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A THEORY OF THIN AEROFOILS.—By H. GLAUERT OF THE ROYAL AIRCRAFT ESTABLISHMENT. PRESENTED BY THE DIRECTOR OF RESEARCH.

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A THEORY OF THIN AEROFOILS.

BY H. GLAUERT of the R.A.E.
Presented by THE DIRECTOR OF RESEARCH.

Reports and Memoranda No. 910. *February*, 1924.
(Ae. 136.)

Summary.—The present report develops a theory of thin aerofoils in two dimensional motion and simple integral expressions are obtained for the angle of incidence and moment coefficient at zero lift. A graphical method of integration is developed which can be used to determine the characteristics of any thin aerofoil. The method is applied successfully to three aerofoil sections and results are also derived for a tail-plane and elevators.

1. *Introduction.*—The general theory of aerofoils in two dimensional motion at small angles of incidence indicates linear relationships between lift coefficient and angle of incidence, and between moment and lift coefficients. Also the slope of these curves does not show any considerable variation between different aerofoil sections and so the problem of determining the characteristics of a given aerofoil section is essentially that of determining the angle of incidence at which the lift vanishes and the corresponding value of the moment coefficient. In addition there is the yet unsolved problem of determining the drag of an aerofoil in two dimensional motion. When these characteristics are known, the behaviour of any finite aerofoil or system of aerofoils in three dimensional motion can be deduced by Prandtl's theory.

The present report develops a method of determining the angle of no-lift and the corresponding moment coefficient for a thin aerofoil of any given shape. A treatment of this problem has been given by Munk in Report No. 142 of the National Advisory Committee for Aeronautics, but the analysis is not quite free from errors. The present report adopts the general principles used by Munk but presents the analysis in a more logical order.

2. *General Analysis.*—The basis of the two dimensional aerofoil theory initiated by Joukowski depends on the possibility of converting a circle in the z plane into the desired aerofoil shape in the ζ plane by means of a suitable conformal transformation. The present analysis will be confined to the case of a thin aerofoil in which it is possible to replace the aerofoil approximately by a curved line which will be the mean of the upper and lower surfaces of the aerofoil. This assumption simplifies the analysis considerably and has been found to lead to very satisfactory results for ordinary aerofoil shapes.

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The starting point of the analysis is the conversion of the circle C (*see* Fig. 1) into the straight line A′B′ by means of the conformal transformation :—

$$\zeta = z + \frac{a^2}{z} \qquad (1)$$

where z and ζ are the complex variables

$$z = x + i\, y$$
$$\zeta = \xi + i\, \eta$$

and a is the radius of the circle C whose centre is at the origin O. The straight line A′B′ obtained by the transformation extends on the real axis between the points $\xi = \pm\, 2a$.

Now consider a thin aerofoil represented by the curved line A′S′B′ which deviates only slightly from the straight line A′B′. The corresponding curve in the z plane, related to the aerofoil by the transformation (1), will be a curve S deviating only slightly from the circle C and intersecting it at the points A and B. The equation of this curve S may be taken to be

$$z' = a\,(1 + r)\; e^{i\theta}$$

where r is a function of θ and represents the difference of radius between the circle C and the curve S.

Since the hypothesis

$$\zeta = z' + \frac{a^2}{z'} \qquad (2)$$

the corresponding point of the aerofoil is

$$\zeta = 2a\,(\cos\theta + i\; r\sin\theta)$$

showing that the ordinate η of the aerofoil at the abscissa $\xi = 2a\,\cos\,\theta$ is

$$\eta = 2\; a\; r\,\sin\,\theta \qquad (3)$$

To solve the problem we require also the transformation from the circle C to the curve S. Now suppose that the point $z = a\; e^{i\phi}$ of the circle corresponds to the point $z' = a\;(1 + r)\; e^{i\theta}$ of the curve, where

$$\phi = \theta + \epsilon \qquad (4)$$

The transformation is then found to be

$$z' = z\left\{1 + i\,\Sigma\,A_n\,\frac{a^n}{z^n}\right\} \qquad (5)$$

where

$$\left.\begin{aligned} r &= \Sigma\,A_n\,\sin\,n\,\theta \\ \epsilon &= -\,\Sigma\,A_n\,\cos\,n\,\theta \end{aligned}\right\} \qquad (6)$$

and

$$A_n = \frac{2}{\pi}\int_0^{\pi} r\,\sin\,n\,\theta\; d\,\theta \qquad (7)$$

The trailing edge of the aerofoil is given by $\theta = \pi$ and so corresponds on the circle to the position $\phi = \pi + \epsilon_0$ where

$$\epsilon_0 = A_1 - A_2 + A_3 - \ldots\ldots$$

To determine ϵ_0 we have formally

$$\epsilon_0 = \frac{2}{\pi}\int_0^\pi (A_1 \sin\theta + A_2 \sin 2\theta + \ldots)(\sin\theta - \sin 2\theta + \ldots)\, d\theta$$

and so ϵ_0 is the imaginary part of

$$\frac{2}{\pi}\int_0^\pi r\,(e^{i\theta} - e^{2i\theta} + \ldots)\, d\theta$$

i.e.,

$$\frac{2}{\pi}\int_0^\pi \frac{r\, e^{i\theta}}{1 + e^{i\theta}}\, d\theta$$

giving finally

$$\epsilon_0 = \frac{1}{\pi}\int_0^\pi r \tan\frac{\theta}{2}\, d\theta = \frac{1}{2\pi a}\int_0^\pi \frac{\eta\, d\theta}{1 + \cos\theta} \quad \ldots\ldots\ldots\ldots(8)$$

3. *Lift of the aerofoil.*—The circulation round the aerofoil is determined by Joukowski's hypothesis that the flow must leave the trailing edge smoothly. This implies that the rear stagnation point on the circle must be situated at the point $\phi = \pi + \epsilon_0$ corresponding to the trailing edge of the aerofoil. If now the undisturbed flow is inclined at angle a to the chord of the aerofoil or to the real axis as in Fig. 1, the complex potential of the flow past the circle will be

$$w = V z e^{i a} - \frac{V a^2}{z} e^{-i a} - \frac{i K}{2\pi} \log z \ldots\ldots\ \ldots\ldots(9)$$

where K is the circulation which must be determined so that the rear stagnation point occurs at the point $\phi = \pi + \epsilon_0$.

Now at the surface of the circle

$$\frac{dw}{dz} = -V e^{i a} + V e^{-i a} e^{-2 i \phi} - \frac{i K}{2\pi a} e^{-i\phi}$$

$$= -2 i V e^{-i\phi}\left\{\sin(a + \phi) + \frac{K}{4\pi a V}\right\} \ldots\ldots\ldots(10)$$

and vanishes for $\phi = \pi + \epsilon_0$ if

$$K = 4\pi a V \sin(a + \epsilon_0) \ldots\ldots\ldots\ldots(11)$$

Thus finally the lift of the aerofoil is

$$L = \rho V K = 4\pi a \rho V^2 \sin(a + \epsilon_0)$$

and since the chord of the aerofoil is $4a$ and the angles a and ϵ_0 can be regarded as small, the lift co-efficient is

$$k_L = \pi(a + \epsilon_0) \qquad (12)$$

The no-lift angle of the aerofoil is, therefore, $\alpha = -\epsilon_0$, where ϵ_0 is determined from the shape of the aerofoil by means of equation (8).

The slope of the lift coefficient against angle of incidence is π per radian or 0·055 per degree in two dimensional motion. The corresponding value for the rectangular aerofoil of aspect ratio 6 is found to be 0·039 per degree by the method of report R. & M. 824. This value refers to a thin aerofoil and may be expected to vary slightly with the actual shape of the aerofoil but it represents a good average value for ordinary aerofoils.

4. *Pitching Moment of the Aerofoil.*—From equations (10) and (11) we deduce that the velocity at the point $\phi = \theta + \epsilon$ of the surface of the circle is

$$
\begin{aligned}
q_c = \left|\frac{dw}{dz}\right| &= 2\,\mathrm{V}\left\{\sin(\alpha + \phi) + \sin(\alpha + \epsilon_0)\right\} \\
&= 2\,\mathrm{V}\left\{\sin\theta + (\alpha + \epsilon)\cos\theta + (\alpha + \epsilon_0)\right\} \quad \ldots\ldots (13)
\end{aligned}
$$

For the corresponding point θ on the surface of the aerofoil we have

$$q = q_c \left|\frac{dz}{d\zeta}\right|$$

where

$$
\begin{aligned}
z' &= z\left\{1 + i\,\Sigma\,\mathrm{A}_n \frac{a^n}{z^n}\right\} \\
\zeta &= z' + \frac{a^2}{z'}
\end{aligned}
$$

Thus

$$
\begin{aligned}
\frac{dz'}{dz} &= 1 - i\,\Sigma\,(n-1)\,\mathrm{A}_n\, e^{-ni\theta} \\
\left|\frac{dz'}{dz}\right| &= 1 - \Sigma\,(n-1)\,\mathrm{A}_n \sin n\theta \\
&= 1 - \frac{d\epsilon}{d\theta} + r
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{d\zeta}{dz'} &= 1 - \frac{a^2}{z'^2} \\
&= 1 - (1 - 2r)\, e^{-2i\theta} \\
&= 2\, e^{-i\theta}\left\{i \sin\theta + r\, e^{-i\theta}\right\} \\
\left|\frac{d\zeta}{dz'}\right| &= 2 \sin\theta\,(1 - r)
\end{aligned}
$$

giving

$$\left|\frac{d\zeta}{dz}\right| = 2\sin\theta\left(1 - \frac{d\epsilon}{d\theta}\right)$$

and

$$q = V\left\{1 + (\alpha + \epsilon)\cot\theta + (\alpha + \epsilon_0)\operatorname{cosec}\theta + \frac{d\epsilon}{d\theta}\right\} \quad . \quad (14)$$

The pitching moment round the origin is now obtained as

$$M_0 = \int_0^{2\pi} 2a^2\,\rho\,q^2 \sin\theta\cos\theta\,d\theta$$

$$= 2a^2\,\rho\,V^2\int_0^{2\pi}\left\{\sin\theta + 2(\alpha+\epsilon)\cos\theta + 2(\alpha+\epsilon_0) + 2\frac{d\epsilon}{d\theta}\sin\theta\right\}\cos\theta\,d\theta$$

where

$$\epsilon = -\Sigma\,A_n\cos n\theta$$

$$\frac{d\epsilon}{d\theta} = \Sigma\,n\,A_n\sin n\theta$$

and on integration

$$Mo = 2\,a^2\,\rho\,V^2\,(2\pi\alpha - \pi A_2 + 2\pi A_2)$$
$$= 2\,\pi\,a^2\,\rho\,V^2\,(2\alpha + A_2)$$

To obtain the moment round the leading edge we must subtract $(2a\mathrm{L})$, and so obtain

$$M = 2\,\pi\,a^2\,\rho\,V^2\,(2\alpha + A_2) - 8\,\pi\,a^2\,\rho\,V^2\,(\alpha + \epsilon_0)$$
$$= 2\,\pi\,a^2\,\rho\,V^2\,(A_2 - 2\alpha - 4\,\epsilon_0)$$

giving the moment coefficient

$$k_m = -\frac{\pi}{4}\,\alpha - \frac{\pi}{2}\,\epsilon_0 + \frac{\pi}{8}\,A_2$$

$$= -\tfrac{1}{4}\,k_L - \frac{\pi}{4}\,\epsilon_0 + \frac{\pi}{8}\,A_2$$

Now put

$$\mu_0 = \frac{\pi}{8}\,A_2 = \tfrac{1}{4}\int_0^{\pi} r\sin 2\theta\,d\theta$$

$$= \frac{1}{4a}\int_0^{\pi}\eta\cos\theta\,d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

and then finally

$$k_m = -\tfrac{1}{4}\,k_L - \frac{\pi}{4}\,\epsilon_0 + \mu_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Thus the slope of the moment coefficient against the lift coefficient is 0·250, but there will actually be small variations in this value with the thickness of the aerofoil. The value of the moment coefficient at no lift is $(\mu_0 - \frac{\pi}{4}\,\epsilon_0)$ and is determined from the shape of the aerofoil by the two integrals (8) and (15).

5. *Graphical Integration.*—In defining the shape of an aerofoil it is customary to measure the ordinates and abscissæ as fractions

of the chord and to take the leading edge as origin. To transform the expressions to this form, we write

$$2x = 1 - \cos\theta$$
$$4ay = \eta$$

obtaining

$$\left.\begin{aligned} \epsilon_0 &= \int_0^1 y\, f_1(x)\, dx \\ \text{where} \quad f_1(x) &= \frac{1}{\pi(1-x)\sqrt{x(1-x)}} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(17)$$

and

$$\left.\begin{aligned} \mu_0 &= \int_0^1 y\, f_2(x)\, dx \\ \text{where} \quad f_2(x) &= \frac{1-2x}{\sqrt{x(1-x)}} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(18)$$

When the form of the aerofoil is a simple mathematical expression, the values of ϵ_0 and μ_0 can be obtained by direct integration. One example of this type is the aerofoil whose shape is defined by the equation

$$y = h\,x\,(1-x)\,(a-x)$$

which represents an aerofoil with reflex curvature towards the trailing edge when a lies between $\frac{1}{2}$ and 1. Direct integration of the expressions (17) and (18) is carried out quite simply in this case by the substitution $x = \sin^2\theta$, and leads to the results

$$\epsilon_0 = \tfrac{1}{8} h\,(4a - 3)$$

$$\mu_0 = \frac{\pi}{64} h$$

so that

$$k_{m_0} = \frac{\pi}{64} h\,(7 - 8a).$$

Thus the aerofoil will have a constant position of the centre of pressure if we take the value $a = \frac{7}{8}$. In report R. & M. 911. aerofoil D has been designed on this basis in the hope of testing the validity of the result.

In general, the form of the aerofoil necessitates the use of a graphical method of integration and for this purpose numerical values of $f_1(x)$ and $f_2(x)$ are given in Table 1. The determination of μ_0 in this manner presents no peculiarities, since $y\,f_2(x)$ is zero at both ends of the aerofoil, although $f_2(x)$ tends to infinity. In the case of ϵ_0, however, the value of $y\,f_1(x)$ generally tends to an infinite value at the trailing edge $x = 1$. This difficulty can be avoided by performing the graphical integration from $x = 0$ to $x = 0{\cdot}95$, and by estimating the additional contribution from $x = 0{\cdot}95$ to $x = 1$ analytically on the assumption that this part of the aerofoil is linear. It can easily be shown that this additional contribution is $2 \cdot 9\ y'$. where y' is the value of the ordinate y at $x = 0 \cdot 95$.

6. *Numerical examples.*—As an example of the method, the details of the calculation in the case of R.A.F. 15 are given in Table 2 and Fig. 2. In Table 2 the ordinates y_1 and y_2 of the upper and lower surfaces are first tabulated, and the fourth column gives the mean ordinate. These values are referred to the standard chord, which is the tangent to the lower surface of the aerofoil, and from them the values of the ordinate y are deduced, referred to the base line joining the leading and trailing edges. This base line makes an angle $0° \cdot 2$ with the chord. The last two columns give the values of $y\ f_1\ (x)$ and $y\ f_2\ (x)$, which are used to prepare the diagrams of Fig. 2. By integration, the part of ϵ_0 up to $x = 0 \cdot 95$ is found to be $0 \cdot 029$, while the additional contribution is $2 \cdot 9 \times 0 \cdot 0030$ or $0 \cdot 009$. Thus, $\epsilon = 0 \cdot 038$ or $2° \cdot 2$, so that the angle of no lift $- 2° \cdot 2$ referred to the base line or $- 2° \cdot 4$ referred to the chord. Also, by integration, the value of μ_0 is found to be $0 \cdot 009$, and so the moment coefficient at no lift is predicted to be $- 0 \cdot 021$.

A comparison with two sets of experimental results is shown in Fig. 3. It will be noticed that the calculated values lie closely on the experimental curves obtained at LV $= 100$, but that the curves obtained at LV $= 10$ show a peculiar and characteristic scale effect at small values of the lift coefficient. It appears that the theory gives a good prediction of the actual values obtained at a reasonably high value of LV. If the only experimental values available were obtained at a low value of LV, it would be necessary to ignore all values below $k_L = 0 \cdot 15$, and to compare the calculated values with the general run of the curves above this point.

This point is confirmed in the case of the aerofoils R.A.F. 14 and R.A.F. 18, for which calculations have also been made. The comparison with experimental data is shown in Figs. 4 and 5. The calculated points again agree excellently with the form of the curves above $k_L = 0 \cdot 15$, but below this point there is the same peculiar scale effect on the model curves. It appears that the method of calculation gives good predictions of the values which may be expected from model tests at a reasonably high value of LV. The method of calculation, however, deals with thin wing sections only, and might need extension or modification before it could be applied to thick sections.

7. *Tail-plane and Elevators.*—The method of analysis can also be applied to the case of a tail-plane and elevator by inserting the values

$$y = \frac{h\ x}{1 - \mathrm{E}} \text{ from } x = 0 \text{ to } (1 - \mathrm{E})$$

$$y = \frac{h\ (1 - x)}{\mathrm{E}} \text{ from } x = (1 - \mathrm{E}) \text{ to } 1,$$

where E is the ratio of elevator chord to total chord.

By direct integration the values of ϵ_0 and μ_0 are found to be

$$\epsilon_0 = \frac{2h}{\pi}\left\{\frac{1}{\sqrt{\mathrm{E}(1-\mathrm{E})}} + \frac{\pi}{2\,\mathrm{E}} - \frac{\operatorname{arc\,cos}\sqrt{\mathrm{E}}}{\mathrm{E}(1-\mathrm{E})}\right\}$$

$$\mu_0 = \frac{h}{2}\left\{\frac{2\mathrm{E}-1}{\sqrt{\mathrm{E}(1-\mathrm{E})}} + \frac{\pi}{2\,\mathrm{E}} - \frac{\operatorname{arc\,cos}\sqrt{\mathrm{E}}}{\mathrm{E}(1-\mathrm{E})}\right\}$$

so that. regarded as an aerofoil, we have the characteristics

$$k_{\mathrm{L}} = \pi\,(\alpha + \epsilon_0)$$

$$k_m = -\tfrac{1}{4}\,k_{\mathrm{L}} - h\sqrt{\frac{1-\mathrm{E}}{\mathrm{E}}}$$

It is more usual. however, to express the lift coefficient of a tail-plane in the form

$$k_{\mathrm{L}} = a_1\,\alpha_1 + a_2\,\alpha_2$$

where α_1 is the angle of incidence of the front part of the tail-plane, and α_2 is the angle between the front and rear parts. In this form the theoretical result is found to be

$$\frac{a_2}{a_1} = 1 - \frac{2}{\pi}\left\{\operatorname{arc\,cos}\sqrt{\mathrm{E}} - \sqrt{\mathrm{E}(1-\mathrm{E})}\right\}$$

and the numerical results deduced from these formulæ are given in Table 3 and Fig. 6. The values of a_2/a_1 found in this way are higher than those which have been deduced from various experimental tests. It seems probable that the sharp angle at the junction of the tail-plane and elevator exerts a harmful effect which is not allowed for by the theory, while, on the other hand, the experimental tests refer to elevators which are divided in the middle and therefore lose a certain part of their efficiency. No fair comparison between theory and experiment is therefore possible, but the theoretical curve probably represents the best possible result which could ever be expected from an elevator on a symmetrical tail plane.

TABLE I.

Graphical determination of ϵ_0 and μ_0.

x = distance from leading edge.
y = ordinate.

$$\epsilon_0 = \int_0^1 y f_1(x)\,dx \qquad f_1(x) = 1/\pi\,(1-x)\sqrt{x(1-x)}$$

$$\mu_0 = \int_0^1 y f_2(x)\,dx \qquad f_2(x) = (1-2x)/\sqrt{x(1-x)}$$

x	$f_1(x)$	$f_2(x)$
0·025	2·09	6·10
0·05	1·54	4·13
0·10	1·18	2·67
0·20	1·00	1·50
0·30	0·99	0·87
0·40	1·08	0·41
0·50	1·27	0
0·60	1·62	−0·41
0·70	2·31	−0·87
0·80	3·98	−1·50
0·90	10·6	−2·67
0·95	29·2	−4·13

In obtaining ϵ_0, graphical integration should be used from $x = 0$ to $x = 0 \cdot 95$. The contribution to ϵ_0 of the part from $x = 0 \cdot 95$ to $x = 1$ may be taken to be $2 \cdot 9\ y_1$, where y_1 is the ordinate at $x = 0 \cdot 95$.

TABLE II.

Calculations for R.A.F. 15.

x	y_1	y_2	$\frac{1}{2}(y_1 + y_2)$	y	$yf_1(x)$	$yf_2(x)$
0	—	—	0·0127	0	0	0
0·025	0·0381	0·0036	0·0208	0·0082	0·017	0·050
0·05	0·0495	0·0012	0·0253	0·0128	0·020	0·053
0·10	0·0601	0·0001	0·0301	0·0177	0·021	0·047
0·20	0·0669	0·0043	0·0356	0·0235	0·024	0·035
0·30	0·0669	0·0084	0·0376	0·0259	0·026	0·023
0·40	0·0645	0·0080	0·0362	0·0248	0·027	0·010
0·50	0·0607	0·0056	0·0332	0·0221	0·028	0
0·60	0·0553	0·0021	0·0287	0·0179	0·029	—0·007
0·70	0·0481	0·0002	0·0242	0·0137	0·032	—0·012
0·80	0·0389	0·0003	0·0196	0·0094	0·037	—0·014
0·90	0·0284	0·0023	0·0153	0·0055	0·058	—0·015
0·95	0·0215	0·0040	0·0127	0·0030	0·087	—0·012
1·00	—	—	0·0095	0	—	0

Angle between chord and base line $= 57° \cdot 3 \times 0 \cdot 0032 = 0° \cdot 2$.

TABLE 3.

Tail-plane and Elevators.

E = ratio of elevator chord to total chord.

E.	ϵ_0/h	$-km_0/$	a_2/a_1
0·05	4·86	4·36	0·282
0·10	3·30	3·00	0·396
0·20	2·19	2·00	0·550
0·30	1·72	1·53	0·661
0·40	1·45	1·22	0·748
0·50	1·27	1·00	0·818
0·60	1·15	0·82	0·876
0·70	1·06	0·65	0·923
0·80	1·00	0·50	0·960
0·90	0·95	0·33	0·986
1·00	1·00	0	1·000

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Fig. 1.

Diagram of Transformation.

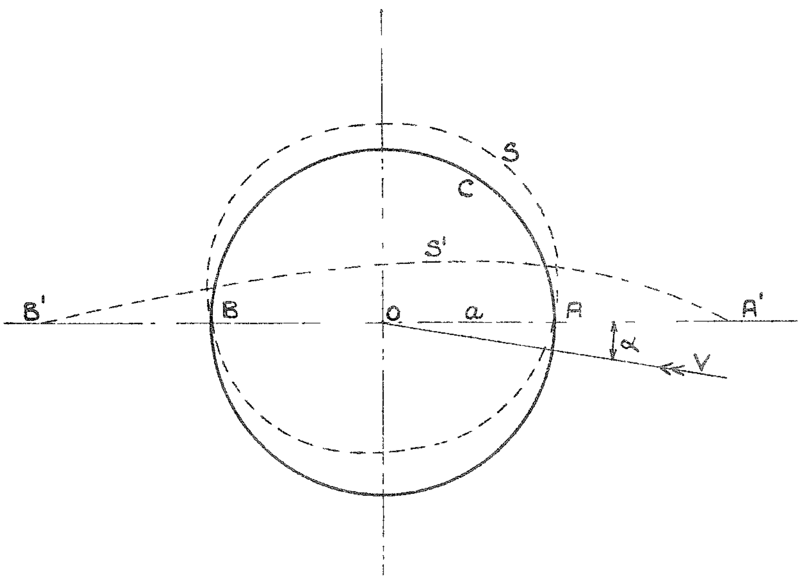

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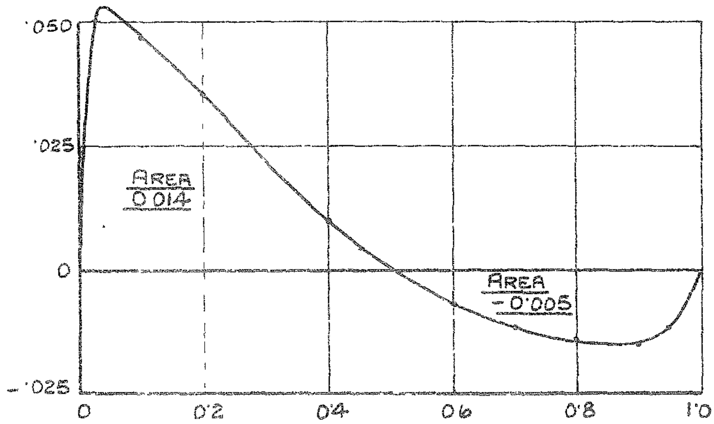
FIG. 2.

CALCULATIONS FOR R.A.F. 15.

CALCULATION OF M_0



CALCULATION OF ϵ_0

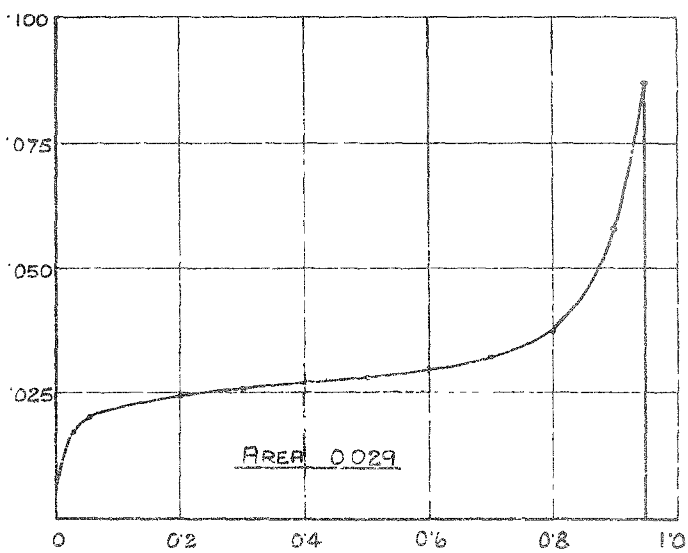
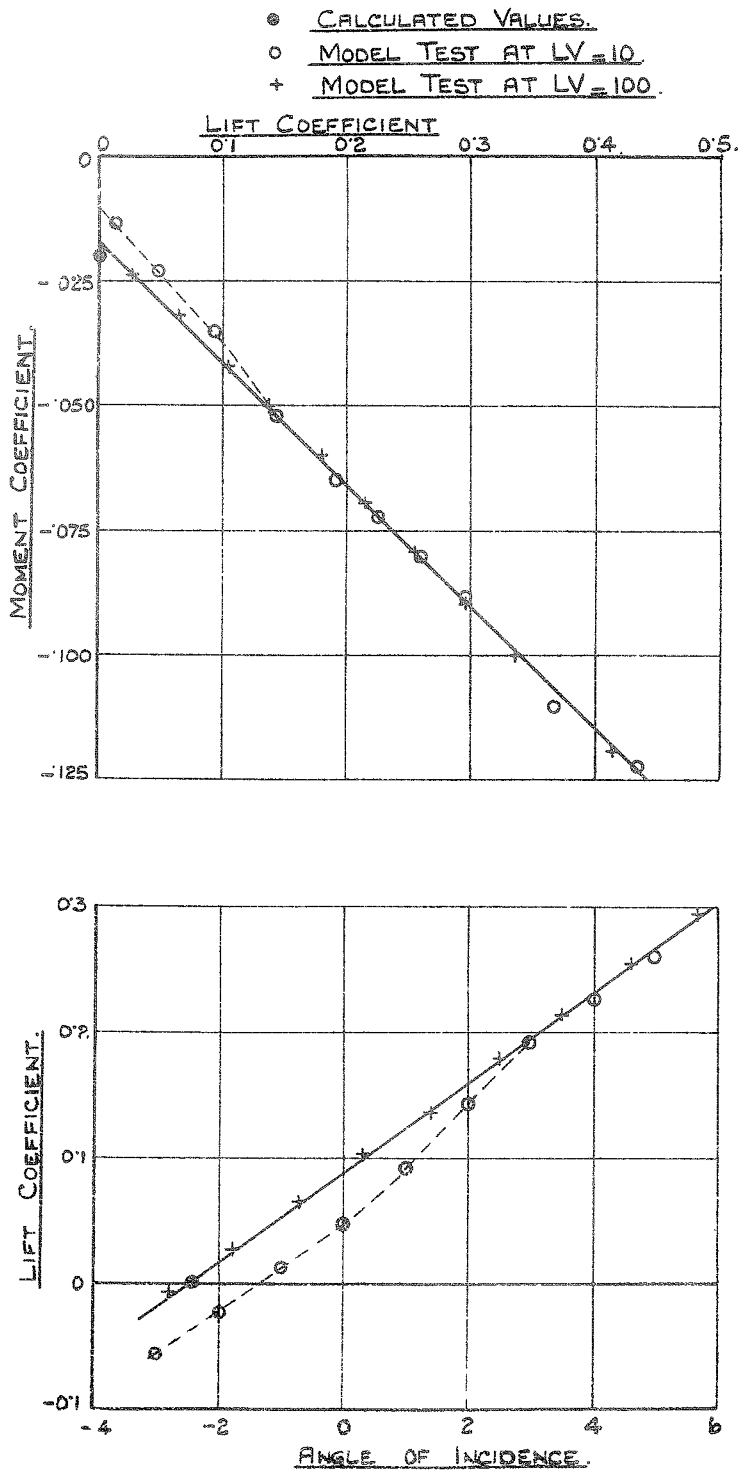


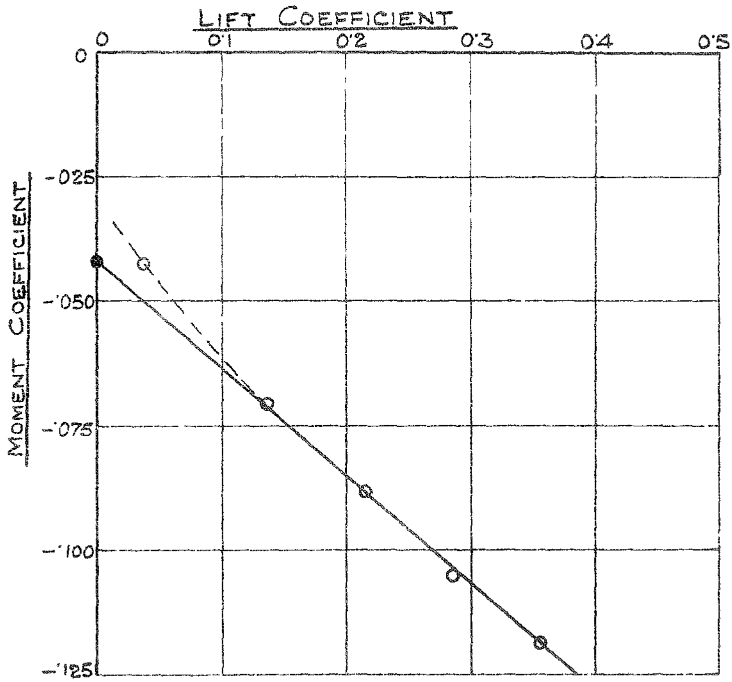
Fig. 3.

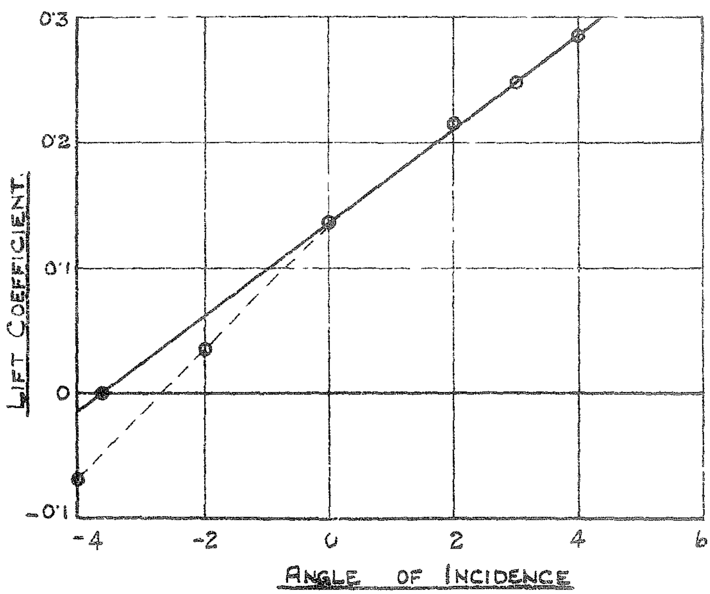
R.A.F. 15. Wing Section.



R.A.F. 14 WING SECTION.

● CALCULATED VALUES
○ MODEL TEST AT LV = 20.

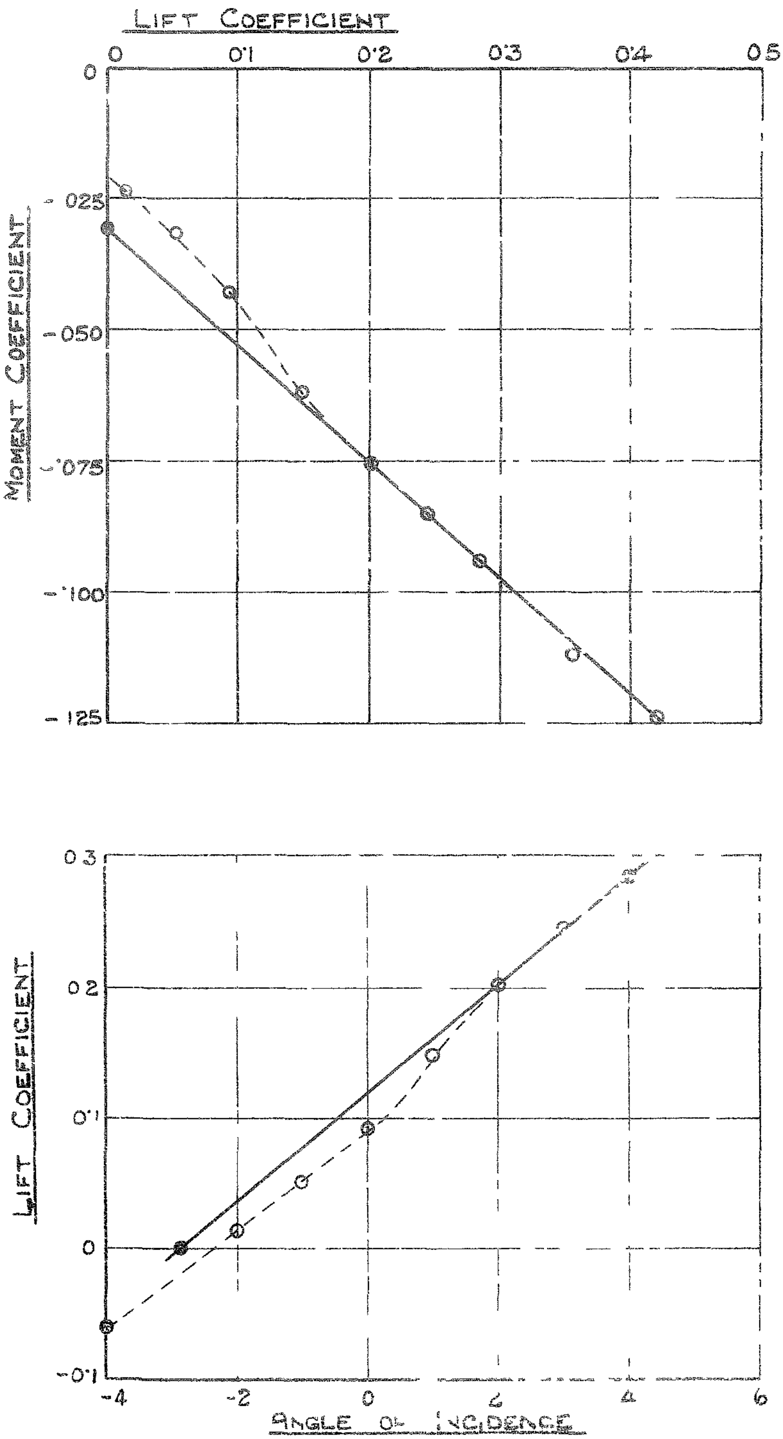




R A F 18 Wing Section

● Calculated Values.

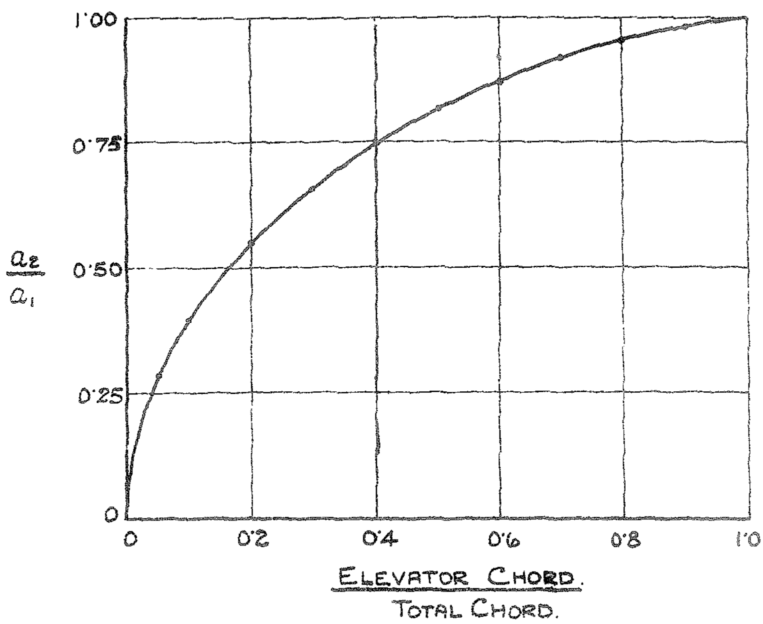
○ Model Test at LV = 15



Tail Plane & Elevator.

$$k_L = a_1 \alpha_1 + a_2 \alpha_2$$

Where α_1 = Angle of Incidence
α_2 = Elevator Angle.



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