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BENDING TESTS OF METAL MONOCOQUE FUSELAGE CONSTRUCTION

By Ralph W. Mossman and Russell G. Robinson

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I. Summary

This thesis presents a study of the bending stress in smooth skin, aluminum alloy, true monocoque fuselage sections of varying ratio of diameter to thickness. The test specimens - circular, thin-walled cylinders - were loaded to give a large bending stress in proportion to the shearing stress, in order to represent the critical section of a fuselage.

The maximum unit bending stress developed in the true monocoque fuselage sections varied with the ratio of diameter to thickness. The results of the tests indicate that 90 per cent of a theoretically derived value for thin-walled circular tubes will be obtained in practice. As a design rule the equation is suggested:

$$F_b = \frac{6000000}{D/t}$$

This type of construction has a relatively high efficiency - 60 to 70 per cent of the breaking load - after the first failure.

The present design rules given in the Department of Commerce Aeronautics Bulletin No. 7-A for the vertical bulkheads

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of a monocoque or semimonocoque fuselage are excessively conservative.

II. Introduction

Monocoque or shell-type fuselages are such fuselages as rely on the strength of the skin to carry either the shear or the loads due to bending moment. They may be divided into three classes - monocoque, semimonocoque, and reinforced shell - and different parts of the same fuselage may be in any of the three classes.

In the true monocoque the only reinforcing members, if any, are vertical bulkheads formed of structural members. The semimonocoque has the skin reinforced by longerons and vertical bulkheads, but has no diagonal web members. The reinforced shell has the skin reinforced by a complete framework of structural members.

The usual type of monocoque and semimonocoque fuselage uses corrugated skin, with the corrugations parallel to the thrust axis, or smooth skin. For the same thickness of sheet the corrugated skin is stronger than the smooth skin, but it is also heavier and the aerodynamic efficiency is not as high. There are also structural difficulties with the corrugated skin that are not present in the smooth skin types.

Smooth skin, true monocoque, fuselage sections were used in this investigation. The curved sheet is the element common to all types of monocoque construction - plain circular or elliptical sections, corrugated circular or elliptical sections, rectangular sections with rounded-off corners, or any of these sections combined with longitudinal stiffeners. Therefore, it was decided to study the thin curved sheet only, as nearly as possible, eliminating stiffening of every kind with the exception of a bulkhead ring. It is realized that this is only the first step toward the complete solution of the more complex sections. The section tested is the most logical first step in that it is the simplest and has only one variable, the D/t ratio. If the relation between the maximum stress and D/t can be determined, the results will be useful in analyzing all sections containing curves.

The cylinders for this work were constructed and donated by The Douglas Aircraft company of Santa Monica. The Engineering Department of the Douglas Company suggested the problem, and their advice, especially that of Mr. A. E. Raymond, was very helpful in all stages of the project.

The testing was done in the Daniel Guggenheim Aeronautic Laboratory at Leland Stanford Junior University under the direction of Professors E. P. Lesley and A. S. Niles.

The purpose of this investigation was to determine:

1. The maximum allowable stress in the true monocoque fuselage sections;

2. How the maximum stress varied with the ratio of diameter to thickness;

3. The type of failure of the section;

4. The percentage of load that the section would carry after first failure; and

5. The action and strength required of the bulkhead ring.

The critical section in a monocoque fuselage is usually the section just to the rear of the wing supports as the total bending moment is normally greatest there. The ratio of bending moment to shear for this section, in the average airplane, is about 120 to 1, comparing inch-pounds to pounds. The important variable in construction is the ratio of diameter to thickness. In a particular design the diameter of the fuselage is fixed within small limits to meet the load carrying requirements and to obtain aerodynamic efficiency. The diameter to thickness ratio and bending strength of the section may vary greatly, depending on the thickness of skin used. The typical bulkhead section used may be open or closed.

The cylinders used in the tests represent the critical section of a fuselage and the loading gives the ratio of bending moment to shear found in a typical airplane. The sheet thickness was varied to give ratios of diameter to thickness which would cover values likely to be used in average designs. An open bulkhead section was used. The cylinders tested give results which can be directly applied in the design of a monocoque fuselage of similar construction.

III. Theory

St. Venant's solution of the bending problem is:

$$M = f_b \times I/y$$

The moment M is a maximum, for a given I/y, when the bending stress f_b, is taken as the maximum tensile stress of the material. One assumption made by St. Venant and used in ordinary design work is that the maximum allowable bending stress or modulus of rupture remains a constant of the order of magnitude of the maximum tensile stress for variations in cross-sectional form. Considering a circular tube whose cross-sectional area is held constant, St. Venant's theory would state that the maximum allowable bending moment varies directly with the I/y of the section, as the latter increases indefinitely. Therefore, the equation:

$$\text{maximum moment} = \text{modulus of rupture} \times I/y,$$

should apply to a monocoque fuselage section in which the I/y is very large due to the disposition of a thin shell of material at a large radius.

This is not the case because "thin" sections are involved. Thin sections fail by elastic instability; therefore their strength depends on the modulus of elasticity and not on the ultimate tensile strength of the material, and the unit stress at failure may vary within wide limits. One of the most common types of sheet failure, due to the lower allowable stresses of thin material, is the buckled or lobed form in which the beam section does not fail as a whole, but crinkles locally. This violates another assumption of St. Venant because he specified that the beam section should not change materially. He considers the stress in any element proportional to its initial distance from the neutral axis, taking no account of the fact that deformations occur which will change this distance and, hence, the stress.

St. Venant's solution is arrived at by neglecting terms of a higher order than the first, such as those which would make the stress proportional to the actual distance of an element from the neutral axis instead of to its original distance before it was strained. This explains why his linear expression does not fit a body in which the cross section can be deformed with relative ease. Mr. R. V. Southwell (Reference 1) shows that in problems involving stability, second order effects must be considered and that is how the results developed by Mr. L. G. Brazier (Reference 2) and used in this work are obtained.

Mr. Brazier considers a thin cylinder of circular cross section in pure bending, assuming general equations for the shape the cross section will assume under bending. He determines this shape by the condition that the potential energy of the stressed tube will be a minimum, and computes the moment the tube will carry. The end conditions are eliminated by considering the section of the tube being investigated to be a portion of a closed toroid. The imposed bending stresses are due and proportional to the curvature c, of the toroid. Instead of paralleling the work of St. Venant by writing the equations of equilibrium, leaving in terms of a higher order, he sets down the displacements v_o, tangentially, and w_o radially, which a section of the tube would suffer according to St. Venant's linear solution. With the tube in this condition he considers a further system of displacements v' and w', their only limitation being that they do not cause the circumference of the tube to lengthen. This is reasonable because, according to Bassett (Reference 3), the energy absorbed by any extensional displacement (compared with a flexural displacement) is mathematically large and therefore precluded. Adding these two systems of displacements and thus having expressions for the distance of any element from the neutral axis in terms of v and w and the longitudinal strain it undergoes, longitudinal strain being proportional to this distance, an integral expression for the total strain energy per unit length of the tube is set up. The dependent variable of this integral, since the displacement w may be expressed in terms of v, is the total tangential displacement v, of any element an angular distance θ from a fixed point on the cross section of the tube. By the calculus of variations this expression for energy is made a minimum and v evaluated. The result is in terms of the undetermined curvature c, of the center line of the tube. The differential coefficients are evaluated by considerations of symmetry and of continuity of the circumference. The displacements found by minimizing the strain energy expression should be the resultant total displacements because they will provide equilibrium, when the curvature c, is evaluated, with the potential or strain energy of the body a minimum. By substituting these displacements in the strain energy integral and performing the integration, the total energy is obtained and if the resulting expression is differentiated with respect to the curvature c, the result is the moment transmitted by the tube in terms of c.* The relation is not the straight line one, $M = E\,I\,c$, of St. Venant's solution. Instead, it is

$$M = \frac{E}{2}\pi r^3 t \left(2c - \frac{3r^4 c^3 (1 - \sigma^2)}{t^2}\right)$$

This equation defines a curve which everywhere gives lower values of M than the St. Venant straight line and whose positive slope decreases to zero and becomes negative as c is increased. Where the slope is zero, the moment developed is a maximum. c is evaluated for this point, substituted back in the moment equation, and the following result obtained for maximum moment:

$$M = \frac{2\sqrt{2}}{9}\,\frac{E\,\pi\,r\,t^2}{\sqrt{1 - \sigma^2}} = 0.99\,\frac{E\,r\,t^2}{\sqrt{1 - \sigma^2}} \qquad (1)$$

where r is the radius of the circular cross section, t is

* See Appendix III.

the wall-sheet thickness, σ is Poisson's ratio, and E the modulus of elasticity for the material. For this moment the displacements are such as to give a quasi-oval shape to the section with a maximum radial displacement of the tension and compression sides of

$$w = \frac{2}{9} r \tag{2}$$

Physically this flattening is accounted for by the components directed toward the center of the tube of both the tension on the outer side of the tube and the compression on the inner side of the tube (outer referring to the side away from the center of curvature of the center line of the tube, inner referring to the side nearest the center of curvature). The component due to the tension or compression at any point is found to be

$$p = E c^2 r t \cos \theta \tag{3}$$

and is checked by the fact that if this normal pressure is applied to the outside of an infinitely long cylindrical shell the distortion of a section is the same as given by the second system of displacements considered for the tube in bending.

Mr. Brazier then attacks the problem by a different method because the allowable bending moment, for the tube which takes the oval sectional form before failure, is derived considering only terms as high as the second order. In order to be sure that terms of a higher order do not reduce the allowable moment further, Southwell's (Reference 1) general theory for shells under combined end thrust and normal pressure is applied, for he arrives at the possibility of a lobed form of distortion. Manifestly, sections of the compression half of a tube in bending can be considered as sections of a thin-walled cylinder with compressive stress varying according to the distance of the section from the neutral axis in the tube resisting bending. The external pressure is evidently the pressure, p, of equation (3) which is the component of the tension or compression caused by curvature of the axis. How to follow the axial loads around the cross section and combine them with the effect of the normal pressure to find the stability conditions and their effect on one another is a difficult problem, but it is conservative to combine p with the axial load at the most stressed point of the compression side which will just satisfy the Southwell relation for collapse into a lobed form. This is done and an equation for the beam curvature, c, found which contains a variable, k, whose positive integral value corresponds to the number of lobes in the deformed section. For the case in which $k = 0$, the cross section remains circular but the diameter varies sinusoidally along the axis. The case $k = 1$ is the pure strut failure described by Euler in which the center line is displaced. Values of k greater than 1 represent cases of local buckling. The equation for c is transferred into one of maximum moment with the following result:

$$M = \eta \frac{E\, r\, t^2}{\sqrt{1 - \sigma^2}} \qquad (4)$$

where the values of η are:

k	η
2	0.643
3	0.788
4	0.843
5	0.872
6	0.893

These values of η are rationally derived.

It will be noted that the form of equations (1) and (4) is the same. According to Mr. Brazier it may reasonably be inferred that the form is adequate and that it will be necessary to call upon experimental results to show whether collapse into a lobed form does intervene before a collapse of the type (1) originally discussed. Since the coefficient is smaller for (4) than for (1), it would seem reasonable that a multilobed failure would occur before that type described by (1). That is to say, the tube must collapse in the manner described as the oval sectional form when condition (1) is reached, but a lobed deformation may occur earlier. The earliest point at which a lobed deformation can occur is obtained by replacing the constant in (1) by the value of η for the number of lobes considered. Further, the results of the experimental work (done by Mr. Brazier) indicate that actually the tube collapses into the three- or four-lobed form but not until the tube reaches the region given by (1).

Tests were made of celluloid cylinders very accurately made to a mean radius of 0.361 inch and wall thickness of 0.0043 inch ($D/t = 168$) which checked equation (1) to a maximum variation of ±1.6 per cent. Their length was 16.7 inches.

Application to aluminum alloy.- Equation (1) can be put into a more easily used form for design purposes by expressing the maximum allowable bending stress in terms of t and r. Remembering that the derivation is for a hollow circular cylindrical section,

$$M = 0.99 \frac{E\ r\ t^2}{\sqrt{1 - \sigma^2}} = F_b \frac{I}{y} = F_b \frac{A}{2} \frac{r^2}{r} = \frac{2\pi\ r\ r^2\ t}{2\ r} = \pi\ r^2\ t\ F_b$$

$$F_b = \frac{0.99\ E\ r\ t^2}{\pi\ r^2\ t\sqrt{1 - \sigma^2}} = \frac{0.99\ E}{\pi\sqrt{1 - \sigma^2}} \frac{t}{r} \qquad (5)$$

Assuming $E = 10{,}000{,}000$* lb. per sq.in., and $\sigma = 0.3$,

$$F_b = \frac{9900000}{\pi\sqrt{1 - 0.09}} \frac{t}{r} = \frac{9900000}{0.954\pi} \frac{t}{r}$$

$$F_b = \frac{3300000}{r/t} = \frac{6600000}{D/t} \qquad (6)$$

Thin-walled cylinders in compression.- Although the bending theory of Reports and Memoranda No. 1081 applies most directly to the problem at hand and though the theory of combined thrust and normal load as given by Southwell has been applied, there are a few other observations from the compression theory which may be of assistance in interpreting experimental results.

*For aluminum alloy sheet (Reference 4).

The critical section will undoubtedly be at the compression section farthest from the neutral axis where the stress is $\frac{M\ y_{max}}{I}$. If this value for the compressive stress on a cylinder is used, at least an inferior limit for the allowable moment will be arrived at because the skin on either side of the critical section is stressed more lightly, would be able to assume any slight additional load which might be shifted to it by the critical section, and thus delay the instability of this central section.

Mr. J. Prescott (Reference 5, p. 542), for the case of a tube under end thrust deforming in the form given by $k = 0$, discusses the effect of end conditions. He states that the effect of the restraint conditions at the end falls to less than 1 per cent at a distance of $7\sqrt{t\ r}$ from the end, and that a column of twice this length could reasonably be regarded as a long tube in this case. Although his investigation does not consider failures of the type described for $k = 2$, it would seem logical that end effects would be of about the same magnitude for the different types of deformation with the possible exception of the Euler strut type of deformation for $k = 1$.

Further (Reference 5, pp. 556-558), Mr. Prescott arrives at an expression for the allowable stress in pure end thrust on thin-walled circular cylinders without any loads normal to the surface. From his absolutely general case, he arrives at the expression:

$$P = E' \frac{t}{r} \frac{m^2 - 1}{m^2 + 1} \sqrt{\frac{1}{3}(1 - \sigma^2)} \qquad (7)$$

in which m has the same meaning and value as the previously discussed k in describing the number of lobes in the deformation. For $m = 2$, which is the oval cross section and therefore the type of failure which takes place at the lowest load according to equation (7),

$$P = \frac{3}{5} E' \frac{t}{r} \sqrt{\frac{1}{3}(1 - \sigma^2)} \qquad (8)$$

Evaluating this for aluminum alloy by using the same assumptions for physical properties as before,

$$P = \frac{3}{5} \times 5,500,000 \frac{t}{r} = 3,300,000 \frac{t}{r} \qquad (9)$$

which exactly agrees with equation (6).

Mr. E. A. H. Love (Reference 6, p. 208), in discussing the effect of end conditions, says that the critical length of a tube (length for which experiments will check theory) is some product of $\sqrt{t\ r}$ and that at a distance from the end, great compared with $\sqrt{t\ r}$, the effect of the end conditions is negligible.

Mr. Southwell says of the lobed deformation "The length of the tube ... is not a matter of great importance in the present problem because the wave length (of the lobe) corresponding to a minimum value of the collapsing pressure is in all cases small, and the strength of any strut of ordinary dimensions will therefore be given by equations (90) or (105)...."

Equation (105) from Southwell is

$$P = E\frac{t}{r}\frac{k^2 - 1}{k^2 + 1}\sqrt{\frac{1}{3}\frac{1}{1 - \sigma^2}} \qquad (10)$$

which corresponds to equation (7) except for the position of the $(1 - \sigma^2)$ term. For aluminum alloy, using $E = 10,000,000$ lb. per sq.in., $\sigma = 0.3$ and $k = 2$ (giving minimum allowable unit end thrust), (10) becomes

$$P = \frac{3}{5} \times 6,050,000\,\frac{t}{r} = 3,630,000\,\frac{t}{r} \qquad (11)$$

which is 10 per cent greater than the value shown by equations (6) and (9).

So little is known about the way in which the stresses due to bending can be compared to those of pure compression, for this type of structure, that it is difficult to judge which of the equations already presented will fit the case of a pure monocoque tube in bending. There is also the added difficulty of predicting what type of failure will occur. Therefore, as suggested by Mr. Brazier, it would be wise to let experiment show which one is most nearly correct, in the hope that it will be an equation which also appears theoretically justified.

Mr. Brazier's tests provided a good experimental check for equation (1) but when his diameter to thickness ratio (168) is compared to that desirable in an airplane fuselage and tested in this work (about 2,000), it is seen that aeronautical engineers are interested in a value as far beyond Mr. Brazier's as his is beyond the value for the ordinary structural sizes. For these reasons, experimental checks would seem to be of vital importance.

IV. Test Cylinders

The test cylinders used were constructed and furnished by the courtesy of The Douglas Aircraft Company of Santa Monica. Figures 1 to 6 inclusive show the construction details and general appearance of the tubes.

The following gives the data on the tubes:

Test	Length	Diameter	Skin	D/t ratio
1	36	36	0.014	2571
2	36	36	0.022	1636
3	36	36	0.032	1125

All the material of the tubes was aluminum alloy, heat-treated. The variation in the ratio of diameter to thickness was obtained by changing the thickness of the skin while keeping the diameter the same. The properties held constant were the length of the tubes, the bulkhead section, the spacing of the bulkhead, and the end flange angles. The cross sections of the bulkhead and flange angles are shown in Figures 5 and 6. The "hat" section bulkhead ring was used as an open section in the test cylinders. It was realized that an open section was less efficient than a closed section because of the free edges, but its use was advised by the Douglas company to facilitate inspection and painting in service.

It was desired to test the bulkhead section separately to determine the per cent of shear carried by the bulkhead and by the skin. Since the bulkhead ring was used as an open section in the test cylinder, the least change in moment of inertia and form factor occurred.

No longitudinal stiffeners were used. The one-piece sheet had the lap joint placed on the top of the tension side so that it would not affect the bulging of the skin.

Figure 1 shows the actual construction details of tubes 2 and 3. Tube 1 differed, in addition to thickness, only in rivet spacing, rivet size, and the number of rivets used. Since these latter variations do not affect the strength of the skin in bending, all three tubes may be considered similar. In tube 1 the 1/8 inch rivets of the double row lap joint had a pitch of 3/8 inch with the rows 3/4 inch apart. The flange angles were attached with a staggered double row of 1/8 inch rivets. The pitch of the joint was 3/4 inch and the rows were 5/16 inch apart. The bulkhead ring was riveted to the skin with a single row of 1/8 inch rivets spaced 3/4 inch apart.

V. Method of Test

Figures 7 to 11 inclusive, show the set-up of the test. The backboard was made of 2 × 12 and 4 × 4 clear surfaced pine, bolted and nailed together. The front end rests on, and the two rear legs are bolted through the floor. The backboard had some deformation under the loads of the test. In testing No. 3 tube, the backboard was pulled out in the center some 5/16 inch at maximum load, but the bottom board was pushed in the same amount. It was considered that a section, near the backboard, through the tube perpendicular to the center line remained in the same plane that it was in before loading, and that the plane rotated about a horizontal line through the neutral axis of the tube.

Surfaced lumber was used and there was a good smooth face to which the aluminum alloy tube was attached. The 1/4 inch bolts attaching the flange of the tube to the backboard were spaced an inch apart on the upper 90 degrees of the flange, and two inches apart on the remainder of the flange. The flange on the outer end of the aluminum alloy tube was bolted to the flange of the galvanized iron tube. Quarter-inch machine screws were used for this joint, the spacing being the same as was used on the flange bolted to the backboard.

With the tripod and galvanized iron tube in place, the test cylinder was subjected to a shear load of 97.5 pounds and a bending moment, at the backboard, of 6260 inch-pounds. The skin of the tube near the backboard was tested by pressing on it by hand. If any bulges were found at this loading, or if any could be formed by very slight external pressure, the bolts through the backboard were tightened or loosened until the skin appeared to be of uniform strength.

The 0.049 inch galvanized iron tube, two feet long, was rolled to a 36 inch diameter and had a 3/4 inch flange turned up on one end. The 1-1/4 × 1-1/4 × 1/4 steel angle compression legs of the tripod and the 1 × 1/4 steel bar tension tie were bolted to the galvanized iron tube. At the points of attachment of the tripod legs, cross braces of 1 × 1/4 flat steel were used to take the radial loads from the legs. The two braces in compression were strengthened with wood strips to prevent bending. In order to keep the galvanized iron tube truly circular, a bulkhead of 1 × 12 pine was cut and attached to the inside of this tube near the outer end. The tripod was used so that the load would be applied 120 inches from the backboard.

The concentrated loads from the legs of the tripod are considered distributed to a planar uniform load where the galvanized iron tube is attached to the aluminum alloy tube. This assumption seems justified because no bulges occurred in the first 18 inch section of the aluminum alloy tube. Since no bulges appeared in the first 18 inch section, this section would also help to distribute the load to the critical section which is the one between the bulkhead ring and the backboard.

Since it was necessary to measure only the change in length of the vertical axis of the bulkhead ring, the micrometer arrangement shown in Figures 10 and 11 was made up. The 1/4 inch rod A, goes through a 9/32 inch hole in the lap joint on the top of the tube, one inch back of the center of the bulkhead. It is attached to the center of the bulkhead, on the bottom, with a small machine screw. The steel micrometer screw C, running in a bakelite nut B, can be adjusted so that there is no play or backlash. The nut is slipped down over the end of the 1/4 inch rod and attached firmly. The scale D, is graduated in 0.025 inch. The head of the micrometer screw is graduated to read 0.001 inch. An electric buzzer was used with the micrometer screw to indicate when the end of the screw came in contact with the tube. One lead from a dry battery was attached to the micrometer screw. The other lead, containing the buzzer, was attached to the aluminum alloy tube.

The loading platform hung on the tripod 120 inches from the backboard and was loaded with bags of shot. The bags of shot were in twenty-five, ten, and five pound sizes. The increment of load was added and a deflection measurement taken after a minute's time. At failure, the platform was allowed to fall about two inches. The load was removed slowly until the platform raised clear of its supports; then more load was added to determine the load that the tube would carry after the first failure.

VI. Results of Test

The three data sheets, found in Appendix I, give complete results of the tests. In brief, they were as follows:

Test	1	2	3
F_b maximum by equation (6)	2567	4034	5867
f_b " at backboard, by test	2150	3890	5570
% of f_b test to F_b of (6)	84	97	95
% of second failure to first failure	67	72	61

The type of failure is shown by Figures 12 to 17 inclusive. In test 1 the first bulges occurred with a shear load of 186.5 pounds. There was one bulge on the bottom, 1/4 inch deep. As the load increased, more bulges formed on either side of the bottom as are shown by the figures. The depth of the bulges just before failure were: bottom bulge 3/4 inch, main left and right side bulges 23/32 inch deep. There were other smaller bulges symmetrically placed. In test 1 there was progressive failure due to the bulges formed at small loads and increasing in size until the tube failed, though there was one rather sudden increase of bulge depth from 1/4 inch to 3/4 inch when the shear became 246.5 pounds.

Tube 2 had only one small bulge in it during the test. This was observed from the start of the test and was probably due to a high spot in the flange angle. This bulge was on the right side of the tube, about 25 degrees down from the horizontal. Just previous to failure this bulge was 5/16 inch deep. No other bulges occurred during the test until failure when all the bulges occurred as the tube failed.

In test 3 no bulges occurred until the failure of the tube.

A negative bulkhead deflection means that the vertical axis of the bulkhead ring shortened; a positive reading indicates that the vertical axis elongated. Just before failure in test 1 the deflection was positive 0.189 inches, in test 2 there was no deflection, and in test 3 the deflection was negative 0.0105 inches. In all cases after failure the bulkhead ring showed considerable positive deflection.

VII. Discussion

The test results would show that the theory of R.& M No. 1081 resulting in equations (1), (5), and (6) can be reasonably applied to pure monocoque construction such as the tests simulated. It is a little difficult, however, to justify the application of these equations because of the small distance between bulkheads, the case $k = 2$ requiring, with the exception of $k = 1$, the longest cylinder for its corresponding lobe formation. On the other hand, applying the statements of Love and Prescott regarding end conditions, it is found that their effect is negligible at distances of three and six inches for tests 1 and 3, respectively, so that, though the bulkhead spacing is small compared to the length of tube used by Brazier in his tests, there is a section of the test cylinder in every case which may act as equations (1) or (4) predict. Figure 12 shows that the lobes nearest the backboard, in every case, are distant from the end by approximately these distances.

The test results show equation (4) to be conservative, as admitted by Mr. Brazier, because of the assumptions in its derivation. Equation (1) would seem to be the one to apply to construction similar to the type investigated because it was derived for bending and, if the effect of end conditions may be neglected, the diameter to thickness ratios of 2571 to 1125 used in this investigation imply a closer approach to the "thin tube" than the ratio of 168 for Mr. Brazier's experiments. Further, for bending, equation (1) gives the smallest moment which will cause failure - that is, for a lobed form when $k = 2$ - and this smallest moment, were any lobed form possible, is checked by the test results. The results show that, except in test 1, a lobed (more than 2) form does not intervene before complete collapse. The presence of the bulkhead would tend to prevent this formation of lobes. Further, these tubes did collapse into at least four lobes but not until the tubes had reached the region given by (1). In one of Mr. Brazier's tests the two-lobed form lasted some two seconds before collapse into the multilobed form but in none of the present tests could the two-lobed form be observed.

That test cylinder No. 1 would not reach the theoretical limit was suspected from the start because:

1. It was imperfect in manufacture, having a flare of from 1/16 to 1/8 inch where the sheet joined the flange.

2. Because of its thinness the sheet was more probably dam-

aged to some extent in handling; and

3. In bolting the cylinder to the backboard any initial strains would have a greater proportional effect and this was observed because it took much more adjusting to eliminate incipient bulges.

Test cylinders 2 and 3 were perfect as nearly as could be observed. Considering cylinder 1 as an exceptionally bad case it would seem that equation (6) would have to be reduced only slightly to take care of manufacturing irregularities in order to use it as a design formula. It is possible that this reduction should vary according to the absolute thickness of the sheet or with the D/t ratio but further tests will be required to establish this assumption.

A design rule for the bulkhead appears less easy to determine and certainly is not possible from considerations of the data of this investigation alone. The results prove, principally, that the design rules for bulkheads as given on page 60 of Bulletin 7-A, Department of Commerce, Aeronautics Branch, are excessively conservative. The bulkhead ring tested separately as a column supported only 150 pounds. Using the rules of Bulletin 7-A and making the least conservative assumptions (column length = $\frac{D}{2}$ = 18 inches), it is found that the bulkhead ring could be used only in a monocoque fuselage where the total shear was no more than 696 pounds.* The shears sustained in the three tests were 306 lb., 781 lb., and 1566 lb., or 44 per cent, 112

*See Appendix II.

per cent, and 225 per cent of the load which the present design rules would permit. If the total deflection of the bulkhead section just before failure is used to get a resultant compressive load on the bulkhead from the load-deflection curve* of the ring, this load is found to be 8 lb., zero pounds, and less than one pound, respectively. The upward (positive) deformation of the top of the bulkhead in test 1 was undoubtedly due to the progressive formation of bulges with loading because, in every case, a positive deformation accompanied complete collapse of the tube into bulges. The change in sign of the deformations throughout the tests can only be accounted for by the different kinds of diagonal tension fields set up in each test. The design of bulkheads would, therefore, seem to depend less directly on the shear at the section and more on the sheet thickness or D/t ratio, and on the bulkhead spacing.

The strength of the test cylinders after a first failure was consistent and relatively good. It is to be understood that the first failure was complete except that the load applied was not allowed to continue to act for a greater distance than about two inches. The average strength after failure was two-thirds that before failure. This result serves no purpose for designing but shows that this type of structure still has considerable value after the failing load has been applied for a short period. A similar condition might come about through a violent maneuver, in which case, if deformations were not too great, the plane

*See Appendix II.

might be able to fly to a landing field.

That the results presented herewith will be of some use in work with corrugated skin is probable. Two possible ways of using them are:

1. Knowing the moment of inertia of a corrugated sheet, the solid sheet of thickness t, which would have the same moment of inertia per unit length can be determined and, by applying a factor perhaps, the allowable unit stress to be applied to the corrugated skin can be found from (6) by using the ratio of fuselage diameter to effective thickness t, or

2. To find the allowable unit stress by using the ratio of radius of curvature of one corrugation to the actual sheet thickness.

Considerations of the upper and lower limits of radius of curvature of one corrugation suggest that the first method would be the better, especially when it is realized that the ratio D/t has to be reduced only to 118 to show a unit stress of 55,000 lb. per sq.in., according to (6). No results of bending tests on corrugated skin were available for comparison and for definite information that will be necessary.

VIII. Conclusion

Although no radial deformation approaching the values of $\frac{2}{9}$ r could be observed, as required by the theory, the theoretical results appear to be applicable. For a pure monocoque section in which the bulkhead spacing is of the order of one-half the diameter, the following design value for maximum allowable bending stress is suggested:

$$F_b = \frac{6000000}{D/t} .$$

This type of construction, though the allowable unit stresses are low for otherwise desirable sections, shows a rather consistently high strength after a failure which has not been allowed to progress very far. An average value of two-thirds the maximum strength was shown.

The present design rules for bulkhead design are ultraconservative and would bear revising. This investigation shows only that this is a fact and does not furnish enough material to show how the design should depend on shear at the section, sheet thickness, diameter to thickness ratio, etc. As an appreciable saving of weight would result in all types of monocoque construction by being able to design bulkhead rings to fit the conditions imposed rather than by the present design rules, it would seem that a systematic investigation of this problem would be well worth while.

Because of the proportionally lower maximum bending stress, as compared to the maximum tensile stress, for pure monocoques of D/t around 2,000, it would seem that the strength-weight ratio could be raised considerably by the use of even light longitudinal stiffeners to delay the formation of bulges and to decrease their size. This method of stiffening would appear to be more efficient than reducing bulkhead spacing. It is well known that the lateral pressure required to keep a long column (comparable to the compression side of the thin sheet) from buckling laterally is slight compared to the end thrust. A few stiffeners, judiciously placed near the most highly stressed element, would undoubtedly increase the efficiency of the section and for this reason would bear further study.

With the same sheet thickness and diameter of section, a corrugated covering, having of necessity a slightly increased weight, would show a greater efficiency than the smooth skin for large D/t ratios. It seems to be an ideal way to incorporate stiffening for the portion having high compressive stresses due to bending. Perhaps a combination of corrugations where compressive stresses are high and smooth skin where they are low, or where tension or shear only occur, would prove practical for construction and if so, should contribute toward decreasing structural weight. The smooth thin curved sheet seems to be a basis for the study of these variations. It is hoped that the results of this investigation will be helpful in their solution.

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Appendix I

Log of Test No. 1

Material: heat-treated aluminum alloy		
Diameter of tube	36	inches
Sheet thickness	.014	"
Ratio of D/t	2571	" 1285
Length of tube	36	"
Weight of tube	10.75	pounds
Weight of loading platform	39	"
Weight of tripod and galvanized iron tube	97.5	"

Load added	Arm	Moment increment	Total load	Total moment	Read	Defl. increment	Total deflection
97.5	64.25	6260	97.5	6260	2.184	.000	.000
39	120	4680	136.5	10940	2.131	.000	.000
					2.121		
					1.068		
25	120	3000	161.5	13940	1.068	.000	.000
25	120	3000	186.5	16940	1.068	.000	.000
25	120	3000	211.5	19940	1.069	.001	+.001
25	120	3000	236.5	22940	1.071	.003	+.004
10	120	1200	246.5	24140	1.149	.078	+.082
10	120	1200	256.5	25340	1.151	.002	+.084
10	120	1200	266.5	26540	1.155	.004	+.088
10	120	1200	276.5	27740	1.158	.003	+.091
10	120	1200	286.5	28940	1.167	.009	+.100
10	120	1200	295.5	30140	1.193	.026	+.126
5	120	600	301.5	30740	held		
5	120	600	306.5	31340	failed		

Load in pounds, moments in inch pounds, deflections and arms in inches.
A positive deflection means a vertical elongation at the bulkhead.

After first failure:
Reduced load on platform to 70 lb. and structure arose.
Held 80 lb. on platform (total moment = 20540 in. lb. or 67 per cent of max.) and then failed again on addition of more load.

After second failure:
Held 25 lb. on platform (total mom. = 13940 in. lb. or 45 per cent of max.)

$I/c = A\ r^2/2\ r = 0.014 \times 36 \times 18 \times \pi/2 = 14.27\ in.^3$.

$\frac{M}{RV} = 5.66$

Maximum f_b as shown by test = Mc/I = 30740/14.27 = 2150 lb. per sq.in.

By equation (6) F_b max. = $\frac{6600000}{2570}$ = 2567 lb. per sq.in.

2150/2567 = 84 per cent.

Log of Test No. 2

Material: heat-treated aluminum alloy		
Diameter of tube	36	inches
Sheet thickness	.022	"
Ratio of D/t	1636	" 818
Length of tube	36	"
Weight of tube	13.75	pounds
Weight of loading platform	39	"
Weight of tripod and galvanized iron tube	97.5	"

Load added	Arm	Moment increment	Total load	Total moment	Read	Defl. increment	Total deflection
97.5	64.25	6260	97.5	6260	1.331	0	0
39	120	4680	136.5	10940	1.331	0	0
50	120	6000	186.5	16940	1.330	-.001	-.001
50	120	6000	236.5	22940	1.330	0	-.001
50	120	6000	286.5	28940	1.330	0	-.001
50	120	6000	336.5	34940	1.330	0	-.001
50	120	6000	386.5	40940	1.330	0	-.001
50	120	6000	436.5	46940	1.330	0	-.001
50	120	6000	486.5	52940	1.330	0	-.001
50	120	6000	536.5	58940	1.330	0	-.001
50	120	6000	586.5	64940	1.330	0	-.001
25	120	3000	611.5	67940	1.331	+.001	0
25	120	3000	636.5	70940	1.331	0	0
25	120	3000	661.5	73940	1.331	0	0
25	120	3000	686.5	76940	1.331	0	0
25	120	3000	711.5	79940	1.331	0	0
25	120	3000	736.5	82940	1.331	0	0
25	120	3000	761.5	85940	1.331	0	0
10	120	1200	771.5	87140			held
10	120	1200	781.5	88340	1.7	+.4	+.4 failed to hold

Load in pounds, moments in inch pounds, deflections and arms in inches.
A positive deflection means a vertical elongation of the bulkhead.

M/Ry · 6.28

After first failure:
Reduced load to 375 lb. and structure arose.
Held 435 lb. on platform (total moment = 63140, or 72 per cent of maximum) and then failed again on addition of more load.

$I/c = A\ r^2/2r = 0.022 \times 36 \times 18 \times \pi / 2 = 22.39\ \text{in.}^3$.

Max. f_b as shown by test = Mc/I = 87140/22.39 = 3890 lb./sq.in.

By equation (6) F_b max. = $\frac{6600000}{1636}$ = 4034 lb./sq.in.

3890/4034 = 97 per cent.

Log of Test No. 3

Material: heat-treated aluminum alloy		
Diameter of tube	36	inches
Sheet thickness	.032	"
Ratio of D/t	1125	" 562
Length of tube	36	"
Weight of tube	11.75	pounds
Weight of loading platform	39	"
Weight of tripod and galvanized iron tube	97.5	"

Load added	Arm	Moment increment	Total load	Total moment	Read	Defl. incr't	Total deflection
97.5	64.25	6260	97.5	6260	1.386	0	0
39	120	4680	136.5	10940	1.386	0	0
345	120	41400	481.5	52340	1.384	-.002	-.002
255	120	30600	736.5	82940	1.381	-.003	-.005
100	120	12000	836.5	94940	1.380	-.001	-.006
100	120	12000	936.5	106940	1.379	-.001	-.007
100	120	12000	1036.5	118940	1.378	-.001	-.008
100	120	12000	1136.5	130940	1.377	-.001	-.009
100	120	12000	1236.5	142940	1.3765	-.0005	-.0095
100	120	12000	1336.5	154940	1.376	-.0005	-.010
100	120	12000	1436.5	166940	1.3755	-.0005	-.0105
100	120	12000	1536.5	178940	1.3755	0	-.0105
20	120	2400	1556.5	181340			held
10	120	1200	1566.5	182540	1.753	+.377	+.367 failed to hold

Load in pounds, moments in inch pounds, deflections and arms in inches.

A positive deflection means a vertical elongation of the bulkhead.

17/Rv = 6.48

After first failure:

Reduced load to 750 lb. and structure arose.

Held 825 lb. on platform (total moment = 109940 or 61 per cent of max.) and then failed again on addition of more load.

Maximum vertical deflection of bulkhead ring observed after failure = 2.060 - 1.386 = +.674 inches.

$I/c = Ar^2/2r = .032 \times 36 \times 18 \times \pi/2 = 32.57$ in.^{3}

Max. f_b as shown by test = $Mc/I = \dfrac{181340}{32.57} = 5570$

By equation (6) F_b max. = $\dfrac{6600000}{1125}$ = 5870 lb. per sq. in.

$\dfrac{5570}{5870}$ = 95 per cent.

APPENDIX II

The bulkhead ring

The cross section of the bulkhead ring is shown in Figure 5. It was an aluminum alloy section, heat treated after bending. The bulkhead ring was tested as a column in the manner indicated by the set-up shown in Figure 18. The bulkhead ring tested was identical with the one used in the test specimens. Rivet holes were drilled for a single row of 1/8 inch rivets spaced 1 inch apart. The rivet holes are in the tension side of the cross section that fails in bending; therefore, having these holes filled with the rivets, as they are in the test cylinders, does not add strength to the critical section of the bulkhead ring. For the same deflection, the shear carried by the bulkhead ring in the test cylinder will be proportional to the value indicated in this test; not equal to it because the skin does increase the moment of inertia of the section, and the shear load, being distributed, would cause relatively less deflection. The remainder of the shear on the test cylinder must be carried by the skin. Due to the complete lateral support of the skin, the load carried by the bulkhead ring may be increased slightly.

The Department of Commerce Aeronautics Bulletin No. 7-A requires that the main bulkheads be strong enough to carry the shear load at their respective sections as columns. The method of determining their strength is the same as for the verticals in a reinforced framework except for the method of computing the length. If they are vertical for more than 50 per cent of the total depth of section, L should be taken as the length of the vertical portion. If they are curved, the length should be taken as not less than 50 per cent of the total depth of section and the maximum unit stress computed from the formula

$$f_b = \frac{P}{A} \pm \frac{P\,e\,y}{I}$$

where e is the maximum offset from a line joining the ends of the assumed effective length to the axis of the member. In this case the maximum allowable unit stress should be that for a straight column of the same length and $c = 2$, no addition being made to allow for the fact that part of the stress is due to bending.

The set-up for testing the bulkhead ring provided lateral support at five points as shown in Figure 18. The base of the bulkhead ring rested on a flat steel plate. A loading platform was hung from the top of the bulkhead ring. The load was added in ten- and five-pound increments. The deflections were measured at the top, along the vertical axis, for each load increment.

The deflection load curve is given in Figure 18. The butt joint joining the two ends of the ring had a cover plate on only one side; hence there was some play in the joint. In testing, the joint was placed where the compression due to the load would force the two ends of the ring together. The reversal of curvature on the lower end of the load-deflection curve is caused by the play in the joint.

The ring held a load of 150 pounds and failed with 5 pounds more load. The total deflection before failure would have been about 3-1/2 to 4 inches. The last measured deflection was 3.273 inches for a load of 145 pounds. The cross sections of the two critical points A and B (Fig. 18), were deformed at failure. The "hat" section flattened out and the free edges turned up toward the neutral axis.

Log of Bulkhead of Ring Test

Material: heat-treated aluminum alloy
Diameter of ring 36 inches
Weight of loading platform 10 pounds
Section No. 104202
Area of cross section A 0.106 in.^{2}
Moment of inertia I 0.0134 in.^{4}
Distance to neutral axis c 0.224 in.

Load	Read	Deflection increment	Total deflection
10	2.154		
20	1.874	0.280	0.280
30	1.691	0.183	0.463
40	1.528	0.163	0.626
50	1.364	0.164	0.790
55	1.285	0.079	0.869
60	1.201	0.084	0.953
65	1.118	0.083	1.036
70	1.031	0.087	1.123
75	0.945	0.086	1.209
80	0.856	0.089	1.298
85	0.765	0.091	1.389
90	0.666	0.099	1.488
95	0.573	0.093	1.581
100	0.475	0.098	1.679
105	0.359	0.116	1.795
110	0.254	0.105	1.900
	0.221 Spacer		
	1.575		
115	1.492	0.116	2.016
120	1.362	0.130	2.146
125	1.203	0.159	2.305
130	1.022	0.181	2.486
135	0.833	0.189	2.675
140	0.544	0.289	2.964
145	0.235	0.309	3.273
150			held
155			failed

Maximum allowable stress by $\frac{P}{A} + \frac{Mc}{I} = \frac{150}{.106} + \frac{150\ (18 + 2)\ .224}{.0134} =$

$= 51,500$ lb. per sq.in.

Load in pounds, deflection in inches.

Analysis of Bulkhead Ring

According to Bulletin 7-A, Department of Commerce

The cross-sectional area considered has the 1/8 inch gap due to holes drilled, as shown in Figure 18. Moment of inertia, area, and position of the neutral axis are obtained graphically by use of an Amsler integrator, from an enlargement of the section.

$I = 0.0134 \text{ in.}^4$

$A = 0.106 \text{ in.}^2$

$\bar{x}$ = neutral axis distance from open base = 0.244 in.
= c for compression

$$\left(\frac{I}{c}\right)_{comp.} \frac{0.0134}{0.224} = 0.599 \text{ in.}^3$$

$$\sigma = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.0134}{0.106}} = \sqrt{0.1264} = 0.356 \text{ in.}$$

Since the member is curved, consider the column length equal to one-half the diameter, and allowable P/A equal to that of a column of this length with end conditions such that c = 2.

L = 18 in. c = 2

L/σ = 50.5 in. ∴ a short column

According to the straight line equation (Reference 4, Table 9:7, page 131), for maximum allowable P/A,

$$P/A = 48{,}000 - 280\ L/\sigma = 48{,}000 - 14{,}100 =$$

33,900 lb. per sq.in.

Equating this to $\frac{P}{A} + \frac{P\ e}{I/c}$ where e, the eccentricity, is

$$e = 18 - \frac{18}{\sqrt{2}} = 18 - 12.72 = 5.28 \text{ in.}$$

$$\frac{P}{0.106} + \frac{5.28\ P}{0.0599} = 33,900$$

$$9.44\ P + 88.2\ P = 33,900$$

$$\text{Allowable} \quad P = \frac{33900}{97.6} = 348 \text{ lb.}$$

Allowable shear at section = 2P = 696 lb.

Appendix III

Energy-Moment Theorem

An important step in the derivation of the expression for the maximum moment that a thin-walled cylinder will develop is based on the fact that the first derivative of the total strain energy per unit length, with respect to the curvature of the elastic axis due to a moment, is equal to the moment. Proof of this statement is included because it is not one of the better known relations.

From the well-known beam theory:

Curvature of beam $= C = \frac{1}{R}$

$$M = \frac{EI}{R} = E\ I\ C \qquad \text{(a)}$$

If W is the total strain energy in a beam deformed by bending,

$$\delta W = \frac{M^2}{2EI}\,\delta x \quad \text{and} \quad W = \int_0^l \frac{M^2}{2EI}\,dx \quad \text{(8.11 Prescott's "Elasticity")}$$

If U is the strain energy per unit length of the beam,

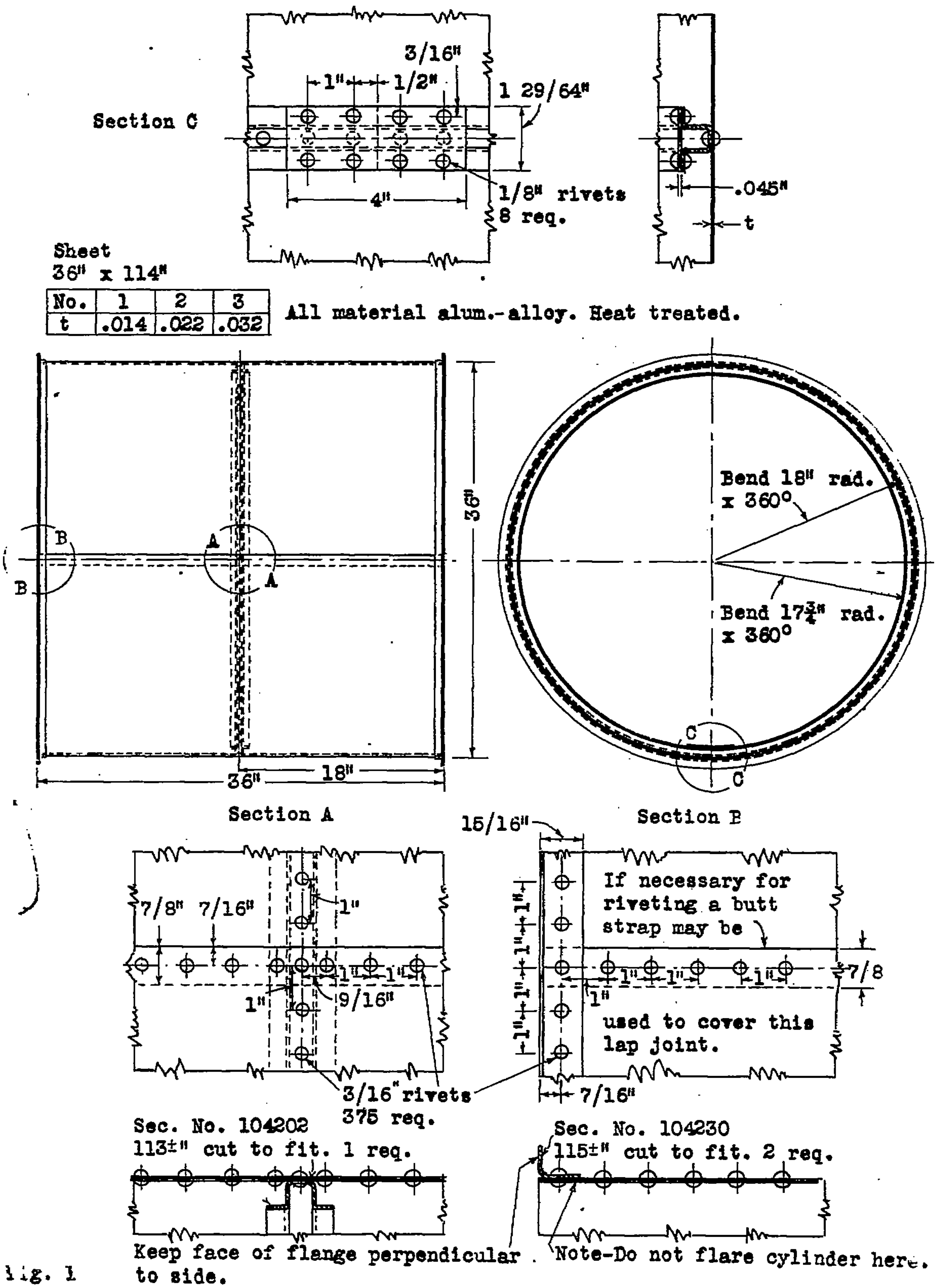
$$U = \frac{W}{l} = \frac{M^2}{2EI} \qquad \text{(b)}$$

Substituting (a) in (b)

$$U = \frac{E^2\ I^2\ C^2}{2EI} = \frac{EIC^2}{2}$$

and

$$\frac{dU}{dC} = \frac{2EIC}{2} = EIC = \text{moment.}$$



No.	1	2	3
t	.014	.022	.032

Fig. 1

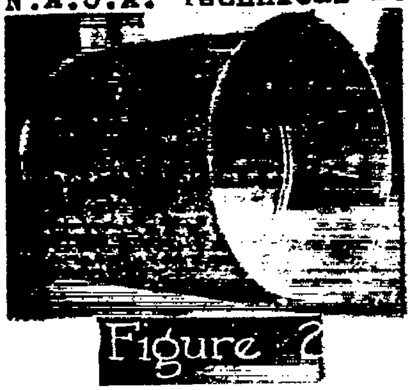
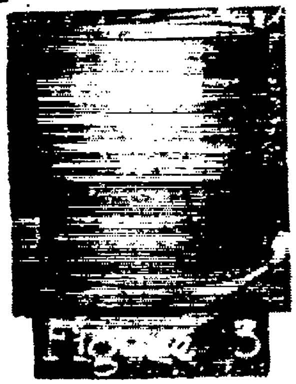


Figure 2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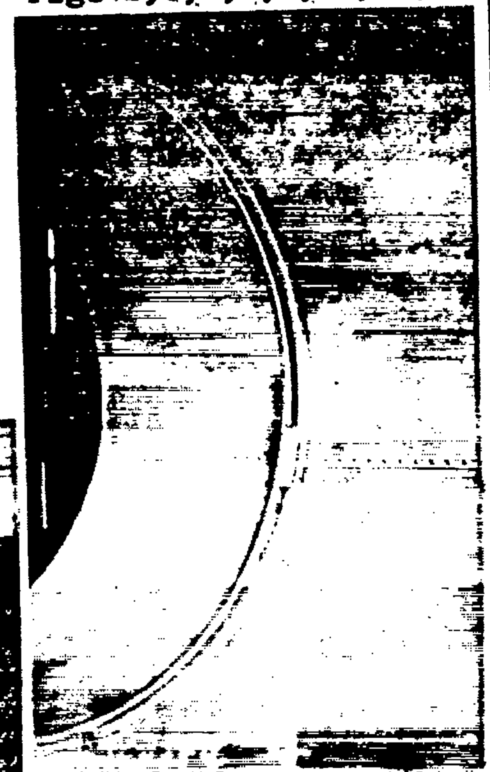

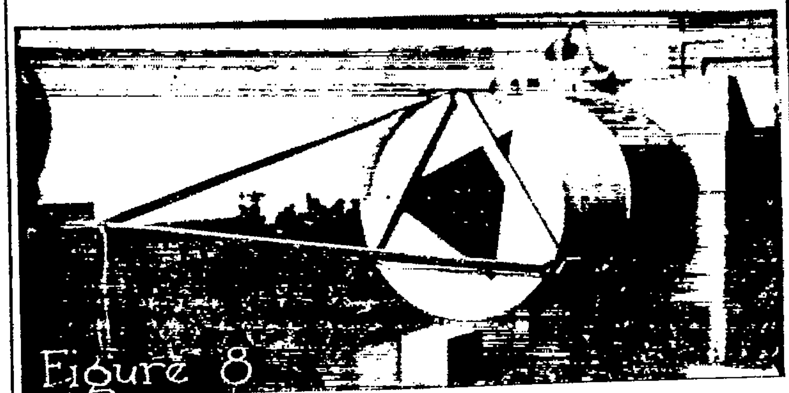
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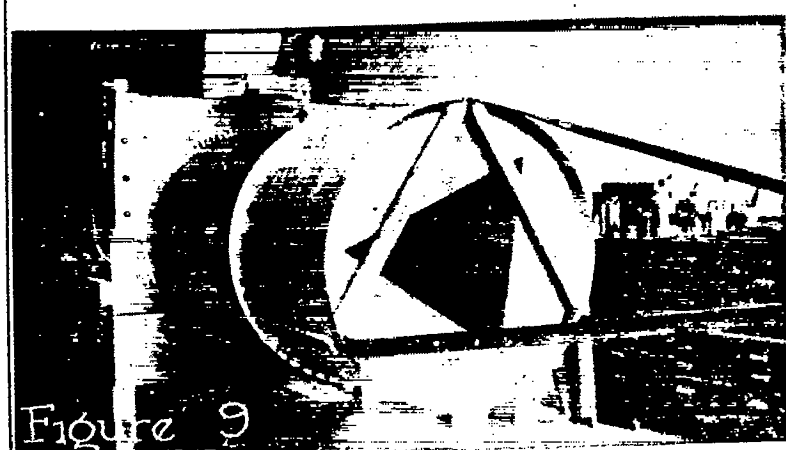
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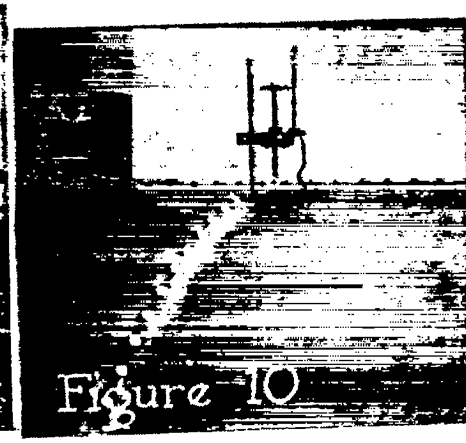


Figure 10

Figure 11

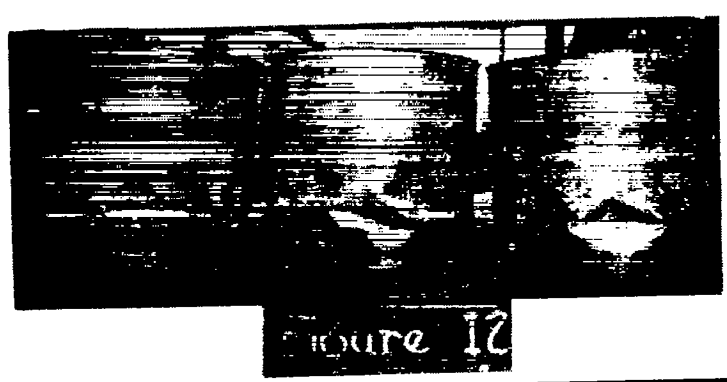
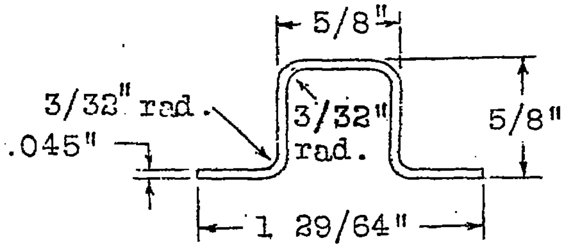
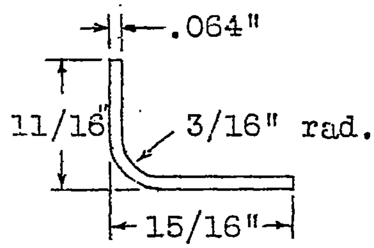


Figure 12



Bulkhead ring. Section 104202

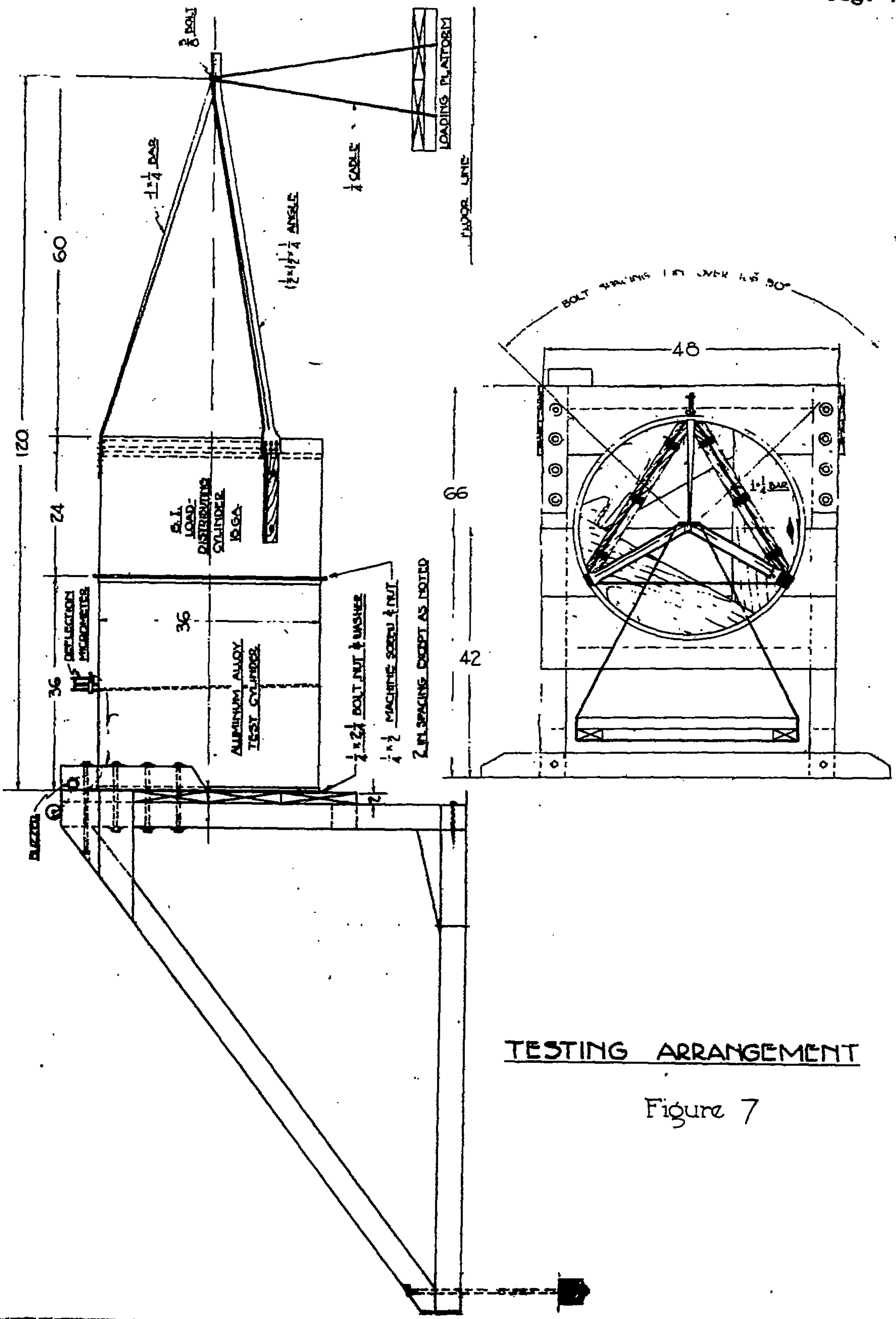
Fig. 5



Flange. Section 104230

Fig. 6

N. B. Material of both sections: Alum-alloy heat treated after bending.



TESTING ARRANGEMENT

Figure 7

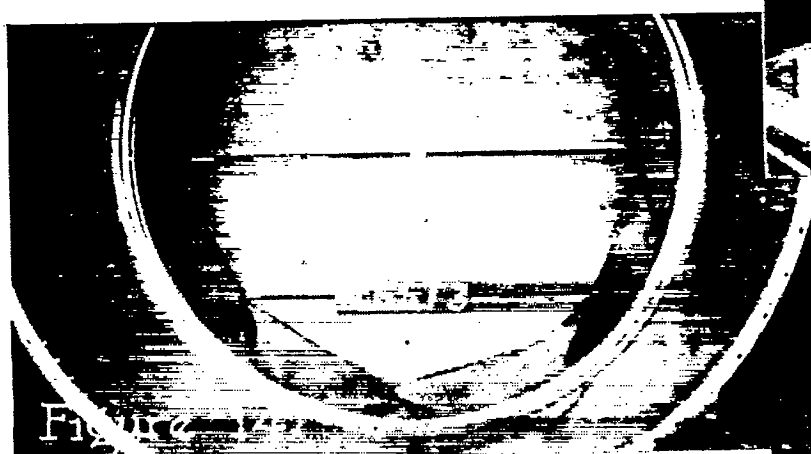


Figure 15

Figure 16

Figure 17

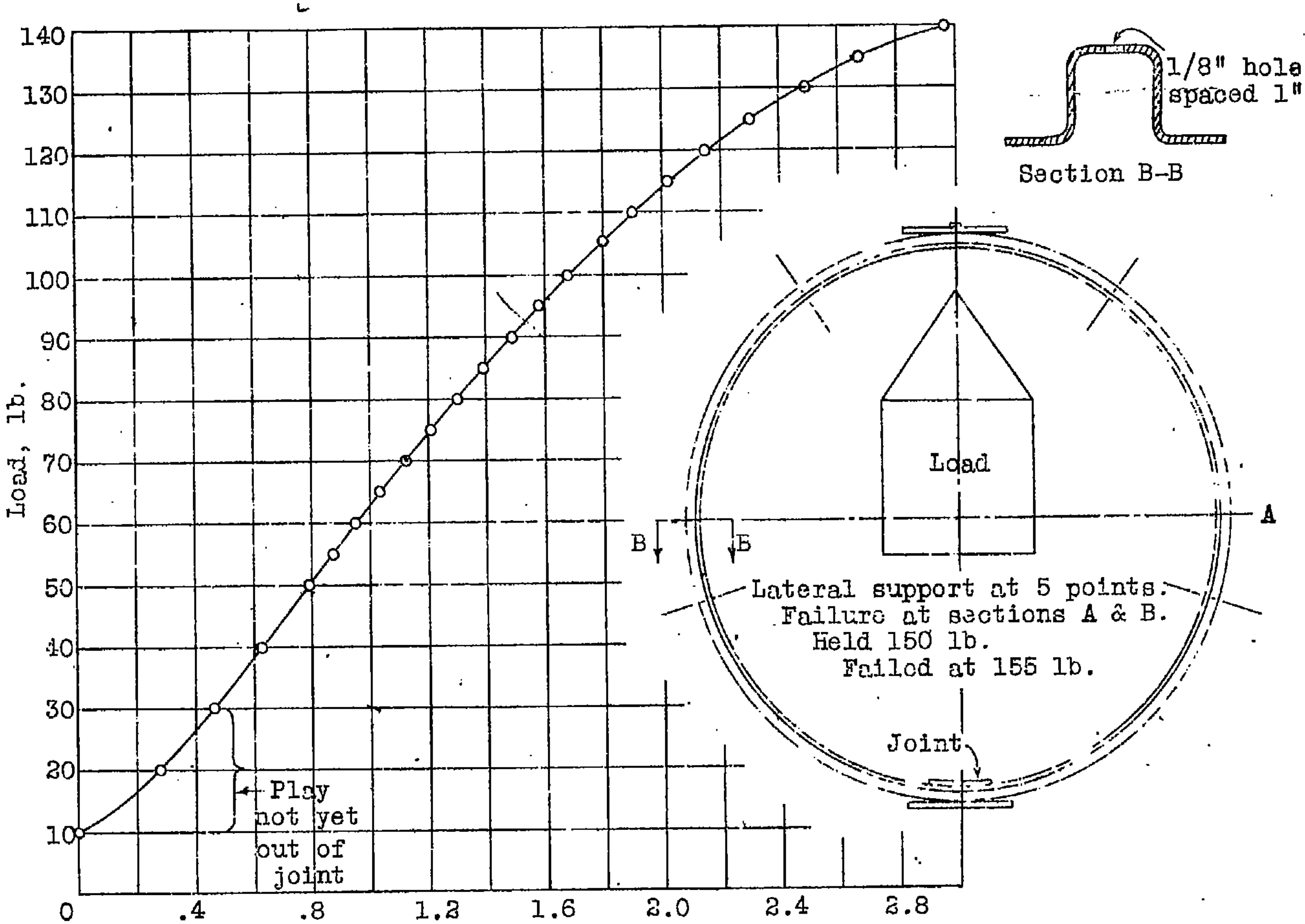


Fig. 18 Load-deflection curve. 36" diameter bulkhead ring.
Section 104202