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AN IMPULSE-MOMENTUM METHOD FOR CALCULATING LANDING-GEAR CONTACT CONDITIONS IN ECCENTRIC LANDINGS

By Robert T. Yntema and Benjamin Milwitzky

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

An impulse-momentum method for determining impact conditions for landing gears in eccentric landings is presented. The analysis is primarily concerned with the determination of contact velocities for impacts subsequent to initial touchdown in eccentric landings and with the determination of the effective mass acting on each landing gear. These parameters determine the energy-absorption requirements for the landing gear and, in conjunction with the particular characteristics of the landing gear, govern the magnitude of the ground loads.

Changes in airplane angular and linear velocities and the magnitude of landing-gear vertical, drag, and side impulses resulting from a landing impact are determined by means of impulse-momentum relationships without the necessity for considering detailed force-time variations. The effective mass acting on each gear is also determined from the calculated landing-gear impulses. General equations applicable to any type of eccentric landing are written and solutions are obtained for the particular cases of an impact on one gear, a simultaneous impact on any two gears, and a symmetrical impact. In addition a solution is presented for a simplified two-degree-of-freedom system which allows rapid qualitative evaluation of the effects of certain principal parameters.

The general analysis permits evaluation of the importance of such initial conditions at ground contact as vertical, horizontal, and side drift velocities, wing lift, roll and pitch angles, and rolling and pitching velocities, as well as the effects of such factors as landing-gear location, airplane inertia, landing-gear length, energy-absorption efficiency, and wheel angular inertia on the severity of landing impacts. A brief supplementary study which permits a limited evaluation of variable aerodynamic effects neglected in the analysis is presented in the appendix.

Application of the analysis indicates that landing-gear impacts in eccentric landings can be appreciably more severe than impacts in symmetrical landings with the same sinking speed. The results also

indicate the effects of landing-gear location, airplane inertia, initial wing lift, side drift velocity, attitude, and initial rolling velocity on the severity of both initial and subsequent landing-gear impacts. A comparison of the severity of impacts on auxiliary gears for tricycle and quadricycle configurations is also presented.

INTRODUCTION

Available literature on the design of aircraft for the landing condition gives relatively little emphasis to the problem of eccentric landings. For example, current design requirements assume the same landing-gear reactions in unsymmetrical impacts as in symmetrical landings. Experience has shown, however, that landing-gear loads in either the initial or some subsequent impact in an eccentric landing can be appreciably greater than the loads in a symmetrical landing with the same sinking speed, particularly when the locations of the landing gears have been chosen without proper regard for the angular inertias of the airplane. The problem of eccentric landings may be especially important in the case of unconventional landing-gear configurations for which little practical design or operating experience is available.

The purpose of the present paper is to investigate the rigid-body dynamics of airplanes during eccentric landings and to evolve a simple analytical method for determining landing-gear contact conditions for successive impacts in such landings. These contact conditions, which include landing-gear contact velocity, effective mass, and airplane attitude at the time of impact, govern the energy-absorption requirements of the landing gear and, in conjunction with the particular characteristics of the landing gear, determine the loads applied to the airplane by the landing gear.

The major portion of the analysis is concerned with the determination of landing-gear contact velocities for impacts subsequent to the initial touchdown. In this part of the analysis, landing-gear impulses and resulting changes in airplane linear and angular velocities are determined by means of impulse-momentum relationships, and the free-body motions of the airplane during the interval between the termination of one impact and the beginning of the next impact are considered. Also presented is an impulse method for determining the effective mass acting on each landing gear during an impact. The application of impulse-momentum relationships in the present analysis eliminates the necessity for considering detailed landing-gear force-time variations but restricts the method to those cases where the impulses on the landing gear or gears in contact with the ground are largely completed prior to the beginning of the next impact.

The analysis is presented in a general form and is applicable to any landing-gear configuration. The treatment permits the investigation of the relative importance of such factors as rate of descent and angular velocities at initial touchdown, side drift, wing lift, wheel spin-up, and landing-gear energy-dissipation efficiency, as well as the longitudinal and lateral locations of the landing-gear units with respect to the rolling and pitching radii of gyration of the airplane. The analysis treats eccentric impacts on one landing gear, simultaneous impacts on any two gears, and symmetrical impacts. Since the terminal conditions for any stage of the motion during a landing represent the initial conditions for the next stage of the motion, the analysis permits the determination of the behavior of the airplane during successive impacts following the initial contact with the ground and also permits a limited evaluation of the stability of the airplane as the landing progresses.

NOMENCLATURE AND SYMBOLS

Coordinate System

The two principle sets of axes employed in the analysis are defined as follows:

Body axes, a,b,c - This coordinate system moves with the airplane, has its origin at the center of gravity of the airplane, and is described as follows:

a-axis parallel to arbitrary fuselage reference line, positive forward

b-axis normal to fuselage plane of symmetry, positive to right

c-axis normal to fuselage reference line in plane of symmetry, positive downward

Space axes, x,y,z - This coordinate system is a fixed system having its origin at a point in the ground plane directly beneath the center of gravity of the airplane at the instant of initial landing impact and is described as follows:

x-axis formed by intersection of ground plane and plane containing a-axis and being perpendicular to ground plane, positive forward y-axis — in ground plane and perpendicular to x-axis, positive to right

z-axis — perpendicular to ground plane, positive upward

Symbols

F_v — vertical landing-gear force, positive upward

F_d — drag force, positive rearward

F_s — side force, positive to right

I_v — vertical landing-gear impulse $\left(\int_{t_o}^{t_t} F_v \, dt\right)$, positive upward

I_d — drag impulse $\left(\int_{t_o}^{t_t} F_d \, dt\right)$, positive rearward

I_s — side impulse $\left(\int_{t_o}^{t_t} F_s \, dt\right)$, positive to right

g — gravitational constant

$\dot{z}$ — vertical velocity of any point on airplane, positive upward

$\dot{y}$ — lateral or side drift velocity of airplane center of gravity, positive to right

$\dot{x}$ — forward velocity of airplane center of gravity, positive forward

$\ddot{z}$ — vertical acceleration of any point on airplane, positive upward

θ — angle of pitch measured between ground plane and a-axis in plane of symmetry, positive nose up

φ — angle of roll measured between ground plane and b-axis in plane perpendicular to a-axis, positive left wing up

α,β,γ — auxiliary angles employed in impulse equations (Defined in terms of attitude angles where they are introduced. See equations (13).)

$\dot{\theta}$	pitching velocity of airplane, positive nose up
$\dot{\varphi}$	rolling velocity of airplane, positive left wing up
E	kinetic energy
W	total weight of airplane at landing
W_e	effective weight acting on given landing gear
M	total mass of airplane at landing (W/g)
M_e	effective mass acting on given landing gear (W_e/g)
r	radius of tire
V	vertical velocity of landing gear at impact
I_{aa}	rolling moment of inertia of airplane about longitudinal (a) axis
I_{bb}	pitching moment of inertia of airplane about lateral (b) axis
ρ_{aa}	rolling radius of gyration of airplane
ρ_{bb}	pitching radius of gyration of airplane
I_W	moment of inertia of one wheel and tire assembly about axle
N	number of wheels attached to given landing gear
K_i	ratio of vertical impulse acting on gear i to sum of vertical impulses on all gears making simultaneous contact with ground
K_L	wing lift factor, ratio of aerodynamic lift force to total airplane weight
K_p	prerotation factor, ratio of prerotation peripheral velocity of wheel to forward velocity of airplane
K_s	ratio of side impulse to vertical impulse
t	time
η_r	energy-dissipation efficiency of landing gear; ratio of impact energy dissipated to initial kinetic energy of impact

A,B,C,D combined constants used in impulse analysis

E combined constants used in equations for effective mass

Subscripts

o initial conditions at beginning of particular landing impulse

t terminal conditions at end of particular impulse, represents initial conditions for free-body portion of analysis

f final conditions for free-body motion, corresponds to initial conditions for next subsequent impact

c dimension measured with shock strut and tire partially compressed

g center of gravity of airplane

T total

m,n identifying integers assigned to each landing-gear unit of aircraft

i landing gear or gears, contact of which initiates given stage of motion

j landing gear or gears, contact of which terminates given stage of motion; j becomes i in next stage of motion

Definitions

Stage of the motion - the interval between the initial contact of a given landing gear and the next impact.

FUNDAMENTAL CONSIDERATIONS

General Considerations for Eccentric Impact

Some of the more important aspects of the problem of eccentric landings can be easily visualized by considering an idealized representation of an airplane contacting the ground on one landing gear. For the purposes of this simplified consideration, the airplane is assumed to have freedom in roll and vertical translation only. It is also assumed that the impacting landing gear does not rebound after contact, that the weight is exactly balanced by the wing lift, that no aerodynamic moments act on the airplane, and that the only forces acting on the landing gear are in the vertical direction. With these assumptions the impact velocity, effective mass, and impact energy for the first and second impacts (denoted by i and j, respectively, in the sketch in fig. 1) can be readily determined from the initial conditions and simple impulse-momentum relationships. Although this idealized system does not fully represent an actual landing of an airplane, the results obtained can be used to illustrate some of the fundamental differences between eccentric and symmetrical landings.

Calculated results for the idealized system are presented in the form of dimensionless ratios in figure 1 which permits comparison of impact conditions for eccentric and symmetrical landings. Figure 1 also illustrates the effects of landing-gear location on impact severity in eccentric landings. The significant parameter for this simplified system is the ratio of the semitread b to the radius of gyration of the airplane in roll ρ_{aa}.

From figure 1 it can be seen that (a) the contact velocity for the second impact V_j in an eccentric landing can be appreciably greater than the initial descent velocity of the airplane V_i if the landing gears are located outboard of the rolling radius of gyration; (b) although the effective mass M_e, which can be considered to act on a given landing gear, is less than half the total mass M for values of $\frac{b}{\rho_{aa}} > 1$, the kinetic energy for the second impact E_j can be appreciably greater than half the total energy of the airplane E_T, because of the increased contact velocity; (c) one of the impacts in an eccentric landing must be at least as severe and, in general, $\left(\text{where } \frac{b}{\rho_{aa}} \neq 1\right)$ will be more severe than each landing-gear impact in a symmetrical landing.

Since these results indicate that impact severity can be appreciably increased in eccentric landings, the foregoing simplified treatment has been broadened to permit consideration of additional factors that can significantly influence the severity of impacts in such landings. The more detailed treatment includes the effects of such factors as freedom to pitch, drag loads, side drift, reduced wing lift, angular velocities and attitude angles at initial contact, and landing-gear energy-dissipation efficiency.

Considerations Leading to Present Type of Analysis

In the general case of an eccentric landing, the motions of the airplane produced by an impact are determined by the time histories of the vertical, drag, and side loads on the landing gear, in conjunction with the geometric, inertia, and aerodynamic characteristics of the airplane. The choice of treatment employed in the present study has been influenced by the fact that the magnitude and variation of the ground reactions cannot, in general, be accurately specified at the present time. Drop-test data do not appear suitable for this purpose since major differences often exist between the results of laboratory and flight tests.

The vertical-force - time relationship during an impact depends largely on the characteristics of the landing gear and on the numerous initial conditions which can vary widely from impact to impact. The prediction of vertical loads is greatly complicated by the simultaneous action of drag and side loads which can greatly increase strut friction.

The drag loads produced by the wheel spin-up process during landing depend on the vertical-load time history, the coefficient of friction, which appears to vary considerably during the spin-up process, the moment of inertia of the wheel, and the radius of the compressed tire. As in the case of the vertical load, the time history of the drag load can be appreciably different for different airplanes. At the present time, because of the lack of information regarding the variation of the friction coefficient, the time history of the drag load cannot be accurately predicted, even if the vertical-load variation were adequately defined.

The prediction of side loads due to yaw during landing is complicated by the large variation in the type of contact between the tire and ground during impact. In the early part of the impact a state of complete skidding exists; following spin-up the wheel is in a state of yawed rolling. The problem is further complicated by the transitory nature of the phenomenon and the absence of either experimental or theoretical information regarding the yawed rolling characteristics of tires at high vertical loads.

In view of the fact that the time histories of ground loads applied to the airplane cannot, in general, be accurately defined, it appeared desirable to develop an analysis which would not require detailed specification of the force-time variations. The present analysis therefore makes use of an impulse-momentum approach since the impulses acting on the landing gear can be more readily described from such simple considerations as the energy-dissipation efficiency or rebound characteristics of the landing gear, the momentum required to spin up the wheels, and the lateral momentum due to side drift of the airplane.

METHOD OF ANALYSIS

In the present treatment the descent velocity and the attitude of the airplane at the instant of initial touchdown are assumed to be known from statistical studies or design requirements or to have been determined by some other means. The analysis is largely concerned with the determination of the landing-gear contact velocities and airplane attitudes for impacts subsequent to the initial contact in eccentric landings. Also treated is the determination of the effective mass which acts on each landing gear during an impact.

The behavior of the airplane between successive impacts is investigated analytically as a problem in rigid-body dynamics since airplane elasticity is considered to have little effect on the over-all motions. In order to avoid the necessity of having to use particular force-time variations to represent the landing-gear reactions, an impulse-momentum approach is used to determine changes in the linear and angular velocities of the airplane during an impact. This part of the problem is treated in a section entitled "Impulse Analysis," in which the changes in the airplane velocities are assumed to occur rapidly without appreciable change in airplane attitude during the impact. The contacting gear is assumed to rebound from the ground with a vertical velocity determined by the contact velocity and the energy-dissipation efficiency of the landing gear. The airplane is thereafter considered a free body under the influence of constant gravitational and wing lift forces. In the section entitled "Free-Body Motion Analysis" the equations defining the translational and rotational motions of the airplane following rebound are set up and integrated, and the vertical and angular velocities and the attitude of the airplane for the next impact are determined. If the contact conditions for impacts subsequent to the second impact are desired, the computation procedure can be repeated with the final conditions for any given stage of the motion as the initial conditions for the next stage.

A section dealing with the calculation of the effective mass which acts on a landing gear during an impact is also presented. In order to eliminate the necessity of assuming that the resultant ground force acting on a landing gear maintains a constant direction throughout an impact, equations for the effective mass in the present treatment are derived on an equal-impulse rather than the usual equal-acceleration basis.

In the organization of each major subdivision of the analysis, the most general case is treated first, following which particular adaptations of the general equations to more specific cases are presented. Assumptions are briefly mentioned wherever they are introduced in the analysis; a more detailed discussion of the manner in which the assumptions influence the calculated results is presented in a separate section.

The following brief outline of the analysis is presented for the convenience of the reader:

I. Impulse Analysis

A. General Impulse Equations for Eccentric Landings

B. Impulse Solution for Impact on Any One Landing Gear

C. Impulse Solution for Simultaneous Impact on Any Two Landing Gears

D. Impulse Solution for Symmetrical Impact

II. Free-Body Motion Analysis

A. General Solution for Motion Following Rebound

B. Free-Body Motion Solution for Symmetrical Landing

III. Simplified Analysis of Airplane Motions for a System with Two Degrees of Freedom

IV. Effective Mass

A. Derivation of Effective Mass for the General Case

B. Effective Mass for Simplified Cases

IMPULSE ANALYSIS

In this section equations are derived for the linear and angular velocities of the airplane at the instant of rebound which terminates the impulse. These terminal conditions serve as the initial conditions for the analysis of the free-body motion following rebound, from which the contact conditions for the next impact are determined.

General Impulse Equations for Eccentric Landings

In the general case, any landing gear or combination of landing gears may contact the ground simultaneously during a landing impact. Thus, to make the impulse equations applicable to the general landing problem, they are set up in terms of subscripts i where this notation indicates that the term or equation applies to any of the landing gears making simultaneous contact during the impact under consideration.

The drag impulse on a particular gear making contact with the ground during a given impact is taken equal to the change in angular momentum of the wheel or wheels attached to the gear as their peripheral velocity is increased from an initial value (zero unless prerotation is considered) to a value equal to the forward ground speed of the airplane at the conclusion of the impulse. Thus, the drag impulse I_{d_i} acting at each landing-gear unit i is defined by

$$I_{d_i} = \int_{t_o}^{t_t} F_{d_i}\,dt = \left[\frac{NI_w}{r_c^2}\left(1 - K_p\right)\right]_i \dot{x}_t \tag{1}$$

Furthermore, the sum of the drag impulses on all the gears in contact with the ground must equal the change in forward momentum of the airplane and thus may be defined in terms of the forward velocities of the airplane at the beginning and end of the impact by

$$\sum_i I_{d_i} = -M\left(\dot{x}_t - \dot{x}_o\right) \tag{2}$$

Combining equations (1) and (2) permits the drag impulse to be written in terms of the forward velocity of the airplane at contact $\dot{x}_o$ as follows:

$$I_{d_i} = \frac{\left[\frac{NI_w}{r_c^2}\left(1 - K_p\right)\right]_i \dot{x}_o}{1 + \frac{1}{M}\sum_i \left[\frac{NI_w}{r_c^2}\left(1 - K_p\right)\right]_i} \tag{3}$$

In computing I_{d_i} the denominator of equation (3) may be taken equal to unity in most practical cases.

The vertical impulse acting on each landing gear contacting the ground during a landing impact may be defined in terms of the change in vertical momentum of the airplane during the impact and the impulse of any unbalanced gravitational forces by

$$I_{v_i} = \int_{t_o}^{t_t} F_{v_i}\,dt = K_i\left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right] \tag{4}$$

where K_i is the ratio of the vertical impulse acting on gear i to the sum of the vertical impulses on all the gears making simultaneous contact with the ground; that is,

$$K_i = \frac{\int_{t_o}^{t_t} F_{v_i}\,dt}{\sum_i \int_{t_o}^{t_t} F_{v_i}\,dt} \tag{5}$$

For any particular landing impact

$$\sum_i K_i = 1 \tag{6}$$

and

$$\sum_i I_{v_i} = \sum_i \int_{t_o}^{t_t} F_{v_i}\,dt = \frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t \tag{7}$$

If the airplane has a side component of velocity during landing, the gears in contact with the ground will be subjected to lateral forces acting in a direction opposite to that of the motion. The sum of these side impulses I_{s_i} must of course equal the change in momentum of the airplane in the lateral direction. Thus,

$$\sum_i I_{s_i} = \sum_i \int_{t_o}^{t_t} F_{s_i}\,dt = \frac{W}{g}\left(\dot{y}_t - \dot{y}_o\right) \tag{8}$$

The maximum side impulse which can be developed during an impact may be expressed in terms of the vertical impulse as

$$\sum_i I_{s_i} = -\frac{\dot{y}_o}{\left|\dot{y}_o\right|} K_s\left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right] \tag{9}$$

where the factor K_s is governed by the cornering characteristics of the tire or the skidding friction coefficient, depending on the angle of yaw, and the factor $\frac{\dot{y}_o}{|\dot{y}_o|}$ is employed to indicate the direction of the side impulse. Similarly for any particular gear

$$I_{s_i} = -\frac{\dot{y}_o}{|\dot{y}_o|} K_i K_s \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + (1 - K_L)Wt_t\right] \tag{10}$$

Since the lateral velocity component in landings with any appreciable side drift is generally as large or larger than the lateral velocity corresponding to the maximum side impulse which can be developed, equations (9) and (10) are employed in the present analysis. Particular cases in which the initial side drift velocity is smaller than that corresponding to the maximum side impulse may be treated by assuming that the initial side drift velocity is reduced to zero during the impact and computing the side impulse on the basis of equation (8).

Since the line of action of the resultant impulse force acting on each impacting gear during a landing does not in general pass through the center of gravity of the airplane, changes in airplane angular momentum will occur during the landing impact. These changes are readily expressed in terms of the landing-gear impulses previously defined by equating impulsive moments to angular-momentum changes. In these moment equations average values for the length of the strut and the compressed-tire radius are used. It is assumed that the angles θ and φ do not change during the impulse of the ground forces and that the respective moments are closely determined by the values of these angles at the time of contact. Changes in these angles during the impulse are normally small; however, even in cases where the angular changes might be comparatively large, this assumption will introduce only minor errors into the calculation of the angular-velocity changes due to the impulsive moments.

Summing pitching moments about the lateral axis b yields

$$\begin{aligned} I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) = \sum_i \Big\{ & I_{v_i}\left(c_{i_c} \sin \gamma_o + a_{i_c} \cos \alpha_o\right) - \\ & I_{d_i}\left(c_{i_c} \cos \theta_o - a_{i_c} \sin \theta_o\right) + I_{s_i}\Big[\left(a_{i_c} + \right. \\ & \left. r_{i_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{i_c} + r_{i_c} \cos \theta_o\right)\sin \gamma_o\Big]\tan \beta_o \Big\} \end{aligned} \tag{11}$$

Similarly, summing rolling moments about the longitudinal axis a gives

$$I_{aa}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) = \sum_i \left\{ I_{v_i}\left[\left(c_{i_c} + r_{i_c} \cos \theta_o\right)\sin \beta_o - b_{i_c} \cos \alpha_o\right] - I_{d_i} b_{i_c} \sin \theta_o - I_{s_i}\left[\left(c_{i_c} + r_{i_c} \cos \theta_o\right)\cos \beta_o + b_{i_c} \tan \beta_o \cos \alpha_o\right]\right\} \tag{12}$$

In the preceding equations β, γ, and α are auxiliary angles as shown in figure 2 and are defined in terms of the airplane attitude angles as follows:

$$\left.\begin{aligned} \beta &= \tan^{-1}(\tan \varphi \cos \theta) \\ \gamma &= \tan^{-1}(\tan \theta \cos \varphi) \\ \alpha &= \cos^{-1}\sqrt{1 - \sin^2\beta - \sin^2\gamma} \end{aligned}\right\} \tag{13}$$

Substituting equations (4) and (10) into (11) and (12) gives

$$I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\sum_i K_i\left(c_{i_c} \sin \gamma_o + a_{i_c} \cos \alpha_o\right) - \sum_i I_{d_i}\left(c_{i_c} \cos \theta_o - a_{i_c} \sin \theta_o\right) - K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\tan \beta_o \sum_i K_i\left[\left(a_{i_c} + r_{i_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{i_c} + r_{i_c} \cos \theta_o\right)\sin \gamma_o\right] \tag{14}$$

$$I_{aa}\left(\dot{\phi}_t - \dot{\phi}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right] \sum_i K_i\left[\left(c_{i_c} + r_{i_c} \cos \theta_o\right) \sin \beta_o - b_{i_c} \cos \alpha_o\right] - \sum_i I_{d_i} b_{i_c} \sin \theta_o + K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right] \sum_i K_i\left[\left(c_{i_c} + r_{i_c} \cos \theta_o\right) \cos \beta_o + b_{i_c} \tan \beta_o \cos \alpha_o\right] \tag{15}$$

The preceding equations can be related by establishing the following kinematic conditions which are governed by the rebound characteristics of the landing gear and the geometry of the airplane relative to the ground. A knowledge of the rebound characteristics of the landing gear permits determination of the rebound velocity $\dot{z}_{i_t}$ of an impacting gear in terms of the contact velocity. This relationship may be expressed by

$$\dot{z}_{i_t} = -\dot{z}_{i_o} \sqrt{1 - \eta_{r_i}} \tag{16}$$

where η_{r_i} is the energy-dissipation efficiency of the landing gear. This parameter can be determined from drop-test data or can be estimated from similar designs.

The contact velocity $\dot{z}_{i_o}$ for any particular landing gear is related to the linear and angular velocities of the airplane at the instant of contact by the equation:

$$\dot{z}_{i_o} = \dot{z}_{g_o} + \dot{\theta}_o(a_i \cos \alpha_o + c_i \sin \gamma_o) - \dot{\phi}_o(b_i \cos \alpha_o - c_i \sin \beta_o) \tag{17a}$$

Similarly the rebound velocity is related to the linear and angular velocities at the end of the impulse by the equation:

$$\dot{z}_{i_t} = \dot{z}_{g_t} + \dot{\theta}_t\left(a_i \cos \alpha_t + c_i \sin \gamma_t\right) - \dot{\phi}_t\left(b_i \cos \alpha_t - c_i \sin \beta_t\right) \tag{17b}$$

The elevation of the airplane center of gravity z_{g_t} at the end of the impulse is given by the equation:

$$z_{g_t} = (c_i \cos \theta_t - a_i \sin \theta_t + r_i)\cos \beta_o + b_i \sin \beta_o \tag{18}$$

This quantity z_{g_t} is not actually used in the impulse analysis but is required in establishing initial conditions for the subsequent free-body motion analysis.

Equations (6), (14), (15), and (17b) form a set of simultaneous equations equal in number to the number of gears concurrently in contact with the ground plus three. For any particular case involving a given number of gears in contact with the ground, these equations can be solved simultaneously to determine the values of $\dot{\theta}_t$, $\dot{\varphi}_t$, and $\dot{z}_{g_t}$ in terms of the geometry at the beginning of the impulse, the geometry at the end of the impulse, the landing-gear energy-absorption efficiency, and the duration of the impulse.

The impulse equations, as written, involve the angles θ_t and φ_t which can be treated as variables by the introduction of additional equations. The added complexity involved in the simultaneous solution of these equations is not considered to be warranted, however, since changes in the attitude angles during the impulse are generally small and only minor trigonometric errors are introduced into the impulse solution by assuming $\theta_t = \theta_o$ and $\varphi_t = \varphi_o$.

The impulse analysis takes into consideration the effects of unbalanced wing lift and gravitational forces and thus the duration of the impulse t_t in the unbalanced-weight terms. The equations, however, do not permit treating t_t as a variable in the general case unless additional simultaneous equations are introduced. On the other hand, when the wing lift is equal to the weight of the airplane, the terms containing t_t vanish from the equations. Since in many landings the difference between the wing lift and the weight is relatively small and the impulse is of short duration, neglect of the term $(1 - K_L)Wt_t$ in the calculation of the changes in linear and angular velocity during the impact appears to be justified.

In certain particular cases, such as those involved in carrier landings or landings of unconventional airplanes, the difference between the wing lift and the weight may be large enough to necessitate consideration of the term $(1 - K_L)Wt_t$ in the impulse solution. In such cases t_t could be determined from an equivalent drop test or estimated from previous experience with similar gears.

Impulse Solution for Impact on One Landing Gear

For the particular case of a landing impact on any one gear m, the equations required to determine changes in airplane motion resulting from the impact can be readily obtained from the general impulse equations previously derived. Since $K_m = 1$ in this case (see equation (6)), equations (17b), (14), and (15) may be written as follows:

$$\dot{z}_{m_t} = \dot{z}_{g_t} + \dot{\theta}_t\left(a_m \cos \alpha_t + c_m \sin \gamma_t\right) - \dot{\varphi}_t\left(b_m \cos \alpha_t - c_m \sin \beta_t\right) \tag{19}$$

$$I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\left\{\left(c_{m_c} \sin \gamma_o + a_{m_c} \cos \alpha_o\right) - K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\left(a_{m_c} + r_{m_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \gamma_o\right]\tan \beta_o\right\} - I_{d_m}\left(c_{m_c} \cos \theta_o - a_{m_c} \sin \theta_o\right) \tag{20}$$

$$I_{aa}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\left\{\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \beta_o - b_{m_c} \cos \alpha_o + K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\cos \beta_o + b_{m_c} \tan \beta_o \cos \alpha_o\right]\right\} - I_{d_m} b_{m_c} \sin \theta_o \tag{21}$$

or more simply as,

$$A_{11}\dot{z}_{g_t} + A_{12}\dot{\theta}_t + A_{13}\dot{\varphi}_t + A_{14} = 0 \tag{22}$$

$$A_{21}\dot{z}_{g_t} + A_{22}\dot{\theta}_t + A_{24} = 0 \tag{23}$$

$$A_{31}\dot{z}_{g_t} + A_{33}\dot{\varphi}_t + A_{34} = 0 \tag{24}$$

where the newly introduced constants are defined by

$$A_{11} = 1$$

$$A_{12} = a_m \cos \alpha_t + c_m \sin \gamma_t$$

$$A_{13} = c_m \sin \beta_t - b_m \cos \alpha_t$$

$$A_{14} = -\dot{z}_{m_t}$$

$$A_{21} = -\frac{W}{g}\left\{c_{m_c} \sin \gamma_o + a_{m_c} \cos \alpha_o - K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\left(a_{m_c} + r_{m_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \gamma_o\right]\tan \beta_o\right\}$$

$$A_{22} = I_{bb}$$

$$A_{24} = I_{d_m}\left(c_{m_c} \cos \theta_o - a_{m_c} \sin \theta_o\right) - I_{bb}\dot{\theta}_o - A_{21}\left[\dot{z}_{g_o} - \left(1 - K_L\right)gt_t\right]$$

$$A_{31} = -\frac{W}{g}\left\{\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \beta_o - b_{m_c} \cos \alpha_o + K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|}\left[\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\cos \beta_o + b_{m_c} \tan \beta_o \cos \alpha_o\right]\right\}$$

$$A_{33} = I_{aa}$$

$$A_{34} = I_{d_m} b_{m_c} \sin \theta_o - I_{aa}\dot{\varphi}_o - A_{31}\left[\dot{z}_{g_o} - \left(1 - K_L\right)gt_t\right]$$

Equations (22), (23), and (24) form a set of simultaneous equations which can be solved for $\dot{\theta}_t$, $\dot{\varphi}_t$, and $\dot{z}_{g_t}$ in terms of known quantities by either matrix or algebraic methods. The following results were obtained by algebraic manipulation:

$$\dot{z}_{g_t} = \frac{A_{24}A_{12}A_{33} - A_{22}\left(A_{14}A_{33} - A_{34}A_{13}\right)}{A_{22}\left(A_{11}A_{33} - A_{31}A_{13}\right) - A_{12}A_{21}A_{33}} \tag{25}$$

$$\dot{\theta}_t = -\frac{A_{21}}{A_{22}} \dot{z}_{g_t} - \frac{A_{24}}{A_{22}} \tag{26}$$

$$\dot{\varphi}_t = -\frac{A_{31}}{A_{33}} \dot{z}_{g_t} - \frac{A_{34}}{A_{33}} \tag{27}$$

In most practical cases θ_t, φ_t, and z_{g_t} can be taken equal to θ_o, φ_o, and z_{g_o}, respectively, and t_t in the unbalanced-weight terms can be assumed equal to zero without introducing appreciable errors into the impulse solution.

Impulse Solution for Simultaneous Impact on Any Two Landing Gears

This section treats an impact in which any two gears contact the ground simultaneously and then rebound with vertical velocities which depend on the contact velocity and the energy-dissipation efficiency of each impacting gear. The equations which follow do not require that the gears contact with identical velocities nor have the same rebound characteristics. If the two gears making initial contact are m and n, impulse equations (6), (17b), (14), and (15) may be written as follows:

$$K_m + K_n = 1 \tag{28}$$

$$\dot{z}_{g_t} + \dot{\theta}_t\left(a_m \cos\alpha_t + c_m \sin\gamma_t\right) - \dot{\varphi}_t\left(b_m \cos\alpha_t - c_m \sin\beta_t\right) = \dot{z}_{m_t} \tag{29}$$

$$\dot{z}_{g_t} + \dot{\theta}_t\left(a_n \cos\alpha_t + c_n \sin\gamma_t\right) - \dot{\varphi}_t\left(b_n \cos\alpha_t - c_n \sin\beta_t\right) = \dot{z}_{n_t} \tag{30}$$

$$\begin{aligned}
I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) = {} & \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\Big\{K_m\left(c_{m_c} \sin\gamma_o + a_{m_c} \cos\alpha_o\right) + K_n\left(c_{n_c} \sin\gamma_o + a_{n_c} \cos\alpha_o\right) - \\
& K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|} K_m\left[\left(a_{m_c} + r_{m_c} \sin\theta_o\right)\cos\alpha_o - \left(c_{m_c} + r_{m_c} \cos\theta_o\right)\sin\gamma_o\right]\tan\beta_o - \\
& K_s \frac{\dot{y}_o}{\left|\dot{y}_o\right|} K_n\left[\left(a_{n_c} + r_{n_c} \sin\theta_o\right)\cos\alpha_o - \left(c_{n_c} + r_{n_c} \cos\theta_o\right)\sin\gamma_o\right]\tan\beta_o\Big\} - \\
& I_{d_m}\left(c_{m_c} \cos\theta_o - a_{m_c} \sin\theta_o\right) - I_{d_n}\left(c_{n_c} \cos\theta_o - a_{n_c} \sin\theta_o\right)
\end{aligned} \tag{31}$$

$$I_{aa}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\left\{K_m\left[\left(c_{m_c} + r_{m_c}\cos\theta_o\right)\sin\beta_o - b_{m_c}\cos\alpha_o\right] + K_n\left[\left(c_{n_c} + r_{n_c}\cos\theta_o\right)\sin\beta_o - b_{n_c}\cos\alpha_o\right] + K_s\frac{\dot{y}_o}{\left|\dot{y}_o\right|}K_m\left[\left(c_{m_c} + r_{m_c}\cos\theta_o\right)\cos\beta_o + b_{m_c}\tan\beta_o\cos\alpha_o\right] + K_s\frac{\dot{y}_o}{\left|\dot{y}_o\right|}K_n\left[\left(c_{n_c} + r_{n_c}\cos\theta_o\right)\cos\beta_o + b_{n_c}\tan\beta_o\cos\alpha_o\right]\right\} - I_{d_m}b_{m_c}\sin\theta_o - I_{d_n}b_{n_c}\sin\theta_o \tag{32}$$

Combining equation (28) with equations (31) and (32) and introducing new constants into equations (29) to (32) results in

$$B_{11}\dot{z}_{g_t} + B_{12}\dot{\theta}_t + B_{13}\dot{\varphi}_t + B_{15} = 0 \tag{33}$$

$$B_{21}\dot{z}_{g_t} + B_{22}\dot{\theta}_t + B_{23}\dot{\varphi}_t + B_{25} = 0 \tag{34}$$

$$B_{31}\dot{z}_{g_t} + B_{32}\dot{\theta}_t + B_{34}\left[\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t\right]K_n + B_{35} = 0 \tag{35}$$

$$B_{41}\dot{z}_{g_t} + B_{43}\dot{\varphi}_t + B_{44}\left[\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t\right]K_n + B_{45} = 0 \tag{36}$$

where

$$B_{11} = B_{21} = 1$$

$$B_{12} = a_m\cos\alpha_t + c_m\sin\gamma_t$$

$$B_{13} = c_m\sin\beta_t - b_m\cos\alpha_t$$

$$B_{15} = -\dot{z}_{m_t}$$

$$B_{22} = a_n\cos\alpha_t + c_n\sin\gamma_t$$

$$B_{23} = -\left(b_n\cos\alpha_t - c_n\sin\beta_t\right)$$

$$B_{25} = -\dot{z}_{n_t}$$

$$B_{31} = -\frac{W}{g}\left\{c_{m_c} \sin \gamma_o + a_{m_c} \cos \alpha_o - K_s \frac{\dot{y}_o}{|\dot{y}_o|}\left[\left(a_{m_c} + r_{m_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \gamma_o\right]\tan \beta_o\right\}$$

$$B_{32} = I_{bb}$$

$$B_{34} = -B_{31} - \frac{W}{g}\left\{c_{n_c} \sin \gamma_o + a_{n_c} \cos \alpha_o - K_s \frac{\dot{y}_o}{|\dot{y}_o|}\left[\left(a_{n_c} + r_{n_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{n_c} + r_{n_c} \cos \theta_o\right)\sin \gamma_o\right]\tan \beta_o\right\}$$

$$B_{35} = I_{d_m}\left(c_{m_c} \cos \theta_o - a_{m_c} \sin \theta_o\right) + I_{d_n}\left(c_{n_c} \cos \theta_o - a_{n_c} \sin \theta_o\right) - B_{31}\left[\dot{z}_{g_o} - \left(1 - K_L\right)gt_t\right] - I_{bb}\dot{\theta}_o$$

$$B_{41} = -\frac{W}{g}\left\{\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\sin \beta_o - b_{m_c} \cos \alpha_o + K_s \frac{\dot{y}_o}{|\dot{y}_o|}\left[\left(c_{m_c} + r_{m_c} \cos \theta_o\right)\cos \beta_o + b_{m_c} \tan \beta_o \cos \alpha_o\right]\right\}$$

$$B_{43} = I_{aa}$$

$$B_{44} = -B_{41} - \frac{W}{g}\left\{\left(c_{n_c} + r_{n_c} \cos \theta_o\right)\sin \beta_o - b_{n_c} \cos \alpha_o + K_s \frac{\dot{y}_o}{|\dot{y}_o|}\left[\left(c_{n_c} + r_{n_c} \cos \theta_o\right) \cos \beta_o + b_{n_c} \tan \beta_o \cos \alpha_o\right]\right\}$$

$$B_{45} = I_{d_m}b_{m_c} \sin \theta_o + I_{d_n}b_{n_c} \sin \theta_o - B_{41}\left[\dot{z}_{g_o} - \left(1 - K_L\right)gt_t\right] - I_{aa}\dot{\varphi}_o$$

If the term $\left[\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t\right]K_n$ in equations (35) and (36) is treated as a new variable, equations (33) to (36) form a set of simultaneous linear equations, from which the following terminal conditions are obtained:

$$\dot{\theta}_t = \frac{C_2C_6 - C_3C_5}{C_1C_5 - C_2C_4} \tag{37}$$

$$\dot{\varphi}_t = -\frac{C_1}{C_2}\dot{\theta}_t - \frac{C_3}{C_2} \tag{38}$$

$$\dot{z}_{g_t} = -\frac{B_{12}}{B_{11}}\dot{\theta}_t - \frac{B_{13}}{B_{11}}\dot{\varphi}_t - \frac{B_{15}}{B_{11}} \tag{39}$$

$$\left[\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t\right]K_n = -\frac{B_{45}}{B_{44}} - \frac{B_{41}}{B_{44}}\dot{z}_{g_t} - \frac{B_{43}}{B_{44}}\dot{\varphi}_t \tag{40}$$

where

$$C_1 = B_{12}B_{21} - B_{22}B_{11}$$

$$C_2 = B_{13}B_{21} - B_{23}B_{11}$$

$$C_3 = B_{15}B_{21} - B_{25}B_{11}$$

$$C_4 = B_{22}\left(B_{31}B_{44} - B_{41}B_{34}\right) - B_{32}B_{44}B_{21}$$

$$C_5 = B_{23}\left(B_{31}B_{44} - B_{41}B_{34}\right) + B_{21}B_{43}B_{34}$$

$$C_6 = B_{25}\left(B_{31}B_{44} - B_{41}B_{34}\right) - B_{21}\left(B_{35}B_{44} - B_{45}B_{34}\right)$$

Solving equation (40) for K_n yields

$$K_n = \frac{-\frac{B_{45}}{B_{44}} - \frac{B_{41}}{B_{44}}\dot{z}_{g_t} - \frac{B_{43}}{B_{44}}\dot{\varphi}_t}{\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t} \tag{41}$$

and from equation (28) K_m is given by

$$K_m = 1 - K_n$$

Impulse Solution for Symmetrical Impact

This section treats symmetrical landings on two identical gears located symmetrically with respect to the airplane center plane as well as symmetrical landings on one gear located in the airplane plane of symmetry. In such landings there is no initial roll angle, rolling velocity, or side velocity - that is, $\varphi_o = 0$, $\dot{\varphi} = 0$, and $K_s = 0$. In the two-wheel case $b_m = -b_n$, $a_m = a_n = a_{mn}$, $c_m = c_n = c_{mn}$, $\eta_{r_m} = \eta_{r_n} = \eta_{r_{mn}}$, $\dot{z}_{m_t} = \dot{z}_{n_t} = \dot{z}_{mn_t}$, and $K_m = K_n$. The foregoing conditions, in conjunction with equation (6), permits equation (17b), (14), and (15) to be written as follows:

$$\dot{z}_{mn_t} = \dot{\theta}_t\left(a_{mn} \cos \theta_t + c_{mn} \sin \theta_t\right) + \dot{z}_{g_t} \tag{42}$$

$$I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) = \left[\frac{W}{g}\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)Wt_t\right]\left(c_{mn_c} \sin \theta_o + a_{mn_c} \cos \theta_o\right) - \left(I_{d_m} + I_{d_n}\right)\left(c_{mn_c} \cos \theta_o - a_{mn_c} \sin \theta_o\right) \tag{43}$$

$$I_{aa}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) = 0 \tag{44}$$

If the subscript mn is considered to represent a single landing gear in the airplane plane of symmetry and $I_{d_{mn}}$ is substituted for $\left(I_{d_m} + I_{d_n}\right)$, the foregoing equations become directly applicable to symmetrical impacts on such gears.

For the case of a symmetrical landing, changes in linear and angular velocity of the airplane during the impulse are completely defined by equations (42) and (43) which can be written in terms of new constants as

$$D_{11}\dot{z}_{g_t} + D_{12}\dot{\theta}_t + D_{13} = 0 \tag{45}$$

$$D_{21}\dot{z}_{g_t} + D_{22}\dot{\theta}_t + D_{23} = 0 \tag{46}$$

where

$$D_{11} = 1$$

$$D_{12} = a_{mn} \cos \theta_t + c_{mn} \sin \theta_t$$

$$D_{13} = -\dot{z}_{mn_t}$$

$$D_{21} = -\frac{W}{g}\left(c_{mn_c} \sin \theta_o + a_{mn_c} \cos \theta_o\right)$$

$$D_{22} = I_{bb}$$

$$D_{23} = -D_{21}\left[\dot{z}_{g_o} - \left(1 - K_L\right)gt_t\right] - I_{bb}\dot{\theta}_o + \left(I_{d_m} + I_{d_n}\right)\left(c_{mn_c} \cos \theta_o - a_{mn_c} \sin \theta_o\right)$$

Solution of equations (45) and (46) gives

$$\dot{z}_{g_t} = \frac{D_{23}D_{12} - D_{13}D_{22}}{D_{11}D_{22} - D_{21}D_{12}} \tag{47}$$

$$\dot{\theta}_t = -\frac{D_{11}}{D_{12}} \dot{z}_{g_t} - \frac{D_{13}}{D_{12}} \tag{48}$$

and, of course,

$$\dot{\varphi}_t = \dot{\varphi}_o = 0 \tag{49}$$

FREE-BODY MOTION ANALYSIS

This section is concerned with the motion of the airplane during the interval between rebound and the next impact. The terminal conditions of the impulse analysis serve as the initial conditions for the free-body analysis, from which the contact conditions for the next impact are determined.

General Solution for Motion Following Rebound

Following a landing impact, the landing gear or gears i rebound from the ground, and the airplane may be considered a free body under the influence of lift and gravitational forces during the interval between the rebound and the subsequent impact on gear j. Throughout this interval a constant average wing lift force is assumed to act vertically through the center of gravity of the airplane and aerodynamic moments are neglected. Thus, since no eccentric forces are involved, the pitching and rolling angular velocities remain constant at the values determined from the impulse calculation.

The vertical acceleration of the airplane with constant lift and gravity forces acting at the center of gravity is defined by

$$\frac{W}{g}\ddot{z}_g = -W\left(1 - K_L\right) \tag{50}$$

Integrating equation (50) between the limits $t = t_t$ and t equal to any time after the rebound and prior to the next impact results in

$$\frac{W}{g}\left(\dot{z}_g - \dot{z}_{g_t}\right) = -W\left(1 - K_L\right)\left(t - t_t\right) \tag{51}$$

Integrating equation (51) in turn and solving for z_g gives

$$z_g = z_{g_t} + \dot{z}_{g_t}\left(t - t_t\right) - \frac{g}{2}\left(1 - K_L\right)\left(t - t_t\right)^2 \tag{52}$$

At the instant of contact of gear j (the next gear to contact) the height of the center of gravity above the ground $z_{g_{f_j}}$ is defined in terms of airplane geometry and attitude by

$$z_{g_{f_j}} = \frac{c_j \cos\theta_{f_j} - a_j \sin\theta_{f_j} + r_j + b_j \tan\varphi_{f_j} \cos\theta_{f_j}}{\sqrt{1 + \tan^2\varphi_{f_j} \cos^2\theta_{f_j}}} \tag{53}$$

Since the pitching and rolling angular velocities remain constant during the interval between rebound and the time of the next impact t_f, the airplane attitude at the instant of contact of the next gear j is given by

$$\theta_{f_j} = \theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right) \tag{54}$$

$$\varphi_{f_j} = \varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right) \tag{55}$$

Substituting equations (54) and (55) into the geometrical equation (53) yields

$$z_{g_{f_j}} = \frac{r_j + \left\{c_j + b_j \tan\left[\varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right)\right]\right\} \cos\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right] - a_j \sin\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right]}{\sqrt{1 + \tan^2\left[\varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right)\right] \cos^2\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right]}} \tag{56}$$

At the time of contact of gear j, equation (52) may be written as

$$z_{g_{f_j}} = z_{g_t} + \dot{z}_{g_t}\left(t_{f_j} - t_t\right) - \frac{g}{2}\left(1 - K_L\right)\left(t_{f_j} - t_t\right)^2 \tag{57}$$

Equating the right-hand members of equations (56) and (57) yields an equation containing only one unknown $\left(t_{f_j} - t_t\right)$ which represents the time interval between the rebound of gear or gears i and the next impact on gear j:

$$z_{g_t} + \dot{z}_{g_t}\left(t_{f_j} - t_t\right) - \frac{g}{2}\left(1 - K_L\right)\left(t_{f_j} - t_t\right)^2 =$$

$$\frac{r_j + \left\{c_j + b_j \tan\left[\varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right)\right]\right\} \cos\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right] - a_j \sin\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right]}{\sqrt{1 + \tan^2\left[\varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right)\right] \cos^2\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right]}} \tag{58}$$

Numerous transcendental terms in the preceding equation (58) prevent an explicit solution for $\left(t_{f_j} - t_t\right)$; numerical solutions for particular cases may be obtained by trial and error or graphical methods if desired. If, however, it is possible to assume, as is generally the case, that the final attitude angles θ_{f_j} and φ_{f_j} are small and, therefore, that transcendental terms may be accurately approximated by the first terms of their respective expansions, equation (58) may be written

$$z_{g_t} + \dot{z}_{g_t}\left(t_{f_j} - t_t\right) - \frac{g}{2}\left(1 - K_L\right)\left(t_{f_j} - t_t\right)^2 = r_j + c_j - a_j\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right] + b_j\left[\varphi_t + \dot{\varphi}_t\left(t_{f_j} - t_t\right)\right] \tag{59}$$

Equation (59) can be readily solved for the desired time interval $\left(t_{f_j} - t_t\right)$ and yields

$$\left(t_{f_j} - t_t\right) = \frac{\dot{z}_{g_t} + a_j\dot{\theta}_t - b_j\dot{\varphi}_t \pm \sqrt{\left(\dot{z}_{g_t} + a_j\dot{\theta}_t - b_j\dot{\varphi}_t\right)^2 + 2g\left(1 - K_L\right)\left(z_{g_t} - c_j + a_j\theta_t - b_j\varphi_t - r_j\right)}}{g(1 - K_L)} \tag{60}$$

In the particular case when $K_L = 1$, the preceding equation yields an indeterminate solution. If it is assumed in equation (59) that $K_L = 1$, however, a finite solution for the time interval in this particular case is obtained as

$$\left(t_{f_j} - t_t\right) = \frac{c_j - a_j\theta_t + r_j + b_j\varphi_t - z_{g_t}}{\dot{z}_{g_t} + a_j\dot{\theta}_t - b_j\dot{\varphi}_t} \tag{61}$$

The quantity $\left(t_{f_j} - t_t\right)$ may be determined from equation (58), (60), or (61), depending on the case being treated. From this result the attitude and position of the airplane at the time of contact of gear j may be determined by means of equations (54), (55), and (53). Examination of the attitude angles thus calculated will reveal whether the assumed landing gear j is actually the next gear to contact or whether some other gear would contact the ground prior to this time. If the calculated attitude angles indicate that the elevation of some other landing gear is lower than that of the gear for which the calculation was made, a new calculation must be made for the gear having the lowest elevation. In general, however, the results of the impulse calculations make it fairly obvious which gear will contact the ground following a given impact.

From the calculated time interval of the free-body motion, the velocity of the center of gravity $\dot{z}_{g_{f_j}}$ at the time of the next impact can be determined by means of the relationship

$$\dot{z}_{g_{f_j}} = \dot{z}_{g_t} - g\left(1 - K_L\right)\left(t_{f_j} - t_t\right) \tag{62}$$

The vertical contact velocity for gear j is given by

$$\dot{z}_{j_f} = \dot{z}_{g_{f_j}} + \dot{\theta}_t\left(a_j \cos \alpha_{f_j} + c_j \sin \gamma_{f_j}\right) - \dot{\varphi}_t\left(b_j \cos \alpha_{f_j} - c_j \sin \beta_{f_j}\right) \tag{63}$$

where β, γ, and α (previously defined by equations (13)) are determined from the results of equations (54) and (55) and $\dot{\theta}_t$ and $\dot{\varphi}_t$ are the angular velocities at the instant of rebound as obtained from the impulse analysis.

The foregoing analysis permits the determination of landing-gear contact conditions for an impact subsequent to the rebound which terminates a preceding impact. As can be seen, some of the equations in the free-body motion analysis involve the attitude angles θ_t and φ_t and the elevation z_{g_t} at the end of the impulse. When the wing lift is nearly equal to the weight, which is the case in most landings, very little error is introduced in the calculated contact conditions for the next impact if in the free-body equations θ_t, φ_t, and z_{g_t} are assumed equal to θ_o, φ_o, and z_{g_o}, respectively. These assumptions are also satisfactory when the wing lift is appreciably different from the weight, if the duration of the impulse is short. A subsequent section of the paper discusses the application of the analysis to cases where the wing lift is considerably different from the weight and the impulse is of long duration.

Free-Body Motion Solution for a Symmetrical Landing

Although the previously developed general solution for the motion following rebound may be used in conjunction with any one of the particular impulse solutions developed, a somewhat simpler free-body solution can be obtained for the case of a symmetrical landing. In a symmetrical landing the roll angle and rolling velocity following rebound are zero, and equation (60) may be written as follows:

$$\left(t_{f_{C_j}} - t_t\right) = \frac{\dot{z}_{g_t} + a_j\dot{\theta}_t \pm \sqrt{\left(\dot{z}_{g_t} + a_j\dot{\theta}_t\right)^2 + 2g\left(1 - K_L\right)\left(z_{g_t} - c_j + a_j\theta_t - r_j\right)}}{g\left(1 - K_L\right)} \tag{64}$$

Equation (56) of the general solution may also be simplified considerably as follows:

$$z_{g_{f_j}} = c_j \cos\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right] - a_j \sin\left[\theta_t + \dot{\theta}_t\left(t_{f_j} - t_t\right)\right] + r_j \tag{65}$$

Values for $\dot{z}_{g_{f_j}}$ and θ_{f_j} in the symmetrical case may be obtained by substitution of $\left(t_{f_j} - t_t\right)$ from equation (64) into equations (62) and (54). With these results the contact velocity for the next impact may be computed by means of equation (63) simplified to the following form:

$$\dot{z}_{j_f} = \dot{z}_{g_{f_j}} + \dot{\theta}_t\left(a_j \cos \theta_{f_j} + c_j \sin \theta_{f_j}\right) \tag{66}$$

SIMPLIFIED ANALYSIS FOR A SYSTEM WITH TWO DEGREES OF FREEDOM

The solutions presented in the impulse and free-body analyses permit the determination of landing-gear impact conditions for several different types of eccentric landings and include the effects of a number of variables which influence the course of such landings. By greatly simplifying the analysis, qualitative results which show the effects of some of the major parameters can be readily obtained in nondimensional form. Such results can be obtained by simplifying the geometry of the airplane, by considering angular motion in the rolling direction only, and by assuming that the initial rolling velocity is zero and that the weight of the airplane is balanced by wing lift throughout the interval between the initial and second impacts.

With the preceding assumptions, impulse and geometric equations describing the initial impact on gear i may be written as follows:

$$I_{v_i} = M\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) \qquad \text{(see equation (4))} \tag{67}$$

$$I_{aa}\dot{\varphi}_t = -I_{v_i} b_i \qquad \text{(see equation (12))} \tag{68}$$

$$\dot{z}_{i_t} = \dot{z}_{g_t} - b_i\dot{\varphi}_t \qquad \text{(see equation (17b))} \tag{69}$$

where the rebound velocity $\dot{z}_{i_t}$ is defined in terms of the initial descent velocity of the airplane and the energy-dissipation efficiency of the impacting gear is defined by equation (16) as

$$\dot{z}_{i_t} = -\dot{z}_{g_o}\sqrt{1 - \eta_{r_i}}$$

Simultaneous solution of the four preceding equations for the conditions at the end of the impulse yields

$$\dot{\varphi}_t = \frac{Mb_i\dot{z}_{g_o}\left(1 + \sqrt{1 - \eta_{r_i}}\right)}{I_{aa} + Mb_i^{\,2}} \tag{70}$$

and

$$\dot{z}_{g_t} = \frac{Mb_i^{\,2}\dot{z}_{g_o}\left(1 + \sqrt{1 - \eta_{r_i}}\right)}{I_{aa} + Mb_i^{\,2}} - \dot{z}_{g_o}\sqrt{1 - \eta_{r_i}} \tag{71}$$

Since the weight of the airplane is assumed to be balanced by wing lift, the angular and linear velocities of the airplane at the time of the next impact are determined by the results of the impulse solution (equations (70) and (71)). Thus the contact velocity of the opposite gear (which is the next gear to contact) may be obtained from these impulse results by

$$\dot{z}_{j_f} = \dot{z}_{g_t} - b_j\dot{\varphi}_t \tag{72}$$

Since the two gears involved are symmetrically located, $b_j = -b_i$, and substitution of equations (70) and (71) into equation (72) yields

$$\dot{z}_{j_f} = \frac{2Mb_i^2\dot{z}_{g_o}\left(1 + \sqrt{1 - \eta_{r_i}}\right)}{I_{aa} + Mb_i^2} - \dot{z}_{g_o}\sqrt{1 - \eta_{r_i}} \tag{73}$$

Simplifying equation (73) and solving for the dimensionless velocity ratio $\dot{z}_{j_f}/\dot{z}_{g_o}$ gives

$$\frac{\dot{z}_{j_f}}{\dot{z}_{g_o}} = \frac{2\left(1 + \sqrt{1 - \eta_{r_i}}\right)}{\dfrac{\rho_{aa}^2}{b_i^2} + 1} - \sqrt{1 - \eta_{r_i}} \tag{74}$$

Equation (74) was used to compute the velocity ratios presented in figure 1. The energy-dissipation efficiency was assumed to be 100 percent $\left(\eta_{r_i} = 1\right)$ in these computations.

EFFECTIVE MASS

Fundamental Concepts

The basic concept of an effective mass involves the representation of a complex inertia system by a single equivalent mass. As applied to landing impacts, the concept is of considerable value since it permits the simulation of eccentric landing impacts by vertical-drop tests of the landing gear and furthermore provides a means for comparing the severity of such impacts analytically.

Formulas for effective mass are conventionally derived on an equal-acceleration basis (reference 1) and involve the assumptions that the attitude of the airplane does not change appreciably during the impact and that the orientation of the resultant ground reaction remains fixed throughout the impact. If these assumptions are substantially correct, a simple vertical impact with such an effective mass will result in the same instantaneous vertical acceleration, vertical impulse, and impact energy as in the corresponding eccentric impact. The first assumption mentioned is in general quite satisfactory, as previously noted in the impulse-analysis section; the second assumption, however, is questionable for actual airplane landings with drag and side forces, since it implies a constant coefficient of friction between the tire and runway, which generally is not correct (references 2 and 3). Since the orientation of the resultant force actually varies with time during a landing, a variable effective mass would be required to produce identical instantaneous accelerations in an equivalent drop test; the use of a constant mass calculated by assuming some arbitrary direction of the resultant force could result in an impact having different values of vertical acceleration, vertical impulse, and impact energy than the eccentric landing being simulated.

In the present paper, which attempts to develop a method for determining contact conditions without making arbitrary assumptions regarding the time histories of the various components of the landing-gear reactions or the direction of the resultant ground force, equations for effective mass are derived on the basis of equal impulse rather than equal acceleration. That is, an average effective mass is determined in such a manner that the total vertical impulse and the change in energy associated with the vertical travel of the landing gear are the same in a vertical impact with the effective mass as in the actual eccentric landing of the airplane, regardless of the direction of the resultant ground force. The average landing-gear vertical accelerations are also identical in both cases, although the instantaneous accelerations are not necessarily exactly the same at all times during the impact. If the resultant ground reaction remains fixed in direction throughout the eccentric impact, the instantaneous accelerations will also be identical and the effective mass determined by the present treatment will be exactly the same as the effective mass obtained on the conventional equal-acceleration basis.

Derivation of Effective Mass for the General Case

In the following derivation of the effective mass for the general case, a simultaneous impact on any landing gear or combination of landing gears is considered and it is assumed that vertical, drag, and side forces act on each gear in contact with the ground.

Effective mass expressed in terms of changes in airplane velocity resulting from a landing impact.- The effective mass acting on each landing gear i in simultaneous contact with the ground can be expressed quite simply in terms of the changes in airplane velocity resulting from the landing impact. Expressions for the vertical impulse on any landing gear in contact with the ground and for the total vertical impulse in an eccentric impact have been given by equations (4) and (7), respectively. It is desired to determine an effective mass which, when acting on a landing gear in a vertical impact, will produce the same vertical impulse and the same rebound velocity of the gear as in the eccentric impact when the initial contact velocities and lift-weight ratios are the same in both cases. The impulse produced by the effective mass is given by

$$I_{v_i} = M_{e_i}\left(\dot{z}_{i_t} - \dot{z}_{i_o}\right) + \left(1 - K_L\right)W_{e_i}t_t \qquad (75)$$

Equating equations (4) and (75) gives an equation for the effective mass acting on gear i

$$M_{e_i} = M\frac{K_i\left[\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + \left(1 - K_L\right)gt_t\right]}{\left(\dot{z}_{i_t} - \dot{z}_{i_o}\right) + \left(1 - K_L\right)gt_t} \qquad (76a)$$

If the wing lift is nearly equal to the weight or the time duration of the impulse is small, as is often the case, equation (76a) becomes simply

$$M_{e_i} = M\frac{K_i\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right)}{\left(\dot{z}_{i_t} - \dot{z}_{i_o}\right)} \qquad (76b)$$

<u>Effective mass expressed in terms of landing-gear impulses</u>.- Although equation (76a) provides the simplest means of computing values of effective mass if results of an impulse solution are available, it is of interest to obtain an expression for the effective mass in terms of values of landing-gear vertical, drag, and side impulses which can be determined from time histories of the landing-gear forces. Such an expression can be obtained by considering the changes in angular momentum during the impact, equations (11) and (12), which can be written as

$$\sum_i I_{v_i}E1_i - \sum_i I_{d_i}E2_i + \sum_i I_{s_i}E3_i = I_{bb}\left(\dot{\theta}_t - \dot{\theta}_o\right) \qquad (77)$$

$$\sum_i I_{v_i}E4_i - \sum_i I_{d_i}E5_i - \sum_i I_{s_i}E6_i = I_{aa}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) \qquad (78)$$

and by making use of the kinematic relationships, equations (17a) and (17b), which define the initial and final velocities of the landing gear i in terms of the angular velocities of the airplane at the beginning and end of the impact

$$\dot{z}_{i_o} = \dot{z}_{g_o} + E_{7_i}\dot{\theta}_o - E_{8_i}\dot{\varphi}_o \tag{79a}$$

$$\dot{z}_{i_t} = \dot{z}_{g_t} + E_{7_i}\dot{\theta}_t - E_{8_i}\dot{\varphi}_t \tag{79b}$$

If small changes in attitude during the impact are neglected, the constants may be defined as follows:

$$E_{1_i} = a_{i_c} \cos \alpha_o + c_{i_c} \sin \gamma_o$$

$$E_{2_i} = c_{i_c} \cos \theta_o - a_{i_c} \sin \theta_o$$

$$E_{3_i} = \left[\left(a_{i_c} + r_{i_c} \sin \theta_o\right)\cos \alpha_o - \left(c_{i_c} + r_{i_c} \cos \theta_o\right)\sin \gamma_o\right]\tan \beta_o$$

$$E_{4_i} = \left(c_{i_c} + r_{i_c} \cos \theta_o\right)\sin \beta_o - b_{i_c} \cos \alpha_o$$

$$E_{5_i} = b_{i_c} \sin \theta_o$$

$$E_{6_i} = \left(c_{i_c} + r_{i_c} \cos \theta_o\right)\cos \beta_o + b_{i_c} \tan \beta_o \cos \alpha_o$$

$$E_{7_i} = a_i \cos \alpha_o + c_i \sin \gamma_o$$

$$E_{8_i} = b_i \cos \alpha_o - c_i \sin \beta_o$$

Combining equations (4) and (76a) and substituting equations (79) yields

$$M_{e_i} = \frac{I_{v_i}}{\dot{z}_{g_t} - \dot{z}_{g_o} + \left(1 - K_L\right)gt_t + E_{7_i}\left(\dot{\theta}_t - \dot{\theta}_o\right) - E_{8_i}\left(\dot{\varphi}_t - \dot{\varphi}_o\right)} \tag{80}$$

Substituting equations (7), (77), and (78) into equation (80) results in the following expression for the effective mass in terms of the landing-gear impulses:

$$M_{e_i} = \frac{I_{v_i}}{\frac{1}{M}\sum_i I_{v_i} + \frac{E7_i}{I_{bb}}\sum_i \left(I_{v_i}E1_i - I_{d_i}E2_i + I_{s_i}E3_i\right) - \frac{E8_i}{I_{aa}}\sum_i \left(I_{v_i}E4_i - I_{d_i}E5_i - I_{s_i}E6_i\right)} \tag{81}$$

or

$$M_{e_i} = \frac{MI_{v_i}}{\sum_i I_{v_i} + \frac{E7_i}{\rho_{bb}{}^2}\sum_i \left(I_{v_i}E1_i - I_{d_i}E2_i + I_{s_i}E3_i\right) - \frac{E8_i}{\rho_{aa}{}^2}\sum_i \left(I_{v_i}E4_i - I_{d_i}E5_i - I_{s_i}E6_i\right)} \tag{82}$$

where

$$\rho_{aa}{}^2 = \frac{I_{aa}}{M}$$

$$\rho_{bb}{}^2 = \frac{I_{bb}}{M}$$

Equations (81) and (82) permit the determination of the effective mass if the impulses acting on all the landing gears contacting the ground simultaneously during an impact are known.

Effective Mass for Simplified Cases

If only one gear contacts the ground and if the drag and side impulses acting on this gear are assumed to be zero, a simplified equation for the effective mass may be written as follows:

$$M_{e_i} = \frac{M}{1 + \dfrac{E_{1_i}E_{7_i}}{\rho_{bb}^{\;2}} - \dfrac{E_{4_i}E_{8_i}}{\rho_{aa}^{\;2}}} \tag{83}$$

Furthermore, if rotational freedom is allowed in the rolling direction only, I_{bb} and therefore ρ_{bb} are infinite, and if the small shock-strut deflections and small angularity effects considered in the foregoing derivation are neglected, the simple effective-mass formula for a system having two degrees of freedom (vertical translation and rolling) is obtained.

$$M_{e_i} = M\left[\frac{1}{1 + \left(\dfrac{b_i}{\rho_{aa}}\right)^2}\right] \tag{84}$$

The effective-mass ratios presented in figure 1 were computed by means of equation (84).

APPLICABILITY AND LIMITATIONS

The impulse-momentum method of determining over-all changes in airplane linear and angular velocities resulting from an eccentric landing impact is employed in the present analysis in order to eliminate the necessity of making arbitrary assumptions regarding the time histories of the landing-gear reactions. The method is considered applicable to the landing-impact problem since landing-gear impulses are generally of short duration and changes in attitude during the impulse of the ground forces are usually small. The present treatment is directly applicable to impacts where one or more landing gears contact the ground simultaneously but requires that the impulses on these gears be largely completed prior to the next impact.

A brief discussion of the applicability and limitations of various aspects of the analysis is presented in the following sections.

Landing-Gear Reactions

In the impulse analysis, changes in airplane linear and angular velocities resulting from vertical, drag, and side impulses on the landing gear were determined. The analysis assumes that, for any given contact velocity, the rebound velocity of an impacting gear is known or can be determined from drop tests or from knowledge of the energy-dissipation efficiency of similar gears. For most conventional landing gears this efficiency lies between approximately 80 and 95 percent. In the case of any particular gear the energy-dissipation efficiency can vary with differences in contact velocity, effective mass, or wing lift. Preliminary calculations indicate that moderate variations in energy-dissipation efficiency have only a secondary effect on critical impact conditions.

It was assumed in the analysis that the drag impulse acting on an impacting gear is independent of the vertical or side impulses and does not depend on some arbitrary tire-ground friction coefficient. Instead the drag impulse is taken equal to the change in angular momentum of the wheels as the peripheral velocity is increased to a value equal to the forward speed of the airplane. This approach is considered valid since in a landing the wheels are generally accelerated up to ground speed prior to rebound and, in most cases, even prior to the attainment of the maximum vertical load.

In the present analysis the side impulse is expressed as a fraction of the vertical impulse. In a landing, of course, the ratio of the side force to the vertical force is not a constant but varies throughout both the wheel spin-up and yawed rolling phases of the impact. Variations of this ratio during wheel spin-up are due to the change in direction of the resultant skidding velocity as the wheel comes up to speed and to variations in the skidding friction coefficient. Variations of this ratio during the yawed rolling phase result from the gradual decrease in yaw angle and the variation in the cornering coefficient of the tire with vertical load (see reference 4). The coefficient K_s used in the present analysis therefore represents an average value for the ratio of the side force to the vertical force which depends on the impact conditions and the tire characteristics. Expressing the side impulse as a fraction of the vertical impulse is appropriate when the lateral momentum of the airplane is sufficiently large that the side drift velocity is not reduced to zero during the impact. For cases where the initial side drift velocity is small, the side impulse may be taken equal to the initial lateral momentum of the airplane in accordance with equation (8).

Time Interval between Impacts

The present treatment, as previously noted, is restricted to landings in which the impulses on the landing gear or gears in contact with the ground are largely completed prior to the next impact. In many cases, of course, when the airplane attitude angles at contact are small, the impulses on the first gear or gears to contact may not be completed before the next impact occurs and the impulses may overlap appreciably. In such cases, however, the contact conditions for subsequent impacts are ordinarily expected to be less severe than in impacts with a greater degree of eccentricity. Since the foregoing analysis is primarily intended for the determination of critical impact conditions, application of the method to impacts with appreciable overlap is not considered in the present paper.

Aerodynamic Effects

Effect of unbalanced weight during impulse.- The impulse analysis with the inclusion of the term $(1 - K_L)Wt_t$ takes into account the effects of unbalanced weight acting during an impulse, which may not be negligible in some cases where the wing lift is appreciably different from the weight and where the time duration of the impulse is fairly large. For such conditions, numerical values of t_t required for the accurate determination of the angular and linear velocities at the end of the impulse, which are initial conditions for the free-body motion analysis, can be obtained from drop-test data or estimated from experience with similar landing gears. In such cases, it may also be desirable to use the values of t_t in conjunction with the computed angular and linear velocities at the end of the impulse to calculate improved values for the geometric parameters θ_t, φ_t, and z_{g_t} which also serve as initial conditions for the free-body motion analysis.

Variable aerodynamic effects.- The present analysis assumes that the aerodynamic forces and moments which act on an airplane during each stage of a landing can be represented by a constant average lift force passing through the center of gravity of the airplane. Although variations in wing lift do, of course, occur during a landing because of changes in vertical velocity and attitude, experimental data indicate that such variations are generally small enough during the relatively short time interval for any given stage of the motion to permit the assumption of a constant average value for the lift factor. Variations in the aerodynamic moments neglected in the analysis may, of course, have some effect on the angular velocities of the airplane during the free-body phase of the motion. For an inherently stable airplane such moments will generally tend to oppose changes in motion resulting from landing-gear impulses. As a result, any differences arising from neglect of the aerodynamic moments should be expected to make the calculated results somewhat conservative since, in such cases, the angular velocities of the airplane will ordinarily be slightly less than the calculations indicate.

In order to permit an order-of-magnitude evaluation of the aforementioned variable aerodynamic effects, a brief supplementary study, which includes the effects of wing damping in roll, unbalanced weight at initial contact, and variations in wing lift during impact, has been made. This analysis, presented in the appendix, is essentially concerned with the changes in airplane motion which result from a landing impact on one gear, where the time duration of the ground impulse is finite and known. In this simplified study the airplane is considered to have freedom in roll and vertical translation only, and angularity effects are neglected. An evaluation of the importance of variable aerodynamic effects by comparison of the results obtained from the supplementary study with those obtained from the foregoing analysis is presented in the following section entitled "Calculated Results and Discussion."

In considering the importance of these variable aerodynamic effects, it should, of course, be borne in mind that the aerodynamic forces and moments during the landing impact are to a certain extent subject to pilot control. This is particularly true of the pitching moment over which the pilot ordinarily has appreciable control and, to a lesser extent, the rolling moment over which the pilot has relatively little control during the landing impact. Thus, piloting technique may serve to modify the motions of the airplane and produce impacts of somewhat greater or less severity than indicated by purely analytical studies.

Small-Angle Approximations

The equations presented in the impulse analysis contain trigonometric functions in the constants which may be simplified by means of the usual first-order approximations when the angles involved are small (say 12° or less), as is normally the case. In the free-body motion analysis, in order to obtain explicit solutions, it was necessary to assume that the angles defining the attitude of the airplane at the end of the free-body phase of the motion are small so that the trigonometric functions of these angles could be represented by the first terms of their respective expansions. These approximations are satisfactory for angles of, say, 12° or less and thus are applicable to landings of most conventional airplanes. If the attitude angles in some particular case are large enough to invalidate these assumptions, trial-and-error solutions may be necessary to calculate the contact conditions accurately. In such cases, however, keeping the angles within the limits of the small-angle approximations may be possible by judicious choice of the reference axes.

CALCULATED RESULTS AND DISCUSSION

In order to investigate the effects of important factors not considered in the simplified treatment previously discussed in connection with figure 1, the more detailed study presented in the analysis sections has been applied to the calculation of landing-gear impact conditions for eccentric landings of a cargo-type airplane having the inertia and geometric characteristics given in table I. The main gears of this airplane are located slightly outboard of the rolling radius of gyration $\left(\frac{b_i}{\rho_{aa}} \approx 1.1\right)$. Most of the results which follow are for eccentric landings in which the first impact occurs on one main gear and the second impact takes place on the opposite main gear. These calculations show the effects of several of the more important approach conditions on impact severity. The importance of variable aerodynamic lift and damping in roll is briefly examined by means of calculations based on the supplementary analysis given in the appendix.

In addition, calculated results are presented which permit comparison of the severity of second impacts on auxiliary gears of a tricycle and quadricycle configuration. These results, which were calculated by means of the more detailed analysis presented in the main portion of the text, are for eccentric landings in which the first impact takes place on a main gear and the second impact occurs on an auxiliary gear.

Impact Severity for Main Gears

Comparison of impacts in eccentric and symmetrical landings.- Contact conditions calculated for eccentric landings in which initial contact takes place on one main gear and the second impact takes place on the opposite main gear are presented in figure 3. In these landings the airplane was assumed to be pitched 3° upward and rolled 7° to the left at the instant of initial contact. The pitching, rolling, and side drift velocities of the airplane were assumed zero at initial contact. To permit comparison, results are also presented for symmetrical landings on both main gears.

A comparison of the vertical velocity for first and second impacts in eccentric landings is given by the curves of figure 3(a). These curves show that the vertical velocity for the second impact in an eccentric landing can be appreciably greater than the contact velocity for the first impact and thus also greater than the contact velocity for each gear in a symmetrical landing with the same initial descent velocity of the center of gravity. It is evident that the greatest increase in vertical velocity for the second impact occurs in landings where the wing lift is less than the weight of the airplane, as might reasonably be expected.

Comparisons of the impact energy for the first and second impacts in an eccentric landing and for each gear in a symmetrical landing are shown in figure 3(b). The values of energy presented in this figure include the kinetic energy, determined by the impact velocity and the effective mass, as well as the potential energy based on the unbalanced effective weight and an assumed mass travel of 1 foot. The curves presented indicate that the energy which must be absorbed by the second gear to contact in an eccentric landing can be considerably greater than the energy per gear in a symmetrical landing with the same initial center-of-gravity descent velocity.

For the particular airplane considered, the energy associated with the initial impact in the eccentric landings is slightly less than the energy per gear in the symmetrical case since the gears are located outboard of the rolling radius of gyration, in which case the effective mass per gear is less than half the airplane mass. If the gears had been located inboard of the rolling radius of gyration, the first impact would have been more severe than each landing-gear impact in the symmetrical case.

Combined effects of damping in roll and variable lift.- A limited evaluation of the combined effects which aerodynamic damping in roll and variable wing lift during impact can have on the contact velocity for second impacts may be obtained from the results presented in table II. The data shown in the portion of the table headed "With aerodynamic effects" were computed by means of the supplementary analysis given in the appendix. These results, which were calculated for two assumed impulse durations, namely 0.3 and 0.4 second, indicate that the choice of the impulse duration has only a minor effect on the contact velocities for the second impact calculated by means of the supplementary analysis. The data shown in the portion of the table headed "Without aerodynamic effects" were taken from those presented in figure 3, which were calculated for an initial roll angle of 7° by means of the analysis presented in the main portion of the paper.

The results of the supplementary analysis correspond to impacts in which the initial angle of roll is equal to the calculated change in roll angle $\left(\varphi_t - \varphi_o\right)$, as is discussed in the appendix. In cases where the supplementary calculations yield values of $\left(\varphi_t - \varphi_o\right)$ approximately equal to 7°, the impact velocities computed by the two methods can be compared to assess the importance of the variable aerodynamic effects. The results computed by the two methods should also be roughly comparable when the calculated values of $\left(\varphi_t - \varphi_o\right)$ are somewhat different from 7° in view of the fact that the angle of roll appears to have only a minor effect on the contact velocities calculated by means of the analysis given in the main portion of the paper.

The results of the supplementary calculations in table II show that the contact velocities for second impacts can be appreciably greater than the initial descent velocity when the wing lift is less than the weight, even if the effects of aerodynamic damping are considered. In landings with the wing lift equal to the weight, however, little or no increase in contact velocity for the second impact is indicated for the airplane under consideration. Other calculations (table III) based on the supplementary analysis, however, indicate that appreciable increases in contact velocity for the second impact can be expected even in landings with wing lift equal to the weight, if the semitread is appreciably greater than the rolling radius of gyration.

Comparison of contact velocities for the second impact calculated with and without consideration of aerodynamic damping and variable wing lift (table II) indicates fairly good agreement for the reduced lift condition but also indicates that neglect of these effects produces somewhat conservative results when the wing lift is equal to the weight. These results are attributed to the fact that the unbalanced-weight impulse $(1 - K_L)Wt_t$, which tends to increase the contact velocity for the second impact, offsets to some extent the effect of damping in roll, which tends to reduce the contact velocity for the second impact. Consequently the impulse-momentum analysis previously presented, which neglects both the unbalanced-weight impulse and the effects of damping in roll, would be expected to yield fairly good results for reduced wing-lift conditions and somewhat conservative results when the wing lift is equal to the weight.

Effect of approach conditions.- Figures 4 to 7 show the effect that side drift, initial roll angle, and an initial rolling velocity can have on the severity of landing-gear impacts in eccentric landings in which the first impact occurs on one main gear and the second impact takes place on the opposite main gear. The results were computed by the method presented in the main portion of this paper which neglects variable aerodynamic effects. Unless otherwise noted, the initial conditions are identical with those for figure 3.

(a) Side drift: The effect of side drift or yaw on the contact velocity for the second impact in an eccentric landing is shown in figure 4. Figure 5 shows the effect of side drift on the impact energy for both the first and second impacts. The ratio of side impulse to vertical impulse for these calculations is given by the value of K_s' where $K_s' = K_s \frac{\dot{y}_o}{|\dot{y}_o|}$; the sign indicates the direction of lateral motion at the instant of touchdown on the left main gear (plus to the right, minus to the left). The value $K_s = 0.6$ was suggested by the design requirements for drift landings. Examination of data on the cornering characteristics of a typical large-airplane tire indicates that this value might correspond to a fairly large yaw angle, on the order of 15°.

As might reasonably be expected in a landing where the first impact occurs on the left main gear, figures 4 and 5 show that the contact velocity and impact energy for the second impact will be increased if the airplane is drifting to the right (K_s' positive) at initial touchdown but will be decreased if the airplane is drifting to the left (K_s' negative). Figure 5 also indicates that the severity of the initial impact is appreciably increased if the airplane is drifting to the left at touchdown but decreased if it is drifting in the opposite direction.

This difference in impact energy results from the variation in effective mass produced by the change in orientation of the resultant force vector.

(b) Initial angle of roll: The variation of second-gear contact velocity with initial angle of roll is shown in figure 6 for landings with an assumed sinking speed of 12 feet per second. The calculations indicate that an increase in initial angle of roll results in a slight increase in the contact velocity for second impacts if the wing lift is equal to two-thirds of the weight of the airplane but produces a slight reduction in contact velocity for the second impact if the wing lift is equal to the weight. In either case, however, the differences in second-gear contact velocity attributable to changes in initial angle of roll are comparatively small.

(c) Initial rolling velocity: The effect of an initial rolling velocity (0.1 radian per second) on the contact velocity for second impacts in eccentric landings is shown in figure 7. The curves show that an initial rolling velocity which reduces the landing-gear contact velocity for the first impact ($\dot{\phi}_0$ positive) increases the contact velocity for the second impact. The calculated increase in second-gear contact velocity is noted to be slightly less than the product of the initial angular velocity (0.1 radian per second) and the semitread, which is equal to the decrease in contact velocity for the first gear to impact.

Impact Severity for Auxiliary Landing Gears

Calculated results which show the effect that location of auxiliary landing gears can have on the severity of second impacts on such gears are shown in figure 8. In this figure impact velocities and energies for second impacts on a forward gear of an airplane equipped with a quadricycle arrangement of gears are compared with similar results for second impacts on the nose gear of the same airplane equipped with a tricycle landing-gear configuration. In both arrangements the main gears have the same position. The initial conditions assumed in these calculations are identical with those used in obtaining the results given in figure 3, except that the airplane was assumed to be rolled 9° to the left instead of 7°, which resulted in the second impact taking place on a forward gear rather than on the opposite main gear.

Comparison of the curves on the right and left sides of figure 8 shows that in similar landings the second impact on a forward gear of the quadricycle arrangement can be appreciably more severe than the second impact on the nose gear of the tricycle. It is of interest to note that the impact energy is appreciably larger for the quadricycle case than for the tricycle even though the effective mass for each auxiliary gear of the quadricycle configuration is appreciably less than the effective mass for the nose gear of the tricycle arrangement. This result is due to the increment in landing-gear contact velocity produced by the rolling velocity of the airplane initiated by the first impact.

In the case of the quadricycle this increment is fairly large because of the outboard location of the auxiliary gears; in the case of the tricycle this increment is negligibly small because of the center-line location of the nose gear. As can also be seen from figure 8, second impacts on an auxiliary gear of the quadricycle configuration would be expected to be appreciably more severe in an eccentric landing than auxiliary gear impacts in a four-point landing; whereas second impacts on the nose gear of the tricycle arrangement should in many cases be less severe in an eccentric landing than nose-gear impacts in a three-point landing.

APPLICATION OF ANALYSIS TO DESIGN

In view of the fact that the foregoing analysis treats a landing condition which is also considered by existing ground-loads requirements, it is desirable to discuss the relationship between the two approaches to the problem. Since current ground-loads requirements have been evolved largely on the basis of past experience, they necessarily include hidden factors which must compensate to some extent for the combined effects of the many conditions which are not rationally considered in detail. For example, the present requirements, as previously noted, specify the same design impact velocities and landing-gear reactions in unsymmetrical landings as in symmetrical landings. Experience and calculations indicate, however, that landing-gear impacts can be appreciably more severe in the unsymmetrical case. On the other hand, the fact that the descent velocities specified by the requirements appear to be appreciably greater than the sinking speeds generally encountered in normal airplane operations may be an indication that the increased impact severity in eccentric landings has not been completely overlooked by the requirements. However, since they do not rationally consider such factors as landing-gear location, airplane inertia, and the various possible combinations of approach conditions, which can greatly affect impact severity, the requirements, although generally permitting the design of reasonably satisfactory landing gears, may unduly penalize some airplanes whereas they may be insufficiently severe for other types.

Since the design conditions specified by the present requirements may include some of the effects of eccentric landings as previously discussed, the use of the specified approach conditions, particularly the descent velocities, as initial conditions in the foregoing analysis would be expected to produce conservative results. The design of landing gears on a completely rational basis, therefore, requires statistical studies of the approach conditions actually encountered in routine operations. A statistical approach to the determination of these conditions is necessary since many unpredictable factors such as piloting technique and atmospheric disturbances largely determine the behavior of the airplane prior to initial contact. Even without adequate statistical data, however, the foregoing analysis permits a limited evaluation of the importance of eccentric landings for particular airplanes and provides a basis for assessing the effects of airplane characteristics and operating conditions on impact severity in such landings.

CONCLUSIONS

An impulse-momentum method has been presented for determining landing-gear contact conditions in eccentric landings. Calculations based on the analysis indicate that:

1. In an eccentric landing either the first or second impact can be appreciably more severe than each landing-gear impact in a symmetrical landing, depending on the location of the landing gears relative to the radii of gyration of the airplane. For given approach conditions increasing the landing-gear tread tends to decrease the severity of the first impact and increase the severity of the second impact; decreasing the landing-gear tread has the opposite effect.

2. The magnitude of the wing lift at the instant of initial contact has an appreciable effect on the severity of the second impact in an eccentric landing. Reductions in wing lift result in increased impact severity, as might reasonably be expected.

3. Side drift velocities can appreciably increase or decrease the severity of successive impacts in an eccentric landing, depending on the direction of side drift.

4. Variations in initial roll angle appear to have only a minor effect on impact severity in eccentric landings.

5. For a given initial rate of descent of the center of gravity, an initial rolling velocity which reduces the contact velocity for the first impact results in an increase in contact velocity for the second impact of almost the same amount.

6. For eccentric landings in which the first impact occurs on one main gear and the second impact occurs on an auxiliary gear, the severity of the second impact can be considerably greater for a quadricycle configuration than for a tricycle arrangement.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., August 28, 1951

APPENDIX

SUPPLEMENTARY ANALYSIS INCLUDING AERODYNAMIC EFFECTS

The main purpose of this supplementary study is to permit an order-of-magnitude evaluation of the effects of variable aerodynamic forces and moments on airplane motions during landing and, in particular, to obtain a rough indication of the error in second-gear contact velocity to be expected from neglect of the following aerodynamic effects which were not included in the impulse-momentum analysis presented in the body of this paper: (a) The effect of aerodynamic moments on the rolling motion of the airplane and (b) the effect of unbalanced weight acting during the impulse of the ground forces, based on the wing lift at initial contact and variations in wing lift during the impact.

For the purposes of the present study, the airplane is assumed to have freedom in roll and vertical translation only and the effects of angularity are neglected. With these assumptions the equations of motion for the airplane, following initial contact on one main landing gear, may be written as follows:

$$M\ddot{z}_g + K_{\dot{z}}\left(\dot{z}_g - \dot{z}_{g_o}\right) = F_v(t) + K_o \tag{A1}$$

$$I_{aa}\ddot{\varphi} + K_{\dot{\varphi}}\dot{\varphi} = -b_i F_v(t) \tag{A2}$$

where $K_{\dot{\varphi}}\dot{\varphi}$ is the aerodynamic damping moment of the wing which resists rolling motion and $K_{\dot{z}}\left(\dot{z}_g - \dot{z}_{g_o}\right)$ is the change in wing lift following initial ground contact due to variations in airplane vertical velocity. The quantities $K_{\dot{\varphi}}$ and $K_{\dot{z}}$ are taken equal to 0.75 times the steady-state values since unsteady-lift conditions exist during the impulse of the ground forces (reference 5). The quantity $K_o = W\left(K_L - 1\right)$ is the unbalanced weight just prior to initial touchdown.

By eliminating $F_v(t)$ from equations (A1) and (A2) and integrating from the time of initial ground contact, $t = 0$, to the time at which the gear making initial contact rebounds, $t = t_t$, the following equation is obtained:

$$M\left(\dot{z}_{g_t} - \dot{z}_{g_o}\right) + K_{\dot{z}}\left(z_{g_t} - z_{g_o}\right) - \left(K_o + K_{\dot{z}}\dot{z}_{g_o}\right)t_t + \frac{K_{\dot{\varphi}}}{b_i}\left(\varphi_t - \varphi_o\right) + \frac{I_{aa}}{b_i}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) = 0 \tag{A3}$$

At the instant of rebound of the gear making initial contact $\left(t = t_t\right)$, the following geometric and kinematic relations can also be written:

$$z_{g_t} = b_i\varphi_t \tag{A4a}$$

$$\dot{z}_{g_t} = b_i\dot{\varphi}_t - \dot{z}_{i_t} = b_i\dot{\varphi}_t - \dot{z}_{g_o}\sqrt{1 - \eta_{r_i}} \tag{A4b}$$

where η_{r_i} is the energy-dissipation efficiency of the impacting gear.

Substituting equations (A4a) and (A4b) and the geometric condition at initial contact $z_{g_o} = b_i\varphi_o$ into equation (A3) yields

$$M\left[b_i\dot{\varphi}_t - \dot{z}_{g_o}\left(1 + \sqrt{1 - \eta_{r_i}}\right)\right] + K_{\dot{z}}b_i\left(\varphi_t - \varphi_o\right) - \left(K_o + K_{\dot{z}}\dot{z}_{g_o}\right)t_t + \frac{I_{aa}}{b_i}\left(\dot{\varphi}_t - \dot{\varphi}_o\right) + \frac{K_{\dot{\varphi}}}{b_i}\left(\varphi_t - \varphi_o\right) = 0 \tag{A5}$$

If, for the purpose of determining the change in roll angle during the impact, $\dot{\varphi}$ is assumed to vary linearly with time (this assumption is discussed at the end of this section), the following simple relationship may be written for the change in roll angle during the impact:

$$\left(\varphi_t - \varphi_o\right) = \frac{t_t}{2}\,\dot{\varphi}_t \tag{A6}$$

Combining equation (A6) with equation (A5) and solving for $\dot{\varphi}_t$ yields

$$\dot{\varphi}_t = \frac{M\dot{z}_{g_o}\left(1 + \sqrt{1 - \eta_{r_i}}\right) + \frac{I_{aa}}{b_i}\,\dot{\varphi}_o + \left(K_o + K_{\dot{z}}\dot{z}_{g_o}\right)t_t}{Mb_i + \frac{I_{aa}}{b_i} + \left(K_{\dot{z}}b_i + \frac{K_{\dot{\varphi}}}{b_i}\right)\frac{t_t}{2}} \tag{A7}$$

If the second impact is assumed to occur at the instant of rebound of the first gear to contact, the vertical velocity for the second gear to contact is given by the simple kinematic relationship:

$$\dot{z}_{j_t} = 2b_i\dot{\varphi}_t - \dot{z}_{g_o}\sqrt{1 - \eta_{r_i}} \tag{A8}$$

The preceding condition exists if the initial angle of roll φ_o is equal to the change in roll angle $\left(\varphi_t - \varphi_o\right)$ determined by equation (A6). Where aerodynamic damping is considered, as in the present analysis, this condition would be expected to produce the largest contact velocities for the second impact.

In order to calculate the contact velocity for the second impact from equation (A8), a value must be known or assumed for the time duration of the impulse t_t required for the calculation of $\dot{\phi}_t$ by equation (A7). Available data indicate that the duration of the vertical impulse is on the order of 0.4 second for airplanes of about the same size as the one for which calculations are presented in the body of this paper. Comparisons in table II of calculated results for impulse durations of 0.4 and 0.3 second indicate that the value used for t_t has only a minor effect on the contact velocities computed by means of equations (A7) and (A8).

In order to evaluate the applicability of the assumption that $\dot{\varphi}_t$ varies linearly with t, which was used as a basis for equation (A6), analytical solutions were obtained for φ_t and $\dot{\varphi}_t$ from equation (A2) for a particular case of a sinusoidal vertical pulse. Setting the change in roll angle $\left(\varphi_t - \varphi_o\right)$ equal to the quantity $K\dot{\varphi}_t t_t$ gave values of the factor K equal to approximately 0.6 for sinusoidal pulses of 0.4- and 0.3-second duration. These values compare fairly well with the value of 1/2 used in equation (A6). Since other computations indicate that variations in the value of K have only a minor effect on the calculated contact velocities for second impacts, the assumption of a linear variation of $\dot{\varphi}$ with time appears reasonable for the purposes of this restricted study.

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TABLE I

CONSTANTS USED IN CALCULATED RESULTS

Weight and inertia constants of the airplane:

Landing weight, $W = 60{,}000$ lb

Mass moment of inertia of the airplane in roll, $I_{aa} = 301{,}900$ slug-ft^2

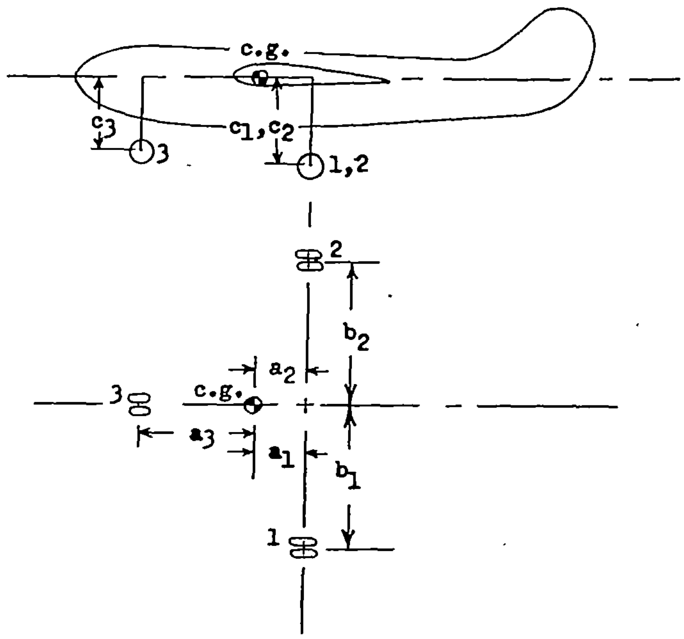
Mass moment of inertia of the airplane in roll, $I_{bb} = 336{,}700$ slug-ft^2

Constants defining the location of the landing gears relative to the airplane center of gravity:

Gear 1 = left main gear

Gear 2 = right main gear

Gear 3 = nose gear



$a_1 = a_2 = -2.928$ ft

$a_{1_c} = a_{2_c} = -3.033$ ft

$a_3 = 13.544$ ft

$a_{3_c} = 12.995$ ft

$c_1 = c_2 = 9.784$ ft

$c_{1_c} = c_{2_c} = 9.189$ ft

$c_3 = 9.542$ ft

$c_{3_c} = 9.346$ ft

$b_1 = b_{1_c} = -14.583$ ft

$b_2 = b_{2_c} = 14.583$ ft

$b_3 = b_{3_c} = 0$

Constants associated with the wheels:

$N_1 = N_2 = N_3 = 2$

$r_1 = r_2 = 1.875$ ft

$r_{1_c} = r_{2_c} = 1.558$ ft

$r_3 = 1.333$ ft

$r_{3_c} = 1.067$ ft

$I_{w_1} = I_{w_2} = 11.84$ slug-ft^2

$I_{w_3} = 1.653$ slug-ft^2

TABLE II

COMPARISON OF CALCULATED RESULTS WITH

AND WITHOUT AERODYNAMIC EFFECTS

Initial conditions		With aerodynamic effects				Without aerodynamic effects		
$\dot{z}_{g_o}$ (fps)	K_L	t_t (sec)	$\dot{\varphi}_t$ (radians/sec)	$\dot{z}_{j_t}$ (fps)	$\varphi_t - \varphi_o$ (deg)	φ_o (deg)	$\dot{\varphi}_t$ (radians/sec)	$\dot{z}_{j_f}$ (fps)
-8	2/3	0.4 .3	0.520 .501	-11.58 -11.02	5.9 4.3	7	-0.444	-12.78
	1	.4 .3	.391 .399	-7.81 -8.05	4.5 3.4	7	-.444	-9.12
-12	2/3	.4 .3	.715 .700	-15.49 -15.05	8.2 6.0	7	-.666	-16.28
	1	.4 .3	.586 .598	-11.72 -12.08	6.7 5.1	7	-.666	-13.68

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TABLE III

EFFECT OF LANDING-GEAR SEMITREAD ON CONTACT VELOCITIES FOR SECOND IMPACTS CALCULATED BY MEANS OF THE SUPPLEMENTARY ANALYSIS WHICH INCLUDES VARIABLE AERODYNAMIC EFFECTS

$\left[\dot{z}_{g_o} = 12.0 \text{ fps}; \quad t_t = 0.4 \text{ sec}\right]$

K_L	b_i	b_i/ρ_{aa}	$\dot{\varphi}_t$	$\dot{z}_{j_t}$	$\varphi_t - \varphi_o$
2/3	12	0.89	0.7	11.4	8.0
	[a]14.58	1.09	.72	15.5	8.3
	16	1.19	.72	17.6	8.3
	18	1.34	.71	20.1	8.1
1	12	.89	.57	8.3	6.5
	[a]14.58	1.09	.59	11.7	6.8
	16	1.19	.59	13.5	6.8
	18	1.34	.58	15.6	6.6

[a]Semitread for cargo airplane described in table I.

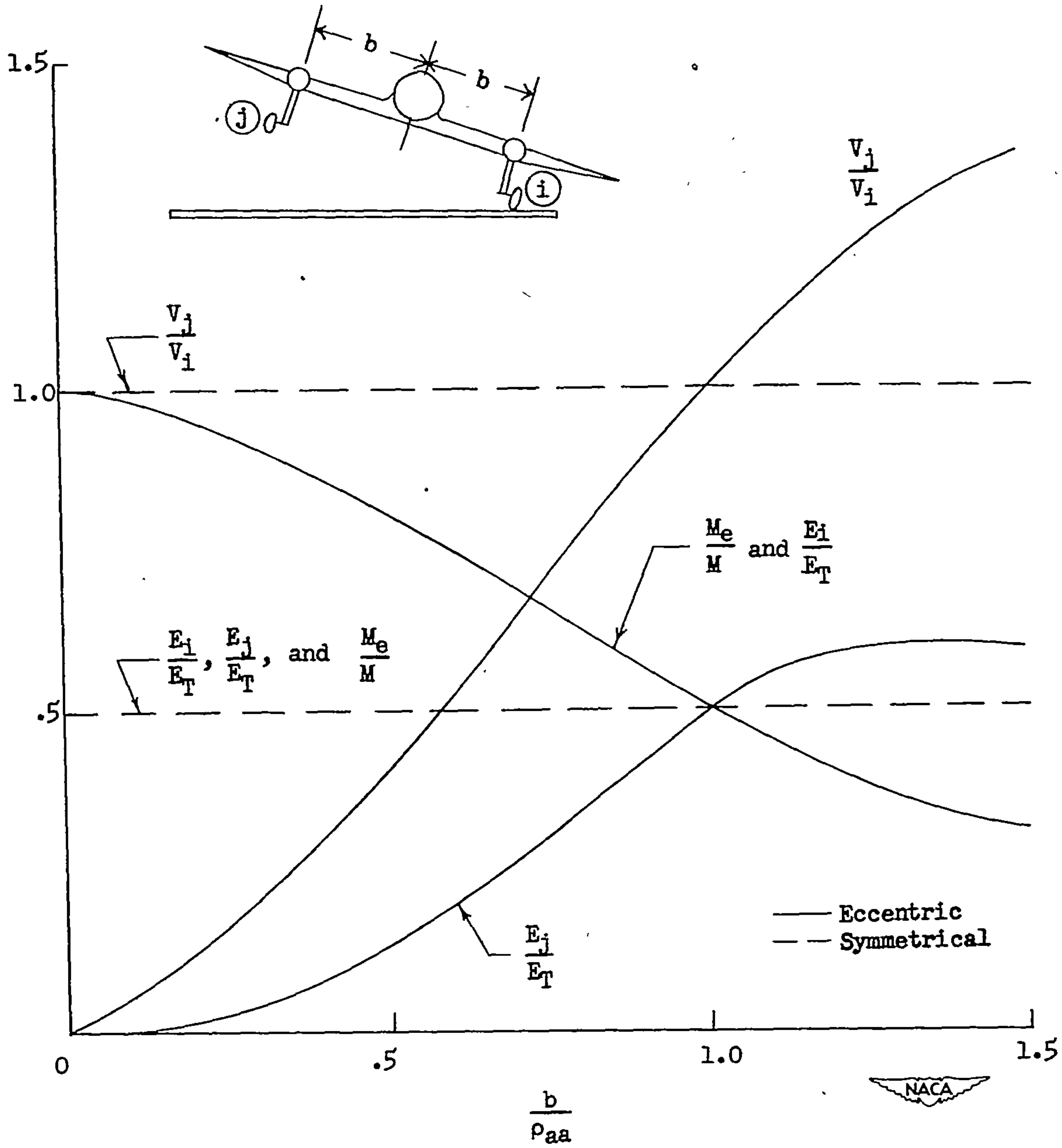


Figure 1.- Comparison of eccentric and symmetrical impacts for an idealized two-degree-of-freedom system.

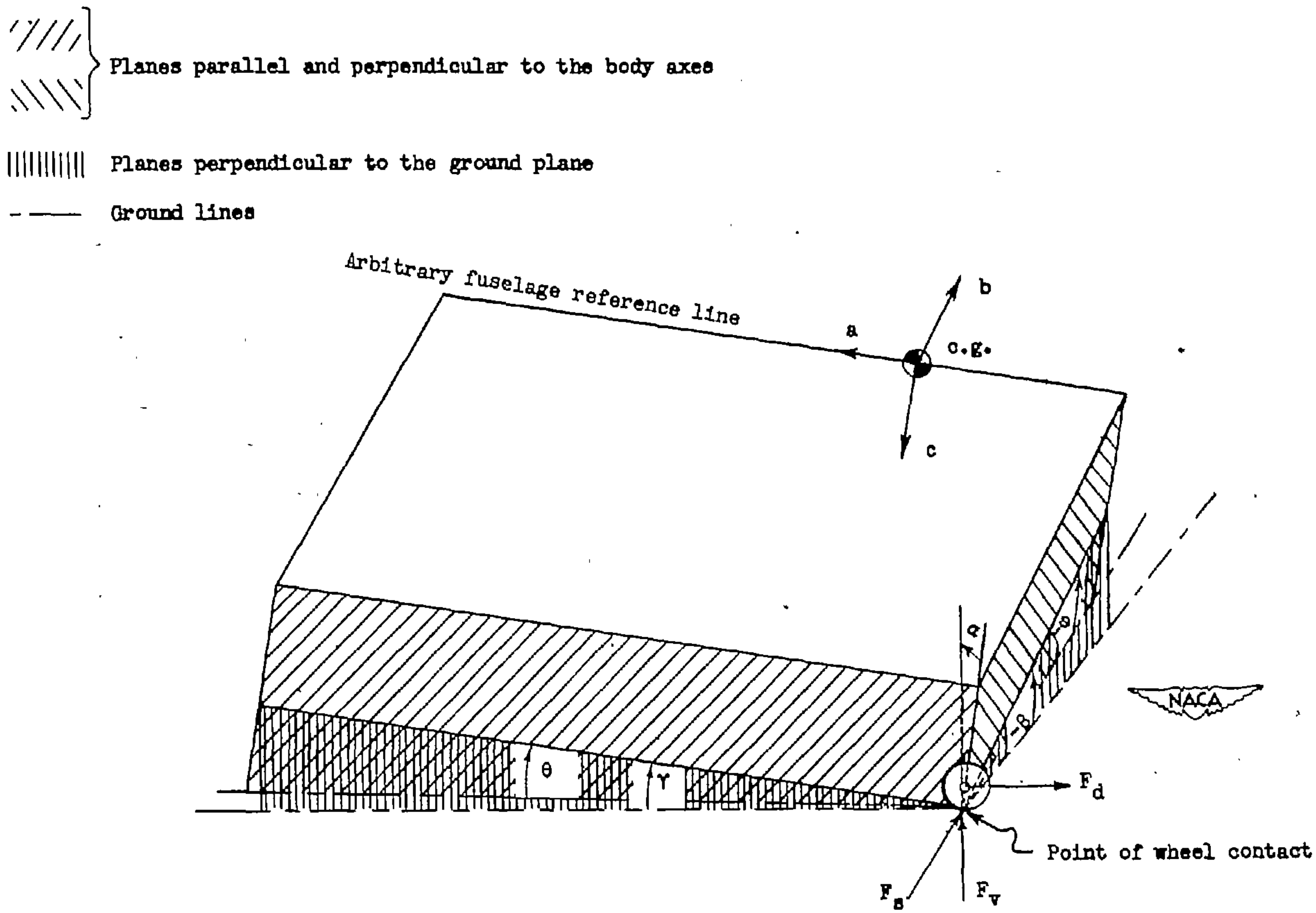
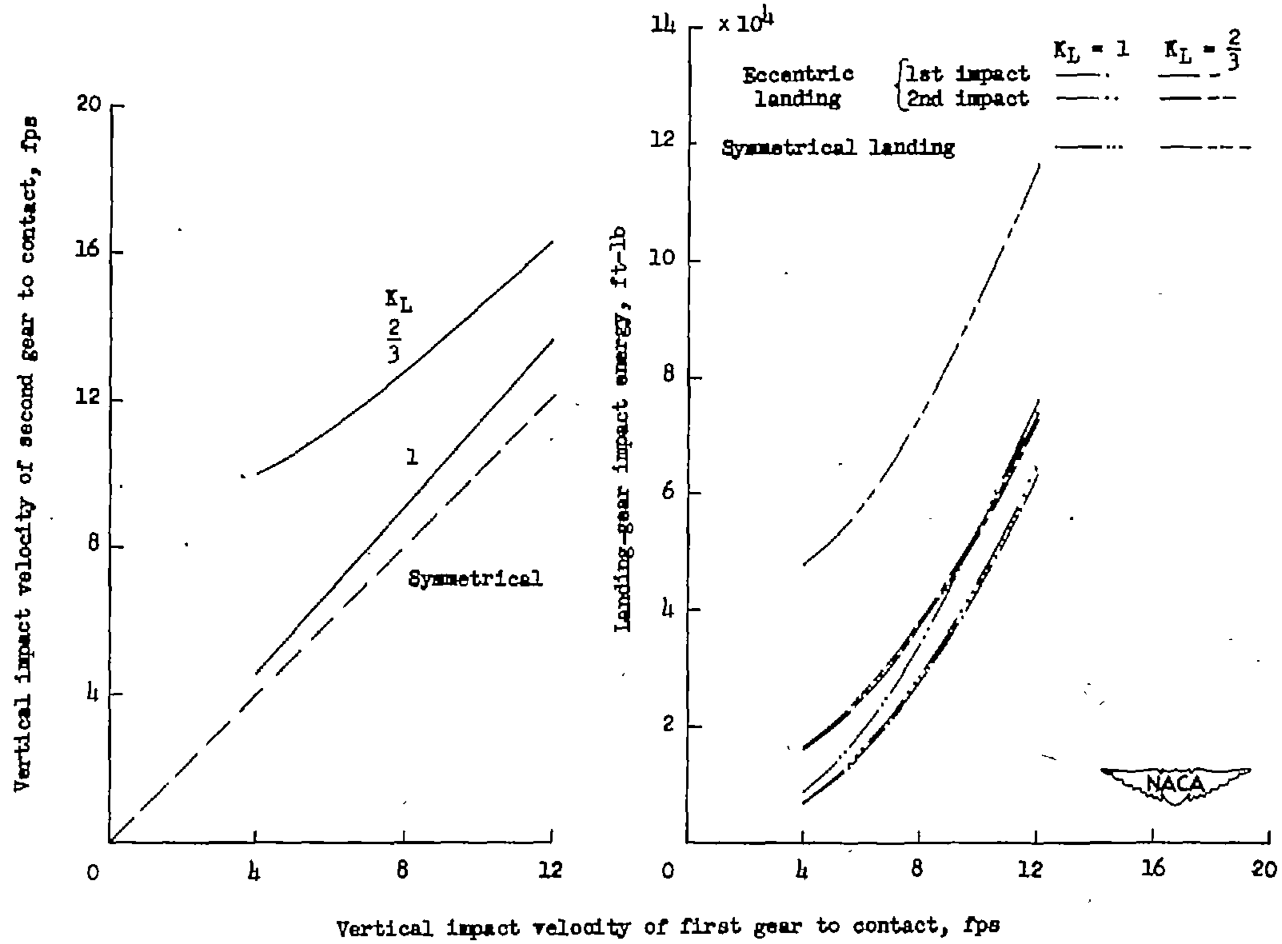


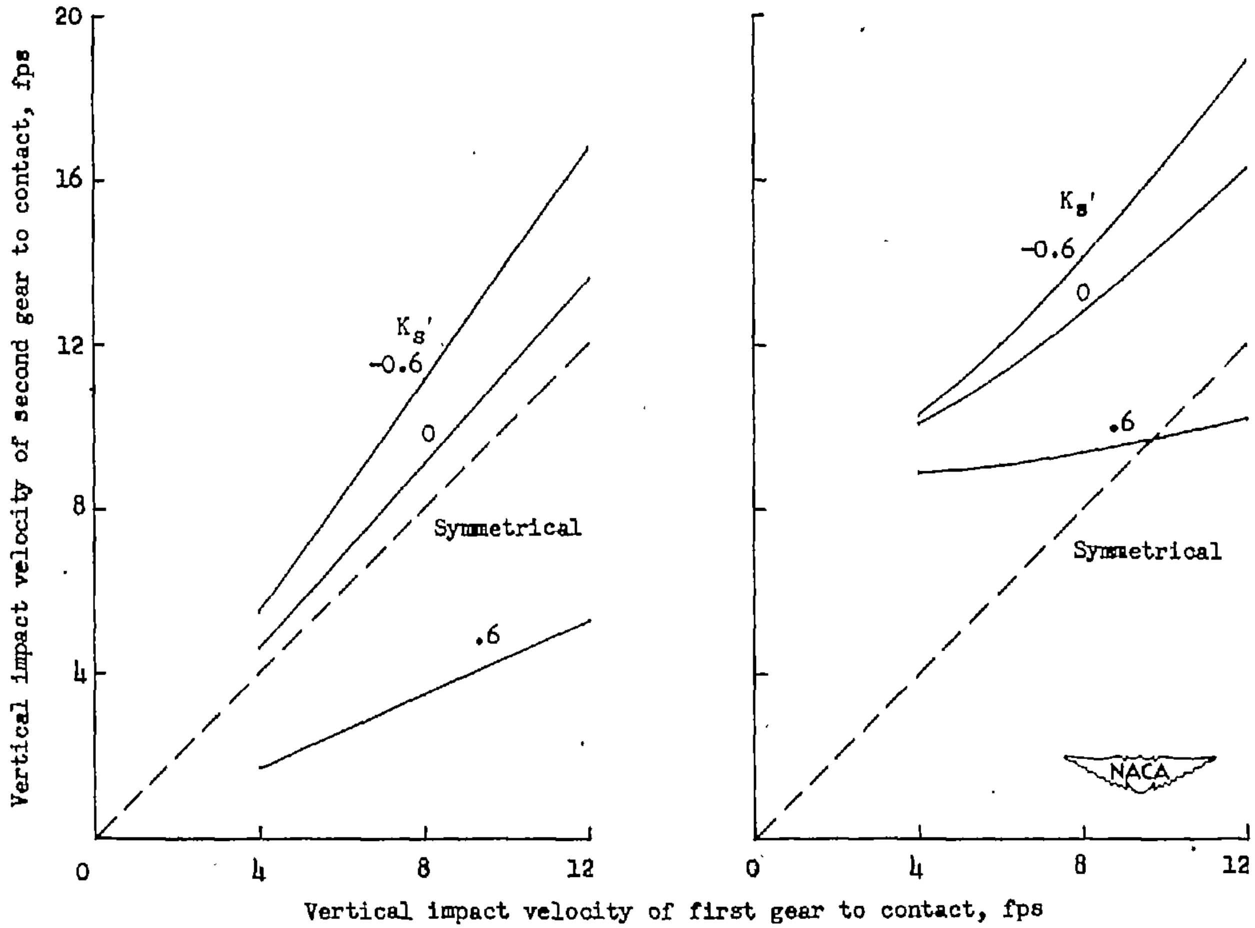
Figure 2.- Schematic representation of an airplane in eccentric landing attitude.



(a) Contact velocity.

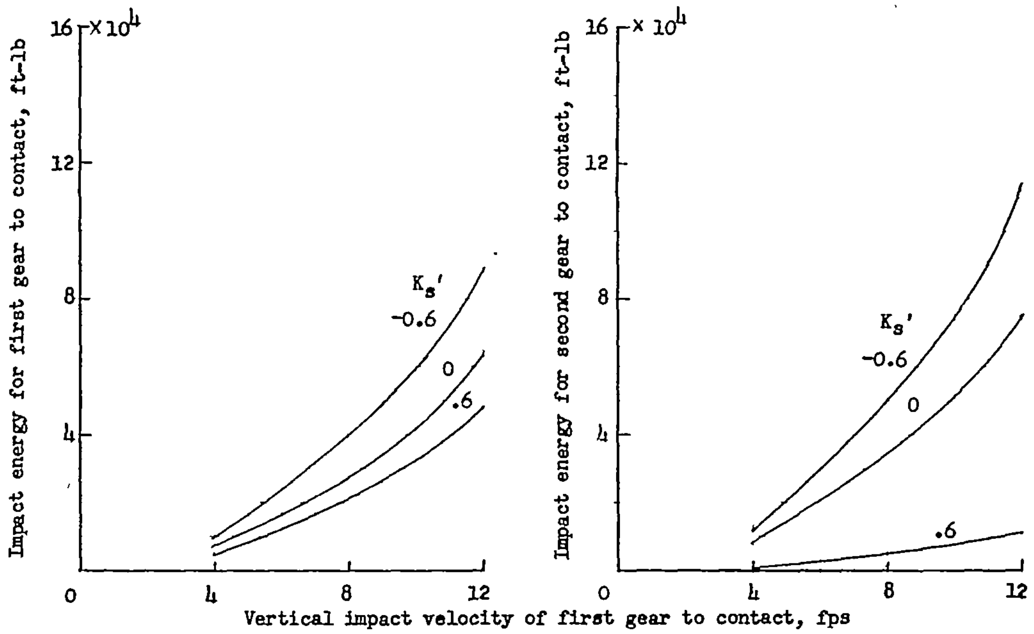
(b) Impact energy.

Figure 3.- Comparison of landing-gear contact velocity and impact energy in eccentric and symmetrical landings. (First impact on one main gear; second impact on opposite main gear.)

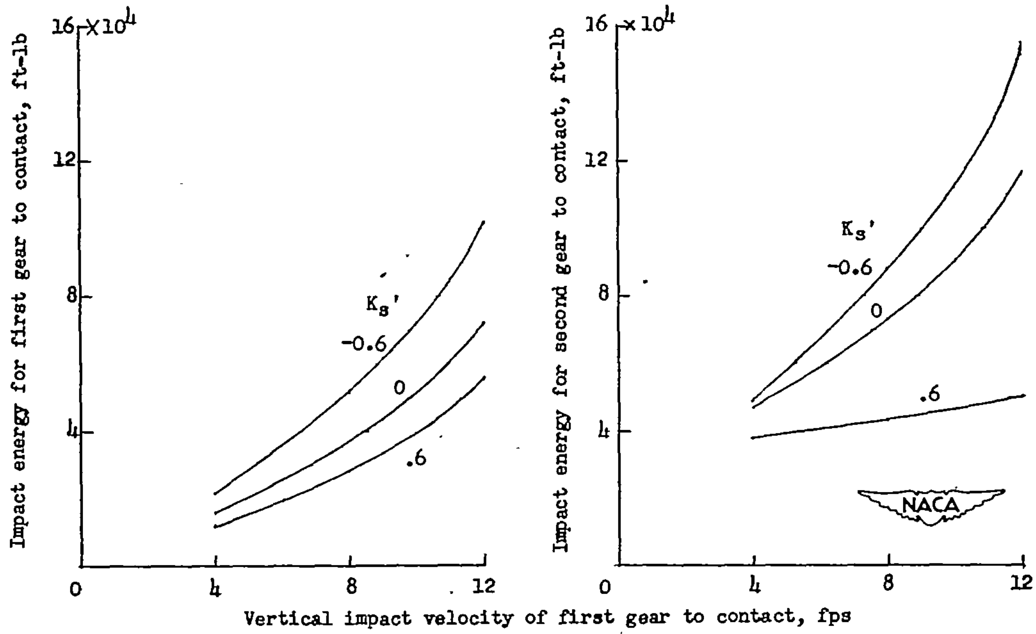


(a) $K_L = 1$. (b) $K_L = \frac{2}{3}$.

Figure 4.- The effect of side drift on landing-gear contact velocity for second impacts in eccentric landings. (First impact on one main gear; second impact on opposite main gear.)



(a) $K_L = 1$.



(b) $K_L = \frac{2}{3}$.

Figure 5.- The effect of side drift on the landing-gear impact energy for first and second impacts in eccentric landings. (First impact on one main gear; second impact on opposite main gear.)

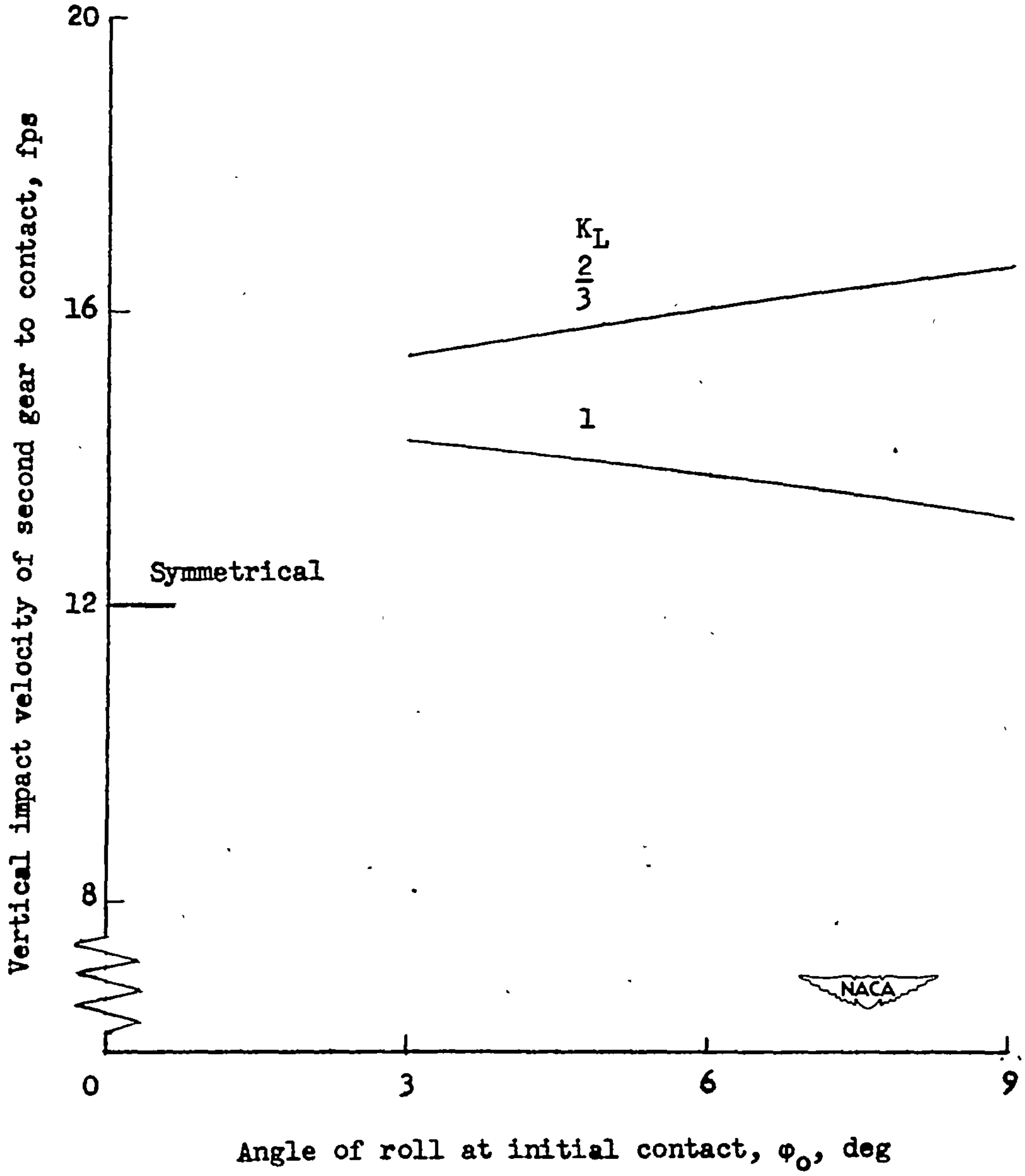
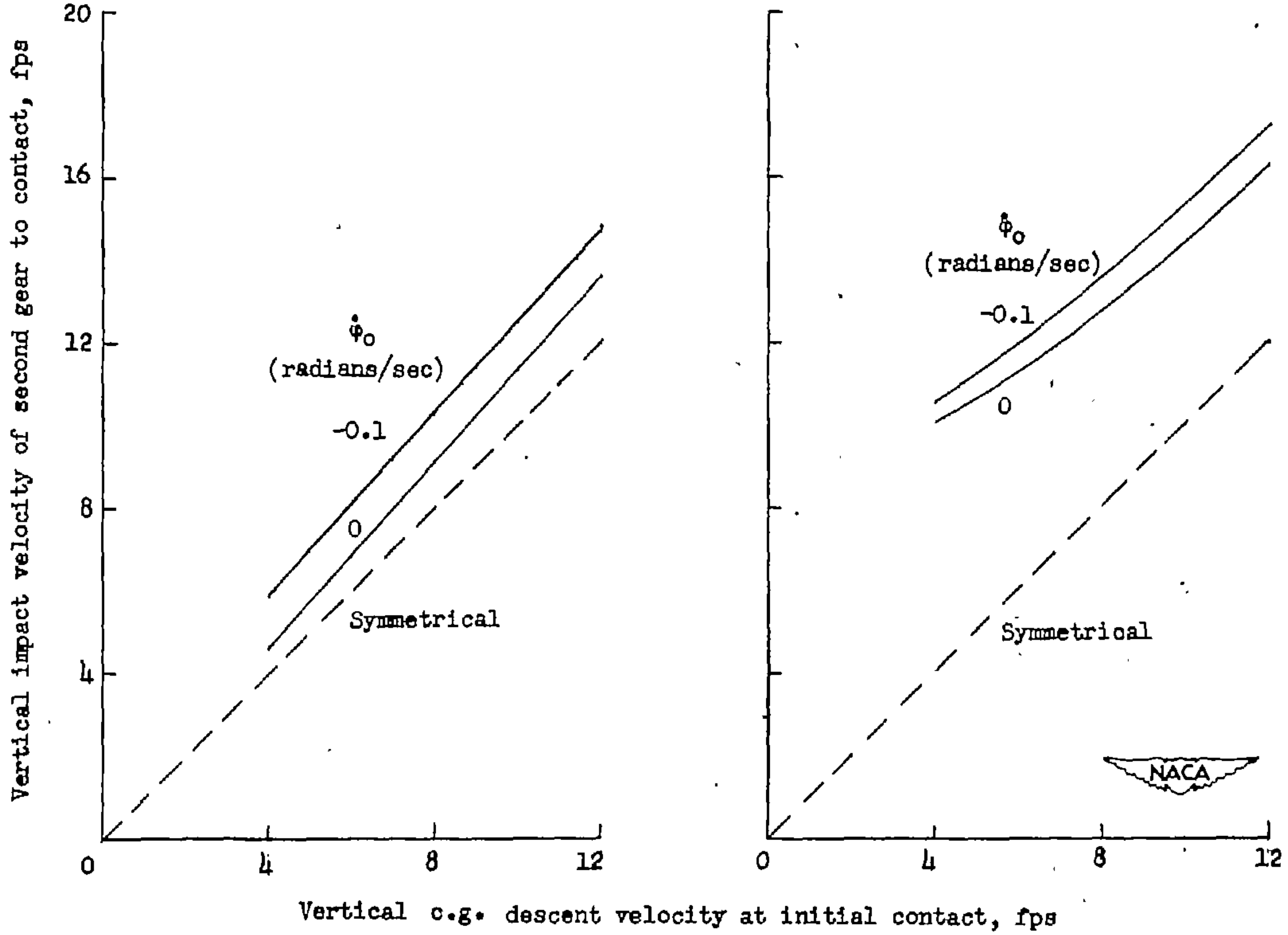
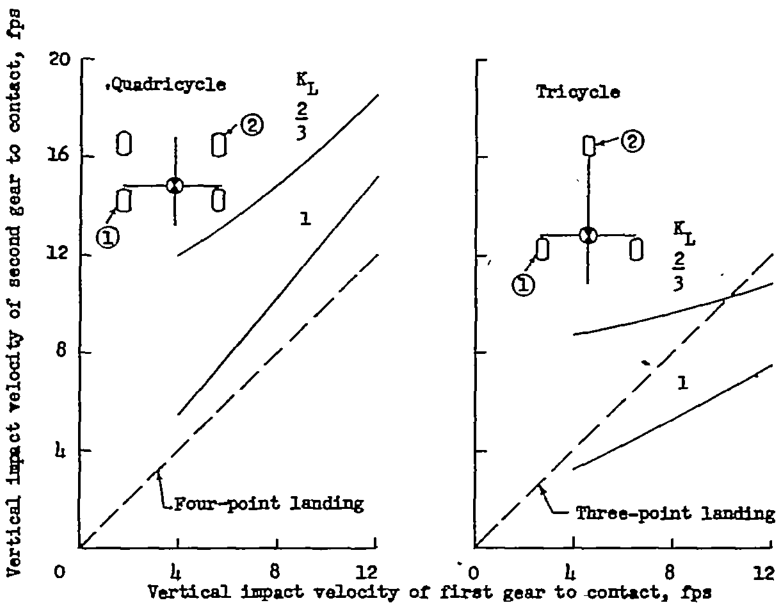


Figure 6.- Effect of initial angle of roll on the contact velocity for second impacts in eccentric landings. (First impact on one main gear; second impact on opposite main gear.)

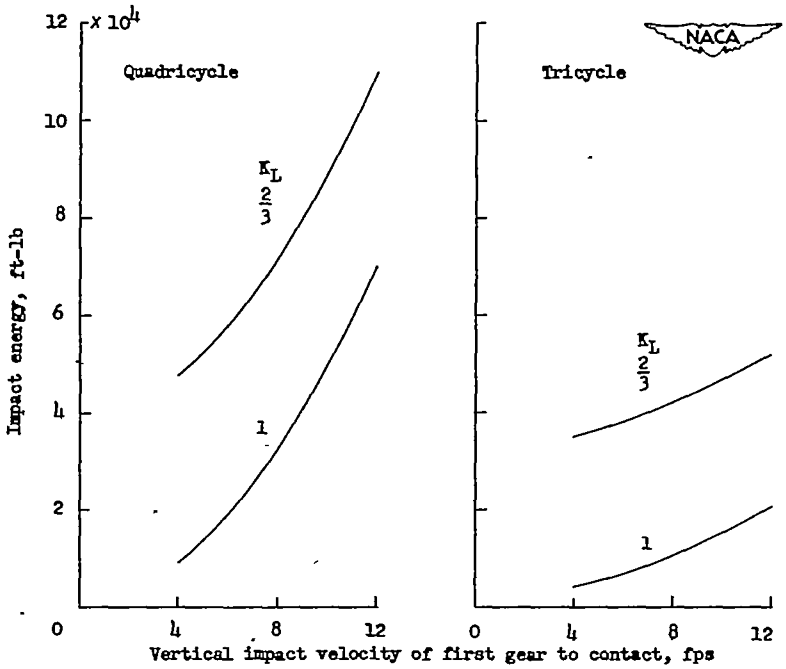


(a) $K_L = 1$.   (b) $K_L = \frac{2}{3}$.

Figure 7.- The effect of an initial rolling velocity on the contact velocity for second impacts in eccentric landings. (First impact on one main gear; second impact on opposite main gear.)



(a) Vertical velocities for second impact on auxiliary gears.



(b) Impact energy for second impact on auxiliary gears.

Figure 8.- Comparison of the severity of landing-gear impacts on auxiliary gears of a quadricycle and tricycle configuration. (First impact on one main gear; second impact on right forward gear of quadricycle or on nose gear of tricycle.)

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