REPORT No. 830

A METHOD FOR DETERMINING THE RATE OF HEAT TRANSFER FROM A WING OR STREAMLINE BODY

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SUMMARY

A method for calculating the rate of heat transfer from the surface of an airfoil or streamline body is presented. A comparison with the results of an experimental investigation indicates that the accuracy of the method is good.

This method may be used to calculate the heat supply necessary for heat de-icing or in ascertaining the heat loss from the fuselage of an aircraft operating at great altitude.

To illustrate the method, the total rate of heat transfer from an airfoil is calculated and compared with the experimental result.

INTRODUCTION

The calculation of the rate of heat transfer from the surface of bodies of aerodynamic shape is a problem for which no explicit solution is known to exist, although considerable attention has been given to the heat transfer from a heated flat plate into both laminar and turbulent flow regimes. It has been shown in reference 1 that the transfer of heat from the surface of a hot plate into an air stream flowing over the the plate is primarily a boundary-layer problem. The extension of heat-transfer theory to permit the calculation of heat flow for an airfoil or streamline body is therefore a matter of calculating the boundary-layer characteristics which may be done by the methods of reference 2. The present report makes use of the results of references 1 and 2 to extend heat-transfer theory to wings and bodies for which pressure distributions may be calculated with accuracy. The derivation of the method is given in the appendix.

A limited experimental investigation of the method was made to determine the accuracy of the method in calculating the local rate of heat transfer into both the laminar and the turbulent boundary layer of an airfoil, and also to obtain a check on the computed total rate of heat transfer for the wing.

It is hoped that this method will facilitate a more accurate determination of the heat losses from wings in designing heat de-icing systems as well as from fuselages in the design of cabin-heating systems for aircraft operating at great altitudes.

SYMBOLS

The symbols used throughout this report and in the appendix are defined as follows:

c	wing chord
c_p	specific heat at constant pressure
h	heat-transfer coefficient, Btu/sq ft, ° F, sec
k	heat conductivity
L	length of streamline body
M_L	local Mach number, ratio of velocity just outside the boundary layer to local velocity of sound
M_0	free-stream Mach number, ratio of velocity of free stream to the velocity of sound in the free stream
q_x	local rate of heat transfer, Btu/sq ft, sec
R_c	Reynolds number based on wing chord (Vc/ν)
R_L	Reynolds number based on body length (VL/ν)
r	radius to surface of streamline body at any point along the axis
r_1	radius to surface of streamline body at point for which boundary layer is being computed
s	distance along the surface from the stagnation point
T	local temperature inside boundary layer, °F, absolute
T_L	local temperature outside boundary layer, ° F, absolute
T_0	free-stream air temperature, ° F, absolute
t	local temperature inside boundary layer, ° F
t_L	local temperature outside the boundary layer, ° F
t_0	free-stream air temperature, ° F
t_p	surface temperature corrected for compressibility, °F
t_s	surface temperature, ° F
t_p-t_0	heat-transfer temperature difference, ° F
U	local velocity just outside the boundary layer
U_1	local velocity just outside the boundary layer at point for which boundary layer is being computed
u	local velocity inside the boundary layer
V	free-stream velocity
x	distance along the chord from the leading edge for an airfoil, or along the axis for a streamline body
y	distance normal to surface
α	angle of attack, degrees
β	eddy heat conductivity
γ	ratio of specific heats
δ_L	heat-transfer characteristic length for a laminar boundary layer
δ_T	heat-transfer characteristic length for a turbulent boundary layer
ϵ	eddy viscosity
ζ	turbulent boundary-layer parameter ($\zeta=\sqrt{\rho U^2/\tau_0}$)
θ	momentum thickness of boundary layer $\left[\theta=\int_0^h \frac{u}{U}\left(1-\frac{u}{U}\right)dy\right]$

μ	absolute viscosity
ν	kinematic viscosity
ρ	air density
σ	Prandtl number ($c_p\mu/k$)
τ_0	surface shear

APPARATUS

Tests to determine the rate of heat transfer from a wing were made in the Ames 7- by 10-foot wind tunnel employing a heated, two-dimensional airfoil model.

DESCRIPTION OF MODEL

The model was a 7-foot chord NACA 65,2–016 airfoil which completely spanned the 7-foot dimension of the wind tunnel (fig. 1). Ordinates are given in table I.

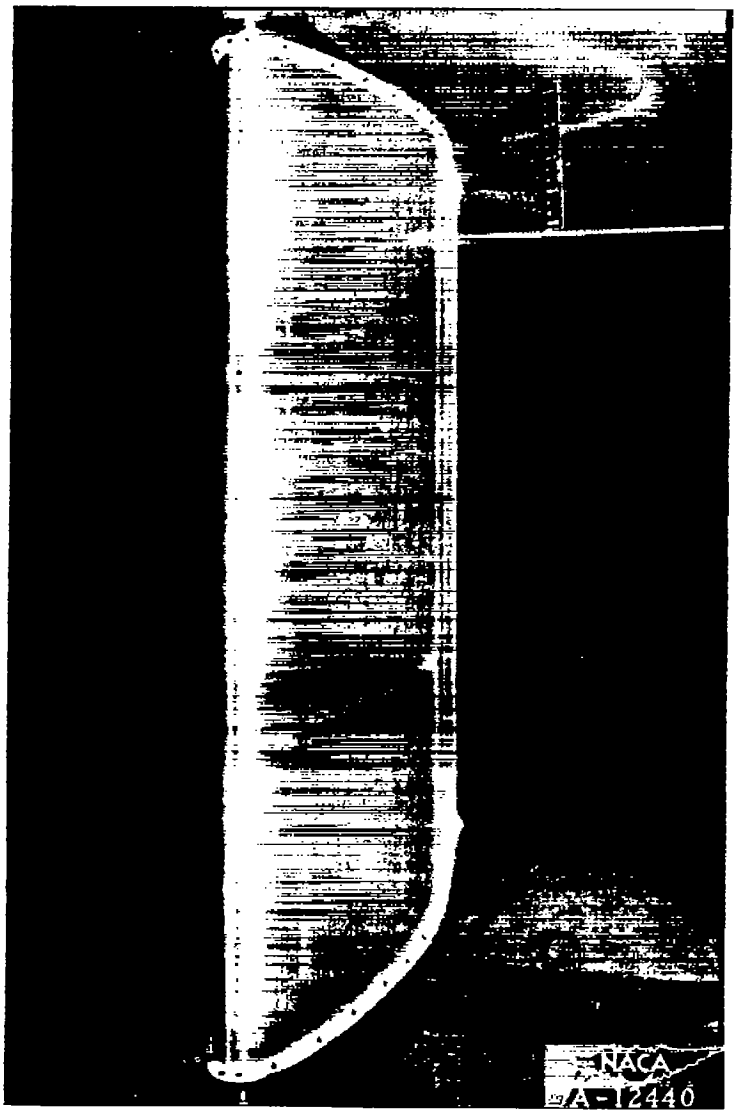

FIGURE 1.—Heated airfoil model installed in the wind tunnel.

The forward portion containing heating lamps was of sheet-aluminum construction, and the trailing section of laminated wood. Construction details are shown in figure 2. Built into the model were 26 pressure orifices and 41 iron-constantan thermocouples. The thermocouples were in three chordwise bands; one at midspan, and two, 4 inches on either side extending to 41 percent chord. The external surface of the heated portion was bare metal and that of the unheated wood part was painted. The entire model was made aerodynamically smooth by chordwise rubbings with successively finer grit sandpaper. All power, temperature, and pressure leads were brought out through the top of the model which was sealed closed to prevent the escape of convected heat.

TABLE I

ORDINATES OF 7-FOOT-CHORD NACA 65, 2–016 AIRFOIL

x Percent Chord	x Inches	y Percent Chord	y Inches
0	0	0	0
.5	.420	1.202	1.010
.75	.630	1.423	1.195
1.25	1.050	1.796	1.509
2.5	2.100	2.057	2.108
5.0	4.200	3.543	2.976
7.5	6.300	4.316	3.625
10	8.400	4.954	4.161
15	12.600	5.958	5.005
20	16.800	6.701	5.629
25	21.000	7.252	6.092
30	25.200	7.645	6.422
35	29.400	7.892	6.629
40	33.600	7.995	6.716
45	37.800	7.938	6.668
50	42.000	7.672	6.444
55	46.200	7.184	6.035
60	50.400	6.495	5.456
65	54.600	5.647	4.743
70	58.800	4.713	3.959
75	63.000	3.738	3.140
80	67.200	2.759	2.318
85	71.400	1.817	1.526
90	75.600	.982	.826
95	79.800	.340	.286
100	84.000	0	0

L. E. Radius: 1.704 Percent c=1.431 Inches

Local unfairness of the model resulted in some minor variations in the pressure distribution, as shown in figure 3, but these were not of sufficient magnitude to induce transition to turbulence.

METHOD OF HEATING

The heated portion of the wing was divided into four compartments by spanwise bulkheads on which the heating elements, nine rows of ordinary 120-volt incandescent lamp bulbs, were mounted (fig. 2). The inside surface of the aluminum skin was painted a dull black to increase the absorption of radiant heat, but all other metal surfaces were bright.

The heat input appropriate to each compartment to give a temperature difference across the boundary layer of 100° F. at a Reynolds number of 13,000,000 was calculated by the method of this report. The size, number, and location of the bulbs within each compartment were such as to give the most nearly uniform skin temperature possible within the practical limitations of the design.

Power was supplied by a direct-current generator equipped with a remote voltage control which permitted a convenient means of adjusting the over-all applied voltage.

Free-stream air temperatures were calculated from average readings of three resistance-type thermometers located in the return passage a short distance ahead of the entrance cone. Adiabatic expansion through the entrance cone was assumed.

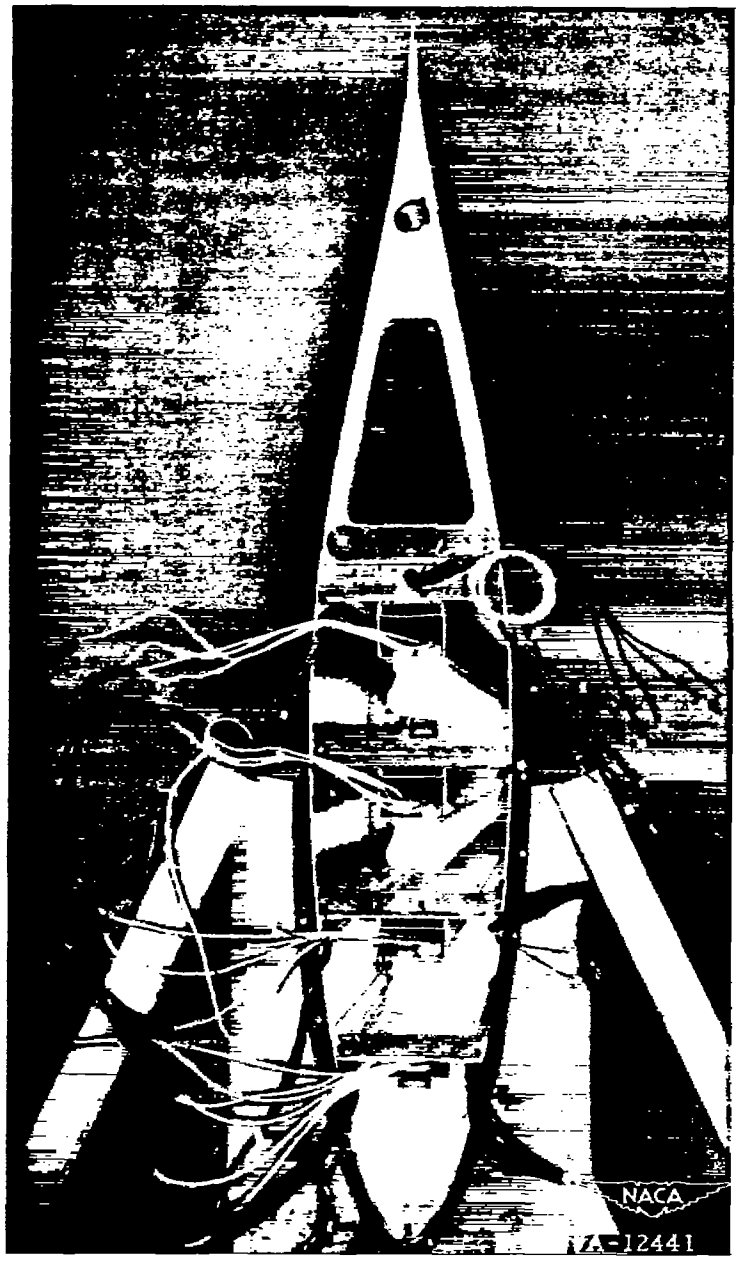

FIGURE 2.—Heated airfoil model showing heating lamps.

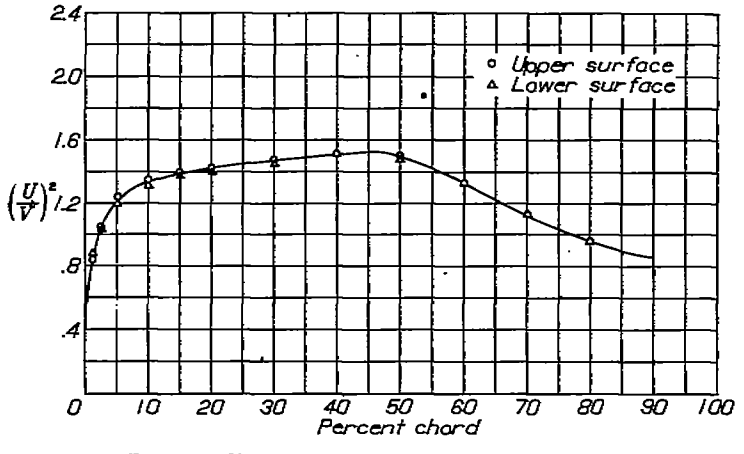

FIGURE 3.—Chordwise pressure distribution. α, 0°; R_c, 11×10⁶.

COMPUTATION OF HEAT TRANSFER

METHOD

The detailed analysis given in the appendix develops the following formulas, by which the local rate of heat transfer into both turbulent and laminar boundary layers may be computed. This method is applicable either to an airfoil or to a streamline body. The local rate for laminar flow is

$$q_x=0.700\,\frac{k}{\delta_L}\,(t_p-t_o) \tag{1a}$$

or the local heat-transfer coefficient is

$$h_x=0.700\,\frac{k}{\delta_L} \tag{1b}$$

and for the turbulent flow,

$$q_x=0.760\,\frac{k}{\delta_T}\,(t_p-t_0) \tag{2a}$$

or the local heat-transfer coefficient is

$$h_x=0.760\,\frac{k}{\delta_T} \tag{2b}$$

Heat transfer from an airfoil.—For the laminar boundary layer of an airfoil, δ_L is computed as in reference 2 from the pressure distribution as follows:

$$\delta_L=c\sqrt{\frac{5.3}{R_c}\left(\frac{V}{U_1}\right)^{9.17}\int_0^{s/c}\left(\frac{U}{V}\right)^{8.17}d\,\frac{s}{c}} \tag{3}$$

For the turbulent boundary layer of an airfoil, δ_T is computed as

$$\delta_T=\frac{\zeta^2 c}{R_c\left(\frac{U_1}{V}\right)} \tag{4}$$

where ζ is the value of the turbulent boundary-layer parameter as determined by a step-by-step solution of the relationship of reference 3, given as

$$\frac{d\zeta}{dx}+\frac{6.13}{U}\,\frac{dU}{dx}=\frac{U}{\nu}\,f(\zeta) \tag{5}$$

The value of $f(\zeta)$ is given in table II as taken from reference 3, and may be plotted on semilogarithmic paper for ease in using.

TABLE II

NUMERICAL VALUES OF $f(\zeta)$

$f(\zeta)=10.411\,\zeta^{-6}\,e^{-0.214\zeta}$

ζ	$10^4 f(\zeta)$	ζ	$10^4 f(\zeta)$
14	221.6	24	1.502
15	130.39	25	.937
16	77.47	26	.585
17	46.11	27	.367
18	27.97	28	.2308
19	16.96	29	.1454
20	10.35	30	.0917
21	6.35	31	.0552
22	3.906	32	.0369
23	2.418	33	.0235

Heat transfer from a streamline body.—For the laminar boundary layer of a streamline body, δ_L is computed from reference 2 as

$$\delta_L=L\sqrt{\frac{5.3}{R_L}\left(\frac{L}{r_1}\right)^2\left(\frac{V}{U_1}\right)^{9.17}\int_0^{s/L}\left(\frac{r}{L}\right)^2\left(\frac{U}{V}\right)^{8.17}d\frac{s}{L}} \tag{6}$$

The turbulent boundary-layer heat-transfer length may be computed as

$$\delta_T=\frac{\zeta^2 L}{R_L\left(\frac{U_1}{V}\right)} \tag{7}$$

where ζ is determined as for an airfoil from the equation

$$\frac{d\zeta}{dx}+\frac{6.13}{U}\frac{dU}{dx}+\frac{2.557}{r}\frac{dr}{dx}=\frac{U}{\nu}f(\zeta) \tag{8}$$

by the step-by-step process mentioned for calculating δ_T for an airfoil.

Compressibility correction.—If the heat flow is to be obtained at free-stream Mach numbers such that the aerodynamic temperature rise is an appreciable portion of the total temperature difference, a correction for aerodynamic heating should be made. The "heat-transfer temperature difference" to be used for a laminar boundary layer is

$$(t_p-t_0)=(t_e-t_0)-0.20M_0^2T_0\left[1-0.13\left(\frac{U}{V}\right)^3\right] \tag{9}$$

and for turbulent flow

$$(t_p-t_0)=(t_e-t_0)-0.20M_0^2T_0 \tag{10}$$

where t_e-t_0 is the desired temperature rise.

The total rate of heat transfer from an airfoil or streamline body may be found as follows:

1. Estimate the location of the transition point by the method of reference 4 for an increasing pressure gradient, or by reference 2 for a falling pressure gradient.

2. Calculations for the laminar region ahead of the transition point.

(a) Compute the values of δ_L along the surface to the transition point by equation (3) for an airfoil, or by equation (6) for a streamline body.

(b) With these values and the desired temperature distribution corrected for compressibility, compute the local rates of heat transfer along the surface by equation (1a).

3. Calculations for the turbulent region behind the transition point.

(a) From the value of δ_L at the transition point compute θ, the momentum thickness as

$$\theta=0.289\ \delta_L$$

Using this value of θ, find the initial value of ζ at the transition point as

$$\zeta=2.557\log_e 4.057\left(\frac{U\theta}{\nu}\right)$$

(b) With this initial value of ζ, calculate the values of ζ along the surface by equation (5) for an airfoil, or by equation (8) for a streamline body. With these values of ζ, compute δ_T along the surface by equation (4) for an airfoil, or by equation (7) for a streamline body.

(c) Using these values of δ_T and the desired temperature difference across the boundary layer corrected for compressibility, compute the local rate of heat transfer along the surface by equation (2a).

4. Integrate these local rates of heat transfer along the chord for both laminar and turbulent regions to obtain the *total* rate of heat transfer.

HEAT-TRANSFER MEASUREMENTS

As mentioned in the Introduction, a limited number of heat-transfer tests were made on the heated wing model to check the accuracy of the theoretical method.

The experimental results are subject to several sources of error. All computed rates of heat transfer were based on the temperature distributions obtained at the center of the span, assuming that the spanwise variation was negligible. This was essentially true except for a small portion at each end of the wing. Precautions were taken to minimize the heat losses at the ends of the wing. These are not believed large since the design of the heating system allowed only slight transfer by convection, and the conduction of heat from the wing to its supports is negligible. Losses due to radiation from the wing have been computed as a maximum of 5 percent for the whole surface heated to 100° F. above the surroundings. This loss is not considered in the heat-transfer data.

For the purpose of computing heat transfer, the chordwise temperature distribution was computed and plotted as heat-transfer temperature difference (t_p-t_0); this is the observed temperature difference corrected for compressibility effect; that is, t_p is the temperature measured by a thermocouple in the skin minus the computed aerodynamic heating temperature rise. The value t_0 is the free-stream air temperature (distinguished from the local temperature just outside the boundary layer of the wing, which will be higher or lower than t_0 due to adiabatic variations caused by the velocity field of the wing).

RESULTS AND DISCUSSION

Heat-transfer tests of the wing were made in two parts, the first concerned with comparing computed and measured values of the local rate of transfer from the airfoil surface to laminar and turbulent boundary layers, and the second with checking the total rate of heat transfer from the airfoil.

The tests to measure the local rate of heat transfer were made at zero lift for two test Reynolds numbers, with free transition to obtain the heat-flow rate into a laminar boundary layer, and with transition fixed at 5-percent chord to determine the flow rate into a turbulent boundary layer. Figure 3 presents the pressure distribution over the wing at zero lift. The experimental procedure consisted in adjusting the heat input so that the skin temperatures were nearly constant along the chord. With this temperature distribution achieved, it was assumed that the second compartment of the wing, extending from 14.6-percent to 26.3-percent chord, was thermally isolated so that no flow of heat occurred in the skin or through the bulkheads. The

power input to this compartment was then measured by means of a voltmeter and an ammeter for comparison with the calculated rate of heat flow.

Figure 4 shows that the desired constant chordwise temperature distribution was attained for the laminar boundary layer, but the data of figure 5 for turbulent flow indicate that while the distribution was nearly constant from 10- to 30-percent chord, covering the region under consideration, the temperatures over the nose were excessively high. This came about through the heating difficulties resulting from the sudden change in heat-transfer coefficient at the point where transition was fixed. This type of distribution may have resulted in some change in the local values of the temperature gradient at the wing surface, though this effect should be small since no appreciable temperature gradient existed over the portion of the surface concerned, the boundries of which are indicated by the dotted lines in the figures.

To obtain the computed values, the variation of the heat-transfer coefficient along the chord was calculated for each case as outlined under the section Method (results plotted in figs. 6 and 7). The heat input into the second compartment

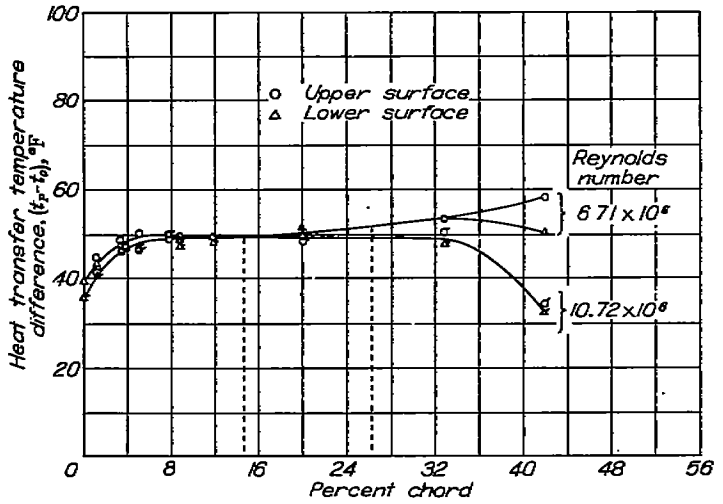

FIGURE 4.—Chordwise distribution of temperature difference, (t_p-t_a). α, 0°; transition free.

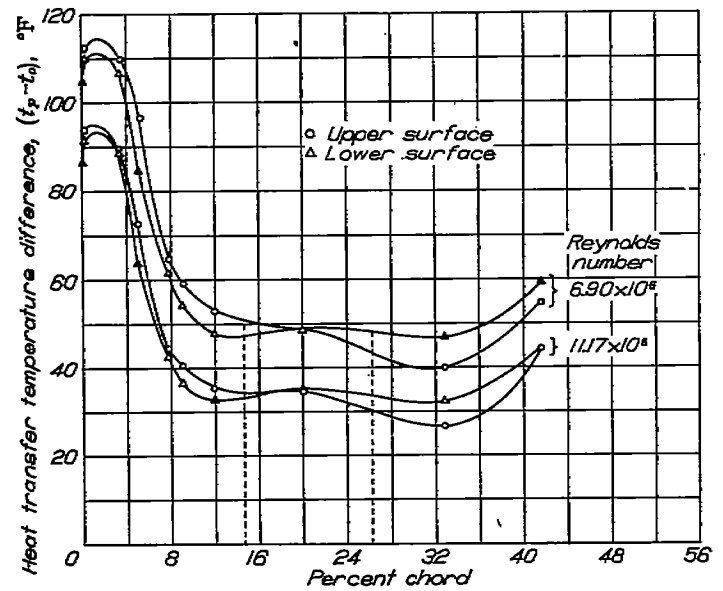

FIGURE 5.—Chordwise distribution of temperature difference, (t_p-t_a). α, 0°; transition fixed at 5 percent chord.

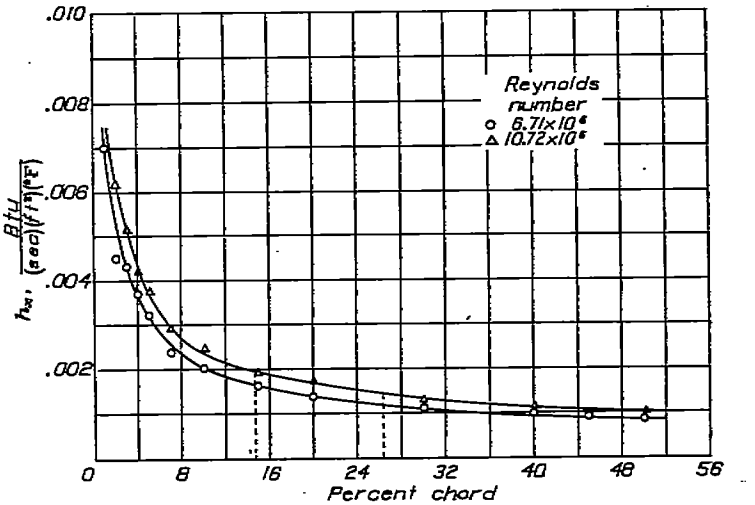

FIGURE 6.—Chordwise distribution of heat transfer coefficient, h_w. α, 0°; transition free.

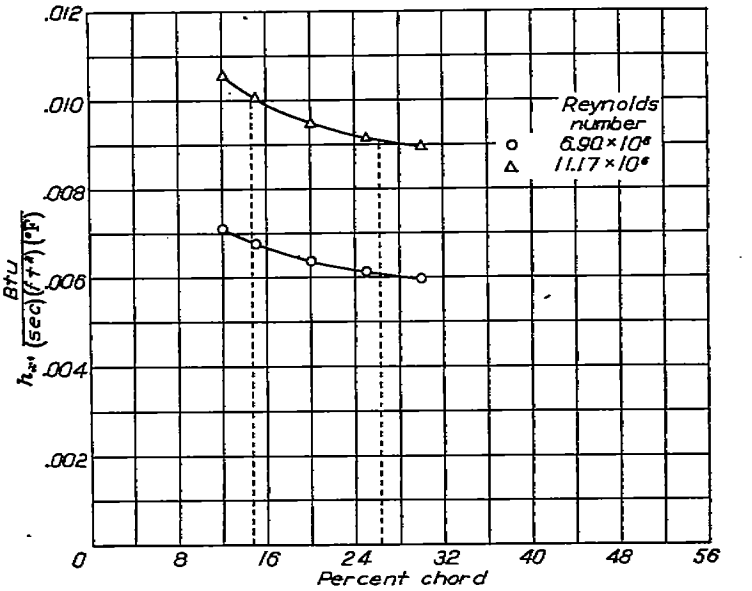

FIGURE 7.—Chordwise distribution of heat transfer coefficient, h_w. α, 0°; transition fixed at 5 percent chord.

was then computed from the heat-transfer coefficient and experimentally measured temperature difference (corrected for compressibility heating effects). Both the measured and the computed values of heat input into the second compartment are listed in the following table:

α (deg)	$R_c \times 10^{-6}$	Boundary layer	Calculated heat transfer (kw)	Measured input (kw)	Error (percent input)
0	6.71	Laminar	0.861	0.875	1.6
0	10.72	do	1.048	1.028	1.9
0	6.90	Turbulent	3.76	3.75	.3
0	11.17	do	4.01	4.01	.0

The agreement is considered quite satisfactory, and is taken to indicate that the method for the computations of the heat transfer coefficient involves no serious errors despite the assumptions involved.

Further tests for the purpose of establishing the validity of the method as regards the *total rate* of heat flow from a wing were made at a lift coefficient of 0.55 and 8.60 million Reynolds number. In addition to a test with free transition,

a second condition simulating the formation of ice near the stagnation point by fixing transition at 5-percent chord on the lower surface was investigated. Heat input in both conditions was maintained at the maximum available from the apparatus, and no attempt was made to achieve a predetermined temperature rise or chordwise distribution. The chordwise temperature distribution obtained, corrected for the effects of compressibility heating, is plotted in figures 8 and 9.

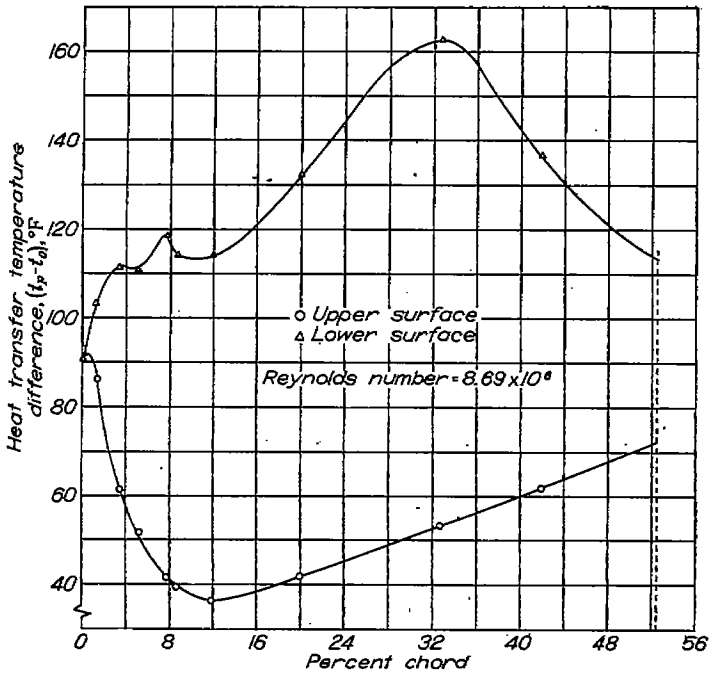

FIGURE 8.—Chordwise distribution of temperature difference, (t_p-t_0). α, 0°; transition free.

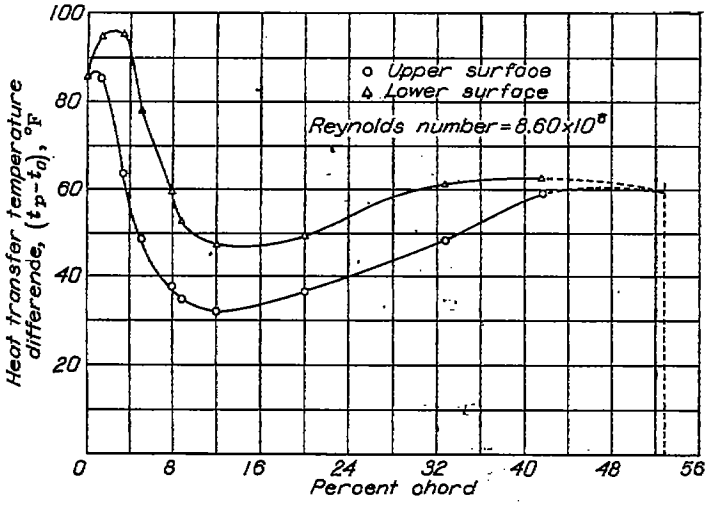

FIGURE 9.—Chordwise distribution of temperature difference, (t_p-t_0). α, 5°; transition fixed at 5 percent chord.

The chordwise variation of heat-transfer coefficient was computed for both the transition-fixed and transition-free conditions, as outlined in Method. Tables III and IV present the computations for the transition-free condition and serve as an illustrative example. The pressure distribution used for these calculations is given in figure 10. The value of heat input was then computed by use of the experimental temperature distributions (corrected for compressibility) and the calculated heat-transfer coefficients of figures 11 and 12. The computed and measured heat input are compared in the table below. Results indicate satisfactory agreement.

TABLE III

COMPUTATION OF HEAT-TRANSFER COEFFICIENT

Airfoil 65,2-016 c_l=0.55 R_c=8.69×10⁶ M_0=0.183
Upper surface Transition at 1% chord $\theta_{1\%c}$=154.5×10⁻⁶ $f_{1\%c}$=18.25

Position $\frac{x}{c}$	Position (ft)	Δx	$\left(\frac{U}{V}\right)$	$\frac{dU/V}{dx}$	$f(\xi)\times10^6$	$\frac{d\xi}{dx}$	$\Delta\xi$	ξ	$10^6\delta_T$	h_x
0.01	0.07		1.61					18.25	166.5	0.0191
		0.07		−0.667	24.6	51.8	3.63			
.02	.14		1.57					21.88	245.4	.0130
		.07		−.572	4.17	10.36	.73			
.03	.21		1.54					22.61	267.1	.0120
		.07		−.443	2.92	7.34	.51			
.04	.28		1.52					23.12	283.2	.0113
		.07		−.300	2.29	5.53	.39			
.05	.35		1.50					23.21	296.9	.0108
		.14		−.270	1.89	4.62	.65			
.07	.49		1.48					24.16	317.4	.0100
		.14		−.185	1.39	3.32	.46			
.09	.63		1.44					24.62	335.3	.00952
		.14		−.133	1.12	2.67	.36			
.11	.77		1.422					24.98	353.4	.00903
		.35		−.069	.949	1.96	.69			
.16	1.12		1.399					25.67	379.5	.00841
		.35		−.046	.685	1.39	.49			
.21	1.47		1.382					26.16	398.0	.00800
		.35		−.0372	.545	1.10	.39			
.26	1.82		1.366					26.55	410.0	.00778
		.35		−.0256	.453	.906	.32			
.31	2.17		1.356					26.87	430.9	.00741
		.35		−.0171	.393	.739	.26			
.36	2.52		1.350					27.13	439.8	.00726
		.35		−.0143	.345	.643	.23			
.41	2.87		1.345					27.36	448.2	.00712
		.35		−.0343	.312	.677	.24			
.46	3.22		1.330					27.60	461.0	.00652
		.35		−.0915	.278	.918	.32			
.51	3.57							27.92	472.1	.00676

TABLE IV

COMPUTATION OF HEAT-TRANSFER COEFFICIENT

Airfoil 65,2-016 c_l=0.55 R_c=8.69×10⁶ M_0=0.183
Lower surface Transition at minimum pressure

$\frac{x}{c}$	$\left(\frac{U}{V}\right)^3$	$\left(\frac{U}{V}\right)^{6.17}$	$\int_0^{x/c}\left(\frac{U}{V}\right)^{6.17}d\frac{x}{c}$	$\left(\frac{U_l}{V}\right)^{6.17}$	$10^6\frac{\delta_L}{c}$	$10^6\delta_L$ (ft)	$10^6 h_x$
0.010	0.0						
.020	.185	0.00100	4×10^{-5}	0.00044	0.745	5.22	56.3
.030	.290	.0063	32×10^{-5}	.0034	.756	5.60	52.0
.040	.370	.0173	144×10^{-5}	.0105	.862	6.04	48.7
.050	.455	.0400	400×10^{-5}	.0270	.951	6.66	44.2
.075	.60	.124	2400×10^{-5}	.096	1.225	8.75	33.7
.100	.71	.247	0.0007	.208	1.401	9.80	30.0
.150	.85	.515	.0254	.475	1.806	12.65	23.2
.200	.92	.712	.0565	.653	2.248	15.73	18.7
.250	1.00	1.00	.1006	1.00	2.473	17.35	16.9
.300	1.07	1.32	.1590	1.36	2.670	18.70	15.7
.350	1.12	1.59	.2310	1.68	2.595	20.27	14.3
.400	1.16	1.83	.3153	1.97	3.13	21.90	13.4
.450	1.17	1.90	.4100	2.10	3.45	24.15	12.2
.500	1.18	1.97	.5080	2.14	3.81	26.62	11.0

α (deg)	$R_c\times10^{-6}$	Boundary layer	Calculated heat transfer	Measured input (kw)	Error (percent input)
5	8.69	Upper surface turbulent. Lower surface laminar.	19.60	23.52	16.7
5	8.60	Upper surface turbulent. Lower surface turbulent.	22.30	23.55	5.3

The experimental temperature distributions for the tests of the total rate of heat transfer show that the heat-transfer temperature difference varies to a marked degree along the chord (figs. 8 and 9). This variation violates one of the assumptions underlying the development of the method; that is, that the temperature difference is constant along the

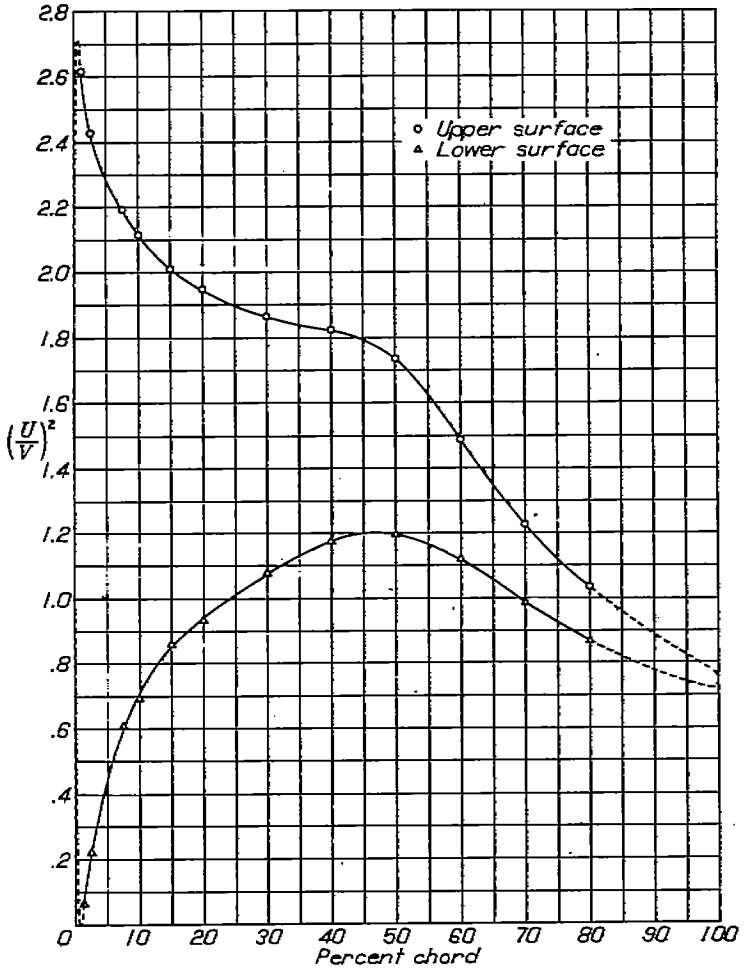

FIGURE 10.—Chordwise pressure distribution. α, 5°; R_c, 8.60×10⁶.

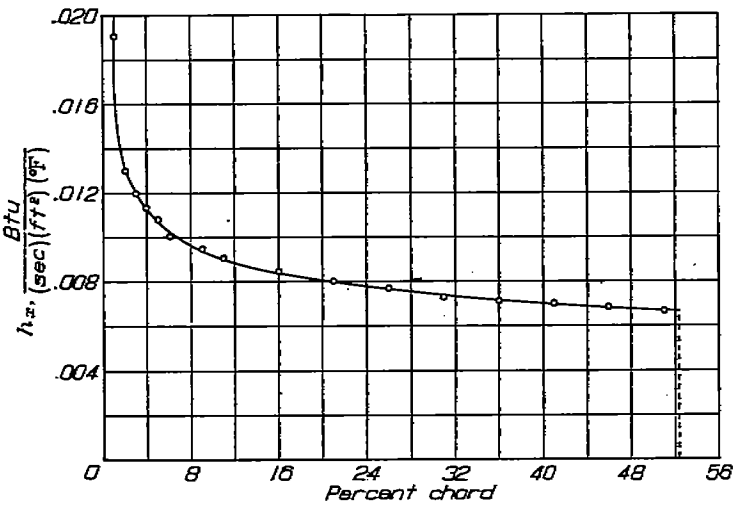

FIGURE 11.—Chordwise distribution of heat transfer coefficient, h_x, for upper surface. α, 5°; transition free and transition fixed at 5 percent chord; R_c, 8.69 and 8.60×10⁶.

chord, which must be true if the transfer of heat at all points along the surface is analogous to the transfer of momentum. To what degree this assumption may be ignored has not been determined analytically, since the problem of considering the variation of temperature along the chord

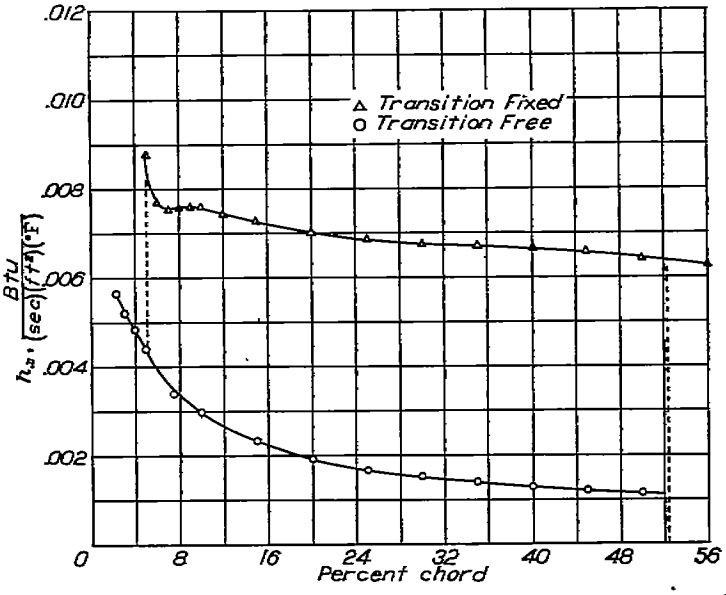

FIGURE 12.—Chordwise distribution of transfer coefficient, h_x, for lower surface. α, 5°; transition free and transition fixed at 5 percent chord; R_c, 8.69 and 8.60×10⁶.

presents difficulties which have so far prevented a solution. The experimental results, however, indicate that the accuracy with which the total rate of heat transfer can be computed is not greatly impaired by the temperature variations experienced. Generalization of this result must await further experimental checks.

The accuracy with which the local rate of heat transfer may be computed in a rising pressure gradient is dependent upon the accuracy with which the surface shear may be determined. Squire and Young's method (reference 3) assumes that the turbulent boundary layer in a rising pressure gradient exhibits the same characteristics as the fully developed turbulent layer of a flat plate. The extent to which the relationship between the surface shear, the momentum thickness, and the local velocity so derived remains valid is shown by the accuracy of the Squire and Young method in determining friction drag. It must be realized, however, that the method will fail if turbulent separation is imminent.

The thickening of the turbulent boundary layer due to the rise in pressure acting on the displaced mass of fluid also is ignored by the assumption that the heat-transfer rate is proportional to the surface shear computed by Squire and Young's method. Actually, the heat capacity of the boundary layer is increased by this thickening which tends to increase the rate of heat flow at the surface. This counteracts the effect of the profile distortion, resulting from the same cause, which reduces the surface shear since it tends to cause separation. But this effect, too, is negligible for all cases where Squire and Young's method may be applied.

In concluding, it must be stated that while the method presented herein is subject to a number of broad assumptions in its development, the experimental evidence presented shows the total rate of heat flow may be calculated with reasonable accuracy.

CONCLUSIONS

The accuracy of the method for determining the rate of heat transfer from an airfoil is shown to be good by the results of a limited experimental investigation. Since the correctness with which the heat transfer can be computed is dependent mainly on the accuracy with which the boundary-layer characteristics may be determined, it is expected that the method possesses the same accuracy for computing heat-transfer rates from a streamline body.

Although the development of the heat-transfer formulas is based on the assumption that the skin temperature remains constant along the surface, the experimental results show that for moderate temperature variations the precision is still good.

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APPENDIX

DERIVATION OF THE METHOD

1. Heat transfer into a laminar boundary layer.—The theory of heat transfer into a laminar boundary layer was first investigated by E. Pohlhausen for the case of incompressible flow along a flat plate maintained at a constant temperature (reference 1). Pohlhausen's solution is developed by solving the differential equation for the temperature boundary layer by using Blasius' solution for the velocity boundary layer.

In order to arrive at a solution for an airfoil in an *incompressible* fluid, it is necessary

(1) to assume that the temperature boundary-layer and the velocity boundary-layer profiles for the airfoil are related in the same manner as for Pohlhausen's solution. (This is true if the temperature of the skin remains constant along the surface and if the thinning of the friction layer in a *favorable* pressure gradient due to the change in pressure acting on the displaced mass of fluid is negligible.)

(2) to calculate the value of $\left(\frac{du}{dy}\right)_{y=0}$ for the velocity boundary layer and then determine $\left(\frac{dt}{dy}\right)_{y=0}$ with the relationship resulting from (1). (The solution of the problem for the temperature of the skin varying along the chord has been prevented because the diffculties so far have been found insurmountable.)

Pohlhausen's expression for the temperature gradient in the boundary layer at the surface of the plate is given as

$$\left(\frac{dt}{dy}\right)_{y=0}=-\frac{1}{2}\,\alpha(\sigma)\,\sqrt{\frac{U\rho}{\mu x}}\,(t_p-t)$$

The function $\alpha(\sigma)$ is the first derivative of Pohlhausen's function defining the temperature boundary layer which, for $\sigma=1$, is equivalent to the second derivative of Blasius' function for the velocity boundary layer. Pohlhausen found that $\alpha(\sigma)$ is accurately given by the relationship

$$\alpha(\sigma)=0.664\sqrt[3]{\sigma}$$

then

$$\left(\frac{dt}{dy}\right)_{y=0}=-0.332\sqrt[3]{\sigma}\,\sqrt{\frac{U\rho}{\mu x}}\,(t_p-t_0)$$

Now, for the Blasius boundary-layer distribution

$$\left(\frac{du}{dy}\right)_{y=0}=0.332\,\sqrt{\frac{U\rho}{\mu x}}\,U$$

so that

$$\left(\frac{dt}{dy}\right)_{y=0}=-\left(\frac{du}{dy}\right)_{y=0}\sqrt[3]{\sigma}\,\frac{(t_p-t_0)}{U}$$

It is now necessary to determine $\left(\frac{du}{dy}\right)_{y=0}$ for the airfoil at any chordwise position. For the Blasius profile

$$\left(\frac{du}{dy}\right)_{y=0}=\frac{0.765}{\delta_L}U$$

where δ_L is the thickness of the boundary layer where $u=0.707\ U$. Substituting in the preceding equation

$$\left(\frac{dt}{dy}\right)_{y=0}=0.765\,\sqrt[3]{\sigma}\,\frac{(t_p-t_0)}{\delta_L}$$

or, taking $\sigma=0.760$ for air, the local rate of heat transfer is

$$q_x=k\left(\frac{dt}{dy}\right)_{y=0}=0.700\,\frac{k}{\delta_L}\,(t_p-t_0)$$

or the heat-transfer coefficient is

$$h_x=0.700\,\frac{k}{\delta_L}$$

The development in reference 5 of an expression for the heat-transfer rate based on Reynolds analogy gives results which are in complete agreement with the above if $\sigma=1$. However, the experimental results of reference 6 indicate that the expression $\sqrt[3]{0.760}$, that is, $\sqrt[3]{\sigma_{\text{air}}}$, properly relates the velocity and temperature gradients in the laminar layer. The two methods give results within 10 percent of each other, which is sufficient for practical cases.

The values of δ_L for laminar flow may be determined both for an airfoil and a streamline body by the method of reference 2.

2. Turbulent boundary layer.—The theory of heat transfer in eddying flow as given by Dryden (reference 7) requires the introduction of several new concepts. If the equations of motion for turbulent flow are written by placing $u=\bar{u}+u'$, $v=\bar{v}+v'$, $w=\bar{w}+w'$, where the bars indicate mean values and the primes indicate fluctuations, and these values substituted into the equations of motion for steady flow, similarity between equations so developed and the steady-flow equations can be shown by introducing a value of eddy viscosity ϵ. Similarly, the concept of eddy heat conductivity, β, is introduced by placing $t=\bar{t}+t'$, in the equations of the temperature field.

These values of eddy viscosity, ϵ, and eddy conductivity, β, however, do not have the same properties as μ and k since they vary from point to point in the flow. Nevertheless, ϵ and β can be shown to vary in the same manner from point to point in the fluid. This is done by introducing Prandtl's concept of a mixing length; that is, a length of path followed by a fluid particle before it becomes lost in the mass of eddying fluid.

It is therefore shown if the shear, $\tau' = -\overline{\rho u' v'}$, then

$$\tau = \epsilon \frac{d\bar{u}}{dy} = \rho l^2 \left| \frac{d\bar{u}}{dy} \right| \left| \frac{d\bar{u}}{dy} \right|$$

or that

$$\epsilon = \rho l^2 \left| \frac{d\bar{u}}{dy} \right|$$

in which the mixing length l varies from point to point in the fluid. Now, if the eddy heat transfer is considered to be $-c_p \overline{v' t'}$, then the eddy heat conductivity is equal to $c_p \rho l^2 \left| \frac{du}{dy} \right|$ or equal to $c_p \epsilon$, provided the mixing length for heat transfer is considered to be the same as the mixing length for the transfer of shearing stress. Dryden states that available experimental data show that the mixing lengths near a wall are closely equivalent for transfer of heat and momentum, but that the relationship falls down, for instance, in the wake of a heated body. Since the present case concerns heat transfer from a wall to eddying flow in a boundry layer, it is believed that this relationship is acceptable.

For turbulent flow, it has been shown that the Prandtl number is equal to unity; that is,

$$\frac{c_p \mu_{turb}}{k_{turb}} = 1$$

If the Prandtl number is unity, then the thermal and dynamic boundary layers have the same profile (reference 6). If we make the same assumptions as in step (1) for the laminar boundary layer, we may write

$$\left(\frac{dt}{dy}\right)_{y=0} = -\left(\frac{du}{dy}\right)_{y=0} \frac{(t_p - t_0)}{U}$$

where U is the velocity outside the friction layer. (This relationship is dependent on the assumption that the temperature along the surface remains constant as for step (1) for the laminar boundary layer, and that the thickening of the boundary layer due to the increasing pressures acting on the displaced mass of fluid and the distortion of the profile thus resulting is negligible.)

Now

$$q_x = -\beta \left(\frac{dt}{dy}\right)_{y=0}$$

but

$$\beta = c_p \epsilon$$

so that

$$q_x = -c_p \epsilon \left(\frac{dt}{dy}\right)_{y=0}$$

or

$$q_x = +c_p \epsilon \left(\frac{du}{dy}\right)_{y=0} \frac{(t_p - t_0)}{U}$$

since the surface shear

$$\tau_0 = \epsilon \left(\frac{du}{dy}\right)_{y=0}$$

$$q_x = c_p \tau_0 \frac{(t_p - t_0)}{U}$$

This is the same formula developed by Reynolds for flow in pipes (reference 7).

So it is seen that the problem of calculating the rate of heat transfer in turbulent flow is primarily a problem of calculating the surface shear along the airfoil. This may be done by the method of reference 3, in which Squire and Young write the relationship

$$\tau_0 = \frac{\rho U^2}{\zeta^2}$$

where

$$\zeta = 2.557 \log_e \left(4.075 \frac{U\theta}{\nu}\right)$$

θ being the momentum thickness of the boundary layer. (This relationship is developed from von Kármán's formula for the skin friction experience by a flat plate with a fully developed turbulent boundary layer. This assumption becomes less and less true as the turbulent boundary-layer profile of an airfoil becomes changed in shape and approaches separation in a steep pressure recovery.) Substituting for τ_0

$$q_x = c_p \frac{\rho U^2}{\zeta^2} \frac{(t_p - t_0)}{U}$$

$$q_x = c_p \frac{\rho U}{\zeta^2} (t_p - t_0)$$

or

$$q_x = c_p \rho \frac{k}{k} \frac{U}{\zeta^2} \frac{\mu}{\mu} \frac{V}{V} \frac{c}{c} (t_p - t_0)$$

$$q_x = \frac{c_p \mu}{k} \frac{k}{c\zeta^2} \frac{Vc}{V} \frac{U}{V} (t_p - t_0)$$

where μ and k are values for uneddying flow. Substituting for $\frac{c_p \mu}{k}$ the value for air, 0.760,

$$q_x = 0.760 \frac{k}{c\zeta^2} R_c \left(\frac{U}{V}\right) (t_p - t_0)$$

or considering $\frac{\zeta^2 c}{R_c \left(\frac{U}{V}\right)} = \delta_T$, a characteristic length for the turbulent boundary layer,
then

$$q_x = 0.760 \frac{k}{\delta_T} (t_p - t_0)$$

and

$$h_x = 0.760 \frac{k}{\delta_T}$$

In calculating δ_T, ζ may be computed by the step-by-step solution of the equations of reference 3 for an airfoil or streamline body.

3. Compressibility effects on heat transfer.—Since the foregoing analysis has been made for incompressible flow, the effect of aerodynamic heating must be dealt with if the heat transfer is to be accurately obtained. The effect of compressibility may be considered, to a first approximation, simply as influencing the heat-transfer temperature difference to be used in the above-developed equations; that is, a part of any desired increase in the skin temperature will result from aerodynamic heating, and this part of the temperature increase involves no expenditure of heat.

The temperature field near the heated surface of an airfoil or body of revolution operating at high Mach numbers may be determined by superposing the heat-transfer temperature field on that due to the friction heating as in reference 8. Eckert (reference 9) has shown that the temperature field due to aerodynamic heating for $\sigma=1$ may be expressed as

$$t=t_L+\frac{\gamma-1}{2}M_L^2T_L\left[1-\left(\frac{u}{U}\right)^2\right]$$

or

$$t=t_0+\frac{\gamma-1}{2}M_0^2T_0-\frac{\gamma-1}{2}M_L^2T_L\left(\frac{u}{U}\right)^2$$

Superposing this on the temperature field for heat transfer, which may be given as

$$t=t_p-(t_p-t_0)\frac{u}{U}$$

the combined temperature field is

$$t=t_p-(t_p-t_0)\left(\frac{u}{U}\right)+\frac{\gamma-1}{2}M_0^2T_0-\frac{\gamma-1}{2}M_L^2T_L\left(\frac{u}{U}\right)^2$$

It is evident that at $y=0$

$$\left(\frac{dt}{dy}\right)_{y=0}=-\frac{(t_p-t_0)}{U}\left(\frac{du}{dy}\right)_{y=0}$$

which indicates that the heat transfer corresponds to the heat-transfer temperature field, so that for compressible flow the only correction necessary is that of correcting the skin temperature for the rise in temperature due to aerodynamic heating.

Eckert has shown that the temperature of the surface due to compressibility effects t_M, is

$$t_M=t_L+\sqrt{\sigma}\frac{\gamma-1}{2}M_L^2T_L$$

$$t_M=t_0+\frac{\gamma-1}{2}M_0^2T_0-\frac{\gamma-1}{2}M_L^2T_L+\sqrt{\sigma}\frac{\gamma-1}{2}M_L^2T_L$$

or

$$t_M=t_0+\frac{\gamma-1}{2}M_0^2T_0\left[1-(1-\sqrt{\sigma})\left(\frac{U}{V}\right)^2\right]$$

The heat-transfer temperature difference for the laminar region, $\sigma=0.760$, is then

$$(t_p-t_0)=(t_e-t_0)-0.2\,M_0^2T_0\left[1-0.13\left(\frac{U}{V}\right)^2\right]$$

and for the turbulent region, $\sigma=1$

$$(t_p-t_0)=(t_e-t_0)-0.2\,M_0^2T_0$$

REFERENCES

1. Pohlhausen, E.: Heat Exchange Between Solid Bodies and Fluids With Small Friction and Low Heat Conductivity. Z. f. a. M. M., vol. I, no. 2, 1921, pp. 115–121.
2. Jacobs, E. N., and von Doenhoff, A. E.: Formulas for Use in Boundary-Layer Calculations on Low-Drag Wings. NACA ACR, Aug. 1941.
3. Squire, H. B., and Young, A. D.: The Calculation of the Profile Drag of Aerofoils. R. & M. No. 1838, British A. R. C., 1937.
4. von Doenhoff, Albert E.: A Method of Rapidly Estimating the Position of the Laminar Separation Point. NACA TN No. 671, 1938.
5. Allen, H. Julian, and Look, Bonne C.: A Method for Calculating Heat Transfer in the Laminar Flow Region of Bodies. NACA Rep. No. 764.
6. Elias, Franz: The Transference of Heat from a Hot Plate to an Air Stream. NACA TM No. 614, 1931.
7. Dryden, Hugh L.: Aerodynamics of Cooling. Vol. VI of Aerodynamic Theory, W. F. Durand, ed., Julius Springer (Berlin), 1936, pp. 238–250.
8. Crocco, Luigi: Transmission of Heat from a Flat Plate to a Fluid Flowing at a High Velocity. NACA TM No. 690, 1932.
9. Eckert, E.: Temperature Recording in High-Speed Gases. NACA TM No. 983, 1941.