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EFFECT OF AUTOMATIC STABILIZATION ON THE LATERAL OSCILLATORY STABILITY OF A HYPOTHETICAL AIRPLANE AT SUPERSONIC SPEEDS

By Leonard Sternfield

Langley Aeronautical Laboratory
Langley Air Force Base, Va.

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SUMMARY

A theoretical investigation has been made to determine the effect of automatic stabilization on the lateral oscillatory stability of a hypothetical supersonic airplane. The investigation included an automatic pilot sensitive to a displacement in either yaw or roll and an automatic pilot sensitive to either the yawing or rolling angular velocity. The calculations were made for each type of automatic pilot acting independently of the others. An idealized control system without lag was assumed for the calculations.

The results of the investigation indicated that all the automatic pilots improved the stability of the original unstable short-period oscillation. The only one of the automatic pilots which resulted in an oscillation that satisfied the NACA and military criterions for satisfactory damping-period relationship, however, is an automatic pilot sensitive to the yawing angular velocity and geared to the rudder so that rudder control is applied in proportion to the angular velocity.

INTRODUCTION

The lateral-stability boundaries calculated in reference 1 indicated that for high-speed airplanes designed with high wing loadings and sweptback wings more directional stability is required for oscillatory stability than for an airplane with a straight wing and lower wing loading. Subsequent lateral stability investigations (references 2 and 3) showed that the directional stability required for oscillatory stability may be reduced if the principal longitudinal axis of the airplane is inclined above the flight path. Additional, unpublished, dynamic-stability calculations on the effect of an automatic pilot which gives control proportional to the angular displacement in either yaw or roll or an automatic pilot which gives control proportional to either the yawing or rolling angular velocity also indicated that the use of the automatic pilot permits a reduction in the directional stability required for oscillatory stability.

These calculations for the effect of the automatic pilot were made at the time the theoretical investigation reported in reference 3 was carried out, but the results of the calculations were not published then because the stabilizing effect that would be obtained by inclining the principal axis above the flight path was believed to be sufficient to obviate the necessity of installing an automatic pilot. Recent lateral-stability analyses of several airplanes have shown, however, that the oscillatory stability is not satisfactory even when the principal axis is inclined above the flight path. The use of an automatic pilot therefore offers another means for improving the oscillatory stability.

SYMBOLS AND COEFFICIENTS

ϕ	angle of roll, radians
ψ	angle of yaw, radians
β	angle of sideslip, radians (v/V)
$r,\dot{\psi}$	yawing angular velocity, radians per second $(d\psi/dt)$
$p,\dot{\phi}$	rolling angular velocity, radians per second $(d\phi/dt)$
v	sideslip velocity along the Y-axis, feet per second
V	airspeed, feet per second
ρ	mass density of air, slugs per cubic foot
q	dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$
b	wing span, feet
S	wing area, square feet
W	weight of airplane, pounds
m	mass of airplane, slugs (W/g)
g	acceleration due to gravity, feet per second per second
μ_b	relative-density factor $(m/\rho Sb)$
η	inclination of principal longitudinal axis of airplane with respect to flight path, positive when principal axis is above flight path at the nose, degrees
γ	angle of flight path to horizontal axis, positive in a climb, degrees
k_{X_0}	radius of gyration in roll about principal longitudinal axis, feet

k_{Z_0}	radius of gyration in yaw about principal vertical axis, feet
K_{X_0}	nondimensional radius of gyration in roll about principal longitudinal axis $\left(k_{X_0}/b\right)$
K_{Z_0}	nondimensional radius of gyration in yaw about principal vertical axis $\left(k_{Z_0}/b\right)$
K_X	nondimensional radius of gyration in roll about longitudinal stability axis $\left(\sqrt{K_{X_0}^2\cos^2\eta + K_{Z_0}^2\sin^2\eta}\right)$
K_Z	nondimensional radius of gyration in yaw about vertical stability axis $\left(\sqrt{K_{Z_0}^2\cos^2\eta + K_{X_0}^2\sin^2\eta}\right)$
K_{XZ}	nondimensional product-of-inertia parameter $\left(\left(K_{Z_0}^2 - K_{X_0}^2\right)\sin\eta\cos\eta\right)$
C_L	trim lift coefficient $\left(\frac{W\cos\gamma}{qS}\right)$
C_l	rolling-moment coefficient $\left(\frac{\text{Rolling moment}}{qSb}\right)$
C_n	yawing-moment coefficient $\left(\frac{\text{Yawing moment}}{qSb}\right)$
C_Y	lateral-force coefficient $\left(\frac{\text{Lateral force}}{qS}\right)$
C_{l_β}	effective-dihedral derivative, rate of change of rolling-moment coefficient with angle of sideslip, per radian $(\partial C_l/\partial\beta)$
C_{n_β}	directional-stability derivative, rate of change of yawing-moment coefficient with angle of sideslip, per radian $(\partial C_n/\partial\beta)$
C_{Y_β}	lateral-force derivative, rate of change of lateral-force coefficient with angle of sideslip, per radian $(\partial C_Y/\partial\beta)$
C_{n_r}	damping-in-yaw derivative due to the airplane, rate of change of yawing-moment coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_n/\partial\frac{rb}{2V}\right)$

ΔC_{n_r}	damping-in-yaw derivative due to the automatic pilot
C_{n_p}	rate of change of yawing-moment coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_n / \partial \frac{pb}{2V}\right)$
C_{l_p}	damping-in-roll derivative due to the airplane, rate of change of rolling-moment coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_l / \partial \frac{pb}{2V}\right)$
ΔC_{l_p}	damping-in-roll derivative due to the automatic pilot
C_{Y_p}	rate of change of lateral-force coefficient with rolling-angular-velocity factor, per radian $\left(\partial C_Y / \partial \frac{pb}{2V}\right)$
C_{Y_r}	rate of change of lateral-force coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_Y / \partial \frac{rb}{2V}\right)$
C_{l_r}	rate of change of rolling-moment coefficient with yawing-angular-velocity factor, per radian $\left(\partial C_l / \partial \frac{rb}{2V}\right)$
C_{n_ψ}	rate of change of yawing-moment coefficient with angle of yaw, per radian $\left(\frac{\partial C_n}{\partial \psi}\right)$
C_{l_ψ}	rate of change of rolling-moment coefficient with angle of yaw, per radian $\left(\frac{\partial C_l}{\partial \psi}\right)$
C_{Y_ψ}	rate of change of lateral-force coefficient with angle of yaw, per radian $\left(\frac{\partial C_Y}{\partial \psi}\right)$
C_{n_ϕ}	rate of change of yawing-moment coefficient with angle of roll, per radian $\left(\frac{\partial C_n}{\partial \phi}\right)$
C_{l_ϕ}	rate of change of rolling-moment coefficient with angle of roll, per radian $\left(\frac{\partial C_l}{\partial \phi}\right)$

C_{Y_ϕ} rate of change of lateral-force coefficient with angle of roll, per radian $\left(\frac{\partial C_Y}{\partial \phi}\right)$

$C_{n_{\delta_r}}$ rate of change of yawing-moment coefficient with rudder deflection, per radian $\left(\frac{\partial C_n}{\partial \delta_r}\right)$

$C_{l_{\delta_r}}$ rate of change of rolling-moment coefficient with rudder deflection, per radian $\left(\frac{\partial C_l}{\partial \delta_r}\right)$

$C_{Y_{\delta_r}}$ rate of change of lateral-force coefficient with rudder deflection, per radian $\left(\frac{\partial C_Y}{\partial \delta_r}\right)$

$C_{n_{\delta_a}}$ rate of change of yawing-moment coefficient with aileron deflection, per radian $\left(\frac{\partial C_n}{\partial \delta_a}\right)$

$C_{l_{\delta_a}}$ rate of change of rolling-moment coefficient with aileron deflection, per radian $\left(\frac{\partial C_l}{\partial \delta_a}\right)$

$C_{Y_{\delta_a}}$ rate of change of lateral-force coefficient with aileron deflection, per radian $\left(\frac{\partial C_Y}{\partial \delta_a}\right)$

$\frac{\partial \delta_r}{\partial \psi}$ control-gearing ratio, rate of change of rudder deflection with angle of yaw

$\frac{\partial \delta_a}{\partial \phi}$ control-gearing ratio, rate of change of aileron deflection with angle of roll

$\frac{\partial \delta_r}{\partial \dot{\psi}}$ control-gearing ratio, rate of change of rudder deflection with yawing angular velocity

$\frac{\partial \delta_a}{\partial \dot{\phi}}$ control-gearing ratio, rate of change of aileron deflection with rolling angular velocity

t time, seconds

s_b nondimensional time parameter based on span (Vt/b)

D_b differential operator $\frac{d}{ds_b}$

R_1	Routh's discriminant for a quintic equation
R_2	Routh's discriminant for a quartic equation
P	period of oscillation, seconds
$T_{1/2}$	time for amplitude of oscillation to change by factor of 2 (positive value indicates a decrease to half amplitude, negative value indicates an increase to double amplitude)
$C_{1/2}$	number of cycles required for amplitude of periodic mode to change by factor of 2 (positive value indicates a decrease to half amplitude; negative value indicates an increase to double amplitude)
A,B,C,D,E,F	coefficients of lateral-stability equations

SCOPE OF INVESTIGATION

The oscillatory-stability boundaries were calculated for a hypothetical airplane to show the effect on the oscillatory stability of an automatic pilot which gives control proportional to the angular displacement in either yaw or roll or an automatic pilot which gives control proportional to either the yawing or rolling angular velocity. The calculations were made for each type of automatic pilot acting independently of the others. An idealized proportional control system in which lag effects were neglected was assumed for the calculations. The relationship of the calculated boundaries to the motion of the aircraft was investigated by determining the period and damping of the oscillatory mode and the damping of the aperiodic mode from the roots of the characteristic lateral-stability equation.

The mass and aerodynamic parameters of the hypothetical airplane in the cruising condition are presented in table I. The results of the calculations presented are based on the assumption that the product of inertia is zero.

EQUATIONS OF MOTION

The linearized equations of motion, referred to stability axes, for the condition of controls fixed are:

$$\left.\begin{aligned}
&\text{Rolling}\\
&2\mu_b\left(K_X{}^2 D_b{}^2\phi + K_{XZ} D_b{}^2\psi\right) = C_{l_\beta}\beta + \tfrac{1}{2}C_{l_p} D_b\phi + \tfrac{1}{2}C_{l_r} D_b\psi\\
&\text{Yawing}\\
&2\mu_b\left(K_Z{}^2 D_b{}^2\psi + K_{XZ} D_b{}^2\phi\right) = C_{n_\beta}\beta + \tfrac{1}{2}C_{n_p} D_b\phi + \tfrac{1}{2}C_{n_r} D_b\psi\\
&\text{Sideslipping}\\
&2\mu_b\left(D_b\beta + D_b\psi\right) = C_{Y_\beta}\dot{\beta} + \tfrac{1}{2}C_{Y_p} D_b\phi + C_L\phi + \tfrac{1}{2}C_{Y_r} D_b\psi\\
&\qquad + \left(C_L \tan\gamma\right)\psi
\end{aligned}\right\} \qquad (1)$$

When $\phi_o e^{\lambda s_b}$ is substituted for ϕ, $\psi_o e^{\lambda s_b}$ for ψ, and $\beta_o e^{\lambda s_b}$ for β in the equations written in determinant form, λ must be a root of the stability equation

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

where

$$A = 8\mu_b{}^3\left(K_X{}^2 K_Z{}^2 - K_{XZ}{}^2\right)$$

$$B = -2\mu_b{}^2\left(2K_X{}^2 K_Z{}^2 C_{Y_\beta} + K_X{}^2 C_{n_r} + K_Z{}^2 C_{l_p} - 2K_{XZ}{}^2 C_{Y_\beta} - K_{XZ} C_{l_r} - K_{XZ} C_{n_p}\right)$$

$$\begin{aligned}
C = \mu_b\Big(&K_X{}^2 C_{n_r} C_{Y_\beta} + 4\mu_b K_X{}^2 C_{n_\beta} + K_Z{}^2 C_{l_p} C_{Y_\beta} + \tfrac{1}{2}C_{n_r} C_{l_p} - K_{XZ} C_{l_r} C_{Y_\beta}\\
&- 4\mu_b K_{XZ} C_{l_\beta} - \tfrac{1}{2}C_{n_p} C_{l_r} - C_{n_p} K_{XZ} C_{Y_\beta} + K_{XZ} C_{n_\beta} C_{Y_p} - K_Z{}^2 C_{Y_p} C_{l_\beta}\\
&- K_X{}^2 C_{Y_r} C_{n_\beta} + K_{XZ} C_{Y_r} C_{l_\beta}\Big)
\end{aligned}$$

$$D = -\frac{1}{4}C_{n_r}C_{l_p}C_{Y_\beta} - \mu_b C_{l_p}C_{n_\beta} + \frac{1}{4}C_{n_p}C_{l_r}C_{Y_\beta} + \mu_b C_{n_p}C_{l_\beta} + 2\mu_b C_L K_{XZ}C_{n_\beta}$$
$$- 2\mu_b C_L {K_Z}^2 C_{l_\beta} - 2\mu_b {K_X}^2 C_{n_\beta}C_L \tan\gamma + 2\mu_b K_{XZ}C_{l_\beta}C_L \tan\gamma + \frac{1}{4}C_{l_p}C_{n_\beta}C_{Y_r}$$
$$- \frac{1}{4}C_{n_p}C_{l_\beta}C_{Y_r} - \frac{1}{4}C_{l_r}C_{n_\beta}C_{Y_p} + \frac{1}{4}C_{n_r}C_{l_\beta}C_{Y_p}$$

$$E = \frac{1}{2}C_L\left(C_{n_r}C_{l_\beta} - C_{l_r}C_{n_\beta}\right) + \frac{1}{2}C_L \tan\gamma\left(C_{l_p}C_{n_\beta} - C_{n_p}C_{l_\beta}\right)$$

If an automatic pilot sensitive to a displacement in yaw is installed in the airplane and rudder control is applied in proportion to the displacement, the stability derivatives C_{n_ψ}, C_{l_ψ}, and C_{Y_ψ} are introduced into equations (1). These derivatives will be functions of $C_{n_{\delta_r}}$, $C_{l_{\delta_r}}$, and $C_{Y_{\delta_r}}$, respectively, and of the control-gearing ratio $\frac{\partial\delta_r}{\partial\psi}$; that is,

$$C_{n_\psi} = C_{n_{\delta_r}}\frac{\partial\delta_r}{\partial\psi}$$

$$C_{l_\psi} = C_{l_{\delta_r}}\frac{\partial\delta_r}{\partial\psi}$$

and

$$C_{Y_\psi} = C_{Y_{\delta_r}}\frac{\partial\delta_r}{\partial\psi}$$

Normally for present-day airplanes, $C_{l_{\delta_r}}$ and $C_{Y_{\delta_r}}$ are small; equations (1) will therefore be simplified by neglecting the terms involving C_{l_ψ} and C_{Y_ψ}. If C_{n_ψ} appears in equations (1), the stability equation becomes

$$A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0$$

where the following terms are added to the terms of the coefficients presented previously:

for C

$$-4{\mu_b}^2{K_X}^2C_{n_\psi}$$

for D

$$\mu_b C_{l_p}C_{n_\psi} + 2\mu_b {K_X}^2 C_{Y_\beta}C_{n_\psi}$$

for E

$$-\frac{1}{2}C_{l_p}C_{Y_\beta}C_{n_\psi} + \frac{1}{2}C_{Y_p}C_{l_\beta}C_{n_\psi}$$

for F

$$C_L C_{l_\beta} C_{n_\psi}$$

If the automatic pilot is made sensitive to a displacement in roll and if aileron control is applied in proportion to the displacement, the derivatives C_{l_ϕ}, C_{n_ϕ}, and C_{Y_ϕ} are introduced into equations (1). These derivatives will be functions of $C_{l_{\delta_a}}$, $C_{n_{\delta_a}}$, and $C_{Y_{\delta_a}}$, respectively, and of the control-gearing ratio $\frac{\partial\delta_a}{\partial\phi}$. The derivative $C_{n_{\delta_a}}$ and $C_{Y_{\delta_a}}$ were assumed to be small; therefore, the derivatives C_{n_ϕ} and C_{Y_ϕ} will be neglected and only the derivative C_{l_ϕ} will be taken into account in equations (1). The coefficients of the stability equation will then include the following additional terms:

for C

$$-4\mu_b{}^2K_Z{}^2C_{l_\phi}$$

for D

$$\mu_b C_{n_r} C_{l_\phi} + 2\mu_b K_Z{}^2 C_{Y_\beta} C_{l_\phi}$$

for E

$$-\frac{1}{2}C_{n_r}C_{Y_\beta}C_{l_\phi} - 2\mu_b C_{n_\beta}C_{l_\phi} + \frac{1}{2}C_{Y_r}C_{n_\beta}C_{l_\phi}$$

for F

$$C_L \tan\gamma\, C_{n_\beta} C_{l_\phi}$$

For the cases of the automatic pilot designed to give control proportional to the yawing or rolling angular velocity, only the derivatives for damping in yaw and roll C_{n_r} and C_{l_p} were assumed to be effectively increased. Actually, the derivatives C_{l_r}, C_{Y_r}, C_{n_p}, and C_{Y_p} are affected by this type of automatic pilot. The effect of the automatic pilot on C_{l_r}, C_{Y_r}, C_{n_p}, and C_{Y_p}, however, was neglected because these derivatives are functions of $C_{l_{\delta_r}}$, $C_{Y_{\delta_r}}$, $C_{n_{\delta_a}}$, and $C_{Y_{\delta_a}}$, respectively, all of which are normally small for present-day airplanes. The derivative C_{n_r} was effectively increased when the rudder was assumed deflected in proportion to the yawing angular velocity, whereas C_{l_p} was effectively increased when the aileron was assumed deflected in proportion to the rolling angular velocity. The expressions for C_{n_r} and C_{l_p} due to the automatic pilot are

$$\Delta C_{n_r} = C_{n_{\delta_r}} \frac{\partial \delta_r}{\partial \dot{\psi}} \frac{2V}{b}$$

and

$$\Delta C_{l_p} = C_{l_{\delta_a}} \frac{\partial \delta_a}{\partial \dot{\phi}} \frac{2V}{b}$$

RESULTS AND DISCUSSION

Figure 1 clearly shows that the probable range of C_{n_β}, C_{l_β} combinations for the hypothetical high-speed airplane is located almost entirely in the oscillatorily unstable region. This boundary represents the case in which the principal longitudinal axis is alined with the flight path; therefore, the product of inertia is zero. The effect of automatic stabilization on the neutral-oscillatory-stability boundary is presented in figures 2 to 5. In each figure the ordinate is the directional-stability derivative C_{n_β} and the abscissa is the stability derivative introduced by the automatic pilot. The curves on each figure represent three different values of the effective-dihedral derivative C_{l_β}. The control-gearing ratio is also plotted as abscissa in figures 2 to 5 to indicate the relation between the control-gearing ratio and the stability derivative introduced by the automatic pilot. The control-gearing ratios $\frac{\partial \delta_r}{\partial \psi}$, $\frac{\partial \delta_a}{\partial \phi}$, $\frac{\partial \delta_r}{\partial \dot{\psi}}$, and $\frac{\partial \delta_a}{\partial \dot{\phi}}$ were calculated on the assumption that

$$C_{n_{\delta_r}} = -0.1$$

and

$$C_{l_{\delta_a}} = -0.1$$

Automatic Pilot Sensitive to Displacement in Yaw

If the automatic pilot is sensitive to a displacement in yaw and rudder control is applied in proportion to the displacement, the derivative C_{n_ψ} is introduced into the equations of motion. The lateral-stability equation now becomes a quintic equation as shown in the previous section entitled "Equations of Motion". The necessary and sufficient conditions for complete stability for a system which results in a quintic stability equation are derived in reference 4 and are also presented in reference 5. A simple derivation of the necessary and sufficient conditions for neutral oscillatory stability and for complete stability is given in the appendix. The necessary and sufficient conditions for neutral oscillatory stability are that

$$R_1 = (BC - AD)(DE - CF) - (BE - AF)^2 = 0$$

and that $BE - AF$ and $BC - AD$ are of the same sign. The necessary and sufficient conditions for complete stability are that the coefficients A, B, D, and F be positive, $R_1 > 0$, and $BE - AF > 0$.

Figures 2(a) to 2(c) show the neutral-oscillatory-stability boundary $R_1 = 0$ and the curve $BE - AF = 0$ plotted as a function of C_{n_β} and C_{n_ψ} or $\frac{\partial \delta_r}{\partial \psi}$ for $C_{l_\beta} = -0.10$, -0.30, and -0.50, respectively. On the shaded side of $BE - AF = 0$, the airplane has at least one mode of motion which is unstable. The significance of the boundary $R_1 = 0$ located on the shaded side of $BE - AF = 0$ is still valid; that is, a stable oscillation becomes unstable upon crossing the boundary. A clearer relationship between the boundary $R_1 = 0$ and the stability of the airplane motion may be obtained from a study of the damping and the period of the oscillation and the damping of the aperiodic modes obtained from the roots of the quintic lateral-stability equation. The results are presented in table II(a) for the case of $C_{l_\beta} = -0.1$ at $C_{n_\beta} = 0.15$ and $C_{n_\beta} = 0.55$ for several values of C_{n_ψ}. For point A in figure 2(a), which corresponds to the point at the values of $C_{l_\beta} = -0.1$, $C_{n_\beta} = 0.15$, and $C_{n_\psi} = 0$, the roots of the stability equation indicate an unstable short-period oscillation, two subsiding aperiodic modes, and a zero root. The zero root means that the airplane is insensitive to displacements in yaw. As C_{n_ψ} is increased negatively from point A to point B, a stable long-period oscillation is introduced in addition to the unstable short-period oscillation and one subsiding aperiodic mode. Passing through the boundary $R_1 = 0$, from point B to point C, causes the stable long-period oscillation to become unstable, but does not appreciably affect the other modes of motion. Two unstable oscillatory modes of motion now exist. As C_{n_ψ} is further increased negatively to point D, the instability of the short-period oscillation is reduced; whereas the instability of the long-period oscillation increases and its period is shortened. Crossing through the boundary $R_1 = 0$ to point E causes the short-period oscillation to become stable; however, as C_{n_ψ} continues to increase negatively, the damping of the short-period oscillation does not improve sufficiently to meet criterions for satisfactory dynamic stability (as described subsequently). This conclusion for large negative values of C_{n_ψ}, where cross-coupling effects are negligible, could be checked by considering $C_{n_\psi} - C_{n_\beta}$ to be the spring constant in the mass-spring dashpot system as expressed by the one-degree-of-freedom equation in yaw

$$\left[2\mu_b K_Z{}^2 D_b{}^2 - \frac{1}{2}C_{n_r} D_b - \left(C_{n_\psi} - C_{n_\beta}\right)\right]\psi = 0$$

An increase in the constant term $C_{n_\psi} - C_{n_\beta}$ of the equation reduces the period of the oscillation, but does not affect the damping.

The results presented in table II(a) for $C_{l_\beta} = -0.1$ and $C_{n_\beta} = 0.55$ when used in conjunction with figure 2(a) indicate that upon crossing through the boundary $R_1 = 0$ the long-period oscillation becomes unstable. As C_{n_ψ} is increased negatively, both the damping and the period of the short-period oscillation vary slightly and in such manner that the number of cycles required to damp to half amplitude $C_{1/2}$ is approximately constant. With increasingly negative values for C_{n_ψ}, the long-period oscillation becomes more unstable and its period is reduced.

The results of these calculations for the hypothetical airplane, therefore, indicate that an automatic pilot sensitive to a displacement in yaw does not substantially improve the stability of the short-period oscillation.

Automatic Pilot Sensitive to Displacement in Roll

If the automatic pilot is sensitive to a displacement in roll and aileron control is applied in proportion to the displacement, the derivative C_{l_ϕ} is introduced into the equations of motion. In the present analysis, level flight is assumed or $\gamma = 0^\circ$ and the stability equation is, therefore, a quartic. The necessary and sufficient conditions for neutral oscillatory stability are that

$$R_2 = BCD - AD^2 - B^2E = 0$$

and that the coefficients B and D are of the same sign (reference 6).

The neutral-oscillatory-stability boundary $R_2 = 0$ is presented in figure 3 for $C_{l_\beta} = -0.10$, -0.30, and -0.50. The complete curves of $R_2 = 0$ for $C_{l_\beta} = -0.30$ and -0.50, however, are not shown in figure 3. Actually, the complete curves for these values of C_{l_β} are similar in shape to the curve for $C_{l_\beta} = -0.10$ and intersect the line of $C_{l_\phi} = 0$ at values of C_{n_β} greater than those plotted in figure 3. The boundaries $R_2 = 0$ in figure 3 indicate that when the oscillation is unstable for $C_{l_\phi} = 0$, oscillatory stability is obtained as C_{l_ϕ} exceeds the value corresponding to a point on the boundary $R_2 = 0$. If the oscillation is stable at $C_{l_\phi} = 0$, oscillatory instability will occur for a limited range of values of C_{l_ϕ} as prescribed by the boundary $R_2 = 0$. Further negative increases in C_{l_ϕ}, however, will again result in oscillatory stability. The damping and period relationship of the oscillatory modes for $C_{l_\beta} = -0.10$ and $C_{n_\beta} = 0.15$ and 0.45 are presented in table II(b). For point A in figure 3, which corresponds to the point at the values of $C_{n_\beta} = 0.15$ and $C_{l_\phi} = 0$, the roots of the stability equation indicate an unstable short-period oscillation and two subsiding aperiodic modes. As C_{l_ϕ} is increased negatively to point B, the two subsiding modes combine to form a stable short-period oscillation. Between points A and B, the boundary for two equal roots occurs, beyond which two oscillations exist (reference 6). Upon passing through this boundary of equal roots, the period of the newly formed oscillation is very long, but it rapidly decreases as indicated by the results of the calculations at point B where the period is approximately 4 seconds. Crossing through the boundary $R_2 = 0$, from point B to point C, causes the original oscillation to become stable. As C_{l_ϕ} continues to increase negatively, the period of one of the oscillations remains constant and its damping decreases slightly whereas the period of the other oscillation decreases and its damping increases slightly. For both oscillations, the number of cycles required to damp to half amplitude $C_{1/2}$ increases as C_{l_ϕ} increases. For large negative values of C_{l_ϕ}, the period and damping of each oscillation may be approximated by the one-degree-of-freedom equations of motion in roll and yaw:

Rolling
$$\left(2\mu_b K_X^2 D_b^2 - \frac{1}{2}C_{l_p}D_b - C_{l_\phi}\right)\phi = 0$$

Yawing
$$\left(2\mu_b K_Z^2 D_b^2 - \frac{1}{2}C_{n_r}D_b - C_{n_\psi}\right)\psi = 0$$

where
$$C_{n_\psi} = -C_{n_\beta}$$

Thus it is seen that as C_{l_ϕ} increases negatively, the system acts as if two independent mass-spring dashpot systems were in operation without any cross-coupling effects. The derivative C_{l_ϕ} acts in the capacity of a spring constant and the period, therefore, varies inversely with C_{l_ϕ}. A comparison of the results in table II(b) for $C_{n_\beta} = 0.15$ and $C_{n_\beta} = 0.45$ at $C_{l_\phi} = -0.05$, -0.10, and -0.20 clearly shows that the oscillation described by the rolling equation is approximately independent of C_{n_β}. For small negative values of C_{l_ϕ}, the equations of motion for the three degrees of freedom involved in lateral motion must be solved simultaneously to determine the period and damping of the modes of motion.

Thus an automatic pilot sensitive to an angular displacement in roll stabilizes the original unstable oscillation and introduces an additional stable oscillatory mode with a shorter period than the original oscillation; however, throughout the range of C_{l_ϕ}, the period and damping relationship of at least one of these oscillations may be objectionable to the pilot.

Automatic Pilot Sensitive to Rate of Displacement

Another type of automatic pilot included in the investigation was one sensitive to either the yawing or the rolling angular velocity. Rudder control was assumed to be applied in proportion to the yawing angular velocity, thereby increasing the damping in yaw derivative C_{n_r}; whereas, aileron control was assumed to be applied in proportion to the rolling angular velocity causing an increase in the damping-in-roll derivative C_{l_p}. For both cases the stability equation is a quartic and the necessary and sufficient conditions for neutral oscillatory stability are similar to those conditions described in the previous discussion of an automatic pilot sensitive to roll.

Automatic pilot sensitive to yawing angular velocity.- Figure 4 shows the neutral-oscillatory-stability boundaries $R_2 = 0$, for $C_{l_\beta} = -0.10$, -0.30, and -0.50. The boundaries indicate that for certain values of C_{n_β} oscillatory stability is obtained provided a definite lower limit of ΔC_{n_r} is exceeded. However, as ΔC_{n_r} is further increased, a critical value is reached beyond which the airplane is unstable. This result is caused by the fact that if the damping in yaw is made sufficiently large, the airplane motion in yaw is restricted. If the motion is then analyzed on the assumption that only two degrees of freedom (roll and sideslip) remain, oscillatory instability will occur for negative values of C_{l_β} (positive effective dihedral). (See reference 3.)

The period and damping relationship for $C_{l_\beta} = -0.10$ and $C_{n_\beta} = 0.15$ for several values of ΔC_{n_r} are presented in table II(c). At point A in figure 4, which corresponds to the airplane without an automatic pilot, a short-period unstable oscillation and two subsiding aperiodic modes are obtained. When the derivative due to the automatic pilot ΔC_{n_r} is introduced or a shift from point A to point B occurs, the instability of the short-period oscillation is improved. Crossing through the boundary $R_2 = 0$ to point C causes the unstable short-period oscillation to become stable. A heavily damped long-period oscillation which is formed from the combination of the two subsiding aperiodic modes also appears for conditions corresponding to point C. As ΔC_{n_r} is increased negatively between point C and point D, the period of the short-period oscillation increases and the damping is improved; thereby greatly reducing the number of cycles required to damp to half amplitude as expressed by the value of $C_{1/2}$. The effect of increasing ΔC_{n_r} negatively on the long-period oscillation is to reduce the period and to decrease the damping. This oscillation with a period of about 7.5 seconds becomes unstable upon passing through the boundary $R_2 = 0$ to point D. For $\Delta C_{n_r} = -44.0$, the roots of the stability equation show that the stable oscillation breaks down into two heavily damped subsiding aperiodic modes and the unstable oscillation becomes more unstable. Because of the comparatively long period of the unstable oscillation, however, pilots might not find this type of instability difficult to control.

Thus the results indicate that an automatic pilot sensitive to the yawing angular velocity and geared to the rudder causes a marked improvement in the original unstable short-period oscillation without introducing any additional modes of motion which might be objectionable to the pilot.

Automatic pilot sensitive to rolling angular velocity.- The neutral-oscillatory-stability boundary $R_2 = 0$ for the airplane equipped with an automatic pilot sensitive to rolling angular velocity is presented in figure 5 for several values of C_{l_β}. The boundaries indicate that oscillatory stability is obtained provided a definite lower limit of ΔC_{l_p} is exceeded. An examination of the period and damping of the oscillation, presented in table II(d) for $C_{l_\beta} = -0.1$ clearly shows that, although the unstable short-period oscillation does become stable as ΔC_{l_p} is increased negatively, any additional damping in roll introduced into the system does not improve the damping of the oscillation sufficiently to result in a satisfactory damping and period relationship. The effect of adding more ΔC_{l_p} into the system is simply to increase the damping of one of the subsiding aperiodic modes while causing a reduction in the damping of the other subsiding aperiodic mode. It is important to note that for some airplane configurations adding more damping in roll into the system will cause the oscillatory mode to become less stable.

Effect of Automatic Stabilization on the Criterions for Satisfactory Damping-Period Relationship of the Oscillatory Mode

The curves that define the satisfactory damping-period relationship of the oscillatory mode as a function of the period are presented in figure 6. The dashed curve represents the NACA criterion (reference 7) and the solid curve represents the military criterion (references 8 and 9). Both curves agree very well with each other for periods greater than 6 seconds whereas for periods from 0.8 to 6 seconds, the military criterion is more stringent than the NACA criterion. For periods smaller than 0.8 of a second, the NACA criterion is more conservative than the military criterion.

The relative merits of the several types of automatic pilots discussed in the present paper can be clearly seen by comparing the damping-period relationship of the oscillatory mode, as affected by the particular type of automatic pilot, with the criterions shown in figure 6. Curves of $\frac{1}{C_{1/2}}$ are presented as a function of the derivative introduced by the automatic pilot for an automatic pilot sensitive to either a displacement in yaw or roll in figure 7 and for an automatic pilot sensitive to either the yawing or rolling angular velocity in figure 8. One curve of $\frac{1}{C_{1/2}}$ is presented for each of the automatic pilots investigated despite the fact that two oscillatory modes may occur, since the airplane motion would be objectionable to the pilot if only one of the oscillatory modes does not satisfy the criterion for the damping-period relationship. An improvement in the original damping-period relationship as each particular type of automatic pilot is introduced is noted in figures 7 and 8 by the fact that $\frac{1}{C_{1/2}}$ changes from a negative to a positive sign. For an automatic pilot sensitive to a displacement in yaw (the solid curve in fig. 7) the values of $\frac{1}{C_{1/2}}$ correspond to points located in the unsatisfactory region of figure 6 according to both criterions. For an automatic pilot sensitive to a displacement in roll (the dashed curve in fig. 7) the oscillatory mode satisfies the NACA criterion for values of C_{l_ϕ} from -0.015 to -0.05. However, for the period of the oscillation during this interval, which ranges from a period of 1.4 to about 2.5 seconds, the military criterion is not satisfied. The values of $\frac{1}{C_{1/2}}$ for the dashed curve in figure 8, which corresponds to an automatic pilot sensitive to the rolling angular velocity, indicate that neither one of the criterions in figure 6 is satisfied. The solid curve in figure 8, which corresponds to an automatic pilot sensitive to the yawing angular velocity, shows that as ΔC_{n_r} increases negatively, the oscillatory mode satisfies the NACA and military criterions. An additional oscillatory mode exists which also satisfies the criterions for the range of ΔC_{n_r} shown in the figure. However, as ΔC_{n_r} continues to increase negatively to a value of approximately −25.0, the value of $C_{1/2}$ becomes negative. (See table II(c).)

Figures 7 and 8 indicate that for this hypothetical supersonic airplane an automatic pilot sensitive to the yawing angular velocity, which effectively increases the derivative C_{n_r}, is the most desirable type of automatic pilot to be used to obtain a satisfactory damping-period relationship of the oscillatory mode.

CONCLUSIONS

The following conclusions were drawn from a theoretical investigation carried out to determine the effect of automatic stabilization on the lateral oscillatory stability of a hypothetical supersonic aircraft:

1. An automatic pilot sensitive to a displacement in either yaw or roll and an automatic pilot sensitive to either the yawing or rolling angular velocity improved the damping-period relationship of the original unstable short-period oscillation.

2. The only one of the several types of automatic pilots investigated which resulted in an oscillation that satisfied the NACA and military criterions for satisfactory damping-period relationship is an automatic pilot sensitive to the yawing angular velocity and geared to the rudder so that rudder control is applied in proportion to the angular velocity.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., December 9, 1948

APPENDIX

DERIVATION OF THE NECESSARY AND SUFFICIENT CONDITIONS FOR NEUTRAL OSCILLATORY STABILITY AND COMPLETE STABILITY OF THE QUINTIC EQUATION

By Leonard Sternfield and Ordway B. Gates, Jr.

The necessary and sufficient conditions for neutral oscillatory stability of the quintic equation are that the coefficients of the stability equation

$$A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0 \tag{A1}$$

satisfy Routh's discriminant set equal to zero

$$R_1 = (BC - AD)(DE - CF) - (BE - AF)^2 = 0$$

and that $BC - AD$ and $BE - AF$ have the same sign. The expression for $R_1 = 0$ can be derived by assuming that the quintic equation has two roots $\lambda = \pm i\omega$, where ω is the angular frequency of the neutrally stable oscillation. This assumption is based on the fact that for the condition of neutral oscillatory stability the real part of the complex root must be zero. If $\lambda = i\omega$ is substituted in the equation (A1) the following two expressions are obtained:

$$A\omega^5 - C\omega^3 + E\omega = 0 \tag{A2}$$

$$B\omega^4 - D\omega^2 + F = 0 \tag{A3}$$

Solving equations (A2) and (A3) simultaneously, thereby eliminating the ω^4 terms, gives the expression

$$\omega^2 = \frac{BE - AF}{BC - AD} \tag{A4}$$

Substituting equation (A4) into either equation (A2) or (A3) results in Routh's discriminant

$$(BC-AD)(DE-CF) - (BE-AF)^2 = 0$$

It is seen from equation (A4) that the expression

$$\omega = \sqrt{\frac{BE - AF}{BC - AD}}$$

defines the angular frequency of the neutrally stable oscillation. The symbol ω represents the frequency of the neutrally stable oscillation only if $BC - AD$ and $BE - AF$ are of the same sign, since ω must have a real value if the root is to represent an oscillation. If $BC - AD$ and $BE - AF$ are of different sign and $R_1 = 0$ is satisfied, ω is an imaginary quantity and the two roots of the quintic equation given by $\lambda = \pm i\omega$ are two real roots equal in magnitude but opposite in sign.

The necessary and sufficient conditions for complete stability of the quintic equation are derived in reference 4 and are presented in a condensed form in reference 5. A much simpler method for obtaining these same conditions is presented in the following analysis.

Assume that the quintic equation

$$A\lambda^5 + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0$$

has the roots

$$\left.\begin{aligned} \lambda_1 &= a_1 + ib_1 \\ \lambda_2 &= a_1 - ib_1 \\ \lambda_3 &= a_2 + ib_2 \\ \lambda_4 &= a_2 - ib_2 \\ \lambda_5 &= a_3 \end{aligned}\right\} \tag{A5}$$

Between the coefficients and roots of the quintic equation the following relationships exist:

$$A = 1 \tag{A6a}$$

$$B = -2(a_1 + a_2) - a_3 \tag{A6b}$$

$$C = a_1^2 + b_1^2 + a_2^2 + b_2^2 + 4a_1a_2 + 2a_3(a_1 + a_2) \tag{A6c}$$

$$\begin{aligned} D = &-2\left[a_1\left(a_2^2 + b_2^2\right) + a_2\left(a_1^2 + b_1^2\right)\right] \\ &- a_3\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 + 4a_1a_2\right) \end{aligned} \tag{A6d}$$

$$\begin{aligned} E = &\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right) \\ &+ 2a_3\left[a_1\left(a_2^2 + b_2^2\right) + a_2\left(a_1^2 + b_1^2\right)\right] \end{aligned} \tag{A6e}$$

$$F = -a_3\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right) \tag{A6f}$$

The requirement for complete stability is that a_1, a_2, and a_3 be negative. Substitution of negative values of a_1, a_2, and a_3 makes all the coefficients positive; therefore, the first condition for complete stability is that all coefficients be positive. From equation (A6f) it can be seen that the F coefficient will be positive only when a_3 is negative. This first condition, therefore, only insures the stability of the root $\lambda_5 = a_3$. Additional requirements are needed to make a_1 and a_2 negative. On the boundary between stability and instability, where $a_2 = 0$, the equations (A6) become

$$\left.\begin{aligned}
A &= 1 \\
B &= -2a_1 - a_3 \\
C &= a_1^2 + b_1^2 + b_2^2 + 2a_3a_1 \\
D &= -2a_1b_2^2 - a_3\left(a_1^2 + b_1^2 + b_2^2\right) \\
E &= \left(a_1^2 + b_1^2\right)b_2^2 + 2a_3a_1b_2^2 \\
F &= -a_3\left(a_1^2 + b_1^2\right)b_2^2
\end{aligned}\right\} \tag{A7}$$

that is, five equations in the four variables a_1, a_3, b_1, and b_2. Elimination of these variables leads to Routh's discriminant

$$R_1 = (BC - AD)(DE - CF) - (BE - AF)^2 = 0$$

which was derived by a much simpler procedure in the preceding analysis of the conditions necessary for neutral oscillatory stability. It is apparent that this discriminant can be equal to zero only if $BC - AD$ and $DE - CF$ are of the same sign. When $a_2 = 0$ the following relationship can be obtained from equations (A7):

$$BC - AD = -2a_1\left(a_3^2 + a_1^2 + b_1^2\right) - 4a_3a_1^2$$

Since a_3 must be negative if all coefficients are positive, $BC - AD$ must be positive if a_1 is negative. Therefore, $DE - CF$ must also be positive if the discriminant is to be equal to zero, which implies that $BC > AD$ and $DE > CF$. Then $C > \frac{AD}{B}$ and $C < \frac{DE}{F}$, also $\frac{DE}{F} > \frac{AD}{B}$; therefore,

$$BE - AF > 0 \tag{A8}$$

Thus, for complete stability a second necessary condition is that $BE - AF$ must be greater than zero.

It can be shown from the simultaneous solution of equations (A7), where $a_2 = 0$, that

$$BE - AF = -2a_1\left[E + \left(B + 2a_1^{\,2}\right)b_2\right]$$

Since the terms within the bracket are all positive, $BE - AF > 0$ if, and only if, a_1 is negative. This verifies the correctness of equation (A8).

In order to determine the sign of Routh's discriminant for a condition of complete stability, it is only necessary to substitute the roots of a completely stable case into equations (A6) and form the discriminant. This substitution establishes the criterion that for complete stability

$$R_1 = (BC - AD)(DE - CF) - (BE - AF)^2 > 0 \tag{A9}$$

In summary, the necessary and sufficient conditions for complete stability of the quintic equation are that all coefficients must be positive, $BE - AF > 0$, and Routh's discriminant must be positive. The conditions obtained as a result of this analysis agree with the conditions as presented in reference 5. The conditions stated in reference 5 are entirely adequate although they do not specifically state that the coefficients C and E must be positive, since the condition that $BE - AF$ must be greater than zero inherently demands that these two coefficients be positive as is evident from the derivation of this condition in the preceding analysis.

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TABLE I.– STABILITY DERIVATIVES AND MASS CHARACTERISTICS

OF HYPOTHETICAL AIRPLANE IN CRUISING CONDITION

W/S, lb/ft^2	40
b, ft	20
ρ, slugs/ft^3	0.0002
V, ft/sec	1465
γ, deg	0
C_L	0.372
μ_b	620
k_{X_0}, ft	2.02
k_{Z_0}, ft	9.64
η, deg	0
C_{l_p}, per radian	−0.197
C_{l_r}, per radian	0.0929
C_{n_p}, per radian	−0.00732
C_{n_r}, per radian	$-1.47\ C_{n_{\beta(tail)}}$
C_{Y_p}, per radian	0
C_{Y_r}, per radian	0
C_{Y_β}, per radian	$-1.33\ C_{n_{\beta(tail)}}$
$C_{n_{\beta(fuselage)}}$, per radian	−0.25
C_{n_β}, per radian	Variable
$C_{n_{\beta(tail)}}$, per radian	$C_{n_\beta} - C_{n_{\beta(fuselage)}}$
C_{l_β}, per radian	Variable

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TABLE II.– PERIOD AND DAMPING OF THE OSCILLATORY MODES

AND DAMPING OF THE APERIODIC MODES

$$\left[C_{l_\beta} = -0.1\right]$$

(a) Automatic Pilot Sensitive to Displacement in Yaw

Points	C_{n_β}	C_{n_ψ}	$\frac{\partial \delta_r}{\partial \psi}$	Oscillatory mode			Aperiodic mode
				P	$T_{1/2}$	$C_{1/2}$	$T_{1/2}$
A	0.15	0	0	3.62	−7.65	−2.11	0.827
				-----	-------	-------	32.7
				-----	-------	-------	∞
B	.15	−.002	.02	3.61	−7.88	−2.18	.805
				59.2	247.0	4.18	------
C	.15	−.0035	.035	3.61	−7.97	−2.20	.814
				44.2	−835.0	−18.9	------
D	.15	−.13	1.3	2.73	−217.0	−78.5	.658
				10.3	−3.75	−.364	------
E	.15	−.15	1.5	2.62	330.0	126.0	.650
				10.0	−3.53	−.353	-------
	.15	−.20	2.0	2.45	68.0	27.8	.637
				9.40	−3.22	−.343	------
	.15	−.40	4.0	1.96	26.7	13.6	.611
				8.65	−2.72	−.314	------
	.55	0	-----	1.95	11.6	5.95	1.06
				-----	-------	-------	58.3
				-----	-------	-------	∞
	.55	−.002	.02	1.95	11.6	5.95	1.05
				88.5	346.0	3.92	------
	.55	−.0035	.035	1.95	11.6	5.95	1.05
				66.8	−825.0	−12.4	------
	.55	−.02	.2	1.93	11.2	5.80	.975
				29.7	−26.5	−.892	------
	.55	−.10	1.0	1.79	10.2	5.70	.830
				15.6	−7.28	−.466	------
	.55	−.20	2.0	1.70	9.7	5.70	.755
				12.5	−4.95	−.395	------
	.55	−.40	4.0	1.50	9.0	6.0	.692
				10.5	−3.71	−.353	------

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TABLE II.– Continued

(b) Automatic Pilot Sensitive to Displacement in Roll

Points	C_{n_β}	C_{l_ϕ}	$\frac{\partial\delta_a}{\partial\phi}$	Oscillatory mode			Aperiodic mode
				P	$T_{1/2}$	$C_{1/2}$	$T_{1/2}$
A	0.15	0	0	3.62 -----	–7.65 ------	–2.11 -------	0.827 32.7
B	.15	–.008	.08	3.15 4.19	–9.24 1.68	–2.93 .401	------ ------
C	.15	–.012	.12	2.56 3.76	9.76 2.57	3.81 .684	------ ------
	.15	–.022	.22	2.07 3.76	3.30 5.42	1.59 1.44	------ ------
	.15	–.050	.50	1.37 3.76	2.69 8.66	1.96 2.30	------ ------
	.15	–.10	1.0	.967 3.76	2.54 10.70	2.63 2.85	------ ------
	.15	–.20	2.0	.681 3.76	2.48 11.70	3.64 3.11	------ ------
	.45	0	0	2.16 -----	18.70 ------	8.66 -------	1.03 53.30
	.45	–.008	.08	2.14 3.51	102.0 1.87	47.70 .533	------ ------
	.45	–.012	.12	2.13 2.87	–40.80 1.75	–19.20 .61	------ ------
	.45	–.022	.22	1.99 2.25	106.60 1.86	53.80 .827	------ ------
	.45	–.040	.40	1.53 2.16	3.03 4.64	1.98 2.15	------ ------
	.45	–.050	.50	1.37 2.17	2.82 5.23	2.06 2.41	------ ------
	.45	–.100	1.0	.964 2.17	2.56 6.47	2.66 2.98	------ ------
	.45	–.200	2.0	.679 2.17	2.49 6.99	3.67 3.22	------ ------

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TABLE II.– Continued

(c) Automatic Pilot Sensitive to Yawing Angular Velocity

Points	C_{n_β}	ΔC_{n_r}	$\frac{\partial \delta_r}{\partial \dot{\psi}}$	Oscillatory mode			Aperiodic mode
				P	$T_{1/2}$	$C_{1/2}$	$T_{1/2}$
A	0.15	0	0	3.62 ------	−7.65 -------	−2.11 -------	0.827 32.70
B	.15	−.733	.05	3.66 ------	−14.96 -------	−4.09 -------	.863 11.60
C	.15	−4.40	.30	3.84 179.90	4.38 1.51	1.14 .0084	------ ------
	.15	−7.33	.50	4.13 16.00	2.11 1.45	.511 .0906	------ ------
	.15	−14.70	1.0	5.68 7.43	.647 3.44	.114 .463	------ ------
	.15	−20.50	1.4	6.69 7.42	.432 17.20	.0646 2.32	------ ------
D	.15	−25.40	1.73	9.54 7.45	.356 −33.20	.0373 −4.47	------ ------
	.15	−44.0	3.0	------ 7.49	------- −4.95	------- −.661	.483 .135

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TABLE II.– Concluded

(d) Automatic Pilot Sensitive to Rolling Angular Velocity

C_{n_β}	ΔC_{l_p}	$\frac{\partial \delta_a}{\partial \dot{\phi}}$	Oscillatory mode			Aperiodic mode
			P	$T_{1/2}$	$C_{1/2}$	$T_{1/2}$
0.15	0	0	3.62	−7.65	−2.11	0.1 7
			----	------	-----	32.70
.15	−0.44	.03	3.59	−20.90	−5.82	.345
			----	------	-----	79.70
.15	−1.17	.07	3.63	39.40	10.90	.172
			----	------	-----	157.50
.15	−1.47	.10	3.65	26.50	7.26	.142
			----	------	-----	189.40
.15	−4.40	.30	3.71	14.70	3.96	.052
			----	------	-----	501.60
.15	−7.33	.50	3.72	13.90	3.74	.032
			----	------	-----	815.50
.15	−14.70	1.0	3.74	13.40	3.58	.016
			----	------	-----	1593.0

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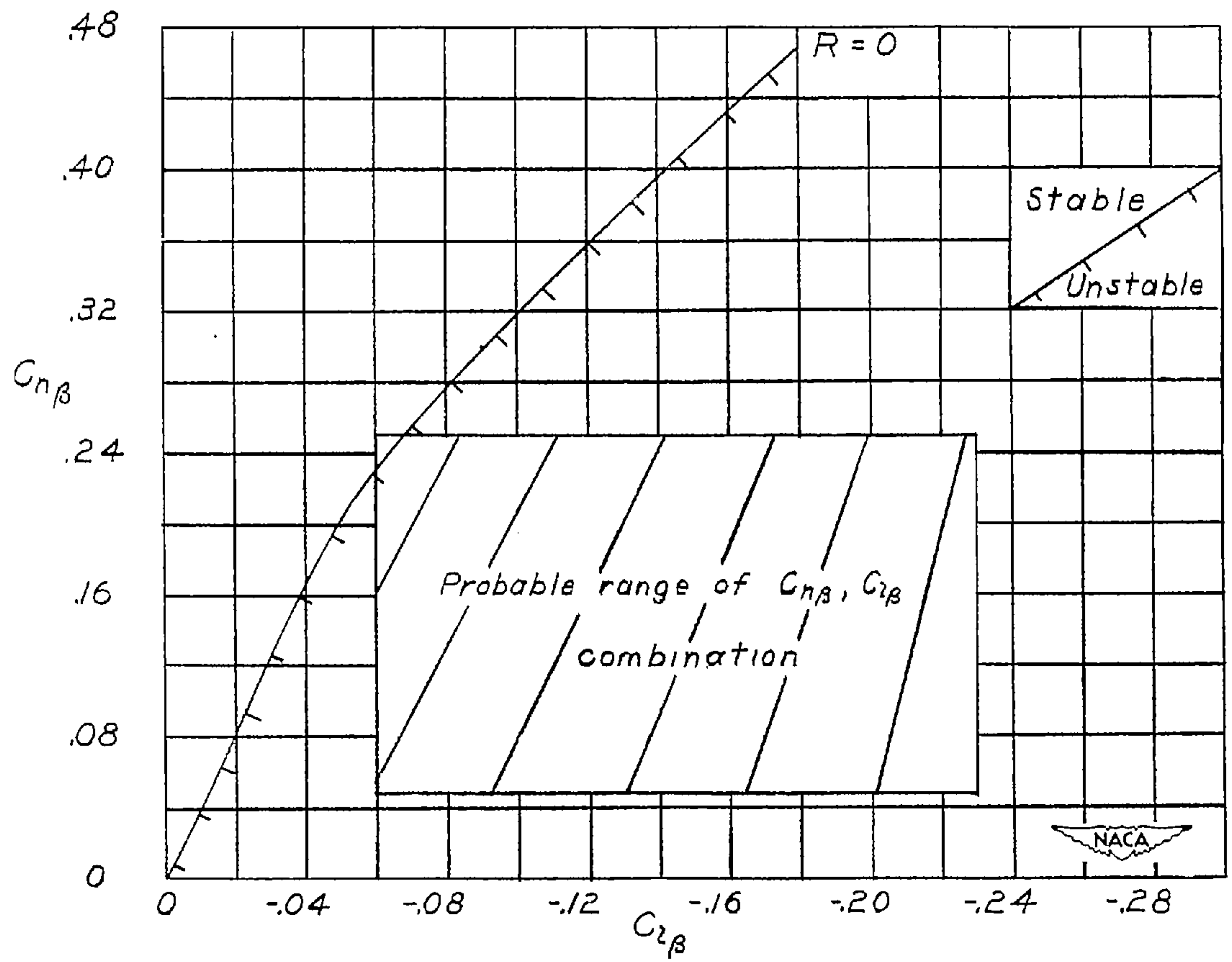
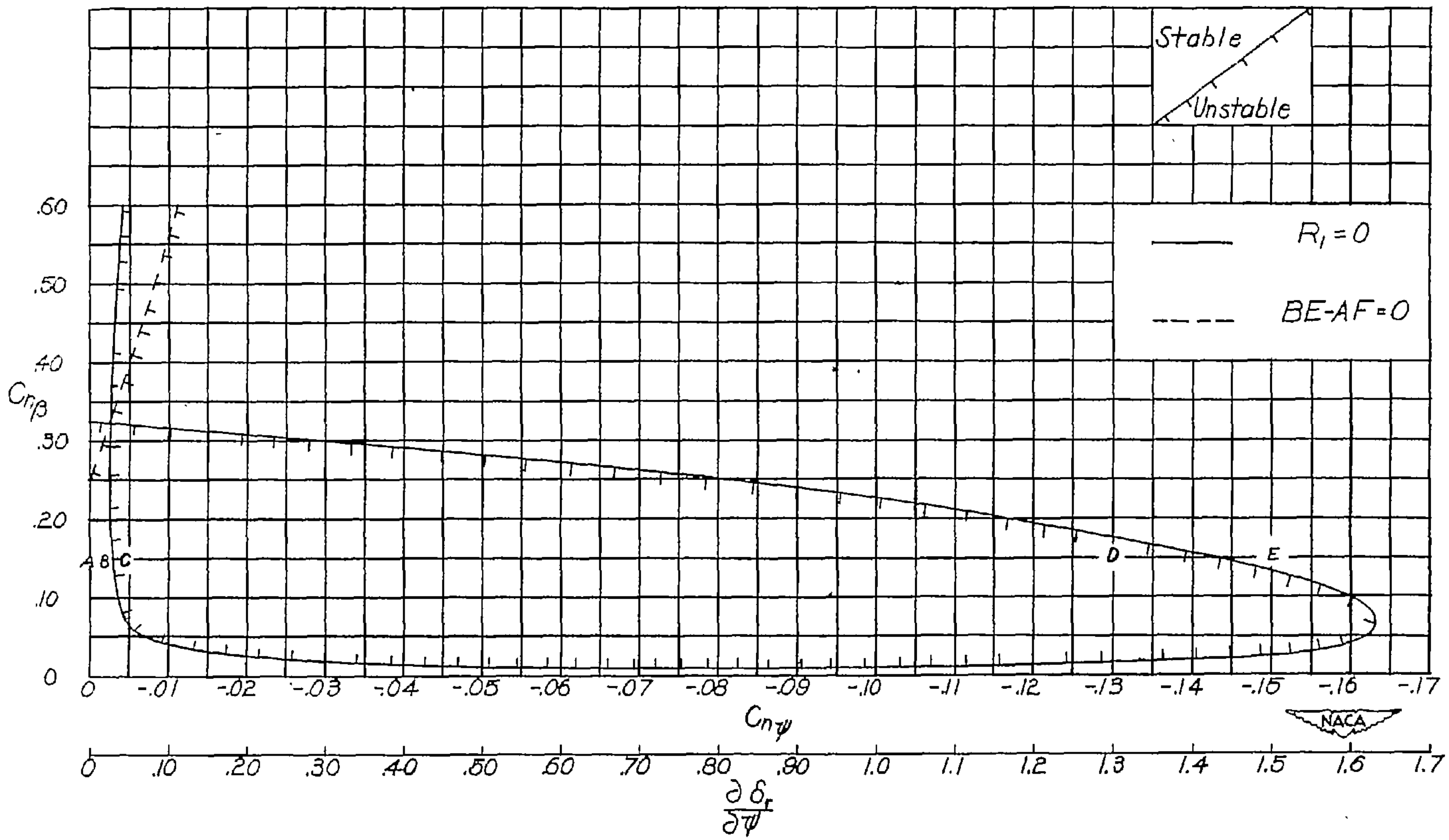
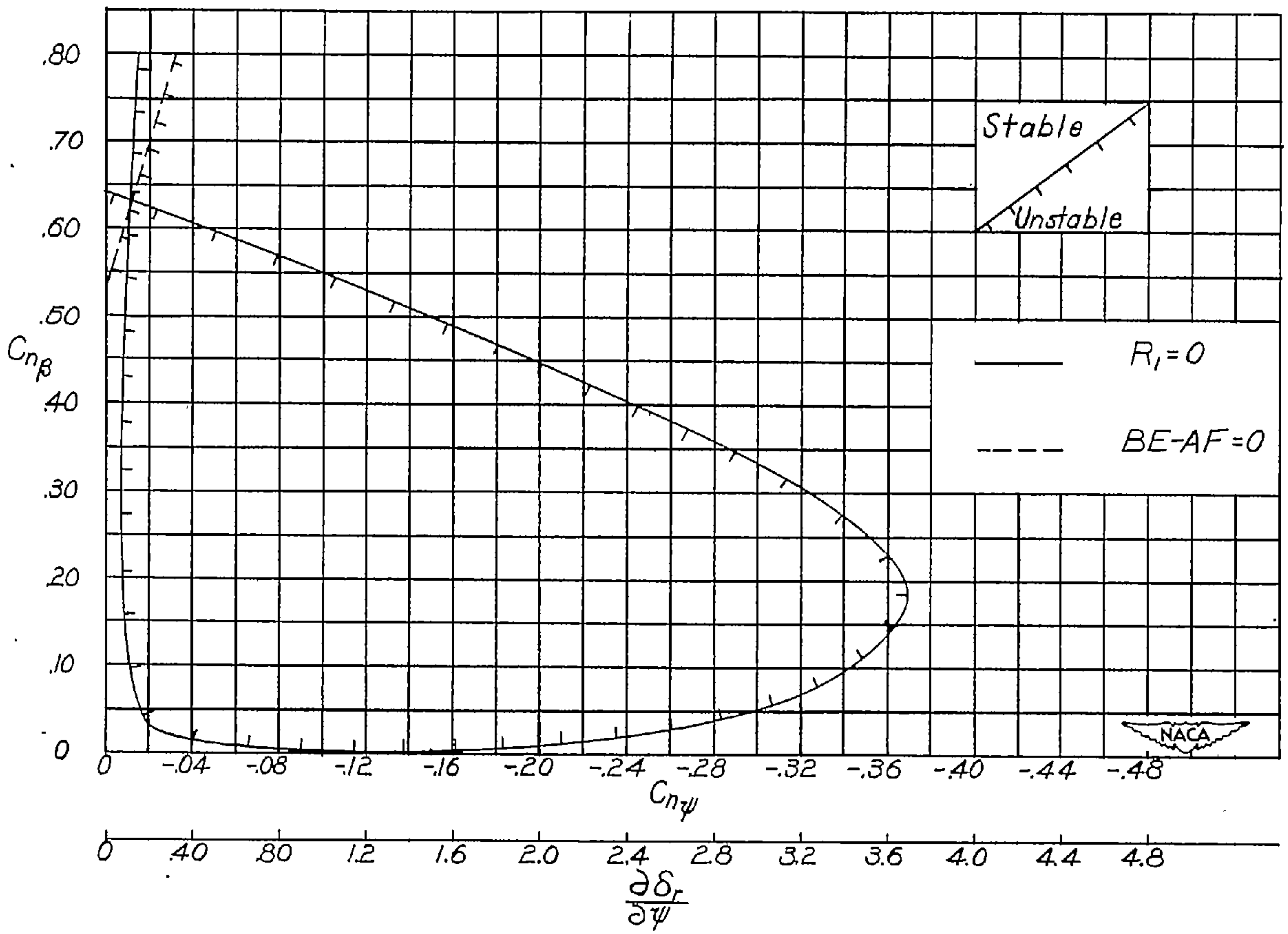


Figure 1.– Neutral-oscillatory-stability boundary for a hypothetical airplane. $\eta = 0^{\circ}$.



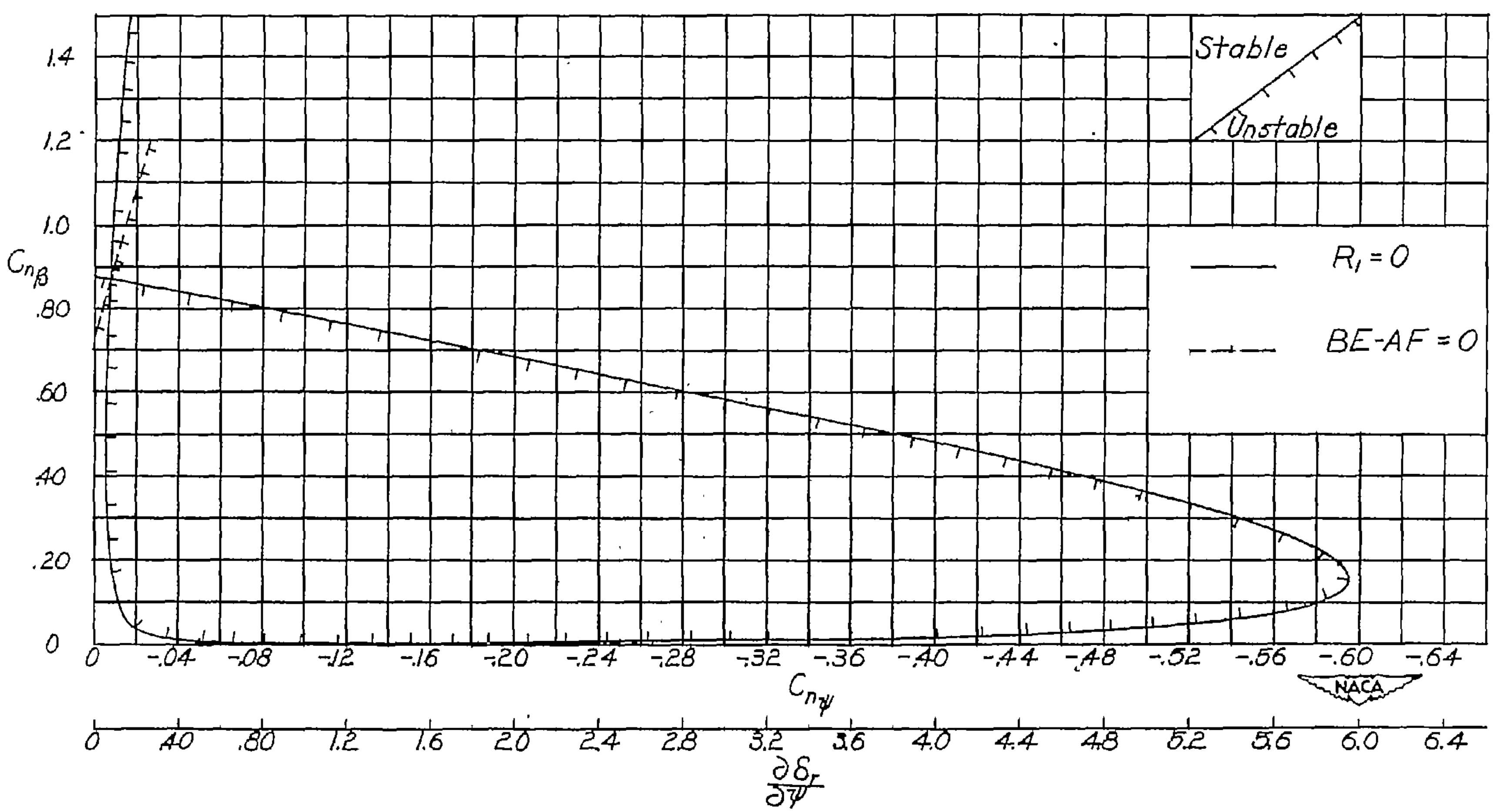
(a) $C_{l_\beta} = -0.10$.

Figure 2.– Effect of C_{n_ψ} on the lateral oscillatory stability.



(b) $C_{l_\beta} = -0.30$.

Figure 2.– Continued.



(c) $C_{l_\beta} = -0.50$.

Figure 2.– Concluded.

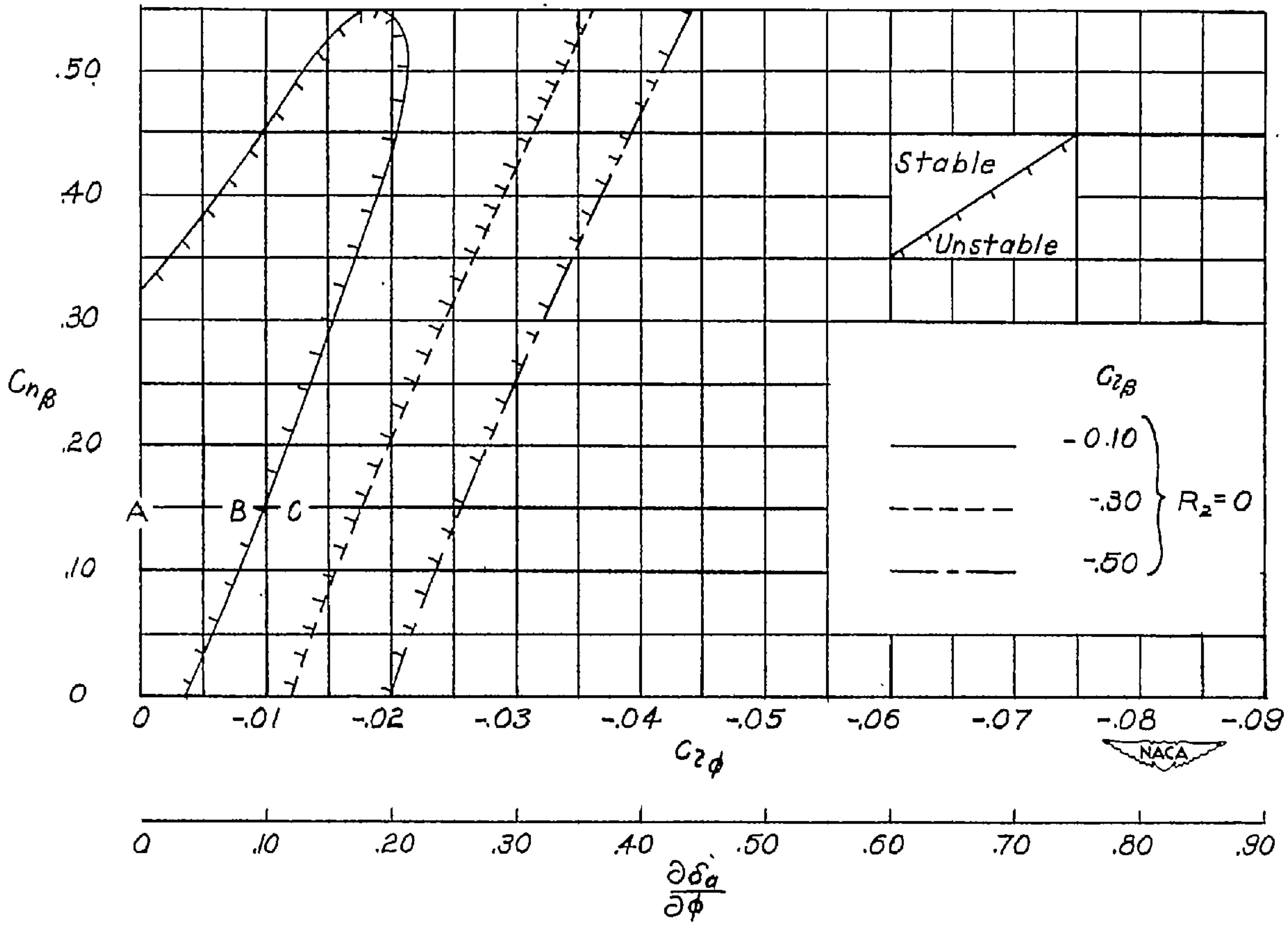


Figure 3.– Effect of C_{l_ϕ} on the lateral oscillatory stability.

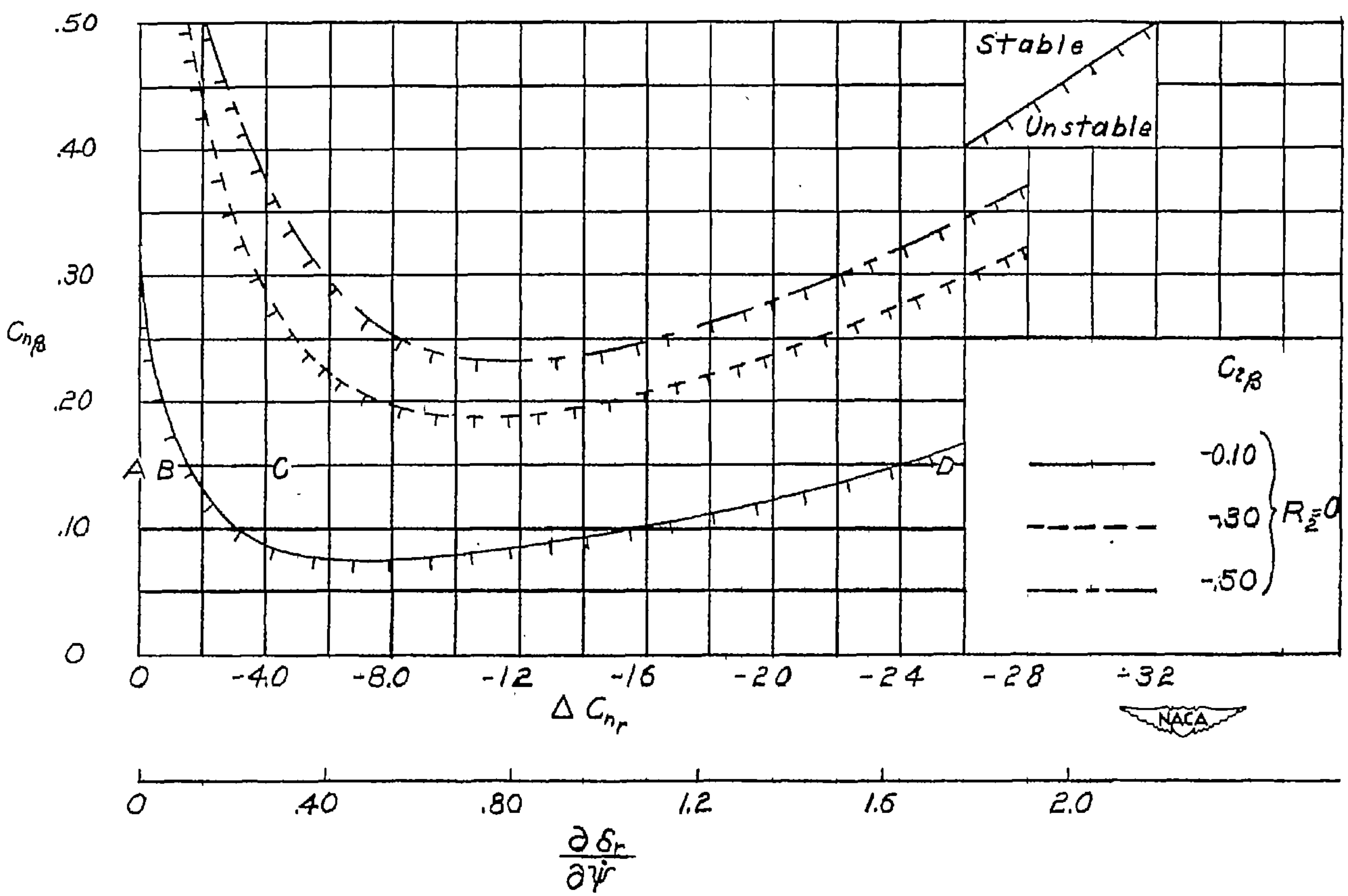


Figure 4.– Effect of ΔC_{n_r} on the lateral oscillatory stability.

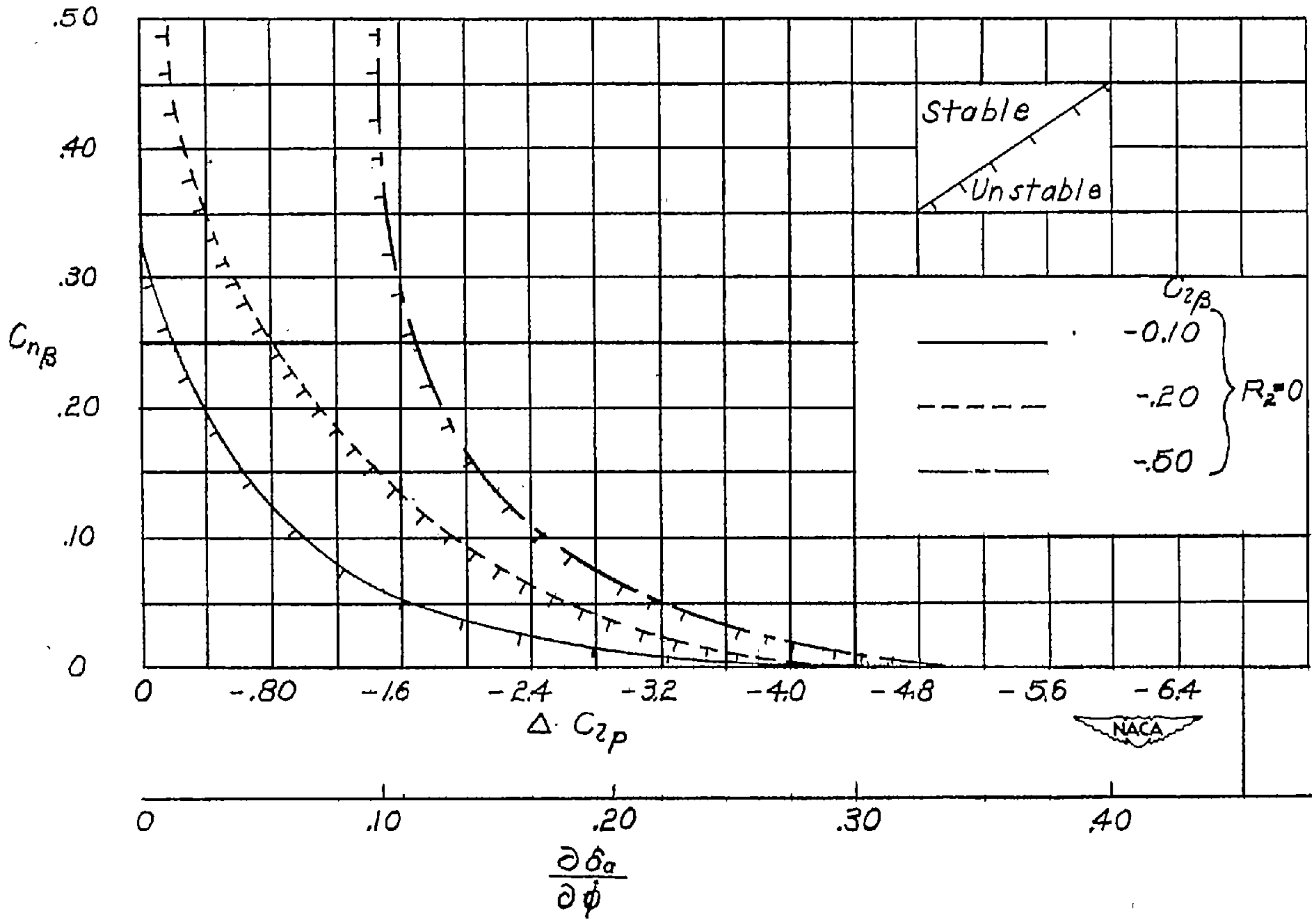


Figure 5.– Effect of ΔC_{l_p} on the lateral oscillatory stability.

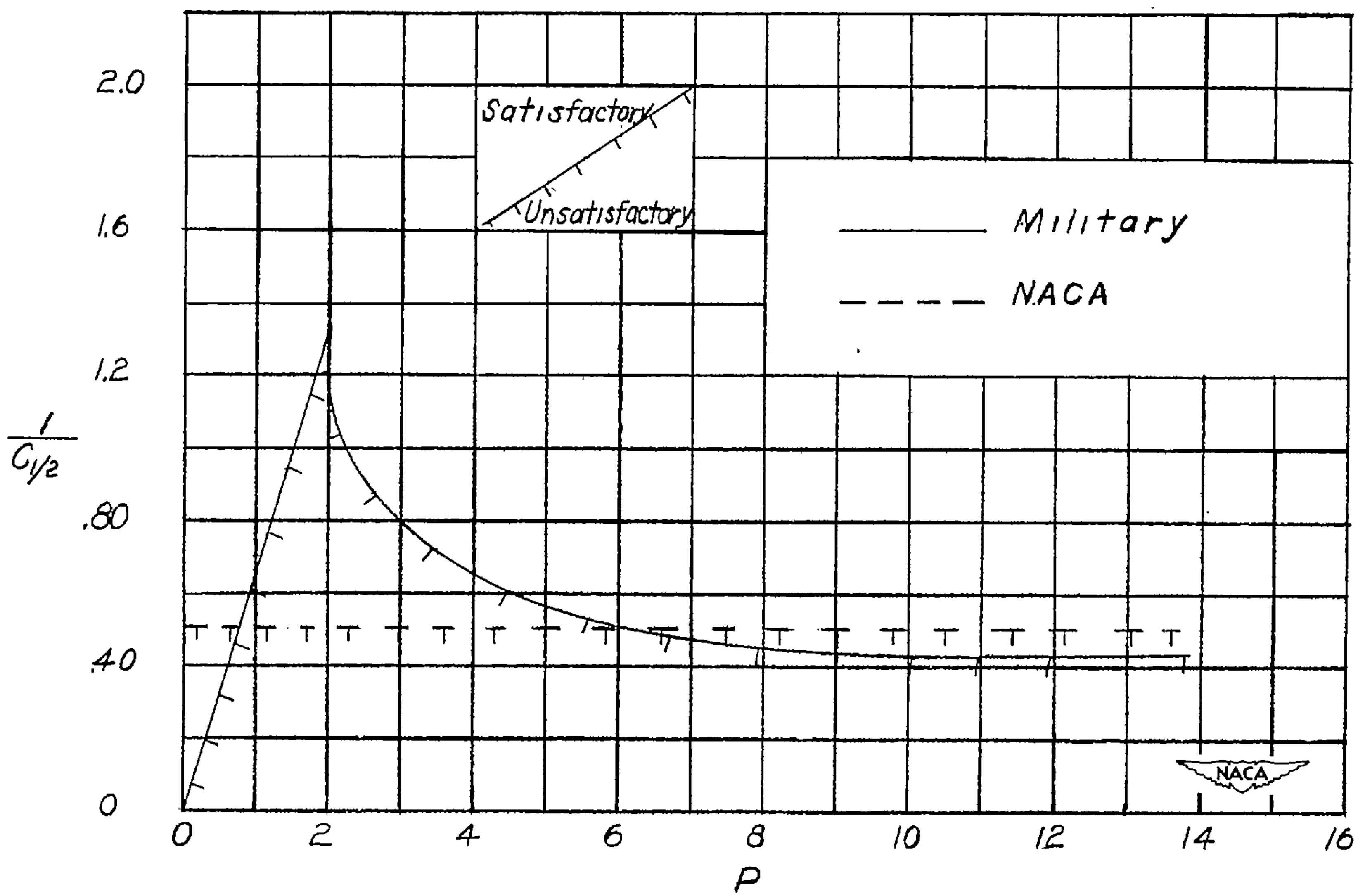


Figure 6.– Criterions for the damping-period relationship of the oscillatory mode.

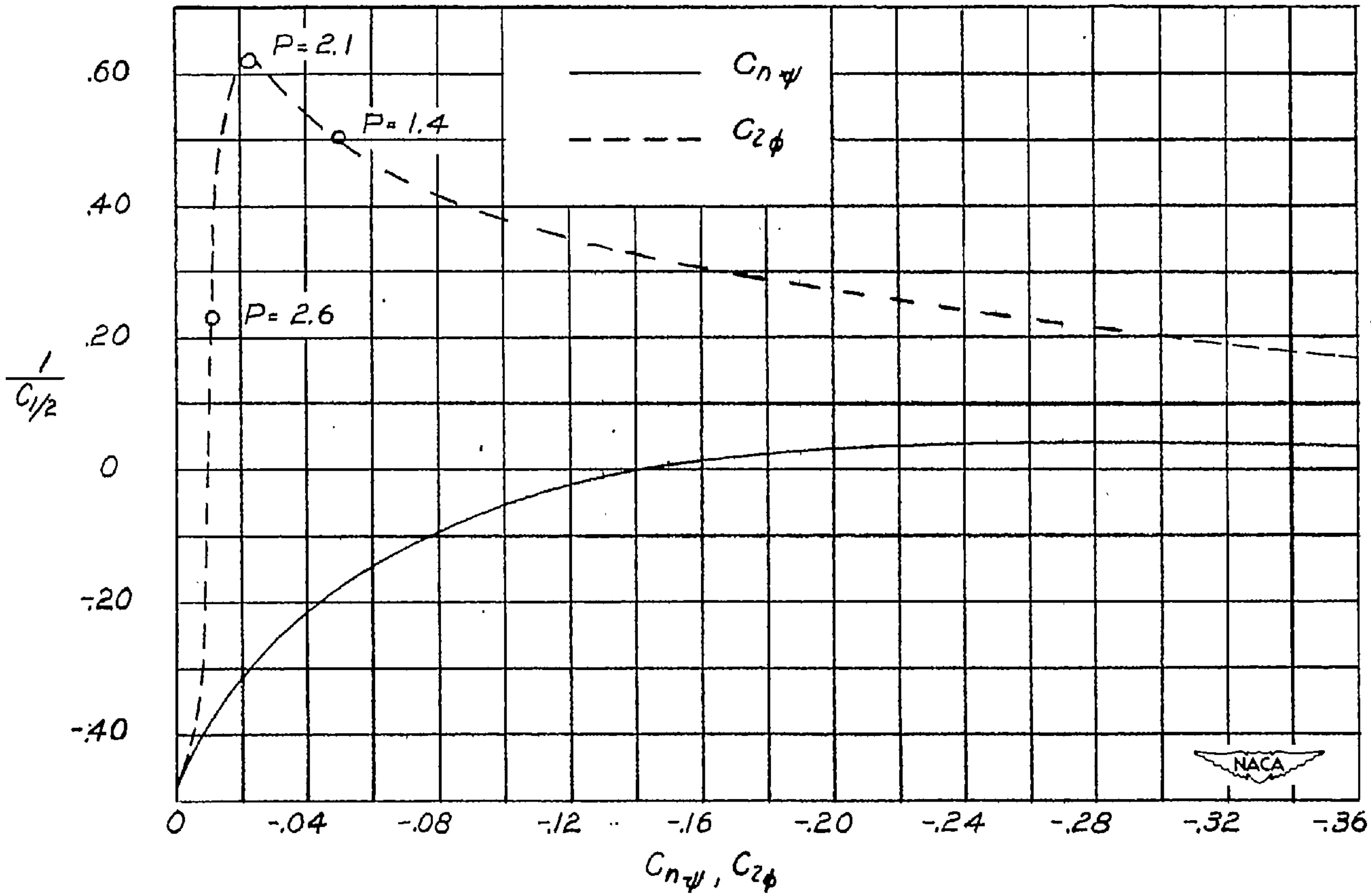


Figure 7.– Effect of C_{n_ψ} and C_{l_ϕ} on the damping-period relationship of the oscillatory mode.

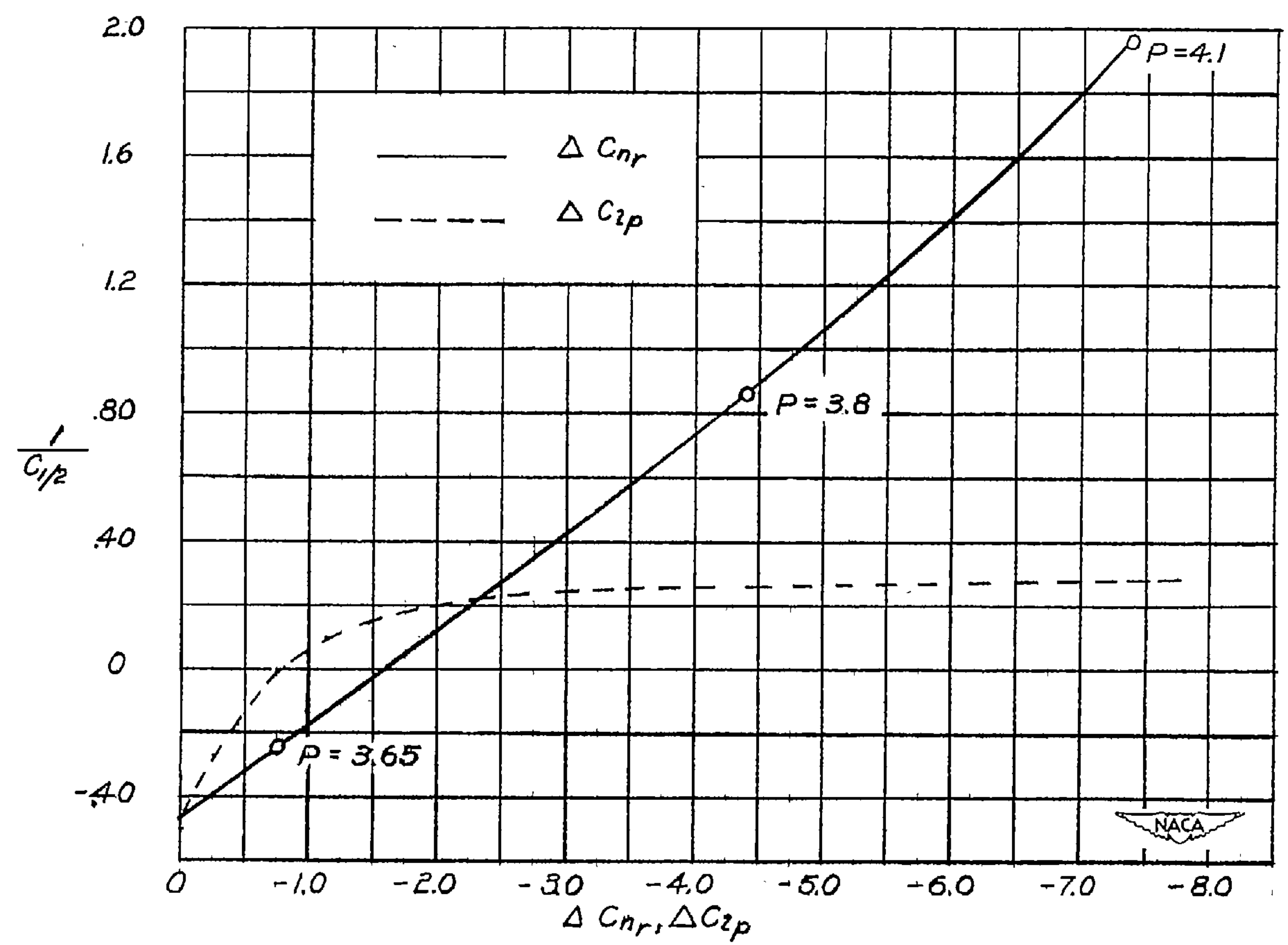


Figure 8.– Effect of ΔC_{n_r} and ΔC_{l_p} on the damping-period relationship of the oscillatory mode.