NASA TECHNICAL NOTE

NASA TN D-7996

NASA TN D-7996

NUMERICAL ANALYSIS AND PARAMETRIC STUDIES OF THE BUCKLING OF COMPOSITE ORTHOTROPIC COMPRESSION AND SHEAR PANELS

Jerrold M. Housner and Manuel Stein

Langley Research Center
Hampton, Va. 23665

AMERICAN REVOLUTION BICENTENNIAL 1776-1976

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1975

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-7996		
4. Title and Subtitle: NUMERICAL ANALYSIS AND PARAMETRIC STUDIES OF THE BUCKLING OF COMPOSITE ORTHOTROPIC COMPRESSION AND SHEAR PANELS		5. Report Date: October 1975
		6. Performing Organization Code
7. Author(s): Jerrold M. Housner and Manuel Stein		8. Performing Organization Report No.: L-10083
9. Performing Organization Name and Address: NASA Langley Research Center, Hampton, Va. 23665		10. Work Unit No.: 505-02-51-01
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, D.C. 20546		13. Type of Report and Period Covered: Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A computer program is presented which has been developed for the combined compression and shear of stiffened variable thickness orthotropic composite panels on discrete springs; boundary conditions are general and include elastic boundary restraints. Buckling solutions are obtained by using a newly developed trigonometric finite-difference procedure which improves the solution convergence rate over conventional finite-difference methods. The classical general shear-buckling results (in terms of universal orthotropic parameters), which exist only for simply supported panels over a limited range of orthotropic properties, have been extended to the complete range of these properties for simply supported panels and, in addition, to the complete range of orthotropic properties for clamped panels. The program has also been applied to parametric studies which examine the effect of filament orientation upon the buckling of graphite-epoxy panels. These studies included an examination of the filament orientations which yield maximum shear or compressive buckling strength for panels having all four edges simply supported or clamped over a wide range of aspect ratios. Panels with such orientations had higher buckling loads than comparable, equal-weight, thin-skinned aluminum panels. Also included among the parameter studies were examinations of combined axial compression and shear buckling and examinations of panels with rotational elastic-edge restraints.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Buckling Orthotropic plates Finite differences Filamentary composite plates	Unclassified – Unlimited Subject Category 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	102	$5.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161

NUMERICAL ANALYSIS AND PARAMETRIC STUDIES OF THE BUCKLING OF COMPOSITE ORTHOTROPIC COMPRESSION AND SHEAR PANELS

Jerrold M. Housner and Manuel Stein
Langley Research Center

SUMMARY

A computer program has been developed for the combined compression and shear of stiffened variable thickness orthotropic composite panels on discrete springs; boundary conditions are general and include elastic boundary restraints. Buckling solutions are obtained by using a newly developed trigonometric finite-difference procedure which improves the solution convergence rate over conventional finite-difference methods. The trigonometric finite-difference procedure introduces two new parameters into the solution. These parameters can be computed by the program or selected by the user. The validity of the program has been substantiated by comparisons with existing solutions, and a program listing, input description, and sample problem are provided.

The classical general shear-buckling results (in terms of universal orthotropic parameters), which exist only for simply supported panels over a limited range of orthotropic properties, have been extended to the complete range of these properties for simply supported panels and, in addition, to the complete range of orthotropic properties for clamped panels. The program has also been applied to parametric studies which examine the effect of filament orientation upon the buckling of graphite-epoxy panels. These studies included an examination of the filament orientations which yield maximum shear or compressive buckling strength for panels having all four edges simply supported or clamped over a wide range of aspect ratios. Panels with such orientations had higher buckling loads than comparable, equal-weight, thin-skinned aluminum panels. Also included among the parameter studies were examinations of combined axial compression and shear buckling and examinations of panels with rotational elastic-edge restraints.

INTRODUCTION

The use of filamentary composite materials in aircraft and space structures offers a potential for weight savings over conventional (all metal) construction. Also, composites introduce added versatility into the design process by allowing the structure to be better tailored to meet the design criteria. One such design criterion is the prevention of compressive and shear buckling in panels of laminated construction. In laminated panels the stiffness properties can be tailored by controlling the filament orientation in each lamina.

A considerable amount of literature exists on the buckling of flat isotropic and orthotropic panels under various boundary conditions. (See refs. 1 to 6.) Few results exist, however, for finite aspect-ratio panels, especially for shear buckling of orthotropic panels. General results for shear buckling, in terms of universal orthotropic parameters, exist only for simply supported panels over a limited range of orthotropic parameters. (See ref. 6.) Several general-purpose computer programs exist which could be employed to obtain results for panels with general boundary conditions under general loading states (refs. 7 to 9). These programs, however, tend to be expensive to use in performing parameter studies; therefore, a program which is suitable for performing parametric buckling studies of orthotropic flat rectangular panels was developed and is employed in this paper.

The present computerized analysis is applicable to the combined compression and shear buckling of stiffened, variable-thickness, flat rectangular orthotropic panels on discrete springs; boundary conditions are general and include elastic boundary restraints. Calculation of the flexural stiffnesses of a laminate from the properties of filament-reinforced laminas is automatically performed. The analysis makes use of a newly developed trigonometric finite-difference procedure. In contrast to conventional (polynomial) finite differences, trigonometric differences take advantage of the sinusoidal form of the buckle pattern to achieve converged solutions with fewer degrees of freedom, hence reducing computer time. The analysis has been validated by many comparisons with solutions in the literature and has been used to produce a variety of additional orthotropic and some isotropic panel results.

The classical general results for the shear buckling of simply supported orthotropic panels are extended in this paper to cover the complete range of orthotropic parameters. Also, the general results for the shear buckling of clamped panels over the complete range of orthotropic parameters have been calculated and are presented herein. In addition, it is of practical interest to present results which consider the effects of filament orientation upon the buckling strength of laminated composite panels. Consequently, parameter studies are presented for graphite-epoxy panels of various aspect ratios, boundary conditions, and in-plane loadings over a wide range of filament orientations, and those orientations which led to maximum buckling loads are identified. Finally, results are presented for the shear buckling of simply supported isotropic panels, each with a central stiffener.

SYMBOLS

a,b dimensions of rectangular plate parallel to X- and Y-axes, respectively

$A^{(r)}$ coefficients defined by equation (C3)

C_x, C_{yx} correction factors defined in equations (B4) and (B5)

D isotropic plate flexural stiffness

D_3 $= D_{12} + 2D_{66}$

$D_{11}, D_{22}, D_{12}, D_{66}$ orthotropic plate flexural stiffnesses

$e_{ij}^{(r)}$ elements of matrix defined by equation (C2)

EI flexural stiffness of discrete stiffener

E_1, E_2 Young's moduli of fibrous reinforced material parallel to fibers and transverse to fibers, respectively

G_{12} shear modulus of fibrous reinforced material

h core thickness of sandwich plate

I_1, I_3 row designations of boundaries ① and ③ (see fig. 2(a))

J_2, J_4 column designations of boundaries ② and ④ (see fig. 2(a))

k_ℓ discrete lateral spring stiffness

k_R uniformly distributed rotational spring stiffness

k_s shear-buckling load coefficient $\dfrac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}}$

k_x, k_y stiffness of rotational springs which resist moments acting about Y- and X-axes, respectively

K_{ij} plate stiffness terms defined by equation (A13)

M,N total number of rows and columns of finite-difference stations, respectively

M_e,N_e total number of rows and columns of finite-difference stations at which equilibrium is satisfied

M_x,M_y,M_{xy} bending moments in plate (see fig. 1)

N_x,N_y,N_{xy} in-plane loads (see fig. 1)

$\overline{N}_x,\overline{N}_y,\overline{N}_{xy}$ shear-buckling stress coefficients $\frac{b^2 N_x}{\pi^2 D_{11}}$, $\frac{b^2 N_y}{\pi^2 D_{11}}$, $\frac{b^2 N_{xy}}{\pi^2 D_{11}}$, respectively

$\hat{N}_x,\hat{N}_y,\hat{N}_{xy}$ buckling parameters $\frac{b^2 N_x}{E_1 t^3\left[1 - \left(\frac{h}{t}\right)^3\right]}$, $\frac{b^2 N_y}{E_1 t^3\left[1 - \left(\frac{h}{t}\right)^3\right]}$, $\frac{b^2 N_{xy}}{E_1 t^3\left[1 - \left(\frac{h}{t}\right)^3\right]}$, respectively

N_{x_o},N_{xy_o} buckling loads for pure axial compression and pure shear, respectively

$\overline{p}$ buckling eigenvalue (see eq. (13))

r_x,r_y,r_{xy} change of $\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$ with $\overline{p}$, respectively (see eq. (13))

R_x,R_{xy} ratio of N_x/N_{x_o} and N_{xy}/N_{xy_o}, respectively

S_{ij} spring-stiffness terms defined by equation (A17)

t total thickness of sandwich plate

$\overline{t}_x,\overline{t}_y,\overline{t}_{xy}$ values of $\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$ when $\overline{p} = 0$

w displacement of panel in positive z-direction

x,y,z panel coordinates shown in figure 1

α_{ij}	curvature terms defined in equation (A14)
β	ratio of panel width to buckle length in an infinitely long panel
$\gamma_1,\gamma_2,\gamma_3$	coefficients defined by equation (7) or (8)
δU	internal virtual work
δV_N	virtual work of in-plane loads
δV_S	virtual work of discrete springs
Δ_x,Δ_y	finite-difference mesh spacings in x- and y-directions, respectively
$\hat{\Delta}_x,\hat{\Delta}_y$	trigonometric finite-difference coefficients as defined in equation (10)
Δ_x^*,Δ_y^*	trigonometric finite-difference terms defined in equation (A24)
θ	filament orientation (see fig. 2(a))
Θ,B	universal orthotropic parameters defined in equations (15) and (16)
λ_x,λ_y	trigonometric parameters defined through equation (10)
ν_{12}	major Poisson ratio relating contraction normal to filament direction to extension parallel to filament direction
$\xi_x,\xi_y,\eta_x,\eta_y$	functions defined by equations (A6) to (A9)
χ_{ij}	twist terms defined in equation (A16)
ψ_{ij}	curvature terms defined in equation (A15)

Comma preceding a subscript denotes differentiation with respect to the subscript.

ANALYSIS

Assumptions

The buckling analysis of linear elastic orthotropic plates has been carried out under the following assumptions:

1. Coupling between bending and extensional deformation is neglected. (In practice this assumption implies a midplane symmetric laminated panel.)

2. Coupling between bending and twisting deformation is neglected. (In practice this assumption implies a balanced laminate.)

3. The deformations of the panel obey the Kirchhoff hypothesis (see ref. 10).

4. The nonlinear strain-displacement relationships used to obtain (linear) buckling equations are

$$e_x = u_{,x} + \frac{1}{2}\left(w_{,x}\right)^2$$

$$e_y = v_{,y} + \frac{1}{2}\left(w_{,y}\right)^2$$

$$\gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y}$$

where e_x, e_y, and γ_{xy} are the strains and u, v, and w are the displacements in x-, y-, and z-directions, respectively.

5. The in-plane loads, N_x, N_y, and N_{xy}, are uniformly distributed along the appropriate edges of the plate.

6. Discrete stiffeners have no torsional stiffness and are symmetrically disposed with respect to the neutral surface of the panel.

Governing Equations

The internal virtual work of the panel during buckling may be expressed as

$$\delta U = \int_0^b \int_0^a \left(M_x \delta w_{,xx} + M_y \delta w_{,yy} + 2M_{xy} \delta w_{,xy}\right) dx\, dy \tag{1}$$

where a and b are the dimensions of the panel parallel to the X- and Y-axes, respectively, and δ is the variational operator. Also,

$$
\left.\begin{aligned}
M_x &= D_{11}w_{,xx} + D_{12}w_{,yy} \\
M_y &= D_{12}w_{,xx} + D_{22}w_{,yy} \\
M_{xy} &= 2D_{66}w_{,xy}
\end{aligned}\right\} \tag{2}
$$

The sign conventions of the bending moments are given in figure 1, and the flexural stiffnesses, D_{11}, D_{12}, D_{22}, and D_{66}, given in reference 11, are about a unique neutral plane which has the property that matrix [B], which represents coupling between bending and extension, is null with respect to this plane. As given by reference 12, the virtual work of the applied in-plane loads is given by

$$
\delta V_N = \int_0^a \int_0^b \left(N_x w_{,x}\delta w_{,x} + N_y w_{,y}\delta w_{,y} + N_{xy} w_{,y}\delta w_{,x} + N_{xy} w_{,x}\delta w_{,y}\right) dy\,dx \tag{3}
$$

where the sign conventions for N_x, N_y, and N_{xy} are shown in figure 1.

In appendix A, equations (A1) to (A3) are expressed in trigonometric finite-difference form (see fig. 2 for finite-difference station layout) and are substituted into the statement of the principle of virtual work, that is,

$$
\delta U = \delta V_N + \delta V_S \tag{4}
$$

where δV_S is the virtual work of the discrete springs. (See appendix A, eq. (A11).) Equation (4) yields the governing equations which are of the following form:

$$
K_{ij} + S_{ij} + N_x\alpha_{ij} + N_y\psi_{ij} + 2N_{xy}\chi_{ij} = 0 \qquad \begin{pmatrix} i = 1, \ldots, M \\ j = 1, \ldots, N \end{pmatrix} \tag{5}
$$

where K_{ij}, S_{ij}, α_{ij}, ψ_{ij}, and χ_{ij} are defined by equations (A13) to (A17) in appendix A.

The numerical technique of trigonometric finite differences and the numerical extraction of the buckling loads N_x, N_y, and N_{xy} from equation (5) are different from those conventionally used and therefore require further discussion.

Numerical Techniques

Trigonometric finite differences.- Conventionally, the central difference approximation for the derivative of a function $f(x)$ at $x = x_0$ is approximated as

$$\frac{df}{dx}(x_0) \approx \frac{1}{\Delta_x}\left[f\left(x_0 + \frac{\Delta_x}{2}\right) - f\left(x_0 - \frac{\Delta_x}{2}\right)\right] \tag{6}$$

The right-hand side of equation (6) is denoted as the conventional finite-difference approximation for the derivative. In the limit as the finite-difference mesh spacing Δ_x approaches zero, the right-hand side of equation (6) expresses the definition of the derivative. If $f(x)$ is parabolic in the neighborhood of x_0,

$$f(x) = \gamma_1 + \gamma_2(x - x_0) + \gamma_3(x - x_0)^2 \tag{7}$$

and it may be readily shown that the approximate expression given by equation (6) becomes an equality. If, however, $f(x)$ is trigonometric about $x = x_0$,

$$f(x) = \gamma_1 + \gamma_2 \sin \frac{\pi(x - x_0)}{\lambda_x} + \gamma_3 \cos \frac{\pi(x - x_0)}{\lambda_x} \tag{8}$$

where λ_x is a wavelength parameter. It may be readily shown that

$$\frac{df}{dx}(x_0) = \frac{1}{\hat{\Delta}_x}\left[f\left(x_0 + \frac{\Delta_x}{2}\right) - f\left(x_0 - \frac{\Delta_x}{2}\right)\right] \tag{9}$$

where

$$\frac{1}{\hat{\Delta}_x} = \frac{\pi}{2\lambda_x \sin\left(\frac{\pi\Delta_x}{2\lambda_x}\right)} \tag{10}$$

The right-hand side of equation (9) is denoted as the trigonometric finite-difference approximation for the derivative. (In a two-dimensional problem a similar set of relationships would be derived for the y-direction, introducing the quantities Δ_y, $\hat{\Delta}_y$, and λ_y.)

The only difference between the right-hand side of equation (9) and that of equation (6) is that in the trigonometric expression $1/\hat{\Delta}_x$ replaces $1/\Delta_x$ of the conventional expression. As λ_x approaches infinity, $\hat{\Delta}_x$ approaches Δ_x and, consequently, the trigonometric difference expression reduces to the conventional expression.

Convergence of trigonometric finite-difference solutions.- Inasmuch as the buckling mode shape is usually trigonometric in nature, the trigonometric finite-difference solution can be made to exhibit a much faster convergence rate than the conventional difference solution by appropriate selection of λ_x and λ_y. This advantage is demonstrated with several isotropic plate examples discussed in appendix B. The convergence rate can also be degraded, however, by an inappropriate choice of λ_x and λ_y. It should be emphasized though, that the selection of λ_x and λ_y does not constrain the buckle mode shape to have wavelengths given by λ_x and λ_y. Rather, the trigonometric solution will always converge to the exact solution if enough degrees of freedom (finite-difference stations) are used.

Selection of trigonometric parameters λ_x and λ_y.- Selecting appropriate values of λ_x and λ_y which improve the convergence rate of solutions is predominantly based upon engineering judgment and experience. One engineering approach which has proven useful is to select λ_x and λ_y based upon the buckle length of infinitely long panels; that is,

$$\frac{\lambda_x}{a} = \frac{b/a}{\beta} \tag{11}$$

$$\frac{\lambda_y}{b} = 1 \tag{12}$$

where β is the wavelength parameter of an infinitely long panel, defined as the ratio of the panel width to the buckle length. The value of β for the combined compression and shear buckling of simply supported and clamped infinite panels may be determined from equations (B2) and (B3) in appendix B. Additional suggestions for the selection of λ_x and λ_y are given in appendix B.

Stability determinant evaluation and eigenvalue extraction.- In this analysis the order of the stability determinant is kept to a manageable size by using the two-dimensional marching procedure outlined in appendix C. This procedure is basically an extension of the one-dimensional procedure used in reference 13. Briefly, the marching procedure successively operates on the equilibrium equations at each finite-difference station to achieve a relatively low-order stability determinant.

In searching for the combined load system which produces buckling, it is convenient to introduce dimensionless stress coefficients, $\overline{N}_x$, $\overline{N}_y$, and $\overline{N}_{xy}$, which may be determined from the dimensional quantities, N_x, N_y, and N_{xy} (fig. 1), by multiplying by the factor $b^2\pi/D_{11}$. It is assumed that $\overline{N}_x$, $\overline{N}_y$, and $\overline{N}_{xy}$ are linear functions of an eigenvalue $\bar{p}$, that is,

$$\left.\begin{aligned}\bar{N}_x &= \bar{t}_x + \bar{p}r_x \\ \bar{N}_y &= \bar{t}_y + \bar{p}r_y \\ \bar{N}_{xy} &= \bar{t}_{xy} + \bar{p}r_{xy}\end{aligned}\right\} \tag{13}$$

This assumption allows some loads to be held constant while others are increased to buckling, or it allows the loads to increase with a fixed proportionality.

To find the lowest value of $\bar{p}$ which makes the stability determinant vanish, a determinant plotting technique is used. In order to increase the speed of the plotting technique, a variable step size is employed. This step size is based upon a numerical parabolic extrapolation of the stability determinant at each step of the determinant plotting procedure.

COMPUTER PROGRAM

A computer program denoted BOP (Buckling of Orthotropic Panels) has been developed for the buckling of flat rectangular orthotropic laminated panels. The program is applicable to panels with compression and/or shear loading, discrete lateral deflection and rotational springs, discrete stiffeners, and general boundary conditions.

The program utilizes trigonometric finite differences to improve the problem convergence and thus requires the selection of λ_x and λ_y. The user has the option of determining and supplying λ_x and λ_y (based upon the discussion in appendix B) or allowing the program to automatically calculate and use values based on equations (11) and (12).

In addition, the user has the option of either (1) supplying the bending stiffnesses of the panel or (2) supplying the elastic moduli, filament orientation, and thickness of each lamina in a laminated panel and allowing the program to calculate the bending stiffnesses. When the second option is chosen, the program prints the flexural stiffness matrix D, defined in reference 11, as well as the laminate Young's moduli, shear modulus, and Poisson's ratios. (The second option may be used independently of the buckling analysis.) A complete description of the program is provided in appendix D.

Results from the computer program have been compared with many classical results for unstiffened isotropic and orthotropic panels under various boundary conditions and with some classical results for stiffened isotropic panels. These comparisons which are discussed in subsequent sections were found to be excellent, thereby indicating the validity of the program.

RESULTS AND DISCUSSION

Shear Buckling of General Orthotropic Panels

From the general fourth-order equation for the shear buckling of orthotropic panels the buckling load coefficient may be expressed as

$$k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}} \tag{14}$$

This coefficient is a function of only two variables

$$\Theta = \frac{\sqrt{D_{11} D_{22}}}{D_3} \tag{15}$$

and

$$B = \frac{b}{a} \sqrt[4]{\frac{D_{11}}{D_{22}}} \tag{16}$$

where $D_3 = D_{12} + 2D_{66}$. (Note that an isotropic panel implies $\Theta = 1$.)

Classically, general shear-buckling results for simply supported finite aspect-ratio panels have been obtained only for values of $\Theta \geqq 1$ (see ref. 6). In figure 3 numerical results for $\Theta < 1$ have been presented. Also, for completeness and comparison purposes numerical results for $\Theta \geqq 1$ are presented. The good agreement between these curves and those of reference 6 indicates the validity of the numerical results from the computer program. General results for the shear buckling of clamped panels, furthermore, do not appear in the literature for any range of Θ with the exception of $\Theta = 1$ (the isotropic case); consequently, numerical results for clamped panels are presented in figure 4.

Both the results for simply supported and clamped panels indicate that the percentage decline in buckling load from $B = 1$ to $B = 0$ decreases as Θ increases. Also, a comparison of figures 3 and 4 shows that the percentage increase in buckling load of clamped panels over simply supported panels increases with increasing Θ. The abrupt changes in slope appearing in these figures are due to changes in mode shape (from symmetric to antisymmetric modes). As anticipated from isotropic results (ref. 1), these abrupt changes are more predominant in clamped panels than in simply supported panels.

Tables 1 and 2 present the shear-buckling load coefficients used in obtaining the general orthotropic panel results of figures 3 and 4. Additionally, the trigonometric difference parameters (the mesh-spacing parameters a/Δ_x and b/Δ_y and the wavelength parameters λ_x/a and λ_y/b) used in obtaining the buckling coefficients are presented in tables 1 and 2.

Shear Buckling of a Simply Supported Panel With a Central Stiffener

Figure 5 presents results for the shear buckling of simply supported isotropic panels each of which contains one central flexural stiffener parallel to either the longer or shorter edges of the panel. As anticipated, the use of a central stiffener always provides an increase in the shear-buckling stress coefficient over that of the unstiffened panels $\left(\frac{EI}{bD} = 0\right)$. The percentage increase over unstiffened panels is greater in square panels than in rectangular panels. In rectangular panels of the same aspect ratio, the percentage increase over unstiffened panels is greater when the stiffeners are parallel to the longer direction than when they are parallel to the shorter direction. The central-stiffener results of figure 5, moreover, are in reasonably good agreement with similar results given in reference 14 for slightly curved panels. This agreement indicates the validity of the computer program for the solution of stiffened panels.

Parametric Studies of Orthotropic Filament Reinforced Panels

Results are presented for the buckling of sandwich panels whose upper and lower skins are of laminated graphite-epoxy construction. Although some of the results in this section could be obtained from general orthotropic curves, such as those of figures 3 and 4, it is of interest to examine the effect of filament orientation upon the buckling load. (The material properties for the graphite-epoxy skins are given in table 3, with their equivalent general orthotropic parameter values Θ and B at various filament orientations.)

In addition to the assumptions listed in the analysis section of this report, it is assumed in this section that

1. The panel is symmetric about the middle surface
2. Each lamina has the same filament orientation θ except for sign
3. The core carries no load and undergoes no transverse shear deformation

As a consequence of these assumptions, it may be shown that the buckling parameters $\hat{N}_x$, $\hat{N}_y$, and $\hat{N}_{xy}$ defined as

$$\hat{N}_x = \frac{b^2 N_x}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17a}$$

$$\hat{N}_y = \frac{b^2 N_y}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17b}$$

$$\hat{N}_{xy} = \frac{b^2 N_{xy}}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17c}$$

depend only on the magnitude of θ, the panel aspect ratio, and the boundary conditions. They do not depend on the thickness of each lamina, the number of laminas, or the core thickness. However, in order for assumption 2 of the analysis section to be reasonable – that is, neglect of bending-twisting coupling – it may be necessary that the ratio of core thickness to total thickness h/t be nearly unity and that the amount of material in either cover oriented in the $+\theta$ and $-\theta$ directions be equal.

The variation of the buckling load with filament orientation for panels of various aspect ratios is presented in figure 6 for axial compression and in figure 7 for shear. The figures indicate that the buckling loads are highly dependent upon filament orientation and that optimum orientations (those which yield a maximum buckling load) may be determined for each aspect ratio. Also, the figures indicate that clamping has a greater effect on compressive buckling than on shear buckling.

An indication of the buckling strength of the epoxy panels as compared to equal-weight aluminum panels is provided by a comparison of the discrete buckling loads appearing on the right-hand ordinate of figures 6 and 7 with the curves in the same figures. These comparable values are valid for thin-skinned sandwich panels which have the same core, of thickness h, as the graphite-epoxy panels, but which have aluminum skins. For all the cases considered, a range of filament orientations exists for which the buckling strength of the graphite-epoxy panels exceeds that of the comparable aluminum panel with the same aspect ratio and boundary conditions. In the case of a clamped square panel in shear, the buckling strength of the graphite-epoxy panel exceeds that of the aluminum panel at all filament orientations.

It should be noted that, if the restriction that each lamina have the same filament orientation $\pm\theta$ is removed, isotropic skins can be produced from groups of three or more laminas (for example, 0, +60, and -60) which will have the same weight as the $\pm\theta$ skins but will yield a higher buckling load for each case shown in figures 6 and 7 and for many other shear and compression loadings. However, this is not necessarily true in all cases; for example, in the transverse compression of long panels (a/b approaching zero), an orthotropic panel with filaments running transversely ($\theta = 0^\circ$) provides a higher buckling load than an equivalent isotropic panel. Furthermore, there are many applications where for various reasons (for example, strength or fabrication criteria) orthotropic panels are preferable to isotropic ones.

In figures 8 to 11 optimum filament orientations are shown for all aspect ratios. The curve of figure 8 was determined from the exact closed-form relationship for the compression of simply supported plates (ref. 6), while the curves of figures 9 to 11 were determined using program BOP. The abrupt changes in the slopes of these curves are caused by changes in the buckling mode shape associated with the optimum filament orientation. Except for figure 8, the location of these abrupt changes has been approximated since it is difficult to determine exactly where they occur.

In the compressive buckling curves (figs. 8 and 9) the optimum filament orientation for small aspect ratio a/b is 0° (parallel to the X-axis or to the direction of compression). This orientation angle rapidly increases at about $a/b = 0.56$ for simply supported panels and at about $a/b = 1.05$ for clamped panels. However, a comparison of the aspect-ratio 1 and 1.1 curves for a clamped panel as shown in figure 6 indicates that the optimum buckling load does not exhibit such a rapid change but decreases slightly as the aspect ratio goes from 1 to 1.1. For higher aspect ratios the optimum orientation oscillates with decreasing excursion about $\pm 45^{\circ}$ and, in general, a practical filament orientation for $a/b > 1$ is $\theta = \pm 45^{\circ}$.

In the case of shear buckling (figs. 10 and 11), the symmetry of the problem requires that the deviation of the optimum filament orientation from 45° for a panel of aspect ratio a/b be equal but opposite to that of a panel with aspect ratio b/a. Also, the peaks of figure 7 are quite flat; that is, they have a large radius of curvature associated with them. Consequently, it was difficult to determine precisely the optimum filament orientations in figures 10 and 11. However, it is reasonable to say from figures 10 and 11 that for large aspect ratios $a/b > 2$, $\theta = \pm 60^{\circ}$ to $\pm 62^{\circ}$ is a practical filament orientation.

Figures 12 and 13 present interaction curves for the buckling of simply supported and clamped panels in combined axial compression and shear for various filament orientations and aspect ratios. The optimum filament orientations (those that correspond to the highest values of the buckling parameters) change according to aspect ratio a/b and the ratio of N_{xy}/N_x. For simply supported panels (fig. 12), when $a/b = 1$, the optimum orientation for all combinations of N_x and N_{xy} is $\theta = \pm 45^{\circ}$. When $a/b = 2$ or 5, the optimum filament orientation for predominantly shear loading is near $\pm 60^{\circ}$ and for predominantly compressive loading is near $\pm 45^{\circ}$. For clamped panels (fig. 13) when $a/b = 1$ the optimum orientation changes from $\theta = \pm 45^{\circ}$ for shear loading to $\theta = 0^{\circ}$ for compression. When $a/b = 2$ or 5, the optimum orientation changes from $\theta = \pm 60^{\circ}$ for pure shear to $\theta = \pm 45^{\circ}$ for pure compression. This behavior was the same as that exhibited by simply supported panels.

A summary of the data from figures 12 and 13 is shown in figure 14, which indicates the banded region in which all the results lie. For orthotropic panels it was found that the band is bounded from below by the following simple relationship given in reference 15 for isotropic panels:

$$R_x + R_{xy}^2 = 1 \tag{18}$$

where

$$\left. \begin{aligned} R_x &= \frac{N_x}{N_{x_o}} \\ R_{xy} &= \frac{N_{xy}}{N_{xy_o}} \end{aligned} \right\} \tag{19}$$

In equations (19), N_{x_o} and N_{xy_o} are the buckling loads for pure longitudinal compression and pure shear, respectively. Consequently, for the orthotropic cases considered, equation (18) is a reasonable conservative approximation for combined longitudinal compression and shear buckling of composite panels.

Figures 15 and 16 contain, respectively, compression and shear-buckling results for graphite-epoxy sandwich panels with nondeflecting edge supports and rotational edge springs for various filament orientations and aspect ratios. The associated boundary conditions are given by equations (A20) to (A22), and the rotational springs were assumed to be uniformly distributed about the panel edges. When the spring stiffness is zero, all four edges are simply supported and, when infinite, all four edges are clamped.

In general, the figures indicate that the buckling load increases sharply as the spring stiffness parameter bk_R/E_1t^3 increases from zero to one, the buckling loads obtaining at least 80 percent of their clamped value when the spring stiffness parameter is one. With further increase in the spring stiffness the buckling loads slowly approach the clamped value, increasing to within at least 10 percent of the clamped value when the spring stiffness parameter is three. Furthermore, the curves for the $\pm45^\circ$ filament orientation generally approached the clamped values most rapidly.

CONCLUDING REMARKS

A computerized analysis has been developed for the combined compression and shear buckling of stiffened orthotropic composite panels on discrete springs. Boundary conditions are general and include elastic boundary restraints. Buckling solutions are obtained by using a newly developed trigonometric finite-difference procedure which increases the solution convergence rate over conventional finite-difference methods, thus allowing problems to be solved with the same accuracy as with conventional differences but with fewer degrees of freedom. The trigonometric finite-difference procedure introduces two new parameters into the solution. These parameters can be internally selected by the program during problem execution or can be selected by the user. The validity of the program has been substantiated by comparisons with many existing known solutions. A program listing, input description, and sample problem are provided.

Using the program, the classical general shear-buckling results (in terms of universal orthotropic parameters), which are available only for simply supported panels over a limited range of orthotropic properties, have been extended to the complete range of these properties for simply supported panels and clamped panels. Results for the shear buckling of isotropic panels with a central stiffener have also been obtained.

The program has been applied to parametric studies which examine the effect of filament orientation upon the buckling of graphite-epoxy sandwich panels. From these studies optimum filament orientations (those which yield maximum buckling loads) were determined within a class of graphite-epoxy sandwich panels for all aspect ratios. In particular, it was found that for shear buckling of high-aspect-ratio panels (greater than two) reasonable filament orientations are between $\pm 60^{\circ}$ and $\pm 62^{\circ}$ while, for axial compression of panels with aspect ratio greater than one, a reasonable filament orientation is $\pm 45^{\circ}$. In addition, interaction curves were determined for the combined axial compression and shear buckling of panels with varying filament orientations. A parabolic interaction relationship previously developed for isotropic infinite strips in combined axial compression and shear provided a reasonably accurate and conservative estimate for the buckling loads of the orthotropic panels considered herein.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
August 1, 1975

APPENDIX A

DEVELOPMENT OF GOVERNING EQUATIONS

For completeness, equations (1) to (3) of the main text are repeated here:

$$\delta U = \int_0^b \int_0^a \left(M_x \delta w_{,xx} + M_y \delta w_{,yy} + 2M_{xy} \delta w_{,xy}\right) dx\, dy \tag{A1}$$

$$\left.\begin{aligned} M_x &= D_{11} w_{,xx} + D_{12} w_{,yy} \\ M_y &= D_{12} w_{,xx} + D_{22} w_{,yy} \\ M_{xy} &= 2D_{66} w_{,xy} \end{aligned}\right\} \tag{A2}$$

$$\delta V_N = \int_0^a \int_0^b \left(N_x w_{,x} \delta w_{,x} + N_y w_{,y} \delta w_{,y} + N_{xy} w_{,y} \delta w_{,x} + N_{xy} w_{,x} \delta w_{,y}\right) dy\, dx \tag{A3}$$

Then, replacing the derivatives in equations (A2) by trigonometric central differences yields

$$\left.\begin{aligned} \left(w_{,xx}\right)_{ij} &= \frac{1}{\hat{\Delta}_x^{\,2}}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right) \\ \left(w_{,yy}\right)_{ij} &= \frac{1}{\hat{\Delta}_y^{\,2}}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right) \\ \left(w_{,xy}\right)_{ij} &= \frac{1}{\hat{\Delta}_x \hat{\Delta}_y}\left(w_{i+1,j+1} - w_{i,j+1} - w_{i+1,j} + w_{ij}\right) \end{aligned}\right\} \tag{A4}$$

where $\hat{\Delta}_x$ and $\hat{\Delta}_y$ are the trigonometric difference coefficients defined by equation (10). The terms $\left(w_{,xx}\right)_{ij}$ and $\left(w_{,yy}\right)_{ij}$ are defined at the full stations denoted by the circles in figure 2(b), while $\left(w_{,xy}\right)_{ij}$ is defined at the half stations denoted by the squares in figure 2(b). Consequently, the indices (i,j) attached to a variable may refer to the variable being evaluated at either full or half stations, depending on the variable.

Introducing equations (A2) and (A4) into equation (A1) and replacing the double integral by a double sum yields

$$\delta U = \Delta_x \Delta_y \sum_{j=1}^{N} \sum_{i=1}^{M} \left\{ \xi_{x_i} \xi_{y_j} \left[\frac{1}{\hat{\Delta}_x^2} M_{x_{ij}} \left(\delta w_{i+1,j} - 2\delta w_{ij} + \delta w_{i-1,j} \right) + \frac{1}{\hat{\Delta}_y^2} M_{y_{ij}} \left(\delta w_{i,j+1} \right. \right. \right.$$

$$\left. \left. \left. - 2\delta w_{ij} + \delta w_{i,j-1} \right) \right] + 2\eta_{x_i} \eta_{y_j} \frac{M_{xy_{ij}}}{\hat{\Delta}_x \hat{\Delta}_y} \left(\delta w_{i+1,j+1} - \delta w_{i,j+1} - \delta w_{i+1,j} + \delta w_{ij} \right) \right\} \tag{A5}$$

where N and M are the total number of finite-difference stations in the x- and y-directions, respectively, and ξ_{x_i}, ξ_{y_j}, η_{x_i}, and η_{y_j} have the following definitions:

$$\xi_{x_i} = \begin{cases} 0 & (i < I_1 \quad \text{or} \quad i > I_3) \\ 1/2 & (i = I_1 \quad \text{or} \quad i = I_3) \\ 1 & (I_1 < i < I_3) \end{cases} \tag{A6}$$

$$\xi_{y_j} = \begin{cases} 0 & (j < J_4 \quad \text{or} \quad j > J_2) \\ 1/2 & (j = J_4 \quad \text{or} \quad j = J_2) \\ 1 & (J_4 < j < J_2) \end{cases} \tag{A7}$$

$$\eta_{x_i} = \begin{cases} 0 & (i < I_1 \quad \text{or} \quad i \geqq I_3) \\ 1 & (I_1 \leqq i \leqq I_3) \end{cases} \tag{A8}$$

$$\eta_{y_j} = \begin{cases} 0 & (j < J_4 \quad \text{or} \quad j \geqq J_2) \\ 1 & (J_4 \leqq j < J_2) \end{cases} \tag{A9}$$

In equations (A6) to (A9), I_1 and I_3 are the row designations of boundaries ① and ③, respectively, and J_2 and J_4 are the column designations of boundaries ② and ④, respectively. (See fig. 2(a).)

Replacing the derivatives in equation (A3) by central trigonometric differences and the double integral by a double sum yields

$$\begin{aligned} \delta V_N = -\Delta_x \Delta_y \sum_{i=1}^{M} \sum_{j=1}^{N} \Bigg\{ & \xi_{y_j} \eta_{x_i} \frac{N_x}{\hat{\Delta}_x^2} \left(w_{i+1,j} - w_{ij}\right)\left(\delta w_{i+1,j} - \delta w_{ij}\right) + \xi_{x_i} \eta_{y_j} \frac{N_y}{\hat{\Delta}_y^2} \left(w_{i,j+1} \right. \\ & \left. - w_{ij}\right)\left(\delta w_{i,j+1} - \delta w_{ij}\right) + \eta_{x_i} \eta_{y_j} \frac{N_{xy}}{4\hat{\Delta}_x \hat{\Delta}_y} \Big[\left(w_{i+1,j} - w_{ij} + w_{i+1,j+1} - w_{i,j+1}\right)\left(\delta w_{i,j+1} \right. \\ & \left. - \delta w_{ij} + \delta w_{i+1,j+1} - \delta w_{i+1,j}\right) + \left(w_{i,j+1} - w_{ij} + w_{i+1,j+1} - w_{i+1,j}\right)\left(\delta w_{i+1,j} - \delta w_{ij} \right. \\ & \left. + \delta w_{i+1,j+1} - \delta w_{i,j+1}\right)\Big]\Bigg\} \end{aligned} \tag{A10}$$

In deriving equation (A10), the first and second terms in the integrand of equation (A3) have been replaced by trigonometric differences evaluated at stations indicated by "x" and "y," respectively, in figure 2(b), while the third and fourth terms have been evaluated at half stations, indicated by squares in figure 2(b), by averaging the derivatives.

The external forces and moments on the panel are those coming from discrete lateral deflection and rotational springs. The virtual work of these forces and moments may be expressed as

$$\begin{aligned} \delta V_S = & \sum_{i=1}^{M} \sum_{j=1}^{N} k_{\ell_{ij}} w_{ij} \delta w_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{k_{x_{ij}}}{\hat{\Delta}_x^2} \left(w_{i+1,j} - w_{ij}\right)\left(\delta w_{i+1,j} - \delta w_{ij}\right) \\ & + \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{k_{y_{ij}}}{\hat{\Delta}_y^2} \left(w_{i,j+1} - w_{ij}\right)\left(\delta w_{i,j+1} - \delta w_{ij}\right) \end{aligned} \tag{A11}$$

where k_ℓ is the spring stiffness associated with a lateral deflection spring and k_x and k_y are stiffnesses associated with rotational springs which resist moments acting about the Y- and X-axes, respectively. The k_ℓ type springs act at full stations, indicated by circles in figure 2(b), while the k_x and k_y type springs act at positions indicated by "x" and "y," respectively, in figure 2(b).

Substituting equations (A5), (A10), and (A11) into the statement of the principle of virtual work, equation (4) yields

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \left(K_{ij} + S_{ij} + N_x \alpha_{ij} + N_y \psi_{ij} + 2N_{xy} \chi_{ij}\right) \delta w_{ij} = 0 \tag{A12}$$

where

$$K_{ij} = \xi_{y_j} \frac{1}{\hat{\Delta}_x^{\,2}}\left(\xi_{x_{i+1}} M_{x_{i+1,j}} - 2\xi_{x_i} M_{x_{ij}} + \xi_{x_{i-1}} M_{x_{i-1,j}}\right) + \xi_{x_i} \frac{1}{\hat{\Delta}_y^{\,2}}\left(\xi_{y_{j+1}} M_{y_{i,j+1}} - 2\xi_{y_j} M_{y_{ij}} + \xi_{y_{j-1}} M_{y_{i,j-1}}\right) + \frac{1}{\hat{\Delta}_x \hat{\Delta}_y}\left(\eta_{x_{i-1}} \eta_{y_{j-1}} M_{xy_{i-1,j-1}} - \eta_{x_{i-1}} \eta_{y_j} M_{xy_{i-1,j}} - \eta_{x_i} \eta_{y_{j-1}} M_{xy_{i,j-1}} + \eta_{x_i} \eta_{y_j} M_{xy_{ij}}\right) \tag{A13}$$

$$\alpha_{ij} = \frac{1}{\hat{\Delta}_x^{\,2}}\left[\xi_{y_j} \eta_{x_i}\left(w_{i+1,j} - w_{ij}\right) - \xi_{y_j} \eta_{x_{i-1}}\left(w_{ij} - w_{i-1,j}\right)\right] \tag{A14}$$

$$\psi_{ij} = \frac{1}{\hat{\Delta}_y^{\,2}}\left[\xi_{x_i} \eta_{y_j}\left(w_{i,j+1} - w_{ij}\right) - \xi_{x_i} \eta_{y_{j-1}}\left(w_{ij} - w_{i,j-1}\right)\right] \tag{A15}$$

$$\chi_{ij} = \frac{1}{4\hat{\Delta}_x \hat{\Delta}_y}\left[\left(w_{i+1,j+1} - w_{ij}\right)\eta_{x_i}\eta_{y_j} - \left(w_{i+1,j-1} - w_{ij}\right)\eta_{x_i}\eta_{y_{j-1}} - \left(w_{ij} - w_{i-1,j-1}\right)\eta_{x_{i-1}}\eta_{y_{j-1}} + \left(w_{ij} - w_{i-1,j+1}\right)\eta_{x_{i-1}}\eta_{y_j}\right] \tag{A16}$$

$$S_{ij} = \frac{1}{\Delta_x \Delta_y} k_{\ell_{ij}} w_{ij} + \frac{1}{\Delta_x \Delta_y \hat{\Delta}_x^2}\left[k_{x_{i-1,j}}\left(w_{ij} - w_{i-1,j}\right) - k_{x_{ij}}\left(w_{i+1,j} - w_{ij}\right)\right]$$

$$+ \frac{1}{\Delta_x \Delta_y \hat{\Delta}_y^2}\left[k_{y_{i,j-1}}\left(w_{ij} - w_{i,j-1}\right) - k_{y_{ij}}\left(w_{i,j+1} - w_{ij}\right)\right] \tag{A17}$$

From equations (A2) and (A4), the moments are related to the displacements as follows:

$$\left.\begin{aligned}
\left(M_x\right)_{ij} &= \left(D_{11}\right)_{ij}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right)\frac{1}{\hat{\Delta}_x^2} + \left(D_{12}\right)_{ij}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right)\frac{1}{\hat{\Delta}_y^2} \\
\left(M_y\right)_{ij} &= \left(D_{22}\right)_{ij}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right)\frac{1}{\hat{\Delta}_y^2} + \left(D_{12}\right)_{ij}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right)\frac{1}{\hat{\Delta}_x^2} \\
\left(M_{xy}\right)_{ij} &= 2\left(D_{66}\right)_{ij}\left(w_{i+1,j+1} - w_{i,j+1} - w_{i+1,j} + w_{ij}\right)\frac{1}{\hat{\Delta}_x \hat{\Delta}_y}
\end{aligned}\right\} \tag{A18}$$

where $\left(M_x\right)_{ij}$ and $\left(M_y\right)_{ij}$ act at the full stations, indicated by circles in figure 2(b), and $\left(M_{xy}\right)_{ij}$ acts at the half stations, indicated by squares in figure 2(b).

Boundary Conditions

<u>All four boundaries free or spring-supported.</u>- If on the plate boundaries no constraints exist on w or its derivatives normal to the boundary, equation (A12) must be valid for all virtual displacements δw_{ij}, thus yielding equation (5) which is repeated here:

$$K_{ij} + S_{ij} + N_x \alpha_{ij} + N_y \psi_{ij} + 2N_{xy} \chi_{ij} = 0 \qquad \begin{pmatrix} i = 1, \ldots, M \\ j = 1, \ldots, N \end{pmatrix} \tag{A19}$$

Equation (A19) represents equilibrium at each finite-difference station with each equilibrium equation containing an array of 13 values of w as depicted in figure 2(b). In solving these equations by the procedure discussed in appendix C, the terms w_{ij} represent the unknowns and equations (A18) are used to determine the moments appearing in the relationship for K_{ij}, equation (A13).

When a difference station lies on the boundary of the plate (that is, $i = I_1$ or $i = I_3$ or $j = J_4$ or $j = J_2$), the corresponding equilibrium equation reduces to the natural boundary condition on the Kirchhoff shear, reference 6. Also, when a difference station lies one finite difference interval off the plate (that is, $i = I_1 - 1$ or $i = I_3 + 1$ or $j = J_4 - 1$ or $j = J_2 + 1$), the corresponding equilibrium equation reduces to the natural boundary condition on the bending moment. Furthermore, when a difference station lies two or more finite-difference intervals off the plate (that is, $i < I_1 - 1$ or $i > I_3 + 1$ or $j < J_4 - 1$ or $j > J_2 + 1$), the corresponding equilibrium equations reduce to the trivial equation $0 = 0$. Consequently, no equilibrium equations exist for these stations.

Edges with nondeflecting lateral supports and rotational springs.- Equation (A19) may be used in approximating the solution of problems with nondeflecting edges; for example, if $w = 0$ on an edge, equation (A19) may be used in conjunction with extremely stiff lateral springs placed along the edge. Alternatively, an edge which is restrained from lateral motion may be handled as a special case, and in so doing the number of computations required for the problem solution is reduced.

The boundary condition for a nondeflecting edge is

$$w = 0 \qquad \text{(on the edge)} \qquad \text{(A20)}$$

If, in addition, uniformly distributed rotational springs act along boundaries ① and ③ (see fig. 2(a)),

$$M_x = k_R w_{,x} \qquad \text{(on the edge)} \qquad \text{(A21)}$$

or, if uniformly distributed rotational springs act along boundary ② or ④,

$$M_y = k_R w_{,y} \qquad \text{(on the edge)} \qquad \text{(A22)}$$

As a result of the foregoing, equation (A20) replaces the boundary condition on the Kirchhoff shear, while the difference form of equation (A21) or (A22) replaces the boundary condition on the edge moment. Furthermore, as an example, equation (A21) on boundary ① becomes

$$\left(M_x\right)_{I_1,j} = \frac{k_R\left(w_{I_1+1,j} - w_{I_1-1,j}\right)}{\hat{\Delta}_x^*} \qquad \text{(A23)}$$

where

$$\frac{1}{\hat{\Delta}_x^*} = \frac{\pi}{2\lambda_x \sin \frac{\pi \Delta_x}{\lambda_x}} \tag{A24}$$

Substituting for M_x from equations (A18) and employing equation (A23) yields

$$\left(M_x\right)_{I_1,j} = \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}\left(w_{I_1+1,j} + w_{I_1-1,j}\right) = \frac{k_R}{\hat{\Delta}_x^*}\left(w_{I_1+1,j} - w_{I_1-1,j}\right) \tag{A25}$$

Then

$$w_{I_1-1,j} = \frac{\left[\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}\right] w_{I_1+1,j}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}} \tag{A26}$$

Substituting into the first of equation (A25) yields

$$\left(M_x\right)_{I_1,j} = \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}\left[1 + \frac{\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}}\right] w_{I_1+1,j} \tag{A27}$$

It is evident from an examination of the first of equation (A25) that equation (A23) is satisfied by setting $w_{I_1-1,j} = 0$ and $\left(D_{11}\right)_{I_1,j} = \left(D_{11}^*\right)_{I_1,j}$ where

$$\left(D_{11}^*\right)_{I_1,j} = \left[1 + \frac{\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x^{\,2}}}\right] \left(D_{11}\right)_{I_1,j} \tag{A28}$$

Similar relationships may be developed for boundaries ②, ③, and ④.

In summary, for a nondeflecting boundary with uniformly distributed rotational springs, equilibrium on the boundary and one station off the boundary are not used. Instead, in the remaining equilibrium equations, w on the boundary and one station off the boundary are set equal to zero and D_{11} on the boundary is set equal to D_{11}^* if the boundary is number ① or ③, and D_{22} on the boundary is set equal to D_{22}^* if the boundary is number ② or ④.

The limiting cases of simply supported or clamped boundaries are readily provided by letting k_R approach zero or infinity, respectively. Hence, for a simply supported boundary

$D_{11}^* = 0$ if the boundary is ① or ③

$D_{22}^* = 0$ if the boundary is ② or ④

and for a clamped boundary

$D_{11}^* = 2D_{11}$ if the boundary is ① or ③

$D_{22}^* = 2D_{22}$ if the boundary is ② or ④

Flexural Stiffeners

The effects of flexural stiffeners are accounted for in a manner similar to that used for nondeflecting supports. At each finite-difference station along the stiffener, $(D_{11})_{ij}$ is replaced by $(\overline{D}_{11})_{ij}$ if the stiffener is parallel to the X-axis and $(D_{22})_{ij}$ is replaced by $(\overline{D}_{22})_{ij}$ if the stiffener is parallel to the Y-axis, where

$$\left.\begin{aligned}(\overline{D}_{11})_{ij} &= (D_{11})_{ij} + \frac{EI}{\Delta_y}\\ (\overline{D}_{22})_{ij} &= (D_{22})_{ij} + \frac{EI}{\Delta_x}\end{aligned}\right\} \tag{A29}$$

and EI is the lateral bending stiffness of the stiffener about the neutral plane of the panel.

APPENDIX A

Summary of Finite-Difference Stations at Which Equilibrium Is Enforced

As a result of the foregoing discussions on free or spring-supported edges and nondeflecting edges, the rows i and columns j at which equilibrium is enforced are, respectively,

$$\left.\begin{aligned} M_e &= I_3 - I_1 + 3 - \text{Twice the number of nondeflecting edges parallel to the Y-axis} \\ N_e &= J_2 - J_4 + 3 - \text{Twice the number of nondeflecting edges parallel to the X-axis} \end{aligned}\right\} \tag{A30}$$

APPENDIX B

TRIGONOMETRIC FINITE DIFFERENCES

Trigonometric finite differences introduce the trigonometric parameters λ_x and λ_y which are not present in conventional finite differences. Consequently, the first purpose of this appendix is to present and demonstrate some effective procedures for selecting values of λ_x and λ_y which results in an improved convergence rate over conventional differences. The second purpose is to point out some of the limitations of trigonometric finite differences.

Selection of λ_x and λ_y

Selection of values of λ_x and λ_y which improve the convergence rate of trigonometric finite-difference solutions over those of conventional finite-difference solutions is predominantly based on engineering considerations and experience. Experience has shown that it is often advantageous to select trigonometric parameters whose ratio is determined on the basis of the infinitely long panel solution as is done in equations (11) and (12), that is,

$$\frac{\lambda_y}{\lambda_x} = \beta \tag{B1}$$

where β is the wavelength parameter of an infinitely long panel, defined as the ratio of the panel width to the buckle length. Imposing equation (B1) on the parameter selection should be reasonable for panels which buckle with more than two half waves along their length.

The value of β may be determined to any degree of accuracy by extending the isotropic results of reference 16. For a panel with its long dimension parallel to the X-axis, first approximations of the buckling eigenvalue $\overline{p}_\infty$ and wavelength parameter β satisfy the following two simultaneous equations for panels whose long sides are simply supported:

$$\left.\begin{aligned}
&\left(\overline{t}_{xy} + \overline{p}_\infty r_{xy}\right)^2 - \frac{9}{4} M_1 M_2 = 0 \\
&\frac{\partial}{\partial\beta}\left(M_1 M_2\right) = 0
\end{aligned}\right\} \tag{B2}$$

and, for panels whose long sides are clamped, $\bar{p}_\infty$ and β satisfy the two simultaneous equations

$$\left.\begin{aligned} &\left(t_{xy} + \bar{p}_\infty r_{xy}\right)^2 - \frac{15}{32}\left(2M_0 + M_2\right)\left(M_1 + M_3\right) = 0 \\ &\frac{\partial}{\partial\beta}\left(M_0 + M_2\right)\left(M_1 + M_3\right) = 0 \end{aligned}\right\} \tag{B3}$$

where

$$M_n = \frac{\pi}{8\beta}\left[\frac{D_{22}}{D_{11}} n^4 + 2\,\frac{D_3}{D_{11}} n^2\beta^2 + \beta^4 - \beta^2\left(\bar{t}_x + \bar{p}_\infty r_x\right) - n^2\left(\bar{t}_y + \bar{p}_\infty r_y\right)\right] \qquad (n = 0,\ 1,\ 2,\ 3)$$

Convergence Behavior

Figures 17(a) to 17(f) illustrate the convergence of trigonometric finite-difference solutions when λ_y/λ_x is fixed on the basis of equation (B1). Results for both simply supported and clamped isotropic panels under either axial compression or shear are shown in these figures. In each case the panel was modeled using an equal number of finite-difference stations in the x- and y-directions. Exact and approximate values for these cases are given in references 1, 6, 16, and 17.

The dashed curve in each of figures 17(a) to 17(f) illustrates the convergence of the conventional difference solution – that is, λ_x and λ_y infinite – while the solid and dash-dot curves illustrate the convergence achieved with some finite values of λ_x. Comparison of the curves indicates that some values of λ_x increase the convergence rate over the conventional rate while other values decrease it. (In those special cases where the buckle shape is exactly a double sine wave, the trigonometric difference solution is exact when λ_x and λ_y are equal to the buckle half wavelength.) Consider though the dash-dot curve of each figure. These curves show the convergence when λ_y is simply taken equal to the panel width and λ_x is taken equal to the buckle length of the infinitely long panel; that is, equations (11) and (12) are applied. Comparison of the dash-dot curves and the dashed curves indicates that equations (11) and (12) provide reasonable values of λ_x and λ_y which improve the solution convergence. As figures 17(a) to 17(f) indicate, however, other values of λ_x/a could be selected which further improve the convergence rate. Such values may be found by making a condensed cross plot of each figure; for example, consider the case of the compression of a square isotropic clamped panel as shown in figure 17(c). For this case, equations (B3) predict $\beta = 1.5$. Then, using program BOP with $\lambda_y/\lambda_x = 1.5$, λ_x/a is varied from 0.25 to 1 for mesh sizes of $a/\Delta_x = b/\Delta_y = 5$ and $a/\Delta_x = b/\Delta_y = 6$; these curves are shown in figure 18. As the mesh spacing is decreased, the curves will approach the exact solution at all values of λ_x/a. However, the two curves cross at $\lambda_x/a = 0.35$ and $\overline{N}_x = 9.75$, which implies that convergence is most rapid at this value of λ_x/a since increasing the mesh size did not change the buckling stress coefficient. It is evident from figure 17(c) that, if such a choice of λ_x were used, convergence would be improved beyond that achieved by selecting λ_x from equation (11).

As further examples, consider the results in table 4 for the shear buckling of the orthotropic panels described in table 3. The values of λ_x and λ_y were determined by making the required cross plots. It is evident by comparing the conventional and trigonometric solutions given in the table that the selected values of λ_x and λ_y provided excellent results.

The additional effort involved in finding better values of λ_x may be justified in problems where convergence would otherwise be extremely slow. It may also be justified in the performance of parameter studies. In such studies some typical problems within the problem class to be studied are chosen; for these, improved values of λ_x are found and then interpolated to yield λ_x for other problems within the study class.

Correction Factors for Equations (11) and (12)

Equations (B2) and (B3) which provide β for equations (11) and (12) do not cover every case; the boundary conditions may not be simply supported or clamped, or it may be inappropriate to use β based on an infinitely long panel. Consequently, equations (11) and (12) must be used with engineering judgment. Some allowance is provided by introducing correction factors C_x and C_{yx} into equations (11) and (12), that is,

$$\frac{\lambda_y}{\lambda_x} = C_{yx}\beta \tag{B4}$$

$$\frac{\lambda_x}{a} = \frac{b}{a}\frac{C_x}{\beta} \tag{B5}$$

A numerical routine which calculates β from equations (B2) or (B3), and then λ_x and λ_y from equations (11) and (12), is used in program BOP. This program is briefly discussed in the main text and is documented in appendix D.

Limitations of Trigonometric Finite Differences

In figure 19 a sketch of the variation with λ_x of the coefficient $1/\hat{\Delta}_x$ as defined by equation (10) is presented. The reader's attention is called to the singularities of

$1/\hat{\Delta}_x$ at $\lambda_x = \frac{\Delta_x}{2}, \frac{\Delta_x}{4}, \frac{\Delta_x}{8}$, etc. In order to avoid these singularities and the rapidly varying behavior of $1/\hat{\Delta}_x$ between them, λ_x and similarly λ_y must be chosen such that

$$\left.\begin{aligned} \lambda_x &> \frac{\Delta_x}{2} \\ \lambda_y &> \frac{\Delta_y}{2} \end{aligned}\right\} \tag{B6}$$

Moreover, if uniformly distributed rotational springs are prescribed on the boundaries in the manner presented in equations (A20) to (A24), then to avoid singularities in $\hat{\Delta}_x^*$ and $\hat{\Delta}_y^*$ choose

$$\left.\begin{aligned} \lambda_x &> \Delta_x \\ \lambda_y &> \Delta_y \end{aligned}\right\} \tag{B7}$$

APPENDIX C

STABILITY DETERMINANT EVALUATION

Since the total number of rows and columns at which equilibrium is enforced is M_e and N_e, respectively, a stability determinant of order $M_eN_e \times M_eN_e$ would result. To produce a stability determinant of smaller size, a marching procedure is employed. This procedure, which is described herein, operates on the equilibrium equations to produce, by a process of successive elimination, a determinant of size $2M_e \times 2M_e$.

The marching procedure takes advantage of the fact that each of the difference equations of equilibrium, equations (5), is linear and homogeneous, with each one containing no more than 13 unknown deflections. For a station (i,j) away from the plate edges

$$I_f + 1 < i < I_\ell - 1$$

$$J_f + 1 < j < J_\ell - 1$$

where I_f and I_ℓ are the first and last rows of finite-difference stations at which equilibrium is prescribed, and J_f and J_ℓ are the first and last columns of finite-difference stations at which equilibrium is prescribed, the 13 unknown deflections form the geometric pattern shown in figure 2(b). It is evident from this pattern that the deflections at stations in column $j + 2$ can be determined by using equilibrium at stations in column j if the deflections in columns $j - 2$, $j - 1$, j, and $j + 1$ are known or prescribed. For equilibrium at stations lying near the edges, however, the geometric pattern of figure 2(b) is reduced. Consequently, equilibrium at stations in the first column J_f may be used to determine the deflections at stations in column $J_f + 2$ if the deflections only in columns J_f and $J_f + 1$ are prescribed, since deflections in columns $J_f - 1$ and $J_f - 2$ do not appear in these equilibrium equations.

Having found the deflections in column $J_f + 2$ from prescribed values in column J_f and $J_f + 1$, equilibrium at stations in column $J_f + 1$ can be used to obtain the deflections in column $J_f + 3$; likewise, equilibrium at stations in column $J_f + 2$ can provide deflections in column $J_f + 4$, etc. Thus, a marching routine is developed from column to column which determines the deflections throughout the panel from prescribed values in the first two columns. It should be noted that equilibrium at stations in the last two columns, $J_\ell - 1$ and J_ℓ, is not used at this stage of the marching procedure.

The evaluation of the stability determinant can now be performed numerically for a given value of the eigenvalue by choosing $2M_e$ linearly independent sets of assumed deflections for the first two columns. These assumed sets are taken as

$$\left[w^{(1)}\right] = \begin{bmatrix}1\\0\\0\\0\\.\\.\\.\\0\end{bmatrix} \qquad \left[w^{(2)}\right] = \begin{bmatrix}0\\1\\0\\0\\.\\.\\.\\0\end{bmatrix} \qquad \left[w^{(3)}\right] = \begin{bmatrix}0\\0\\1\\0\\.\\.\\.\\0\end{bmatrix}, \ldots, \left[w^{(2M_e)}\right] = \begin{bmatrix}0\\0\\0\\0\\.\\.\\.\\1\end{bmatrix} \tag{C1}$$

where each column contains $2M_e$ values. By marching across the plate with the rth set of these assumed values, deflections throughout the plate $w_{ij}^{(r)}$ are determined. However, the equilibrium equation at stations in the last two columns will not, in general, be satisfied by any of these assumed sets. Therefore, consider the column matrix

$$\left\{e^{(r)}\right\} = \begin{bmatrix} e^{(r)}_{I_f,J_\ell-1} \\ . \\ . \\ . \\ e^{(r)}_{I_\ell,J_\ell-1} \\ e^{(r)}_{I_f,J_\ell} \\ . \\ . \\ . \\ e^{(r)}_{I_\ell,J_\ell} \end{bmatrix} \tag{C2}$$

where each element of the matrix represents the value of the left-hand side of an equilibrium equation at a station in columns $J_\ell - 1$ or J_ℓ for the rth assumed set and would be identically zero if the assumed deflections were exact. The total solution is a linear superposition of all the assumed sets, that is,

$$w_{ij} = \sum_{r=1}^{2M_e} A^{(r)} w_{ij}^{(r)} \qquad \begin{pmatrix} I_f \leqq i \leqq I_\ell \\ J_f \leqq j \leqq J_\ell \end{pmatrix} \tag{C3}$$

Correspondingly, the total contribution to equilibrium at columns $J_\ell - 1$ and J_ℓ for all assumed sets of deflections is

$$[e] = \sum_{r=1}^{2M_e} A^{(r)} \left\{e^{(r)}\right\} \tag{C4}$$

The coefficients $A^{(r)}$ are determined by enforcing equilibrium at stations in the last two columns which leads to

$$[e] = 0 \tag{C5}$$

or

$$\left[e^{(1)} \middle| e^{(2)} \middle| \; . \; . \; . \; \middle| e^{(r)} \middle| \; . \; . \; . \; \middle| e^{(2M_e)}\right] \begin{bmatrix} A^{(1)} \\ A^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ A^{(r)} \\ \cdot \\ \cdot \\ \cdot \\ A^{(2M_e)} \end{bmatrix} = 0 \tag{C6}$$

For a nontrivial solution of equation (C6) the determinant of the coefficients must vanish, resulting in

$$|e| = 0 \tag{C7}$$

and it is clear from equation (C6) that $|e|$ is of order $2M_e \times 2M_e$.

APPENDIX D

COMPUTER PROGRAM

The computer program BOP (Buckling of Orthotropic Panels) was written in FORTRAN IV on a SCOPE 3.1 system modified for Langley Research Center and executes and loads with a field length of 60000 octal locations. The program is applicable to the combined compression and shear of stiffened, variable-thickness, flat rectangular orthotropic panels on discrete springs; boundary conditions are general and include elastic boundary restraints. A description of the input, an example problem showing input and output, and a program listing are provided.

Input Description

For each case the input consists of a single identification card and a Namelist BUCKLE as follows:

ISTIFF,ISTEP,IX,JX,MSHAPE,MA,NOMAT,TH,AT,MATYPE,E1,E2,U1,G12,IBC,AKR,D1, D2,D12,D66,DS1,XA,XB,AKL,AKX,AKY,NUPRIT,EI,IORIENT,LOC,TX,TY,TXY,RX,RY, RXY,P1,DELP,PFIN,TEST,MR,NC,X,Y,DS2,DS12,DS66

Many of the input variables have associated default values as will be indicated in the following descriptions:

Control parameters

ISTIFF = 1	no preprocessing of laminate properties – execute for buckling (only)
= 2	preprocess and execute for buckling
= 3	preprocess only – do not execute for buckling

DEFAULT:ISTIFF = 2

ISTEP = 1	program automatically varies the input step size, DELP
= 2	step size fixed and equal to DELP

DEFAULT:ISTEP = 1

IX = 1	output of intermediate results
= 2	output of intermediate results suppressed
JX = 1	output of flexural stiffnesses at each finite-difference station
= 2	output of flexural stiffnesses suppressed

DEFAULT:IX = JX = 2

MSHAPE = 1 compute mode shape

= 2 do not compute mode shape

DEFAULT:MSHAPE = 2

Laminate and lamina properties (Required if ISTIFF = 2 or 3)

MA number of laminas in the laminate

NOMAT number of different materials comprising the laminate

TH a one-dimensional array in which the ith element of the array corresponds to the filament orientation (as measured from the X-axis in degrees) in the ith lamina

AT a one-dimensional array in which the ith element of the array corresponds to the thickness of the ith lamina

MATYPE a one-dimensional array in which the ith element is the number designation of the material in the ith lamina

E1 a one-dimensional array in which the jth element of the array corresponds to the Young's modules parallel to the fibers in the jth material

E2 a one-dimensional array specifying the Young's modulus transverse to the fibers

U1 a one-dimensional array specifying Poisson's ratio ν_{12} in each lamina

G12 a one-dimensional array specifying the shear modulus in each material

Boundary conditions

IBC a one-dimensional array of four elements in which the ith element refers to the ith boundary (see fig. 2(a)); four options are available at each boundary

IBC(I) = 1 nondeflecting lateral support with uniform rotational springs on edge I

= 2 simple support on edge I

= 3 clamped on edge I

= 4 free on edge I

= 5 other boundary conditions – set by user through appropriate input of D1, D2, D12, and D66

AKR — a one-dimensional array in which the ith element of the array corresponds to the uniformly distributed rotational spring stiffness per unit length of boundary on the ith boundary; required if any boundary has IBC = 1

Laminate flexural stiffnesses (Required if ISTIFF = 1)

D1 — a two-dimensional array in which the (i,j)th element of the array corresponds to the value of $(D_{11})_{ij}$

D2 — similar to D1, but specifying $(D_{22})_{ij}$

D12 — similar to D1, but specifying $(D_{12})_{ij}$

D66 — similar to D1, but specifying $(D_{66})_{ij}$

DS1 — reference value of D_{11}

Plate geometry

XA = a — dimension parallel to X-axis (fig. 2(a))

XB = b — dimension parallel to Y-axis (fig. 2(a))

Discrete springs

AKL — a two-dimensional array in which the (i,j)th element corresponds to $(k_{\ell})_{ij}$

AKX — similar to AKL but referring to $(k_x)_{ij}$

AKY — similar to AKL but referring to $(k_y)_{ij}$

Discrete flexural stiffeners

NUPRIT — number of stiffeners

EI — a one-dimensional array whose ith element specifies the flexural stiffness of the ith stiffener about the neutral plane of the panel

IORIENT — a one-dimensional array whose ith element specifies whether the stiffener is parallel to X- or Y-axis

= 1 stiffener parallel to X-axis

= 2 stiffener parallel to Y-axis

LOC — a one-dimensional array whose ith element gives the row or column location of the ith stiffener

DEFAULT:NUPRIT = 0; EI, IORIENT and LOC need not be input

Applied in-plane loads

In-plane loads are assumed to be uniform over the boundary to which they are applied and are increased to buckling according to the relationships prescribed by equations (13); therefore, the user inputs

TX = $\bar{t}_x$

TY = $\bar{t}_y$

TXY = $\bar{t}_{xy}$

RX = r_x

RY = r_y

RXY = r_{xy}

Eigenvalue search parameters

P1 — starting value of $\bar{p}$. If P1 < 0., the program will calculate P1 from equation (B2) or (B3) according to the relation,

$$P1 = ABS(P1)*PBAR \tag{D1}$$

where PBAR is $\bar{p}_\infty$ from equation (B2) or (B3).

DEFAULT:P1 = 0.9*PBAR

DELP — increment of ($\bar{p}$); if P1 < 0., DELP = 0.1*PBAR; if ISTEP = 1, DELP is automatically varied during the eigenvalue search

PFIN — maximum value of $\bar{p}$ during the eigenvalue search

TEST — eigenvalue accuracy

DEFAULT:1. $\times 10^{-3}$

Trigonometric finite-difference data

MR — number of rows of finite-difference stations interior to the plate – not including boundaries

NC — number of columns of finite-difference stations interior to the plate – not including boundaries

Note: The marching procedure requires NC $\geqq$ 4

X = λ_x/a

Y = λ_y/b

Note: If the user inputs $X \leq 0$, the program automatically calculates a new value of X and Y according to the relationship expressed by equations (B4) and (B5); that is,

$$X = ABS(X)*XB/BETA/XA \tag{D2}$$

$$Y = ABS(Y) \tag{D3}$$

where the input magnitudes of X and Y (that is, ABS(X) and ABS(Y)) replace C_x and C_{yx} in equations (B4) and (B5). Also, in equation (D1), BETA = β, and β is calculated from equation (B2) or (B3).

When ISTIFF = 1 and the evaluation of X and Y is chosen, the user must also input

DS2	average or typical value of D_{22}
DS12	average or typical value of D_{12}
DS66	average or typical value of D_{66}

DEFAULT: Calculation of X and Y using equations (D2) and (D3) where ABS(X) and ABS(Y) are set equal to unity.

Example Problem

Consider the shear buckling of a 12-inch by 3-inch clamped sandwich panel which has as its lay-up, 45/-45/45/-45/CORE/-45/45/-45/45. The core thickness is 0.0605 inch and each lamina of the skins is graphite-epoxy with a thickness of 0.0055 inch.

Sample Input

```
THIS IS A FREE FIELD IDENTIFICATION CARD
$BUCKLE TX=.0,TY=.0,TXY=.0,RX=.0,RY=.0,RXY=1.,
XA=12,XB=3.,MR=12,NC=6,IBC=4*3,NOMAT=2,E1=2.10E7,1.,E2=2.39E6,1.,
U1=.31,.2,G12=6.5E5,1.,MA=9,MATYPE=4*1,2,4*1,
AT=4*.0055,.0605,4*.0055,TH=45.,-45.,45.,-45.,.0,-45.,45.,-45.,45.
$
```

Sample Output

```
INPUT FOR CASE

 THIS IS A FREE FIELD INDENTIFICATION CARD




                                          111111111111***** Y
                                          4          2
                                          4          2
                                          4          2
                                          4          2
                                          4          2
                                          4          2
                                          333333333333
                                          *
                                          *
                                          *
                                          *
                                          *

                                          X


     ISTIFF=2    ISTEP=1         IX=2    JX=2          TEST=   1.00000000E-03

     XA=   1.20000000E+01      XB=   3.00000000E+00      ASPECT RATIO=   4.00000000E+00

               MR=12      NC= 6

     TX=   0.                  TY=   0.                  TXY=   0.

     PI= -9.00000000E-01       DELP=   1.00000000E-01    PFIN=   1.00000000E+02

     X= -1.00000000E+00        Y=  1.00000000E+00

     RX=   0.                  RY=   0.                  RXY=   1.00000000E+00
```

LAMINATED PLATE PROPERTIES

MATERIAL KIND	E1	E2	U1	GXY
1	2.10000000E+07	2.39000000E+06	3.10000000E-01	6.50000000E+05
2	1.00000000E+00	1.00000000E+00	2.00000000E-01	1.00000000E+00

LAYER NO.	MAT. KIND	THICK	THETA
1	1	5.50000000E-03	4.50000000E+01
2	1	5.50000000E-03	-4.50000000E+01
3	1	5.50000000E-03	4.50000000E+01
4	1	5.50000000E-03	-4.50000000E+01
5	2	6.05000000E-02	0.
6	1	5.50000000E-03	-4.50000000E+01
7	1	5.50000000E-03	4.50000000E+01
8	1	5.50000000E-03	-4.50000000E+01
9	1	5.50000000E-03	4.50000000E+01

****

CAUTION -- COUPLING BETWEEN EXTENSION AND BENDING MAY BE SIGNIFICANT
IF THE FOLLOWING FOUR VALUES ARE NOT ALL EQUAL.
IF THIS IS THE CASE, THE RESULTS SHOULD BE USED WITH DESCRETION

4.76775528E-17   7.15163293E-17   8.80102296E-17   5.97234077E-17

****

A MATRIX

3.05215229E+05	2.48015179E+05	0.
2.48015179E+05	3.05215229E+05	0.
0.	0.	2.43655139E+05

B MATRIX

1.45519152E-11	2.18278728E-11	8.00355338E-11
2.18278728E-11	2.18278728E-11	7.27595761E-11
7.27595761E-11	7.27595761E-11	1.45519152E-11

D MATRIX

5.31653046E+02	4.32016589E+02	4.69571561E+01
4.32016589E+02	5.31653046E+02	4.69571561E+01
4.69571561E+01	4.69571561E+01	4.24421785E+02

OVERALL LAMINATE PROPERTIES

```
EX=  9.92156003E+05  EY=  9.92156003E+05  GXY=  2.33162813E+06
NUXY=  8.12591100E-01  NUYX=  8.12591100E-01
```

```
***********************************************************************

    P=  1.30776204E+01  B=  9.57954000E-01  F= -1.83044904E-08G=  3.83577069E-03

    PROGRAM HAS COMPUTED AND USED X=  2.60972865E-01
    AND Y=  1.00000000E+00 BASED ON THE INFINITE PLATE WAVE LENGTH

    BASED ON AN INFINITELY LONG PLATE THE BUCKLING STRESS COEFFICIENTS ARE
    NXBAR=  0.
    NYBAR=  0.
    NXYBAR=  1.30776204E+01
    AND THE STRAINS ARE
    STRNX=  0.
    STRNY=  0.
    STRNXY=  3.12923870E-02

PROGRAM WILL NOW CONTINUE WITH FINITE ASPECT RATIO SOLUTION

***********************************************************************
```

BOUNDARY CONDITIONS

BOUNDARY NO. 1 IS CLAMPED

BOUNDARY NO. 2 IS CLAMPED

BOUNDARY NO. 3 IS CLAMPED

BOUNDARY NO. 4 IS CLAMPED

NXBAR	NYBAR	NXYBAR	DETERMINANT
0.	0.	1.17698583E+01	6.54187653E+12
0.	0.	1.19006345E+01	2.18161022E+12
0.	0.	1.20314107E+01	2.37244168E+11
0.	0.	1.20473676E+01	1.33435159E+11
0.	0.	1.20687996E+01	3.35545245E+10
0.	0.	1.20761369E+01	9.37018457E+09
0.	0.	1.20793082E+01	0.

BUCKLING LOADS PER UNIT OF LENGTH ALONG BOUNDARY EDGE

NX= 0. NY= 0. NXY= 7.04251216E+03

```
(XB**2)*NX/(PI**2)/(T**3) =   0.
(XB**2)*NY/(PI**2)/(T**3) =   0.
(XB**2)*NXY/(PI**2)/(T**3) =   5.62757767E+06
```

```
(XB**2)*EPSILONX/(T**2) =  -0.
(XB**2)*EPSILONY/(T**2) =  -0.
(XB**2)*EPSILONXY/(T**2) =  -1.19105540E+01
```

APPENDIX D

```
      PROGRAM BOP(INPUT=201,OUTPUT=201,TAPE5=INPUT,TAPE6=OUTPUT,        100000
     1TAPE7=201)                                                        200000
C                                                                       300000
C  NASA - LANGLEY RESEARCH CENTER - PROGRAM                             400000
C                                                                       500000
C     THE PROGRAM FINDS THE BUCKLING LOADS (COMPRESSIVE OR SHEAR) OF ORTHOTROPIC  600000
C     PANELS WHOSE BOUNDARY CONDITIONS ON EACH EDGE MAY BE EITHER SIMPLY  700000
C     SUPPORTED, CLAMPED-OR ELASTICALLY CONSTRAINED BY ROTATIONAL SPRINGS.  800000
C     THE ORTHOTROPIC ELASTIC PROPERTIES OF THE PANEL MAY BE EITHER DIRECTLY  900000
C     SUPPLIED BY THE USER OR MAY BE INTERNALLY PREPROCESSED BY THE PROGRAM  1000000
C     FOR LAMINATED PANELS.                                            1100000
C                                                                      1200000
C     LAMINATE DEFINITIONS                                             1300000
C     ********************                                             1400000
C                                                                      1500000
C     MA=NUMBER OF LAMINAS IN THE LAMINATE                             1600000
C     NOMAT=NUMBER OF DIFFERENT TYPES OF MATERIAL                      1700000
C     TH=FIBER ORIENTATION (IN DEGREES)  MEASURED FROM X AXIS          1800000
C     MATYPE=TYPE OF MATERIAL IN EACH LAMINA                           1900000
C                                                                      2000000
C     BOUNDARY CONDITIONS                                              2100000
C     ********************                                             2200000
C                                                                      2300000
C     IBC=1, ROTATIONAL SPRING SUPPORT                                 2400000
C     IBC=2, SIMPLE SUPPORT                                            2500000
C     IBC=3, CLAMPED SUPPORT                                           2500001
C     IBC=4, FREE BOUNDARY                                             2600000
C     IBC=5, BOUNDARY CONDITION ENTERED THRU D MATRICES                2700000
C     AKR=ROTATIONAL SPRING STIFFNESS PER UNIT LENGTH OF BOUNDARY      2800000
C                                                                      2900000
C     CONTROL OPTIONS                                                  3000000
C     ***************                                                  3100000
C                                                                      3200000
C     ISTIFF=1, DO NOT PREPROCESS, BUT EXECUTE FOR BUCKLING            3300000
C     ISTIFF=2, PREPROCESS AND EXECUTE FOR BUCKLING (DEFUALT)          3400000
C     ISTIFF=3, PREPROCESS ONLY, DO NOT EXECUTE FOR BUCKLING           3500000
C     MSHAPE=1, EXECUTE FOR BUCKLING AND EXECUTE MODE SHAPE OPTION     3600000
C     MSHAPE=2, DO NOT EXECUTE MODE SHAPE OPTION                       3700000
C     MSHAPE=3, DO NOT EXECUTE FOR BUCKLING, BUT DO EXECUTE MODE SHAPE OPTION.  3800000
C     ITSEP=1, PROGRAM AUTOMATICALLY VARIES THE INITIAL STEP SIZE,DELP. (DEFAULT  3900000
C     ISTEP=2, STEP SIZE FIXED AND EQUAL TO DELP                       4000000
C                                                                      4100000
```

```
             C                                                                                  4200000
             C                                                                                  4300000
000003             DIMENSION A(56,56),W(32,32),CASE(8),PP(200),DET(200)                         4400000
000003             DIMENSION D1(32,32),D2(32,32),D12(32,32),D66(32,32) ,IBC(4),AKS(4)           4500000
                  1,TH(100),AT(100),MATYPE(100),E1(10),E2(10),U1(10),G12(10)                    4600000
                  2,EI(5),IORIENT(5),LOC(5),B(56,1),IPIVOT(56)                                  4700000
                  * ,AKR(4),WK(56)                                                              4800001
000003             EQUIVALENCE (AKR,AKS)                                                        4900000
             C                                                                                  5000000
             C                                                                                  5100000
             C                                                                                  5200000
000003             COMMON/LAYER/NOMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                              5300000
000003             COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),                 5400000
                  1ZIY(32),ETAX(32),ETAY(32)                                                    5500000
000003             COMMON/SPRING/AKL(32,32),AKX(32,32),AKY(32,32)                               5600000
000003             COMMON/XY/TX,TY,TXY,RX,RY,RXY,PI,ADB,DS1,DS2,DS12,DS66,                      5700000
                  1IBC2,AKS2,XB,EX,EY,EXY,GXY,TT                                                5800000
000003             NAMELIST/BUCKLE/ISTIFF,IX,JX,TX,TY,TXY,RX,RY,RXY,                            5900000
                  1 XA,XB,MR,NC,X,Y,TEST,                                                       6000001
                  2 AKR,IBC,D1,D2,D12,D66,P1,DELP,PFIN,                                         6100000
                  3 NOMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                                          6200000
                  4,NUPRIT,EI,IORIENT,LOC,MSHAPE,ISTEP                                          6300000
                  5,AKL,AKX,AKY,DS1,DS2,DS12                                                    6400000
000003            ABSCISA(X3,X2,X1,Y3,Y2,Y1)=X3+((Y1*Y3-Y2*Y3)*(X3-X1)*(X2-X3))/(Y1*00000026   6600000
                  1Y2*(X1-X2)+Y1*Y3*(X3-X1)+Y2*Y3*(X2-X3))                      00000027        6700000
             C                                                                                  6800000
             C     SET DEFAULTS                                                                 7000000
000032             NUPRIT=0                                                                     7100000
000033             X=-1.                                                                        7200000
000035             Y=1.                                                                         7300000
000036             MSHAPE=2                                                                     7400000
000037             ISTIFF=2                                                                     7500000
000040             IX=JX=2                                                                      7600000
000042             ISTEP=1                                                                      7700000
000043             AKS(2)=.0                                                                    7800000
000044             P1=-.9                                                                       7900001
000046             PFIN=100.                                                                    8000000
000047             INCX=0                                                                       8100000
000050             ICNT=1                                                                       8200000
000051             DELP=.1                                                                      8300000
000052             P=1.                                                                         8300001
000053             TEST=1.E-3                                                                   8300002
000055             DO 9 I=1,32                                                                  8400000
```

```
000056            DO 9 J=1,32
000057          9 AKL(I,J)=AKX(I,J)=AKY(I,J)=.0
            C       END OF DEFUALT LIST
            C
000070          1 IF(ICNT .GT. 1)GO TO 2
000074          7 READ(5,5001) CASE
000102            IF(EOF,5) 4,5
000105          4 PRINT 8997
000111            STOP
000113          5 PRINT 5032
000117            WRITE(6,5000) CASE
000125            WRITE(7,5000) CASE
000133            PRINT 5025
000137            READ BUCKLE
000142            WRITE(7,8998) MR,NC,X,Y
000156       8998 FORMAT(5X,*MR=*,I2,* NC=*,I2,*   X=*,E16.8,*   Y=*,E16.8/)
000156            ADB=XA/XB
000160            WRITE(7,3999) TH(1),ADB
000170       8999 FORMAT(/,5X,*TH(1)=*,E16.8,5X,*ASPECT RATIO=*,E16.8/)
000170            ADB2=ADB*ADB
000172            EL=MR+1
000174            B=NC+1
000176            PIS=PI
000200            DELPS=DELP
000201            PFINS=PFIN
000203            XSS=X
000204            PRINT 5021,ISTIFF,ISTEP,IX,JX,TEST
000222            IF(ISTIFF .EQ. 3 .OR. ISTEP .EQ. 3)GO TO 98
000231            PRINT 5022,XA,XB,ADB,MR,NC
000246            PRINT 5023,TX,TY,TXY,P1,DELP,PFIN,X,Y,RX,RY,RXY
000300            GO TO 97
000301         98 PRINT 5038
000305         97 CONTINUE
000305            CALL PREP(D1,D2,D12,D66,DS1,DS2,DS12,DS66,IX,JX,MR,NC,TT,EX,EY,EXY,GXY
                 1,GXY,IBC,ISTIFF)
000330            MR1=MRTOT
000332            NC1=NCTOT
000333        105 CONTINUE
000333            PI=3.14159265359
000335            PI2=PI*PI
000336            IBC2=IBC(2)
000337            AKS2=AKS(2)
000341            IF(INCX .EQ. 0)GO TO 249
```

```
000342       2 X=XSTART+(ICNT-1)*XDELTA                                     12700000
000350         IF(X .GT. XSTOP)GO TO 7                                      12800000
000353         IF(ICNT .EQ. 1)CALL AUTOXY(XZ,YZ,BETA,INCX)                  12900000
000357         Y=AD9*X*DELTA                                                13000000
000362         ICNT=ICNT+1                                                  13100000
000363         PRINT 8998,MR,NC,X,Y                                         13200000
000377         WRITE(7,8998) MR,NC,X,Y                                      13300000
000413         GO TO 168                                                    13400000
000414     249 IF(X .LE. 0. .OR. PI .LT. 0.)CALL AUTOXY(X,Y,BETA,INCX,P)    13500001
000427     168 CONTINUE                                                     13600000
000427         PRINT 5010                                                   13700000
000433         S2X=SIN(.5*PI/X/EL)                                          13800000
000441         SX=SIN(PI/X/EL)                                              13900000
000446         S2Y=SIN(.5*PI/Y/B)                                           14000000
000454         SY=SIN(PI/Y/B)                                               14100000
000461         IF(NUPRIT .EQ. 0)GO TO 113                                   14400000
000462         PRINT 5027                                                   14500000
000465         DO 112 I=1,NUPRIT                                            14600000
000467         EH=EI(I)                                                     14700000
000471         IF(IORIENT(I) .EQ. 2)GO TO 114                               14800000
000473         EPS=XA/EL                                                    14900000
000475         EI(I)=EH/EPS/DS1                                             15000000
000500         R=1.+EI(I)                                                   15100000
000502         CALL SET(D1,1,IBPOS1,IBPOS3,LOC(I),R)                        15200000
000506         GO TO 112                                                    15300000
000507     114 EPS=XB/B                                                     15400000
000511         EI(I)=EH/EPS/DS1                                             15500000
000515         R=DS2+EI(I)                                                  15600000
000517         CALL SET(D2,2,IBPOS4,IBPOS2,LOC(I),R)                        15700000
000523     112 PRINT 5026,I,IORIENT(I),LOC(I),EH,EI(I)                      15800000
000544     113 CONTINUE                                                     15900000
000544         CALL BC(X,Y,EL,B,AKS,DS1,DS2,DS12,D1,D2,D12,D66,MR,NC,IBC,   15900001
              * XA,XB)                                                      15900002
000565         IF(JX .EQ. 2)GO TO 125                                       16000000
000567         PRINT 5016                                                   16100000
000573         PRINT 5015, ((D1(I,J),J=1,NC1),I=1,MR1)                      16200000
000612         PRINT 5017                                                   16300000
000616         PRINT 5015, ((D2(I,J),J=1,NC1),I=1,MR1)                      16400000
000635         PRINT 5018                                                   16500000
000641         PRINT 5015, ((D12(I,J),J=1,NC1),I=1,MR1)                     16600000
000660         PRINT 5019                                                   16700000
000664         PRINT 5015, ((D66(I,J),J=1,NC1),I=1,MR1)                     16800000
000703     125 CONTINUE                                                     16900000
```

```
000703            IF(ISTIFF .EQ. 3)GO TO 84                                          17000000
            C                                                                        17100000
            C     BEGIN BUCKLING ANALYSIS                                            17200000
            C                                                                        17300000
000705            IF(X .GE. 100. .OR. Y .GE. 100.)GO TO 122                          17400000
            C     EDL=EHATX/EHATY*ADB                                                17500000
000716            SX=SIN(.5*PI/EL/X)                                                 17600000
000724            XSX=2.*X*SX/PI                                                     17700000
000727            EDL=X/Y*ADB*SX/SIN(.5*PI/B/Y)                                      17800000
000737            EDL1=2.*X/Y*ADB*SX/SIN(PI/B/Y)                                     17900000
000751            EDE=2.*SX/SIN(PI/EL/X)                                             18000000
000760            GO TO 123                                                          18100000
000760       122  EDE=1.                                                             18200000
000762            XSX=1./EL                                                          18300000
000764            EDL=EDL1=ADB*B/EL                                                  18400000
000767       123  MR2=2*(MR1-4)                                                      18500000
000771            PIEL=PI**2                                                         18600000
000773            TXSS=TX*PIEL                                                       18700000
000774            TYSS=TY*PIEL                                                       18800000
000776            TXYSS=TXY*PIEL                                                     18900000
000777            IF(MSHAPE .EQ. 3)GO TO 84                                          19000000
001001       120  P1=P1S*PIEL                                                        19100000
001003            DELP=DELPS*PIEL                                                    19200000
001005            PFIN=PFINS*PIEL                                                    19300000
001006            IF(P1 .GT.0.)GO TO 129                                             19300001
001011            P1=ABS(P1)*P                                                       19300002
001012            DELP=.01*P*PIEL                                                    19300003
001015       129  CONTINUE                                                           19300004
001015            P1B=P1                                                             19400000
001017            PRINT 61                                                           19500000
001022      7700  L=0                                                                19600000
001023            LC=1                                                               19700000
001024      7702  CONTINUE                                                 00000108  19800000
001024            P1=P1B*ADB2                                                        19900000
001026            TXS=TXSS*ADB2+RX*P1                                                20000000
001032            TYS=TYSS*ADB2+RY*P1                                                20100000
001035            TXYS=TXYSS*ADB2+RXY*P1                                             20200000
001041        77  CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE,  20300000
                 1 2,IBC)                                                            20400000
001062      4002  L=L+1                                                              20500000
001064            CALL DEUPPS(A,MR2,CDET,56)                                         20600000
001067            DET(L)=CDET                                                        20700000
001071            PP(L)=P1B/PIEL                                                     20800000
```

```
001073          TXS=TXS/PIEL/ADB2                                              20900000
001076          TYS=TYS/PIEL/ADB2                                              21000000
001077          TXYS=TXYS/PIEL/ADB2                                            21100000
001101       80 PRINT 6,TXS,TYS,TXYS,DET(L)                                    21200000
001115          IF(L.LT.3)GO TO 90                                             21300000
001120          AD1=DET(L)*DET(L-1)                                            21400000
001122          AD2=DET(L)*DET(L-2)                                            21500000
001124          IF(ABS(DELP) .LT. PP(L)*TEST*PIEL)GO TO 155                    21600001
001131          IF(ISTEP .EQ. 2)GO TO 154                                      21700000
001133          IF(LC .EQ. 21)GO TO 155                                        21800000
001135          DELP=DET(L)*DELP/(DET(L-1)-DET(L))                             21900000
001141          IF(DELP .LT. 0. .AND. LC .GT. 5)GO TO 155                      22000000
001152          IF(DELP .GT. 10.*DELPS*PIEL)DELP=10.*DELPS*PIEL                22100000
001157          IF(ABS((PP(L-1)-PP(L))/PP(L)).GT. .01 .AND. LC .EQ. 1)GO TO 154 22200000
001174          IF(LC .GE. 15)GO TO 156                                        22300000
001176          DEL1=(PP(L)-PP(L-1))*PIEL                                      22400000
001202          DEL2=(PP(L-1)-PP(L-2))*PIEL                                    22500000
001204          DEL1P2=DEL1+DEL2                                               22600000
001206          FD=(DET(L)-DET(L-1))/(PP(L)-PP(L-1))/PIEL                      22700000
001213          FDD=(DEL1*DET(L-2)-DEL1P2*DET(L-1)+DEL2*DET(L))/(.5*DEL1*DEL2  22800000
              1 *DEL1P2)                                                       22900000
001225          DELP=-DET(L)/FD - FDD*(DET(L)**2)/(8.*FD**3)                   23000000
001235          IF(ABS(DELP) .LT. PP(L)*1.E-6)GO TO 155                        23100000
001241          LC=LC+1                                                        23200000
001243          GO TO 154                                                      23300000
001243      156 DELP=.03*PP(L)*PIEL                                            23400000
001246          LC=LC+1                                                        23500000
001250      154 IF(L .GT. 100)GO TO 1                                          23600000
001254          IF(AD1 .GE. .0 .AND. AD2 .GE. .0)GO TO 90                      23700000
001263      155 CONTINUE                                                       23800000
001263          CISSA=ABSCISA(PP(L),PP(L-1),PP(L-2),DET(L),DET(L-1),DET(L-2))  23900000
001270          P=CISSA                                                        24000000
001271          DD=0.                                                          24100000
001272          TXS=TXSS/PIEL+RX*P                                             24200000
001275          TYS=TYSS/PIEL+RY*P                                             24300000
001300          TXYS=TXYSS/PIEL+RXY*P                                          24400000
001304          PRINT 6,TXS,TYS,TXYS,DD                                        24500000
001317          WRITE(7,6)TXS,TYS,TXYS,DD                                      24600000
001333          WRITE(7,6) P,DD                                                24700000
001343          TXS=TXS*PI*PI*DS1/(XB*XB)                                      24800000
001347          TYS=TYS*PI*PI*DS1/(XB*XB)                                      24900000
001351          TXYS=TXYS*PI*PI*DS1/(XB*XB)                                    25000000
001353          PRINT 5014,TXS,TYS,TXYS                                        25100000
```

```
001365          WRITE(7,5014)TXS,TYS,TXYS
          C
          C     COMPUTATION OF USEFUL BUCKLING PARAMETERS
          C
001377          IF(ISTIFF .NE. 2)GO TO 130
001401          FA=XB*XB/PI2/(TT**3)
001405          TXI=TXS*FA
001406          TYI=TYS*FA
001410          TXYI=TXYS*FA
001411          PRINT 5030, TXI,TYI,TXYI
001423          FA=XB*XB/(TT**3)
001426          EXT=FA*(1./EX)*(TXS-EXY/EY*TYS)
001434          EYT=FA*(1./EY)*(TYS-EXY/EX*TXS)
001442          EXYT=.5*FA*TXYS/GXY
001446          EXT=-EXT
001447          EYT=-EYT
001450          EXYT=-EXYT
001451          PRINT 5031, EXT,EYT,EXYT
001462          PRINT 5032
001466      130 IF(MSHAPE .EQ. 1)GO TO 87
001470          GO TO 85
001471       90 IF(PIB .GT. PFIN)GO TO 85
001475          PIB=PIB+DELP
001476          GO TO 7702
001477       85 PI=PIS
001501          DELP=DELPS
001502          PFIN=PFINS
001504          X=XSS
001505          GO TO 1
          C
          C     COMPUTE MODE SHAPES
          C
001506       84 P=PI
001510          IF(MSHAPE .EQ. 2)GO TO 1
001512       87 TXS=(TXSS+RX*PIEL*P)*ADB2
001517          TYS=(TYSS+RY*PIEL*P)*ADB2
001524          TXYS=(TXYSS+RXY*P*PIEL)*ADB2
001530          CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE,
               1 2,IBC,AKL,LOCX,LOCY,NK)
001554          DO 115 I=1,MR2
001556      115 B(I,1)=-A(I,MR2)
001565          MR2M1=MR2-1
001567          CALL GELIM(56,MR2M1,A,1,B,IPIVOT,0,WK,IERR)
```

```
001577          B(MR2,1)=1.                                                  29500000
001601          DO 116 I=1,MR                                                29600000
001603          W(I+2,3)=B(I)                                                29700000
001605      116 W(I+2,4)=B(I+MR)                                             29800000
001611          CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE, 29900000
               1 1,IBC)                                                      30000000
001631          PRINT 5028                                                   30100000
001635          WM=0.                                                        30200000
001636          DO 170 I=3,MRP2                                              30300000
001640          DO 170 J=3,NCP2                                              30400000
001641          W1=ABS(W(I,J))                                               30500000
001645      170 WM=AMAX1(W1,WM)                                              30600000
001654          WMI=1./WM                                                    30700000
001656          DO 171 I=2,MRP3                                              30800000
001657          DO 172 J=2,NCP3                                              30900000
001660      172 W(I,J)=W(I,J)*WMI                                            31000000
001666          PRINT 5029, (W(I,J),J=2,NCP3)                                31100000
001702      171 PRINT 5033                                                   31200000
001711          GO TO 85                                                     31300000
001711        6 FORMAT(2X,E16.8,3X,E16.8,4X,E16.8,10X,E16.8)                 31400000
001711       61 FORMAT(///,9X,*NXBAR*,15X,*NYBAR*,13X,*NXYBAR*,17X,*DETERMINANT*/) 31600001
001711     5000 FORMAT(////15H INPUT FOR CASE///1X8A10////)                  31700000
001711     5001 FORMAT(8A10)                                       00000087  31800000
001711     5002 FORMAT(1X8A10)                                               31900000
001711     5010 FORMAT(//,20X,19HBOUNDARY CONDITIONS )                       32000000
001711     5011 FORMAT(/,5X,*BOUNDARY NO. *,I1,* HAS A ROTATIONAL SPRING SUPPORT O 32100000
               1F MAGNITUDE *,E16.8/)                                        32200000
001711     5012 FORMAT(/,13X,*BOUNDARY NO. *,I1,* IS SIMPLY SUPPORTED*,/)    32300000
001711     5013 FORMAT(/,13X,*BOUNDARY NO. *,I1,* IS CLAMPED*,/)             32400000
001711     5014 FORMAT(//,15X,*BUCKLING LOADS PER UNIT OF LENGTH ALONG BOUNDARY ED 32500000
               1GE*,/,5X,*NX=*,E16.8,5X,*NY=*,E16.8,5X,*NXY=*,E16.8/)        32600000
001711     5015 FORMAT(/,(3E20.8))                                           32700000
001711     5016 FORMAT(//,30X,*D1*)                                          32800000
001711     5017 FORMAT(//,30X,*D2*)                                          32900000
001711     5018 FORMAT(//,30X,*D12*)                                         33000000
001711     5019 FORMAT(//,30X,*D66*)                                         33100000
001711     5021 FORMAT(5X,*ISTIFF=*,I1,3X,*ISTEP=*,I1,10X,*IX=*,I1,3X,*JX=*,I1, 33300001
               * 9X,*TEST=*,F16.8//)                                         33300002
001711     5022 FORMAT(5X,*XA=*,E16.8,5X,*XB=*,E16.8,5X,*ASPECT RATIO=*,E16.8// 33400000
               1,15X,*MR=*,I2,5X,*NC=*,I2//)                                 33500000
001711     5023 FORMAT(5X,*TX=*,E16.8,5X,*TY=*,E16.8,7X,*TXY=*,E16.8//       33600001
               1,5X,*P1=*,E16.8,5X,*DELP=*,E16.8,5X,*PFIN=*,E16.8//          33700000
               * ,5X,*X=*,E16.8,6X,*Y=*,F16.8//,5X,*RX=*,E16.8,5X,*RY=*,E16.8,7X, 33800001
```

```
                   3*RXY=*,E16.8//)
001711      5024 FORMAT(/,1X,100(1H*))
001711      5025 FORMAT(//,30X,12(1H1),5(1H*),1X ,1HY,/,6(30X,1H4,10X,1H2,/),30X,
                 1 12(1H3),/,3(30X,1H*,/),/,30X,1HX,////)
001711      5026 FORMAT(1X,I2,3X,I1,3X,I2,4X,E16.8,2X,E16.8)
001711      5027 FORMAT(//,30X,*STIFFENER DATA*,/,*NO.*,2X,*IORIENT*,2X,*LOC*,10X,
                 1 *EI*,12X,*EI/EPS/D1*,/)
001711      5028 FORMAT(//,40X,*MODE SHAPE*,/)
001711      5029 FORMAT(1X,8(E15.7,1X))
001711      5030 FORMAT(//,2X,28H(XB**2)*NX/(PI**2)/(T**3) = ,E16.8/,
                 1          2X,28H(XB**2)*NY/(PI**2)/(T**3) = ,E16.8/,
                 2          2X,29H(XB**2)*NXY/(PI**2)/(T**3) = ,E16.8//)
001711      5031 FORMAT(//,2X,26H(XB**2)*EPSILONX/(T**2) = ,E16.8/,
                 1          2X,26H(XB**2)*EPSILONY/(T**2) = ,E16.8/,
                 2          2X,27H(XB**2)*EPSILONXY/(T**2) = ,E16.8/)
001711      5032 FORMAT(1H1)
001711      5033 FORMAT(//)
001711      5035 FORMAT(/,2X,*WARNING,  A BUCKLING VALUE MAY HAVE BEEN SKIPPED*,/)
001711      5038 FORMAT(//,10X,*ISTEP=3 OR ISTIFF=3 - PREPROCESS ONLY*,//)
001711      8997 FORMAT(//*                     STOPPED ON EOF,5              *)      00000032
001711           END
```

```
              SUBROUTINE ARRAY(D1,D2,D12,D66,EDL,W,A,TX,TY,TXY,MR,NC,XSX,EDL1,
             1 EDE,MS,IFC)
000024        DIMENSION IBC(4)
000024        DIMENSION D1(32,32),D2(32,32),D12(32,32),D66(32,32)
000024        DIMENSION W(32,32),A(56,56)
000024        COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),
             1ZIY(32),ETAX(32),ETAY(32)
000024        COMMON/SPRING/AKL(32,32),AKX(32,32),AKY(32,32)
000024        WXX(K,L)=W(K+1,L)-2.*W(K,L)+W(K-1,L)
000042        WYY(K,L)=(W(K,L+1)-2.*W(K,L)+W(K,L-1))*EDLS
000057        WXY(K,L)=(W(K+1,L+1)-W(K+1,L)-W(K,L+1)+W(K,L))*EDL
000075        WX(K,L)=ZIY(L)*ETAX(K)*(W(K+1,L)-W(K,L))
000112        WY(K,L)=ZIY(L)*ETAX(K-1)*(W(K,L+1)-W(K,L))*EDLS
000127        WXY2(K,L)=((W(K+1,L+1)-W(K,L))*ETAX(K)*ETAY(L)-(W(K+1,L-1)-W(K,L))
             1*ETAX(K)*ETAY(L-1) -(W(K,L)-W(K-1,L-1))*ETAX(K-1)*ETAY(L-1)
             2 +(W(K,L)-W(K-1,L+1))*ETAX(K-1)*ETAY(L))*EDLP*.25
000173        TW(K,L)=(TX*(WX(K,L)-WX(K-1,L)) + TY*(WY(K,L)-WY(K,L-1))
             1 +2.*TXY*WXY2(K,L))*XSX**2
             2 +(AKL(K,L)*W(K,L)*XSX**2+AKX(K-1,L)*(W(K,L)-W(K-1,L))
             3 -AKX(K,L)*(W(K+1,L)-W(K,L)) +(AKY(K,L-1)*(W(K,L)-W(K,L-1))
             4-AKY(K,L)*(W(K,L+1)-W(K,L)))*EDLS)*EDL
000304        XM(K,L)=D1(K,L)*WXX(K,L)+D12(K,L)*WYY(K,L)
000327        YM(K,L)=D2(K,L)*WYY(K,L)+D12(K,L)*WXX(K,L)
000352        XYM(K,L)=2.*D66(K,L)*WXY(K,L)
000366        XXM(K,L)=XM(K+1,L)-2.*XM(K,L)+XM(K-1,L)
000413        YYM(K,L)=(YM(K,L+1)-2.*YM(K,L)+YM(K,L-1))*EDLS
000441        XYXYM(K,L)=(XYM(K,L)-XYM(K,L-1)-XYM(K-1,L)+XYM(K-1,L-1))*EDL
000474        EKE(K,L)=XXM(K,L)+2.*XYXYM(K,L)+YYM(K,L)+TW(K,L)
000525        EDLP=EDL1*EDE
000530        EDLS=EDL**2
000531        IROW=MRTOT-2
000533        ICOL=NCTOT-4
000535        IF(MS .EQ. 1)GO TO 35
000537        NNC=ICOL+1
000540        NCC=ICOL+2
000542        DO 10 J=1,NCTOT
000543        DO 10 I=1,MRTOT
000544        W(I,J)= 0.0
000550     10 CONTINUE
000555        MMN=0
000556        DO 40 N=3,4
000557        DO 40 M=3,IROW
```

```
000560          MN=0
000561          MMN=MMN+1
000563          DO 39 I=3,IROW
000564       39 W(I,3)=W(I,4)=.0
000572          W(M,N)=1.0
          C
          C     MARCHING PROCEDURE
          C
000576       35 DO 30 J=3,ICOL
000600          DO 30 I=3,IROW
000601          YDM=-XXM(I,J)-2.*XYXYM(I,J)-TW(I,J)
000621          YMOM=YDM/EDLS+2.*YM(I,J)-YM(I,J-1)
000636          YK=(YMOM-D12(I,J+1)*WXX(I,J+1))/D2(I,J+1)
000653          IF(I .EQ. IBPOS1 .OR. I .EQ. IBPOS3) YK=YMOM/D2(I,J+1)
000665          W(I,J+2)=YK/EDLS+2.*W(I,J+1)-W(I,J)
000676       30 CONTINUE
000703        1 FORMAT(/,2(1X,7E18.8/))
000703          IF(MS .EQ. 1)RETURN
000706          DO 20 J=NNC,NCC
000710          DO 20 I=3,IROW
000711          MN=MN+1
000713          NUM=NC/2
000715          CF=10.**NUM
000720          A(MN,MMN)=EKE(I,J)/CF
000732       20 CONTINUE
000737       40 CONTINUE
000743        2 FORMAT(/,4(1X,5E20.8/))
000743          RETURN
000744          END
```

```
                SUBROUTINE DEUPPS(A,N,DET,MAX)

000007          DIMENSION A(MAX,N)
000007          DET=1.
000010          NN = N-1

           C    PIVOT SEARCH

000012          DO 560 I=1,NN
000013          CAVM = 0.
000014          II = I
000015          DO 105 J=II,N
000016          SWAP = A(J,II)
000022          CAVA=ABS(SWAP)
000024          IF(CAVM.GE.CAVA) GO TO 105
000027          IROW = J
000030          CAVM = CAVA
000031      105 CONTINUE
000034          IF(CAVM.EQ.0.) GO TO 720

           C    ROW INTERCHANGE

000035          IF(IROW.EQ.II) GO TO 203
000037          DET = -DET
000040          DO 200 L=II,N
000041          SWAP = A(IROW,L)
000045          A(IROW,L) = A(II,L)
000053          A(II,L) = SWAP
000056      200 CONTINUE
000060      203 SWAP = A(II,II)
000064          DET = DET*SWAP

           C    NORMALIZE PIVOT ROW

000065          K = II+1
000067          DO 350 L=K,N
000070      350 A(II,L) = A(II,L)/SWAP

           C    ELIMINATION

000077          DO 550 LI=K,N
000101          SWAP = A(LI,II)
```

```
000105          DO 520 L=K,N                                    47300000
000107          A(I1,L) = A(I1,L)-A(II,L)*SWAP                  47400000
000121     520  CONTINUE                                        47500000
000124     550  CONTINUE                                        47600000
000126     560  CONTINUE                                        47700000
000131          GO TO 730                                       47800000
000131     720  DET=0.                                          47900000
000132          GO TO 750                                       48000000
000132     730  DET = DET*A(N,N)                                48100000
000136    750   RETURN                                          48200000
000137          END                                             48300000
```

```
                    SUBROUTINE BC(X,Y,EL,B,AKS,DS1,DS2,DS12,D1,D2,D12,D66,MR,NC,IBC,
                   1 XA,XB)
              C
              C     COMPUTES VALUES OF PLATE STIFFNESSES SUCH THAT THE APPROPRIATE
              C     BOUNDARY CONDITIONS ARE SET
              C
              C     OPTIONS
              C     IBC=1  ROTATIONAL SPRING SUPPORT
              C     IBC=2  SIMPLE SUPPORT
              C     IBC=3  CLAMPED SUPPORT
              C     IBC=4  FREE BOUNDARY
              C     IBC=5, BOUNDARY CONDITIONS NOT SET BY THIS SUBROUTINE
              C
000024              DIMENSION AKS(4),IBC(4),D1(32,32),D2(32,32),D12(32,32),D66(32,32)
000024              COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),
                   1ZIY(32),ETAX(32),ETAY(32)
000024              PI=3.14159265359
000025              SX=SIN(PI/X/EL)
000034              SY=SIN(PI/Y/B)
000044              S2X=SIN(.5*PI/X/EL)
000055              S2Y=SIN(.5*PI/Y/B)
000066              IBP3M1=IBPOS3-1
000070              IBP2M1=IBPOS2-1
000072              IBP4P1=IBPOS4+1
000073              IBP2M1=IBPOS2-1
000074              DO 150 M=1,4
000076              L=IBC(M)
000101              GO TO(151,152,153,154,150)L
000112          151 PRINT 5011,M,AKS(M)
000126              IF(M .EQ. 2 .OR. M .EQ. 4)GO TO 160
000141              GAMMAX=AKS(M)*XA/DS1
000144              EHATX=4.*X*S2X**2/PI/SX
000150              CX=(1.-.5*GAMMAX*EHATX)/(1.+.5*GAMMAX*EHATX)
000160              IF(M .EQ. 3) GO TO 170
000162          181 R=DS1*(1.-CX)/DS1
000165              CALL SET(D1,2,IBPOS4,IBPOS2,IBPOS1,R)
000172              GO TO 150
000176          170 R=DS1*(1.-CX)/DS1
000201              CALL SET(D1,2,IBPOS4,IBPOS2,IBPOS3,R)
000206              GO TO 150
000212          160 GAMMAY=AKS(M)*XB/DS2
000216              EHATY=4.*Y*S2Y**2/PI/SY
```

```
000223           CY=(1.-.5*GAMMAY*EHATY)/(1.+.5*GAMMAY*EHATY)                    52600000
000233           IF(M .EQ. 4)GC TO 180                                           52700000
000235       183 R=DS2*(1.-CY)/DS1                                               52800000
000241           CALL SET(D2,1,IBPOS1,IBPOS3,IBPOS2,R)                           52900000
000245           GO TO 150                                                       53000000
000251       180 R=DS2*(1.-CY)/DS1                                               53100000
000255           CALL SET(D2,1,IBPOS1,IBPOS3,IBPOS4,R)                           53200000
000261           GO TO 150                                                       53300000
000265       152 PRINT 5012,M                                                    53400000
000273           CX=CY=1.                                                        53500000
000276           GO TO(181,183,170,180)M                                         53600000
000311       153 PRINT 5013,M                                                    53700000
000317           CX=CY=-1.                                                       53800000
000322           GO TO(181,183,170,180)M                                         53900000
000335       154 PRINT 5014,M                                                    54000000
000343           GO TO (201,202,203,204)M                                        54100000
000357       201 DO 221 J=IBPOS4,IBPOS2                                          54200000
000361           D2(IBPOS1,J)=.5*D2(IBPOS1,J)*(1.-D12(IBPOS1,J)/D1(IBPOS1,J)     54300000
                1 *D12(IBPOS1,J)/D1(IBPOS1,J))                                   5440000C
000377           D1(IBPOS1,J)=.0                                                 54500000
000403       221 D12(IBPOS1,J)=.0                                                54600000
000407           GO TO 150                                                       54700000
000410       202 DO 223 I=IBPOS1,IBPOS3                                          54800000
000412           D1(I,IBPOS2)=.5*D1(I,IBPOS2)                                    54900000
000416           D2(I,IBPOS2)=.5*D2(I,IBPOS2)                                    55000000
000421       223 D12(I,IBPOS2)=D12(I,IBPOS2)*.5                                  55100000
000427           GO TO 150                                                       55200000
000427       203 DO 225 J=IBP4P1,IBP2M1                                          55300000
000431           D2(IBPOS3,J)=.5*D2(IBPOS3,J)*(1.-D12(IBPOS3,J)/D2(IBPOS3,J)     55400000
                1 *D12(IBPOS3,J)/D1(IBPOS3,J))                                   55500000
000447           D1(IBPOS3,J)=.0                                                 55600000
000452       225 D12(IBPOS3,J)=.0                                                55700000
000456           GO TO 150                                                       55800000
000457       204 DO 227 I=IBPOS1,IBPOS3                                          55900000
000461           D1(I,IBPOS4)=.5*D1(I,IBPOS4)                                    56000000
000465           D2(I,IBPOS4)=.5*D2(I,IBPOS4)                                    56100000
000470       227 D12(I,IBPOS4)=.5*D12(I,IBPOS4)                                  56200000
000475       150 CONTINUE                                                        56300000
000477           DO 260 I=1,MRTOT                                                56400000
000501           ZIX(I)=ETAX(I)=1.                                               56500000
000505       260 IF(I .LT. IBPOS1 .OR. I .GT. IBPOS3)ZIX(I)=ETAX(I)=.0           56600000
000525           ZIX(IBPOS1)=.5                                                  56700000
000527           ZIX(IBPOS3)=.5                                                  56800000
```

```
000530          ETAX(IBPOS3)=.0                                                    56900000
000531          DO 261 J=1,NCTOT                                                   57000000
000533          ZIY(J)=ETAY(J)=1.                                                  57100000
000537      261 IF(J .LT. IBPOS4 .OR. J .GT. IBPOS2)ZIY(J)=ETAY(J)=.0             57200000
000557          ZIY(IBPOS4)=.5                                                     57300000
000561          ZIY(IBPOS2)=.5                                                     57400000
000562          ETAY(IBPOS2)=.0                                                    57500000
000563     5011 FORMAT(/,13X,*BOUNDARY NO. *I1,* HAS A ROTATIONAL SPRING SUPPORT O 57600000
               1F MAGNITUDE *,F16.8/)                                              57700000
000563     5012 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS SIMPLY SUPPORTED*,/)           57800000
000563     5013 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS CLAMPED*,/)                    57900000
000563     5014 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS FREE*,/)                       58000000
000563          RETURN                                                             58100000
000564          END                                                                58200000
```

```
                 SUBROUTINE PREP(D1,D2,D12,D66,DS1,DS2,DS12,DS66,IX,JX,MR,NC,TT,EX,        58300000
                1EY,EXY,GXY,IBC,ISTIFF)                                                    58400000
          C                                                                                58500000
          C      THIS SUBROUTINE PREPROCESSES THE ORTHOTROPIC PROPERTIES OF A LAMINATE     58600000
          C                                                                                58700000
000026           DIMENSION Q(10,4),E1(10),E2(10),U1(10),G12(10),DD(3,3),IBC(4),            58800000
                1AT(100),TH(100),MATYPE(100),D1(32,32),D2(32,32),D12(32,32),               58900000
                2D66(32,32)                                                                59000000
000026           COMMON/LAYER/NUMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                           59100000
000026           COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),              59200000
                1ZIY(32),ETAX(32),ETAY(32)                                                 59300000
000026           IBPOS1=2+IBC(1)/4                                                         59400000
000031           IBPOS2=NC+3+IBC(4)/4*2                                                    59500000
000040           IBPOS3=IBPOS1+MR+1                                                        59600000
000043           IBPOS4=2+IBC(4)/4*2                                                       59700000
000051           MRTOT=MR+4+IBC(1)/4   +IBC(3)/4                                           59800000
000055           NCTOT=NC+4+IBC(2)/4*2+IBC(4)/4*2                                          59900000
000067         1 FORMAT(//,10I5//)                                                         60200000
000067           IF(ISTIFF .EQ. 1)GO TO 160                                                60300000
000072           DO 165 I=1,MRTOT                                                          60400000
000073           DO 165 J=1,NCTOT                                                          60500000
000074       165 D1(I,J)=D2(I,J)=D12(I,J)=D66(I,J)=.0                                      60600000
000115           TT=0.                                                                     60700000
000116       100 PRINT 5003                                                                60800000
000122           PRINT 5006                                                                60900000
000126           DO 110 K=1,NOMAT                                                          61000000
000133           PRINT 5007,K,E1(K),E2(K),U1(K),G12(K)                                     61100000
000150           ANVXY=U1(K)                                                               61200000
000152           ANVYX=ANVXY*E2(K)/E1(K)                                                   61300000
000155           ANU=1./(1.-ANVXY*ANVYX)                                                   61400000
000160           Q(K,1)=E1(K)*ANU                                                          61500000
000163           Q(K,2)=E2(K)*ANU                                                          61600000
000165           Q(K,3)=ANVXY*E2(K)*ANU                                                    61700000
000170       110 Q(K,4)=G12(K)                                                             61800000
000200           PRINT 5008                                                                61900000
000203           DO 115 J=1,MA                                                             62000000
000210           PRINT 5009,J,MATYPE(J),AT(J),TH(J)                                        62100000
000223       115 TT=TT+AT(J)                                                               62200000
000234           ZL=.5*TT                                                                  62300000
000236           CALL EVAL(AT,TH,MATYPE,ZL,MA,Q,DD,IX,JX,EX,EY,EXY,GXY)                    62400000
000255           DS1=DD(1,1)                                                               62500000
000262           DS2=DD(2,2)                                                               62600000
```

```
000263          DS12=DD(1,2)                                        62700000
000264          DS66=DD(3,3)                                        62800000
000266          DO 140 KR=IBPOS1,IBPOS3                             62900000
000270          DO 140 KC=IBPOS4,IBPOS2                             63000000
000272          D1(KR,KC)=DD(1,1)/DS1                               63100000
000276          D2(KR,KC)=DD(2,2)/DS1                               63200000
000301      140 D12(KR,KC)=DD(1,2)/DS1                              63300000
000310          IB2M1=IBPOS2-1                                      63400000
000312          IB3M1=IBPOS3-1                                      63500000
000313          DO 150 KR=IBPOS1,IB3M1                              63600000
000315          DO 150 KC=IBPOS4,IB2M1                              63700000
000317      150 D66(KR,KC)=DD(3,3)/DS1                              63800000
000330          RETURN                                              63900000
000331      160 DS1I=1./DS1                                         64000000
000333          DO 170 I=IBPOS1,IBPOS3                              64100000
000335          DO 170 J=IBPOS4,IBPOS2                              64200000
000337          D1(I,J)=D1(I,J)*DS1I                                64300000
000343          D2(I,J)=D2(I,J)*DS1I                                64400000
000346          D12(I,J)=D12(I,J)*DS1I                              64500000
000350      170 D66(I,J)=D66(I,J)*DS1I                              64600000
000357          IB1M1=IBPOS1-1                                      64700000
000361          IB2P1=IBPOS2+1                                      64800000
000363          IB3P1=IBPOS3+1                                      64900000
000364          IB4M1=IBPOS4-1                                      65000000
000365          DO 180 I=1,IB1M1                                    65100000
000367          DO 180 J=1,NCTOT                                    65200000
000370          D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0                65300000
000405      180 CONTINUE                                            65400000
000411          DO 181 I=IB3P1,MRTOT                                65500000
000413          DO 181 J=1,NCTOT                                    65600000
000414          D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0                65700000
000431      181 CONTINUE                                            65800000
000435          DO 182 J=1,IB4M1                                    65900000
000437          DO 182 I=IBPOS1,IBPOS3                              66000000
000441          D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0                66100000
000456      182 CONTINUE                                            66200000
000463          DO 183 J=IB2P1,NCTOT                                66300000
000465          DO 183 I=IBPOS1,IBPOS3                              66400000
000467          D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0                66500000
000504      183 CONTINUE                                            66600000
000511     5003 FORMAT(//,30X,26HLAMINATED PLATE PROPERTIES )       66700000
000511     5004 FORMAT(//,20X,35HMID-PLANE USED AS REFERENCE SURFACE )  66800000
000511     5005 FORMAT(//,20X,5HZREF=,F16.8)                        66900000
```

```
000511     5006 FORMAT(//,1X,13HMATERIAL KIND,8X,3H E1,15X,3H E2,13X,5H U1   ,12X,     67000000
                13HGXY)                                                                  67100000
000511     5007 FORMAT(7X,I2,7X,E16.8,3(1X,E16.8))                                      67200000
000511     5008 FORMAT(//,1X,9HLAYER NO.,3X,9HMAT. KIND,6X,5HTHICK,12X,5HTHETA)        67300000
000511     5009 FORMAT(5X,I2,9X,I2,5X,E16.8,1X,E16.8)                                   67400000
000511          RETURN                                                                   67500000
000512          END                                                                      67600000
```

```
          SUBROUTINE EVAL(T,TH,IL,ZL,NL,Q,DD,IX,JX,EX,EY,EXY,GXY)            67700000
000020    DIMENSION T(100),TH(100),IL(100),DD(3,3),Q(10,4),AA(3,3),BB(3,3),  67800000
         2QB(3,3)                                                            67900000
000020    O3=1./3.                                                           68000000
000022    TT=0.                                                              68100000
000023    DO 6 IV=1,3                                                        68200000
000024    DO 6 JV=1,3                                                        68300000
000025    AA(IV,JV)=0.                                                       68400000
000030    BB(IV,JV)=0.                                                       68500000
000032  6 DD(IV,JV)=0.                                                       68600000
000040    DO 20  J=1,NL                                                      68700000
000042    K=IL(J)                                                            68800000
000044    THETA=TH(J)*3.14159265359/180.                                     68900000
000047    CALL TRANS(Q,THETA,QB,K,IX,JX)                                     69000000
000054    HJ=ZL-TT                                                           69100000
000062    HJP1=HJ-T(J)                                                       69200000
000065    HSQ=.5*(HJ**2-HJP1**2)                                             69300000
000070    TT=TT+T(J)                                                         69400000
000073    DO 20 IV=1,3                                                       69500000
000074    DO 20 JV=1,3                                                       69600000
000075    AA(IV,JV)=AA(IV,JV)+QB(IV,JV)*T(J)                                 69700000
000104    BB(IV,JV)=BB(IV,JV)+QB(IV,JV)*HSQ                                  69800000
000111 20 CONTINUE                                                           69900000
000120    ZREF11=BB(1,1)/AA(1,1)                                             70000000
000122    ZREF22=BB(2,2)/AA(2,2)                                             70100000
000124    ZREF12=BB(1,2)/AA(1,2)                                             70200000
000126    ZREF33=BB(3,3)/AA(3,3)                                             70300000
000130    PRINT 9                                                            70400000
000133    PRINT 8, ZREF11,ZREF22,ZREF12,ZREF33                               70500000
000147    PRINT 9                                                            70600000
000153    ZL=ZL-ZREF11                                                       70700000
000161    TT=.0                                                              70800000
000161    DO 30 J=1,NL                                                       70900000
000163    K=IL(J)                                                            71000000
000165    THETA=TH(J)*3.14159265359/180.                                     71100000
000170    CALL TRANS(Q,THETA,QB,K,IX,JX)                                     71200000
000175    HJ=ZL-TT                                                           71300000
000203    HJP1=HJ-T(J)                                                       71400000
000206    HCUBE=O3*(HJ**3-HJP1**3)                                           71500000
000212    TT=TT+T(J)                                                         71600000
000215    DO 30 IV=1,3                                                       71700000
000216    DO 30 JV=1,3                                                       71800000
```

```
000217      30 DD(IV,JV)=DD(IV,JV)+QB(IV,JV)*HCUBE                              71900000
000236         PRINT 3                                                          72000000
000241         PRINT 2,((AA(IV,JV),IV=1,3),JV=1,3)                              72100000
000257         PRINT 4                                                          72200000
000263         PRINT 2,((BB(IV,JV),IV=1,3),JV=1,3)                              72300000
000301         PRINT 5                                                          72400000
000305         PRINT 2,((DD(IV,JV),IV=1,3),JV=1,3)                              72500000
000324         ANUXY=AA(1,2)/AA(2,2)                                            72600000
000326         ANUYX=AA(1,2)/AA(1,1)                                            72700000
000333         EX=AA(1,1)*(1.-ANUXY*ANUYX)/TT                                   72800000
000335         EY=AA(2,2)*(1.-ANUXY*ANUYX)/TT                                   72900000
000342         EXY=ANUXY*EY                                                     73000000
000344         GXY=AA(3,3)/TT                                                   73100000
000346         PRINT 7,EX,EY,GXY,ANUXY,ANUYX                                    73200000
000364       2 FORMAT(//,3(1X,E16.8,2X,E16.8,2X,E16.8/))                        73300000
000364       3 FORMAT(//,25X,8HA MATRIX)                                        73400000
000364       4 FORMAT(//,25X,8HB MATRIX)                                        73500000
000364       5 FORMAT(//,25X,3HD MATRIX)                                        73600000
000364       7 FORMAT(//,15X,*OVERALL LAMINATE PROPERTIES*,//,* EX=*,E16.8,     73700000
              1 *  EY=*,E16.8,*  GXY=*,E16.8,/,* NUXY=*,E16.8,*  NUYX=*,E16.8//)  73800000
000364       3 FORMAT(/,5X, *CAUTION -- COUPLING BETWEEN EXTENSION AND BENDING MA  73900000
              1Y BE SIGNIFICANT*,/,5X,*IF THE FOLLOWING FOUR VALUES ARE NOT ALL E  74000000
              2QUAL.*,/,5X,*IF THIS IS THE CASE. THE RESULTS SHOULD BE USED WITH    74100000
              3DESCRETION*,//,4(2X,E16.8),/)                                    74200000
000364       9 FORMAT(1X,76(1H*),/)                                             74300000
000364         RETURN                                                           74400000
000365         END                                                              74500000
```

```
              SUBROUTINE TRANS(Q,THETA,QB,K,IX,JX)
000011        DIMENSION T(3,3),TI(3,3),QS(3,3),ST(3,3),QB(3,3),Q(10,4)
000011        SN=SIN(THETA)
000016        CS=COS(THETA)
000024        T(1,1)=TI(1,1)=T(2,2)=TI(2,2)=CS**2
000036        T(1,2)=TI(1,2)=T(2,1)=TI(2,1)=SN**2
000050        T(1,3)=TI(2,3)=2.*CS*SN
000055        T(3,1)=TI(3,2)=-SN*CS
000061        T(2,3)=TI(1,3)=-T(1,3)
000065        T(3,2)=TI(3,1)=-T(3,1)
000071        T(3,3)=TI(3,3)=CS*CS-SN*SN
000076        QS(1,1)=Q(K,1)
000100        QS(2,2)=Q(K,2)
000102        QS(1,2)=QS(2,1)=Q(K,3)
000106        QS(3,3)=2.*Q(K,4)
000111        QS(1,3)=QS(3,1)=QS(2,3)=QS(3,2)=0.
000122        IF(IX .EQ. 1)PRINT 1,((QS(I,J),J=1,3),I=1,3)
000145        IF(IX .EQ. 1)WRITE(7,1) ((QS(I,J),J=1,3),I=1,3)
000170        DO 5 IM=1,3
000172        DO 5 IS=1,3
000173      5 ST(IM,IS)=QB(IM,IS)=0.
000204        DO 10 IM=1,3
000205        DO 10 IS=1,3
000206        DO 10 IR=1,3
000207     10 ST(IM,IS)=ST(IM,IS) + QS(IM,IR)*T(IR,IS)
000230        DO 20 IR=1,3
000231        DO 20 IS=1,3
000232        DO 20 IM=1,3
000233     20 QB(IR,IS)=QB(IR,IS)+TI(IR,IM)*ST(IM,IS)
000254        DO 30 IM=1,3
000255     30 QB(IM,3)=.5*QB(IM,3)
000262        IF(IX .EQ.1)PRINT 1,((T(I,J),J=1,3),I=1,3),((TI(I,J),J=1,3),I=1,3)
000317        IF(IX .EQ. 1)PRINT 1,((QB(I,J),J=1,3),I=1,3)
000346        IF(IX .EQ. 1)WRITE(7,1) ((QB(I,J),J=1,3),I=1,3)
000375      1 FORMAT(3(3X,E16.8))
000375        RETURN
000376        END
```

```
                  SUBROUTINE AUTOXY(X,Y,B,INCX,P)                              78300001
000010            COMMON/XY/TX,TY,TXY,RX,RY,RXY,PI,ADB,DS1,DS2S,DS12,DS66,      78400000
                 1IBC2,AKS2,XB,EX,EY,EXY,GXY,TT                                78500000
000010            PRINT 53                                                     78500001
000013            D12S=(DS12+2.*DS66)/DS1                                      78600000
000017            DS2=DS2S/DS1                                                 78700000
000020            IC=1                                                         78800000
000021            P=.1                                                         78900000
000026            B=1.                                                         79000000
000027            DP=DB=1.                                                     79100000
000031            PID8=PI/8.                                                   79200000
            C                                                                  79300000
            C     SIMPLE SUPPORTS, IK=1                                        79400000
            C     CLAMPED SUPPOSTS, IK=2                                       79500000
            C                                                                  79600000
000032            IF(IBC2 .EQ. 3 .OR. AKS2*XB/DS2 .GT. 100.)IK=2               79700000
000047          1 CX=TX+P*RX                                                   79800000
000052            CY=TY+P*RY                                                   79900000
000055            CXY=TXY+P*RXY                                                80000000
000060            AM1=PID8*(DS2+2.*D12S*B*B+B**4-B*B*CX-CY)                    80100000
000072            AM1B=PID8*(4.*D12S*B+4.*B*B*B-2.*CX*B)                       80200000
000102            AM1BB=PID8*(12.*B*B-2.*CX+4.*D12S)                           80300000
000110            AM1P=PID8*(-B*B*RX-RY)                                       80400000
000113            AM1BP=-PI*.25*RX*B                                           80500000
000117            AM2=PID8*(16.*DS2+8.*D12S*B*B+B**4-B*B*CX-4.*CY)             80600000
000132            AM2B=PID8*(16.*D12S*B+4.*B**3-2.*CX*B)                       80700000
000142            AM2BB=PID8*(12.*B*B+16.*D12S-2.*CX)                          80800000
000150            AM2P=PID8*(-B*B*RX-4.*RY)                                    80900000
000154            AM2BP=-PI*.25*RX*B                                           81000000
000160            IF(IK .EQ. 2)GO TO 20                                        81100000
000162            F=(CXY*B)**2-2.25*AM1*AM2                                    81200000
000167            G=2.*CXY*CXY*B-2.25*(AM1B*AM2+AM2B*AM1)                      81300000
000175            FB=G                                                         81400000
000177            FP=2.*CXY*RXY*B*B-2.25*(AM1P*AM2+AM2P*AM1)                   81500000
000207            GB=2.*CXY*CXY-2.25*(AM1BB*AM2+AM2BB*AM1+2.*AM1B*AM2B)        81600000
000221            GP=-2.25*(AM1BP*AM2+AM2BP*AM1+AM1B*AM2P+AM2B*AM1P)+4.*CXY*B*RXY 81700000
000235            GO TO 25                                                     81800000
000235          2 AM0=PID8*(B**4-B*B*CX)                                       81900000
000241            AM0B=PID8*(4.*B**3-2.*CX*B)                                  82000000
000246            AM0BB=PID8*(12.*B*B-2.*CX)                                   82100000
000252            AM0P=-PID8*B*B*RX                                            82200000
000255            AM0BP=-.25*PI*RX*B                                           82300000
```

```
000260          AM3=PID8*(81.*DS2+18.*D12S*B*B+B**4-B*B*CX-9.*CY)               82400000
000273          AM3B=PID8*(36.*D12S*B+4.*B**3-2.*B*CX)                          82500000
000303          AM3BB=PID8*(12.*B*B+36.*D12S-2.*CX)                             82600000
000311          AM3P=PID8*(-B*B*RX-9.*RY)                                       82700000
000316          AM3BP=-.25*PI*RX*B                                              82800000
000321          F=(CXY*B)**2-(.21972656)*(2.*AM0+AM2)*(AM1+AM3)                 82900000
000333          G=2.*CXY*CXY*B-(.21972656)*((2.*AM0B+AM2B)*(AM1+AM3)            83000000
               1+(2.*AM0+AM2)*(AM1B+AM3B))                                      83100000
000352          FB=G                                                            83200000
000354          FP=2.*CXY*RXY*B*B-(.21972656)*((2.*AM0P+AM2P)*(AM1+AM3)         83300000
               1+(2.*AM0+AM2)*(AM1P+AM3P))                                      83400000
000374          GB=-(.21972656)*((2.*AM0BB+AM2BB)*(AM1+AM3)+2.*CXY*CXY          83500000
               1+(2.*AM0+AM2)*(AM1BB+AM3BB)+2.*(2.*AM0B+AM2B)*(AM1B+AM3B))      83600000
000420          GP=-(.21972656)*((2.*AM0BP+AM2BP)*(AM1+AM3)+(2.*AM0+AM2)        83700000
               1*(AM1BP+AM3BP)+(2.*AM0B+AM2B)*(AM1P+AM3P)+(2.*AM0P+AM2P)*(AM1B  83800000
               2 +AM3B))+4.*CXY*B*RXY                                           83900000
000455       25 IF(ABS(DP) .LT. 1.E-5 .AND. ABS(DB) .LT. 1.E-5)GO TO 10         84000000
000467          AJACOBI=FP*GB-FB*GP                                             84100000
000472          DP=(-F*GB+G*FB)/AJACOBI                                         84200000
000476          DB=(-FP*G+F*GP)/AJACOBI                                         84300000
000502          P=P+DP                                                          84400000
000503          B=B+DB                                                          84500000
000505           LC=LC+1                                                        84600000
000506          IF(LC .GT. 50)RETURN                                            84700000
000512          GO TO 1                                                         84800000
000513       10 PRINT 101,P,B,F,G                                               84900000
000527          WRITE(7,101) P,B,F,G                                            85000000
000546      101 FORMAT(    5X,*P=*,E16.8,2X,*B=*,E16.8,2X,*F=*,E16.8,*G=*,E16.8) 85100000
000546          IF(INCX .EQ. 1)RETURN                                           85200000
000554          IF(X .GT. 0.)GO TO 60                                           85200001
000557          X=1./B/ADB*ABS(X)                                               85300000
000562          Y=ABS(Y)                                                        85400000
000563          PRINT 51,X,Y                                                    85500001
000573          WRITE(7,51) X,Y                                                 85500002
000606       60 COEF=PI*PI*DS1/(XB*XB)                                          85500003
000611          ANX=CX*COEF                                                     85600000
000613          ANY=CY*COEF                                                     85700000
000615          ANXY=CXY*COEF                                                   85800000
000616          STRNX=(ANX/EX-EXY*ANY/EY)/TT                                    85900000
000624          STRNY=(ANY/EY-EXY*ANX/EX)/TT                                    86000000
000630          STRNXY=ANXY/GXY/TT                                              86100000
000632          WRITE(7,52) CX,CY,CXY,STRNX,STRNY,STRNXY                        86400000
000652          PRINT 52,CX,CY,CXY,STRNX,STRNY,STRNXY                           86500000
```

```
                 SUBROUTINE SET(S,N,LL,LU,IF,R)
000011           DIMENSION S(32,32)
000011           IF(N .EQ.2)GO TO 10
000013           DO 20 IR=LL,LU
000014        20 S(IR,IF)=R
000022           RETURN
000023        10 DO 30 IC=LL,LU
000025        30 S(IF,IC)=R
000033           RETURN
000034           END
```

REFERENCES

1. Gerard, George; and Becker, Herbert: Handbook of Structural Stability. Part I – Buckling of Flat Plates. NACA TN 3781, 1957.
2. Becker, Herbert: Handbook of Structural Stability. Part II – Buckling of Composite Elements. NACA TN 3782, 1957.
3. Johns, D. J.: Shear Buckling of Isotropic and Orthotropic Plates – A Review. R. & M. No. 3677, British A.R.C., 1971.
4. Lekhnitskii, S. G. (S. W. Tsai and T. Cheron, transl.): Anisotropic Plates. Gordon and Breach Sci. Publ., c.1968.
5. Ashton, J. E.; and Whitney, J. M.: Theory of Laminated Plates. Progress in Materials Science Series, Volume IV, Technomic Pub. Co., Inc., c.1970.
6. Timoshenko, Stephen P.; and Gere, James M.: Theory of Elastic Stability. Second ed. McGraw-Hill Book Co., Inc., c.1961.
7. Almroth, B. O.; Brogan, F. A.; Meller, E.; and Petersen, H. T.: User's Manual for STAGS. LMSC-D358197, Lockheed Missiles & Space Co., Jan. 1974.
8. McCormick, Caleb W., ed.: The NASTRAN User's Manual (Level 15). NASA SP-222(01), 1972.
9. Reed, D. L.: Laminated Sandwich Panel Analysis. FZM-5590, Convair Aerospace Div., General Dynamics, Oct. 1, 1971.
10. Fung, Y. C.: Foundations of Solid Mechanics. Prentice-Hall, Inc., c.1965.
11. Ashton, J. E.; Halpin, J. C.; and Petit, P. H.: Primer on Composite Materials: Analysis. Technomic Pub. Co., Inc., c.1969.
12. Libove, Charles; and Batdorf, S. B.: A General Small-Deflection Theory for Flat Sandwich Plates. NACA Rep. 899, 1948. (Supersedes NACA TN 1526.)
13. Housner, J. M.; and Stein, Manuel: Flutter Analysis of Swept-Wing Subsonic Aircraft With Parameter Studies of Composite Wings. NASA TN D-7539, 1974.
14. Stein, Manuel; and Yaeger, David J.: Critical Shear Stress of a Curved Rectangular Panel With a Central Stiffener. NACA TN 1972, 1949.
15. Stowell, Elbridge Z.; and Schwartz, Edward B.: Critical Stress for an Infinitely Long Flat Plate With Elastically Restrained Edges Under Combined Shear and Direct Stress. NACA WR L-340, 1943. (Formerly NACA ARR 3K13.)

16. Batdorf, S. B.; Schildcrout, Murry; and Stein, Manuel: Critical Combinations of Shear and Longitudinal Direct Stress for Long Plates With Transverse Curvature. NACA TN 1347, 1947.

17. Budiansky, Bernard; and Hu, Pai C.: The Lagrangian Multiplier Method of Finding Upper and Lower Limits to Critical Stresses of Clamped Plates. NACA Rep. 848, 1946. (Supersedes NACA TN 1103.)

TABLE 1.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES SIMPLY SUPPORTED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED

Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$	Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$	No. of mesh points in x- and y-directions		Wavelength ratios used in trigonometric differences		Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}}$
		a/Δ_x	b/Δ_y	λ_x/a	λ_y/b	
0.2	1.0	9	9	0.56	0.56	26.28
	.8	9	9	.56	.60	21.43
	.6	9	9	.56	.80	17.33
	.5	9	9	.50	.90	15.36
	.4	11	11	.50	1.00	13.77
	.2	13	13	.35	1.00	11.55
	[a]0	---	---	---	---	10.87
.4	1.0	9	9	.56	.56	15.78
	.8	9	9	.56	.60	12.98
	.6	9	9	.56	.80	10.86
	.5	9	9	.50	.90	9.93
	.4	11	11	.50	1.00	9.29
	.2	15	8	.30	1.00	8.21
	[a]0	---	---	---	---	7.72
.6	1.0	9	9	.56	.56	12.21
	.8	9	9	.56	.60	10.11
	.6	9	9	.56	.80	8.67
	.5	9	9	.50	.90	8.09
	.4	11	11	.50	1.00	7.73
	.2	15	8	.25	1.00	6.71
	[a]0	---	---	---	---	6.53
.8	1.0	9	9	.56	.56	10.40
	.8	9	9	.56	.60	8.66
	.6	9	9	.56	.80	7.57
	.5	9	9	.50	.90	7.10
	.4	11	11	.50	1.00	6.80
	.2	15	8	.25	1.00	6.02
	[a]0	---	---	---	---	5.79
1	1.0	9	9	.56	.56	9.31
	.8	9	9	.56	.60	7.68
	.6	9	9	.56	.80	6.91
	.4	11	11	.50	1.00	6.22
	.2	15	8	.23	1.00	5.49
	[a]0	---	---	---	---	5.33

[a]For $B = 0$, k_s was calculated by using equations (B2).

TABLE 1.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES SIMPLY SUPPORTED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED – Concluded

Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$	Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$	No. of mesh points in x- and y-directions		Wavelength ratios used in trigonometric differences		Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$
		a/Δ_x	b/Δ_y	λ_x/a	λ_y/b	
1.25	1.0	9	9	0.56	0.56	8.43
	.8	9	9	.56	.60	7.08
	.6	9	9	.56	.80	6.38
	.4	11	11	.50	1.00	5.75
	.2	15	8	.22	1.00	5.09
	.1	25	9	.13	1.00	5.05
	[a]0	---	---	---	---	4.96
1.667	1.0	9	9	.56	.56	7.54
	.8	9	9	.56	.60	6.37
	.6	9	9	.56	.80	5.85
	.4	11	11	.50	1.00	5.26
	.2	15	8	.22	1.00	4.72
	.1	22	8	.13	1.00	4.68
	[a]0	---	---	---	---	4.60
2.5	1.0	9	9	.56	.56	6.65
	.8	9	9	.56	.60	5.66
	.6	9	9	.56	.80	5.32
	.4	11	11	.50	1.00	4.77
	.2	15	8	.22	1.00	4.32
	.1	22	8	.13	1.00	4.33
	[a]0	---	---	---	---	4.17
5	1.0	9	9	.56	.56	5.74
	.8	9	9	.56	.60	4.94
	.6	9	9	.56	.80	4.78
	.4	11	11	.50	1.00	4.27
	.2	15	8	.22	1.00	3.90
	.1	22	8	.13	1.00	3.86
	[a]0	---	---	---	---	3.75
∞	1.0	9	9	.56	.56	4.83
	.8	9	9	.56	.60	4.22
	.6	9	9	.56	.80	4.25
	.4	11	11	.50	1.00	3.76
	.2	15	8	.22	1.00	3.47
	[a]0	---	---	---	---	3.30

[a]For $B = 0$, k_s was calculated by using equations (B2).

TABLE 2.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES CLAMPED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED

Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$	Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$	No. of mesh points in x- and y-directions		Wavelength ratios used in trigonometric differences		Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$
		$a/\Delta x$	$b/\Delta y$	λ_x/a	λ_y/b	
0.2	1.0	9	9	1.00	1.0	32.56
	.8	9	9	.80	1.0	26.31
	.6	9	9	.60	1.0	22.21
	.4	11	11	.40	1.0	18.91
	.2	17	9	.31	1.0	17.34
	.1	25	9	.15	1.0	17.31
	[a]0	---	---	---	---	17.13
.4	1.0	9	9	1.10	1.1	21.63
	.8	9	9	.90	1.0	17.92
	.6	9	9	.60	1.0	15.43
	.4	11	11	.40	1.0	13.62
	.2	17	9	.25	1.0	12.64
	.1	25	9	.13	1.0	12.89
	[a]0	---	---	---	---	12.51
.6	1.0	9	9	1.10	1.1	17.86
	.8	9	9	.90	1.0	14.89
	.6	9	9	.60	1.0	13.06
	.4	11	11	.40	1.0	11.60
	.2	15	8	.22	1.0	10.64
	.1	25	9	.13	1.0	10.95
	[a]0	---	---	---	---	10.69
.8	1.0	9	9	1.10	1.1	15.94
	.8	9	9	.90	1.0	13.34
	.6	9	9	.60	1.0	11.84
	.4	11	11	.40	1.0	10.55
	.2	17	9	.24	1.0	9.99
	.1	25	9	.13	1.0	10.16
	[a]0	---	---	---	---	9.63
1	1.0	9	9	1.20	1.2	14.81
	.8	9	9	1.00	1.0	12.44
	.6	9	9	.60	1.0	11.08
	.4	11	11	.40	1.0	9.89
	.2	17	9	.22	1.0	9.27
	.1	25	9	.12	1.0	9.11
	[a]0	---	---	---	---	8.99

[a]For $B = 0$, k_s was calculated by using equations (B3).

TABLE 2.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES CLAMPED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED – Concluded

Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$	Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$	No. of mesh points in x- and y-directions		Wavelength ratios used in trigonometric differences		Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$
		a/Δ_x	b/Δ_y	λ_x/a	λ_y/b	
1.25	1.0	9	9	1.20	1.2	13.87
↓	.8	9	9	1.00	1.0	11.68
	.6	9	9	.60	1.0	10.46
	.4	9	9	.40	1.0	9.39
	.2	15	8	.22	1.0	8.80
	.1	22	8	.12	1.0	8.98
	[a]0	---	---	---	---	8.45
1.667	1.0	9	9	1.20	1.2	12.91
↓	.8	9	9	1.00	1.0	10.90
	.6	9	9	.60	1.0	9.80
	.4	9	9	.40	1.0	8.86
	.2	15	8	.22	1.0	8.34
	.1	22	8	.12	1.0	8.58
	[a]0	---	---	---	---	7.93
2.5	1.0	9	9	1.20	1.2	11.93
↓	.8	9	9	1.00	1.0	10.11
	.6	9	9	.60	1.0	9.07
	.4	9	9	.40	1.0	8.31
	.2	15	8	.22	1.0	7.84
	.1	25	9	.12	1.0	8.12
	[a]0	---	---	---	---	7.32
5	1.0	9	9	1.20	1.2	10.94
↓	.8	9	9	1.00	1.0	9.31
	.6	9	9	.60	1.0	8.33
	.4	9	9	.40	1.0	7.74
	.2	15	8	.22	1.0	7.33
	.1	25	9	.12	1.0	7.66
	[a]0	---	---	---	---	6.72
∞	1.0	9	9	1.20	1.2	9.92
↓	.8	9	9	1.00	1.0	8.48
	.6	9	9	.60	1.0	7.57
	.4	11	11	.40	1.0	6.97
	.2	15	8	.22	1.0	6.79
	.1	25	9	.12	1.0	7.17
	[a]0	---	---	---	---	6.11

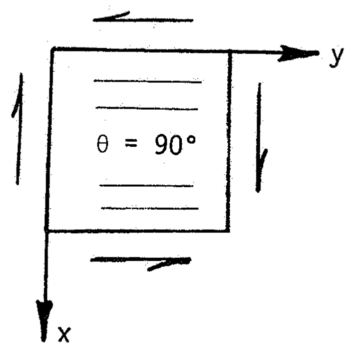
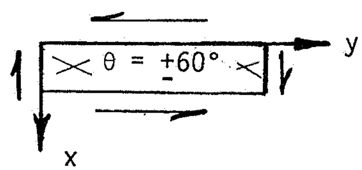
[a]For $B = 0$, k_s was calculated by using equations (B3).

TABLE 3.- MATERIAL PROPERTIES OF GRAPHITE-EPOXY SKINS WITH THEIR EQUIVALENT ORTHOTROPIC PARAMETERS AT VARIOUS FILAMENT ORIENTATIONS

$\left[E_1 = 145\ \mathrm{GN/m^2}\ (21 \times 10^6\ \mathrm{psi});\quad E_2/E_1 = 0.1138;\right.$
$\left.G_{12}/E_1 = 0.03095;\quad \nu_{12} = 0.31\right]$

Filament orientation, $\pm\theta$, deg	$\Theta = \dfrac{\sqrt{D_{11}D_{22}}}{D_3}$	$\dfrac{a}{b} B = \sqrt[4]{\dfrac{D_{11}}{D_{22}}}$
0	3.50	1.722
30	.511	1.389
45	.415	1.000
60	.511	.720
90	3.50	.581

TABLE 4.- COMPARISON OF CONVENTIONAL AND TRIGONOMETRIC FINITE DIFFERENCES FOR ORTHOTROPIC PANELS

Problem description	Degrees of freedom		$\bar{N}_{xy}$		λ_x/a	λ_y/λ_x
	M_e	N_e	Conventional	Trigonometric		
Shear buckling of a clamped, square, graphite-epoxy panel	4	4	56.03	----	---	---
	6	6	48.43	42.84	0.55	1
	8	8	45.65	----	---	---
	12	12	43.79	----	---	---
	20	20	42.90	----	---	---
						
Shear buckling of a simply supported 5 × 1 graphite-epoxy panel	20	10	20.40	19.17	0.21	1
	29	13	19.70	----	---	---
	40	15	19.39	----	---	---
	50	20	19.22	----	---	---
						

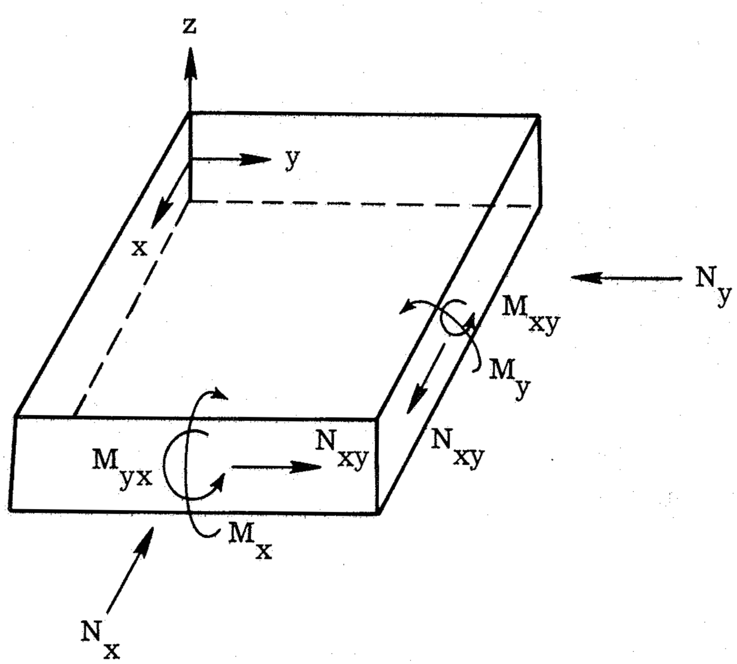
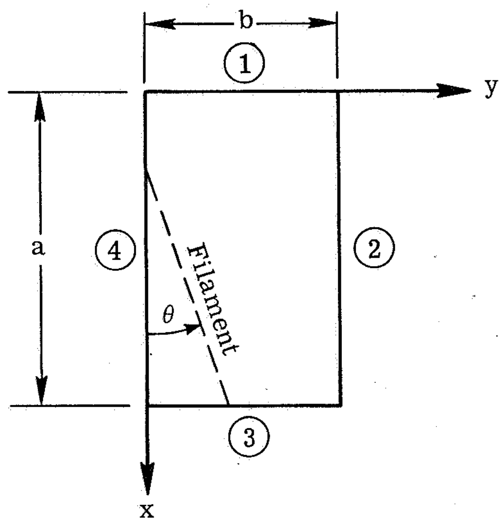
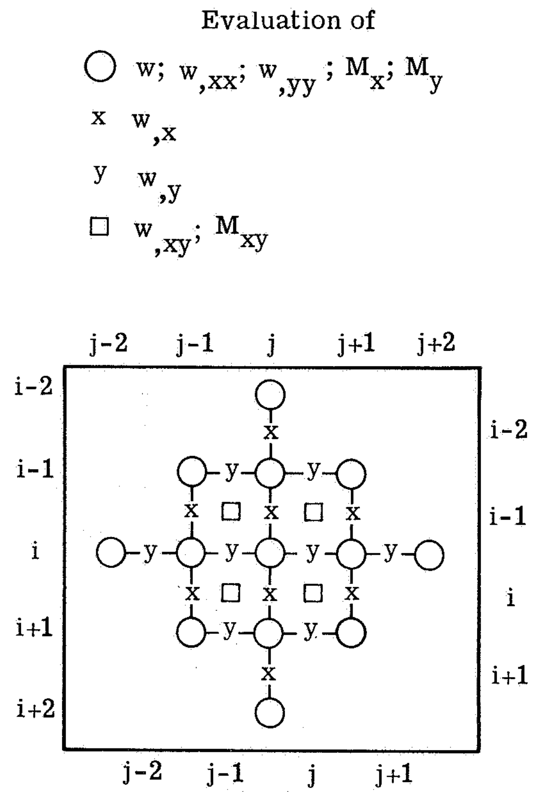


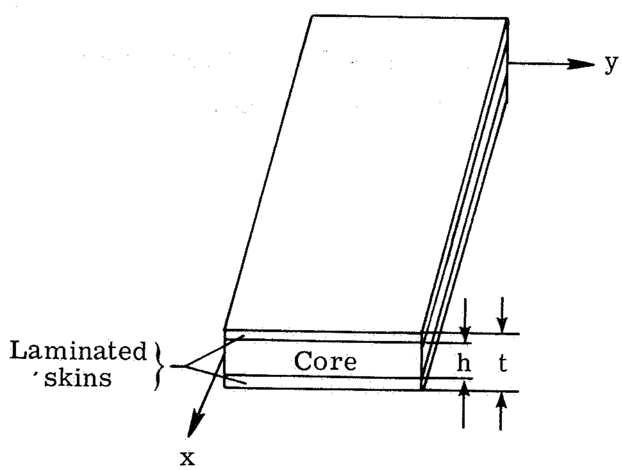
Figure 1.- Stress resultants acting upon an element of the plate.



(a) Panel geometry and boundary designation.



(b) Finite-difference station layout and designation.



(c) Sandwich panel.

Figure 2.- Geometrical and numerical configurations.

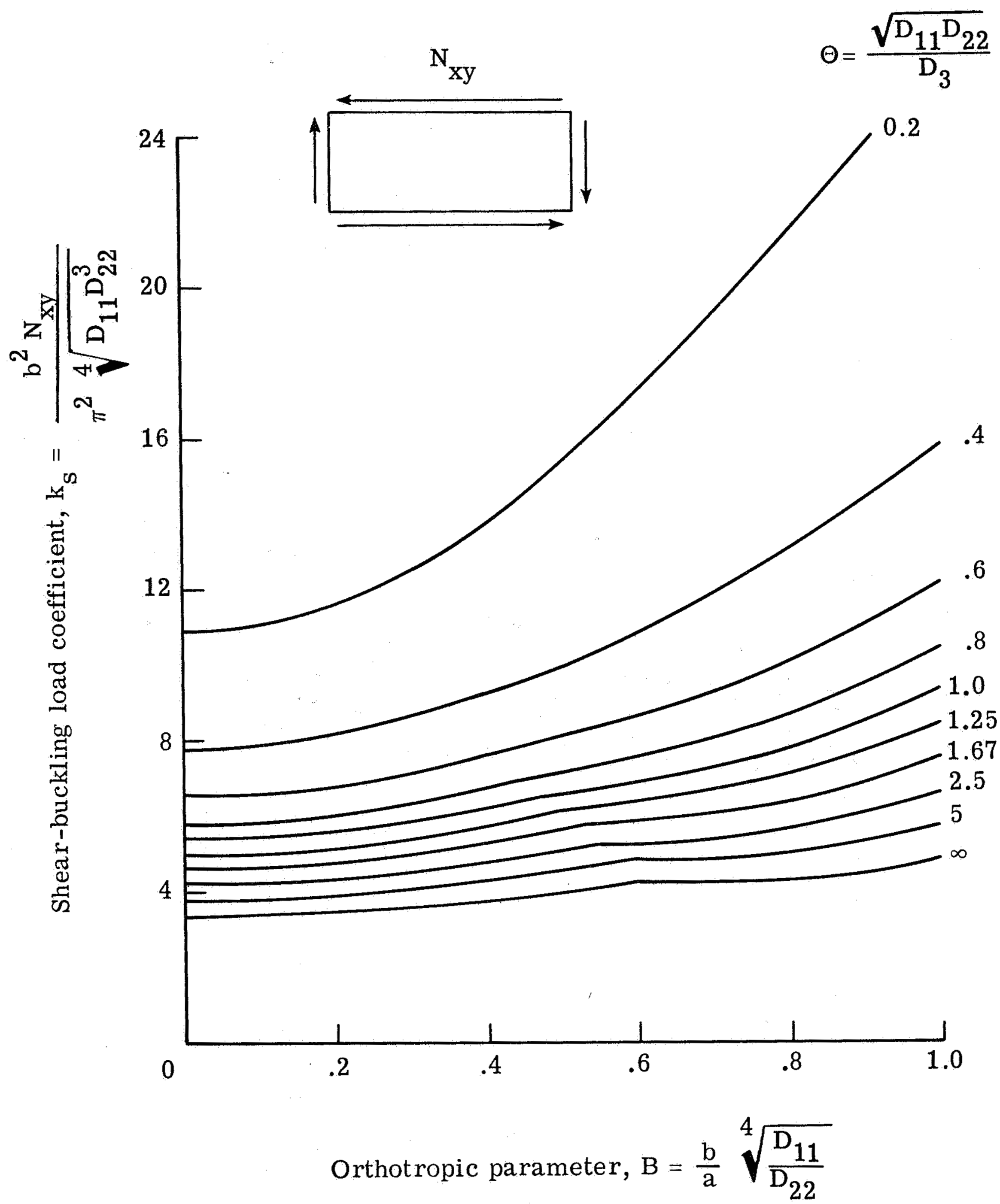


Figure 3.- Shear-buckling load coefficients for rectangular orthotropic plates with all edges simply supported.

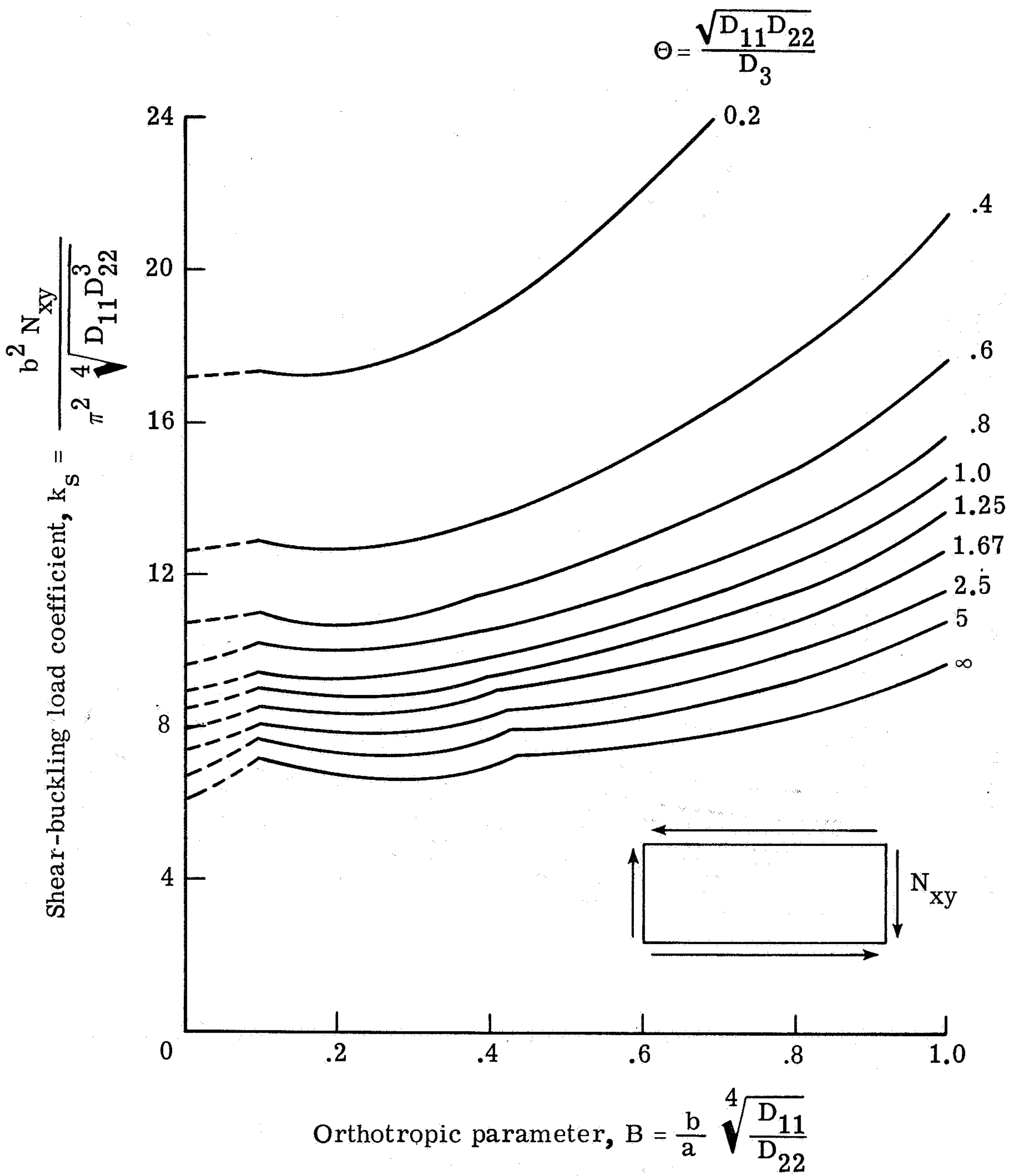


Figure 4. - Shear-buckling load coefficients for rectangular orthotropic plates with all edges clamped.

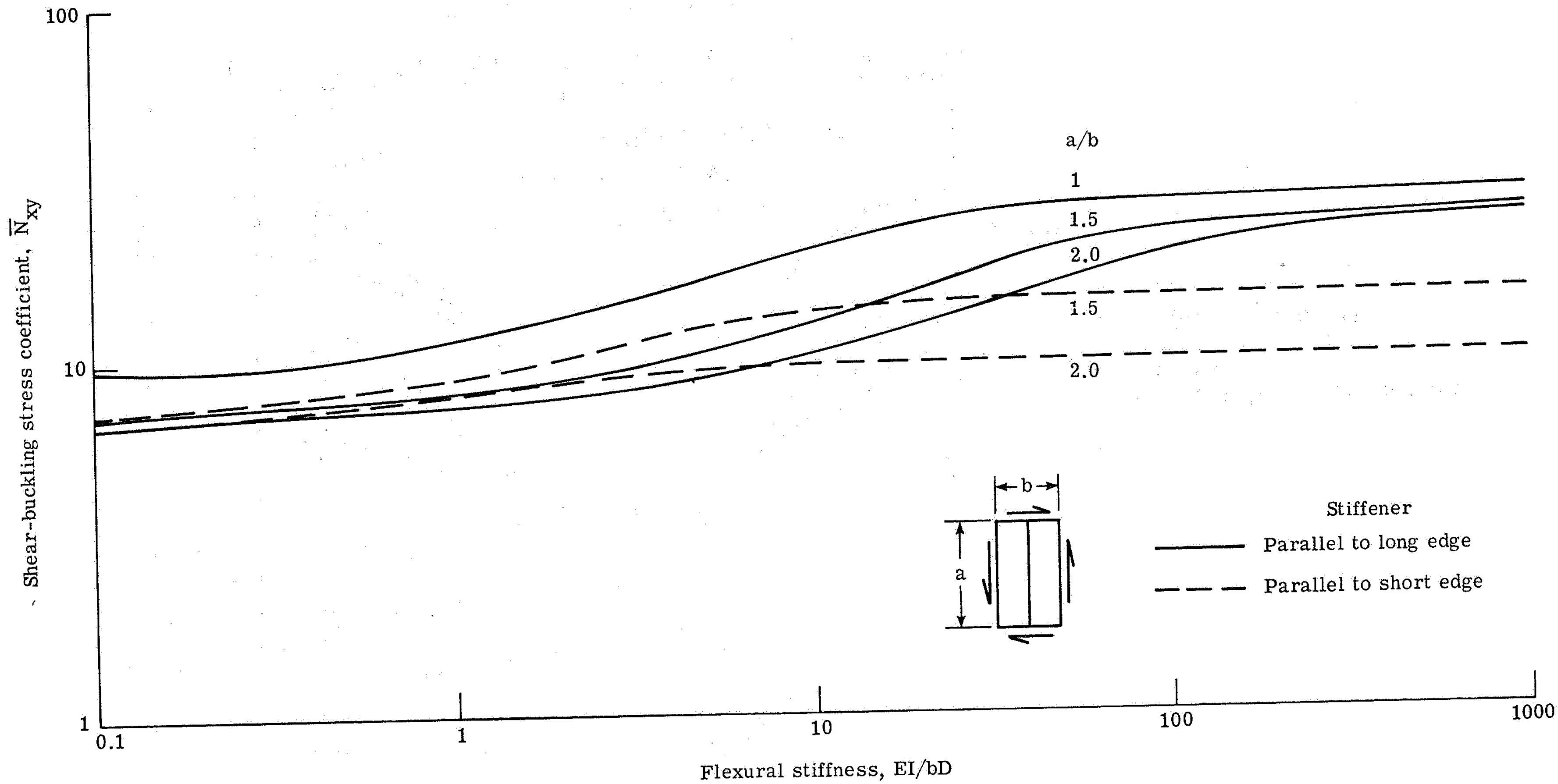


Figure 5. - Shear buckling of simply supported isotropic panels each with a central stiffener.

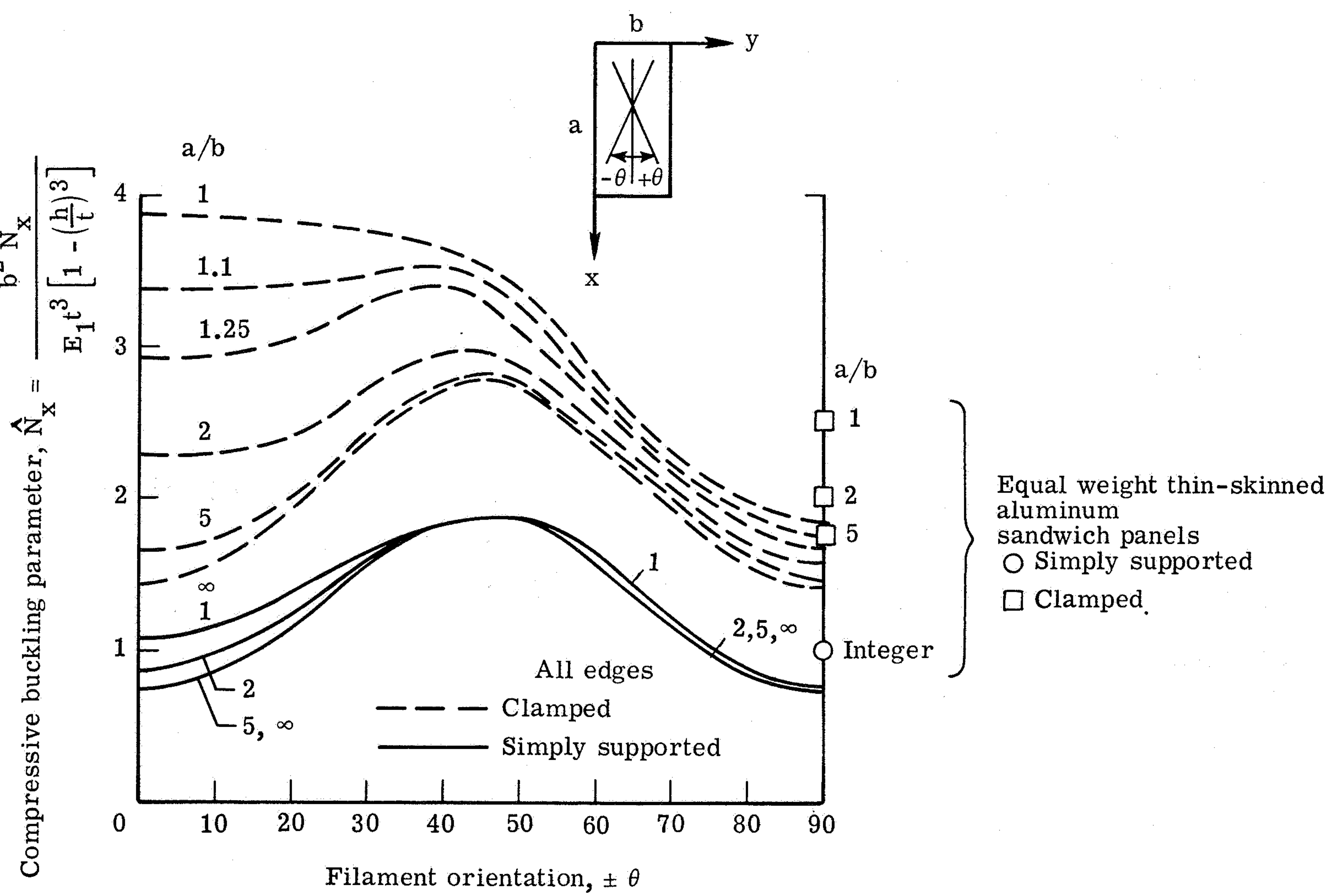


Figure 6.- Variation of compressive buckling load with filament orientation for panels of various aspect ratios.

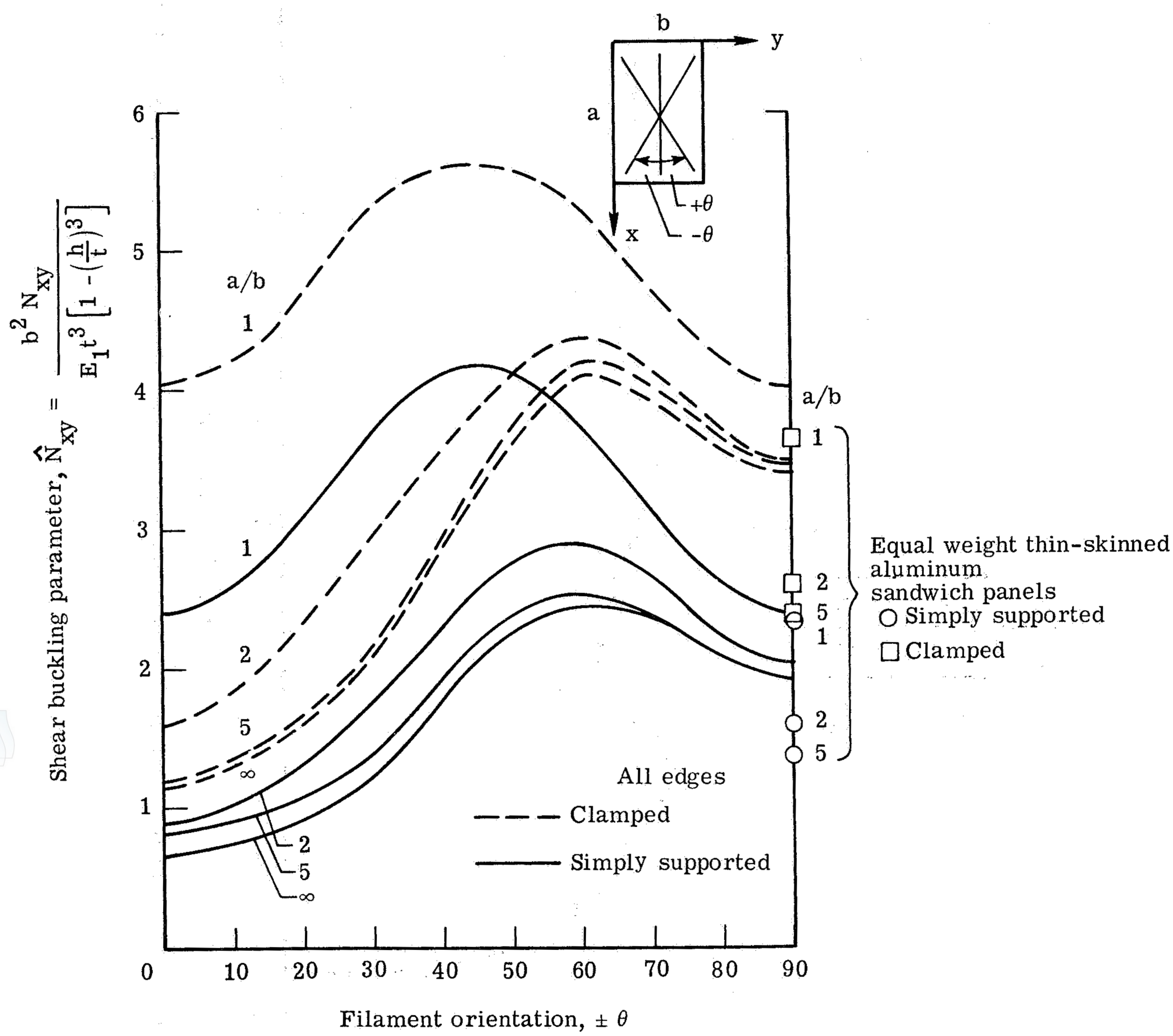


Figure 7.- Variation of shear buckling load with filament orientation for panels of various aspect ratios.

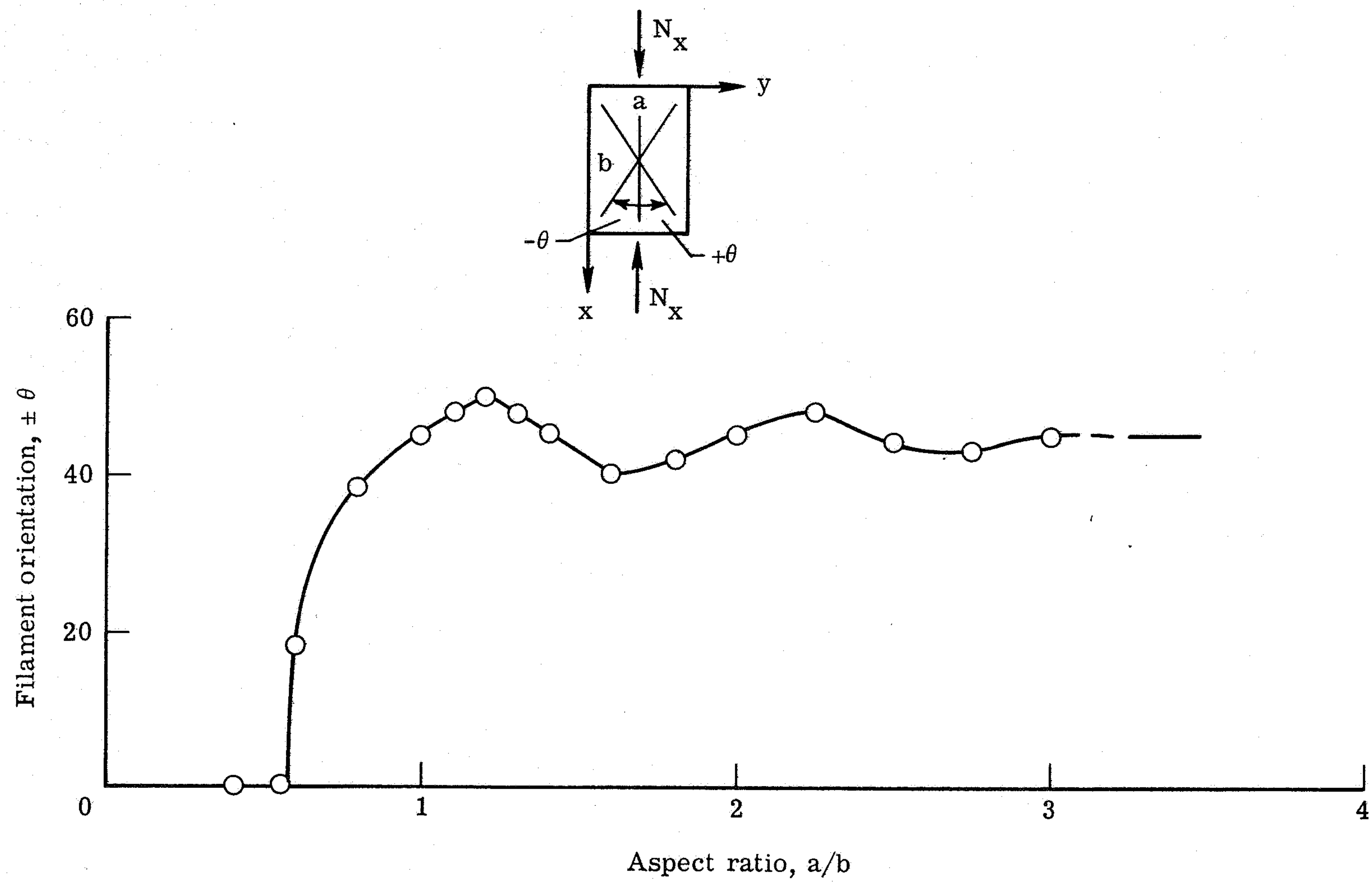


Figure 8.- Optimum filament orientation for the compressive buckling of a simply supported panel.

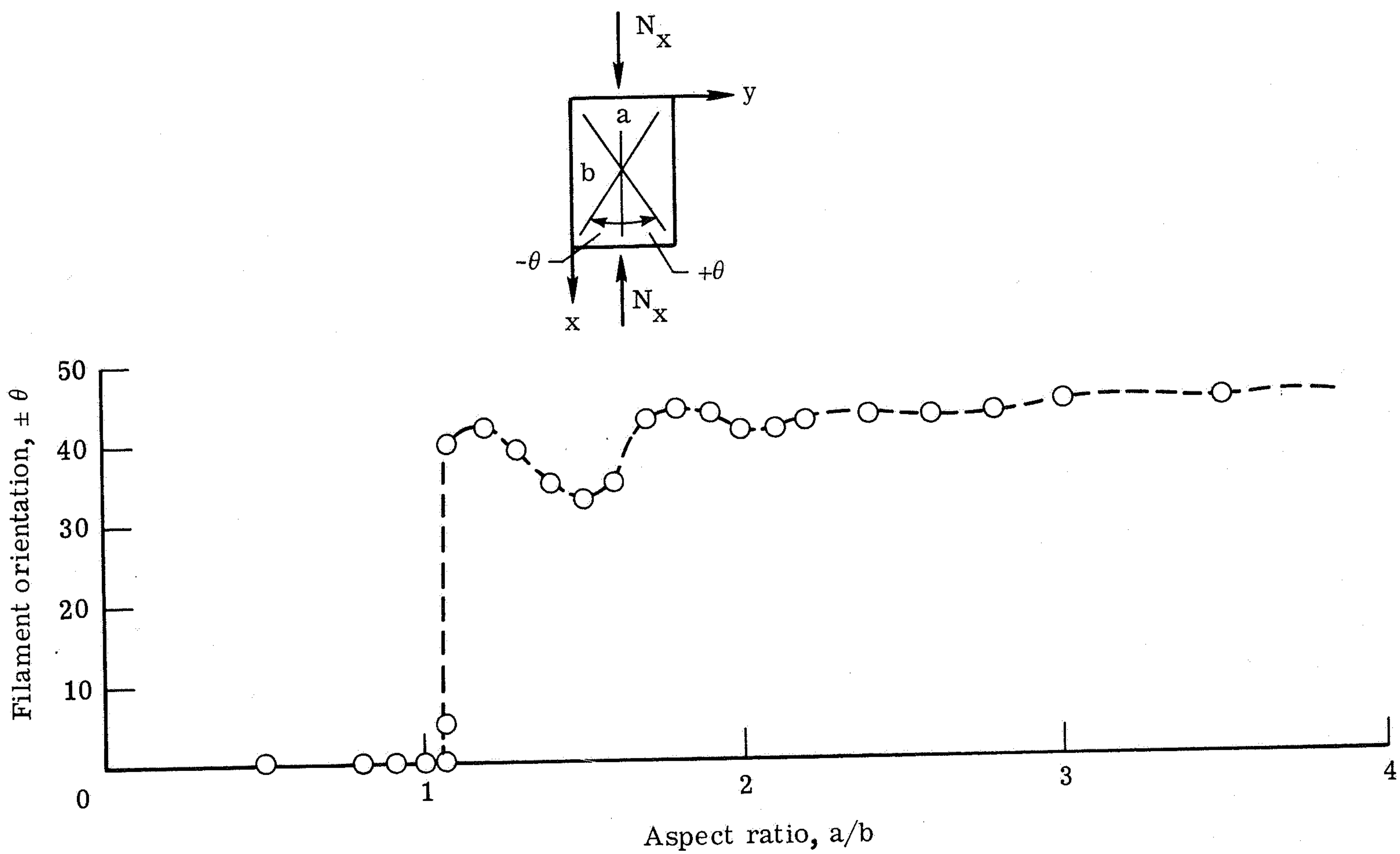


Figure 9.- Optimum filament orientation for the compressive buckling of a clamped panel.

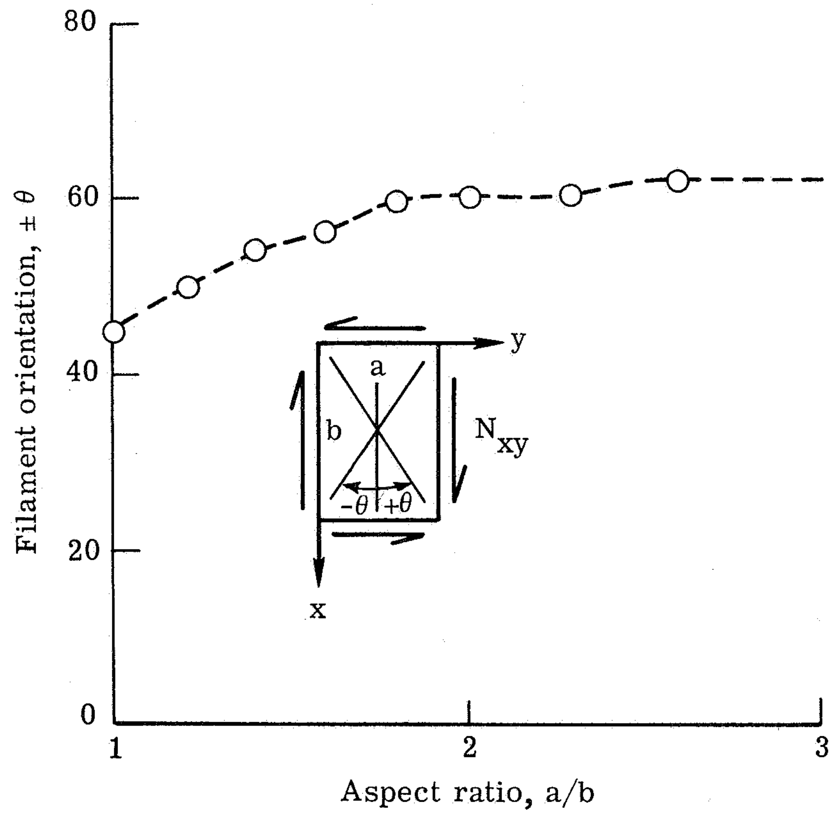


Figure 10.- Optimum filament orientation for the shear buckling of a simply supported panel.

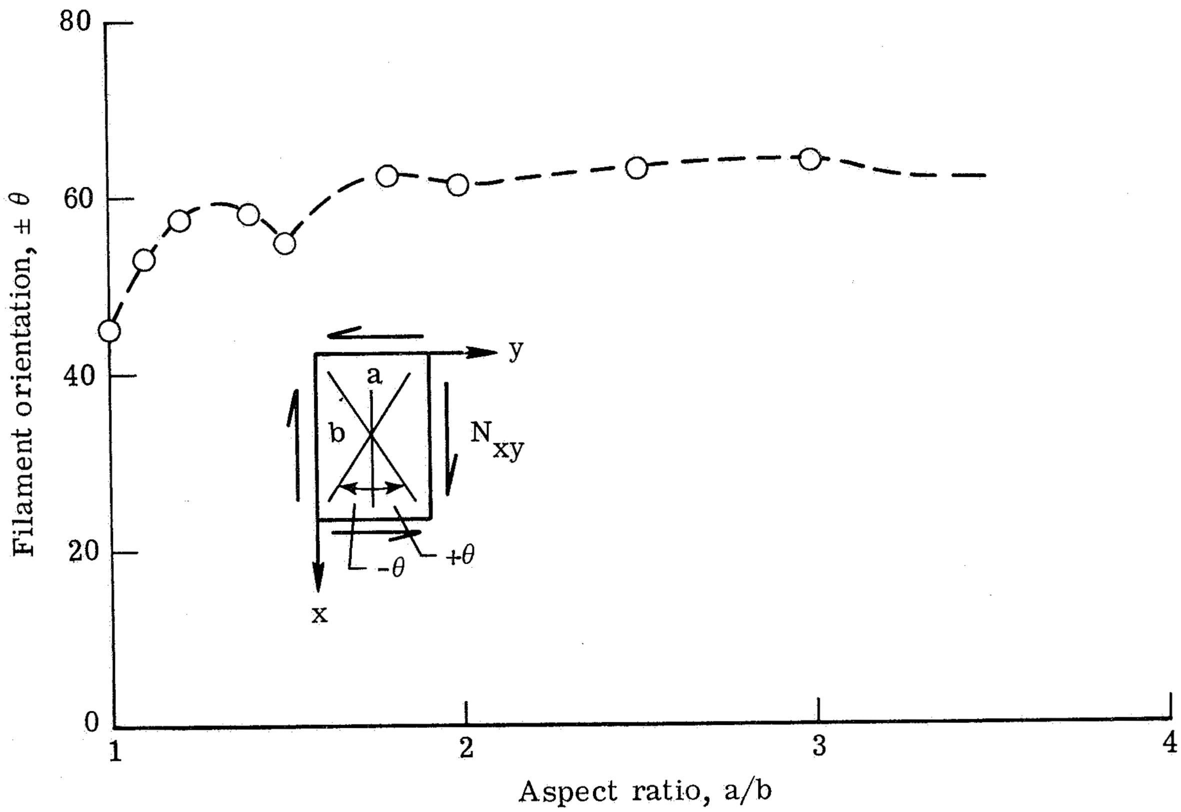
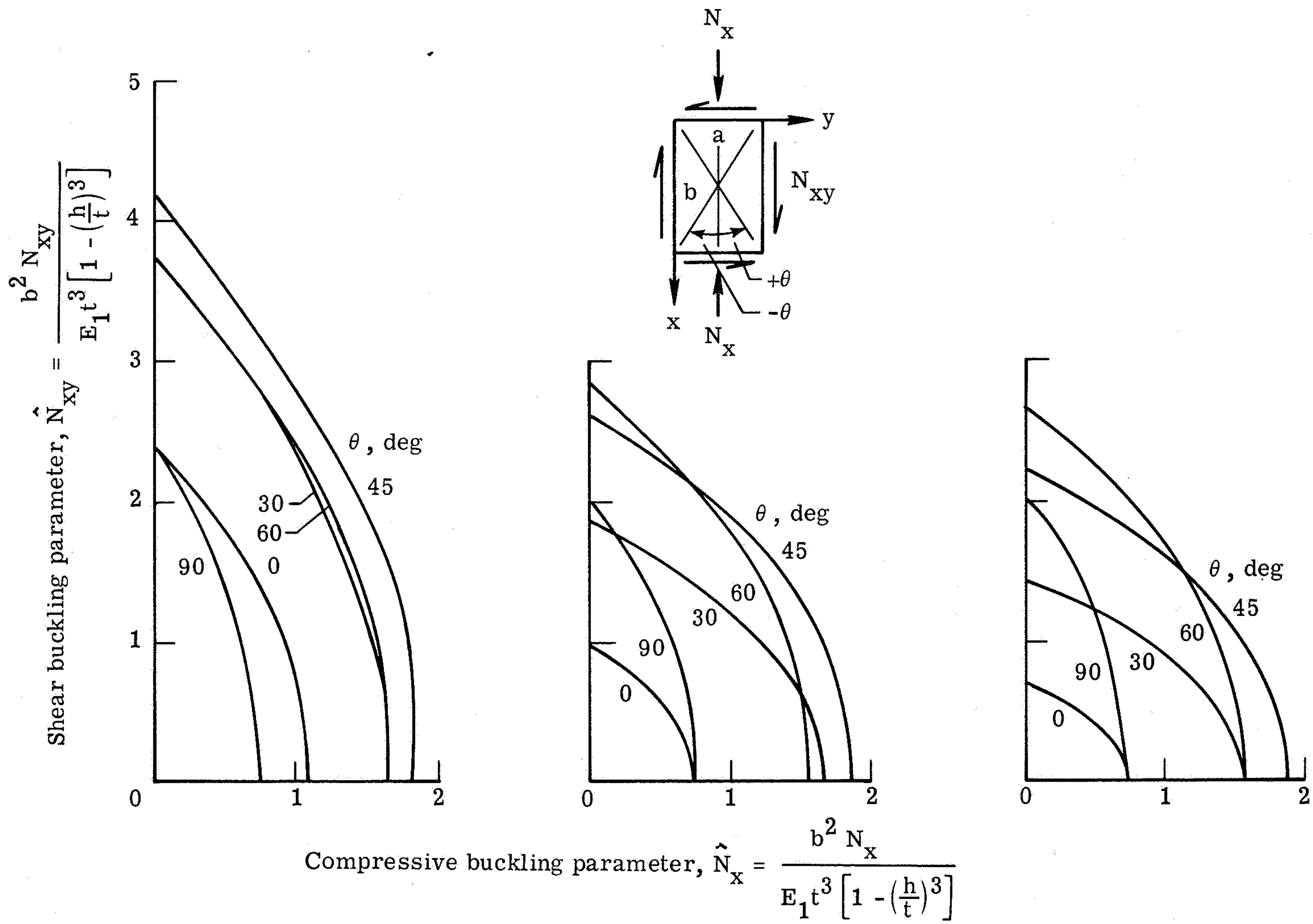


Figure 11.- Optimum filament orientation for the shear buckling of a clamped panel.

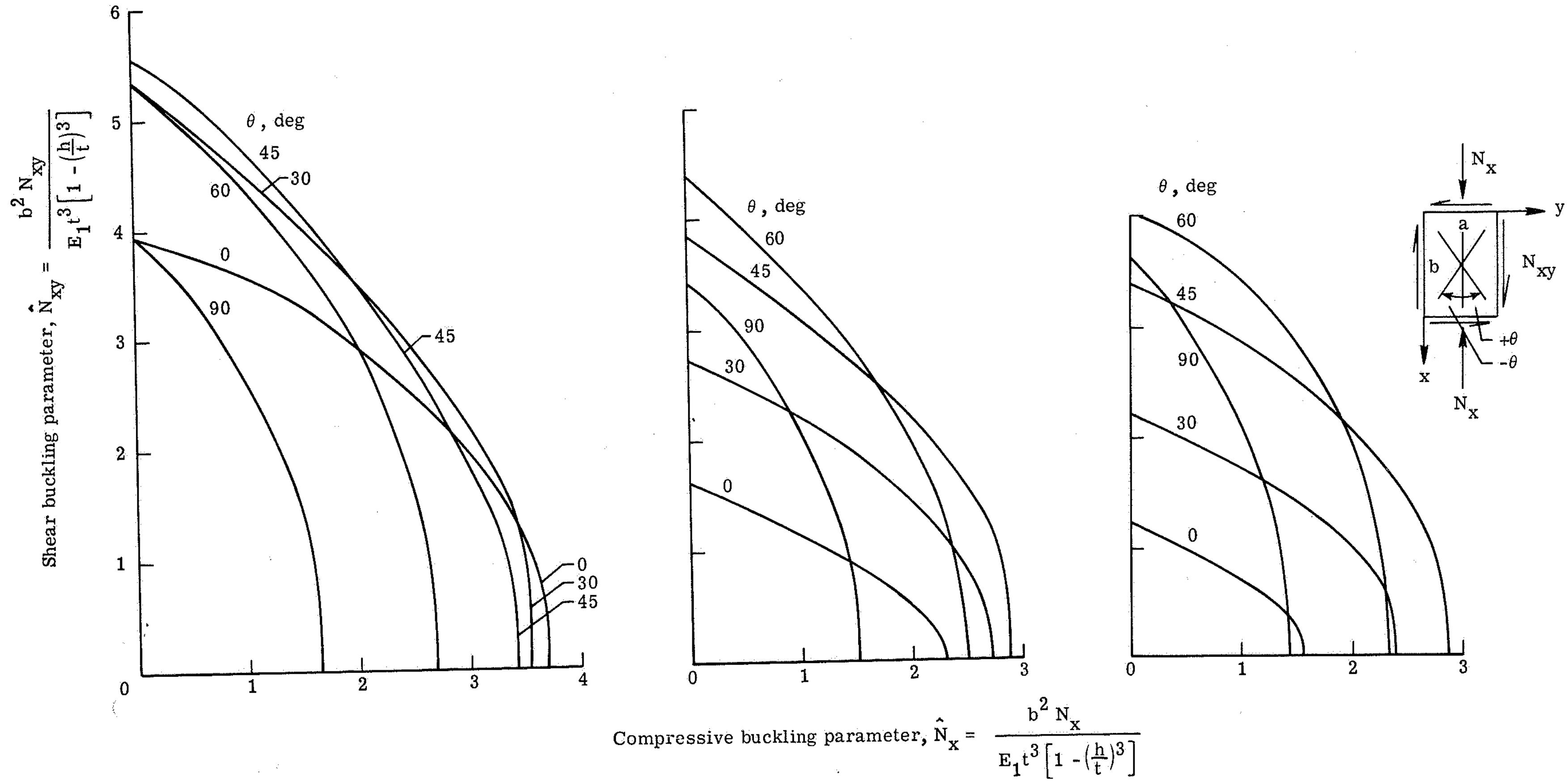


(a) a/b = 1.

(b) a/b = 2.

(c) a/b = 5.

Figure 12.- Combined axial compression and shear of simply supported panels.



(a) a/b = 1.

(b) a/b = 2.

(c) a/b = 5.

Figure 13.- Combined axial compression and shear of clamped panels.

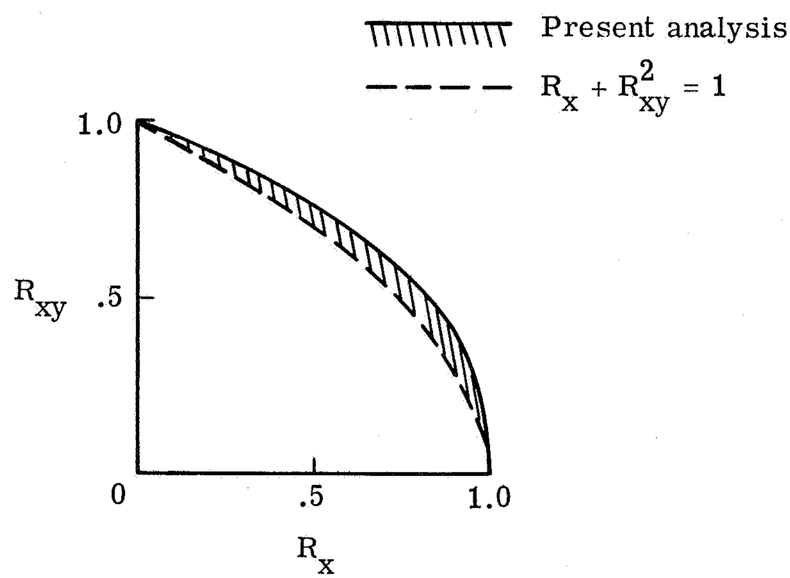
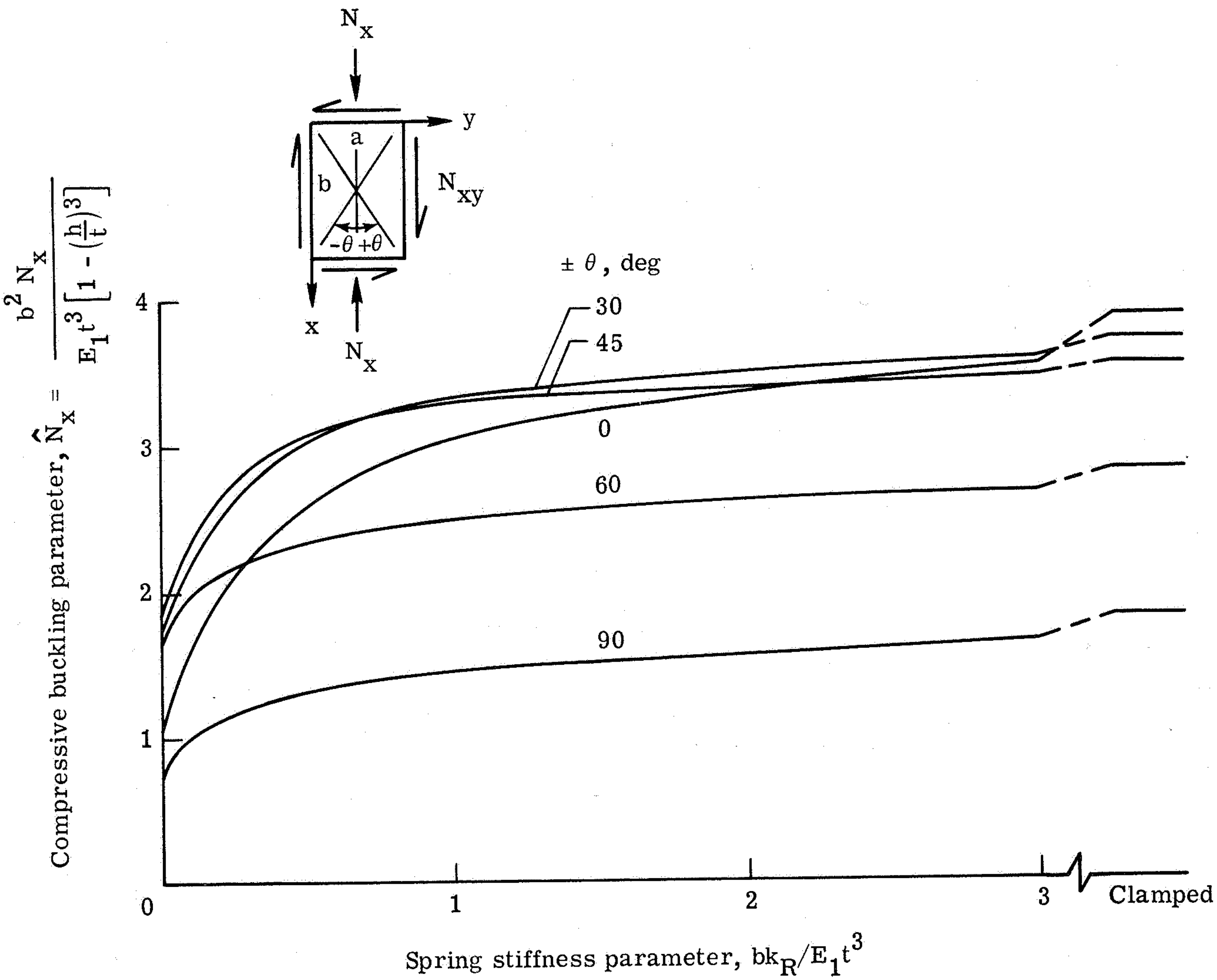
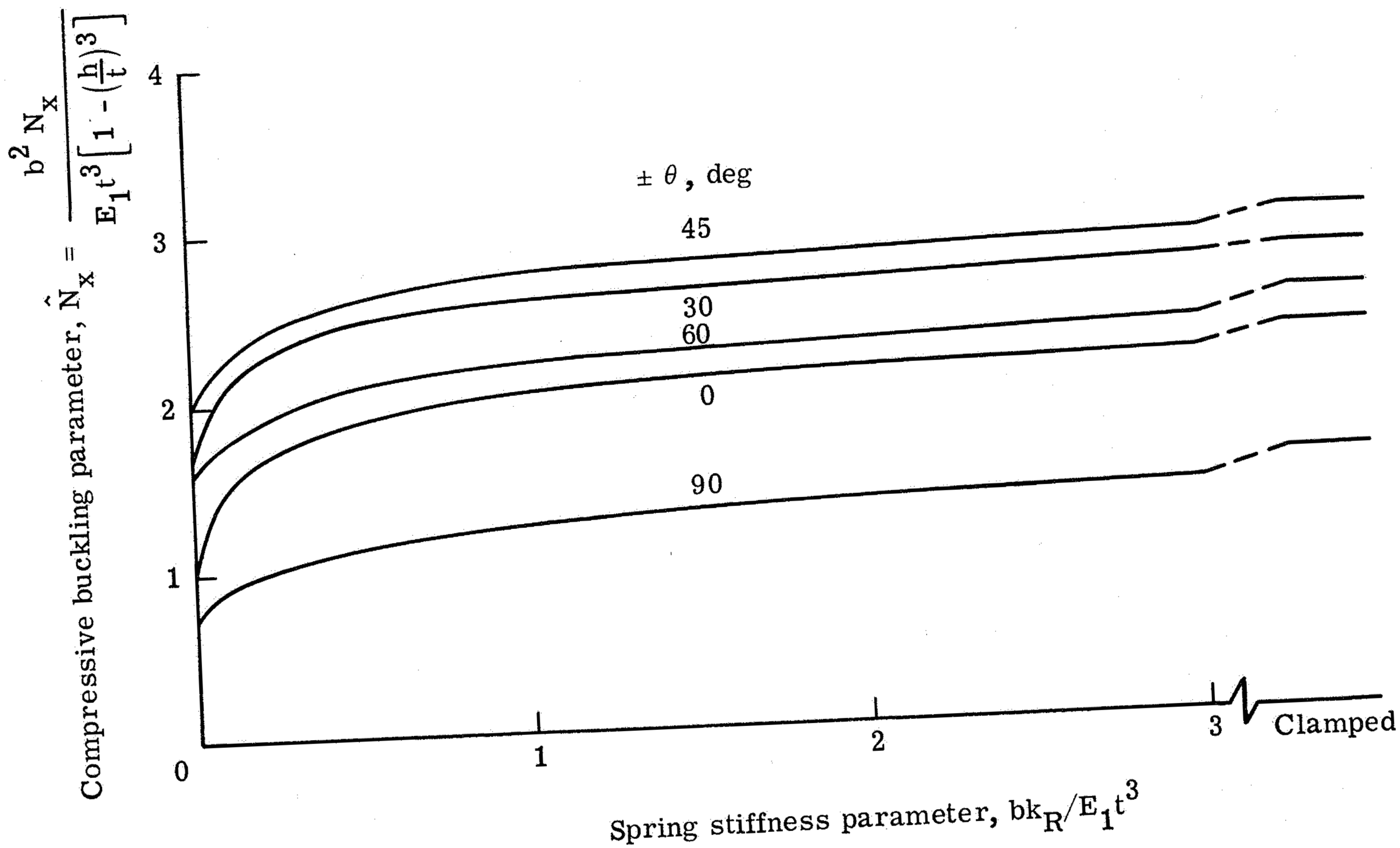


Figure 14.- Summary of combined axial compression and shear-buckling results for simply supported and clamped panels.



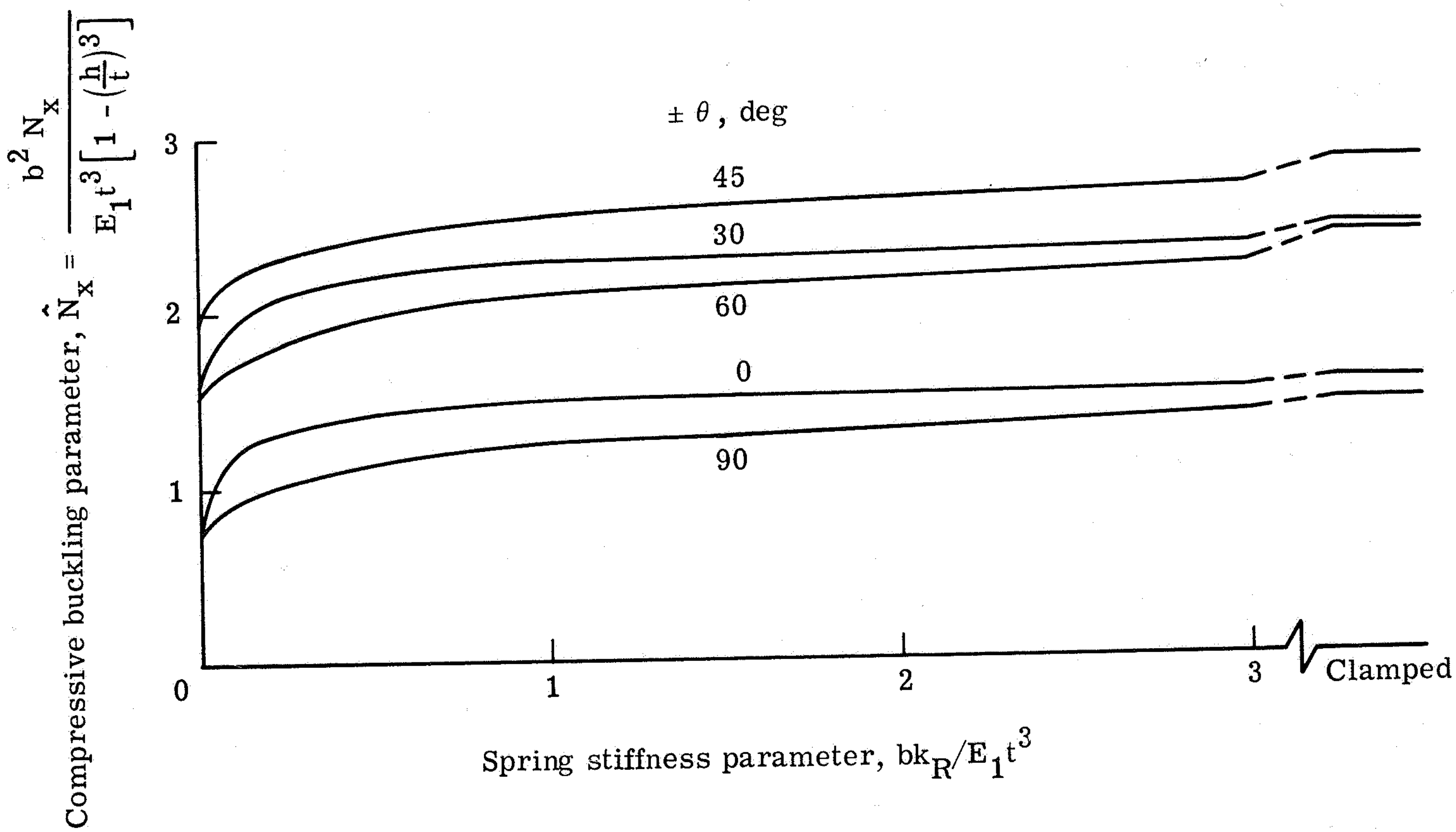
(a) a/b = 1.

Figure 15.- Variation of compressive buckling parameter with rotational edge spring stiffness.



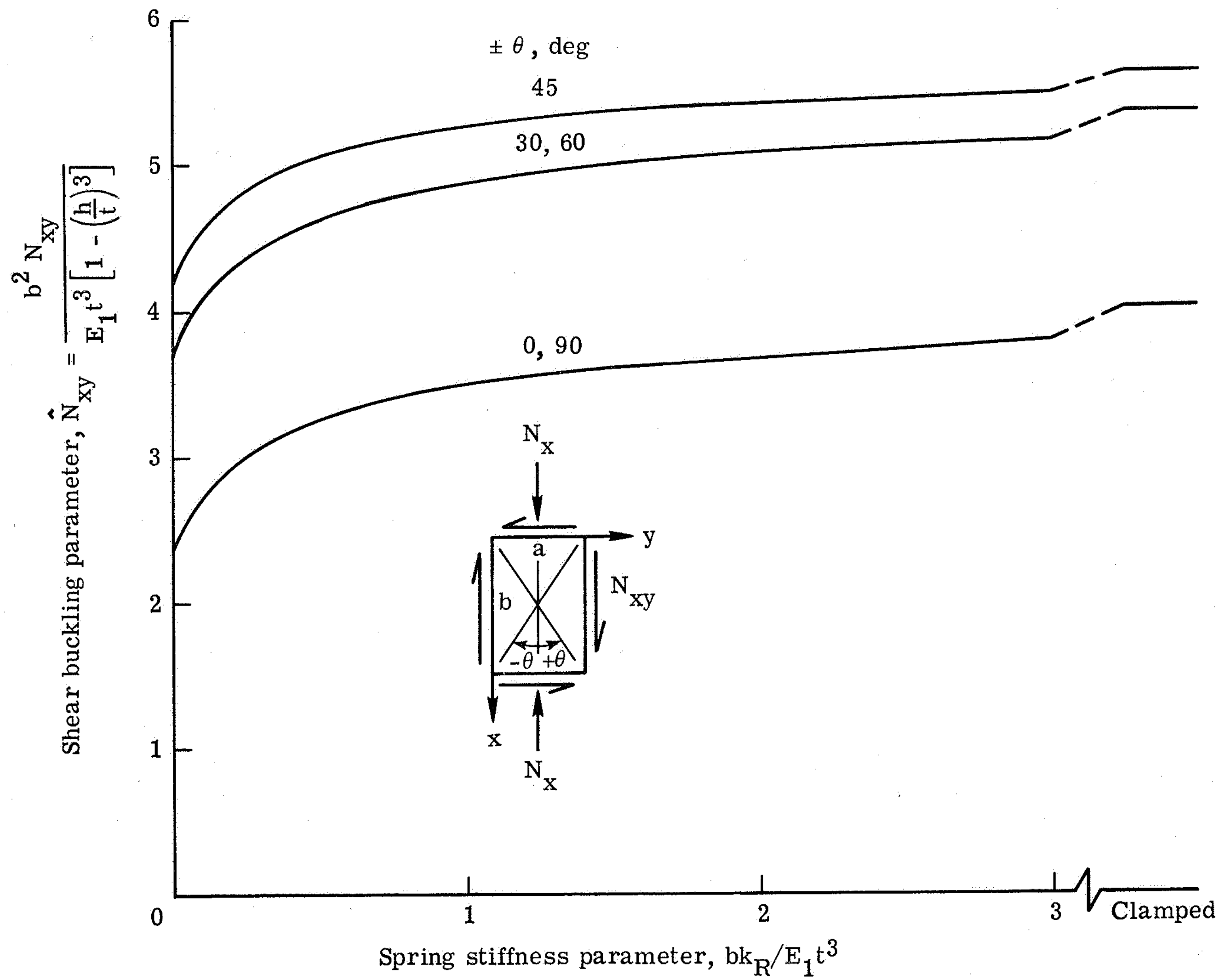
(b) $a/b = 2$.

Figure 15.- Continued.



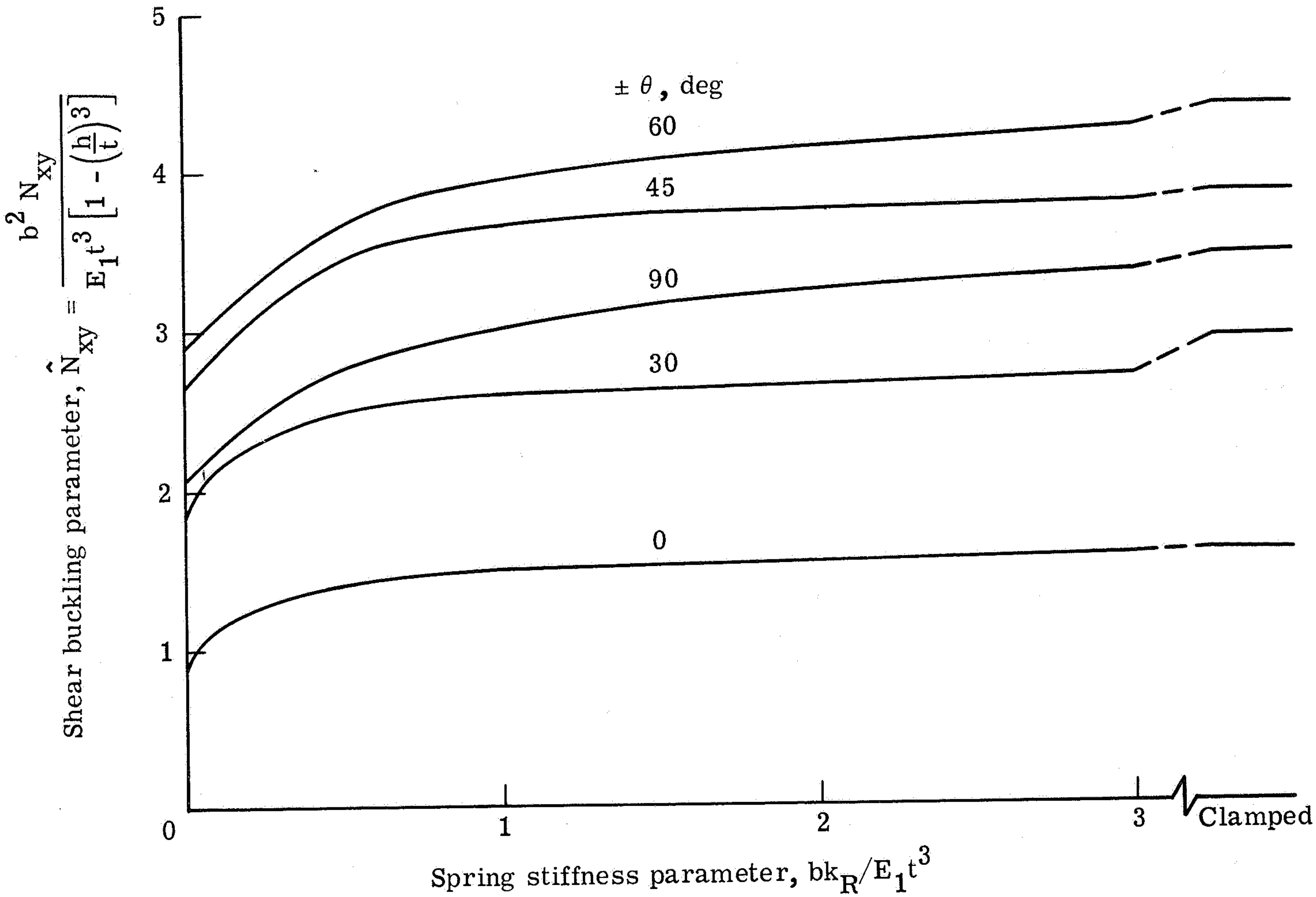
(c) a/b = 5.

Figure 15.- Concluded.



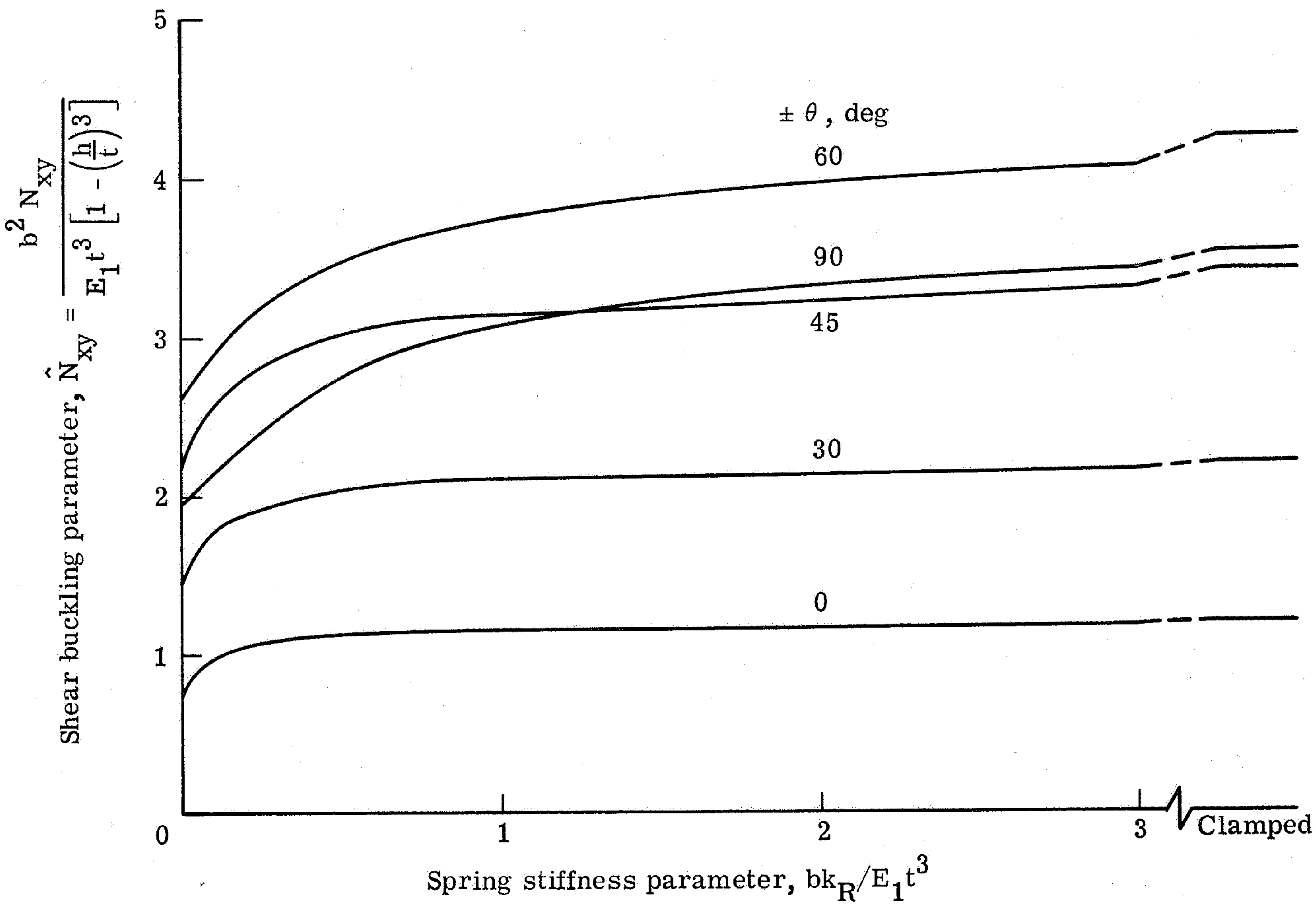
(a) a/b = 1.

Figure 16.- Variation of shear buckling parameter with rotational edge spring stiffness.



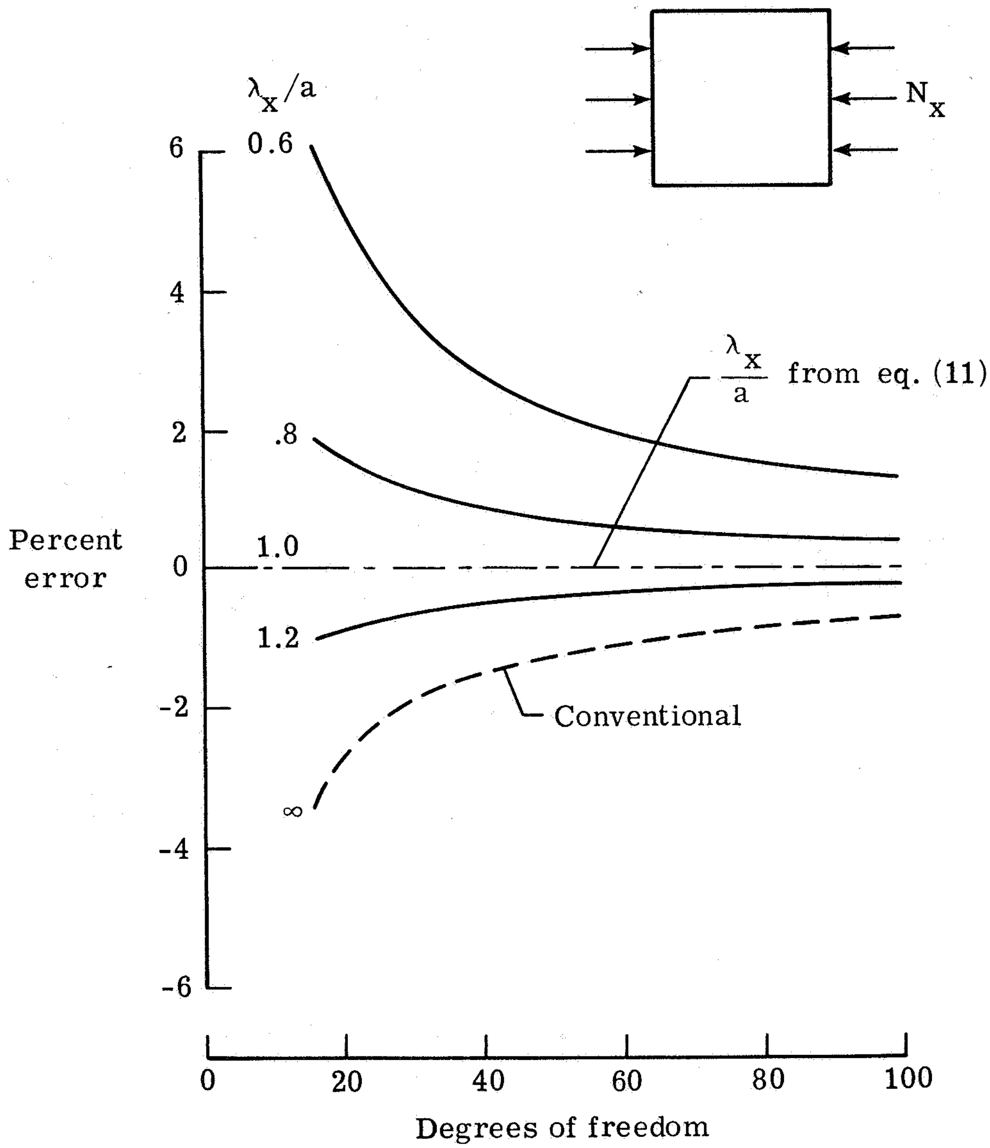
(b) a/b = 2.

Figure 16.- Continued.



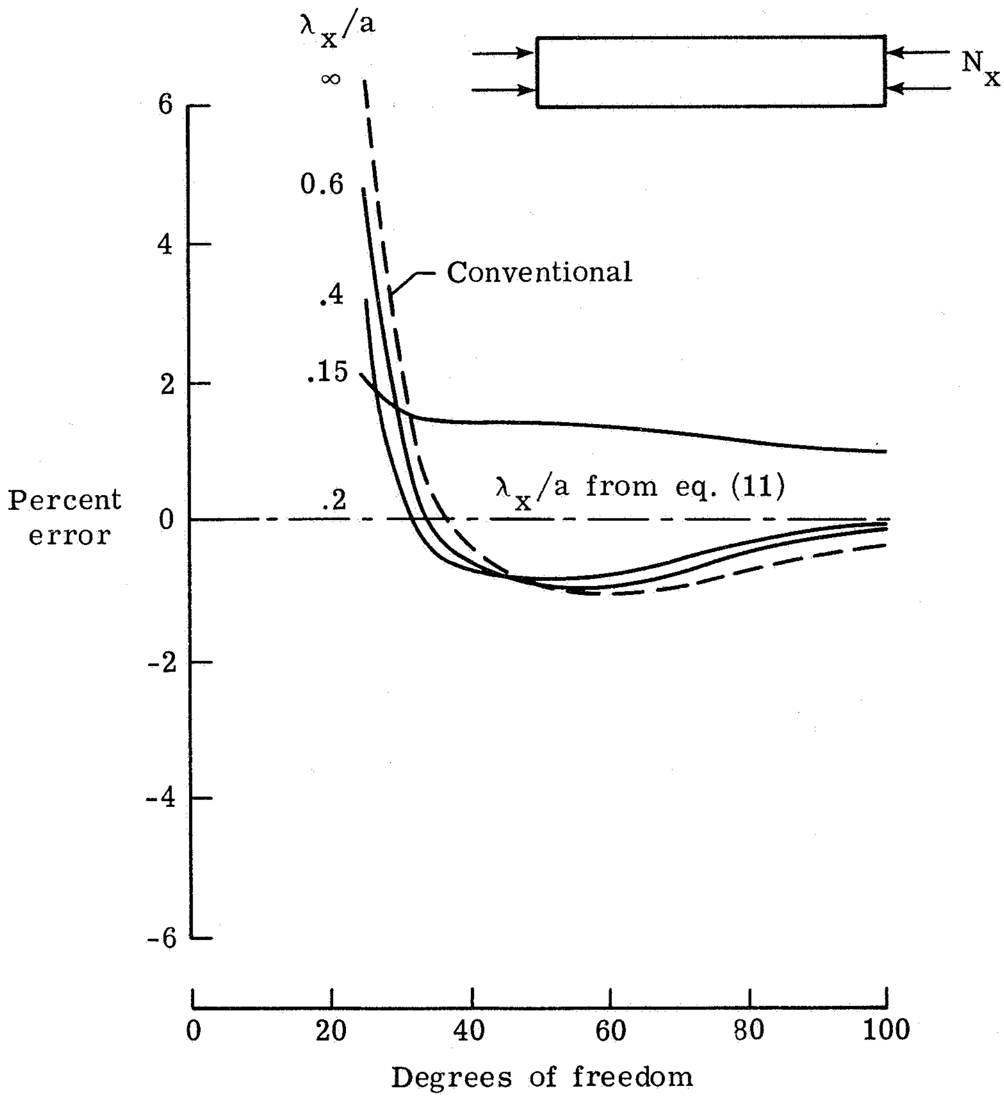
(c) a/b = 5.

Figure 16.- Concluded.



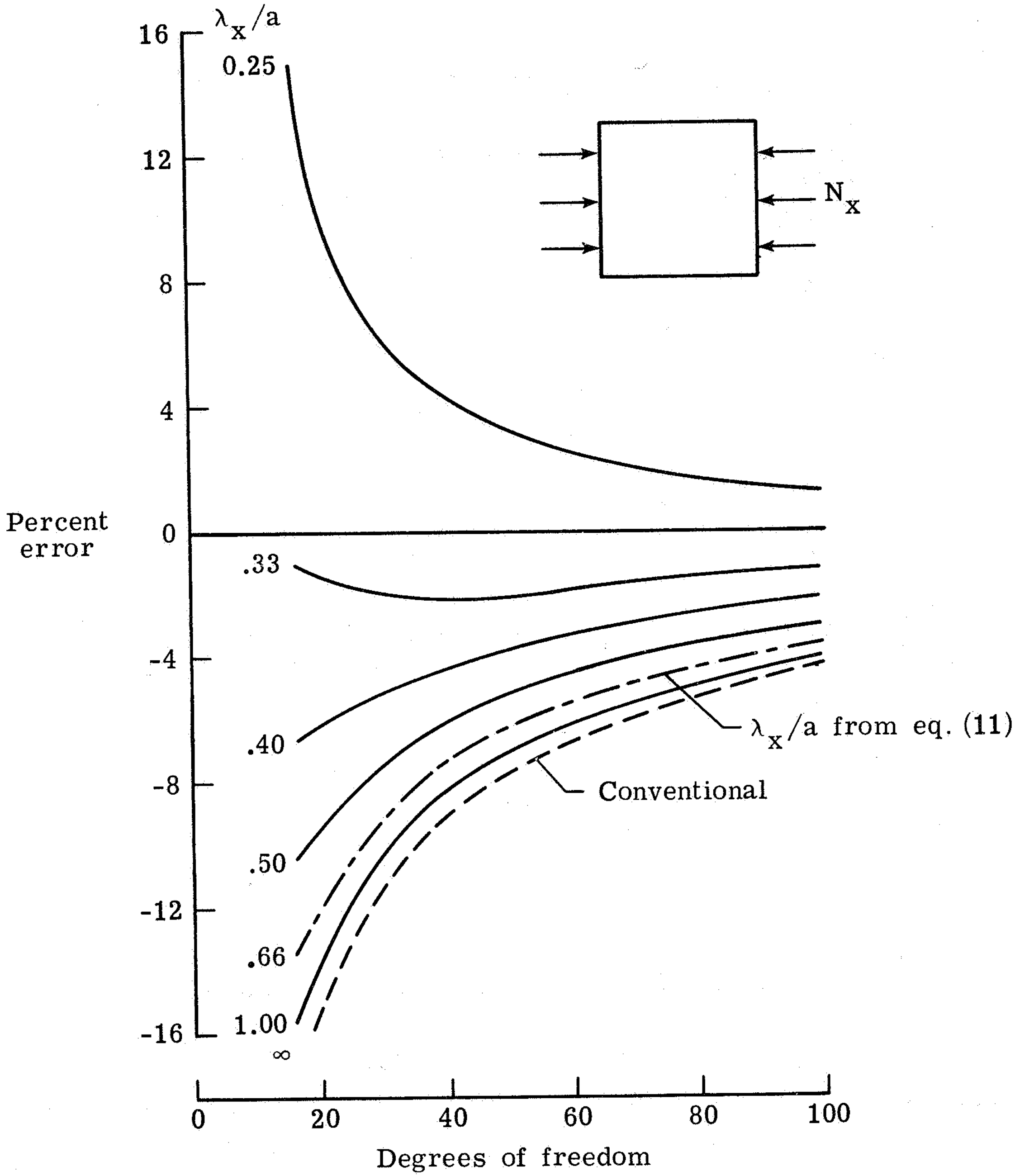
(a) Convergence for the compressive buckling of simply supported isotropic square panels. $\lambda_y/\lambda_x = \beta = 1.$ From reference 6, $\overline{N}_x = 4.0.$

Figure 17.- Convergence characteristics of trigonometric finite differences.



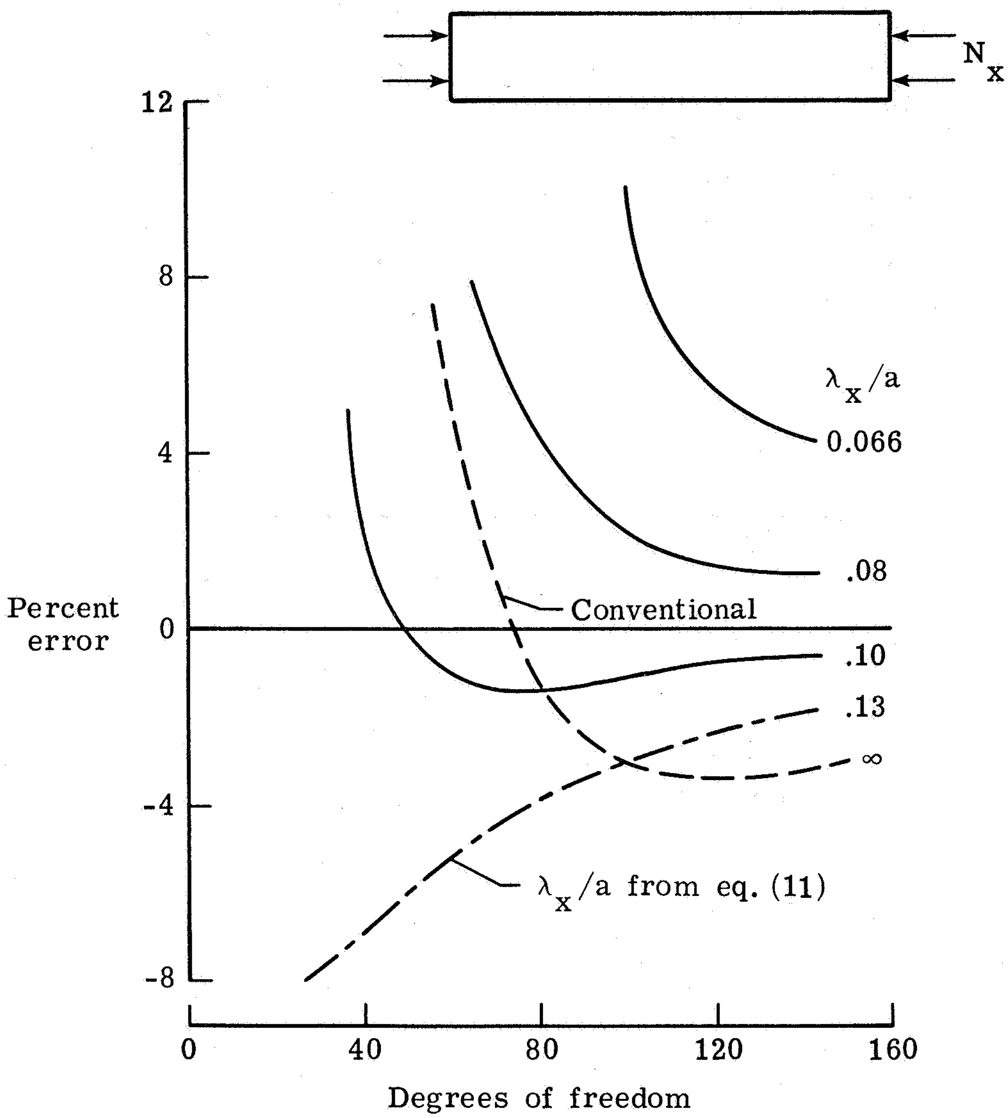
(b) Convergence for the compressive buckling of 5 × 1 simply supported isotropic panels. $\lambda_y/\lambda_x = \beta = 5$. From reference 6, $\overline{N}_x = 4.0$.

Figure 17.- Continued.



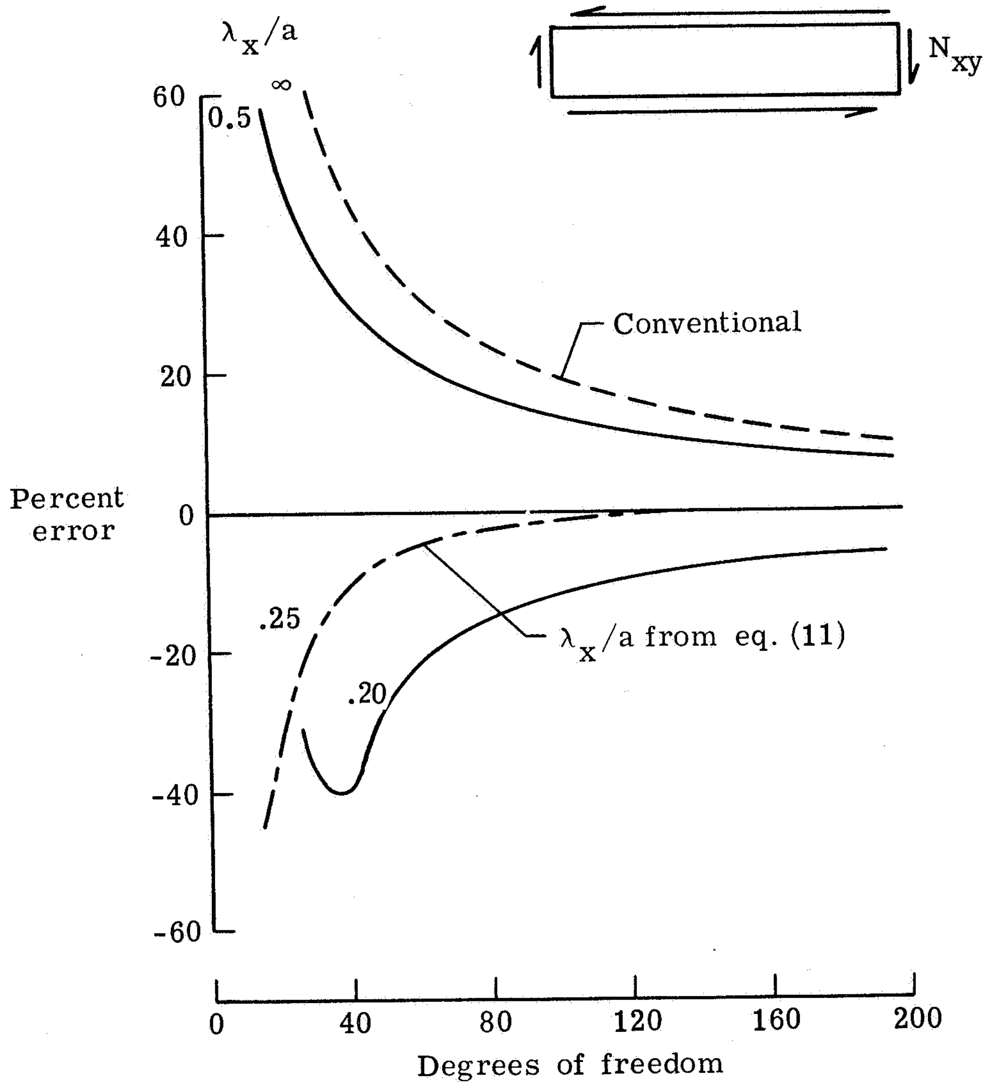
(c) Convergence for the compressive buckling of clamped square isotropic panels. $\lambda_y/\lambda_x = \beta = 1.5$. From reference 17, $\overline{N}_x \approx 10.074$.

Figure 17.- Continued.



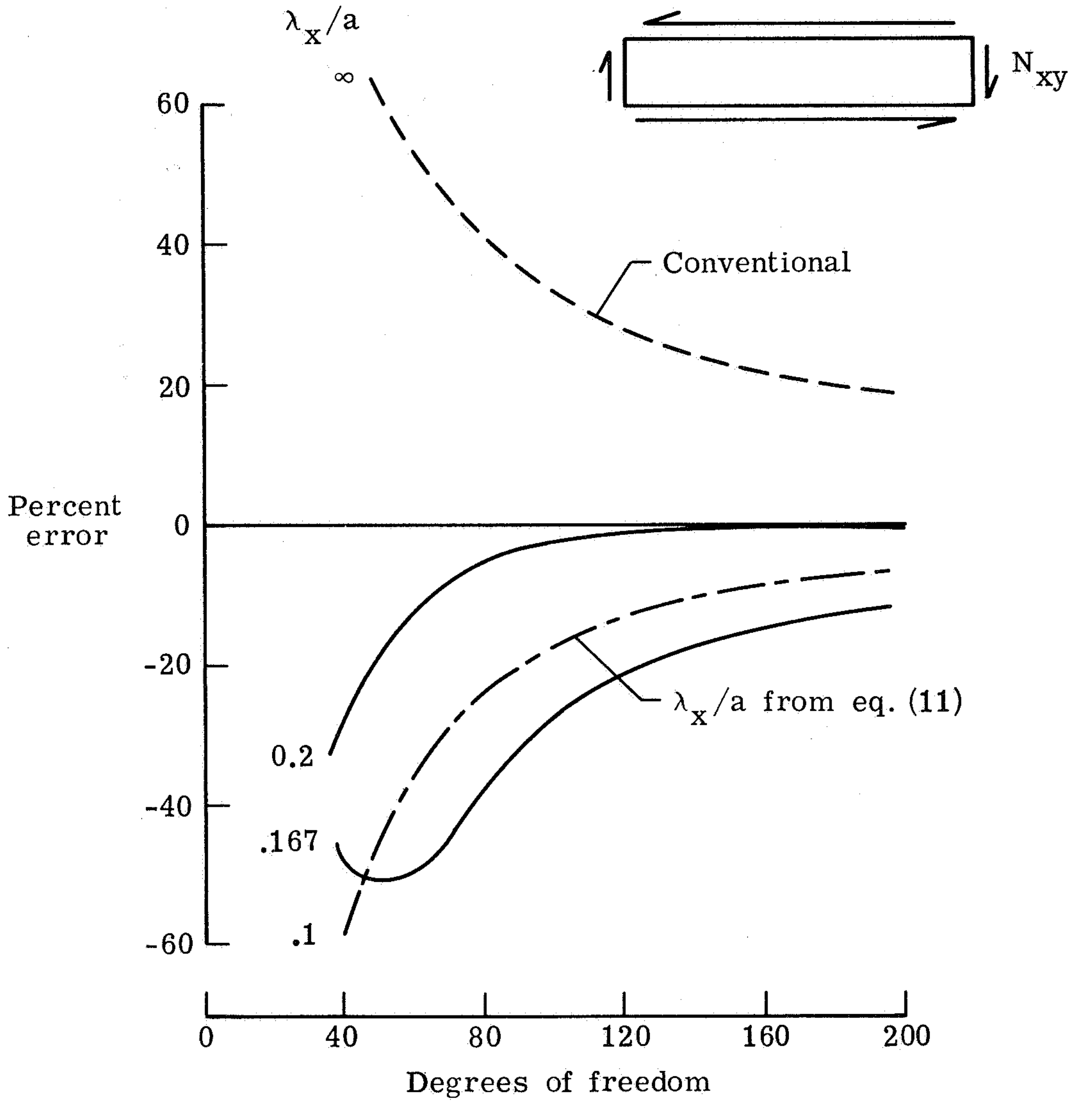
(d) Convergence for the compressive buckling of isotropic clamped 5 × 1 panels. $\lambda_y/\lambda_x = \beta = 1.5$. From reference 16, $\overline{N}_x \approx 7.0$.

Figure 17.- Continued.



(e) Convergence for the shear buckling of isotropic simply supported 5 × 1 panels. $\lambda_y/\lambda_x = \beta = 0.8$. From reference 1, $\overline{N}_{xy} \approx 5.55$.

Figure 17.- Continued.



(f) Convergence for the shear buckling of isotropic clamped 5 × 1 panels.
$\lambda_y/\lambda_x = \beta = 1.2$. From reference 1, $\bar{N}_{xy} \approx 9.3$.

Figure 17.- Concluded.

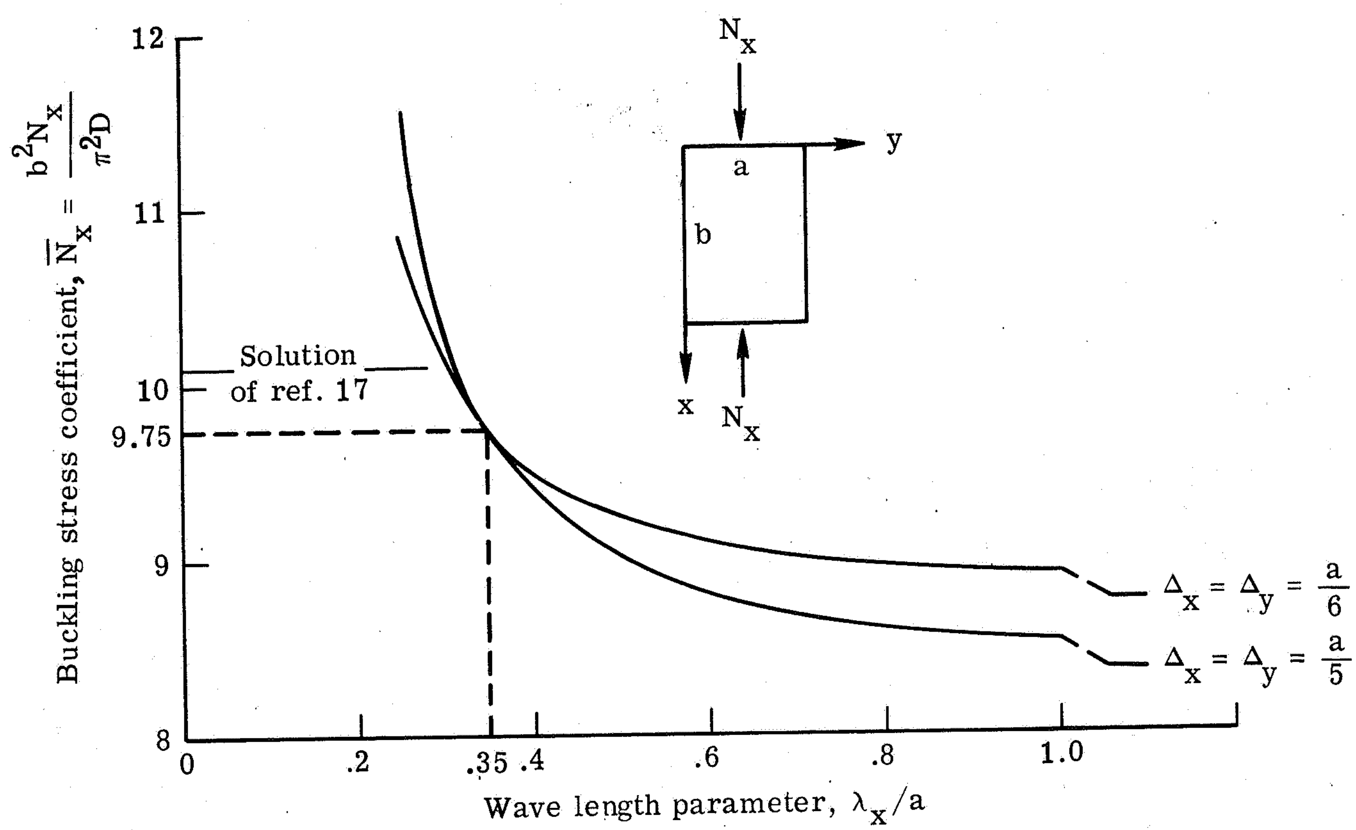


Figure 18.- Variation of buckling stress coefficient $\overline{N}_x$ with trigonometric wavelength parameter λ_x/a for the compression buckling of a clamped square isotropic panel.

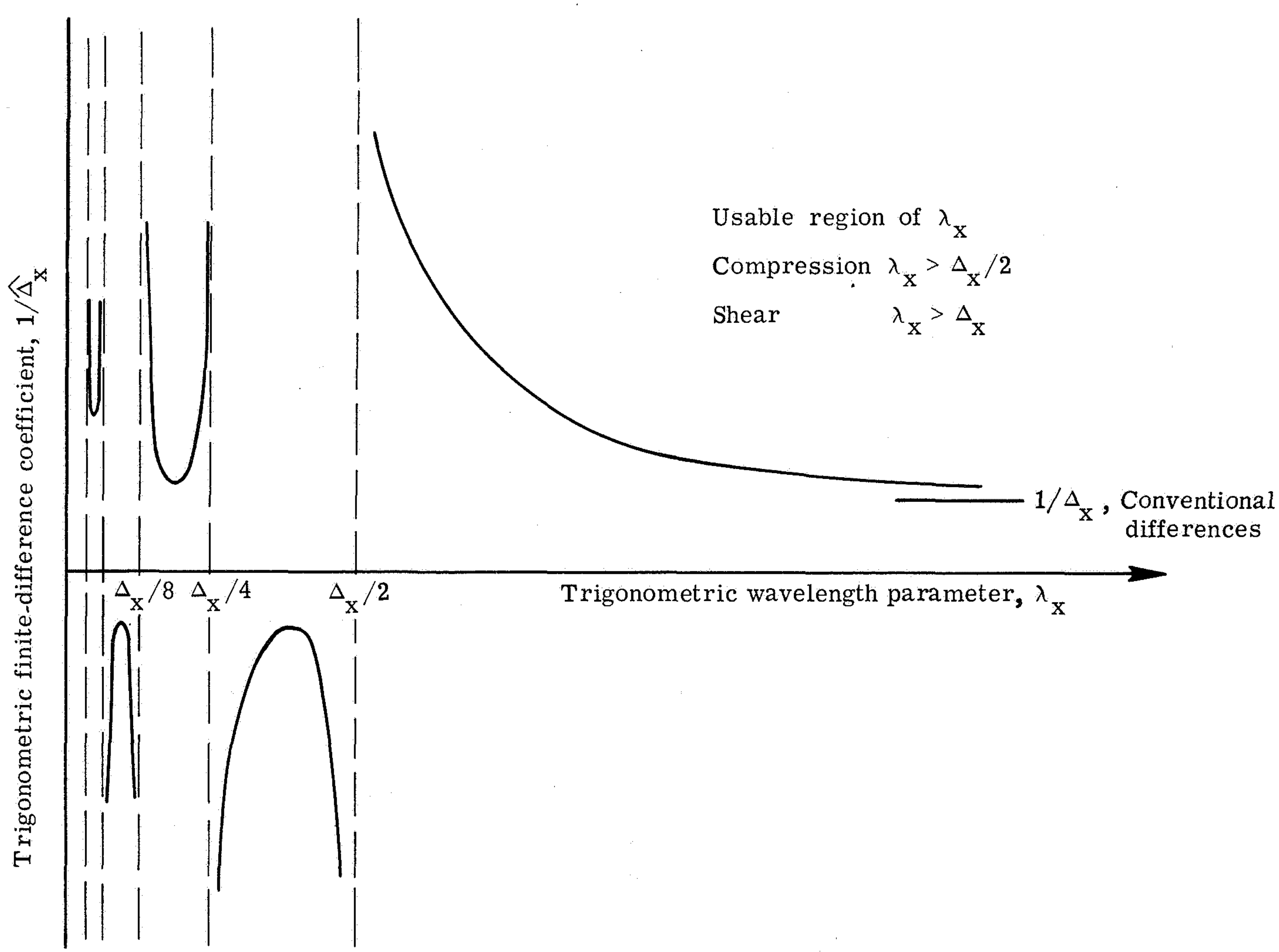


Figure 19.- Variation of $1/\hat{\Delta}_x$ with trigonometric wavelength parameter λ_x.

NASA-Langley, 1975 L-10083