NASA
SPACE VEHICLE
DESIGN CRITERIA
(STRUCTURES)

NASA SP-8083

DISCONTINUITY STRESSES IN METALLIC PRESSURE VESSELS

NOVEMBER 1971

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FOREWORD

NASA experience has indicated a need for uniform criteria for the design of space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment
Structures
Guidance and Control
Chemical Propulsion

Individual components of this work will be issued as separate monographs as soon as they are completed. A list of all published monographs in this series can be found at the end of this document.

These monographs are to be regarded as *guides* to the formulation of design requirements and specifications by NASA Centers and project offices.

This monograph was prepared under the cognizance of the Langley Research Center. The Task Manager was W. C. Thornton. The authors were F. L. Rish and L. Kovalevsky of North American Rockwell Corporation. A number of other individuals assisted in developing the material and reviewing the drafts. In particular, the significant contributions made by the following are hereby acknowledged: H. P. Adam and M. B. Harmon of McDonnell Douglas Corporation; M. Dublin of General Dynamics Corporation; L. Hall of Boeing; M. Kural of Lockheed Missiles & Space Company; L. Salter of NASA George C. Marshall Space Flight Center; J. H. Starnes, Jr. of NASA Langley Research Center; and V. Svalbonas of Grumman Aerospace Corporation.

NASA plans to update this monograph periodically as appropriate. Comments and recommended changes in the technical content are invited and should be forwarded to the attention of the Structural Systems Office, Langley Research Center, Hampton, Virginia 23365.

November 1971

For sale by the National Technical Information Service, Springfield, Virginia 22151 – Price $3.00

GUIDE TO THE USE OF THIS MONOGRAPH

The purpose of this monograph is to provide a uniform basis for design of flightworthy structure. It summarizes for use in space vehicle development the significant experience and knowledge accumulated in research, development, and operational programs to date. It can be used to improve consistency in design, efficiency of the design effort, and confidence in the structure. All monographs in this series employ the same basic format – three major sections preceded by a brief INTRODUCTION, Section 1, and complemented by a list of REFERENCES.

The STATE OF THE ART, Section 2, reviews and assesses current design practices and identifies important aspects of the present state of technology. Selected references are cited to supply supporting information. This section serves as a survey of the subject that provides background material and prepares a proper technological base for the CRITERIA and RECOMMENDED PRACTICES.

The CRITERIA, Section 3, state *what* rules, guides, or limitations must be imposed to ensure flightworthiness. The criteria can serve as a checklist for guiding a design or assessing its adequacy.

The RECOMMENDED PRACTICES, Section 4, state *how* to satisfy the criteria. Whenever possible, the best procedure is described; when this cannot be done, appropriate references are suggested. These practices, in conjunction with the criteria, provide guidance to the formulation of requirements for vehicle design and evaluation.

CONTENTS

DISCONTINUITY STRESSES IN METALLIC PRESSURE VESSELS

1. INTRODUCTION

Metallic pressure vessels are containers fabricated from metal and designed to hold liquids or gases under pressure. All pressure vessels contain discontinuities that can be described as (1) abrupt deviations or changes in shell geometry, thickness, material properties, loads, or temperatures; (2) openings or cutouts in the pressure vessel surface; or (3) geometric irregularities resulting from variations in the manufacture of structural parts. These discontinuities may cause high local stresses, which in turn can cause pressure vessels to fail.

The consequences of discontinuity stress failures may range from leakage, bursting, and detrimental deformation to catastrophic failure of the vessel and loss of the mission. Examples of past failures involving discontinuity problems include the following:

- During hydrostatic proof test, a spherical pressure vessel for cryogenic application failed catastrophically at 99 percent of the intended proof pressure. The failure was caused by the stresses produced by an approximately 50-percent weld mismatch at the boss fitting.

- A toroidal tank containing thrust-vector-control injection fluid under pressure failed below the design pressure because of excessive stresses in the area where the support structure joined the pressure vessel. The relative deflections of the two components were incompatible.

- A steel motor case failed as a result of high stresses at a nozzle junction caused by a mismatch of the two components.

- A fuel tank test ended in failure at less than half of the design pressure because of a discontinuity in the surface contour in the form of a flat spot which caused excessive local stresses.

This monograph presents the state of the art, criteria, and recommended practices for the theoretical and experimental analyses of discontinuity stresses and their distribution in metallic pressure vessels for space vehicles. The applicable types of pressure vessels include propellant tanks ranging from main load-carrying integral tank structure to small auxiliary tanks, storage tanks, solid propellant motor cases, high-pressure gas bottles, and pressurized cabins. The overall design of pressure vessels is not discussed in this monograph except where it relates to determination of the discontinuity stresses. Nonmetallic pressure vessels, inflatable structures, pressure-operated deployable structures, piping, and tubing are excluded.

The major sources of discontinuity stresses are discussed, including deviations in geometry, material properties, loads, and temperature. The advantages, limitations, and disadvantages of various theoretical and experimental discontinuity-analysis methods are summarized. Guides are presented for evaluating discontinuity stresses so that pressure-vessel performance will not fall below acceptable levels.

Critical parameters to be considered in determining pressure-vessel discontinuity stresses are as follows:

- The variations in meridional tangent to the middle surface
- The variations of the radii of curvature of the middle surface
- The variations in shell thickness
- The variations of Young's modulus, Poisson's ratio, and thermal coefficient of expansion of the material
- The longitudinal and circumferential variation of load (forces and moments) including concentrated loads (pipe connections, etc.)
- Deflections and rotations resulting from temperature changes, including differential expansion
- Geometric misalignment (mismatch)
- Discontinuous load paths such as cutouts (doors, windows, pipes, etc.)

In practice, pressure vessels are usually sized in at least two steps using conservative estimates of loads from the mission load/temperature/pressure/time-history. Initially, vessel member sizes are obtained from a membrane analysis and then refined – for example, by a bending analysis – to account for the locally increased stresses at discontinuities. Meridional and circumferential distributions of the discontinuity stresses are investigated to locate critical stress areas and determine whether the structure is adequate for these stress magnitudes and distributions.

The following standard analytical methods are used for determining stresses resulting from geometric or material-property discontinuities:

- In special cases, *standard equations* that account primarily for the effect of joining one geometric shape to another

- The *equating of end deflections and rotations* of pressure-vessel structural elements to account for geometric changes and for material property changes. The structural elements are then "assembled" analytically with equations of continuity and equilibrium

- *Finite-element modeling* of the pressure vessel followed by force or displacement analysis techniques to obtain the solution, generally with a computer program

- *Finite-difference techniques* to solve the governing differential equations of the pressure vessel, generally with a computer program

- *Numerical integration* for solution of linear or nonlinear problems, generally with a computer program

Methods for experimentally indicating discontinuity stress levels and distributions include two- and three-dimensional photoelasticity, photoelastic coatings, brittle coatings, and strain gages. These methods or combinations of methods are often used to verify the analysis of complex discontinuities. In some cases, these are the only methods to determine the feasibility of a pressure-vessel design approach.

This monograph is related to many other published or planned monographs in this series that treat problems of concern to designers and analysts of metallic pressure vessels. These areas include: buckling of thin-walled shells (refs. 1 to 3); propellant slosh loads (ref. 4); slosh suppression (ref. 5); fracture control (ref. 6); and compartment venting (ref. 7). Design criteria monographs on testing include references 8 to 10. In addition, other monographs are planned in related areas such as fatigue, design factors, windows and hatches, and nonmetallic pressure vessels.

2. STATE OF THE ART

Metallic pressure vessels contain regions where abrupt changes in geometry, material properties, or loading occur. These regions are known as discontinuity areas, and the stresses associated with them are known as discontinuity stresses.

Various configurations of pressure vessels with various types of discontinuities and their solutions (refs. 11 to 66) are shown in table I. Thermal effects sometimes complicate the solutions, but computer programs can usually account for these effects. In addition, some commonly encountered discontinuities such as the "Y"-ring joint where a skirt attaches to a tank, are too complex for closed-form solution; they are normally solved with an appropriate computer program. Analytical methods and computer programs (using the finite-element technique) also have been developed to analyze complex discontinuities such as cutouts with local reinforcements, discrete stringers, and fittings. For discontinuities with nonlinear behavior, various finite-difference and numerical-integration techniques are available. The experimental methods (strain gages, photoelastic and brittle coatings, and two- and three-dimensional photoelastic techniques) provide solutions where theoretical methods are in doubt or nonexistent.

2.1 Analysis of Discontinuity Stresses

Both theoretical and experimental stress analyses are performed to evaluate discontinuities in pressure vessels. Loads and loading combinations to be encountered during the service life of the vessel are required for these analyses. Derivation of these loadings is beyond the scope of this document, but information on the subject may be found in references 4, 5, and 67 to 73. Experimental stress analyses are used to substantiate the theoretical analyses and to determine detail stresses when theoretical analyses are not available.

2.1.1 Theoretical Analysis

Since metallic pressure vessels consist of shell structures, the analysis of these vessels is based on the theory of shells. This theory, an approximation made within the theory of elasticity, is concerned with the stresses and deformations of thin elastic bodies under applied loads and temperatures. These are determined by several methods, including those described in references 20 and 26. Of special importance are the various linear theories, such as first- and second-order-approximation shell theory, shear-deformation shell theory, specialized theories of shells of revolution, and membrane shell theory, which are discussed in the literature (refs. 18, 27, and 74 to 78) and compared in reference 26.

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS

(Note: Blank space indicates no closed-form or experimental solution available)

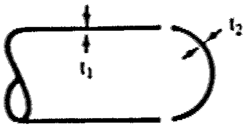
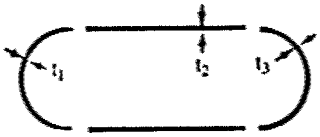
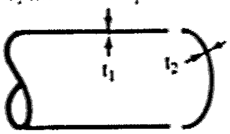
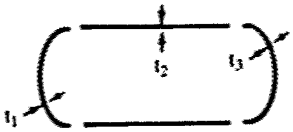
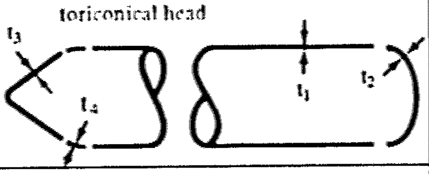
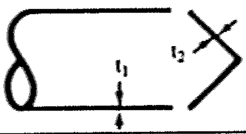
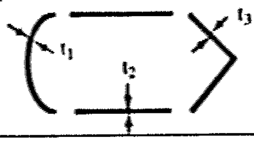
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part I – Common middle-surface discontinuities				
1. Long cylinder – hemispherical head 	11, 12, 17, 18	13 with 14	15, 16	19
2. Short cylinder – hemispherical head 	11		15	
3. Long cylinder – ellipsoidal head 	11, 18			
4. Short cylinder – ellipsoidal head 	11, 20			
5. Long cylinder – torispherical and/or toriconical head 	17, 19		21	19
6. Long cylinder – conical heads 	11, 17			
7. Short cylinder – conical and ellipsoidal heads 	11			

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Continued

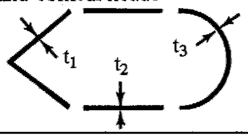
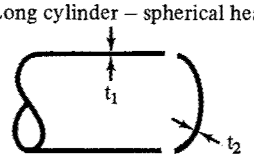
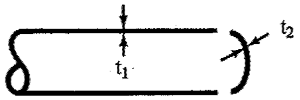
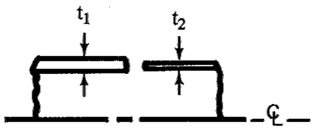
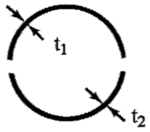
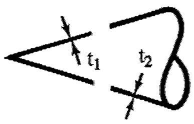
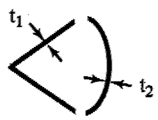
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part I – Common middle-surface discontinuities – continued				
8. Short cylinder – hemispherical and conical heads 	11			
9. Long cylinder – spherical head 	17			
10. Long cylinder – cassinian head 	22			22
11. Change in thickness – cylinder 	6, 11, 17	13		23
12. Change in thickness – sphere 	17	14		
13. Change in thickness – cone 	17			
14. Cone sphere 	17			

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Continued

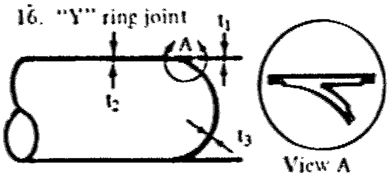
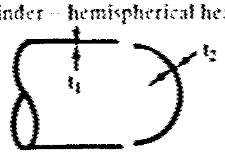
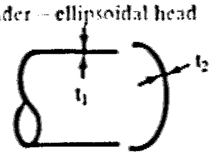
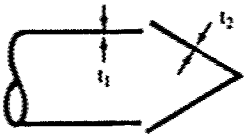
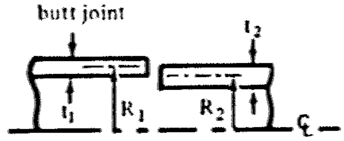
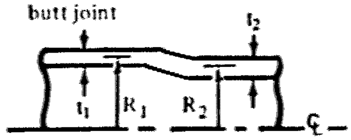
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part I – Common middle-surface discontinuities – concluded				
15. Junction of multiple shells 	24		25	
16. "Y" ring joint 	26 (method)			
Part II – Eccentric middle-surface discontinuities				
17. Cylinder – hemispherical head 	27-axisym-metrical/ mismatch (method) 28 (method only)	13 with 14		
18. Cylinder – ellipsoidal head 	28 method only			
19. Cylinder – conical head 	28–method only			
20. Mismatch – cylinder unfilleted butt joint 	11-axisym-metrical/ mismatch 29-local mismatch	13-axisym-metrical/ mismatch 31-local mismatch		30, 33
21. Mismatch – cylinder filleted butt joint 		13-axisym-metrical mismatch		30

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Continued

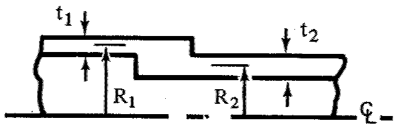
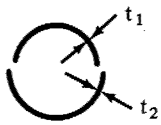
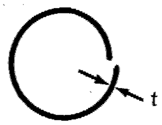
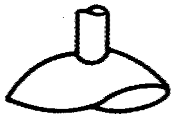
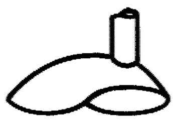
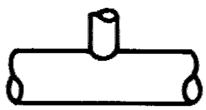
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part II – Eccentric middle-surface discontinuities – concluded				
22. Mismatch – cylinder lap joint 		13-axisym- metrical mismatch		
23. Weld-sinkage joint in cylinders and spheres 	32		32	32
24. Mismatch – spheres 	27-axisym- metrical/ mismatch (method only) 28-method only	14-influence coefficients only		
25. Mismatch – cylinder longitudinal joint 	29			
Part III – Intersecting shapes				
26. Sphere-cylinder 	33 to 36		16, 37 to 42	43, 44
27. Nonradial nozzle in sphere 				45
28. Cylinder-cylinder at 90° 	36, 46 to 50		51	43, 44, 52, 53

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Continued

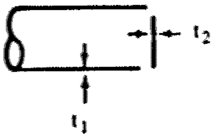
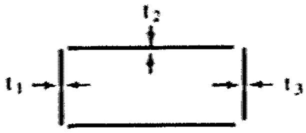
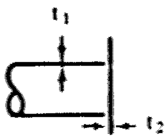
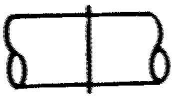
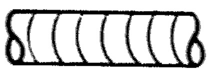
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part III – Intersecting shapes – concluded				
29. Cylinder-cylinder at 45° 	54 (theoret-ical)			45
30. Long cylinder – flat head 	11, 17		55	
31. Short cylinder – flat head 	11			
32. Cylinder – flanged ends 	12			
33. Cylinder – ring 	11			
34. Cylinder – equidistant rings 	11, 56		57	
Part IV – Other discontinuities				
35. Reinforced opening in sphere 	58			44, 59

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Continued

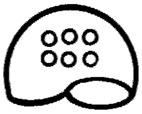
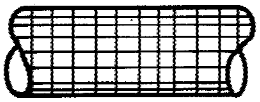
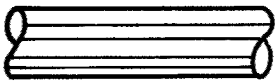
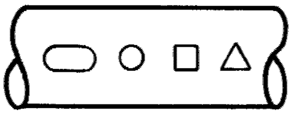
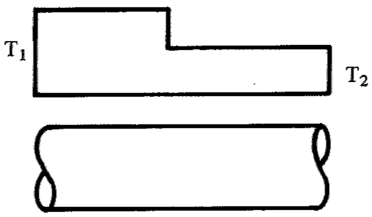
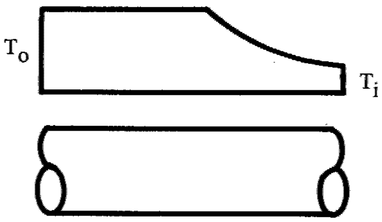
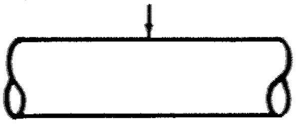
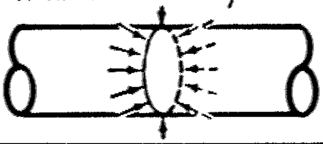
Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part IV – Other discontinuities continued				
36. Multiple holes in spherical shells 	50, 60			
37. Waffle stiffening (zero/90 deg) 	26			
38. Longitudinal stiffening 	61			
39. Cylinders with cutouts 	60, 62			60
40. Cylinder with different temperature 	63			
41. Cylinder with varying temperature 	64			

TABLE I. – REFERENCES FOR DISCONTINUITIES IN PRESSURE VESSELS – Concluded

Description	Linear elastic solution (Ref no.)	Nonlinear elastic solution (Ref no.)	Elastic-plastic solution (Ref no.)	Experimental solution (Ref no.)
Part IV – Other discontinuities - concluded				
42. Cylinder with concentrated load 	12			65, 66
43. Cylinder with radial loads (pressure on circumferential line) 	12			

In the application of these theories to the analysis of pressure vessel discontinuities, direct solution of the governing differential equations has two distinct disadvantages: first, it requires knowledge of a variety of sophisticated techniques for solution of ordinary or partial differential equations; second, a given form of analytical solution is invariably limited to shells of simple geometric shape, such as cylindrical, conical, or spherical, which are subjected to simple loadings.

However, the shell theories are the basis for many other techniques and routines which are in common use today. For example, a well-known technique for the solution of discontinuity problems is based on the force method (refs. 26 and 75). A complicated shell or multi-shell structure may be divided into elements of various shapes. To restore continuity between elements, forces and deformations due to discontinuities must be accounted for by analysis. For example, if a shell of revolution is separated into elementary shells such as cylinders, cones, and spheres, it is necessary to determine the magnitude of the stress resultants between the elementary shells. This may be done by considering that redundant moments and shears act at the junctions. At any junction, the displacements and rotations caused by the redundant loads are superimposed on those due to internal pressure. The requirement of continuity (i.e., that the total deflections and total rotations be equal at the junctions) yields a set of equations at each junction relating the redundants to the internal pressure and the geometry of the adjacent free bodies. Useful formulas for stress resultants are presented in references 26, 74, 75, and 79. The discontinuity stresses can be determined by several methods, all of them based on the above described theories.

Pressure vessel analysis, as generally used, does not automatically include the damping effects on the discontinuity shears and moments that are attributed to the meridional-stress resultant (ref. 80). The analytical solution for the coupled stress resultants is nonlinear and, in some cases, may be unnecessary for the verification of the structural design. For other geometries and pressures, however, the nonlinear effect is more significant and may either increase or decrease the computed stress, compared with the linear analysis, depending upon the mismatch and nonlinearity parameters (refs. 32, 81, and 82). The nonlinear coupling effect is shown schematically in figure 1, adapted from reference 82.

An example of the coupled-stress resultant for a cylinder-bulkhead-skirt, Y-ring junction stress distribution is shown in figures 2 and 3. The figures show the distributions of longitudinal and circumferential stresses at the inner and outer surfaces of the cylinder, dome, and skirt, and the effects of meridional-tension stiffening. The effects are more pronounced for shells with larger radius-to-thickness ratios and for increasing pressures.

The following are typical methods and techniques employed for determination of discontinuity stresses and deformations.

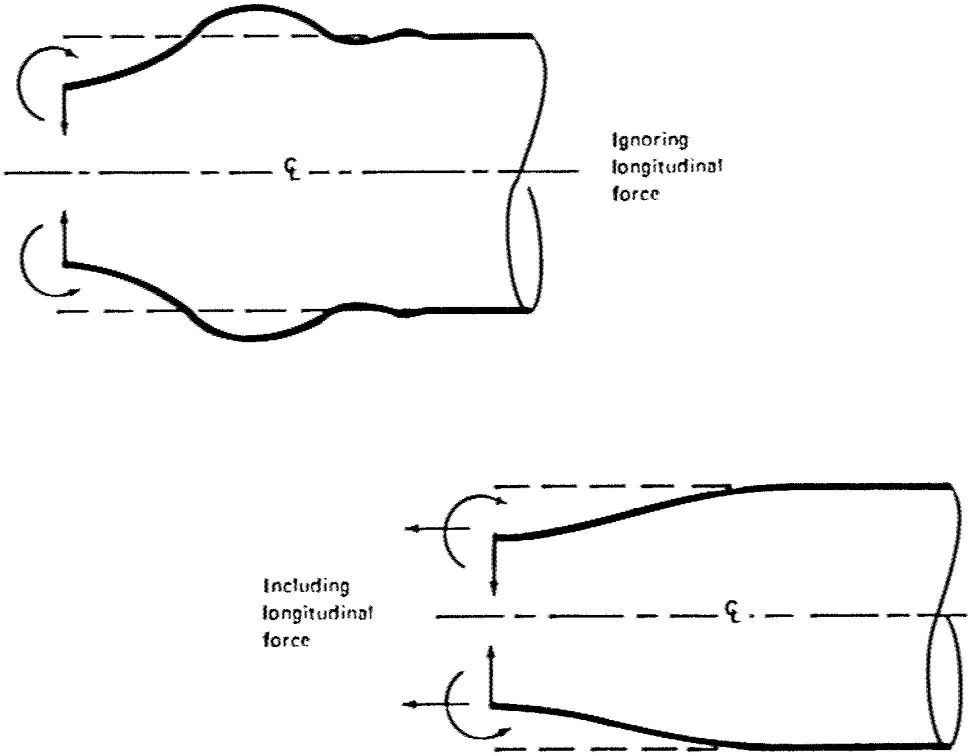


Figure 1.–Effect of longitudinal load on deflection (From ref. 82).

2.1.1.1 Membrane Analysis

Membrane analysis treats the deformation of shells neglecting bending. For discontinuity analyses of aerospace pressure vessels, the membrane theory is commonly used in conjunction with bending theory. In this application the membrane solutions are designated as primary solutions, and bending solutions are designated as secondary solutions.

Discussion of the linear membrane theory is found in references 18, 20, 27, 74, and 76. Nonlinear membrane theory, which treats geometrical nonlinearity in the strain-displacement sense, is discussed in references 83 and 84.

2.1.1.2 Bending Analysis

Bending theories are used for simple shells of revolution containing one or two axisymmetrical discontinuities. Linear bending theories are discussed in references 18, 20, 27, 74, and 76, and nonlinear bending theories in references 83 to 85.

A significant class of problems is one in which local loading produces stress concentrations only in the proximity of the loaded zone and in which deformations are

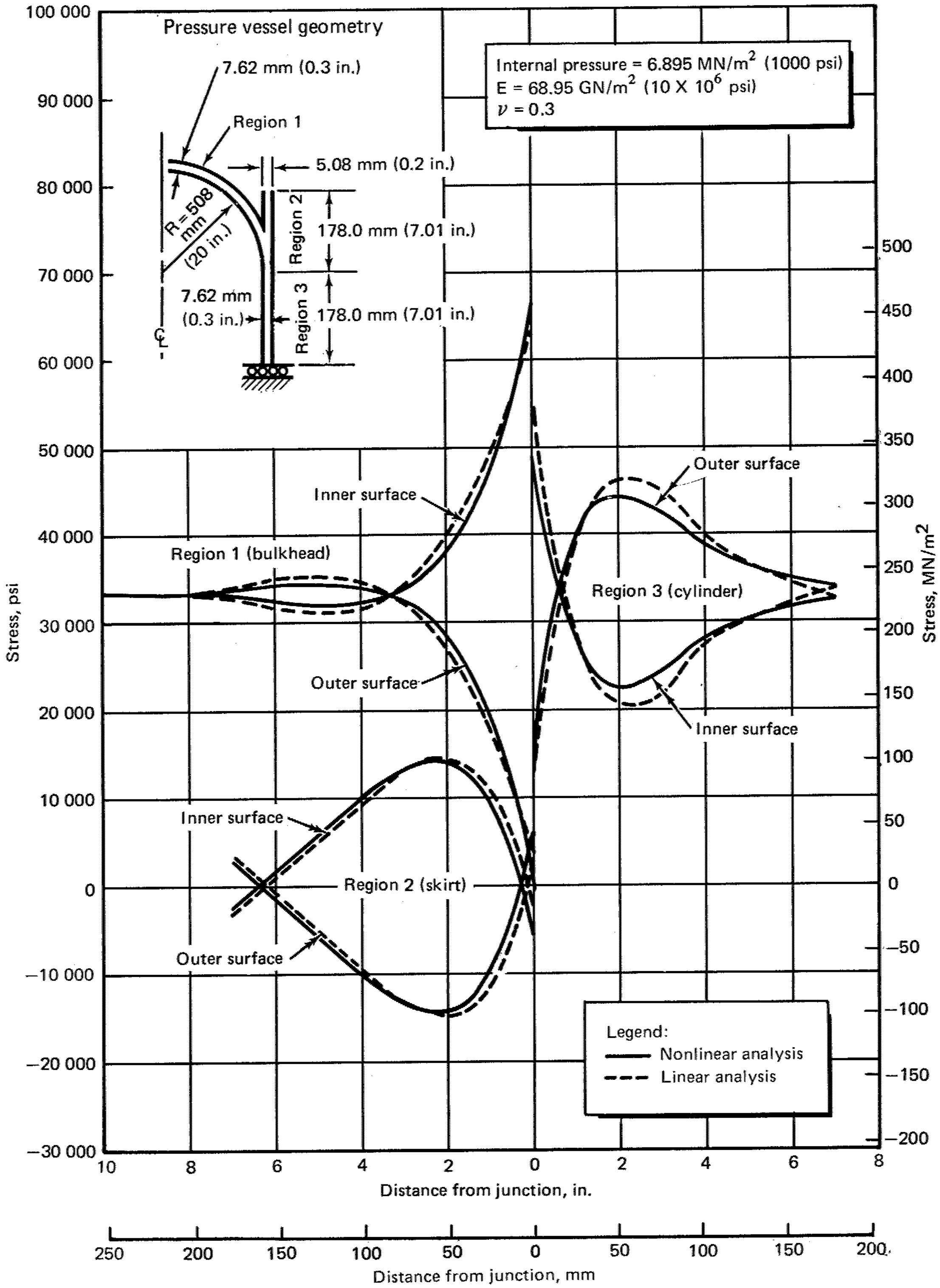


Figure 2. – Meridional stresses at discontinuity.

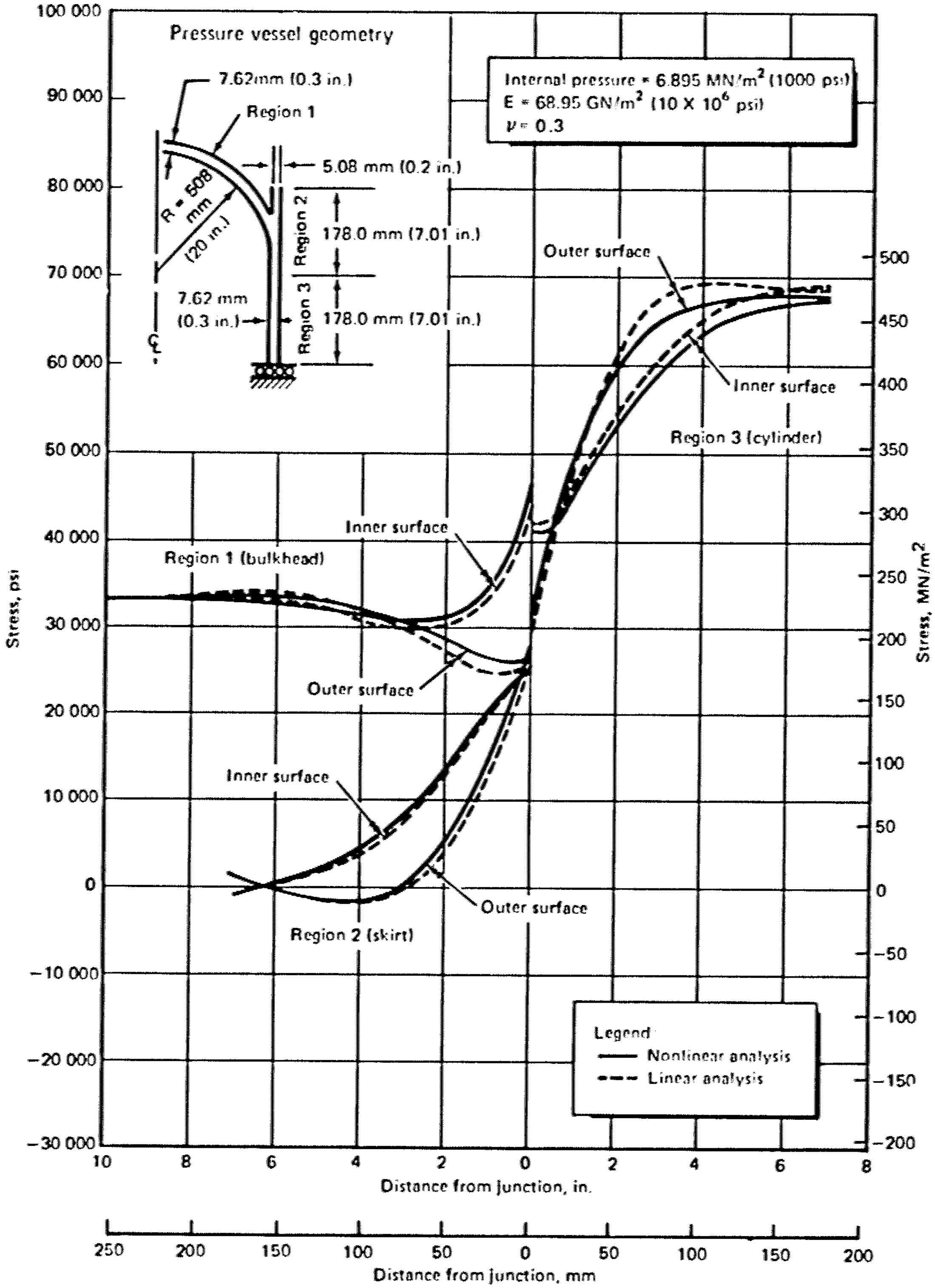


Figure 3. – Circumferential stresses at discontinuity.

small. In references 28, 35, 36, and 86, the following cases are treated: (1) internally pressurized spherical shell with a rigid insert, (2) axially loaded insert, (3) ring load, (4) uniformly loaded spherical cap with fixed ends, (5) external moment on an insert, and (6) tangentially loaded insert. Influence coefficients for shells of revolution with variable wall thicknesses are considered in many references (e.g., ref. 85).

While many pressure vessels are designed on the basis of the simplest form of shell theory – linear membrane theory – application of this theory to the torus leads to a discontinuity of displacements at the crown, an obviously unacceptable result. The use of shell-bending theory, on the other hand, yields physically reasonable results. Simple membrane theory yields stresses which differ from the bending results. For certain cases, the difference may be 20 percent or more. For thinner toroidal shells, even linear bending theory is not sufficiently accurate and nonlinearity must be introduced (refs. 87 and 88).

2.1.1.3 Force Method

The force method is commonly used for determination of discontinuity stresses (refs. 18, 26, and 75). It is based upon use of a library of existing solutions for single shells which are combined and interacted to obtain a solution for a more complicated shell. Any of the available solutions may be used, depending on the required accuracy. For example, a multishell may be separated into elements such as cylinders, cones, and spheres, for which the membrane and bending solutions are known. The interaction of the elements leads to the determination of all the discontinuity stresses and deformations. When the junctions are sufficiently distant from one another, the discontinuity stresses and deformations of one junction do not affect the discontinuities at another (ref. 75). In such cases, a large system of linear, algebraic equations is replaced by two or three equations for each junction. The solution is numerically simple. This is one of the most useful methods in practice because of its simplicity. The use of computer programs may be avoided in many cases.

2.1.1.4 Displacement Method

The displacement method, described in reference 75, is applicable to pressure vessels which are multishells of revolution. The method is analogous to the force method.

The pressure vessel is assumed to be separated into elements with fixed boundaries; the primary solution consequently yields fixed end moments and zero displacements at boundaries. End rotation and end displacements are then introduced at the boundaries in terms of variable unknowns, causing additional end moments and shears. The compatibility and equilibrium conditions for each junction lead to the determination of the values for rotations and displacements at every junction. The moments and shears can then be determined. The method is excellent for the case of shells of revolution with rotationally symmetrical loading.

2.1.1.5 Iterative Method

The method is based on an iteration procedure (ref. 75) similar to the well known Hardy Cross method for rigid-frames analysis. It is applicable to multishells that are rotationally symmetrical in geometry, loading, and material. It is based on linear elasticity (small-deflection theory). Since this method uses stiffness coefficients, it is useful for rotationally symmetrical multishells for which the stiffness coefficients are known. It is not applicable, however, for single shells. For small-order systems, this is an excellent method to be used with the slide rule.

2.1.1.6 Finite-Element Method

The finite-element method (refs. 89 and 90) is based upon mathematically modeling the pressure vessel structure as an assemblage of finite elements connecting nodal locations as shown in figure 4, which uses triangular elements. Virtually no limitation is imposed on the geometry of the structure because of the extensive library of elements available, such as lineal, triangular, quadrilateral, tetrahedral, pentahedral, hexahedral, triangular and quadrilateral torus, and isoparametric elements. In the analysis of pressure vessels that are bodies of revolution, it is advantageous to use finite elements that are themselves shells of revolution (e.g., conical frustra joined at nodal circles) or combined shells and bodies of revolution. The displacement at every point in the structure is described in terms of a set of arbitrary deformations usually at the end or mid side point of these segments, and the actual loading of the structure is replaced by a set of equivalent loads at these nodal points. These loads are equivalent in the sense that the work done by these loads during an increment of deformation approximates the work of the actual loading.

For each segment, relationships are derived for the forces at the ends of the segment as functions of the arbitrary deformations at the ends of the segment. These can be written in matrix form.

The force-displacement equation in terms of the stiffness matrix for the entire shell is obtained by adding the equations for the individual elements. The total stiffness matrix, as assembled from the constituent element matrices, is singular and cannot be inverted; however, the inclusion of boundary conditions permits a solution for the deflections. The stresses in the shell are then determined by the additional elastic relationships. The fundamental quantities needed initially by this method are the stiffness and stress matrices of the shell.

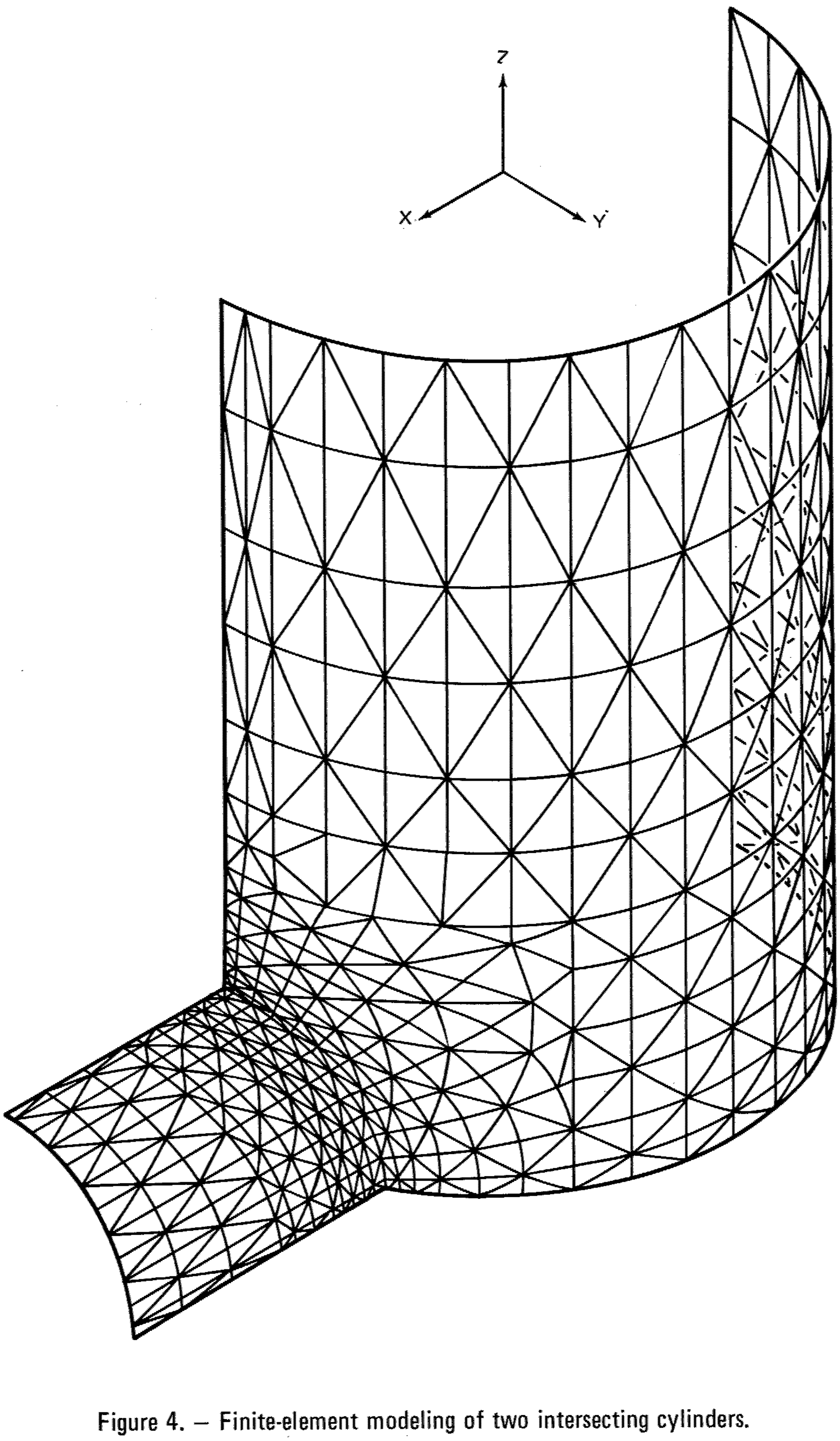


Figure 4. – Finite-element modeling of two intersecting cylinders.

The finite-element method may also be formulated in terms of flexibility matrix expressions. A description of this method is found in reference 91. At present, this can be appraised as a most useful, accurate, general, and powerful method, if digital computer programs are used.

2.1.1.7 Finite-Difference Method

Before numerical computer-based solutions of real problems dealing with complex continua can be solved, it is necessary to limit their infinite degrees of freedom to a finite, if large, number of unknowns. Such a process of discretization was first successfully performed by the now well-known method of finite differences (ref. 92). This method is based on the replacement of differential equations by the corresponding finite-difference equations. The most useful applications of this technique are the method of successive approximations and the relaxation method. Additional information can be found in references 93 to 97.

The accuracy of the finite-difference approach is governed by the number of subintervals into which the shell is divided. If the mesh is too coarse, the so-called truncation errors in the finite-difference approximations to the derivatives are large, but roundoff errors are small. If, on the other hand, the mesh is too fine, the truncation errors in the derivatives are small, but the round-off errors are large. Most investigators have concluded that it is best to vary the mesh size to provide a fine mesh where rapid variations are expected to occur in the results and a coarse mesh in portions where smooth behavior is expected. However, a fine mesh does not necessarily yield good convergence.

2.1.1.8 Numerical-Integration Method

In the numerical-integration method (ref. 98), the governing equations of a given two-point boundary-value problem are reduced to first-order equations and various schemes are used to integrate these equations with a digital computer. A solution which also satisfies the conditions prescribed at the final point is found by making two arbitrary choices for the force quantities at the initial point and then interpolating between the resulting solutions to obtain the required conditions at the final point. Some choice is available in the selection of the stepwise-integration process.

The numerical-integration method may be difficult to apply to the equations of shell theory because the shell boundary effects are highly localized (ref. 99). If, for example, a cylindrical shell is longer than a certain characteristic length which depends primarily on its radius, thickness, and Poisson's ratio, no interaction occurs between the conditions at its two ends. In addition, there may be a loss of accuracy due to round-off error. In reference 99, these difficulties are avoided by dividing "long" shell segments into smaller parts.

2.1.1.9 Orthotropic Analysis

If the pressure vessel is of orthotropic construction, appropriate theories must be used (refs. 26 and 80). Unfortunately, the closed-form analysis of orthotropic shells is limited to the available solutions (membrane and edge loadings) for elementary shells (such as cylinders, cones, spheres, shallow spheres, circular plates, etc.). Orthotropic solutions for toroids and Cassinian domes are unknown. Most available solutions are for shells which have rotationally symmetrical geometry and loading. Some solutions for homogeneous anisotropic shells are presented in reference 80; in particular, shells constructed from an orthotropic material with the axes of elastic symmetry rotated with respect to lines of principal curvature are considered. For a circular cylindrical shell under axially symmetrical loading, solutions in terms of edge moments and shears are obtained in reference 80. The influence of axial load is also considered. Many current computer programs are capable of yielding numerical solutions to problems of orthotropic analysis; the Appendix lists some of them.

2.1.1.10 Plastic Analysis

Closed-form methods for plastic analysis have been developed for special cases of pressure vessels such as spheres and cylinders (ref. 100). The analysis is long and generally requires a computer program. An approximate solution is presented in reference 15 for the pressure-versus-radius relationship for cylinders with hemispherical caps. The solution is based upon the deformation theory of plasticity, together with the assumption of a circular profile for the deformed cylinder generator. The influence of material hardening is investigated using the Ludwik strain-hardening law as being representative of the stress-strain behavior of most ductile materials. The von Mises yield criterion (refs. 15 and 101) is used.

For cylinders with rigid ends (ref. 15). the solutions are approximate and are restricted to those cases in which instability occurs before the cylinder "barrels" into a sphere; therefore, the results are limited to cases in which the length-to-diameter ratio is greater than about 1. For shorter lengths, instability occurs after the shell becomes spherical, and this method is not applicable (ref. 100).

The influence of end restraint on the burst strength is evaluated by comparing the results obtained in an end-condition study. For cylinders with a length-to-diameter ratio larger than 2, the effect of end restraint, as represented by spherical heads or rigid caps, upon the burst strength is small, amounting to less than 13 percent. For length-to-diameter ratios smaller than 2, the effect of end restraint becomes significant.

The rigidly capped cylinder is considerably stronger than the hemispherically headed one; and both shells with end restraint are considerably stronger than the infinitely long cylinder.

Several recently developed computer programs are capable of handling plastic analyses of discontinuities; the Appendix lists a number of them.

A comparison of all these analytical methods is given in table II (ref. 102).

2.1.1.11 Applicability of Computer Programs

In the last few years, tremendous efforts have been directed toward using computers to analyze discontinuity stresses in pressure vessels (ref. 98). The methods used in computer programs range from finite-difference, numerical-integration, and finite-element techniques to direct applications of existing analytical solutions (ref. 103). Most Government and industry organizations dealing with shell analysis possess computer programs that can handle many of the pressure-vessel problems. A comprehensive list of these programs is presented in the Appendix. These programs offer solutions for the following kinds of analytical problems:

- Stress distributions around discontinuities, joints, and openings for axisymmetrical, nonuniform-thickness shells

- Stress distribution at junction of components of thick vessels

- Thermal stresses

Most of these programs, however, are limited to the elastic behavior of the structure and are also confined to shell structures with axisymmetrical geometry.

Computer programs are available for the analysis of pressure vessels consisting of orthotropic shells, layered shells, shells with discontinuous middle surfaces, unsymmetrically loaded shells, thick shells, shells with branches, and shells under various thermal and mechanical load arrangements. No single program stands out as better than the rest; selection of the best program would seem to be governed only by the end use, since so many are available.

There are several disadvantages to digital computer methods. It is common experience to find that a program which is declared to be running by its originators will not necessarily run successfully elsewhere without considerable further effort. A computer of adequate capacity must be available to the analyst. The output of a computer is often a vast array of numbers, and this situation sometimes obscures trends that might

TABLE II. – COMPARISON OF THEORETICAL METHODS

Method	Application	Advantages	Limitations
Membrane analysis (2.1.1.1)	General method applicable where bending can be neglected. Used in combination with bending analysis of discontinuities	Simplicity	Cannot be used alone when bending has to be considered. Cannot predict stresses and deformations due to concentrated loadings
Bending analysis (linear) (2.1.1.2)	Simple cases of shell of revolution with multiple axisymmetrical or asymmetrical discontinuities. Useful in combination with membrane theory for analysis of discontinuities	Useful for the cases in which loading produces stress concentrations in the proximity of the loaded zone and in which deformations are small	Can be used only for simple problems. Application to the torus, for example, is unacceptable
Force method (2.1.1.3)	Common practical method for determination of discontinuity stresses in branched shells and multishells	Simple to apply with slide rule or computer	Limited to shell of revolution with linear characteristics only. Prerequisite – existence of tabulated influence coefficients for deformations at junction due to unit edge loadings
Displacement method (2.1.1.4)	Applicable to pressure vessels which are multishells of revolution	Simple to apply with a slide rule for small-order uncoupled systems	Limited to shells of revolution with linear characteristics and with axisymmetrical loading. Stiffness influence coefficients are not readily available for some shells of revolution
Iterative method (2.1.1.5)	Applicable to rotationally symmetrical multishells	Only slide rule is needed for performing calculations. Analysis is straightforward and systematic	Limited to shells of revolution with axisymmetrical loading. Linear characteristics required
Finite-element method (2.1.1.6)	Very general application for shells and multishells. Arbitrary loading and geometry. Arbitrary material properties. Both linear and nonlinear capabilities. Ideal for various intersections of shells. Applicable also for cutouts	Numerous computer programs in existence make utilization of this method practical for complex multishells. Generality of method permits application to wide variety of complex discontinuity problems	Computer program with adequate library of elements must exist or be developed. Each program has own limitations. Practical for complex multishells; for simple pressure vessels use of the programs may be more costly than use of closed-form solutions

TABLE II.–COMPARISON OF THEORETICAL METHODS–Concluded

Method	Application	Advantages	Limitations
Finite-difference method (2.1.1.7)	Variously shaped symmetrical and nonsymmetrical shells and multishells with symmetrical and nonsymmetrical loadings. Linear and nonlinear characteristics of geometry and materials. Applicable also for intersection of shells, cutouts, etc.	This method is more developed for nonlinear analysis than finite-element method. Usually requires less computer time. Numerous computer programs are in existence (but not as many as for finite-element method)	Finite-difference method is currently applied only to simple types of discontinuities. Not generally applicable to highly redundant complex discontinuities such as irregularly shaped, reinforced cutouts. Generally must utilize computer program for solution
Numerical-integration method (2.1.1.8)	Analysis of shells of revolution subjected to symmetrical and nonsymmetrical loads	For some applications, programming simpler and more efficient than for other methods. There is good control of numerical accuracy since segmenting allows variable integration lengths	Since boundary effects are highly localized, it is difficult to apply this method at discontinuities, as described in Section 2.1.1.8. This method can be applied only to problems which can be reduced to a one-dimensional mathematical form.
Orthotropic analysis (2.1.1.9)	Applicable to shells with stiffness properties which vary in the meridional and circumferential directions such as pressure vessels with closely spaced stiffeners, waffles, etc	Required to determine accurately discontinuity stresses in orthotropic pressure vessels	Not all available computer programs have this capability
Plastic analysis (2.1.1.10)	Developed only for special cases of cylinders and spheres. Problems of bursting for cylinders with hemispherical caps. Problem of material hardening. Cylinders with rigid ends	Solves elastoplastic problems not amenable to other methods	Analysis is long and requires computer programs. Developed for limited problems only

be obvious from an algebraic formula. Errors in the solutions are thus often difficult to detect. Automated plotting and other visual displays of results tend to remedy this problem; however, these devices have often been neglected in computer program development. The use of interactive graphics with the finite-element-analysis method is becoming more frequent. Further information can be found in reference 104.

2.1.2 Experimental Analysis

Experimental stress analyses are performed to substantiate theoretical analyses (ref. 23) and to determine detail stresses when theoretical analyses are not available. Sometimes any theoretical analysis is questionable for very complex pressure vessel discontinuities. For example, experimental methods are available and have been applied to pressure vessels with complicated discontinuities, such as cutouts, reinforcements, and fittings. An experimental analysis may be conducted during preliminary design in order to evaluate design feasibility and the resulting discontinuities before building hardware.

A number of experimental methods are used (ref. 105), including electrical-resistance strain gages, brittle coatings, two- and three-dimensional photoelasticity, photoelastic coatings, and plastic model tests. The distribution of stresses, strains, and displacements in pressure vessels with discontinuities is currently determined by a number of important experimental methods. In some cases, the experimental methods are the only ones available to determine the feasibility of a pressure-vessel design approach. A comparison of these methods is shown in table III.

2.1.2.1 Electrical-Resistance Strain Gage

A strain gage is used to measure the linear surface deformation (strain) occurring in a structure over a given gage length as the structure is loaded (refs. 106 to 108). This definition covers the range of instruments, including linear scales, and the precise optical and electrical strain gages now available.

The standard electrical-resistance strain gage can record up to 4-percent elongation. Special large-elongation gages can record up to 10-percent elongation. Room-temperature gages are usable from 256 to 322K (0 to 120°F). Low- and high-temperature gages allow strains to be measured over a range from 21K (-423°F) to about 866K (1100°F). For other than room-temperature tests, the gage or circuitry is usually temperature-compensated. Accuracy depends upon the installation and the recording system. If the installation and recording system are good, the accuracy may reach 5 percent. Accuracy can be quite poor, however, when measuring peak stresses at sharp discontinuities, since the strain gage integrates the elongation over its entire length. This limitation can often be overcome by the use of very short gages. Gages can be obtained with lengths as short as 0.381 mm (0.015 in.).

TABLE III. – COMPARISON OF EXPERIMENTAL METHODS

Method	Application	Advantages	Limitations
Strain gages:			
Room-temperature	Most commonly used experimental method of strain determination. Generally, applicable to all types of pressure vessels. Temperatures range 256 to 322K (0 to 120° F)	Give continuous and repeatable measurement of strains in local areas as actual pressure vessel is loaded (model not required). Quite accurate (±5 percent) with good instrumentation. Used on actual pressure vessel. Nondestructive method	Not good in high-strain gradients. In any significant quantity, relatively expensive. Must be applied on smooth, clean prepared surface. Locations must be accessible for installation of gage and for routing of wires. Only good for local area strain determination
Low-temperature	Temperature range down to 21K (−423° F) (liquid hydrogen)	Measure strains at low temperatures	Accuracy is controversial. No one good calibration method defined for low temperatures. Readings must be temperature-compensated
High-temperature bonded	Temperature range up to 589K (600° F)	Measure strains at high temperatures	High-temperature cure of strain gage installation at test temperature desirable. Readings must be temperature-compensated. Accuracy depends upon particular installation
High-temperature welded	Temperature range up to 811 to 866K (1000 to 1100° F)	Measure strains at high temperatures	Accuracy is controversial. No one good calibration method defined for high temperatures. Must consider eccentricity of gage with respect to mounting surface
High-elongation	Measurement of high elongations	Measure elongations up to 10 percent	Time at load must be limited to avoid creep in bonding material

TABLE III.–COMPARISON OF EXPERIMENTAL METHODS–Concluded

Method	Application	Advantages	Limitations
Brittle coatings	Materials are available for room-temperature (e.g., Stresscoat) and high-temperature applications (ceramics). Determine gross strain distributions, magnitudes, and directions. Best used for preliminary work to determine areas for application of strain gages	Shows maximum tension strain locations and direction. Pick up very local stresses. Relatively inexpensive to apply. Used on actual pressure vessel but also can be used on plastic model with good results, since higher strains occur at lower loads in the model. Techniques available to measure residual stresses (destructive method)	Normally measures tension strains only on visible surface. Can be applied to evaluate compression. Must be applied to fairly smooth surface. Accuracy about 20 percent. Time of load must be limited (~30 sec) to avoid creep. Humidity and temperature of laboratory and load rate must be controlled for quantitative results. Tank material must be compatible with carbon disulfide (Stresscoat). Requires experienced operator to apply uniform coating and observe cracks
Photoelasticity, two- and three-dimensional	Design study investigations. (e.g., size of fillet radius studies.) Can be used for elastic analysis of three-dimensional parts with any loading at room temperature	Gives an overall description of stresses and strains and their variations. Locates high strain gradients and gives complete stress distributions as required to verify any analytical solution. Parts or designs can be analyzed before production. Gives continuous picture of stress distributions in three-dimensional analysis (also internal stresses if required)	Must build plastic model of structure. Fabrication of model requires precision, especially if model scaled down. Requires experienced personnel and special equipment. Requires environmental control of laboratory. Room-temperature testing for elastic stress determination only. Accuracy between 5 and 10 percent. Reduced scaling may be a problem if the wall of the model becomes too thin to machine
Photoelastic coatings	Apply photoelastic material to actual pressure vessel surface	Gives an overall description of strain state and its variations at infinite set of points, especially high strain or stress gradients. Poisson's ratio effect is not a problem. Uses actual pressure vessel	Accuracy approximately ±20 percent. Best results when applied to flat surface; must be molded on curved surface. Results give two-dimensional strain fields on illuminated surface only. Requires special equipment and trained and experienced personnel. At high pressures readings must be taken remotely to maintain adequate safety of personnel

2.1.2.2 Brittle Coating Method

The brittle coating is sprayed over the area of interest on the surface of the actual pressure vessel. When the coating is dry, the pressure vessel is loaded. Initial cracking in the coating is detected and related to the maximum surface strains on the one visible side of the pressure vessel. The accuracy of the strain measurement is approximately 20 percent. With this method it is possible to find the directions and distribution of discontinuity strains and the location of the peak strains. These results can be used to position strain gages accurately for precise strain measurements. This method is relatively inexpensive, and an experienced technician can apply the coating and observe the cracks. With special ceramic coatings this method can be used at high temperatures or in a high humidity environment, but it has been used mostly at room temperature. Loading is usually applied at a controlled rate because the coating material has a tendency to creep. Applications of brittle coatings are being broadened with the development of new materials and techniques (e.g., ceramic coatings for high temperature). Further information on brittle coatings may be found in references 109 and 110.

2.1.2.3 Photoelastic Methods

Photoelastic methods are based upon the principle that polarized light passing through a birefringent plastic material is modified according to the stress distribution present. The effect of the principal stresses acting at some point in the photoelastic model changes the velocities of the components of the light that is propagated through the photoelastic material. It has been established that for a given material at a given temperature, and for light of a given wavelength, the phase difference is proportional to the differences in the principal stresses, and to the thickness of photoelastic material.

Photoelastic analysis requires a minimal amount of optical equipment (polariscope). Two- and three-dimensional photoelastic analyses require that transparent birefringent plastic models be made geometrically similar to the actual structure to be studied. The fabrication of two-dimensional models is somewhat delicate, since care must be taken not to induce residual stresses (apparent strains) in the edges during the machining or routing process. For three-dimensional analysis, the model is stress-frozen, carefully sliced, and then analyzed as in two-dimensional photoelasticity. The method is applicable only for elastic stresses at room temperature. Accuracy depends on the precision with which the model is fabricated, the correspondence of the model to the actual structure, and the simulation of loadings. The typical accuracy is from 5 to 10 percent, although skilled personnel can sometimes achieve better results. A three-dimensional photoelastic analysis is a relatively expensive method; however, it may be the only method which can measure complicated stress distributions, thereby preventing a failure. Further information is available in references 108, 111, and 112.

2.1.2.4 Photoelastic Coating Method

The photoelastic coating method is similar in principal to the photoelastic method. A sheet of photoelastic material is bonded to the surface of the actual pressure vessel. Upon loading the vessel, photoelastic interference fringes are formed in the coating with reflected polarized light and indicate the difference in principal stresses. Photoelastic coating methods are usually performed at room temperature, but some coatings are used in a controlled high-temperature environment. Relatively inexpensive equipment and trained personnel are required. Accuracy may be less than that of the photoelastic transparent-model method. Further information can be found in reference 108.

2.1.2.5 Combined Methods

Sometimes it is advantageous to use a combination of methods. For example, if the pressure vessel configuration is complex, the brittle coating method may be used first to locate the peak stresses and their principal stress directions. Then, where it is desirable, electrical-resistance strain gages may be mounted on the locations of the peak strains to obtain more accurate results at discontinuities, except in regions where very high strain gradients occur (ref. 109).

2.1.2.6 Newer Methods

New methods being developed for the experimental determination of discontinuity stresses include the techniques of holography, in which coherent light reflected from a material under strain reveals in its fringe pattern the location and degree of the strain (stress) (refs. 113 and 114), and x-ray diffraction, in which deformation of the surface crystal lattice is revealed and can be correlated with strain (stress) (ref. 115).

2.2 Other Considerations

Having determined the discontinuity stresses, the designer must also consider material properties, flaws, and allowables. For example, the presence of a flaw at a discontinuity can seriously degrade structural integrity. These subjects are beyond the scope of this document but are covered in references 6, 29, 31, 81, 82, 101, and 116 to 121.

Verification of the structural integrity of pressure vessels with discontinuities normally involves testing the entire pressure vessel. The final verification of pressure vessels with discontinuities prior to service is obtained by proof and burst tests. Specifications for testing pressure vessels with discontinuities are not well defined in the literature. Most aerospace companies prepare and document their own test specifications in the form of company reports. These documents usually have some internal distribution, but little or no external circulation. Good testing practices are described in references 6 and 8 to 10.

3. CRITERIA

The design of metallic pressure vessels for space vehicles shall minimize the number and magnitude of discontinuities. The magnitudes of stress, and deflection when pertinent, of all discontinuities shall be determined for critical loading conditions by theoretical or experimental analysis, or both, with sufficient accuracy to permit an adequate assessment of the structural integrity of the pressure vessels.

3.1 Loads

Accurate critical loading conditions shall be supplied as an input to the analysis of metallic pressure vessels with discontinuities. The input loads shall include, but not be limited to, pressure, dead weight, inertial, dynamic, acoustic, cyclic, aerodynamic, and thermal loads.

3.2 Theoretical Analysis

Stresses and deflections due to discontinuities in pressure vessels shall be determined with acceptable methods of analysis. The discontinuity analysis shall include the effects of offset (mismatch), peaking (angle mismatch), change in thickness, junctures, branches, openings, and attachments. When a theoretical analysis is questionable, it shall be substantiated by experimental analysis or test.

All material properties or characteristics used in the analysis of discontinuity regions of pressure vessels shall be taken from reliable sources of data or be adequately substantiated by tests.

3.3 Experimental Analysis

An experimental analysis shall be conducted to determine discontinuity stresses when a theoretical analysis produces results that have not been substantiated by empirical evidence from similar configurations, when the theoretical approach is new, or when no theoretical analysis has been conducted. Acceptable experimental-analysis techniques shall be employed. Critical loading conditions and their combined actions, if applicable, shall be accurately simulated. The results of the experimental analysis shall either verify the results of the theoretical analysis or be used in lieu of theoretical analysis.

4. RECOMMENDED PRACTICES

Although some discontinuities are always present in all metallic pressure vessels, it is highly desirable to reduce their number and magnitude to a minimum. To meet this goal it is recommended that pressure vessel designs minimize discontinuities and lend themselves to ease of fabrication, to continuous quality control, and to thorough inspection procedures during fabrication. The cross-sectional area of openings for interconnecting elements that penetrate the pressure vessel wall should be minimized. Where localized loads are introduced, causing local discontinuities in stress distribution and deformations in a pressure vessel, a ring or pad or other reinforcement structure is recommended to distribute the loads into the shell and, consequently, to reduce the magnitude of the discontinuity stresses and deformations. For the same reason, designs which introduce loads normal to the shell should be avoided, if possible. Where this type of design cannot be avoided, a suitable analysis for the resulting discontinuity stresses should be performed.

A theoretical and/or experimental stress analysis should be performed for every metallic pressure vessel and should include stresses resulting from internal pressure, ground and flight loads, and thermal gradients. The analysis of stresses resulting from internal pressure should include primary membrane stresses and secondary bending and membrane stresses that result from design discontinuities and allowable design deviations. The stress analysis should include the effects of discontinuities in thickness, contour, material properties, loadings, and temperature; nonlinear effects should be accounted for (ref. 32). Discontinuities such as openings (windows, doors, and hatches), fittings, and weldments should be accounted for in the analysis using theoretical and/or experimental methods such as those discussed in Section 2. A complete elastic analysis of the discontinuity stresses at the weldments is recommended. It is also recommended that each pressure vessel be analyzed as if it contained a flaw and that the degrading effect of the flaw be evaluated (ref. 6).

Allowance should be made for residual stresses in the analysis when they are deemed to be significant. Although estimates of residual stresses are difficult to obtain without special investigation, they may be estimated by consideration of the deformation that occurs during manufacturing processes, weld shrinkages, etc. There are also experimental techniques using brittle coatings or photoelastic coatings to determine local residual stresses. The area of interest is coated and a small hole is drilled in the area. Cracks or fringes in the coating in the immediate area may indicate the magnitude of the residual stresses which are relieved by the hole. These techniques result in local destruction of the part. However, the nondestructive x-ray technique can be used in some cases (e.g., for parts small enough to be examined by this method in the laboratory).

4.1 Loads

The loads for metallic pressure vessels depend upon the application, intended use, and environment. Related documents provide useful information for selection of loadings used in the discontinuity analysis. Recommended documents include published NASA design criteria monographs such as references 4 to 6, and 67 to 73.

4.2 Theoretical Analysis

The following types of theoretical analysis are recommended for the determination of discontinuity stresses in pressure vessels:

- Elastic membrane analysis to size the basic pressure vessel shell
- Plastic membrane analysis to evaluate strain-hardening effects
- Elastic bending analysis to determine elastic stresses at geometrical discontinuities
- Plastic bending analysis to examine the redistribution of the stresses

Table I indicates common pressure vessel configurations with discontinuities for which solutions exist. References to sources where the recommended solutions can be found are included in the table. Table IV represents a collection of typical shell elements, such as cylinders or spheres, and recommends references to the sources where the primary (usually membrane) and secondary solutions (bending) may be found.

When the configuration is not shown in table IV, an approximate solution should be obtained in the following way. The primary solution should be obtained with standard membrane equations; and the secondary solution should be obtained by locally approximating the bulkhead as a spherical, cylindrical, or conical shell as described in references 26, 28, and 75. This type of substitution provides a fairly accurate approximation of the local discontinuity stresses.

When the configuration to be analyzed is too complex to be treated with the analytical methods shown in tables I or IV, the analysis should usually be performed with one of the numerical techniques; representative computer programs are listed in table V in the Appendix.

Finite-difference methods or numerical-integration techniques should be used for pressure vessels which are shells of revolution containing axisymmetrical discontinuities.

TABLE IV. – AVAILABLE SOLUTIONS FOR SHELLS OF REVOLUTION (REFERENCES)

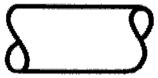
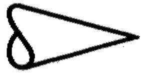
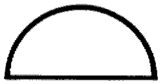
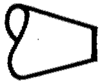
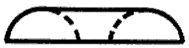
Description	Solution (Ref no.)		Description	Solution (Ref no.)	
	Primary (a)	Secondary (b)		Primary (a)	Secondary (b)
1. Long cylinder 	18, 25, 26, 28, 64, 74, 75	18, 25, 26, 28, 64, 75,	7. Pointed shell 	25, 26, 74	
2. Short cylinder 	26, 34, 75, 80	26, 75, 80	8. Cone 	25, 26, 28, 74, 75, 80	26, 28, 75, 80
3. Hemisphere 	18, 25, 26, 28, 64, 74, 75, 76, 80	18, 25, 26, 28, 64, 75, 76, 80	9. Truncated cone 	26, 74, 75	26, 75, 80
4. Truncated hemisphere 	26, 64, 75	26, 75	10. Cassinian 	25, 26, 74	
5. Ellipsoid 	18, 25, 26, 74		11. Paraboloid 	26, 74	
6. Toroid 	25, 26, 28, 74, 76	28	12. Cycloid 	26, 74	

TABLE IV. – AVAILABLE SOLUTIONS FOR SHELLS OF REVOLUTION (REFERENCES) – Continued

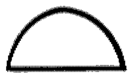
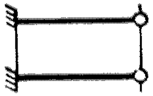
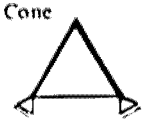
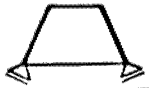
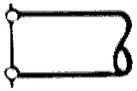
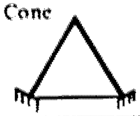
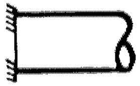
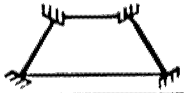
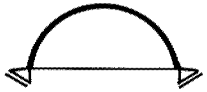
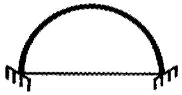
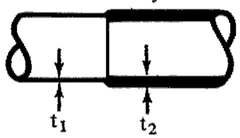
Description	Solution (Ref no.)		Description	Solution (Ref no.)	
	Primary (a)	Secondary (b)		Primary (a)	Secondary (b)
13. Catenaroid 	26, 74		19. Cylinder 	26, 75	26, 75
14. Circular plate 	26, 122	26, 122	20. Cylinder 	26, 75	26, 75
15. Circular plate with hole 	26, 28, 122, 123	26, 28, 122	21. Cone  for special case see no. 8	26, 75	26, 75
16. Shallow sphere 	75		22. Truncated cone  for special case see no. 9	26, 75	26, 75
17. Cylinder 	26, 75	26, 75	23. Cone 	26, 75	26, 75
18. Cylinder 	26, 75	26, 75	24. Truncated cone 	26, 75	26, 75

TABLE IV.– AVAILABLE SOLUTIONS FOR SHELLS OF REVOLUTION (REFERENCES) – Concluded

Description		Solution (Ref no.) Primary (a)	Solution (Ref no.) Secondary (b)
25. Hemisphere 	for special case see no. 3	26, 75	26, 75
26. Hemisphere 		18, 26, 75	18, 26, 75
27. Truncated hemisphere 	for special case see no. 4	26, 75	26, 75
28. Cylinder with discontinuity 		26, 75	26, 75

Description	Solution (Ref no.) Primary (a)	Solution (Ref no.) Secondary (b)
29. Circular ring 	25, 26, 122	25, 26

Legend

a —△— b Simply supported boundary, free to move along a–b

Pinned boundary

Fixed boundary

Free boundary

[a]Solution in most cases is the membrane solution

[b]Solution in all cases is the bending solution.

Finite-element techniques should be used for the above cases as well as for mathematically modeling complex pressure vessel discontinuities such as cutouts, reinforcements, and fittings. A sufficiently detailed representation should be made of the highly stressed areas in the regions of discontinuities to obtain accurate results. The relative proportion of adjacent individual components in the structural model must be chosen with care to minimize extreme variations in stiffness or flexibility which result in loss of accuracy. Unless the equivalent of IBM 360 double-precision arithmetic is used, one should not allow the ratio of numerical values between diagonal elements in the elastic matrix to exceed 1:1000 (ref. 124). Several computer programs of interest are listed in the Appendix.

A word of caution: a program which is declared to be running by its originators will not necessarily run elsewhere by someone unfamiliar with the program without considerable further effort.

New developments in analytical techniques should be closely monitored; a number of highly promising techniques are under development, including the use of interactive graphics.

4.3 Experimental Analysis

In some cases, the discontinuities are so complex that experimental analyses provide the only means available to determine the magnitudes and distributions of the discontinuity stresses (refs. 107 and 112).

Strain gages, brittle coatings, photoelasticity, and photoelastic coating methods, or a combination of these methods, should be used to verify any questionable theoretical analysis that has not been proven by previous experiments or successful vehicle flights, since the accuracy of this analysis depends on the adequacy of the theoretical method and the mathematical model of the discontinuity.

Strain gages should be used when the locations of the peak stresses are known and the stress gradient field does not have abrupt deviations. When geometry permits, back-to-back gages should be used, especially if bending is involved. Brittle coatings should be used when the locations of peak stresses are unknown and must be determined to locate strain gages precisely. Use of a photoelastic model should be used for complex stress fields if the test environment can be controlled and if skilled personnel and proper equipment are available. This method is recommended especially in the early design stages when the strain gage method cannot be used and also when peak stresses are very localized and could be missed by the strain gage method. The photoelastic coating method should be used for the measurement of rapidly varying stresses and for analysis of the full-stress field. It avoids problems of fabricating a model; the experimental analysis is performed with the actual metallic pressure vessel. This method is recommended where hardware is already fabricated and when theoretical analysis has not been previously verified.

4.4 Material Properties at Discontinuities

Material-property data are published and may be obtained from applicable Government or company specifications. For commonly used materials in the aerospace industry, the basic source for material properties should be a NASA-approved source of data or MIL-HDBK-5B (ref. 116). When reliable data are not available, appropriate coupon tests should be run and material properties developed using the approved methods required in reference 116.

APPENDIX

COMPUTER PROGRAMS

A number of computer programs (ref. 125) applicable to pressure vessels are summarized in table V (adapted from ref. 125). In those cases where no formal name has been attached to the code, it is listed in the table by the name of its developer. The organization or principal investigator responsible for the development of the code is listed in the second column. The third column contains the name of the agency(ies) that sponsored the development effort. A brief description of each code is presented in the fifth column. The last column presents geometries and constructions. The status of the code in 1971 is designated by one of three letters which indicates the following:

D = Currently under development

P = Considered proprietary by developing organization

A = Available outside of developing organization

TABLE V.–SUMMARY OF COMPUTER CODES

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
SABOR 1	MIT	SAMSO	A	F E, (1D), linear, static, axisymmetric loads	Layered, orthotropic, or composite shell. Meridional variation of material properties or discrete ring stiffeners. Shells with internal branches. Multishells
SABOR 3	MIT	SAMSO	A	F E, (1D), linear, static, asymmetric loads	Same as SABOR 1
SABOR 4	MIT	SAMSO	A	F E, (1D), linear, static, asymmetric loads (Uses improved elements)	Same as SABOR 1
SABOR 5	MIT	SAMSO	A	F E, (1D), linear, static, asymmetric loads (Uses further improved elements)	
STACUSS 1	MIT	SAMSO	A	F E, (2D), linear, static, curved-shell elements	
DRASTIC 2	MIT	SAMSO	A	Numerical utilization package for use with SABOR 3	
DRASTIC 5	MIT	SAMSO	A	Improved numerical integration package for use with SABOR 5	Layered, branched shells
PETROS 2	MIT	BRL	D	F D, (2D), nonlinear, inelastic, transient response cone, cylinder panel	
SALORS	NASA/LRC	NASA/LRC	A	F D, (1D), linear, static, asymmetric loads, nonlinear symmetric loads, bifurcation buckling, prestressed modal vibration	Layered, orthotropic, or composite shell. Meridional variation of material properties or discrete ring stiffeners. Shells with internal branches. Multishells
BALOR	NASA/LRC	NASA/LRC	A	Early bifurcation, buckling version of SALORS	Same as SALORS

TABLE V.–SUMMARY OF COMPUTER CODES–Continued

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
VALOR	NASA/LRC	NASA/LRC	A	Early modal vibration version of SALORS	(see SALORS, previous page)
SCHAEFFER	NASA/LRC	NASA/LRC	A	F D, (1D), linear, static, asymmetric loads	Layered, anisotropic, or composite. Material properties may vary meridionally. Discrete ring stiffeners. Internal branches
STEPHENS/ FULTON	NASA/LRC	NASA/LRC	A	F D, (1D), nonlinear, static, and transient response, axisymmetric loads	Composite or stiffened walls. Multishells and branched shells
KALNINS	A. Kalnins	AFFDL	A	N I, (1D), linear, static, modal vibration, bifurcation buckling, nonsymmetric loads	Layered, anisotropic, or composite walls. Multishells, internal branches
STARS II	Grumman	NASA/MSFC	A	N I, (1D), static, linear, asymmetric, nonlinear symmetric	Thin shells of revolution, isotropic and orthotropic, monocoque, sandwich, reinforced sheet (stringers, rings, waffle) or sandwich
REPSIL	BRL	BRL	D	F D, (2D), nonlinear inelastic, transient response cone, cylinder panel	Elasto-plastic shell. Utilizes von Mises' yield criterion
BALL	Dynamic Sciences	NASA/LRC	A	F D, (1D), nonlinear, static, asymmetric loads	Layered, anisotropic, or composite shell. Meridional variation of material, properties or discrete ring stiffeners. Multishells, internal branches
SAMIS	Philco-Ford	JPL	A	F E, (2D), linear static, linear elastic transient response	Cylindrical shell with and without cutouts. Complicated shell configurations. Space frames, trusses, plates. Orthotropic properties
STARDYNE	Mechanics Research	Mechanics Research	A	F E, (2D), linear, static, dynamic. Two-dimensional dynamic response	

TABLE V.–SUMMARY OF COMPUTER CODES–Continued

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
EASE	Engineering Analysis	Engineering Analysis	A	F E, (2D), linear, static	Very useful for intersecting cylinders and other shells
SOR	Space Div. North American Rockwell	NASA/MSC	A	F D, (1D), linear, static, asymmetric loads, nonlinear symmetric	Axisymmetric multishell and branched shells. Arbitrary material: sandwich, stiffened or orthotropic walls
COHEN	Structures Research Association	Structures Research Association	A	N I, (1D), static bifurcation buckling, linear asymmetric, nonlinear axisymmetric	Layered, anisotropic, or composite multishell with branches. Meridional variation of material. Discrete ring stiffeners
BOSOR 3	LMSC	NSRDC, SAMSO	A	F D, (1D), static, eigenvalue linear nonsymmetric, nonlinear symmetric (1D)	Composite shells, stiffened shells
STAGS	LMSC	(AFFDL, NSRDC) NASA/LRC	D	F D, (2D), linear and nonlinear inelastic static. Finite-difference formulation, nonlinear elastic collapse of a cylinder with noncircular cross section	Shells, cutouts
STAR	LMSC	LMSC, SAMSO	A	F D, (2D), nonlinear, inelastic, transient response	Single orthotropic material. Cones and cylinders with cutouts
WASP	LMSC	LMSC	D	F E, (1D), linear, static, thick shell, asymmetric loads	Arbitrary geometry, wide variety of construction
SMERSH	Kaman Nuclear	Kaman Nuclear	D	F D, (2D), nonlinear, inelastic, transient response. Finite difference formulation	Single shells
SCARS	Sandia	Sandia	D	F D, (2D), nonlinear, inelastic, transient response	Single shells
SLADE	Sandia	Sandia	D	F E, (2D), linear, static	Shells with cutouts

TABLE V.–SUMMARY OF COMPUTER CODES–Continued

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
SNAP	Lockheed-Huntsville	LMSC	D	F E, (2D), linear, static, eigenvalue	Rectangular cutouts
FORMAT	Douglas Aircraft	AFFDL	A	F E, (2D), linear, static, eigenvalue	Arbitrary geometry, wide variety of construction
ASTRA	Boeing	Boeing	P	F E, (3D), static eigenvalue, nonlinear, elastic linear dynamic response. Two- and three-dimensional finite elements. Shell structures. Considering also thermal effect. Nonlinearity considered in static case	Very general. Every kind of structure and every type of construction
NASTRAN	Computer Sciences Corp.	NASA/GSFC/LRC	A	F E, (3D) linear, nonlinear considered as piecewise linear, thermal effects, static, dynamic, direct and modal transient response, direct and modal frequency response, real and complex eigenvalues, buckling	Very general. Every kind of structure and every type of construction
ASKA	H. Argyris, University of Stuttgart	North American Rockwell	P	F E (3D), static and dynamic analyses, geometric and material nonlinear analyses. Based on matrix-displacement method	Structural shells and multishells with various wall construction (sandwich, stiffened, etc). Elastic, plastic, large deflection, stability
MINI-ASKA	Univ. of Arizona	H. Kamal	P	F E, (3D), linear, static, eigenvalue	
SNASOR	Texas A&M	NASA/MSC & Sandia	A	F E, (1D), nonlinear, static, asymmetric loads	Shells of revolution with arbitrary material properties
REXBAT 5	LMSC	LMSC	D	F E, (2D), linear, static, eigenvalue	Rectangular cutouts, beams, orthotropic bending

TABLE V.–SUMMARY OF COMPUTER CODES–Continued

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
NARSAMS	Space Div. North American Rockwell	Space Div. North American Rockwell	P	F E, (2D), nonlinear, static, piecewise linear deflection distribution assumption. Finite elements	Almost any geometry and type of construction can be handled
EPSOR	LMSC	LMSC	A	F D, (1D), nonlinear, inelastic, static axisymmetric loads	Nonlinearity in material. Plasticity considered
MARCAL	Brown Univ.	NSRDC	D	F E, (2D), nonlinear, inelastic, dynamic, eigenvalue, transient response	Modeled with bar, beam column. Doubly-curved shell of revolution
GIRLS I	LMSC	SAMSO	A	F D, (1D), nonlinear, inelastic, transient response. Only circumferential variation in response	Inelastic material behavior. Strain hardening and strain rate effect
GIRLS II	LMSC	SAMSO	A	F D, (1D), nonlinear, inelastic, transient response axisymmetric, arbitrary loadings	Inelastic material behavior. Strain hardening and strain rate effect. Axisymmetric shells
UNIVALVE II	Sandia	Sandia	A	F D, (1D), nonlinear, inelastic, transient response	Inelastic material behavior. Strain hardening and strain rate effect. Beams, rings, arches (structures with in-plane deformation)
DYNASOR	Texas A&M	NASA/MSC, Sandia	A	F E, (1D), nonlinear, dynamic, asymmetric loads	Shells of revolution, arbitrary properties of materials. Stiffness and shell thickness equal in circumferential direction. Orthotropic properties
STRICKLIN	Texas A&M	Sandia	A	F E, (1D), nonlinear, static SOR with circumferentially varying stiffness, asymmetric loads	Composite wall, circumferential variation of the shell wall thickness

TABLE V.–SUMMARY OF COMPUTER CODES–Concluded

Computer code	Developing organization	Funding organization	Status	Type of code	Geometry/construction
ELAS	JPL	NASA	A	F E, linear, static, circumferentially and meridionally varying stiffness	Very general program. Arbitrary structure. Arbitrary material (including stiffened walls, sandwich walls, etc.
WILSON	LMSC	LMSC	P	F D, (2D), linear (bilinear material representation), static, axisymmetric shells	Composite shells, beams, axisymmetric solids
3019	GD/Convair Aerospace Division	NASA/LeRC	A	F E, (2D), nonlinear, static, axisymmetric loads, and/or temperatures	Monocoque, axisymmetric, circular cylinders, other shells of revolution by approximation, and can include discrete ring stiffeners and internal branches
P1580	GD/Convair Aerospace Division	NASA/LeRC	A	Series solution, (2D), linear, static, concentrated loading	Monocoque, isotropic, long, circular cylindrical thin-walled shells
ROARK	United Computing Systems	United Computing Systems	A	Closed-form solutions. Teletype with dial-in and log-on	Simple shells of revolution, isotropic monocoque

Abbreviations:

F E = Finite Element; F D = Finite Difference; N I = Numerical Integration; SOR = Shell of Revolution; SAMSO = Air Force Space & Missile Systems Organization; BRL = Ballistic Research Laboratory; LMSC = Lockheed Missiles & Space Company; AFFDL = Air Force Flight Dynamics Laboratory; NSRDC = Naval Ship Research & Development; (1D) = One Dimensional (Method Applied Only to Problems Which Can Be Reduced to One-Dimensional Mathematical Form); (2D) = Two Dimensional; (3D) = Three Dimensional

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