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THE IMPACT ON SEAPLANE FLOATS DURING LANDING

By Th. von Karman
Aerodynamical Institute of the
Technical High School, Aachen

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Introduction

In order to make a stress analysis of seaplane floats, and especially of the members connecting the floats with the fuselage, it is of great importance to determine the maximum pressure acting on the floats during landing. A series of experiments have been carried out, both for model and full-scale floats. Results of model and full-scale experiments can be found in the English Reports and Memoranda Nos. 583, 683 and 926 (Reference 1). In Germany, investigations have been made with full-size floats, mounted on a large impact pendulum.* Recently in America the actual maximum pressures occurring during landing were obtained by means of a series of indicators located in the bottom of the float, each of which was set at a different maximum pressure reading (Reference 2). When I first learned of these investigations during my last visit to America, I tried to develop theoretically a formula for the maximum pressures during landing so that experimental results might be applied to different bodies and different velocities. I found that the formula checked very well with the experimental results, and present herewith the development of the formula.

*As yet unpublished.

Derivation of the Impact Formula

The problem may be treated approximately in the following manner. We consider a horizontal cylindrical body with a wedge-shaped under surface as it strikes a horizontal surface of water, and calculate the force acting between the body and the water.

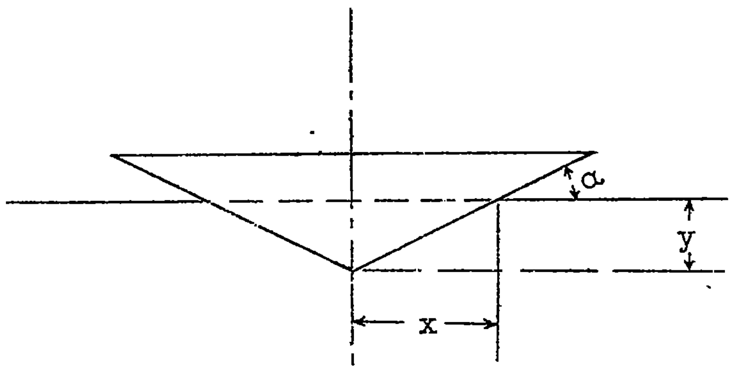

Fig.1

Let W = weight of the body per unit length, α = angle of inclination of the under surfaces (Fig. 1), with the horizontal, y = the vertical distance through which the body travels in the time t, and $2x = 2y \cot \alpha$ = the corresponding breadth of the part of this body in the water. We proceed to calculate the force by application of the momentum theorem. The original momentum of the body is distributed at the time t between the body and the water. That part of the momentum already transferred to the water at the time t depends on x and can be approximated as follows. It is known that if a long plate of width $2x$ is accelerated in a fluid, its inertia is increased by the amount $\rho x^2 \pi$ where ρ represents the density of the fluid. In other words, the apparent increase of the mass of the plate is equal to the mass of fluid contained in a circular cylinder of diameter equal to the width of the plate. But in the case of the seaplane float entering the water, the imaginary cylinder of fluid first mentioned will be composed half of water and half of air. In other words, the effect of the float will be, on the one hand, to accelerate the water particles before it, and on the other hand, to suck in the air behind it. But the effect of the air is negligible compared with that of the water, therefore we assume but half of the apparent increase of mass. On account of the wedge-shaped bottom of the float, the real value is probably a trifle smaller.

We write, therefore, for the total momentum, where v = downward velocity at time t,

$$M = \frac{W}{g} v + \frac{1}{2} x^2 \rho \pi v$$

which must be equal to the original momentum

$$M = \frac{W}{g} v_0$$

v_0 = velocity at moment of first contact.

To be sure, there is also the effect of buoyancy, which serves to decrease the momentum; but for floats with under surfaces not too sharply inclined, the effect may be neglected.

Setting

$$v = \frac{dy}{dt} = \tan\alpha \ \frac{dx}{dt}$$

we obtain

$$\frac{W}{g}\frac{dx}{dt}\tan\alpha\left(1 + \frac{\rho\,\pi\,g\,x^2}{2\,W}\right) = \frac{W}{g}\,v_o,$$

where

$$\gamma = \rho\,g$$

or

$$\frac{dx}{dt}\left(1 + \frac{\gamma\,\pi\,x^2}{2\,W}\right) = v_o\cot\alpha$$

as the law giving the relation between sinking velocity and depth.

Writing this in the form

$$\frac{dx}{dt} = \frac{v_o\cot\alpha}{1 + \dfrac{\gamma\,\pi\,x^2}{2\,W}},$$

$$V = \frac{V_o}{1 + \dfrac{\pi\rho g x^2}{2W}}$$

it is easy to calculate

$$\frac{d^2x}{dt^2} = \frac{d}{dx}\left[\frac{1}{2}\left(\frac{dx}{dt}\right)^2\right],$$

which gives

$$\frac{d^2x}{dt^2} = \frac{v_o^2\cot^2\alpha}{\left(1 + \dfrac{\gamma\,\pi\,x^2}{2\,W}\right)^3}\left(-\frac{\gamma\,\pi\,x}{W}\right).$$

Now we easily obtain the expression for the force of impact, as

$$P = \frac{W}{g}\frac{d^2y}{dt^2} = \frac{v_o^2\cot\alpha}{\left(1 + \dfrac{\gamma\,\pi\,x^2}{2\,W}\right)^3}\,\rho\pi x$$

and the average pressure as

$$p = \frac{P}{2x} = \frac{\rho\, v_o^2}{2}\ \frac{\pi \cot\alpha}{\left(1 + \frac{\gamma\, \pi\, x^2}{2\, W}\right)^3}$$

The pressure is evidently a maximum in the middle of the float and at the moment of first contact, therefore

$$p_{max} = \frac{\rho\, v_o^2}{2}\ \pi \cot\alpha$$

The term $\frac{\rho\, v_o^2}{2}$ represents the dynamic pressure corresponding to the impact velocity v_o, and the term $\pi \cot\alpha$ is the theoretical factor of increase. For different values of the angle of inclination of the under surfaces α, we obtain the following table.

α	0	5°	10°	15°	20°	25°	30°	45°
$\pi \cot\alpha$	∞	32.00	17.82	11.72	8.63	6.64	5.44	3.14

Comparison with Experiments

The value of the angle α in the American measurements was 20°; this gives for several values of the sinking velocity v_o = 2, 3, 4, m/s.

v_o	2 m/s	3 m/s	4 m/s	
$\gamma \frac{v_o^2}{2\ g}$	0.02	0.045	0.08	atm.
$\pi \cot\alpha\ \gamma \frac{v_o^2}{2\ g}$	0.173	0.388	0.69	atm.

The actual measurements gave a maximum value of about 0.5 atm. Unfortunately, however, the downward velocity was not measured directly, therefore we must calculate an approximate value from the landing speed and the probable value of the gliding angle. The landing speed was 20-25 m/s and the gliding angle about 1 : 6, giving a downward velocity of about 3.3 - 4.1 m/s. It is seen, therefore, that the formula checks well with the results of these experiments.

It is interesting to notice that waves may cause pressure of the same order. For instance, if a seaplane is lifted by a wave through a height of 50-80 cm, it will fall with a velocity of about 3-4 m/s, giving rise to a pressure of 0.4-0.7 atmospheres.

The Flat-Bottomed Float

It remains to consider the limiting case of the flat-bottomed float, where $\alpha = 0$. In this case the formula gives an infinite impact pressure, since the water has been assumed incompressible; therefore the compressibility must be taken into consideration. It is possible to obtain an approximate value for the maximum pressure occurring when a flat body strikes against the surface of a fluid in the following manner. The propagation of the momentary increase of pressure in a fluid takes place at the speed of sound in the fluid, designated by c. Therefore the mass of fluid accelerated in the time dt is $\rho F c\, dt$, where F = the surface of the fluid struck by the body. Since the velocity of this mass is increased from zero to v_0 in the time dt, the force acting is $\rho F c v_0$ and the pressure is

$$p = \frac{\rho\, v_o^2}{2} \left(\frac{2\, c}{v_o}\right).$$

Thus the pressure appears as a factor $\frac{2c}{v_o}$ times the stagnation pressure. Following are values of the pressure for several downward velocities, with $c = 1450$ m/s for water.

v_o (m/s)	2	3	4
$\frac{2c}{v_o}$ =	1450	967	725
p (atm.)	29	43.5	58

These values of the pressure are doubtless higher than would arise in an actual airplane, since the elastic yielding of the whole construction is probably greater than that of the water under compression. It is evident, however, that a flat-bottom in float construction is unfavorable, especially for the connecting members, since the struts and bracings are highly stressed, and the magnitudes of the stresses are difficult to estimate.

Further Investigation

These calculations leave much room for further development. For one thing, the variation of the pressure with time could be studied as well as the distribution of maximum pressure over the bottom of the float. Moreover, the apparent increase of mass could be more exactly calculated, taking into consideration the true shape of the float, and the pressure distribution over the length of the float obtained. A thorough theoretical treatment of the subject together with systematic experiments would be of great value in improving seaplane design.

Translation by
Frank L. Wattendorf,
Aachen University.

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