STRENGTH TESTS OF THICK-WALLED DURALUMIN CYLINDERS IN COMBINED TRANSVERSE SHEAR AND BENDING

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By Eugene E. Lundquist

SUMMARY

This report is the fourth of a series presenting the results of strength tests on thin-walled cylinders and truncated cones of circular and elliptic section; it includes the results obtained from combined shear and bending tests on 100 thin-walled duralumin cylinders of circular section with ends clamped to rigid bulkheads. The tests show that as the ratio of moment to shear varies from small to large values the failure changes from a shear to a bending type. In the report a chart is presented that shows the corresponding changes in strength.

INTRODUCTION

As part of an investigation of the strength of stressed-skin structures for aircraft, the National Advisory Committee for Aeronautics in cooperation with the Army Air Corps; the Bureau of Aeronautics, Navy Department; the National Bureau of Standards; and the Bureau of Air Commerce has made an extensive series of tests on thin-walled duralumin cylinders and truncated cones of circular and elliptic section. In these tests the absolute and relative dimensions of the specimens were varied to study the types of failure and to establish useful quantitative data in the following loading conditions: torsion, compression, bending, and combined loading.

The first three reports of this series (references 1, 2, and 3) present the results obtained in the torsion, the compression, and the pure-bending tests of cylinders of circular section. This report presents the results obtained in tests of cylinders of circular section in combined transverse shear and bending.
MATERIALS

The duralumin (Al. Co. of Am. 17ST) used in these tests was obtained from the manufacturer in sheet form with nominal thicknesses of 0.011, 0.016, and 0.022 inch. The properties of this material as determined by the National Bureau of Standards from specimens selected at random are given in references 1 and 2. Since all the test cylinders failed by elastic buckling of the walls at stresses considerably below the yield-point stress, the modulus of elasticity $E$, which was substantially constant for all sheet thicknesses, is the only important property of the material that need be considered. For all cylinders an average value of $E$ ($10.4 \times 10^6$ pounds per square inch) was used in the analysis of the results.

SPECIMENS

The test specimens were right circular cylinders of 7.5- and 15.0-inch radii with lengths ranging from 3.75 to 15.0 inches. The cylinders were constructed in the following manner. A duralumin sheet was first cut to the dimensions of the developed surface. The sheet was then wrapped about and clamped to end bulkheads. (See figs. 1 to 4, inclusive.) With the cylinder thus assembled, a butt strap 1 inch wide and of the same thickness as the sheet was fitted, drilled, and bolted in place to close the seam. In the assembly of the specimen care was taken to avoid having either a looseness of the skin (soft spots) or wrinkles in the walls when finally constructed.

The end bulkheads, to which the loads were applied, were each constructed of two steel plates one-quarter inch thick separated by a plywood core 1-1/2 inches thick for the bulkheads of 7.5-inch radius and 3-1/2 inches thick for the bulkheads of 15.0-inch radius. These parts were bolted together and turned to the specified outside diameter. Steel bands approximately one-quarter inch thick were used to clamp the duralumin sheet to the bulkheads. These bands were bored to the same diameter as the bulkheads.
APPARATUS AND METHOD

The thickness of each sheet was measured to an estimated precision of ±0.0003 inch at a large number of stations by means of a dial gage mounted in a special jig. In general, the variation in thickness throughout a given sheet was not more than 2 percent of the average thickness. The average thicknesses of the sheets were used in all calculations of radius/thickness ratio and stress.

A photograph of the loading apparatus used in the tests is shown in figure 1. Different ratios of moment to shear were obtained by placing the jack at different distances from the column. In this way it was possible to study the transition from failure by shear at small ratios to failure by bending at large ratios of moment to shear. In all cases the cylinder when mounted for tests had the seam and butt strap located on the extreme-tension fiber. Loads were applied by the jack in increments of about 1 percent of the estimated load at failure.

DISCUSSION OF RESULTS

As far as is known there is no theoretical treatment of the stability of the walls of a thin-walled cylinder in combined transverse shear and bending. Consequently, as an aid to the interpretation of the results of the tests herein considered, some of the important factors will be discussed.

From purely physical considerations it is clear that the magnitude of the shear $V$ and the moment $M$ relative to the size of the cylinder should be considered in the analysis of the test results. Consequently, $V$, $M$, and $r$ (where $r$ is the radius of the cylinder) have been combined to form a nondimensional term $\frac{M}{rV}$ that is descriptive of the loading condition. Physically, the term $\frac{M}{rV}$ is the distance from the section under investigation to the resultant shear force in terms of the radius of the cylinder. (See fig. 5.)

\[
\frac{M}{rV} = \frac{V(d - x)}{rV} = \frac{d - x}{r}
\]
If it is assumed that the ordinary beam theory applies, as was done in the analysis of the results of the pure-bending tests on thick-walled cylinders (reference 3), it follows that before buckling occurs the compressive stress on the extreme fiber and the shearing stress at the neutral axis are, respectively

\[ f_b = \frac{M}{\pi r^2 t} \]  
\[ (1) \]

and

\[ f_v = \frac{V}{\pi r t} \]  
\[ (2) \]

In these equations \( t \) is the thickness of the cylinder wall.

If equation (1) is divided by equation (2) the following relation is obtained:

\[ \frac{f_b}{f_v} = \frac{M}{rv} \]  
\[ (3) \]

Thus, a particular value of \( \frac{M}{rv} \) is descriptive of a definite stress condition as well as a definite loading condition in the same manner that torsion, compression, and pure bending are descriptive of definite stress conditions, and hence definite loading conditions. In the analysis of the results of the tests, the variation of the bending stress at failure with \( \frac{M}{rv} \) is studied for each of the following groups of cylinders tested. (For the tabulated data, see tables I and II.)

<table>
<thead>
<tr>
<th>Group</th>
<th>( r )</th>
<th>( l/r )</th>
<th>( r/t )</th>
<th>Nominal sheet thickness</th>
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<tr>
<td></td>
<td>Inches</td>
<td></td>
<td></td>
<td>Inch</td>
</tr>
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<td>1</td>
<td>7.5</td>
<td>1.0</td>
<td>323 - 366</td>
<td>0.022</td>
</tr>
<tr>
<td>2</td>
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<td>1.0</td>
<td>452 - 490</td>
<td>0.016</td>
</tr>
<tr>
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<td>0.5</td>
<td>586 - 670</td>
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</tr>
<tr>
<td>4</td>
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<td>1.0</td>
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<tr>
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<td>2.0</td>
<td>581 - 688</td>
<td>0.011</td>
</tr>
<tr>
<td>6</td>
<td>15.0</td>
<td>1.0</td>
<td>647 - 746</td>
<td>0.022</td>
</tr>
<tr>
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<td>1.0</td>
<td>932 - 980</td>
<td>0.016</td>
</tr>
<tr>
<td>8</td>
<td>15.0</td>
<td>1.0</td>
<td>1293 - 1455</td>
<td>0.011</td>
</tr>
</tbody>
</table>
From figure 2 it will be noted that failure always occurs over an area of the cylinder and not at some particular station between transverse bulkheads. It will be further noted from figure 5 that the bending stress varies linearly between bulkheads. Thus, instead of plotting the bonding stress at failure against \( \frac{M}{rV} \) as calculated at only one station, it is desirable to plot these values for all stations along the length of the cylinder. This method amounts to plotting the bending-stress diagram with \( \frac{M}{rV} \) as the abscissa scale.

On figure 6 are plotted bending-stress diagrams for each test cylinder with ordinates of stress \( f_b \) divided by the modulus of elasticity. An inspection of this figure, together with the photographs of the types of failure (figs. 2, 3, and 4), reveals a transition from a shear type of failure at small values of \( \frac{M}{rV} \) to a bending type of failure at large values of \( \frac{M}{rV} \). In the following discussion separate consideration will be given to bending failure, shear failure, and the transition from bending to shear failure.

Bending failure (large values of \( \frac{M}{rV} \)).— At large values of \( \frac{M}{rV} \), failure occurs by a sudden collapse of the outermost compression fibers in the same manner as in the pure-bending tests reported in reference 3. (See figs. 2 and 4.) It is therefore reasonable to suppose that at these values the bending strength of a thin-walled cylinder should approach the strength of a cylinder of the same dimensions in pure bending.

For comparison of the present results with the results of the pure-bending tests reported in reference 3, lines a and b have been drawn on figure 6 representing the upper and lower limits of the strength in pure bending. These limiting values represent the dispersion of the results of the pure-bending tests and were obtained for cylinders of the average radius/thickness ratio in each group by interpolation of the results plotted in figure 5 of reference 3.
Upon reference to figure 6 it will be noted that, in general, the bending-stress diagrams plot between lines a and b at large values of \( \frac{M}{rV} \). Since slight imperfections in the cylinders cause wide variations in the bending strength (reference 3), the few diagrams that plot outside the band established by lines a and b probably represent cylinders in which the imperfections were greater or less than those of the cylinders tested in pure bending.

Shear failure (small values of \( \frac{M}{rV} \)) - At small values of \( \frac{M}{rV} \) failure occurs in shear by the formation of diagonal shear wrinkles on the sides of the cylinders. (See figs. 2 and 3.) It is therefore reasonable to suppose that at these values the shear strength of a thin-walled cylinder should be closely related to the strength of a cylinder of the same dimensions in torsion (pure shear).

For comparison of the present results with the results of the torsion tests reported in reference 1, lines c and d have been drawn on figure 6 representing the probable upper and lower limits for shear failure. These lines were obtained by plotting the equation

\[
\frac{f_B}{E} = \frac{S_s}{E} \frac{M}{rV}
\]

Equation (4) is obtained from equation (3) by transposing terms, dividing by \( E \), and substituting \( S_s \) for \( f_v \), where \( S_s \) is the shearing stress at failure for a thin-walled cylinder of the same dimensions in torsion (pure shear, reference 1 or 4). Thus, the value of \( \frac{f_B}{E} \) as given by equation (4) is the critical compressive strain on the extreme fiber when failure occurs in shear, provided that the shearing stress at the neutral axis when failure occurs is the same as the shearing stress at failure for a cylinder of the same dimensions in torsion. The lines c and d for shear failure in figure 6 are shown for the two values of \( S_s \) calculated as outlined in reference 1 for the largest and smallest radius/thickness ratio for each group of cylinders.
Inspection of figure 6 shows that in some cases the bending-stress diagrams at very low values of \( \frac{M}{rV} \), corresponding to shear failure, plot above lines c and d. This fact indicates that the transverse shearing stress on the neutral axis at failure is higher than the shearing stress at failure in torsion. In order to obtain the quantitative relation existing between the two values, \( \frac{f_v}{S_s} \) is plotted against \( \frac{M}{rV} \) in figure 7 for each of the tests. It is seen that as \( \frac{M}{rV} \to 0 \) the ratio \( \frac{f_v}{S_s} \) approaches a value between 1.20 and 1.38. Thus, if \( S_v \) is the shearing stress on the neutral axis at failure in pure transverse shear and \( S_s \) is the shearing stress at failure for a cylinder of the same dimensions in torsion, \( S_v \) and \( S_s \) may be related by the following approximate equation

\[
S_v = 1.25 \ S_s
\]  

(5)

Transition from shear to bending failure (intermediate values of \( \frac{M}{rV} \).)-- It can be seen by reference to figure 6 and figures 2, 3, and 4 that the transition from shear to bending failure is not always as abrupt as the intersection of lines a and b with lines c and d might indicate. At the intermediate values of \( \frac{M}{rV} \) the transition from failure by shear to failure by bending is accompanied by a slight reduction in strength. (See groups 3, 4, and 5 of fig. 6 in particular.) The following discussion is offered as a possible explanation of the transition.

When an elastic body is subjected to one type of loading such as torsion, pure bending, compression, or any other loading, it has in general a definite resistance to that loading at which elastic failure occurs and this resistance is ordinarily different for each type of loading. If such a body should be subjected to two or more different types of loading simultaneously, it cannot offer as great a resistance to either type of loading as if that type of loading were acting alone. In such a case the following approximation may be used.
\[
\frac{f_1}{S_1} + \frac{f_2}{S_2} + \ldots + \frac{f_n}{S_n} = 1 \tag{6}
\]

where \( S_1, S_2, \ldots, S_n \) are the critical stress values for different types of loading acting alone on the body, and \( f_1, f_2, \ldots, f_n \) are the allowable stress values for those same types of loading when acting simultaneously.

Since a cylinder under combined transverse shear and bending has varying stress conditions around its periphery, the application of equation (6) is made in the following manner. The bending stress at any point \( \theta \) degrees above the neutral axis is

\[
f_b = \frac{M_r \sin \theta}{\pi r^3 t} = \frac{M}{\pi r^3 t} \sin \theta \tag{7}
\]

The longitudinal shearing stress at this same point is

\[
f_V = \frac{V_2 t r^2 \cos \theta}{2t} = \frac{V \cos \theta}{\pi r t} \tag{8}
\]

It is very probable that certain elements of the cylinder reach a critical state of stress before others and that these latter than take a greater proportional share of the load. It is assumed, however, that collapse of the cylinder occurs when all elements have reached such stress conditions that for some fiber the following equation holds

\[
\frac{f_V}{S_V} + \frac{f_b}{S_b} = 1 \tag{9}
\]

Because of the variation of stress around the periphery

\[
\frac{f_V}{S_V} + \frac{f_b}{S_b} = U = f(\theta) \tag{10}
\]

The location in the cylinder of the element \( \theta_m \) for which \( U \) is a maximum is obtained by setting the derivative equal to zero. Thus, substitution of the values for \( f_V \)
and \( f_b \) given by equations (7) and (8) in equation (10) gives

\[
U = \frac{M \sin \theta}{\pi r^2 t S_b} + \frac{V \cos \theta}{\pi r t S_V}
\]  

(11)

and the derivative is

\[
\frac{dU}{d\theta} = \frac{M \cos \theta}{\pi r^2 t S_b} - \frac{V \sin \theta}{\pi r t S_V} = 0
\]

from which

\[
\theta_m = \tan^{-1} \left( \frac{M}{rV} \frac{S_V}{S_b} \right)
\]  

(12)

Failure is assumed to occur when \( U = 1 \) for the element \( \theta_m \) degrees from the neutral axis on the compression side of the cylinder. With these substitutions, equation (11) becomes

\[
1 = \frac{M \sin \theta_m}{\pi r^2 t S_b} + \frac{V \cos \theta_m}{\pi r t S_V}
\]  

(13)

The solution of this equation for \( M \) and \( V \), remembering that

\[
\tan \theta_m = \frac{M}{rV} \frac{S_V}{S_b}
\]

gives

\[
M = \pi r^2 t S_b \sin \theta_m
\]  

(14)

\[
V = \pi r t S_V \cos \theta_m
\]  

(15)

The strength of a cylinder in pure bending and pure transverse shear is, respectively

\[
M = \pi r^2 t S_b
\]  

(16)

\[
V = \pi r t S_V
\]  

(17)

Since \( \sin \theta_m \) and \( \cos \theta_m \) can never exceed unity, equations (14) and (15) show that the presence of shear reduces the bending strength and, conversely, that the pres-
ence of bending reduces the strength in shear. Because equations (14) and (15) are related, both having been derived from equation (13), only one of them need be used to measure the strength of a cylinder in combined transverse shear and bending.

In order to show the effect of shear upon bending in the most effective manner, it is desirable to express the strength of a cylinder under combined transverse shear and bending as a percentage of the strength in pure bending. The curves of figure 8, derived from equations (14) and (16), show this relation as a function of the ratios \( \frac{M}{rV} \) and \( \frac{S_b}{S_v} \).

In figure 6 the full-curved lines were obtained from figure 8, using in one case the value of \( \frac{S_b}{S_v} \) corresponding to lines a and c, and in the other case the value of \( \frac{S_b}{S_v} \) corresponding to lines b and d. An inspection of the figures indicates that these two curves represent quite well the limits of the experimental data plotted.

In order to use the curves of figure 8 in design, it is necessary to know the loading condition \( \frac{M}{rV} \) and to be able to predict the values of \( S_b \) and \( S_v \) for the cylinder. If these three quantities are known, the maximum allowable moment and/or stress on the extreme fiber can be read from the chart as a percentage of that for pure bending. The strength in shear then need not be investigated because its effect has been taken into account by a reduced bending strength.

When checking the strength of any section between adjacent bulkheads, the largest value of \( \frac{M}{rV} \) in that section should be used to enter the chart of figure 8. This procedure tends toward conservatism and is certainly justified by the wide scattering of the test data.

Wrinkles.—In the preceding paragraphs it has been shown that the strength of a thin-walled cylinder in combined transverse shear and bending can be correlated with the strength of a cylinder of the same dimensions in tor-
sion and pure bending, depending upon whether $\frac{M}{rV}$ is small or large, respectively. It would therefore appear that the size of the shear wrinkles that form on the sides of the cylinder and the bending wrinkles that form near the extreme fiber on the compression half of the cylinder would be the same size, respectively, as the wrinkles for torsion and pure bending. Consequently, experimental values of $\kappa$, as defined by the equation

$$\kappa = \frac{2\pi r P}{\lambda_{o}}$$

where $\lambda_{o}$ is the wave length of a wrinkle in the direction of the circumference, are compared with the corresponding values of $\kappa$ for torsion and pure bending. From Table II, where the comparison is made, it will be noted that values of $\kappa$ as calculated for the shear and bending wrinkles compare very well with those tabulated for cylinders of corresponding dimensions in torsion and pure bending, respectively.

CONCLUSIONS

1. For large values of $\frac{M}{rV}$ failure occurred in bending by a sudden collapse of the compression half of the cylinder. The stress on the extreme fiber as calculated by the ordinary beam theory and the size of the wrinkles that formed were both equal to their respective values for a cylinder of the same dimensions in pure bending.

2. For small values of $\frac{M}{rV}$ failure occurred in shear by the formation of diagonal wrinkles on the side of the cylinder. The size and form of the wrinkles at failure were the same as those that occurred at failure for a cylinder of the same dimensions in torsion (pure shear). As $\frac{M}{rV}$ approached zero, the shearing stress on the neutral axis at failure as calculated by the ordinary beam theory was approximately 1.25 times the allowable shearing stress in torsion.

3. At intermediate values of $\frac{M}{rV}$ there was a transition from failure by bending to failure by shear that was
accompanied by a reduction in strength. For use in calculating the strength of thin-walled cylinders in combined transverse shear and bonding, a chart is presented that allows for this reduction in strength.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics
Langley Field, Va., March 4, 1935.

REFERENCES


TABLE I

RESULTS OF COMBINED TRANSVERSE SHEAR AND BENDING TESTS

For all cylinders, $E = 10.4 \times 10^6$ lb. per sq.in. Tabulated values of $f_b$, $f_b/E$, and $M/rV$, taken at bulkhead supported on column. (See fig. 1.)

Group 1

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>$t$</th>
<th>$r/t$</th>
<th>$V$</th>
<th>$M$</th>
<th>$f_V$</th>
<th>$f_b$</th>
<th>$M/rV$</th>
<th>$f_b/E$</th>
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<th>$f_V$</th>
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### N.A.C.A. Technical Note No. 523

**Group 4**

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<td>0.000198</td>
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<td>1415</td>
<td>515</td>
<td>14500</td>
<td>672</td>
<td>1940</td>
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<td>0.000187</td>
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<tr>
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<td>1364</td>
<td>305</td>
<td>15100</td>
<td>588</td>
<td>2172</td>
<td>4.10</td>
<td>0.000209</td>
<td>Failure</td>
</tr>
<tr>
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<td>1815</td>
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<tr>
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<td>0.0105</td>
<td>1428</td>
<td>105</td>
<td>12330</td>
<td>212</td>
<td>1660</td>
<td>7.83</td>
<td>0.000160</td>
<td>Failure</td>
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</table>
TABLE II

COMPARISON OF EXPERIMENTAL VALUES OF $k$ FOR BENDING, TORSION,
AND COMBINED TRANSVERSE SHEAR AND BENDING TESTS

(Values of $k$ for torsion tests obtained from fig. 7 of reference 1.
Values of $k$ for pure bending tests obtained from table I of reference 3.)

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>$l/r$</th>
<th>$t = 0.011$ (in.)</th>
<th>$t = 0.016$ (in.)</th>
<th>$t = 0.022$ (in.)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Torsion (pure shear)</td>
<td>Pure bending</td>
<td>Transverse shear and bending</td>
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<tr>
<td>7.5</td>
<td>0.5</td>
<td>21</td>
<td>15-16</td>
<td>21</td>
</tr>
<tr>
<td>7.5</td>
<td>1.0</td>
<td>16-17</td>
<td>10-13</td>
<td>16-17</td>
</tr>
<tr>
<td>7.5</td>
<td>2.0</td>
<td>11-12</td>
<td>9-11</td>
<td>12-13</td>
</tr>
<tr>
<td>15.0</td>
<td>1.0</td>
<td>18</td>
<td>11-13</td>
<td>17-19</td>
</tr>
</tbody>
</table>
Figure 1.— Loading apparatus used in combined transverse shear and bending tests.
Fig. 2

$\frac{M}{IV} = -0.53$ to $1.47$

$\frac{M}{IV} = 0.88$ to $2.88$

$\frac{M}{IV} = 1.47$ to $3.47$

$\frac{M}{IV} = 2.39$ to $4.39$

$\frac{M}{IV} = 4.64$ to $8.64$

$\frac{M}{IV} = 6.85$ to $8.85$

Figure 2. - Side view of circular cylinders after failure in combined transverse shear and bending tests, cylinders of group 5
NACA Technical Note No. 523

Fig. 3

$\frac{l}{r} = 0.87$

$\frac{l}{r} = 2.0$

Fig. 3

Cylinders after failure in torsion (fig. 4, reference 1).

$\frac{l}{r} = 3.0$
Figure 4.—Cylinders after failure in pure bending (fig. 2, reference 3).
Figure 5.—Shear, moment, and bending-stress diagrams for a cylinder in combined transverse shear and bending.
Figure 6a.- Bending-stress diagrams for circular cylinders in combined transverse shear and bending.

Figure 6b.- Bending-stress diagrams for circular cylinders in combined transverse shear and bending.
Figure 6c.- Bending-stress diagrams for circular cylinders in combined transverse shear and bending.

Figure 6d.- Bending-stress diagrams for circular cylinders in combined transverse shear and bending.

Figure 6e.- Bending-stress diagrams for circular cylinders in combined transverse shear and bending.
Figures 6f, 6g, 6h, 8-Bending-stress diagrams for circular cylinders in combined transverse shear and bending.

Figure 8.-Chart for bending strength of thin-walled cylinders subjected to combined transverse shear and bending.