NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1654

SOUND FROM DUAL-ROTATING AND MULTIPLE

SINGLE-ROTATING PROPELLERS

By Harvey H. Hubbard

Langley Aeronautical Laboratory Langley Field, Va.



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SUMMARY

Sound measurements for static conditions in the tip Mach number range 0.37 to 0.89 are presented for three different dual-rotating-propeller configurations and one combination of two single-rotating propellers operating side by side in the same plane of rotation. The results obtained are compared with a theoretical analysis of the problem and excellent agreement is found.

The sound from a four-blade dual-rotating propeller was found to fluctuate approximately between that of a two-blade and a four-blade single-rotating propeller when the propellers are absorbing the same power at the same tip speeds; the amount of fluctuation was found to depend on the angle of overlap with respect to the observer. By correct phasing of the components of a dual-rotating propeller in flight, the sound reaching the ground in a given direction can be reduced by a small amount.

Mutual interference has been detected in the form of sound near the axis of rotation for dual-rotating propellers. The magnitude of the sound resulting from this interference has been found to vary directly as the power and as the cube of the tip speed.

INTRODUCTION

The problem of propeller noise of large airplanes necessitates an understanding of the way in which the noise fields from multiple propellers add up. Since very few sound measurements have been reported on dual-rotating propellers and multiple single-rotating propellers, extension of the theory for predicting the sound and an experimental check of this theory seemed desirable.

A solution for the noise from a single-rotating propeller in which the air forces are steady has been given by Gutin in reference 1. For dual-rotating propellers, the mutual interference between them results in a periodic variation of air loads on the blades. An exact solution of the problem taking this fact into account is difficult, particularly since the load variation has not been accurately determined.



Static tests were made for a series of dual-rotating propellers and one combination of two single-rotating propellers operating side by side in the same plane. Data obtained were compared with an elementary theoretical analysis expressing the noise from a dual-rotating propeller as the sum of the noise from each of two single-rotating propellers and neglecting mutual interference.

SYMBOLS

$\mathtt{P}_{\mathtt{l}_{\mathtt{m}}}$	maximum sound pressure of mth-harmonic referred to one single-rotating propeller, dynes per square centimeter
P _l ' _m	maximum sound pressure of mth-harmonic referred to single blade, dynes per square centimeter
P _{lmp}	maximum sound pressure of mth-harmonic for two single-rotating propellers having arbitrary phase angle, dynes per square centimeter
$p_{l_{m_1}}$	instantaneous value of sound pressure of mth-harmonic of sound from single-rotating propeller, dynes per square centimeter
n	number of blades
x	blade numbers (1, 2, 3, etc.)
	•
D	order of harmonic
$\mathbf{m} = \mathbf{q}\mathbf{n}$	order of harmonic
_	order of harmonic $Bessel \ function \ of \ order \ qn \ and \ argument \\ qn \overline{\frac{V}{C}} \ sin \ \beta$
m = qn	Bessel function of order qn and argument
$\mathbf{m} = \mathbf{q}\mathbf{n}$ $\mathbf{J}_{\mathbf{q}\mathbf{n}} \left(\mathbf{q}\mathbf{n}_{\mathbf{q}}^{\mathbf{r}} \sin \beta \right)$	Bessel function of order qn and argument $qn\frac{V}{C}\text{sin}\beta$
$m = qn$ $J_{qn}(qn \frac{V}{c} \sin \beta)$ M_t	Bessel function of order qn and argument $qn\frac{V}{c}\sin\beta$ tip Mach number of blade (rotation only).
$m = qn$ $J_{qn}(qn\frac{V}{c} \sin \beta)$ M_{t} P_{H}	Bessel function of order qn and argument $qn\frac{V}{c}\sin\beta$ tip Mach number of blade (rotation only) . horsepower supplied to propeller
$m = qn$ $J_{qn}(qn\frac{V}{c} \sin \beta)$ M_{t} P_{H} J	Bessel function of order qn and argument $qn\frac{V}{C}\sin\beta$ tip Mach number of blade (rotation only) . horsepower supplied to propeller advance ratio



k	integer
Z · .	spacing between planes of rotation of dual- rotating propellers, inches
α	blade phase angle, degrees
β	angle from propeller axis of rotation (zero in front), degrees
θ	blade angle of propeller at 0.75 radius, degrees
Ψ	angle of overlap $(\alpha/2)$, degrees
Υ	velocity of section at 0.8 radius, feet per second
c	velocity of sound, feet per second
в	distance from propeller, feet
Subscripts:	
F	front
R	rear
max .	maximum

A bar placed over a symbol indicates a vector quantity.

SOUND THEORY

Experiments reported in this paper indicate that the noise from a dual-rotating propeller at a given point in space is a function of the position at which the blades overlap each other. The experiments further indicate that the magnitude of the noise and the frequency spectrum are such as might be obtained from the summation of the noise fields of two single-rotating propellers.

In the following elementary analysis the periodic variation of the air loads on the blades is neglected. The noise of a dual-rotating propeller is expressed in terms of the noise from two single-rotating propellers operating with the same direction of rotation but with arbitrary phase angle between them.



The analysis can be extended to include the case of two or more propellers operating separately as in a multiengine airplane. This analysis consists simply of adding the pressure fields at a point in space and taking into account the proper phase relations as determined by the phase of the propellers and the difference in distances to the observer.

It can be shown for a given single-rotating propeller that $p_{l_m} = np_{l_m}^{}$ for all values of m which are integral multiples of the number of blades n and that $p_{l_m} = 0$ for all other values of m. (See appendix.)

Gutin indicates that the noise from a propeller is independent of direction of rotation. If the phase angle of two oppositely rotating blades (fig. l(a)) is the same with respect to the observer $(\alpha_A = \alpha_B)$, the sound from each propeller is the same at the observer. In the following analysis, propeller B operating in the clockwise direction is replaced by an equal propeller B' operating in the counterclockwise direction with phase angle α with respect to the observer (see fig. l(b)); the sound from propeller B' is added to that of propeller A also operating in the counterclockwise direction with zero phase angle. The axis of overlap as indicated in the figure is the position where the dual-rotating blades cross.

Since the sound pressure at any point in space is the vectorial sum of the sound pressures of all the blades, it is merely necessary to determine the proper phase relation of the sound of each harmonic of each blade and to add them vectorially.

When the two equal propellers of figure 1(b) are considered, the sound of the mth-harmonic of propeller B' is out of phase by angle ma with the sound of the mth-harmonic from propeller A at the observer's station. Hence,

$$\overline{p_{l_{m_{\underline{T}}}}} = p_{l_{\underline{m}}}(\cos 0^{\circ} + j \sin 0^{\circ} + \cos m\alpha + j \sin m\alpha)$$
 (1)

Since the absolute value of $\overline{p_{l_{m_T}}}$ equals the square root of the sum of the squares of the real and imaginary components,

$$p_{l_{m_T}} = p_{l_m} \sqrt{(\cos 0^{\circ} + \cos m\alpha)^2 + (\sin 0^{\circ} + \sin m\alpha)^2}$$



By expanding the preceding expression and combining like terms,

$$p_{l_{\underline{m}\underline{T}}} = 2p_{l_{\underline{m}}} \sqrt{\frac{1 + \cos m\alpha}{2}}$$

or

$$p_{\underline{l}_{\underline{m}_{\underline{T}}}} = 2p_{\underline{l}_{\underline{m}}} \left| \cos \frac{m\alpha}{2} \right| \tag{2}$$

For two propellers having equal numbers of blades, the sound cancels for all values of $m\alpha = k$, where k is an odd integer. For all even values of k or for k=0, the sounds of the two propellers add in phase.

The foregoing analysis indicates that for the case of two propellers of two blades each, operating in the same direction, $p_{lm_{\rm I}}=0$ for all odd values of m since each propeller has an even number of blades. If $\alpha=0^{\rm O}$, the sound of the propellers add in phase. If $\alpha=\frac{\pi}{2}$, the sound pressures of the harmonics $m=2,6,10,14,\ldots$ are eliminated; but the pressures of the harmonics $m=4,8,12,16,\ldots$ are added. This statement merely says that the sound from 2 two-blade propellers $90^{\rm O}$ out of phase is the same as for a four-blade propeller.

Since the two propellers operating in the same direction with phase angle α are equivalent to a dual-rotating propeller with angle of overlap Ψ equal to $\alpha/2$ as shown in figure 1(b), the following statement is evident: A four-blade dual-rotating propeller (two blades in each component) is equivalent in sound output to a single two-blade single-rotating propeller absorbing the same power at the same tip speed when the phase angle of overlap is equal to zero with respect to the observer. The same propeller is equivalent to a four-blade single-rotating propeller when the angle of overlap is at $\pi/4$ or 45° with respect to the observer.

Figure 2 has been calculated by using equation (2) and values of $qnJ_{qn}\left(qn\frac{V}{c}\sin\beta\right)$ from reference 2. This figure shows the calculated sound pressures of each of the first four harmonics and the total sound pressure for a four-blade dual-rotating propeller at $\beta=120^{\circ}$ and an effective Mach number of 0.5 (V is evaluated at 0.8 radius) for values of ψ from 0° to 90° . The figure illustrates the amplitude modulation of each harmonic of sound emitted by the propeller and, also, the fluctuation of the total sound pressures as a function of the angle of overlap.



A comparison of theoretical sound pressures of two-blade and four-blade single-rotating propellers at $\beta=120^{\circ}$ (reference 2) shows that the difference in rotational-noise-intensity levels varies from approximately 4 decibels at $M_t=0.9$ to approximately 12 decibels at $M_t=0.5$. Thus, for a four-blade dual-rotating propeller the rotational sound pressure is approximately 4 to 12 decibels less (depending on the tip Mach number) when the blades overlap at 45° than when they overlap at 0° with respect to the observer. This difference indicates the desirability of fixing the phase relations of a dual-rotating propeller in order to take advantage of the directional characteristics of the emitted sound.

APPARATUS AND METHODS

Static tests were conducted for the measurement and analysis of the noise emission for three different dual-rotating propellers and one combination of two single-rotating propellers operating side by side in the same plane of rotation. The dual-rotating propellers tested are designated the 2-0-2, 2-0-3, and 2-0-4 configurations, in which the first digit in the designation denotes the number of blades in the front component, the second digit denotes the number of countervanes, and the third digit denotes the number of blades in the rear component. Related tests were also conducted for comparison of the sound emission of pusher and tractor single-rotating propellers absorbing the same power and for different strut clearances.

All propeller configurations were made up of NACA 4-(3)(08)-03 blades. This NACA designation is defined and the blade-form curves for the blades are given in reference 3.

The blade angles for the front and rear components of all dualrotating combinations were set so that each absorbed approximately the same power while running singly.

Two 200-horsepower, water-cooled, variable-speed electric motors were used to drive the test propellers. In the dual-rotating setups the motors were placed end to end and facing each other, one driving a pusher propeller and the other driving a tractor propeller. (See fig. 3.) Provisions were made to vary the spacing between propellers by moving the motor mounts. Total power input to the drive motors in all tests was measured directly by means of a wattmeter. These readings were corrected by means of motor-efficiency charts to determine actual power input to the propellers.

Line voltage on one of the drive motors was reduced to give a maximum slip of approximately 20 rpm at a motor speed of 4800 rpm. This reduction permitted a study of the effect of relative blade position on the sound emission of multiple propellers. The sum of the voltages from tachometer generators mounted on each motor shaft was recorded to give an instantaneous picture of the blade positions.

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relative to each other. On the oscillograph records shown in this paper, relative blade positions are indicated by the trace marked "blade phase."

Total sound pressures were measured with a General Radio Company sound level meter, type 759, which was calibrated against an arbitrary sound standard. Permanent records for the analysis of propeller noise spectrums were obtained by recording the output of a Hewlitt-Packard harmonic wave analyzer by means of a Heiland oscillograph recorder, type A 400-R. The analyzer which was modified to include an automatic scanning device had a band width of 100 cycles. The pickup used with the analyzer was a Western Electric moving-coil pressure-type microphone.

The microphone was always placed at ground level and was at a distance of 30 feet from the propeller hub for single-rotating propellers and 30 and 60 feet from a point midway between the propeller hubs for dual-rotating and multiple propellers, respectively. Data were taken at values of β from 0° to 180° at rotational speeds of 1000 to 4800 rpm. All tests were run on days when wind velocities were low in order to get consistent data.

RESULTS AND DISCUSSION

Static tests were run for three dual-rotating configurations and one combination of two single-rotating propellers operating side by side in the same plane of rotation in order to compare the results with the theory given in this paper. Quantitative and qualitative records of the noise emission from multiple propellers are presented in the figures. An attempt is also made to evaluate the effects of strut interference for pusher propellers and mutual interference between the blades of a dual-rotating configuration.

Frequency Spectrums

Figure 4 gives a comparison of the frequency spectrums for the same tip speeds of two single-rotating propellers with a four-blade dual-rotating configuration made up of a tractor two-blade propeller and a pusher two-blade propeller and with a combination of two single-rotating propellers operating side by side. Thus, if several two-blade propellers are being operated simultaneously, the combined frequency spectrum should be the same as for a single-rotating two-blade propeller with the exception that the amplitude of each frequency will fluctuate.

The effect of blade phase angle on the amplitude of the various harmonics of sound from a 2-0-2 propeller is shown in figure 5. The actual blade phase angle is not known for each of these records, but they were taken for blade-phase differences of about 90°. Figure 5 merely illustrates that the amplitude of each frequency varies greatly;



this variation depends on the overlap angle of the blades. Succeeding figures show the extent of this variation.

Figure 6 illustrates the manner in which the amplitude of the fundamental frequency of a 2-0-2 propeller varies when compared with the amplitudes of two single-rotating propellers. For the two single-rotating propellers the amplitude of the fundamental frequency remains quite steady with time regardless of the blade positions relative to the observer. On the other hand, the fundamental frequency of the 2-0-2 propeller seems to be amplitude modulated to an extent dependent on the relative blade positions.

This modulation effect is further shown in figure 7 in which the second, third, fourth, and fifth harmonics of the same 2-0-2 propeller are pictured. Records taken during these tests have indicated clearly the modulation of all rotational noise frequencies up to the eleventh harmonic. For the records shown in figure 7 maximum and minimum points on the blade-phase trace indicate a blade phase of 0° which occurs when the blades are parallel to a line drawn from the microphone to the propeller hub. Figure 7 shows that the sound varies from a value near zero to a maximum, as predicted in the theory and calculated in figure 2. Although the exact phase of the blades is not known, the sound is known to be a maximum when the blades overlap in the plane of the microphone. Figure 7 also shows that the number of maximums and minimums per blade slip revolution equals the value of m. Thus for m = 6, six minimums and six maximums occur. Since the acoustical degrees are m times the mechanical degrees, as is shown in the theory (see fig. 2), the maximums occur at all multiples of $m\alpha = k\pi$ when k is even.

Records shown in figure 8 of the first four harmonics of sound from 2 two-blade propellers operating side by side in the same plane indicate that this arrangement is just a special case of propellers operating in multiple. The amplitude of each harmonic depends on the relative blade position as it did for the dual-rotating configurations and, also, on the difference in distances from the observer to each propeller.

For a dual-rotating configuration only those frequencies are modulated which are integral multiples of the fundamental frequencies of both the front and rear components. This phenomenon is illustrated in figure 9 which shows the sound emission of a 2-0-3 propeller. Only the frequency for m = 6, which is an integral multiple of the fundamental frequencies of both the two-blade and three-blade propellers, is modulated. The amplitude of the fundamental frequencies of both front and rear components are nearly constant because neither is an integral multiple of the other. This same point is further illustrated in figure 10 which shows similar records of two different harmonics of sound from a 2-0-4 propeller.

The records in figures 9 and 10 substantiate Gutin's theory which predicts that for a given tip speed, thrust, and power the pressure



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omplitude of a given fraguency is the same for each of two manuallines

amplitude of a given frequency is the same for each of two propellers even though the propellers have different numbers of blades.

Total Sound Pressures

In the case of a 2-0-2 propeller all rotational noise frequencies have been shown to be modulated as in figure 2. Figure 2 also indicates that the total-noise level should fluctuate between the intensity levels for 2-0-0 and 4-0-0 propellers operating at the same tip speeds and power. At an effective Mach number of 0.5, figure 2 indicates a fluctuation of total-sound-pressure levels equivalent to approximately 7 decibels at $\beta = 120^{\circ}$.

°Figure 11 shows the recorded total—sound—pressure variation for a 2-0-2 propeller at an effective Mach number of 0.62 at $\,\beta=105^{\rm O}_{\cdot}$. The maximum value is approximately twice the minimum value, therefore, this variation is equivalent to a 6-decibel difference in sound—pressure levels. Since the operating conditions were comparable, this magnitude of fluctuation appears consistent with that predicted in figure 2.

Figure 12 further illustrates the manner in which sound from a 2-0-2 propeller fluctuates. Total—sound—pressure levels are shown for the tip Mach number range of 0.37 to 0.89 and for spacings between the front and rear components of 6.75, 11.75, 18, and 24 inches. An attempt was made to record maximum and minimum sound readings at each speed in order to obtain a comparison of sound data for the 2-0-0 and 4-0-0 propellers at the same tip speeds and power. Magnitudes of the sound measurements for the 2-0-2 propeller are consistently larger than would be expected for a tractor installation. The data points, however, do fall within the limits estimated for single—rotating pusher propellers. Since the test setup consisted of both a pusher component and a tractor component, the data presented in figure 12 seem to confirm the theory.

These tests also indicate that the spacing between components of a dual-rotating propeller has no apparent effect on the sound, at least in the vicinity of $\beta=105^{\circ}$. However, the sound near $\beta=0^{\circ}$ is shown in the section entitled "Mutual Interference" to be affected noticeably by the spacing.

In an actual dual-rotating-propeller installation the overlap axis is fixed and an observer would hear a steady sound, the intensity level of which would fluctuate according to his position relative to the overlap axis. For convenience in these experiments the microphone position was fixed and the overlap axis was made to move relative to it. This arrangement was accomplished by causing one of the motors to run at a slightly different rotational speed (see "Apparatus and Methods").



· Strut Interference

Since the experimental setup used was a combination of a pusher propeller and a tractor propeller, an attempt has been made to evaluate the difference in the noise produced by each of these at the same tip speed and power. For the same strut clearances in figure 13 this difference appears to be about 3 decibels over a wide range of tip speeds. If the strut clearance is reduced as indicated in figure 13, a large increase in sound results. Figure 14 gives a clearer picture of what happens to the amplitudes of the various frequencies as strut clearance is reduced. All frequencies seem to become stronger, but the greatest increase in amplitude appears in the higher harmonics of the rotational noise.

In nearly all records of figure 7 showing modulation of the various frequencies for a 2-0-2 propeller, complete cancellation as would be expected from the theory does not occur. This discrepancy can partly be accounted for by the difference in intensity levels of the sounds from the front and rear components. Likewise, in figure 12 the total sound intensities for the 2-0-2 propeller are higher than would be expected for a complete tractor installation. The theory indicates that the random data points in figure 12 should fall between the two-blade curve as an upper limit and the four-blade curve as a lower limit. From the data presented in figure 12 it is evident that a complete tractor installation would have more nearly checked the theory.

Mutual Interference

An attempt has been made to evaluate the effect of mutual interference for a 2-0-2 propeller at various spacings of the front and rear components. At a value of $\beta=105^{\circ}$ the data presented in figure 12 do not indicate the presence of any additional noise which might be caused by mutual interference between blades. Measurements were also made on the axis of rotation where rotational noise is a minimum and where this disturbance was expected to appear for the harmonics of m=4, 8, 12, . . . which are harmonics of the blade-passage frequency. These data are shown in figures 15 and 16.

When the two components are running separately, the individual rotational-noise frequencies have small pressure amplitudes at $\beta=0^{\circ}$, as illustrated by figure 15(a) and 15(b). In figure 15(c), which shows a corresponding sound spectrum for a 2-0-2 propeller at $\beta=0^{\circ}$, $m=\frac{1}{4}$ is greatly reinforced by a strong steady sound equal in frequency to the blade-passage frequency.

Note that noise due to mutual interference appears to be directional with its maximum along the axis of rotation much the same as vortex noise. This noise is dependent on the spacing of the components of a dual-rotating propeller. At a spacing of 12 inches the noise was very weak if it existed at all; whereas at $6\frac{3}{4}$ inches the noise became predominant at $\beta = 0^{\circ}$.



At a spacing of $6\frac{3}{4}$ inches the pressure amplitude of the fundamental frequency of this blade—interference noise at $\beta=0^{\circ}$ varies directly as the power input to the propeller and as the cube of the tip speed (fig. 16(a) and 16(b)). The rotational noise of the same frequency, on the other hand, can be shown to vary approximately as the sixth power of the tip speed even at angles near $\beta=0^{\circ}$.

These tests were run at a very low value of J, and hence from figure 19 of reference 4 the lift variation along the blades can be seen to be very small. For a larger value of J the effect of blade interference might be much greater.

CONCLUSIONS

Sound studies for static conditions of dual-rotating propellers and multiple single-rotating propellers in the tip Mach number range 0.37 to 0.89 indicated the following conclusions:

- 1. The total sound pressure emitted by a four-blade dual-rotating propeller with equal number of blades in each component varies between the limits of the sound pressure from a two-blade single-rotating propeller as an upper limit and the sound pressure from a four-blade single-rotating propeller as the lower limit, at the same tip speeds and with the same power absorption.
- 2. The maximum sound pressures from a dual-rotating propeller occur at the axis of overlap. Tests for a four-blade dual-rotating propeller over a wide speed range indicated approximately a 6-decibel difference between the maximum and minimum sound-pressure measurements.
- 3. The fundamental frequency of the mutual interference noise for a dual-rotating propeller is equal to the blade-passage frequency. Its pressure amplitude is a maximum when the angle from propeller axis of rotation equals 0° and varies directly as the power at that angle and as the cube of the tip speed. Near the plane of rotation mutual interference noise appears to be negligible.
- 4. As the strut clearance for pusher propellers is decreased, the total sound emitted is increased. Amplitudes of all harmonics are increased but the greatest increase occurs in the higher harmonics.
- 5. The theory presented for predicting sound emission of multiple propellers is adequate for multiple single-rotating propellers at all values of the angle from propeller axis of rotation and for dual-rotating propellers for all values of this angle except those values near 0°.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., April 6, 1948



APPENDIX

INSTANTANEOUS SOUND PRESSURE

The instantaneous value of sound pressure of the mth-harmonic of sound from a single-rotating propeller is given as the vector summation of the sound from n-blades equally spaced

$$p_{l_{m_1}} = p_{l_m} \sum_{m=1}^{n} \sin \left(m \omega t + \frac{x}{n} 2 \pi m \right)$$

When m is an integral multiple of n and x is an integer, the following may be seen from inspection

$$p_{l_{m_1}} = np_{l_m}^{t} \sin m\omega t$$

and

$$p_{1_{m}} = \left(p_{1_{m_{i}}}\right)_{max} = np_{1_{m}}$$

Hence, the total sound in the mth-harmonic from an n-blade propeller is n times the sound from one blade.

When m is not an integral multiple of n, it may be shown that $p_{l_{m,i}} = 0$ as follows:

$$\sum_{x=1}^{n} \sin \left(mx + \frac{x}{n}2\pi m\right) = \sin mx + \sum_{x=1}^{n} \cos x2\pi \frac{m}{n}$$

$$+ \cos mx + \sum_{x=1}^{n} \sin x2\pi \frac{m}{n}$$

$$= -\sin mx + \sin mx + \sum_{x=0}^{n} \cos x2\pi \frac{m}{n}$$

$$+ \cos mx + \sum_{x=1}^{n} \sin x2\pi \frac{m}{n}$$



From reference 5, page 81, if m is not an integral multiple of n,

$$\sum_{x=1}^{\underline{n}} \sin x2\pi \underline{\overline{n}} = \frac{\sin n\pi \underline{\overline{n}} \sin (n+1)\pi \underline{\overline{n}}}{\sin \pi \underline{\overline{n}}} = 0$$

$$\frac{\sum_{x=0}^{n} \cos x2\pi \frac{m}{n} = \frac{\cos n\pi \frac{m}{n} \sin (n+1)\pi \frac{m}{n}}{\sin \pi \frac{m}{n}} = 1$$

Hence, if m is not an integral multiple of n,

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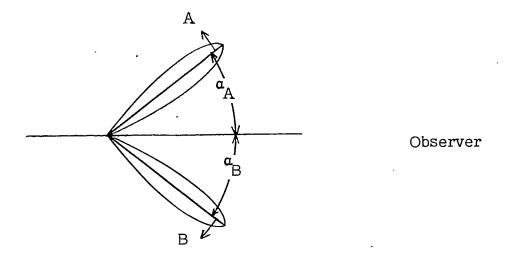
$$p_{\underline{l}_{\underline{m}_{\underline{i}}}} = -\sin mwt + \sin mwt = 0$$



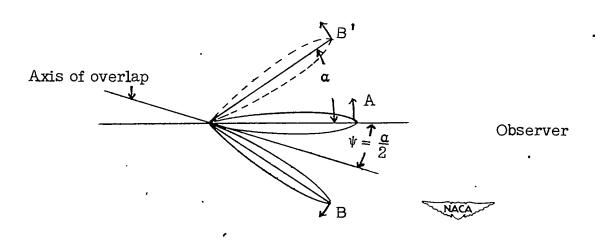
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(a) Two dual-rotating propellers operating in same plane of rotation.



(b) Two single-rotating propellers operating in same plane of rotation with phase angle α .

Figure 1.- Propeller configurations used in sound-theory presentation.

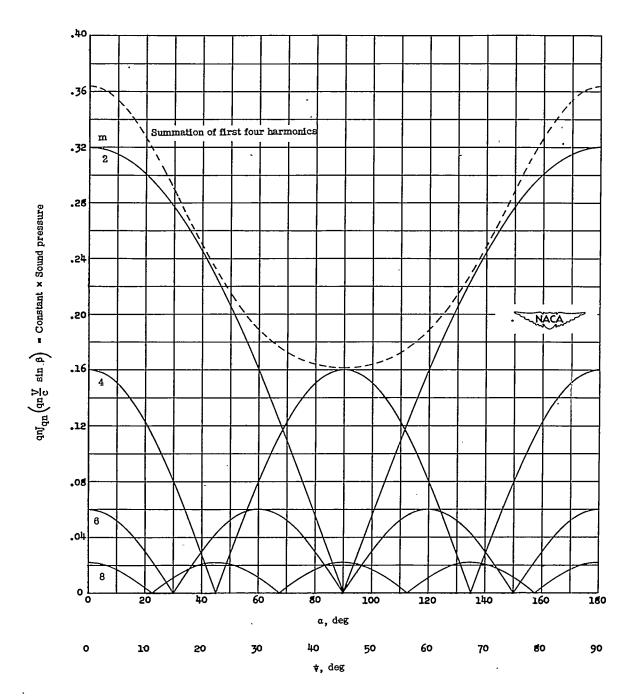


Figure 2.- Calculated sound pressures for four-blade dual-rotating propeller with arbitrary phase angle or dual rotation. $\beta = 120^{\circ}$; $\frac{V}{c} = 0.5$. (For values of $qnJ_{qn}(qn\frac{V}{c}\sin\beta)$ see fig. 2 of reference 2.)



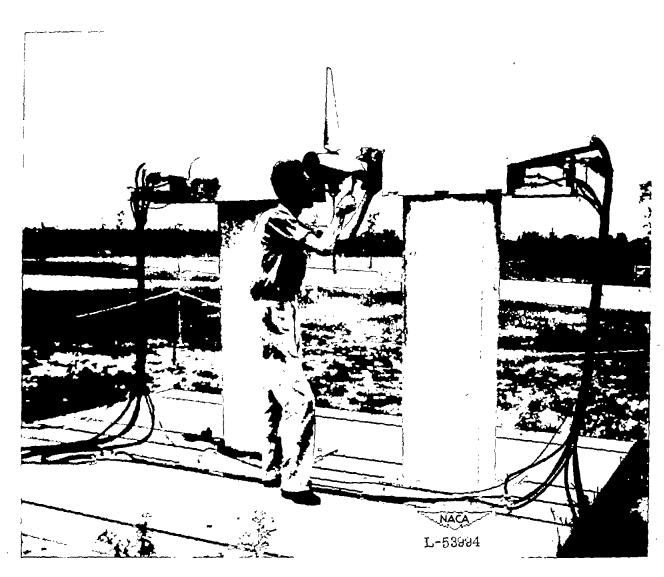
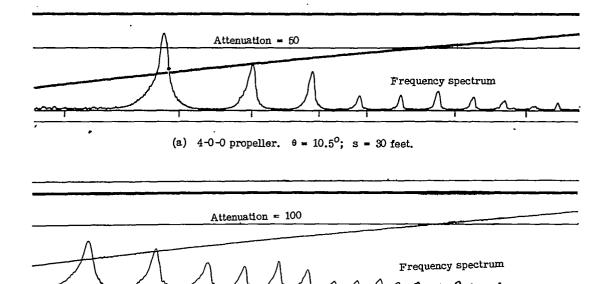
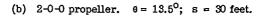


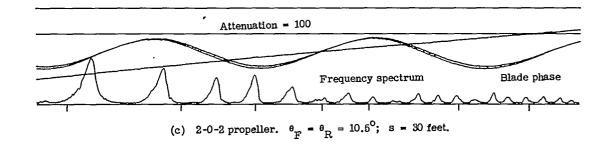
Figure 3.- Setup for studying sound emission of dual-rotating propellers.

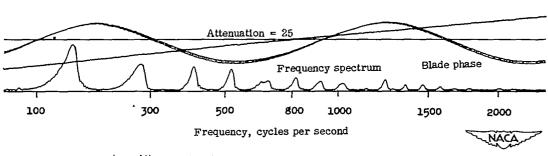








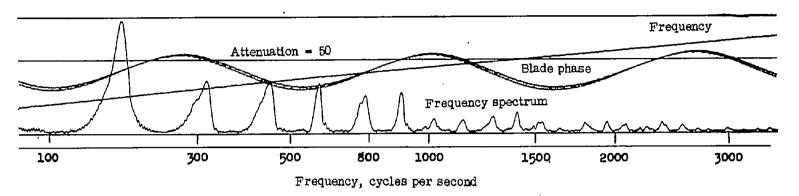




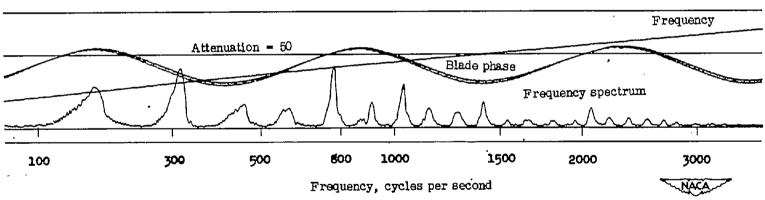
(d) Two 2-0-0 propellers operating side by side. $\theta = 10.5^{\circ}$; s = 60 feet.

Figure 4.- Frequency spectrums of four different propeller combinations absorbing approximately the same power. N=4400 rpm; $\beta=90^{\circ}$.





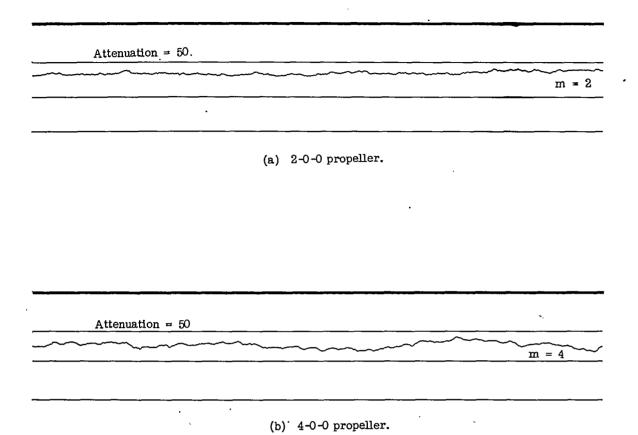
(a) Random record.



(b) The blade phase angle is changed from that of figure 5(a) by approximately 90°.

Figure 5.- Frequency spectrums showing the effect of blade phase angle for a 2-0-2 propeller. N = 4400 rpm; $\beta = 90^{\circ}$; $\theta_{\rm F} = \theta_{\rm R} = 10.5^{\circ}$; Z = 6.75 inches.





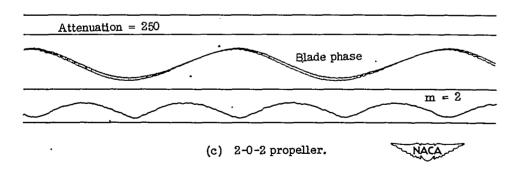
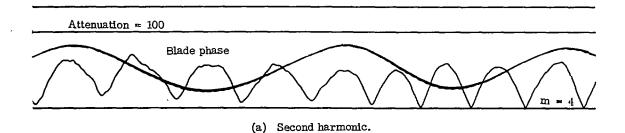
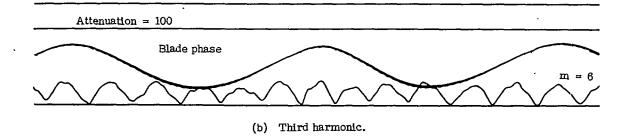
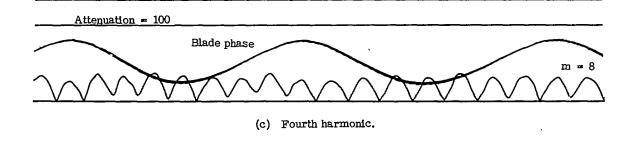


Figure 6.- Amplitude variation of the fundamental rotational noise frequency of three different propellers. N=4400 rpm; $\beta=90^{\circ}$.







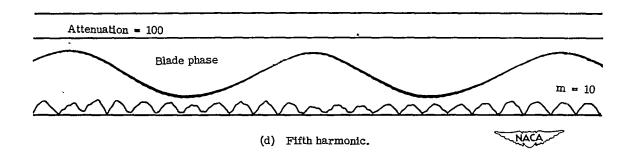
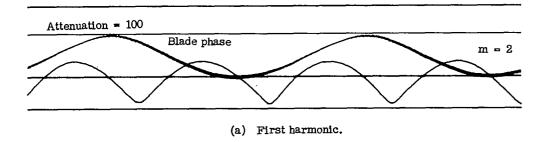
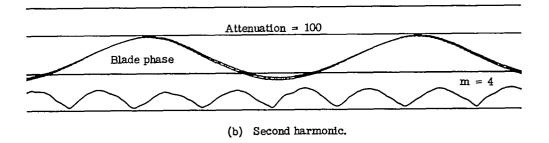
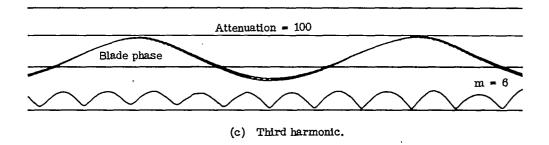


Figure 7.- Amplitude variation of four different harmonics of sound from a 2-0-2 propeller. N = 4400 rpm; β = 90°; Z = 12 inches; $\theta_{\rm F}$ = $\theta_{\rm R}$ = 10.5°.









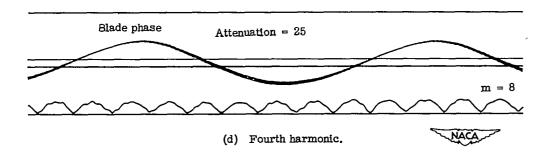
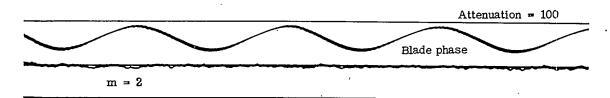
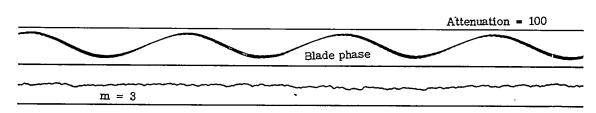


Figure 8.- Amplitude variation of four different harmonics of sound from 2 two-blade propellers operating side by side and absorbing equal power at nearly the same rotational speed. N = 4400 rpm; $\beta = 90^{\circ}$; $\theta = 10.5^{\circ}$.

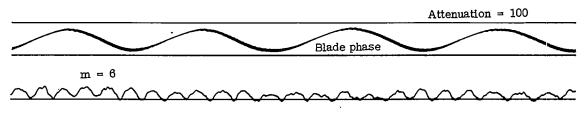




(a) First harmonic of front component.



(b) First harmonic of rear component.

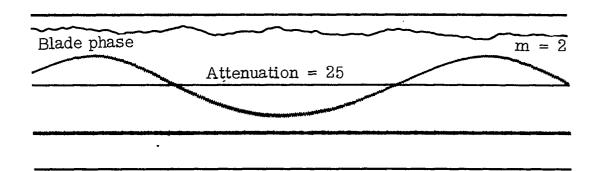


(c) Third harmonic of front component modulated by second harmonic of rear component.

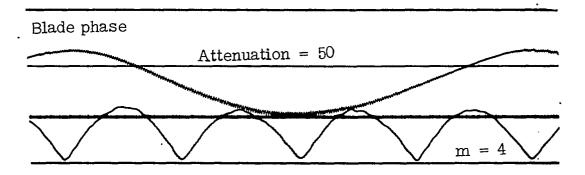


Figure 9.- Amplitude variation of three harmonics of sound from a 2-0-3 propeller. N = 4400 rpm; β = 90°; Z = 6.75 inches; θ _F = 13.5°; θ _R = 12°.





(a) First harmonic of front component.



(b) First harmonic of rear component modulated by second harmonic of front component.



Figure 10.- Amplitude variation of two harmonics of sound from a 2-0-4 propeller. N = 3000 rpm; β = 90°; Z = 12 inches.

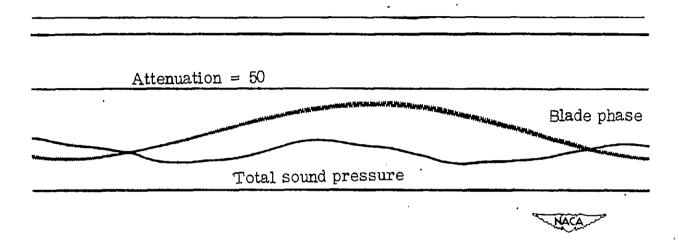


Figure 11.- Variation of amplitude of total noise emission from a 2-0-2 propeller. $N = 3800 \text{ rpm}; \quad \beta = 105^{\circ}; \quad Z = 12 \text{ inches}; \quad \frac{V}{c} = 0.62.$



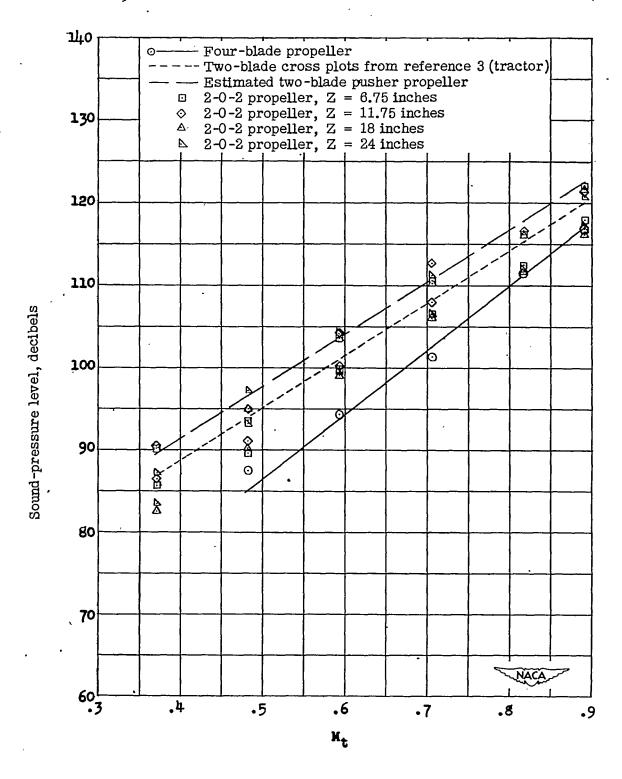


Figure 12.- Comparison of sound emission of 2-0-2 propeller at four different spacings with three single-rotating propellers absorbing the same power at approximately equal tip speeds. $\beta = 105^{\circ}$; s = 30 feet.

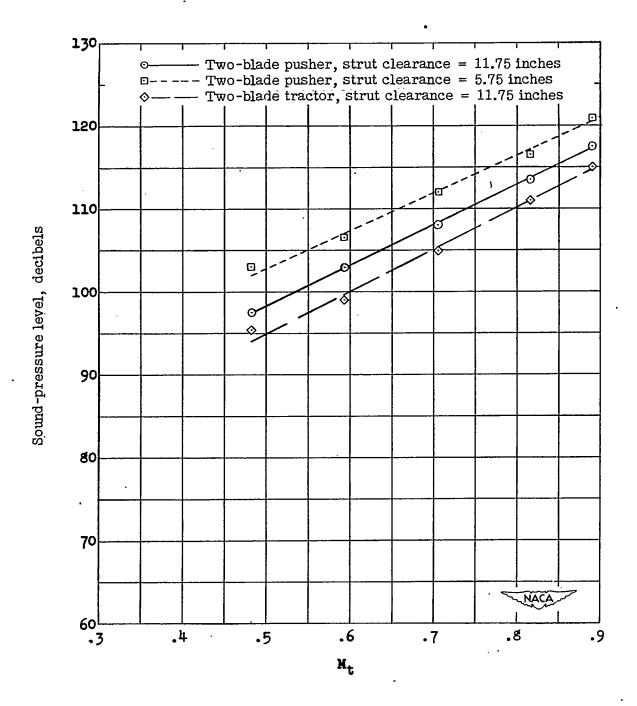
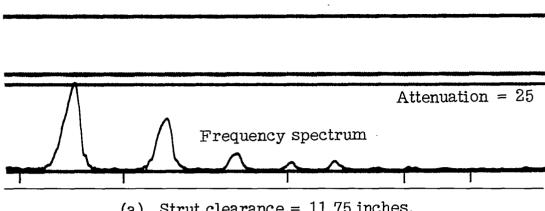
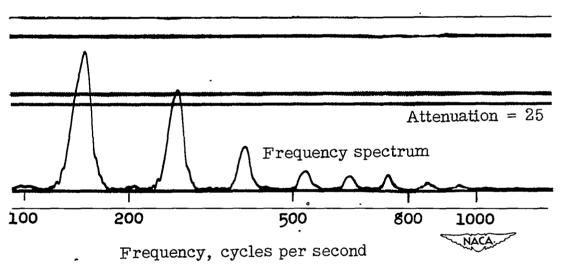


Figure 13.- Effect of strut interference on the sound emission of two-blade propellers. $\beta = 90^{\circ}$; s = 30 feet.



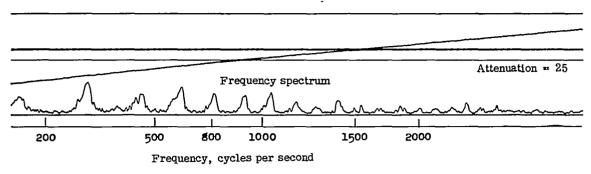


(a) Strut clearance = 11.75 inches.

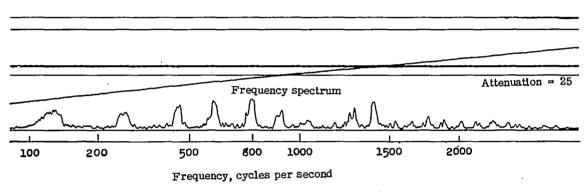


(b) Strut clearance = 5.75 inches.

Figure 14.- Effect of strut clearance on the amplitude of rotational noise for a 2-0-0 pusher propeller. Both records are at the same attenuation for comparison. N = 4400 rpm; $\beta = 90^{\circ}$.



(a) 2-0-0 propeller (tractor). $\theta = 10.5^{\circ}$.



(b) 2-0-0 propeller (pusher). $\theta = 10.5^{\circ}$.

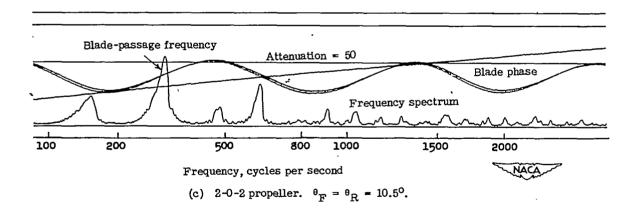
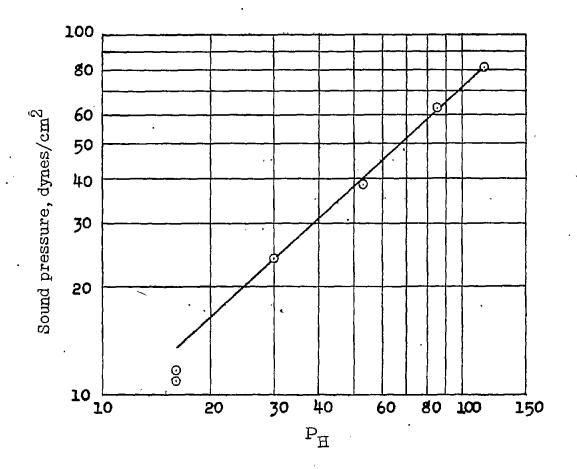
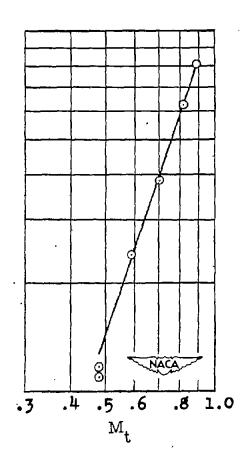


Figure 15.- Frequency spectrums of a 2-0-2 propeller and each of its components taken separately showing the effect of blade interference. N=4400 rpm; $\beta=0^{\circ}$; Z=6.75 inches.





(a) Effect of power.

(b) Effect of tip Mach number.

Figure 16.- Mutual-interference-noise variation at $\beta=0^{\circ}$ as a function of tip speed and power. Z=6.75 inches. (Same frequency as m=4.)