the combustion space, increasing the temperature and thus the exhaust velocity of the gases, and improving the operation of the rocket.

* Here, as in points 4-7, rocket means rocket engine. – Editor's note.

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11) Calculation for rockets that simultaneously eject gaseous and solid combustion products:
   a) axial thrust in the case when the heat from the solid combustion products is partly transferred to the gases and the solid particles lag behind the gases;
   b) approximate determination of the velocity difference between the gaseous and the solid combustion products for different gas densities;
   c) approximate and precise determination of the time that gases and solid combustion products stay in the rocket;
   d) approximate determination of the temperature drop of the solid particles with their time of stay in the rocket;
   e) exact calculations for this rocket; differential equations for the determination of density, temperature, pressure, and velocity of the gases, temperature and velocity of the solid particles, temperature of the walls and of the cooling liquid for various cross sections of the rocket nozzle. Solution of this system of differential equations;
   f) composition of combustion products for a given relative content of metal in the fuel;
   g) change in the cost of spaceships for different amounts of solid combustion products and, in general, for different types of fuel;
   h) calculations of rockets for fireworks;
   i) numerical examples:
      α) rocket operating on hydrogen, magnesium, and oxygen;
      β) rocket operating on gasoline, magnesium, and oxygen.
12) Consumption of fuel in a rocket for different efficiencies with a given flight velocity if:
   a) oxygen is taken along;
   b) oxygen is supplied by the atmosphere.
13) Calculation of rockets operating on atmospheric air and on some fuel. Rocket with a piston or turbine compressor. Rocket with injector (Mêlot's system):
   a) theoretical calculation;
   b) numerical examples:
      α) hydrogen rocket;
      β) gasoline rocket.
14) Calculation of two-or multi-stage rockets according to Professor Oberth's design.
15) On the results of Professor Goddard's experiments and comparison of his results with theory.

V. Theory of interplanetary flight
1) Methods applicable to travel in interplanetary space:
   a) rocket-propelled flight;
   b) flight with the aid of a rocket mounted in combination with an engine actuating an airplane propeller. Method of Tsander;
   c) other methods.
2) Methods applicable to travel from one planet to another starting from a position in interplanetary space:
   a) rocket-propelled flight;

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b) flight with the aid of mirrors moved by light pressure;
c) flight with the aid of a rocket linked with a concave mirror which increases the rocket propulsion;
d) flight with the aid of the devices described in V2b and V2c, where the effect of the mirrors is increased by large concave mirrors transmitting a beam of concentrated, almost parallel, solar rays from outside onto the mirror of the spaceship;
e) flight with the aid of combined mirrors;
f) flight with the aid of a system of mirrors and prisms from which light cannot re-emerge;
g) application of an apparatus converting solar rays into low-velocity cathode rays:
 α) flight made possible by pressure of cathode rays emitted by the spaceship;
 β) flight under pressure of cathode rays transmitted from outside in a parallel beam;
h) flight with the aid of solenoids containing iron dust; electrical current flows through the solenoids; a beam of condensed rays from outside or direct solar light falls on the dust; light pressure propels the solenoid connected to the spaceship;
i) flight with the aid of spheres charged with static electricity and repelled by the celestial bodies (Sun, planets, their moons, etc.), provided they are charged. Ditto in case of attraction;
j) flight by other methods.

3) Interplanetary stations; their significance, position for various purposes, probable design.
4) On the results of calculations of rocket flight by K. E. Tsiolkovskii and Professor H. Oberth. Studies of Professor V. P. Vetchinkin and W. Hohmann.
5) Calculation and practical density curves; pressure and composition of the Earth's atmosphere; atmospheric temperature:
 a) derivation of formulas;
 b) adiabatic variation of atmospheric temperature;
 c) case when the temperature at high altitudes is almost absolute zero;
 d) case when the temperature is equal to 180° abs from an altitude of about 33 km;
 e) case intermediate to c) and d);
 f) from observations of Professor Fesenkov;
 g) after Wegener.
6) Calculation of the ascent of a spaceship in the atmosphere (flight velocity $\gg 8$ km/sec). Theory:
 a) engine-propelled flight;
 b) rocket-propelled flight;
 c) flight with the aid of an engine and a rocket simultaneously;
 d) numerical examples:
  α) ascent with a constant-power engine, diagrams;
  β) ascent with an engine whose power may increase strongly for a short time;

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γ) ascent with a rocket, diagrams.

7) Calculation of the descent of a spaceship from interplanetary space to the Earth:
   a) nature of descent;
   b) determination of maximum limiting flight velocities for which the centrifugal force is negligible;
   c) calculation of gliding descent neglecting in the first approximation the centrifugal force in the lift equation, subsequent more exact calculation;
   d) calculation of the gliding descent in c) in the case when the temperature of the upper atmospheric layers is constant;
   e) calculation of gliding descent allowing in the equation of lift for the centrifugal force on the airplane;
   f) flight at constant altitude at the expense of the kinetic energy of the flying machine at high flight velocities:
      α) general case;
      β) case when the wing-resistance coefficient R is constant;
      γ) case when $R:A$ is constant (where A is the lift coefficient of the airplane);
   g) flight at constant altitude over any circle at the expense of the kinetic energy of the airplane;
   h) fall in the atmosphere, allowing for the Earth's curvature, at a small angle to the horizontal;
   i) fall in the atmosphere neglecting the Earth's curvature, sloping flight.
   j) constant-velocity motion over a circular arc;
   k) approximate determination of the zone surrounding the terrestrial globe which allows safe re-entry at high velocities ($\geqslant 8$ km);
   l) other cases of descent to Earth:
      α) straight-line gliding descent;
      β) combined and other descents.
   m) numerical examples to points c) to k);
   n) the temperature of a spaceship during a gliding descent on Earth.

8) Calculations of flight trajectories, additional velocities and times required for trips to other planets:
   a) case when the planet of destination is at the aphelion of the trajectory. Determination of the minimum additional velocity at which it is at all possible to reach the given planet, transformation of the formula for exact calculations;
   b) case when the terrestrial globe is at the perihelion of the flight trajectory;
   c) general case;
   d) determination of trajectory elements ensuring minimum flight time for a given additional velocity;
   e) numerical examples to points a) and b). Flights to all planets and to some asteroids;
   f) numerical examples to points a) to d), flights to Mars, Venus, Neptune, and Mercury.

9) Choice of time of departure from Earth, time of arrival on another planet allowing for the use of economically feasible velocity:

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a) theoretical calculation;
b) numerical examples: flights to Mars and Venus.

10) Determination of time required to reach a given small distance from a given planet:
a) theoretical calculation;
b) numerical example.

11) Determination of the distance from a given planet at which the attraction of the Sun and attraction of the planet are in a given ratio:
a) theoretical calculation;
b) numerical examples for all the planets, the Moon, and some asteroids.

12) Correction of trajectory when approaching the planet with the purpose of ensuring landing at a desired spot. Determination of magnitude and direction of the most economical additional velocity:
a) theoretical calculation;
b) numerical example.

13) Change of flight trajectory about the Sun under the influence of the planets. The advantage in increasing or decreasing the flight velocity:
a) velocity triangles before and after the completion of a revolution;
b) increment or decrement in kinetic energy of the spaceship after revolution about a planet;
c) maximum change in the spaceship's kinetic energy;
d) maximum change in the spaceship's velocity;
e) numerical examples for all planets.

14) Orbiting the Moon with the purpose of accelerating or decelerating the spaceship. Maximum change in velocity.

15) On the advisability of accelerating the flight by a rocket at the moment when the flight velocity is high, numerical example, acceleration near the trajectory perihelion.

16) Determination of flight trajectories in space ensuring return to the Earth after n revolutions of the spaceship about the Sun:
a) after a whole number of years;
b) after a fractional number of years;
c) numerical data.

17) Determination of flight trajectories before and after orbiting a given planet. Numerical example: revolution about Mars. Particular case: returning to Earth after orbiting Mars.

VI. Calculations for rocket flights, through outer space around a larger part of the terrestrial globe. Minimum velocities to be given to a rocket to ensure a given range of flight. Numerical data.

VII. Calculations for flights in outer space with the aid of light pressure:
1) magnitude of light pressure;
2) thickness of sheets admissible in flight in comparison with the practicable thickness;
3) surface extent of the sheets required in flight:
a) flight to another planet with the aid of a disk made of very

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thin sheets catching the solar rays normal to its surface. Flight trajectory – a conic trajectory. Flight paths and times, attainable velocities, weight of disks;

b) flights to other planets along spiral trajectories, with the solar rays falling obliquely onto the rotating disk connected with the spaceship. Trajectories and flight times, attainable velocities, weight of disks;

c) orbiting planets along spiral trajectories with the purpose of gradually increasing the flight velocity by the pressure of solar rays falling on the rotating disks connected with the spaceship;

d) stresses in the rotating disks under the influence of centrifugal force and light pressure. Numerical example;

e) numerical example to points a), b) and c).

4) Calculation of pressure exerted by light on a combined mirror:

a) case when light falls first on one mirror, then on another which is protected from the side of light by an obliquely fixed mirror. Generation of light pressure directed to a light source. Numerical example. Use of this design for flights to other planets along spiral trajectories;

b) case when light passes through a prism into a system of three or more reflecting surfaces and cannot re-emerge from it. Direction and magnitude of the resulting light pressure. On the use of this design in flight and in solar engines. Numerical example.

5) Calculation of large concave mirrors transmitting a beam of concentrated, almost parallel, solar rays to the structures described in points 3 and 4; times and velocities of flight attainable with the given design. Travel to other solar systems.

6) Calculation of solenoids containing a cloud of iron dust; direct or alternating currents flow through the solenoid, and solar rays fall on the dust, propelling, together with dust, the solenoid which is connected to the spaceship.

VIII. Apparatus for the conversion of solar rays into low-velocity cathode rays. Flight with the aid of cathode-ray pressure.

IX. Calculations of charged spheres repelled by the planets and the Sun (which are assumed charged) or attracted by them. Stresses in the material of these spheres. Forces of repulsion or attraction. Attainable velocities. Methods of charging the spheres.

1) Extraction of electricity from the near-planetary medium.
2) Charging by solar rays (analogy with polar auroras).
3) Charging by electrical machines.

X. Design and calculation of a spaceship and of its engine.

1) Design of an engine actuating the propeller of a spaceship. Oxygen-petrol engine with variable filling and adjustable maximum pressure of gases. High working pressure of gases (200 atm). High variability of power. Engine operation independent of atmospheric pressure. Connection between weight and working pressure. Cylinder cooling. Detonation. Variant: use of the combustion chamber of a rocket for generating combustible gases for the engine.

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2) Size of propellers for high-speed flight in rarefied air.
3) Design and weight of the rocket of a spaceship.
4) Design and weight of the container for the molten combustible metal.
5) Apparatus for spraying metallic components to be used as rocket fuel. Analogy with compression of coal dust in boilers.
6) Design of airplane components to be melted or sprayed and of the actuating mechanism:
   a) drums with generators of various shapes spinning cables which move airplane components, and the method of their actuation;
   b) connections ensuring automatic disconnection of the moving cables;
   c) design of mobile wings;
   d) design of mobile rudders;
   e) design for moving the engine;
   f) design for moving parts of the airplane body;
   g) design for raising the undercarriage;
   h) jointed fuel containers;
   i) partially jointed rocket;
   j) cooling system for the components of the design.
7) Guidance and control of the spaceship in interplanetary space:
   a) small lateral rockets controlled by a gyroscope, or flipping over of special steering rockets;
   b) displacement of gyroscope-controlled masses;
   c) gyroscopes;
   d) other methods.
8) General outline of the calculations for a spaceship; initial and final weight; size of engine, rocket, and container; wing area; specific features in the calculation of wings and body undercarriage; mechanisms for drawing-in of components. Cooling of small wings and other parts of a small airplane for landing.

XI. Greenhouses light enough to be carried by a rocket and the closed cycle necessary to sustain life in airtight quarters in a spaceship, in an interplanetary station, on the Moon, and on another planet possessing an atmosphere.

1) Amount of oxygen indispensable for breathing, amount of exhaled carbon dioxide, production of oxygen by a greenhouse, utilization of carbon dioxide by the greenhouse, designs for both partitioning and connecting the greenhouse with the living quarters.
2) Amount and composition of the daily food.
3) Growing plants in pure oxygen and
   a) in fertilized, crushed charcoal;
   b) in nutrient liquid;
   c) in quarters where nutrient liquid is sprayed with an atomizer;
   d) in case when nutrient liquid is drop-fed to the roots.
4) Preparation of fertilizers and nutrient solutions; application of the method of aeration of sewage.
5) Experiments carried out until now.
6) First experiments carried out by the author.

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7) Design and calculation of airtight suits.
8) Utilization of wastes for feeding domestic fowl, fish, and animals.

XII. **Approximate outline of theoretical and experimental investigations of materials and structures suitable for interplanetary travel. Conditions for new life in interplanetary space.**

1) Investigations bearing on the engine; pumping of oxygen into high-pressure container (200 and more atmospheres); construction of a small experimental engine (approximately 20 hp)*.
2) Testing metals and other materials of high calorific capacity and high strength, adequate specific elongation, and (for metals) low melting point, which are easily powdered. High, average, and low temperature, also very low temperatures.
3) Investigations bearing on the rocket: safe combustion of fuel in pure oxygen, application of compound fuel giving both gaseous and solid combustion products, measurement of the propelling force, temperature, pressure, and velocity of gases and solid particles, rockets utilizing external atmospheric air for combustion, and rockets with artificial heating of fuel and combustion products, experimental lifting dismountable rockets.
4) Investigation of the design of spaceships, construction and launching of airplane models which can be put together in the air.
5) Testing of respiration apparatus and of airtight suits. Prolonged use of latter.
6) Investigation of greenhouses light enough to be carried by a rocket.
7) Mounting engines and rockets in airplanes, ensuring higher ceiling for airplanes, increasing flight velocity at high altitudes, escape to space.
8) Scientific investigations of the atmosphere, cosmic rays, cosmic dust, electrostatic charge of the Earth, etc.
9) Investigations bearing on the production of very thin sheets of various materials, and on their rotation about an axis perpendicular to their plane; on light pressure on simple and combined mirrors, on a system of prisms and mirrors.
10) Investigations bearing on solenoids containing a cloud of iron dust.
11) Investigations bearing on the design of apparatus emitting low-velocity cathode rays and on the deflection of dust jets by this apparatus, also on the penetrating power of small particles moving with very large velocities.
12) Preparation of a system of concave mirrors or lenses converting slightly divergent (0.5°) solar light into a concentrated beam of more parallel light.
13) Investigations on the usefulness of apparatus for conversion of solar energy into other types of energy for interplanetary purposes.
14) Theoretical investigation of enormous telescopes to be mounted in interplanetary space or on the Moon.

* Tsander called this engine "Astron". – Editor's note.

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15) Investigations and tests of various measuring apparatus and devices necessary in a spaceship and on other planets.
16) Investigation of methods suitable for extraction of water and oxygen from the soil of the Moon under different assumptions.
17) Theoretical investigation of the advantages and the disadvantages of the life:
    a) in an interplanetary station;
    b) on the Moon, moons of other planets, asteroids, or Mercury (all of which have no atmosphere);
    c) on other planets possessing atmospheres, under different assumptions.
18) A more exact analysis of flight trajectories, additional velocities, and initial weight of spaceships:
    a) assuming the orbits of both planets to be ellipses;
    b) under the influence of two celestial bodies.
19) Other investigations.

XIII. Brief survey of results achieved.

XIV. Prospects for the future:
1) the near future;
2) the distant future.

F. A. Tsander

Appendix 4

OUTLINE OF THE BOOK

"CALCULATIONS OF JET ENGINES AND THEIR COMBINATIONS WITH OTHER ENGINES"

A. Preface.

B. Introduction.

C. Jet engines for which the combustible oxygen is carried along.

I. Determination of the combustion temperature with varying oxygen content:
   a) with varying heat capacities and neglecting dissociation;
   b) with varying heat capacities and allowing for dissociation.

II. Determination of nozzle cross section, velocity, pressure, density, and temperature at various points of the nozzle; thrust for a given consumption of mixture, thermal efficiency, using either *iS* and *TS* diagrams, or perfect gas formulas.

III. Determination of the quantity of heat transmitted through the walls, wall temperature:
   a) for the combustion chamber;
   b) for the nozzle;
   c) for the liquid-oxygen container;
   d) for the liquid-oxygen evaporator.

IV. Determination of the increase in temperature and decrease in pressure of gases as the result of their friction at the walls.

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V. Efficiency of jet engines.
VI. Cost of fuel and oxygen. The economic aspects of the use of jet engines.
VII. Determination of the combustion chamber's size.
VIII. Constructive design of simple liquid-jet engines for various purposes.
IX. Powder rockets, specific calculations.
X. Use of cyclic processes in jet engines for increasing their efficiency:
  a) practical calculation of direct and inverse cone (double cone);
  b) multistage cycles of double cones;
  c) cone shapes for various circular cycles;
  d) determination of the most expedient cycle for a given purpose;
  e) controlling and triggering devices for jet engines;
  f) structural data for various purposes;
  g) use of double (direct and inverse) cones in the capacity of:
    1. wind tunnels;
    2. jet pumps.

D. Jet engines utilizing the atmospheric air and jet compressors.

I. Types of jet engines utilizing the atmospheric air.
II. Calculation of a jet engine utilizing atmospheric air and of a jet compressor in which the low pressure of gases inside the apparatus is maintained by a special method and the combustion products do not mix with air;
  a) without secondary utilization of heat for cooling the inverse cone;
  b) with secondary utilization of this heat.
III. Calculation of a jet engine utilizing atmospheric air in which the combustion products mix with air without producing a shock or with a small shock:
  a) without secondary utilization of heat;
  b) with secondary utilization of heat.
IV. Calculation of multistage jet engines.

E. Calculation of jet engines emitting simultaneously solid and gaseous combustion products (Jet engines operating partially or entirely on metallic fuel, powder, and chemical engines).

I. Axial thrust and thermal efficiency as a function of the ratio of solid to gaseous particles.
II. Heat transfer from solid oxides to the gaseous products of their combustion.
III. General thermodynamic calculations.
IV. Determination of resistance to flight of compound rockets made up of a central one and a cluster of lateral rockets.
V. Structural design of metallic-fuel rockets.
VI. Air rockets with metallic or chemical fuel.
VII. Powder air rockets and jet engines.

F. Combination of jet engines with ordinary ones, and the use of jet compressors instead of centrifugal compressors.

G. Combination of turbines with jet engines.

H. Jet-propulsion flight.

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