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CRITICAL STRESS FOR AN INFINITELY LONG FLAT PLATE

WITH ELASTICALLY RESTRAINED EDGES UNDER

COMBINED SHEAR AND DIRECT STRESS

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NACA

WASHINGTON

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SUMMARY

A simple interaction curve is presented for evaluating the conditions of combined shear and direct stress under which an infinitely long, flat plate with equal elastic restraints against rotation along the edges will become unstable. The theoretical work that led to the interaction curve is presented in the form of appendixes.

INTRODUCTION

In the design of stressed-skin structures, consideration must sometimes be given to the critical stresses for a sheet under a combination of shear and direct stress. The upper surface of a wing in flight, for example, may be subjected to combined shear and compressive stress and the lower surface to combined shear and tensile stress simultaneously. Under this condition the upper surface may buckle at a lower compressive stress than if the shear were not present and the critical shear stress for the lower surface will be increased by the presence of the tensile stress. The purpose of the present report is to specify the combination of shear and direct stress under which such surfaces may be expected to buckle.

INTERACTION CURVE

For the case of flat sheet between identical stiffeners, the combination of shear stress $\tau$ and compressive stress $-\sigma_x$ that will cause buckles to appear in the sheet has been exactly solved in appendix A. The values

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of $\sigma_x$ and $\tau$ may be given in the form (reference 1, pp. 355 and 359)

$$\sigma_x = \frac{k_c \pi^2 D}{b^2 t}$$

$$\tau = \frac{k_s \pi^2 D}{b^2 t}$$

where

D       flexural stiffness of sheet per unit length

$$\left[ \frac{E t^3}{12(1-\mu^2)} \right]$$

E       modulus of elasticity

$\mu$       Poisson's ratio

b       width of sheet between stiffeners

t       thickness of sheet

$k_c, k_s$    coefficients dependent upon the restraint supplied
        by the stiffeners

Appendix A describes the intricate relationship that exists between $k_c$ and $k_s$. As is usual with exact solutions of buckling problems, this solution is very laborious to apply.

The possibility of great simplification in the computation of the critical stresses is indicated by the results of the application of an energy method presented in appendix B. In this appendix it is shown that if the restraint supplied to the sheet by the stiffeners is independent of the wave length of the buckles, the following relation exists:

$$R_c + R_s^2 = 1 \tag{1}$$

where

$R_s$   ratio of shear stress when buckling occurs in combined shear and direct stress to shear stress when buckling occurs in pure shear

$R_c$   ratio of compressive stress when buckling occurs in combined shear and direct stress to compressive stress when buckling occurs in pure compression

Inasmuch as design charts for determining the critical stresses under pure shear and pure compression for flat sheet with equal elastic edge restraints have been presented in references 2 and 3, equation (1) may be easily applied to the combined loading of such sheet. In an actual structure, however, the restraint is usually dependent upon the wave length of the buckles, and the formula must therefore be shown to hold with sufficient accuracy in this case also.

Equation (1) was tested as described in appendix C, by use of exact values of the critical stresses obtained by the method of appendix A. The results are shown in figure 1, in which the curve represents the values obtained by the combined loading formula of equation (1) and the plotted points are for the exact values computed by the method of appendix A and listed in table 1. In some cases, the restraint was assumed to be independent of the wave length; whereas, in others, a relationship between restraint and wave length typical of a sturdy stiffener (reference 4) was assumed. Some values were also computed for tension in the sheet ($k_c$ negative). All values are seen to lie on the curve or close to it, indicating that equation (1) holds with sufficient accuracy for engineering purposes in all cases.

## CONCLUSIONS

It is concluded that the values of combined shear and direct stress at which an infinitely long, flat plate supported along the edges with equal elastic restraints against rotation will become unstable may be determined for practical engineering purposes by the equation

$$R_c + R_s^2 = 1$$

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where

$R_c$    ratio of direct stress when buckling occurs in com-
bined shear and direct stress to compressive stress
when buckling occurs in pure compression. Tension
is regarded as negative compression.

$R_s$    ratio of shear stress when buckling occurs in com-
bined shear and direct stress to shear stress when
buckling occurs in pure shear

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# APPENDIX A

## SOLUTION BY DIFFERENTIAL EQUATION

The exact solution for the critical stress at which
buckling occurs in a flat rectangular plate subjected to
combined shear and compression forces in its own plane
may be obtained by solving the differential equation that
expresses the equilibrium of the buckled plate. The plate
is assumed to be infinitely long, and equal elastic re-
straints against rotation are assumed to be present along
the two edges of the plate.

Figure 2 shows the coordinate system used. The dif-
ferential equation for equilibrium of a flat plate under
shear and direct stress in the direction of one axis is
(reference 1, p. 305)

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] + 2\tau t\frac{\partial^2 w}{\partial x \partial y} + \sigma_x t\frac{\partial^2 w}{\partial x^2} = 0 \qquad (A1)$$

where

D    flexural stiffness of plate per unit length $\left[\frac{Et^3}{12(1-\mu^2)}\right]$

E    modulus of elasticity

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$\mu$    Poisson's ratio

t    thickness of plate

w    deflection of plate at $(x,y)$ from unstressed position

$\tau$    uniformly distributed shear stress

$\sigma_x$    uniformly distributed compressive stress in the direction of $x$

It is known that the critical stresses are of the form (reference 1, pp. 355 and 359)

$$\left.\begin{array}{l} \sigma_x = \dfrac{k_c \pi^2 D}{b^2 t} \\[3mm] \tau = \dfrac{k_s \pi^2 D}{b^2 t} \end{array}\right\} \qquad (A2)$$

where $b$ is the width of the plate and $k_c$ and $k_s$ are constants which, for an infinitely long plate, depend only upon the elastic restraints along the edges of the plate. Substitution of expressions (A2) in equation (A1) gives

$$\frac{\partial^4 w}{\partial x^4} + 3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{2\pi^2 k_s}{b^2} \frac{\partial^2 w}{\partial x \partial y} + \frac{\pi^2 k_c}{b^2} \frac{\partial^2 w}{\partial x^2} = 0 \quad (A3)$$

If the plate is infinitely long in the x-direction, all displacements must be periodic in $x$, and the deflection surface may be taken in the form

$$w = Y e^{i \frac{\pi x}{\lambda}} \qquad (A4)$$

where $Y$ is a function of $y$ only and $\lambda$ is the half wave length of the buckles in the x-direction.

Substitution of expression (A4) into the differential equation (A3) gives as the equation which determines $Y$

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$$\frac{d^4Y}{dy^4} - \frac{2\pi^2}{\lambda^2}\frac{d^2Y}{dy^2} + \frac{\pi^4}{\lambda^4}Y + \frac{2\pi^3 i k_s}{b^2\lambda}\frac{dY}{dy} - \frac{\pi^4 k_c}{b^2\lambda^2}Y = 0 \qquad (A5)$$

A solution of equation (A5) is

$$Y = e^{im\frac{y}{b}}$$

when $m$ is a root of the characteristic equation

$$m^4 + 2\left(\frac{\pi b}{\lambda}\right)^2 m^2 - 2\pi^2\left(\frac{\pi b}{\lambda}\right)k_s m + \left(\frac{\pi b}{\lambda}\right)^4 - \pi^2\left(\frac{\pi b}{\lambda}\right)^2 k_c = 0 \qquad (A6)$$

Except for the substitution of $\left(\frac{\pi b}{\lambda}\right)^4 - \pi^2\left(\frac{\pi b}{\lambda}\right)^2 k_c$ for $\left(\frac{\pi b}{\lambda}\right)^4$

equation (A6) is identical with equation (A6) of reference
2, in which equation (A1) of this appendix was solved with
$k_c = 0$. With this change, all the results obtained in
that appendix are applicable here. The stability crite-
rion for combined compression and shear is therefore for-
mally the same as for shear alone as given by equation
(A19) of reference 2. This criterion is

$$2\alpha\beta\left(4\gamma^2 - \frac{\epsilon^2}{4}\right)\left(\cosh 2\alpha \cos 2\beta - \cos 4\gamma\right)$$

$$-\left[4\gamma^2\left(\beta^2 - \alpha^2\right) - \left(\beta^2 + \alpha^2\right)^2 - \left(4\gamma^2 - \beta^2 + \alpha^2\right)\frac{\epsilon^2}{4}\right]\sinh 2\alpha \sin 2\beta$$

$$+ \epsilon\left[\alpha\left(4\gamma^2 + \alpha^2 + \beta^2\right)\cosh 2\alpha \sin 2\beta\right.$$

$$\left.+ \beta\left(4\gamma^2 - \alpha^2 - \beta^2\right)\sinh 2\alpha \cos 2\beta - 4\alpha\beta\gamma \sin 4\gamma\right] = 0 \qquad (A7)$$

where the relation between $k_s$ and $\alpha$, $\beta$, and $\gamma$ is

the same as in reference 2, equation (A23)

$$k_s = \frac{8\gamma \left(\alpha^2 + \beta^2\right)}{\pi^2 \left(\dfrac{\pi b}{\lambda}\right)} \qquad \text{(A8)}$$

but in the present report

$$\left. \begin{array}{l} \alpha = \sqrt{\gamma^2 + \dfrac{1}{4}\left(\dfrac{\pi b}{\lambda}\right)^2 + \sqrt{4\gamma^4 + \left(\dfrac{\pi b}{\lambda}\right)^2 \gamma^2 + \dfrac{\pi^2}{16}\left(\dfrac{\pi b}{\lambda}\right)^2 k_c}} \\[4ex] \beta = \sqrt{-\gamma^2 - \dfrac{1}{4}\left(\dfrac{\pi b}{\lambda}\right)^2 + \sqrt{4\gamma^4 + \left(\dfrac{\pi b}{\lambda}\right)^2 \gamma^2 + \dfrac{\pi^2}{16}\left(\dfrac{\pi b}{\lambda}\right)^2 k_c}} \end{array} \right\} \text{(A9)}$$

The restraint coefficient $\epsilon$ is a measure of the relative resistance to rotation of the restraining element at the edge of the plate; it is discussed more fully in reference 2.

The procedure for evaluating $k_s$, after values of $k_c$ and an expression for $\epsilon$ have been chosen, is as follows: A value of $b/\lambda$ is assumed; a series of values of $\gamma$ are taken until one is found which, together with the corresponding values of $\alpha$ and $\beta$ as computed from equations (A9), satisfies equation (A7); $k_s$ is then computed from equation (A8). Another value of $b/\lambda$ is assumed, a new set of values of $\gamma$, $\alpha$, and $\beta$ are found that satisfy the stability criterion, and a new value of $k_s$ is computed. The entire process is repeated until by plotting $k_s$ against $b/\lambda$ the minimum value of $k_s$ is found. In the case where $\epsilon$ is a function of $b/\lambda$, $\epsilon$ must be re-evaluated each time a different value of $b/\lambda$ is assumed. This minimum value of $k_s$ and the chosen value of $k_c$, when inserted in equations (A2), give a critical combination of shear and direct stress.

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# APPENDIX B

## SOLUTION BY ENERGY METHOD FOR CONSTANT  $\epsilon$

In appendix B of reference 2, an energy solution was given for the stability of a plate under pure shear by use of oblique coordinates. This solution may be extended to cover the case of combined shear and compression loading by simply adding to the work done by the shear force an additional term expressing the work done by the compressive force.

Figure 3 shows the coordinate system and the plate dimensions. The procedure is to evaluate the terms in the equation

$$T_c + T_s = V_1 + V_2 \tag{B1}$$

where

$T_c$   work done by the compressive force per half wave length

$T_s$   work done by the shear force per half wave length

$V_1$   strain energy in the plate per half wave length

$V_2$   strain energy per half wave length in the elastic restraining members assumed to be present along the edges of the infinitely long plate

The deflection surface is taken to be the same as in reference 2, equation (B2); that is,

$$w = w_0 \left[ \frac{\pi \epsilon}{2} \left( \frac{y^2}{b_1^2} - \frac{1}{4} \right) + \left( 1 + \frac{\epsilon}{2} \right) \cos \frac{\pi y}{b_1} \right] \cos \frac{\pi x}{\lambda} \tag{B2}$$

where  $\epsilon$  is the restraint coefficient defined as in reference 2. The values of  $T_s$,  $V_1$,  and  $V_2$  may be taken directly from equations (B3), (B4), and (B5) of reference 2:

$$T_s = w_0^2 \frac{\pi^2 b_1 t \sin \phi}{2\lambda} \left[ \left( \frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2} \right) \epsilon^2 + \left( \frac{1}{2} - \frac{4}{\pi^2} \right) \epsilon + \frac{1}{2} \right] \tau \tag{B3}$$

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$$V_1 = w_0^2 \frac{\pi^4 D}{4 b_1 \lambda \cos^3 \phi} \left\{ \left(\frac{b_1}{\lambda}\right)^2 \left[ \left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2} \right] \right.$$

$$+ \left(\frac{\lambda}{b_1}\right)^2 \left[ \left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2} \right]$$

$$\left. + 2(1 + 2 \sin^2 \phi) \left[ \left(\frac{5}{24} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2} \right] \right\} \quad (B4)$$

$$V_2 = w_0^2 \frac{\pi^2 D \lambda \epsilon}{2 b_1^3 \cos^3 \phi} \quad (B5)$$

The work $T_c$ done by the compressive stress $\sigma_x$ is given
by

$$T_c = \frac{\sigma_x t}{2} \int_{-\frac{b_1}{2}}^{\frac{b_1}{2}} \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \left(\frac{\partial w}{\partial x}\right)^2 dx\, dy \cos \phi$$

By use of the assumed expression (B2) for $w$, $T_c$ is
found to be

$$T_c = w_0^2 \frac{\pi^2 b_1 t \cos \phi}{4 \lambda} \left[ \left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2} \right]\sigma_x \quad (B6)$$

When equations (B3), (B4), (B5), and (B6) are sub-
stituted in equation (B1), the result is

$$\sigma_x = k_c \frac{\pi^2 D}{b^2 t} - 2\tau \tan \phi \quad (B7)$$

where

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$$k_C = \frac{\left(\frac{b}{\lambda}\right)^2}{\cos^4\phi} + \frac{C_1}{\left(\frac{b}{\lambda}\right)^2} + C_2 \frac{1 + 2\sin^2\phi}{\cos^2\phi}$$

$$C_1 = \frac{\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{2}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}}$$

$$C_2 = 2 \frac{\left(\frac{5}{24} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} + \frac{1}{8} - \frac{2}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}}$$

The angle of inclination $\phi$ and the ratio $b/\lambda$ will adjust themselves to make $\sigma_x$ a minimum; that is,

$$\frac{\partial \sigma_x}{\partial \left(\frac{b}{\lambda}\right)} = 0 \qquad\qquad (B8)$$

$$\frac{\partial \sigma_x}{\partial \phi} = 0 \qquad\qquad (B9)$$

If the shear stress $\tau$ is considered as a given constant, and the restraint coefficient $\epsilon$ is independent of $b/\lambda$, the values of $b/\lambda$ and $\phi$ resulting from the operations indicated by equations (B8) and (B9) are:

$$\left(\frac{b}{\lambda}\right) = \sqrt[4]{C_1} \cos\phi \qquad\qquad (B10)$$

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$$\phi = \tan^{-1} \frac{\tau}{\left(2\sqrt{C_1} + 3C_2\right)\left(\frac{\pi^2 D}{b^2 t}\right)} \tag{B11}$$

Inasmuch as the value of $b/\lambda$ given by equation (B10) and the value of $\phi$ given by equation (B11) are the values which make the stress $\sigma_x$ a minimum, it is necessary to substitute them in equation (B7) to obtain the critical compressive stress. When this substitution is made and the result expressed in nondimensional form, it is found that

$$\frac{\sigma_x}{\left(2\sqrt{C_1} + C_2\right)\left(\frac{\pi^2 D}{b^2 t}\right)} = 1 - \frac{\tau^2}{\left(2\sqrt{C_1} + 3C_2\right)\left(2\sqrt{C_1} + C_2\right)\left(\frac{\pi^2 D}{b^2 t}\right)^2} \tag{B12}$$

In order to identify the denominators in equation (B12), it is necessary to refer to the energy solutions for pure compression and for pure shear. From equation (B14) of reference 3, it is found that

$$k_{pc} = \left(\frac{b}{\lambda}\right)^2 + \frac{C_1}{\left(\frac{b}{\lambda}\right)^2} + C_2$$

where $k_{pc}$ is the value of $k_c$ for pure compression; or, $\varepsilon$ being assumed constant and $k_{pc}$ minimized with respect to $b/\lambda$,

$$k_{pc} = 2\sqrt{C_1} + C_2 \tag{B13}$$

From equation (B6) in reference 2, it is found that

$$k_{ps} = \frac{1}{\sin 2\phi}\left[\frac{\left(\frac{b}{\lambda}\right)^2}{\cos^2\phi} + \frac{C_1 \cos^2\phi}{\left(\frac{b}{\lambda}\right)^2} + C_2(1 + 2\sin^2\phi)\right]$$

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where $k_{ps}$ is the value of $k_s$ for pure shear. If the value of $\epsilon$ is assumed constant and $k_{ps}$ is minimized with respect to $b/\lambda$ and $\phi$, $k_{ps}$ becomes

$$k_{ps} = 2 \sqrt{C_1 + 2 \sqrt{C_1}\, C_2 + \frac{3}{4}\, C_2^2}$$

or

$$k_{ps}^2 = 4\left(C_1 + 2\sqrt{C_1}\, C_2 + \frac{3}{4}\, C_2^2\right) = \left(2\sqrt{C_1} + 3C_2\right)\left(2\sqrt{C_1} + C_2\right) \quad \text{(B14)}$$

Substitution of relations (B13) and (B14) in equation (B12) gives the combined loading formula

$$\frac{\dfrac{\sigma_x}{\pi^2 D/b^2 t}}{k_{pc}} = 1 - \frac{\left(\dfrac{\tau}{\pi^2 D/b^2 t}\right)^2}{k_{ps}^2}$$

or

$$\frac{k_c}{k_{pc}} + \left(\frac{k_s}{k_{ps}}\right)^2 = 1 \quad\quad\quad \text{(B15)}$$

where $k_c$ and $k_s$ are written for $\dfrac{\sigma_x}{\pi^2 D/b^2 t}$ and $\dfrac{\tau}{\pi^2 D/b^2 t}$, respectively. Equation (B15) may also be written in the form

$$R_c + R_s^2 = 1 \quad\quad \text{(see equation (1))}$$

where

$$R_c = \frac{k_c}{k_{pc}}$$

$$R_s = \frac{k_s}{k_{ps}}$$

These definitions of $R_c$ and $R_s$ are equivalent to the definitions given in terms of stresses under sections Interaction Curve and Conclusions in the body of this report.

## APPENDIX C

### TEST OF INTERACTION FORMULA

The theory of appendix B, although based upon an approximate deflection surface for the buckled plate, indicates a simple algebraic relationship between $k_c$ and $k_s$, when the restraint coefficient $\epsilon$ is independent of the wave length $\lambda$, of the form

$$\frac{k_c}{k_{pc}} + \left(\frac{k_s}{k_{ps}}\right)^2 = 1 \tag{C1}$$

where

$k_{pc}$ value of $k_c$ when $k_s = 0$ (pure compression)

$k_{ps}$ value of $k_s$ when $k_c = 0$ (pure shear)

The exact solution found in appendix A provides the means by which equation (C1) may be tested. An exact numerical check could not be expected because some error would be introduced in reading the charts for $k_{pc}$ and $k_{ps}$ and in the graphical work involved in obtaining $k_s$; the test should reveal, however, whether the formula is sufficiently accurate for engineering computations.

Equation (C1) was tested for the following values of $\epsilon$:

$$\epsilon = 0$$

$$\epsilon = 10$$

$$\epsilon = \infty$$

$$\epsilon = \frac{5}{\left(\frac{\lambda}{b}\right)^2} + \frac{5}{\left(\frac{\lambda}{b}\right)^4}$$

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The first three values of $\epsilon$ are independent of the wave length, and are therefore representative of the values of $\epsilon$ upon which equation (C1) is based. The final value of $\epsilon$ varies with the wave length in a manner typical of a sturdy stiffener (reference 4), and this variation is therefore representative of a practical case.

For each value of $\epsilon$ a value of $k_c$ was chosen, and the associated value of $k_s$ required to cause the plate to buckle was computed as described in appendix A. Four cases were computed with tension on the plate ($k_c$ negative). The values of $k_{pc}$ and $k_{ps}$ were then read from their respective charts in references 2 and 3. For the case of constant $\epsilon$, the values of $k_{pc}$ and $k_{ps}$ were read at the minimums of their appropriate $\epsilon$-curves. (The values of $b/\lambda$ at which the plate buckles under pure compression, pure shear, and under the combined loading are, of course, all different.) For the case of variable $\epsilon$, it is necessary to shift from one $\epsilon$-curve to another, in conformity with the assumed relationship between $\epsilon$ and $\lambda$, until minimum values of $k_{pc}$ and $k_{ps}$ are found.

The results of the numerical test of the interaction formula are shown in table I and in figure 1. The final column of table I indicates the value of the left-hand members of equation (C1). For the cases in which $\epsilon$ is independent of $\lambda$ the difference in this value from unity is only a fraction of a percent, except when $R_c$ has a large negative value, in which case the difference is 2 to 5 percent; the values in the last column of table I will be more in error for these values of $R_c$ because their computation involves subtracting quantities of the same order of magnitude. For the case in which $\epsilon$ varies with $\lambda$ the difference from unity is 1 or 2 percent. The validity of the interaction formula is therefore established for all engineering purposes.

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# TABLE I

## TEST OF ACCURACY OF INTERACTION FORMULA
[Values computed by the method of appendix A]

| $k_c$ | $k_{pc}$ | $R_c = \dfrac{k_c}{k_{pc}}$ | $k_s$ | $k_{ps}$ | $R_s = \dfrac{k_s}{k_{ps}}$ | $R_c + R_s^2$ |
|---|---|---|---|---|---|---|
| | | | $\epsilon = 0$ | | | |
| 4.00 | | 1.0000 | 0 | | 0 | 1.0000 |
| 3.00 | | .7500 | 2.682 | | .5027 | 1.0027 |
| 2.10 | 4.00 | .5250 | 3.690 | 5.335 | .6917 | 1.0034 |
| 1.00 | | .2500 | 4.623 | | .8665 | 1.0008 |
| 0 | | 0 | 5.335 | | 1.0000 | 1.0000 |
| -30.00 | | -5.0000 | 13.997 | | 2.4362 | .9468 |
| | | | $\epsilon = 10$ | | | |
| 5.605 | | 1.0000 | 0 | | 0 | 1.0000 |
| 5.00 | | .8921 | 2.380 | | .3274 | .9993 |
| 2.80 | | .4996 | 5.115 | | .7073 | .9999 |
| 1.30 | 5.605 | .2319 | 6.357 | 7.270 | .8744 | .9965 |
| 0 | | 0 | 7.270 | | 1.0000 | 1.0000 |
| -1.30 | | -.2319 | 8.052 | | 1.1076 | .9949 |
| -6.00 | | -1.0705 | 10.418 | | 1.4330 | .9830 |
| | | | $\epsilon = \infty$ | | | |
| 6.98 | | 1.0000 | 0 | | 0 | 1.0000 |
| 5.40 | | .7736 | 4.272 | | .4757 | .9999 |
| 3.49 | | .5000 | 6.347 | | .7068 | .9996 |
| 1.80 | 6.98 | .2579 | 7.737 | 8.98 | .8616 | 1.0002 |
| 0 | | 0 | 8.980 | | 1.0000 | 1.0000 |
| -18.00 | | -2.5788 | 15.953 | | 1.8879 | .9854 |
| | | | $\epsilon = \dfrac{5}{\left(\dfrac{\lambda}{b}\right)^2} + \dfrac{5}{\left(\dfrac{\lambda}{b}\right)^4}$ | | | |
| 6.119 | | 1.0000 | 0 | | 0 | 1.0000 |
| 5.50 | | .8998 | 2.315 | | .3363 | 1.0119 |
| 3.00 | 6.119 | .4903 | 5.018 | 6.883 | .7290 | 1.0217 |
| 1.00 | | .1634 | 6.328 | | .9194 | 1.0087 |
| 0 | | 0 | 6.883 | | 1.0000 | 1.0000 |

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(a) Shear and compression

(b) Shear and tension
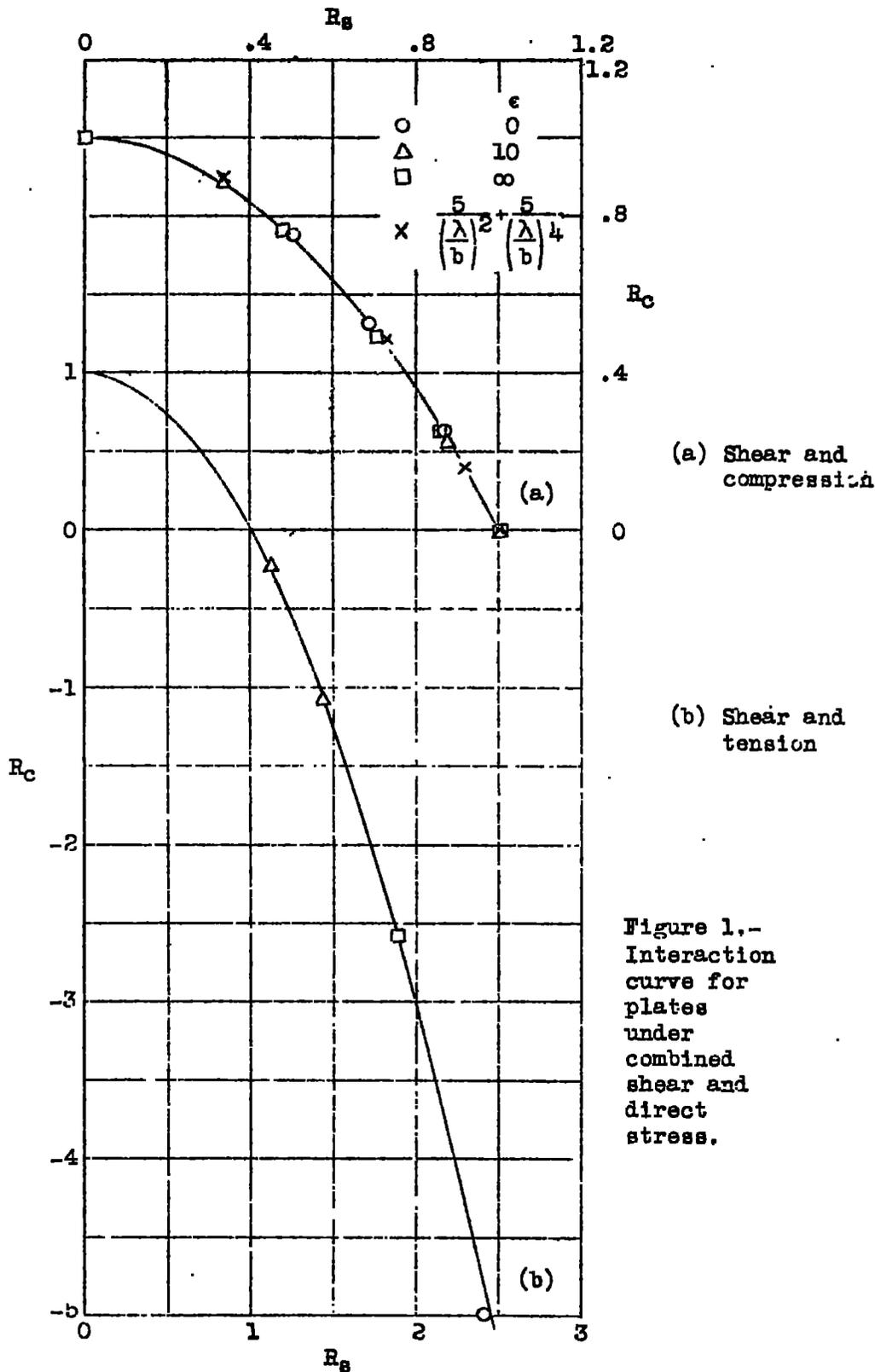
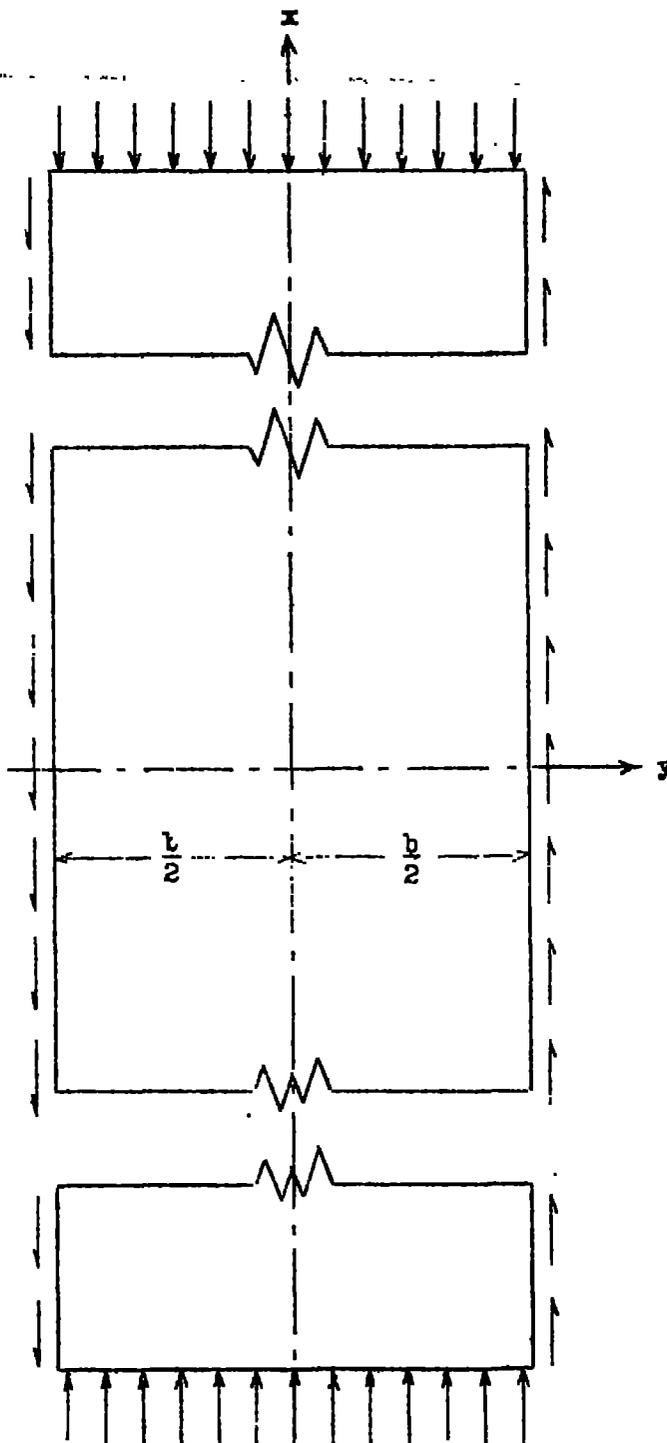Figure 1.-
Interaction
curve for
plates
under
combined
shear and
direct
stress.

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Figure 2.- Infinitely long rectangular plate under combined
shear and compression; coordinate system used in
appendix A.

Fig. 3

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Figure 3.- Oblique coordinate system used in appendix B.