TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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EFFECT OF AEROFOIL ASPECT RATIO ON THE SLOPE OF THE LIFT CURVE.

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January, 1922.

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Introduction.

One of the most important characteristics of an aerofoil is the rate of change of lift with angle of attack, $\frac{dC_L}{d\alpha}$. This factor determines the effectiveness of a tail plane in securing static longitudinal stability. The following application of the Göttingen formulas in calculating the variation of $\frac{dC_L}{d\alpha}$ with aspect ratio should therefore be of interest to many aeronautical engineers.

Variation of $\frac{dC_L}{d\alpha}$ with Aspect Ratio.

The relation between the angles of attack at which a given lift coefficient obtains for two aerofoils of the same section but of different aspect ratio is expressed by the equation:

$$(\alpha_1 - \alpha_2) = \frac{C_L}{\pi}\left(\frac{S_1}{b_1^2} - \frac{S_2}{b_2^2}\right) \times 57.3 \quad . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

where S is the area, b the span and C_L the absolute lift coefficient defined by $L = C_L \times \frac{1}{2}\,\rho\,S\,V^2$. This formula, due to Dr. Prandtl and Dr. Munk of Göttingen University, has been checked by tests and found reliable. A verification by Dr. Prandtl may be found in "Ergebnisse der Versuchsanstalt zu Göttingen" (1921, p.51 et seq.

If the value of $\frac{dC_L}{d\alpha}$ be known for an aerofoil section at a given aspect ratio the value for any other aspect ratio may be calculated from (1) by the method illustrated in Fig. 1. For the average aerofoil $\frac{dC_L}{d\alpha}$ is substantially constant over an angular range of, say 10°, or more. Assuming $\frac{dC_L}{d\alpha}$ to be constant with the value thus defined, the angular range corresponding to an increase in the lift coefficient from zero to any value C_L is

$$\alpha_1 = C_L / \left(\frac{dC_L}{d\alpha}\right)_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

For the same section in other aspect ratio, C_L will obtain at the angle α_2 defined by equation (1)
or

$$\alpha_2 = \alpha_1 - (\alpha_1 - \alpha_2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

The value of $\frac{dC_L}{d\alpha}$ corresponding to this aspect ratio is

$\left(\frac{dC_L}{d\alpha}\right)_2 = \frac{C_L}{\alpha_2}$ as shown by Fig. 1.

Illustration of Method.

Assume that it is desired to find $\frac{dC_L}{d\alpha}$ for an aerofoil of aspect ratio 2.5 when from test data it is known that $\frac{dC_L}{d\alpha} = .072$ for the same section at aspect ratio 6. For convenience, take $\Delta C_L = 0.10$, then

$$\alpha_1 = \frac{\Delta C_L}{\left(\frac{dC_L}{d\alpha}\right)_1} = \frac{0.10}{.072} = 1.390^\circ$$

$$(\alpha_1 - \alpha_2) = \frac{C_L}{\pi}\left[\frac{S_1}{b^2_2} - \frac{S_2}{b_2}\right] \times 57.3$$

$$= \frac{0.10}{\pi}\left[\frac{1}{6} - \frac{1}{2.5}\right] \times 57.3$$

$$= -0.427^\circ$$

$$\alpha_2 = \alpha_1 - (\alpha_1 - \alpha_2)$$

$$= 1.390^\circ + 0.427^\circ$$

$$= 1.817^\circ$$

$$\therefore \left(\frac{dC_L}{d\alpha}\right)_2 = \frac{0.10}{1.817} = .055$$

Application.

For the convenience of the engineer a set of curves calculated by this method are given in Fig. 1. Also, the observed value of $\frac{dC_L}{d\alpha}$ for a few standard aerofoils are given in Table 1. It is of interest to note that the observed values of $\frac{dC_L}{d\alpha}$ for the same aerofoil at various aspect ratios follow the calculated curves closely. For application, reference is made to N.A.C.A. Report #96, "Statical Longitudinal Stability of Airplanes," in which the effect of $\frac{dC_L}{d\alpha}$ is treated.

Aerofoil Section	Aspect ratio	$\frac{dC_L}{d\alpha}$	Aerofoil Section	Aspect ratio	$\frac{dC_L}{d\alpha}$
USA - 15	6	.077	M 80	6	.074
" 16	6	.076	M-6	6	.068
" 27	6	.072	RAF-6 (M.I.T.)	5	.070
USATS - 2	6	.070	"	6	.075
" 5	6	.072	"	7	.077
Durand - 13	6	.074	"	8	.079
Sloane	6	.075	"	9	.082
RAF - 14	6	.078	RAF6a - (NPL)	6	.071
" 15	6	.070	"	13	.078
16	6	.074	Göttingen 164	2	.050
19	6	.094	"	3	.060
Albatros	6	.076	"	4	.067
Göttingen 173	6	.075	"	5	.072
" 227	5	.074	"	6	.074
" 242	5	.078			
" 255	5	.072			
" 256	5	.070			
" 322	5	.076			
" 344	6	.078			

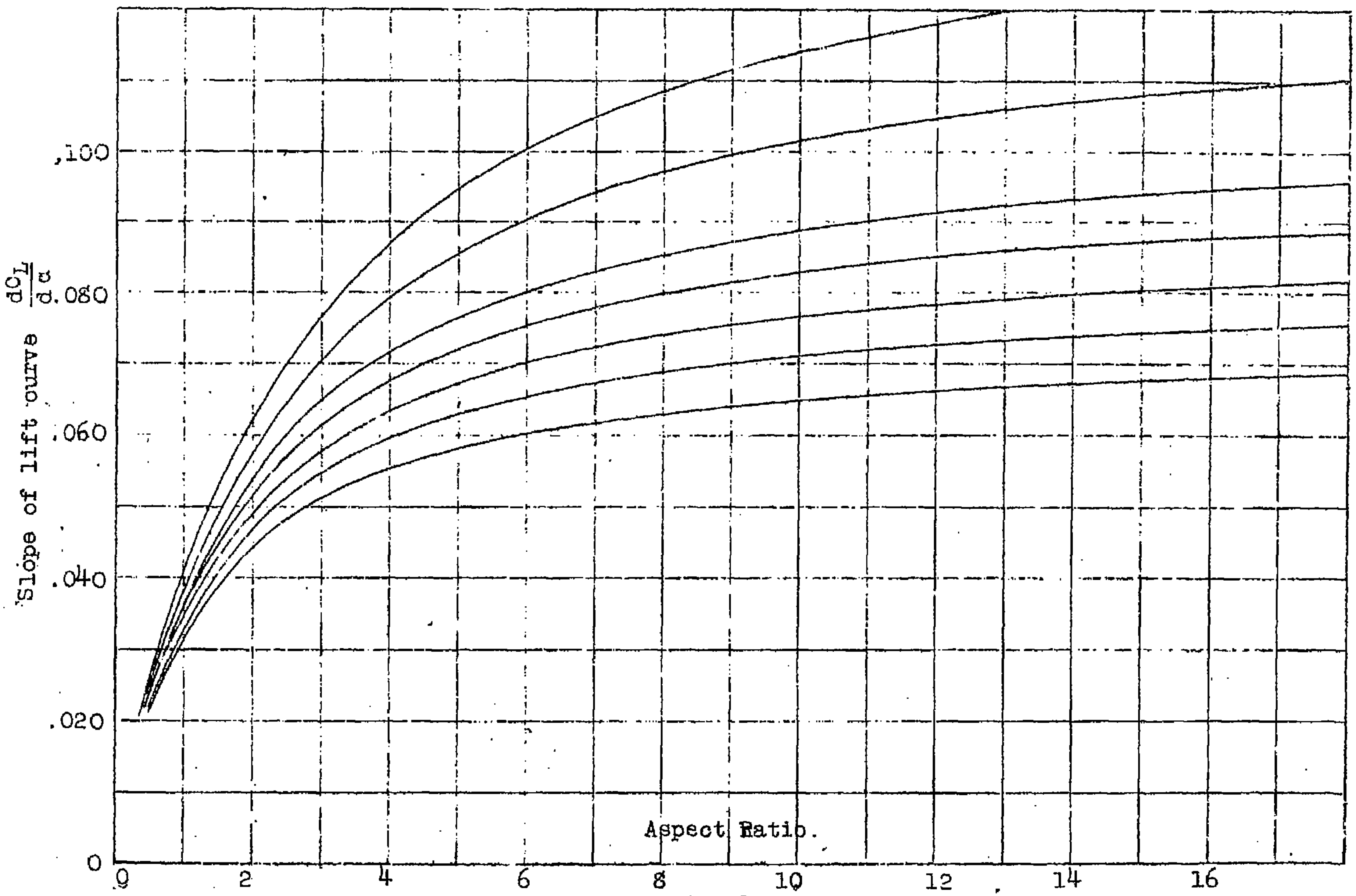
Slope of lift curve $\frac{dC_L}{d\alpha}$
.100
.080
.060
.040
.020
0
Aspect Ratio.
0
2
4
6
8
10
12
14
16

Fig. 1.