R. & M. No. 3672

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MINISTRY OF DEFENCE
(AVIATION SUPPLY)

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Possible Losses in Airspeed during Turning Manoeuvres in Gusty Air

By W. J. G. PINSKER

Aero/Flight Dept., R.A.E., Bedford

With an Appendix by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1971

PRICE £1·20 NET

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Summary.

It is normally assumed that the direction in which a body of turbulent air is traversed in flight is not significant in relation to the aircraft, and that this is also true in non-rectilinear flight. In this report it is demonstrated that, contrary to the results obtained using this general assumption, aircraft can experience losses or gains in airspeed when performing turns in gusty air, which persist when the turbulence ceases, and can be substantially larger than the gust speeds responsible for their generation. The results have important consequences to flight safety in low-level operations at relatively low speeds.

*Replaces R.A.E. Technical Report 70 021—A.R.C. 32 211.

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1. *Introduction.*

When discussing the effect on aircraft flight of non-stationary atmospheric conditions it is usual to distinguish between two separate aspects.

Steady wind is basically considered to be of no consequence to aircraft control and in fact not detectable by the pilot as long as he is not concerned with the aircraft's progress in relation to the ground. During take-off and landing, where flight must be co-ordinated with the runway, winds and especially cross-winds, create control problems which are well understood, but once away from the immediate vicinity of the runway, a steady wind only affects navigation.

Turbulence, on the other hand, continuously disturbs the aircraft from its trimmed condition and may demand pilot action to maintain proper control. However, turbulence is a random phenomenon with the general characteristic that disturbances cancel one another in the long run, leaving only the underlying average or mean wind as a persistent effect. Provided the aircraft is originally in trim and is statically and dynamically stable, the pilot may expect the basic undisturbed flight condition to be maintained and quickly restored once the turbulence or an individual gust has ceased. The main consequence of turbulence is therefore to disturb the aircraft about an essentially maintained mean flight condition.

One of the fundamental hypotheses on which practically all treatment of flight through turbulence is based, is the Taylor theorem which states that in considering turbulence effects on aircraft, space and time are interchangeable. One of the consequences of this assumption is, for instance, that the spatial direction in which random turbulence is traversed is irrelevant and another that this applies equally to non-rectilinear flight.

The present study is concerned especially with an investigation of the validity of this last assumption and it will be shown that when turning manoeuvres are performed in ordinary gusts an aircraft can experience disturbances in airspeed which are not normally taken into account when assessing the effects of turbulence on flight. There are two principal new observations. Firstly, that speed disturbances, which normally would be considered to be symmetric with respect to a mean wind speed and direction, can have cumulative and perpetuated effects on the aircraft, and secondly that the final loss or gain in speed resulting from this phenomenon can be significantly larger than the magnitude of the gusts generating it.

The consequences emerging from this study are perhaps most relevant to flight at very low altitude and at relatively low speed, where a deficiency in airspeed cannot be readily restored by a deliberate dive and where the stall margin, although by normal standards adequate, becomes insufficient to cover major speed deviations. This may be particularly important for general aviation operations requiring frequent turning manoeuvres as in crop-spraying, search and surveillance.

The ideas reported in this report were originally stimulated by an attempt to explain an accident to a Canberra aircraft which stalled during a turn in low level flight through fairly severe turbulence. The airspeed at which this occurred was, however, too high to make a stall readily plausible as the sole explanation, unless one could either assume gross piloting errors or some other mechanism not previously understood. The theories developed here can of course not claim to be a reconstruction of this or other similar accidents but they would certainly assist in making these more explicable.

The main objective of this report is rather to bring this mechanism to the notice of those concerned with flight safety so that suitable provisions can be made in the formulation of safety rules for rough air flying, especially near the ground.

2. *Flight through Non-Stationary Air.*

In order to assist the discussion of the case of manoeuvering flight through turbulent air it might be useful first to consider briefly the mechanism of flight in steady wind.

2.1. *Flight in Steady Wind.*

When considering the mechanics of flight in a moving atmosphere we have to consider three relevant velocities, namely

$\bar{V}_W$ the velocity of the wind which we defined here in relation to the ground

$\bar{V}_a$ the velocity of the aircraft in relation to the air (airspeed)

$\bar{V}_g$ the velocity of the aircraft in relation to the ground (ground speed).

These quantities must be generally treated as vectors and we can then relate the three by the kinematic relationship

$$\bar{V}_g = \bar{V}_a + \bar{V}_W \tag{1}$$

as illustrated in Fig. 1. From this simple relationship derive the well known rules of air navigation. Being of a rather trivial nature we shall not pursue this subject in any detail but since it has some relevance to our later analysis it may be expedient to remind ourselves of a specific case, namely that of an aircraft performing a steady banked turn at constant airspeed in a steady wind. This is illustrated in Fig. 2 showing both the track of the aircraft over ground and the variation of ground speed with time. Although, and indeed because in this manoeuvre airspeed is constant, ground speed will fluctuate periodically, being at a minimum $(V_a - V_W)$ when the aircraft is heading into wind at points (A) amd (E) and at a maximum $(V_a + V_W)$ when it flies with the wind (C). If the pilot cannot see the ground, say if he flies above a cloud layer, he will only be aware of airspeed and of his track in relation to the air and therefore perceive the manoeuvre as describing a perfect circle.

2.2. *Flight through Horizontal Gusts.*

Equation (1) expresses a fundamental kinematic relationship and applies therefore at any instant of flight and hence also in unsteady conditions. Even if we specify $V_W(t)$, equation (1) contains two unknowns and cannot be solved without making either some further assumptions (V_a = const. for instance, as in the example illustrated in Fig. 2) or more generally by introducing the aircraft dynamics in a rigorous manner. It is not the intention here to show what happens to a particular aircraft in manoeuvering flight through gusts but to consider more generally the fundamental difference in aircraft speed response to fore and aft components of turbulence when the spatial relationship between gust direction and aircraft heading is properly accounted for. This is more clearly demonstrated if one makes simplifying assumptions with regard to the aircraft dynamics. These assumptions are designed to eliminate the effects of the aerodynamic characteristics of a particular aircraft from the analysis without of course offending against basic physical principles, and are:

(i) The aircraft is a perfect windvane, aligning itself instantaneously with the relative airflow. In other words we assume sideslip and incidence changes induced by gusts to be zero throughout. This removes the moment equations from the analysis and permits us to ignore gust components normal to the flight path.

(ii) The pilot maintains constant height by appropriate elevator control.

(iii) The fore and aft forces acting on the aircraft are in equilibrium throughout the manoeuvre. If (ii) is satisfied, gravity components are not involved and the above condition then means simply drag = thrust.

The last of these assumptions can of course be questioned as being perhaps inappropriate in an investigation concerned specifically with airspeed response. It must be understood that calculations made under these assumptions, such as presented as illustrations in this report, will only be able to indicate what *can* happen to an aircraft and not what necessarily *will* happen. Assumption (iii) contains two implications, first that the changes in airspeed introduced by gust do not by themselves change the balance between drag and thrust and secondly that the pilot does not control the throttle in an effort to counteract the speed changes. The requirement that drag = thrust = const. is essentially satisfield—for fixed throttle position—when the aircraft flies at or near minimum-drag speed. In this context it is worth recalling that on a given aircraft, minimum drag speed increases with the square root of normal acceleration n, as can be seen from Fig. 3. This has the effect that an aircraft flying at a speed well above the $V_{D_{\min}}$ appropriate to straight level flight will operate much closer to or even below the appropriate minimum drag speed in manoeuvres with $n > 1$.

More important perhaps is the credibility of the assumption that throttle is not used in the face of major speed losses. It must be remembered that we are discussing here manoeuvering in gusty conditions and close to the ground. In such conditions the ASI will continuously fluctuate and the pilot will only react to these indications if (a) he has time to watch the indicator and (b) if he is aware that a particular indication reflects a downward trend in speed and not just a momentary fluctuation. It may not be unreasonable to assume that in such circumstances considerable time may pass before the pilot reacts with an appropriate throttle adjustment.

Nevertheless one must expect that the assumption that $D = T$ becomes dubious when prolonged manoeuvres are considered, and that the results obtained by ignoring aerodynamic effects and throttle control may then give exaggerated answers. However, the main purpose of this report is to draw attention to a significant difference in the consequence of horizontal gusts to aircraft speed deviations between rectilinear flight and turning flight and this comparison should not be affected by assumptions which affect both equally.

If we accept the assumption that thrust = drag and that the aircraft maintains level flight by suitable pilot's control, then it follows that variations in airspeed are only induced by variations in windspeed.

As we restrict the discussion here to considerations of airspeed (V_a) and hence to what is normally termed fore and aft gusts only, one would conclude that only the component of windspeed in the direction of aircraft heading (V_X) need be taken into account. Following conventional practice the differential equation describing airspeed variations in level flight is

$$\frac{dV_a}{dt} = \frac{dV_X}{dt} + \frac{T-D}{m_a}. \tag{2}$$

The assumptions we had made about thrust and drag, namely that $T = D$ removes the last term and we are left with

$$\frac{dV_a}{dt} = \frac{dV_X}{dt}. \tag{3}$$

If we apply equation (2) or, when the assumptions $(T = D) = 0$ is justified, equation (3) to rectilinear flight, there is no difficulty and the solution of these equations leads to results in broad agreement with common experience.

All that is required is a knowledge of V_X either as a function of time or of space, i.e. knowledge of the component of windspeed V_W in the direction of the fixed heading of the aircraft. We shall consider this case first before proceeding to considerations of turning flight, where we shall find that application of equation (2) can lead to difficulties.

2.3. *Rectilinear Flight through Horizontal Gusts.*

In rectilinear flight, heading ψ is constant, and if the aircraft is neutrally stable with respect to airspeed variations; equation (3) applies and we observe that airspeed varies only in response to and in direct proportion to, variations in fore and aft wind velocity V_X. Formally this result follows from integration of equation (3):

$$V_a = V_{a_0} + \int_0^t \frac{dV_X}{dt}\,dt = V_{a_0} + (V_X - V_{X_0}) \tag{4}$$

where suffix 0 denotes an initial condition at an arbitrarily chosen datum $t = 0$.

Equation (4) reflects the generally accepted observation that if an aircraft having neutral speed stability is maintained in level flight by suitable elevator control, its airspeed will vary directly as the component of turbulence in the direction of the aircraft track. A direct implication of this result is that ground speed remains constant in this situation. This assumption is in fact implicit in the formulation of equation (3). For ground speed to vary there must be some force acting on the aircraft and not just a change of wind. Changes in height, thrust or drag will of course lead to changes in ground speed and in consequence to changes in airspeed additional to those described by equation (4).

Let us now consider some simple examples:

(a) An aircraft is trimmed in level flight in still air. At 'A' in Fig. 4a it meets a tailgust ΔV_W which reduces airspeed by ΔV_W. As we had assumed that the aircraft does not respond dynamically to this change in speed and that the pilot maintains height constant by appropriate elevator control, ground speed will not change in the gust encounter. At 'B' the gust ceases and airspeed returns to its original value. The passage through the gust has not left any permanent effect on the flight condition.

This is again shown by a second example in Fig. 4b. Here the aircraft is assumed to fly in a mean head-wind. According to equation (1) ground speed V_g will be less than airspeed V_a. The tailgust at 'A' (or equivalent reduction in headwind) again reduces airspeed as in the previously discussed case, and when the gust ceases at 'B', the original airspeed is also restored.

The argument can be readily extended to flight through general random turbulence, the principal observation remains that when the wind returns to its mean value, i.e. to its 'steady' value, airspeed will also then return to its trimmed value. No permanent effect is suffered by the aircraft in its passage through a body of turbulent air and the changes in airspeed during the gust encounter are no greater than the gust velocities

This is the generally accepted picture of the effect of turbulence on airspeed. It is strictly only valid insofar as the assumption of neutral speed stability applies and if height is maintained constant; in practice the aircraft will, of course, be subject to aerodynamic effects and these will modify this very simple picture. Nevertheless even then the broad conclusion of the above analysis will still apply.

The observations made so far may appear fairly trivial. They have been spelled out in detail, to contrast them with the behaviour of the aircraft in manoeuvering flight which we shall find to be significantly different.

2.4. *Manoeuvering Flight through Horizontal Gusts.*

If an aircraft is manoeuvered in the horizontal plane, e.g. if it performs turning manoeuvres, heading ψ is no longer constant. In this case we have to enquire carefully into the meaning or indeed into the applicability of equation (2). Wind and turbulence are of course properties of the atmosphere which exist in space irrespective of the presence and direction of an aircraft penetrating it. We can define turbulence therefore in terms of spatial components, in the horizontal plane for instance we may choose an earth orientated reference system and resolve windspeed into a northerly component V_N and an easterly component V_E as illustrated in Fig. 5. We make north the datum for aircraft heading ψ. Hence the component of windspeed in the direction of heading is

$$V_X(\mathrm{t}) = V_N(\mathrm{t}) \cos \psi(\mathrm{t}) + V_E(\mathrm{t}) \sin \psi(\mathrm{t})\,. \tag{5}$$

Differentiating this expression we obtain

$$\frac{dV_X}{dt}(t) = V_N(t)\frac{d\cos\psi}{dt} + \cos\psi(t)\frac{dV_N}{dt} + V_E(t)\frac{d\sin\psi}{dt} + \sin\psi(t)\frac{dV_E}{dt}. \tag{6}$$

Let us now apply this to the case illustrated in Fig. 2, namely the aircraft performing a steady turn in a steady westerly wind, i.e. $V_E = \text{const.} < 0$. Equation (6) reduces to

$$\frac{dV_X}{dt} = V_E\frac{d\sin\psi}{dt}. \tag{7}$$

Assuming that it was possible to apply equation (3) to this case we would get by integration

$$V_a(t) = V_{a_0} + V_E \sin\psi(t). \tag{8}$$

This result would suggest that in steady turning flight in a steady wind, airspeed varies periodically with heading ψ. This is clearly false, since we know from experience that airspeed remains constant in this manoeuvre and that ground speed varies. In fact equation (2) is in effect the equation of motion relative to the ground incorporated in (3) is the assumption that ground speed is constant, and that is why its application has led to the above result. Equation (2) is therefore clearly inadequate in other than rectilinear flight. In order to derive a generally valid form of the differential equation describing the fore and aft motion of the aircraft we have to start from first principles and consider more carefully the appropriate Euler equation and be rigorous in the interpretation of the velocity components u, v and w in this equation. This derivation is presented in Appendix A. This leads to a formulation of the differential equation defining the fore and aft motion of the aircraft as

$$\frac{dV_a}{dt} = \frac{X}{m_a} + \frac{dV_X}{dt} - q(w_a - w_W) + r(v_a - v_W) \tag{9}$$

instead of equation (2). Here X is the total of the external forces (aerodynamic and gravitational) acting on the aircraft, V_X, w_W and v_W are components of wind velocity in the direction opposite to the X, Z and Y axis of the aircraft, V_a, w_a and v_a are the corresponding components of aircraft velocity in relation to the air and q and r are pitch and yaw rate respectively.

The reader will note that the definitions of the various velocities used in this analysis are chosen to agree with the usual sign conventions used in meteorology, navigation and aircraft dynamics respectively. In this combination, not normally met in aircraft stability investigations, the sign of gust components is opposite to that normally used in pure stability work.

Introducing the wind velocity components as defined in Fig. 5, and again the simplifying assumptions about the aircraft dynamics equation (9) is shown in Appendix A to lead to:

$$\frac{dV_a}{dt} = \cos\psi(t)\frac{dV_N}{dt} + \sin\psi(t)\frac{dV_E}{dt}. \tag{10}$$

Now we see that changes in airspeed can only be induced by changes in windspeed, i.e. in V_N or V_E, but *not* by changes in relative windspeed resulting merely from changes in heading ψ. We note immediately that in a co-ordinated turn in a steady wind, airspeed will remain constant and this, of course, agrees with common experience.

Equation (10), or the complete form given in equation (9), is the key to the main argument developed on the following pages. This is concerned with turns in gusty air. It will lead to results which at first sight may appear to contradict customary expectation, to results which may have serious consequences in an important area of flight safety.

In the simplified form given in equation (10) we can readily integrate to get:

$$V_a(t) = V_{a_0} + \int_0^t \cos\psi(t) \frac{dV_N}{dt}\, dt + \int_0^t \sin\psi(t) \frac{dV_E}{dt}\, dt \,. \tag{11}$$

If we specify a manoeuvre as a function $\psi(t)$, and also the time history of the two horizontal components of turbulence $V_N(t)$ and $V_E(t)$, equation (11) allows us then to calculate the variation in airspeed experienced by the aircraft during this manoeuvre.

First we shall consider a few simple examples, where the answer can in fact be deduced from generally understood principles without recourse to formal mathematical analysis. This will then be followed by considerations of flight through more complex forms of turbulence.

Fig. 6 shows an aircraft flying initially on a northerly track in still air. At 'A' it meets a tailgust ($V_N = -V_W$) and at the same time enters a turn to starboard. At B the turn is terminated and at the same time the gust ceases. The aircraft proceeds then on an easterly course in still air. The corresponding time histories of airspeed, windspeed and ground speed are also shown. We note at once that after passing through what was a shortlived gust, the aircraft has 'permanently' lost airspeed equivalent to the magnitude of the gust velocity. We also know that, had the aircraft flown on a straight course, it would have emerged from the same gust with its original speed V_{a_0}. How does this happen? The example of Fig. 6 has been deliberately chosen to permit the time history to be constructed from portions of flight in steady wind, to which we already have the answer. The reduction of V_a at the end of the initial straight segment by the tailgust is identical to that already demonstrated, e.g. in Fig. 4a. Under the assumption of neutral speed stability, which we had made, the aircraft will now proceed at this lower speed until something happens to change it again. The fact that it now enters a turning manoeuvre does of course not change this. The 90° turn takes place in a steady southerly wind and can therefore be simply visualised as a portion of the manoeuvre illustrated in Fig. 2. Common experience tells us that airspeed remains constant in this manoeuvre. When the aircraft emerges at 'B' from the gust, the cessation of the gust appears to the aircraft now as a sidegust from port. The aircraft will respond to this by weathercocking through the appropriate yaw angle until $\beta = 0$, but this will have no effect on speed, which is still $(V_a - V_W)$. It is readily seen that it clearly is irrelevant whether the aircraft now goes into straight flight or continues turning. The natural speed stability of the aircraft, perhaps assisted by the pilot increasing engine thrust, will of course eventually ensure that airspeed return to the desired trimmed condition. What is important, however, is that this does not happen simply as a result of the disappearance of the gust. It is obvious that the effect demonstrated in Fig. 6 does not depend on the precise coincidence between turn initiation or termination, and the instant at which the gust is met.

In Fig. 7 we consider another variation on the same theme, the aircraft now performing a turn through 180° during the whole of which the gust persists. On emerging from the gust, the termination of what the aircraft originally met as a tailgust is now perceived as a further tailgust with the result that the aircraft emerges with a deficiency in airspeed equal to twice the magnitude of the gust velocity V_W. The effect on the aircraft of what is clearly a single gust of extended but limited duration, is now greater than the magnitude of gust velocity itself, a result which certainly at first sight seems unexpected. It should be noted that in such a relatively prolonged manoeuvre the assumption that the pilot fails to apply corrective throttle to assist speed recovery is perhaps becoming less plausible, but still not impossible.

Another interesting case is shown in Fig. 8, where just before initiating the turning manoeuvre the aircraft meets a gust blowing in a north-westerly direction. It is quite simple to calculate the relevant speeds for this case if we remember that it is the component of windspeed in the direction of flight at any instant, that matters. We see that, when the aircraft re-enters still air after having completed the turn, airspeed has dropped by $2 \times V_W \sin 45° = 1{\cdot}41\ V_W$, i.e. by an amount significantly larger than the magnitude of the gust itself. It is readily shown that the particular case with the gust blowing at 45° to the original track is the worst possible condition arising from the coincidence of a single gust and a 90° turn.

Using the same procedure it is now possible to construct any conceivable sequence of gusts and manoeuvres. A particularly severe case of speed loss is shown in Fig. 13, assuming the worst possible combination of gust variations during a 180° turn. In this case we assume that during the first half of the manoeuvre a gust V_W blows in the northwesterly direction and veers to north east without changing its magnitude as the aircraft passes through 90°. We note that when the aircraft re-enters still air after completing this manoeuvre it has lost $4 \times V_W \times \sin 45° = 2{\cdot}83\, V_W$, i.e. speed equivalent to nearly 3 times the gust velocity. Such a result would be inconceivable within the context of conventional treatment of flight through gusts.

For clarity of the argument, we have used step gusts for our examples. In these cases it was possible to derive the answers from well understood principles without any recourse to mathematics. However, the step gust is a somewhat unreal oversimplification and to broaden the discussion the more general case of manoeuvering flight through sinusoidal turbulence is now considered.

2.5. *Turning Manoeuvres through Sinusoidal Gusts in the Horizontal Plane.*

The general formulation of the problem is shown in Fig. 10. The aircraft flies through turbulence consisting of sinusoidal variations in windspeed both in the northerly and easterly direction with frequency

$$\omega = \frac{2\pi}{\lambda} V_{a_m} \tag{12}$$

where λ is the turbulence wavelength and V_{a_m} is the average airspeed during the manoeuvre. The relative phasing of the two gust components to each other and also in relation to the aircraft turning manoeuvre is treated as arbitrary and this introduces then a probability consideration.

The aircraft itself is assumed to be initially in straight flight on a northerly heading and enters a steady turn at $t = 0$ which is terminated at $t = t_M$ when the aircraft has changed heading by $\Delta\psi = \psi_M$. The manoeuvre duration t_M is expressed as the air distance D flown along the track of the turn

$$D = t_M\, V_{a_m}\,. \tag{13}$$

During the initial straight portion of flight ($t < 0$) airspeed would fluctuate sinusodally about an initial mean speed V_{a_0}, during the turning manoeuvre there would be a general shift of this mean according to the mechanism described earlier, and when the aircraft eventually resumes straight flight at the new heading, there would be a shift of the aircraft's mean speed, upon which are again superimposed sinusoidal variations. As we are here only concerned with the difference between what happens in straight flight and in manoeuvering flight, the analysis considers only the shift ΔV_F as a result.

Mathematically the solution to this problem involves a great deal of tedious but conventional algebra, which is not here repeated. The final results are given in Fig. 11, 12 and 13.

In Fig. 11 the results are shown of the maximum speed shift (loss or gain) which an aircraft with neutral speed stability could suffer, in the absence of any action by the pilot, when performing turns through 90°, 180° and 360° heading change in the presence of sinusoidal gusts of wavelength λ and amplitude $\overline{V}_N$ or $\overline{V}_E$. The speed shifts due to northerly gusts and easterly gusts are shown separately and the results are plotted against the ratio D/λ i.e. of manoeuvre distance to gust wavelength. The values shown are the maxima possible at each relative frequency if the gust sinusoid has the worst possible phasing in the relation to the manoeuvre. Since gust phase is arbitrary there is an equal probability that it may lie anywhere within the range $-\pi < \varepsilon < +\pi$ one can then calculate the probability of the resulting speed shift to be greater than a given fraction of the maxima for each D/λ presented in Fig. 11. This probability function is shown as an insert in Fig. 11. Since there is an even chance for the aircraft to emerge with either a loss or gain in mean speed, the probability function approaches only a value of 50 per cent from either side. Fig. 11 shows that the greatest speed shift for a given gust amplitude occurs if the relative gust frequency is near 1, i.e. if the manoeuvre distance D roughly coincides with the gust wavelength λ. There are further 'resonance' conditions approximately at multiples of this frequency. Also we see that the magnitude of the possible speed shifts ΔV_F increases with the heading change ψ_M. To illustrate the progressive nature of the phenomenon we have calculated results for turning manoeuvres of up to 360° heading change, i.e. for a full circle. It should be stressed that in a manoeuvre of such duration the assumptions made about lack of aircraft response and pilot's interference become rather implausible so that the actual numerical results obtained for this case should be treated as of mere academic interest.

In Fig. 12, results are shown for the case where at each frequency both northerly gusts and easterly gusts, each in the worst possible phase, are acting simultaneously. This result is simply obtained by adding the two components shown separately in Fig. 11.

Obviously the speed shift possible in this case, which represents perhaps a more realistic gust model is now substantially greater than when each contribution is treated separately. On the other hand, since there is now a joint probability involved of phases of the two gust components to be in a particular relation to each other and to the manoeuvre, the corresponding probabilities are lower than those applicable to the results of Fig. 11. These joint probabilities have been calculated with the results shown in Fig. 13. The curves labelled $\Delta V_{F_{max}}$ are the maxima given in Fig. 12. Since this result depends on both gust components being in precisely the worst possible phase, the associated probability is strictly 0 per cent. The other curves in this figure show the amount of speed loss or gain which will be exceeded with probability P = 20 per cent, 40 per cent, and 80 per cent. Since there is equal probability of either losing or gaining speed, the probability of *losing* a given amount of speed is therefore half that noted in Fig. 13.

In a manner similar to this it would be possible to add Fourier components of a turbulence pattern made up of discrete frequency components. It should be noted however, that this process cannot be easily extended to allow manipulation by power spectral techniques. The appropriate treatment by this technique will be discussed in Section 2.6 and Appendix B.

Let us now illustrate the use of these data by an example. We consider a light aircraft flying at 125 knots mean speed performing turns through 180° at $2g$ normal acceleration. From Fig. 14 the manoeuvre distance D = 2400 ft. Gusts of 8 knots amplitude and wavelength λ = 2800 ft exist in the airspace traversed by the aircraft. Thus $D/\lambda = 0{\cdot}855$. From Fig. 16
$\Sigma\, \Delta V_{F_{max}} = 4{\cdot}5 \times 8 = 36$ knots. From Fig. 13 we obtain

P	$\dfrac{\Delta V_F}{V_G}$	V_F
20%	3·7	±29·5 knots
40%	2·05	±16·4 knots
80%	0·7	±5·6 knots

In other words of 10 turning manoeuvres flown in these conditions, 2 could result on average with the aircraft emerging with speed being at least 29·5 knots higher or lower than on entry i.e. 1 with speed at least 29·5 knots lower if no extra thrust is applied to restore airspeed. In 4 out of 10 manoeuvres, speed would change by at least 16·4 knots, or in 2 cases this amount of speed may be lost on average etc. The assumption made here that turbulence can be represented by discrete sinusoidal components is not entirely realistic but more so than the step gusts considered earlier. A still more plausible gust structure based on a power spectral model will be discussed in Section 2.6 and Appendix B.

These results and Figs. 11–13 confirm what was already apparent from the earlier consideration of manoeuvres through step gusts, namely that in turning manoeuvres through gusty air, an aircraft can experience cumulative gains or losses in airspeed, which in the most adverse circumstances can be several times greater than the magnitude of the gusts responsible for this phenomenon. This process is clearly seen to be cumulative, i.e. the larger the heading change, the greater is the possible final speed deficiency. This result must be qualified, however, by noting that the assumption made here about the response characteristics of the aircraft become more dubious for long duration manoeuvres.

It is also apparent that the spatial orientation and correlation of turbulence play an important role in this, and realistic statistical assessment of the practical significance of the problem cannot be made without more detailed information on the spatial structure of gusts, especially those experienced close to the ground where the present problem has the most important consequences.

2.6. *Turning Manoeuvres in Random Turbulence.*

In Appendix B, J. G. Jones applies statistical methods to the treatment of turning flight through horizontal turbulence. The two normal components of turbulence $V_N(t)$ and $V_E(t)$ are defined by uncorrelated power spectra having identical mean square intensities σ^2. Apart from this, the assumptions made with respect to the aircraft and to the manoeuvre are identical to those used in Section 2.5. The Appendix is self-contained and its content need not be repeated here in any detail. The relevant definitions are given in Fig. 15 and a result computed for one particular case (turbulence scale $L = 1{,}000$ ft, aircraft at an initial mean speed $V_0 = 250$ ft/sec and turning with 1·85 g normal acceleration) is shown in Fig. 16. The two dotted lines represent the contributions to the variance $\sigma_{V_a}^2$ of airspeed error produced separately by the northerly and the easterly component of turbulence. The full line is the sum of these two contributions, i.e. the variance of airspeed error due to general turbulence in the horizontal plane. It can be seen how the statistical probability of airspeed to differ from the original mean speed increases progressively as the turning manoeuvre proceeds. In straight flight, as shown by the results for $t < 0$, airspeed would simply fluctuate as the component of turbulence along the flight path, i.e. $\sigma_{V_a}^2 = \sigma^2$. If the manoeuvre were terminated at a given heading $\psi_m = \Omega\, t_M$, the variance of airspeed error would then cease to grow and stay near the level reached at that instant. It should be noted that the airspeed error computed by this statistical process is composed of two distinct components, one of which is the general fluctuation directly arising from the fluctuating turbulence, and a second one which is the cumulative airspeed change produced by the process examined in the earlier parts of this report.

Jones' solution is:

$$\frac{\sigma_{V_a}^2(\psi)}{\sigma^2} = \frac{1}{1+\left(\pi\dfrac{L}{D_\pi}\right)^2}\left[2\pi\frac{L}{D_\pi}\psi + \frac{\left\{\left(\pi\dfrac{L}{D_\pi}\right)^2-1\right\}^2}{1+\left(\pi\dfrac{L}{D_\pi}\right)^2}\right.$$

$$\left. + \frac{2\pi\dfrac{L}{D_\pi}}{1+\left(\pi\dfrac{L}{D_\pi}\right)^2} e^{-\left(\frac{\psi}{\pi\frac{L}{D_\pi}}\right)} \left\{2\pi\frac{L}{D_\pi}\cos\psi \left[\left(\pi\frac{L}{D_\pi}\right)^2-1\right]\sin\psi\right\}\right]$$

where $\sigma_{V_a}(\psi)$ = rms airspeed error when the aircraft has turned through ψ

σ = rms gust velocity of the horizontal turbulence

L = scale length of turbulence

D_π = distance travelled by aircraft along the arc of a turn through 180°

ψ = heading change during turn in relation to initial straight flight direction.

This expression has been evaluated for a representative range of the relevant parameters with the result shown in Fig. 18. It is seen that within this range the ratio L/D does not materially affect the answer and we get a result which has fairly general validity. We see that when the aircraft has turned through 180°, the rms airspeed change is approximately $1\frac{3}{4}$ times the speed variation directly attributable to turbulence, and after a complete turn this ratio is approximately $2\frac{1}{4}$.

These results again confirm the general observation made earlier on the basis of less sophisticated analysis.

3. *Discussion:*

In straight flight through turbulence, gusts are known to lead to fluctuations in airspeed but not, if the aircraft is stable, to a divergence in flight conditions. On the other hand if the aircraft performs turning manoeuvres, shortlived gusts or general random turbulence may lead to quite severe speed deviations which tend to increase during the turn.

Eventually, of course, the natural stability of the aircraft and pilot's intervention will restore the original flight condition. The effects discussed here will therefore be significant mainly in turning manoeuvres, which are completed in a time which is short by comparison to, or at least of the same order as the time constant of the aircraft's relevant speed stability mode, and the time for pilot's reaction and throttle adjustments to take effect. To put this problem into perspective, the distance travelled by an aircraft in completing a 180° turn is plotted in Fig. 14 against speed and applied normal acceleration. It is seen that certainly at the lower speeds, these manoeuvres require relatively little time, so that the assumptions made in the simple analysis here of ignoring natural aircraft speed recovery are not unjustified.

If an aircraft is performing turning manoeuvres of this kind at low speeds in gusty weather, it is therefore exposed to the danger of larger and more persistent speed changes than would be predicted by a conventional treatment of gusts. This phenomenon can have serious consequences if the aircraft is flown at a speed allowing only a modest stall margin, because it might find itself then for a fairly long period of time in a flight condition where this stall margin is substantially reduced by a loss of speed and might subsequently stall due to encounters with further gusts (especially vertical gusts) and/or further manoeuvres.

There are perhaps three types of flight operation where a proper recognition of this hazard is essential for the formulation of prudent safety requirements.

First the military aircraft engaged in low level flying. With the current emphasis on low level penetration as a defence against radar detection, a number of aircraft are designated for this role and hence for intense low level practice, for which they were not originally designed and stressed. As a result, fatigue considerations tend to dominate the flying rules and one finds these aircraft being operated in rough weather at speeds below the level advisable in rough air. This means in practice that instead of operating in a condition where the chances of gust-induced stalls and gust-induced structural failure are equal, these aircraft are now flying in a condition where the risk of stalling is much greater (and perhaps unacceptably so) than that of breaking up under gust loads. An aircraft already prejudiced in its flight safety by such practice is of course made even more vulnerable to a stall if the mechanism described above is exposing it to the possibility of inadvertent speed losses potentially much greater than conventional gust-response theory would suggest. Some protection would be obtained by restricting manoeuvres in rough air, but the better policy would be to maintain a rough airspeed properly reflecting flight safety, accepting the effects on the fatigue life of the aircraft, or minimising them by restricting this type of flying where possible, to modest turbulence only.

Another form of flying to which the above results seem relevant is display flying of the low speed variety. This involves essentially the type of turning manoeuvre which we had found to put the aircraft at special risk. In calm weather this is safe in the hands of a skilled pilot, but he may not appreciate fully the allowances required to cope with the consequences of the coincidence of manoeuvres and gusts in gusty weather conditions.

Another form of flight operation involving frequent manoeuvering close to the ground is crop spraying and allied commercial activities. In this class of aircraft, the risks seem even greater than for the military aircraft because of their inherently lower speeds, which means that with these the absolute speed margin is therefore of greater relative consequence. Crop spraying and much survey work furthermore requires a meandering flight pattern with frequent and sharp 180° turns. This clearly means that for an aircraft engaged in such activity the odds of experiencing the hazard described in this report must be greatly increased. It is understood that stalls during these 'procedure turns' are a common cause of disaster for this class of aircraft.

4. *Recording of Gust Velocities in Flight.*

The mechanism governing gust response in manoeuvering flight discussed in the foregoing analysis has also some interesting repercussions on the recording of turbulence data in flight. If conventional procedures are used for data reduction, an aircraft experiencing the gust encounter illustrated in Fig. 11 would record this as a succession of two tailgusts of magnitude V_W, and it would appear to the user of this data that the atmosphere had undergone a permanent shift in mean speed. This is certainly not what Fig. 10 represents. This result would, incidentally also be obtained if the aircraft responds to drag changes during the manoeuvre and these are allowed for in the analysis in the conventional manner.

The same problem arises also with respect to the other horizontal component of turbulence, the sidegust in the direction at right angles to the flight path. Analysing the gust encounter of Fig. 10 for instance, it is readily seen that this would appear as a step gust at B again appearing as an apparently permanent shift of lateral wind velocity, whereas in reality the aircraft has only passed through a gust of finite duration.

Gust data are of course not normally recorded in flight involving sharp turns and the problem posed here may not have much general significance. Nevertheless, it is important for those engaged in such work to be aware of this particular effect and recognise it when and where it is relevant.

The proper equation to be used to avoid such errors in the analysis is equation (A–20) of Appendix A.

5. *Conclusions.*

It has been shown that when performing turning manoeuvres in turbulent air, it is possible for an aircraft to suffer quasi-permanent losses (or gains) in airspeed which persist even when the turbulence has ceased. Moreover, these speed changes can be substantially larger than the magnitude of the gust velocities responsible for their generation. Neither of these effects occur in rectilinear flight.

This mechanism does not appear to have been previously appreciated and requires to be seriously considered in the formulation of airworthiness requirements, and in the conduct of flying involving manoeuvering at low altitude and low airspeed.

The same phenomenon could also lead to physically misleading results when gust data are derived from flight records taken in flight involving turning manoeuvres.

Equations are presented which permit rigorous analysis of manoeuvering flight in turbulent air. These are equally valid for aircraft response calculations and for the analysis of gust data obtained in flight.

LIST OF SYMBOLS

D	Air distance traversed in turns
D_π	Air distance traversed in 180° turn
g	Gravitational acceleration
L	Lift
m_a	Aircraft mass
P	Probability
p	Rate of roll
q	Rate of pitch
R	Radius of turn or rolling moment
t	Time
t_M	Time to complete turning manoeuvre
V	Speed
V_a	Airspeed, i.e. speed of aircraft in relation to air
V_W	Wind speed
V_N	Northerly component of wind speed
V_E	Easterly component of wind speed
V_X	Component of wind speed tangential to aircraft track
V_g	Ground speed, i.e. speed of aircraft in relation to ground
V_0	Initial speed
ΔV_{a_F}	Increment in mean airspeed experienced during turning manoeuvre
$\left.\begin{array}{l}\bar{V}_N\\ \bar{V}_E\\ \bar{V}_G\end{array}\right\}$	Amplitudes of sinusoidal gusts components
v	Sideslip velocity
v_a	Sideslip velocity of aircraft in relation to steady air
v_W	Component of wind velocity in the direction of the negative y-axis of aircraft
w	Vertical velocity
w_a	Vertical velocity of aircraft in relation to steady air
w_W	Component of wind velocity in the direction of the (negative) z-axis of aircraft
ψ	Aircraft heading
ψ_M	Total heading change in turning manoeuvre
Ω	Spatial gust frequency (ft/rad)
λ	Gust wavelength (ft)
ω	Gust frequency (rad/sec)
σ	rms
σ_{V_a}	rms airspeed variability
σ_G	rms gust velocity

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APPENDIX A

The Euler Equation Describing the Fore and Aft Motion of Aircraft in Turning Flight Through Turbulence.

We are concerned here only with the fore and aft motion of the aircraft and hence only the X-force equation is required. This conventionally reads[2]

$$\frac{X}{m_a} = \dot{u}+q\,w+r\,v \tag{A.1}$$

this equation is framed with reference to an earth fixed reference system and in consequence all the variables in it are referred to earth axes. In particular this applies to u, v and w which are aircraft velocities with respect to ground. These quantities are related to the corresponding velocities referred to air by the relationships

$$\dot{u} = \frac{dV_a}{dt}-\frac{dV_X}{dt} \tag{A.2}$$

$$w = w_a - w_W\,. \tag{A.3}$$

$$v = v_a - w_W \tag{A.4}$$

where V_X, w_W and v_W are the components of wind velocity in the direction opposite to the X, Z and Y axis of the aircraft.

The definition used here for the wind velocities conforms with usual practice in flight dynamics analysis, where a positive wind of gust velocity is considered to be additive to the corresponding aircraft velocity component.

The velocities V_a, w_a and v_a are the components of the aircraft motion in relation to air. We note that the relationship expressed in equation (A.2) is that we had used earlier in equation (2).

The complete Euler equation for the aircraft flying in a non-stationary atmosphere is then

$$\frac{X}{m_a} = \frac{dV_a}{dt}-\frac{dV_X}{dt}+q\,(w_a-w_W)-r\,(v_a-v_W) \tag{A.5}$$

which we rearranged into the usual form of a differential equation:

$$\frac{dV_a}{dt} = \frac{X}{m_a}+\frac{dV_X}{dt}-q\,(w_a-w_W)+r\,(v_a-v_W)\,. \tag{A.6}$$

This equation is of unrestricted validity and the proper form to use for any work in which spatial orientation of winds is to be properly accounted for. Similar expressions can be readily derived for the remaining five Euler equations.

We return now to a more restricted problem and reintroduce the simplifying assumptions made in the main test. These are that $X = 0$ and that sideslip, i.e. $v_a = 0$. Strictly, we cannot eliminate w_a, i.e. incidence from the analysis, even with the assumptions made, because these (for instance that the pilot maintains constant height by appropriate elevator control) imply controlled changes in w_a i.e. incidence. It can be readily shown, however, that the $(q\,w_a)$ terms does not materially affect the main theme of this investigation and we shall therefore ignore it for the present. Hence equation (A.6) reduces to

$$\frac{dV_a}{dt} = \frac{dV_X}{dt} + q\,w_W - r\,v_W\,. \tag{A.7}$$

Now we have to relate the variables contained in equation (A.7) to the kinematic relationship involved in turning flight and introduce the appropriate wind components.

If $\dot{\psi}$ is the turning rate, and the turn is properly coordinated ($\beta = 0$) we can resolve this into the aircraft orientated components q and r according to Fig. 19a as:

$$q = \dot{\psi} \sin\phi \tag{A.8}$$

$$r = \dot{\psi} \cos\phi\,. \tag{A.9}$$

To obtain the wind velocity components v_W and w_W we have to find first a relationship between these and the radial component V_R of the horizontal wind. From Fig. 19b

$$v_W = V_R \cos\phi \tag{A.10}$$

$$w_W = -V_R \sin\phi\,. \tag{A.11}$$

Now we form the products ($q\,w_W$) and ($r\,v_W$) and add them in the way in which they appear in equation (A.7)

$$q\,w_W - r\,v_W = -V_R\,\dot{\psi} \sin^2\phi - V_R\,\dot{\psi}\cos^2\phi \tag{A.12}$$

which reduces to

$$q\,w_W - r\,v_W = -V_R\,\dot{\psi}\,. \tag{A.13}$$

Now we require a relationship between the earth orientated wind velocity components V_N and V_E and the aircraft orientated components V_R and V_X. From Fig. 5 these are obtained as:

$$V_R = -V_N \sin\psi + V_E \cos\psi \tag{A.14}$$

and

$$V_X = V_N \cos\psi + V_E \sin\psi\,. \tag{A.15}$$

Differentiating equation (A.15) gives

$$\frac{dV_X}{dt} = V_N \frac{d\cos\psi}{dt} + \cos\psi\,\frac{dV_N}{dt} + V_E \frac{d\sin\psi}{dt} + \sin\psi\,\frac{dV_E}{dt} \tag{A.16}$$

and this gives finally:

$$\frac{dV_X}{dt} = -V_N\,\dot{\psi}\sin\psi + \cos\psi\,\frac{dV_N}{dt} + V_E\,\dot{\psi}\cos\psi + \sin\psi\,\frac{dV_E}{dt}\,. \tag{A.17}$$

Substituting equations (A.13), (A.14) and (A.17) into the simplified Euler equation (A.7) we obtain:

$$\frac{dV_a}{dt} = -V_N\,\dot{\psi}\sin\psi + \cos\psi\,\frac{dV_N}{dt} + V_E\,\dot{\psi}\cos\psi + \sin\psi\,\frac{dV_E}{dt} + V_N\,\dot{\psi}\sin\psi - V_E\,\dot{\psi}\cos\psi \tag{A.18}$$

or:

$$\frac{dV_a}{dt} = \cos\psi\,\frac{dV_N}{dt} + \sin\psi\,\frac{dV_E}{dt}\,. \tag{A.19}$$

Equation (A.19) is of course only valid within the specific assumptions made here with respect to aircraft stability and piloting technique. If we introduce again the additional terms contained in the full Euler equations we get the general form

$$\frac{dV_a}{dt} = \frac{X}{m_a} + \cos\psi\,\frac{dV_N}{dt} + \sin\psi\,\frac{dV_E}{dt} - q\,w_a + r\,v_a\,. \tag{A.20}$$

APPENDIX B

Effect of a Turning Manoeuvre on Airspeed Changes of a Neutrally Speed-Stable Aircraft in a Field of Random Atmospheric Turbulence.

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B.1. *Introduction.*

In this Appendix we consider the problem in terms of a frequently used stationary Gaussian random process representation of the turbulence. The manoeuvre is as illustrated in Fig. 15. For $t < 0$ the aircraft flies in a straight line in a northerly direction. At $t = 0$ it commences a circular turn to the east with angular velocity Ω rad sec^{-1}. Using the nomenclature of Fig. 8 the gust components V_N and V_E are represented, to simplify the analysis, by uncorrelated processes, each with mean square intensity σ^2. The lack of correlation between V_N and V_E in the model is not a realistic assumption, but it will not affect the predicted trends in the particular problem under consideration. We denote by $V_N(t)$ and $V_E(t)$ the turbulence velocity components acting at the position of the aircraft at time t. For flight in a straight line Taylor's theorem is usually invoked to relate the variation of gust velocity in space with the variation of gust velocity, in time, as experienced by the aircraft. In the situation illustrated in Fig. 15 the relationship between $V_N(t)$ and $V_E(t)$ and the random turbulence field in space is more complex than in the case of flight in a straight line; for instance, for a complete turn of 360° we return to the spatial starting point. However, if we restrict attention to the case of turns of not greater than 180°, with moderately large radius of curvature, it is plausible to describe $V_N(t)$ and $V_E(t)$ statistically in the same way as in flight in a straight line. Thus we can take the power spectrum of the (uncorrelated) stationary random processes $V_N(t)$ and $V_E(t)$ to be

$$\Phi(\omega) = \sigma^2 \frac{T}{\pi} \frac{1}{1+(T\omega)^2} , \tag{B.1}$$

where T is the time taken for the aircraft to traverse a distance equal to the scale length of the turbulence, and ω is frequency in rad sec^{-1}. Φ then has the dimension of (velocity)^2 per rad sec^{-1}.

We take the airspeed changes of the aircraft to be given by equation (10):

$$\frac{dV_a}{dt} = \frac{dV_N}{dt}\cos\psi + \frac{dV_E}{dt}\sin\psi . \tag{10}$$

For the sake of clarity we recapitulate the assumptions for which we understand equation (10) to be valid. We assume that

(i) The controlled aircraft is neutrally speed stable, both in straight flight and during the turn, as far as airspeed changes due to turbulence are concerned.

(ii) No additional airspeed changes are introduced due to the turn itself (it being assumed that the pilot pilot makes an appropriate thrust change).

The consequences of these assumptions are as follows. In the straight flight part of the manoeuvre the fluctuations in V_a are identical with $V_N(t)$. V_a thus fluctuates about its mean value with mean square intensity σ^2. If $V_N(t)$ increases, and then decreases again, V_a will increase, and then decrease again. In the turning flight path part of the manoeuvre, however, the changes in V_a due to $V_N(t)$ not only fluctuate with $V_N(t)$, but in addition acquire a random 'drift', or low frequency component. For instance, if $V_N(t)$ increases in the region of $\psi = 0$ and then decreases again in the region $\psi = \pi/2$, V_a will have a corresponding increase near $\psi = 0$ but not a corresponding decrease near $\psi = \pi/2$, as $V_N(t)$ is a side-gust in this region. There is thus, in turning flight, no tendency for any particular realisation of $V_a(t)$ to fluctuate about the original mean.

We will treat the problem statistically in terms of the amplitude probability distribution of airspeed as a function of time. Since V_N and V_E are assumed to be Gaussian processes, it follows from equation (10) that the amplitude distribution of V_a, for any value of t, is Gaussian. It is thus defined by its variance, $\sigma^2_{V_a}$. In the straight flight part of the manoeuvre we have

$$\sigma_{V_a}^2(t) = \sigma_{V_N}^2 = \sigma^2. \tag{B.2}$$

In the turning part of the manoeuvre the random 'drift' component causes $\sigma_{V_a}^2(t)$ to increase with time. Note that the 'fluctuations' in V_a do not increase in intensity. It is the addition of the random low frequency 'drift' that causes the variance of the amplitude probability distribution of V_a to increase with time.

B.2. *Effect of* $V_N(t)$.

In this section we evaluate the time variation of $\sigma_{V_a}^2$ due to $V_N(t)$.

For the manoeuvre illustrated in Fig. 22 we have, from equation (10),

$$\frac{dV_a}{dt} = \frac{dV_N}{dt} F(t), \tag{B.3}$$

where

$$F(t) = \begin{cases} 1 & , \quad t < 0 \\ \cos \Omega t, & t \geqslant 0. \end{cases} \tag{B.4}$$

In order to simplify the resulting equations, we introduce the definitions:

$$x(t) = \frac{dV_a}{dt} \tag{B.5}$$

$$y(t) = V_a(t) \tag{B.6}$$

$$N(t) = \frac{dV_N}{dt}. \tag{B.7}$$

Then from equation (B.3)

$$x(t) = F(t)\ N(t). \tag{B.8}$$

Thus the (non-stationary) auto-correlation function of $x(t)$ can be written in terms of 'expected value' E:

$$\begin{aligned} R_{xx}(t_1, t_2) &= E\{x(t_1)\ x(t_2)\} \\ &= E\{F(t_1)\ N(t_1)\ F(t_2)\ N(t_2)\} \\ &= F(t_1)\ F(t_2)\ \{N(t_1)\ N(t_2)\} \\ &= F(t_1)\ F(t_2)\ R_{NN}(t_1, t_2). \end{aligned} \tag{B.9}$$

From equation (B.7) and the assumed spectral form, equation (B.1), it can be shown that

$$R_{NN}(t_1, t_2) = \frac{2\sigma^2}{T} \left\{ \delta(t_1 - t_2) - \frac{1}{2T} e^{-\frac{|t_1 - t_2|}{T}} \right\}. \tag{B.10}$$

Further, from equation (B.5) and (B.6):

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$= \int_{0}^{\infty} x(t-\tau)\, d\tau\,. \tag{B.11}$$

Thus we have for the cross-correlation

$$R_{yx}(t_1, t_2) = E\{y(t_1)\; x(t_2)\}$$

$$= E\; \left\{ x(t_2) \int_{0}^{\infty} x(t_1-\tau) \right\}\; d\tau$$

$$= \int_{0}^{\infty} E\{x(t_2)\; x(t_1-\tau)\}\, d\tau$$

$$= \int_{0}^{\infty} R_{xx}(t_1-\tau, t_2)\, d\tau\;\;. \tag{B.12}$$

Similarly, it can be shown that

$$R_{yy}(t_1, t_2) = \int_{0}^{\infty} R_{yx}(t_1, t_2-\tau)\, d\tau\,. \tag{B.13}$$

By symmetry, the mean of the distribution of the changes in V_a is equal to zero. Thus the variance of the distribution of the changes in V_a, due to the effect of V_N, is

$$\sigma^2_{V_a}(t) = E\{y^2(t)\}$$

$$= R_{yy}(t, t)$$

$$= \int_{0}^{\infty} R_{yx}(t, t-\tau)\, d\tau$$

(from equation (B.13))

$$= \int_{0}^{\infty} \left\{ \int_{0}^{\infty} R_{xx}(t-\tau', t-\tau)\, d\tau' \right\}\; d\tau$$

(from equation (B.12))

$$= \int_0^\infty \left\{ \int_0^\infty F(t-\tau')\; F(t-\tau)\; R_{NN}(t-\tau', t-\tau)\, d\tau' \right\} \; d\tau \tag{B.14}$$

(from equation (B.9)).

The required variance can now be obtained by substituting from equation (B.10) and evaluating the double integral in equation (B.14). The final result is as follows:

For $t < 0$,

$$(\sigma^2_{V_a}(t))_N = \sigma^2$$

for $t \geqslant 0$,

$$(\sigma^2_{V_a}(t))_N = \frac{\sigma^2}{1+\Omega^2 T^2}\left[\Omega T \sin \Omega t \cos \Omega t + \Omega^2 t T + \frac{1}{1+\Omega^2 T^2}(\cos^2 \Omega t - \Omega^2 T^2 \sin^2 \Omega t) \right.$$
$$\left. + \frac{2\Omega^2 T^2 e^{-t/T}}{1+\Omega^2 T^2}(\cos \Omega t - \Omega T \sin \Omega t) + \frac{\Omega^4 T^4}{1+\Omega^2 T^2} \right]. \tag{B.15}$$

B.3. *Effect of $V_E(t)$.*

The analysis for the variance of the amplitude distribution of changes in V_a due to the easterly component of turbulence $V_E(t)$ is exactly analogous. Clearly, for the initial straight flight path of the manoeuvre there is no effect of V_E on V_a. The full result is as follows:

for $t < 0$,

$$(\sigma^2_{V_a}(t))_E = 0$$

for $t \geqslant 0$

$$(\sigma^2_{V_a}(t))_E = \frac{\sigma^2}{1+\Omega^2 T^2}\left[-\Omega T \sin \Omega t \cos \Omega t + \Omega^2 t T + \frac{1}{1+\Omega^2 T^2}(\sin^2 \Omega t - \Omega^2 T \cos^2 \Omega t) \right.$$
$$\left. + \frac{2\Omega T e^{-t/T}}{1+\Omega^2 T^2}(\sin \Omega t + \Omega T \cos \Omega t) - \frac{\Omega^2 T^2}{1+\Omega^2 T^2} \right]. \tag{B.16}$$

B.4. *Effect of Summing Components.*

Since V_N and V_E are assumed to be uncorrelated the total $\sigma^2_{V_a}$ is obtained by adding the components $(\sigma^2_{V_a})_N$ and $(\sigma^2_{V_a})_E$. Equations (B.15) and (B.16) thus give:

for $t < 0$,

$$\sigma^2_{V_a}(t) = \sigma^2$$

for $t \geqslant 0$,

$$\sigma_{V_a}^2(t) = \frac{\sigma^2}{1+\Omega^2 T^2}\left[2\,\Omega^2\, t\, T + \frac{(\Omega^2 T^2 - 1)}{1+\Omega^2 T^2} + \frac{2\,\Omega\, T e^{-t/}}{1+\Omega^2 T^2}\{2\,\Omega\, T\cos\Omega\, t - (\Omega^2 T^2 - 1)\sin\Omega\, t\}\right]. \quad \text{(B.17)}$$

For large t the oscillatory terms become negligible and $\sigma_{V_a}^2(t)$ increases linearly with t (Fig. 16).

B.5. *Illustrative Examples.*

As an example we consider the case of an aircraft flying with an unperturbed airspeed of 250 ft sec^{-1}. We suppose the scale length of turbulence to be $L = 1{,}000$ ft, and the circumference of the turning circle to be $8L = 8{,}000$ ft. Then $T = 4$ sec, and $\Omega = \pi/16$ rad sec^{-1}. The time taken to traverse a semi-circle, $\psi = \pi$, is 16 sec. Substituting these values into equations (B.15) to (B.17), gives the results illustrated in Fig. 16.

In order to clarify the significance of the variance $\sigma_{V_a}^2$ we have illustrated a typical time variation of V_a due to V_N in Fig. 17. For $t < 0$ the airspeed V_a fluctuates about its initial mean value with mean square variation σ^2. Thus for $t < 0$, due to V_N we have $(\sigma_{V_a}^2)_N = \sigma^2$ (Fig. 16). For $t \geqslant 0$, dV_a/dt is given by equation (10). As ψ approaches $\pi/2$, dV_a/dt tends to zero and the fluctuation in V_a decreases in magnitude. Thus the value of the variance $(\sigma_{V_a}^2)_N$ at $\pi/2$ (point A in Fig. 16) is simply a measure of the spread of 'random offsets' illustrated in Fig. 21. As ψ increases beyond $\pi/2$ the fluctuations begin to grow again (Fig. 17) and the variance $(\sigma_{V_a}^2)_N$ increases rapidly.

The variation of V_a due to V_E can be understood in a similar manner. For $t < 0$ there is no variation of V_a due to V_E and thus $(\sigma_{V_a}^2)_E = 0$. For $t \geqslant 0$, dV_a/dt is given by equation (10) and thus, as ψ approaches $\pi/2$, the fluctuations in V_a due to V_E grow in magnitude. As ψ increases beyond $\pi/2$ the fluctuations decrease again until at $\psi = \pi$ (point B in Fig. 16) the value of the variance $(\sigma_{V_a})_E$ is again a measure of the spread of 'random effects'.

As can be seen from Fig. 18 the overall effect is a monotonic increase of $\sigma_{V_a}^2$ with t, growing from a value of σ^2 at $t = 0$, through a value of $2\sigma^2$ at approximately 120°, to a value of just over $3\sigma^2$ at $\psi = \pi$. Again, it is emphasised that this increase in variance does not indicate any increase in the size of the 'fluctuations' but the addition of a low frequency 'drift', which has been expressed in terms of 'random offsets', not present in straight flight.

Thus, whereas in straight flight the absence of corrections to airspeed changes due to random turbulence can lead to no more than random changes in airspeed with variance equal to the mean square value of the turbulence fluctuations, in turning flight changes in airspeed due to turbulence, if not corrected, can grow to much larger values.

B.6. *Final Remarks.*

It should be noted that the effect described here is only significant if the distance traversed by the aircraft during the manoeuvre is at least of the order of the turbulence scale length. This is due to a property of the power spectrum (B.1) which is, that only over distances of this order does the occurrence of a change of wind velocity in one direction significantly increase the probability of a subsequent change in the opposite direction. Over distances small compared with the scale length a fluctuation causing an increase of airspeed, for example, is just as likely to be followed by another in the same sense as by one causing a decrease of airspeed. Under these conditions the airspeed fluctuations in straight flight and in turning flight would be identical.

Measurements of horizontal turbulence at low altitude (up to 1,000 ft) indicate a scale length of the order of 1,000 ft, but a power spectrum shape (more low frequency energy) which would lead to rather smaller manoeuvre effects than predicted by the turbulence mode, equation (B.1).

A thorough statistical assessment of the practical significance of the effect of turning manoeuvres on airspeed fluctuations due to gusts thus depends upon the acquisition of more extensive knowledge of the scale lengths of horizontal turbulence, and the related joint probabilities of gusts in opposite directions.

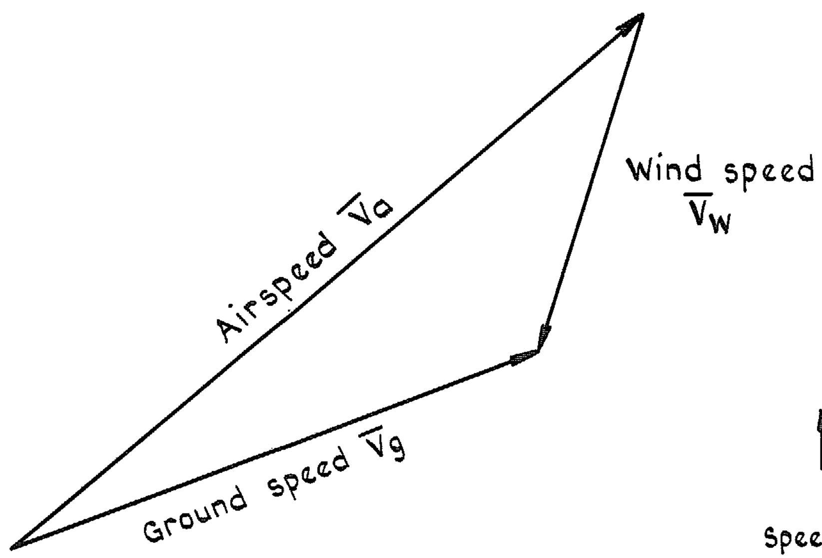


FIG. 1. Relationship between airspeed, wind speed and ground speed.

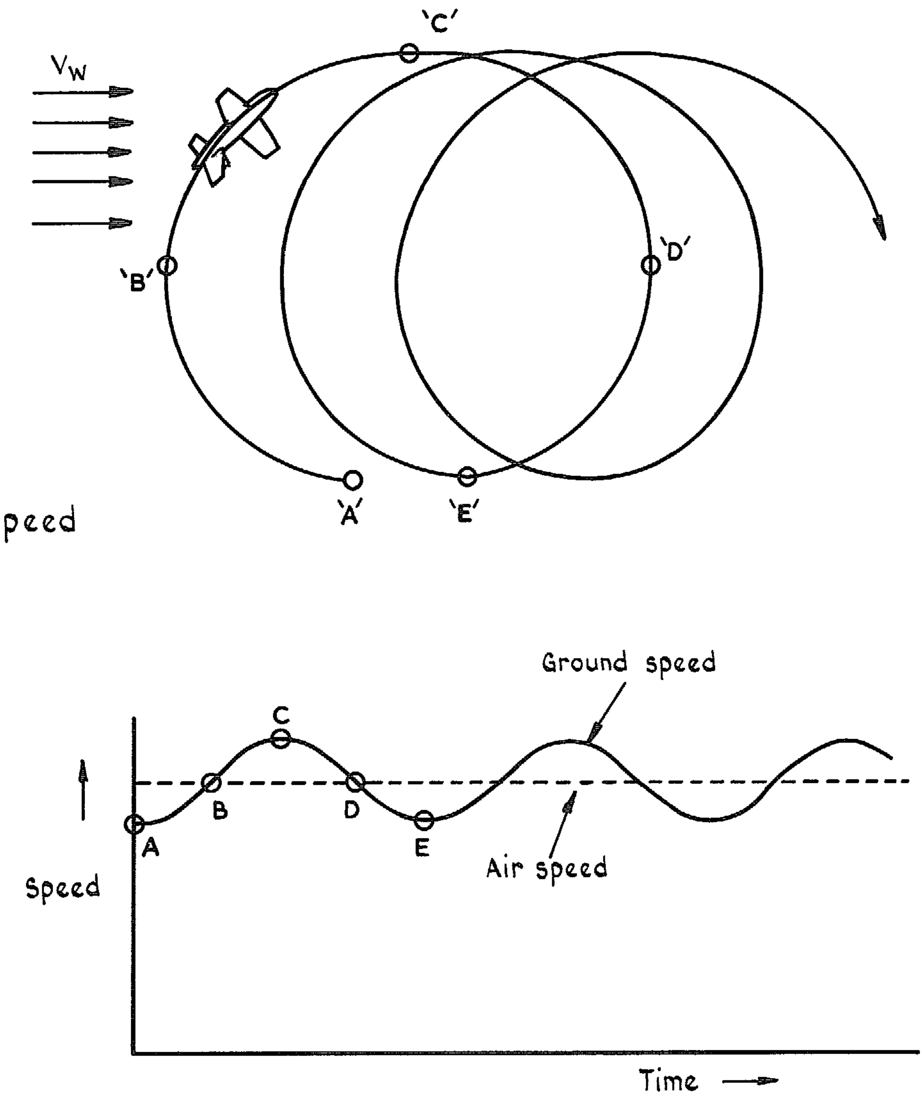


FIG. 2. Ground track and time history of ground speed of an aircraft in steady turning flight in a steady wind V_W.

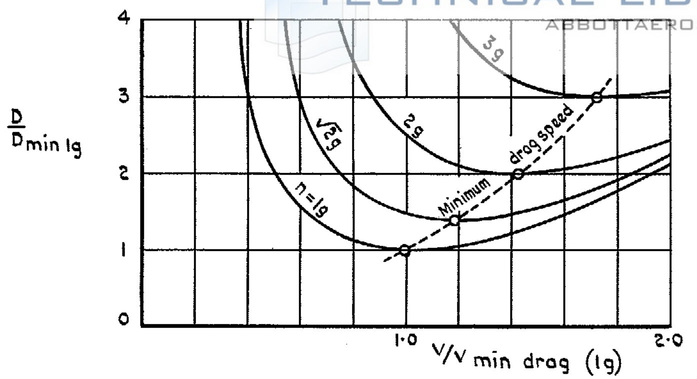
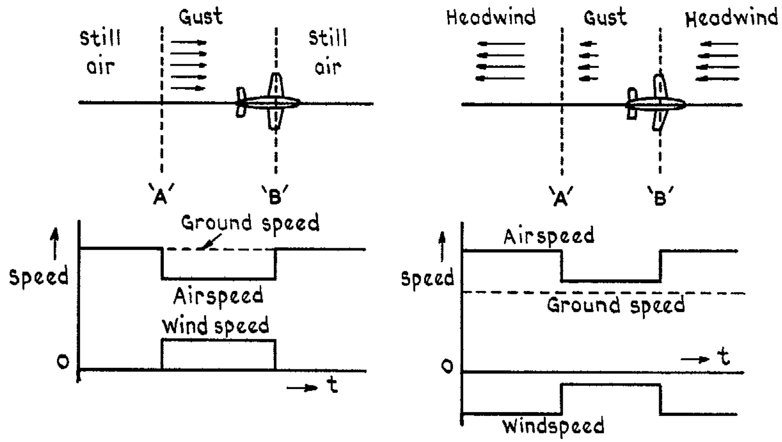


FIG. 3. Change of level-flight drag with speed and normal acceleration in turns.



(a) In still air

(b) In headwind

FIG. 4a & b. Rectilinear flight through tail gust of aircraft with neutral speed stability and elevator controlling height constant.

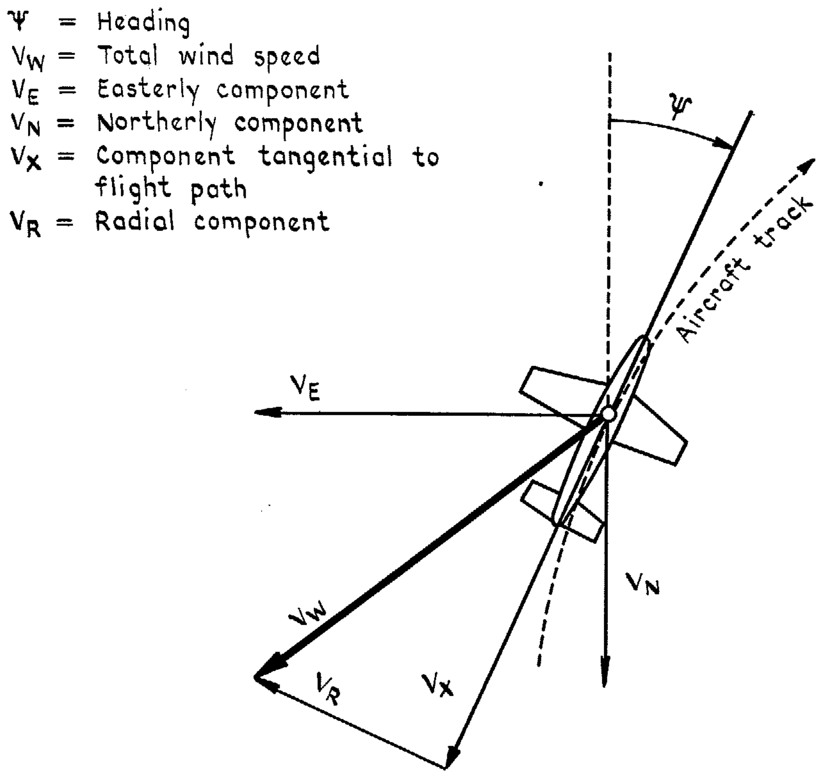


FIG. 5. Resolution of wind velocity into earth orientated components and aircraft orientated components.

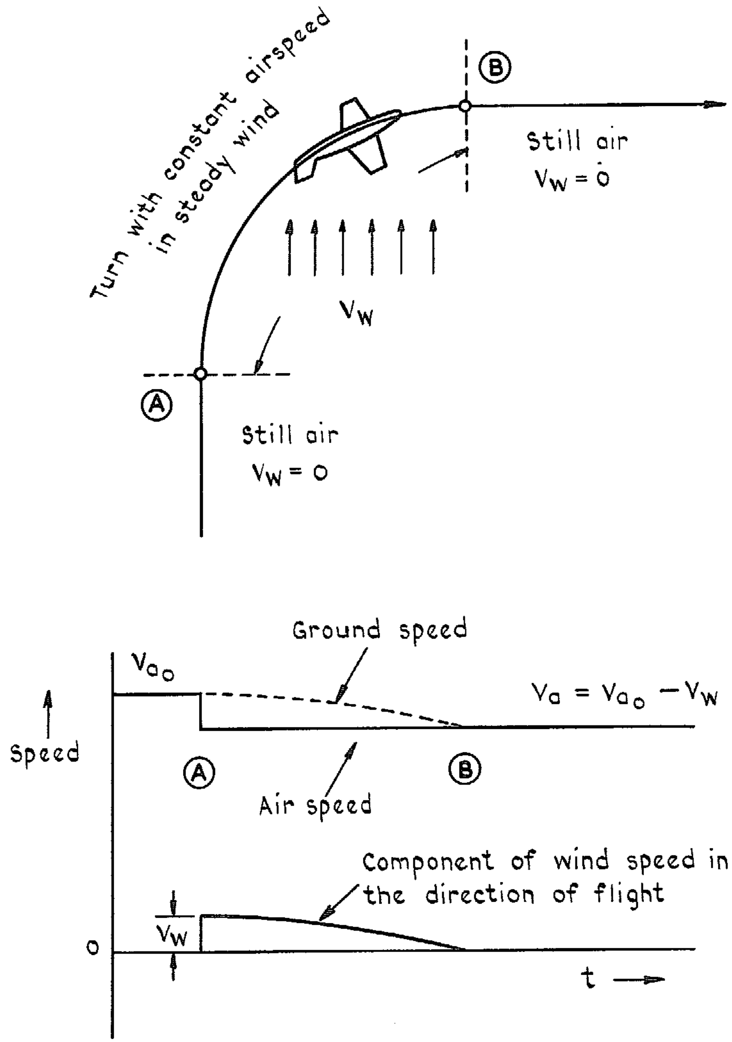


FIG. 6. Change of airspeed and groundspe md in a 90° turn coinciding with a gust blowing in the direction of the initial track.

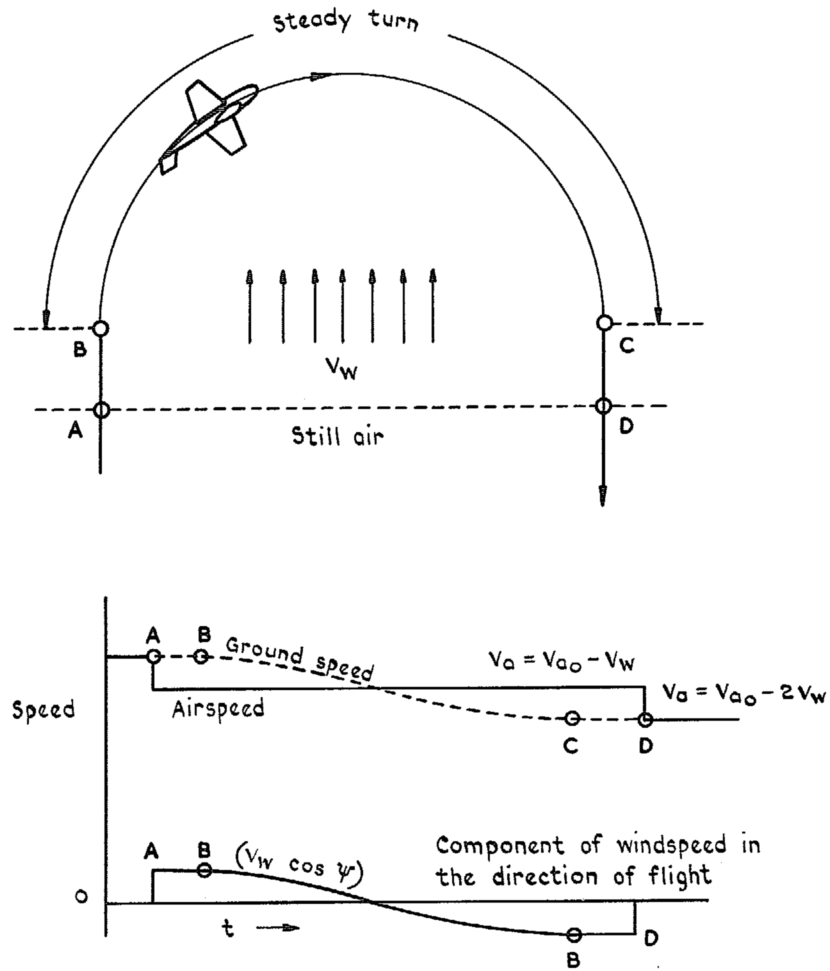


FIG. 7. 180° turn in gust persisting throughout manoevre.

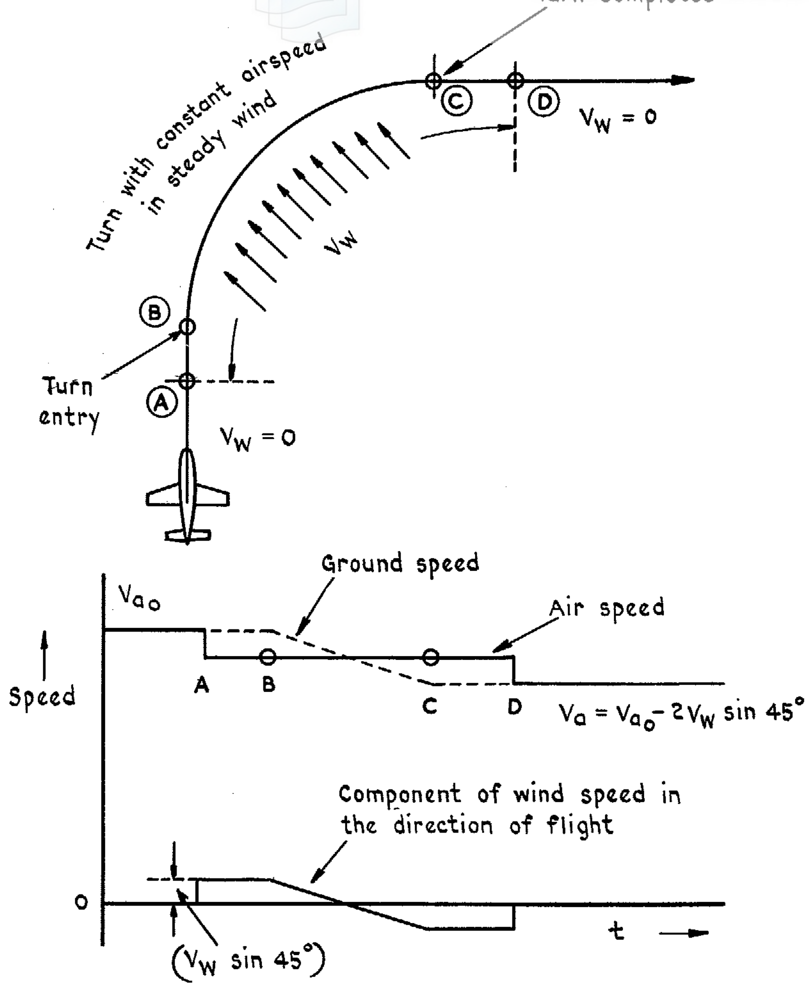


FIG. 8. Change of airspeed and groundspeed in a 90° turn coinciding with a gust blowing in a particularly unfavourable direction.

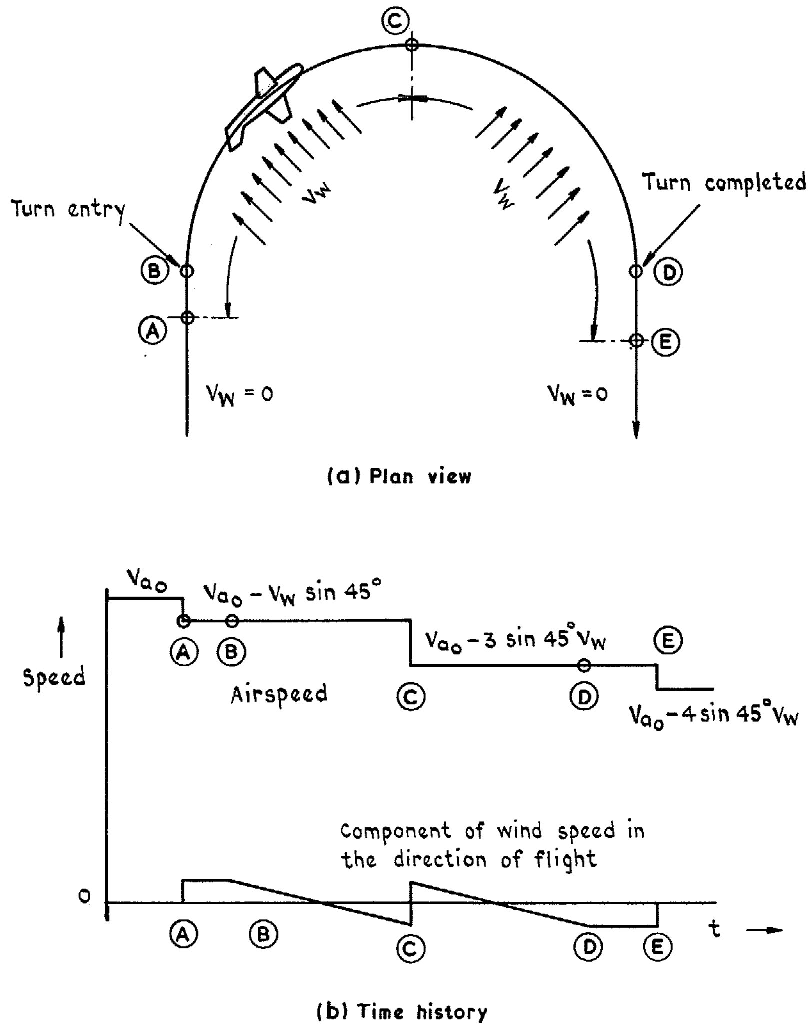


FIG. 9a & b. Turn through 180° coinciding with particularly unfavourable gust sequence.

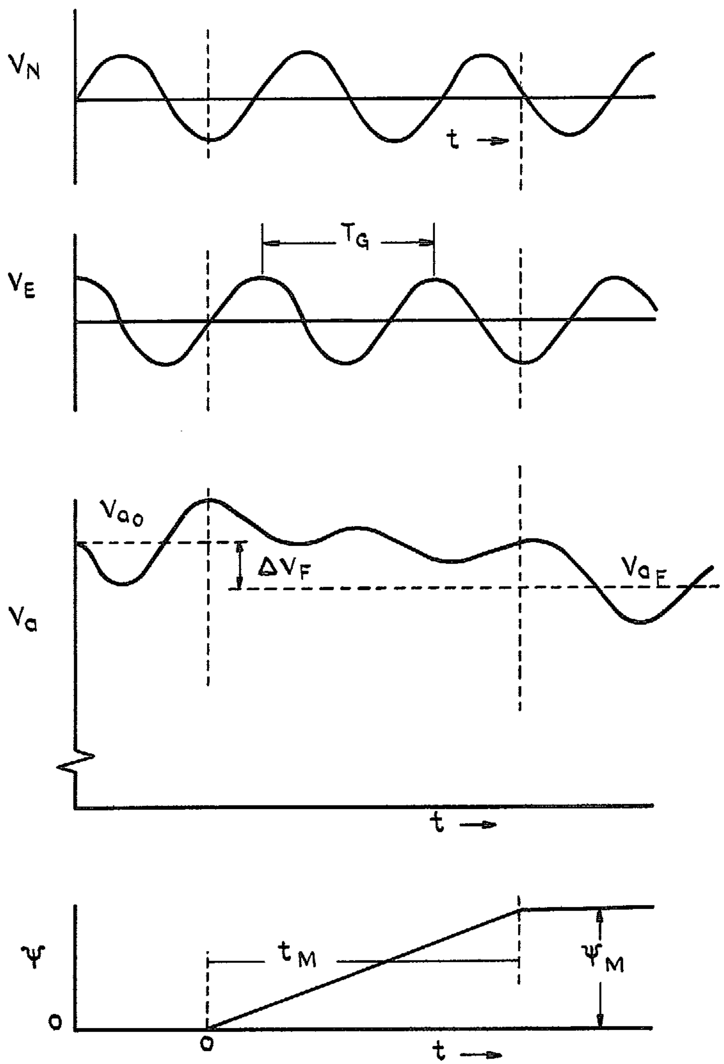


FIG. 10. Turning flight through sinusoidal turbulence.

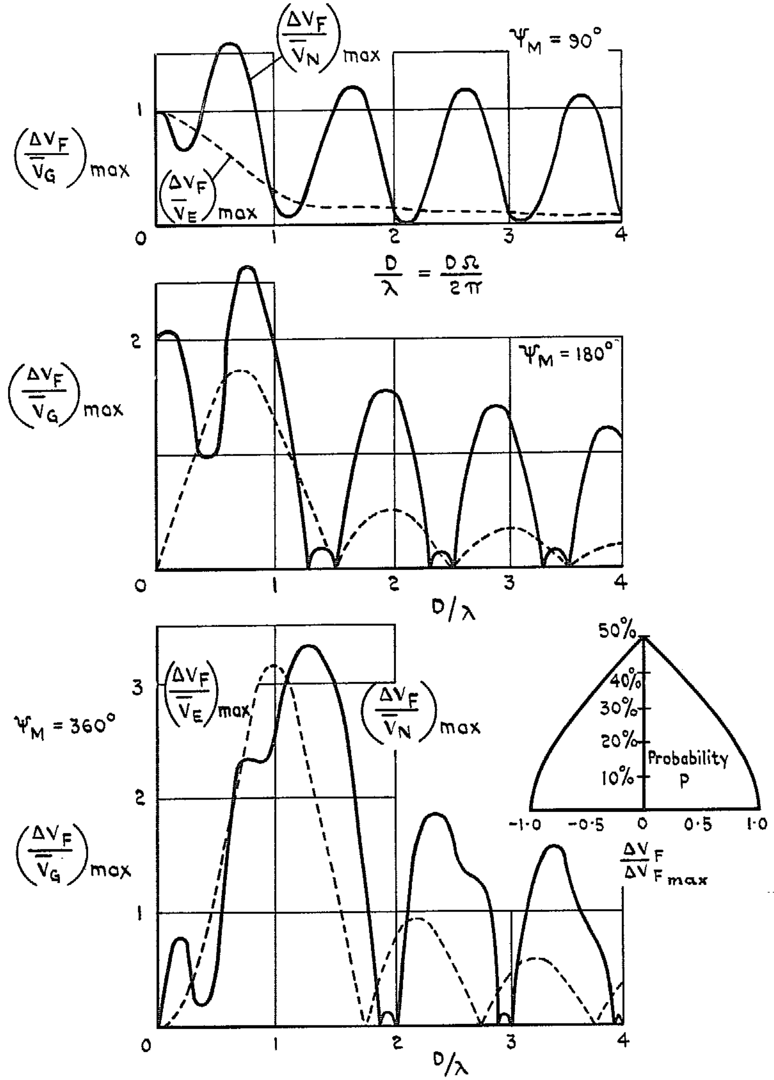


FIG. 11. Maximum possible loss or gain in mean speed when turning through ψ_M in the presence of sinusoidal gusts with amplitude $\overline{V}_N$ or $\overline{V}_E$ and wavelength λ.

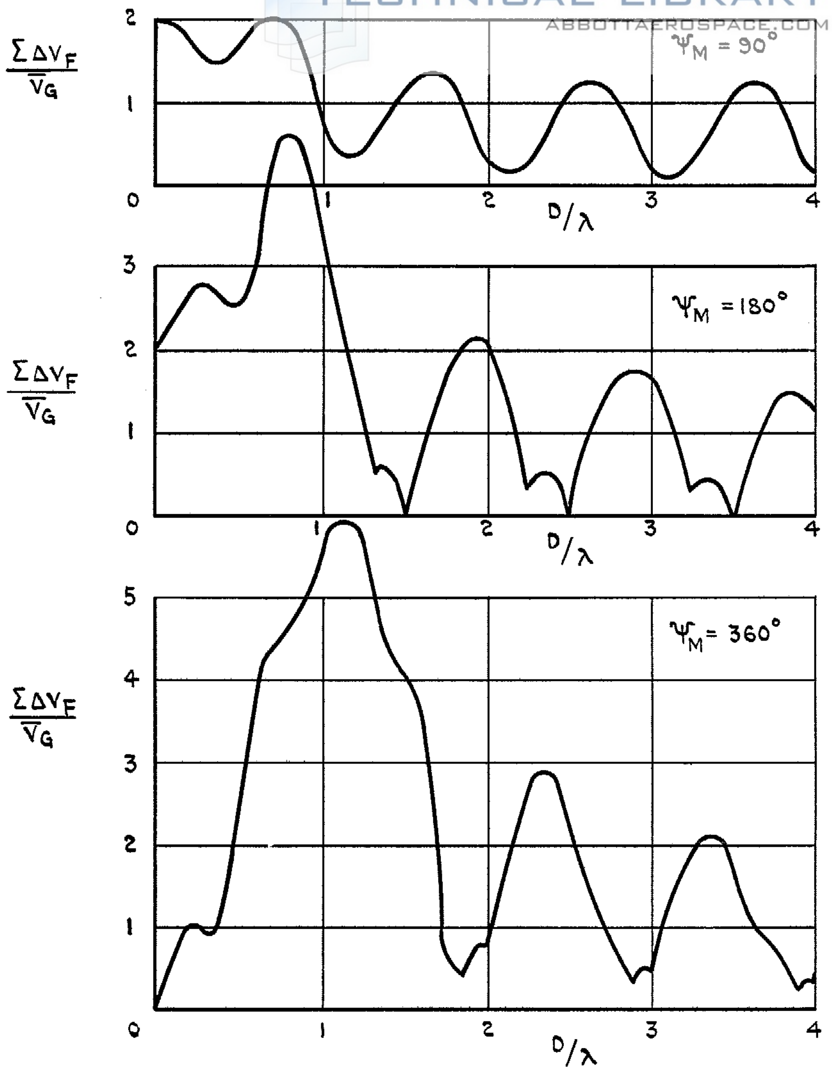


FIG. 12. Maximum possible loss or gain in mean airspeed when turning through ψ_M in northerly and easterly gusts with amplitude $\overline{V}_G$.

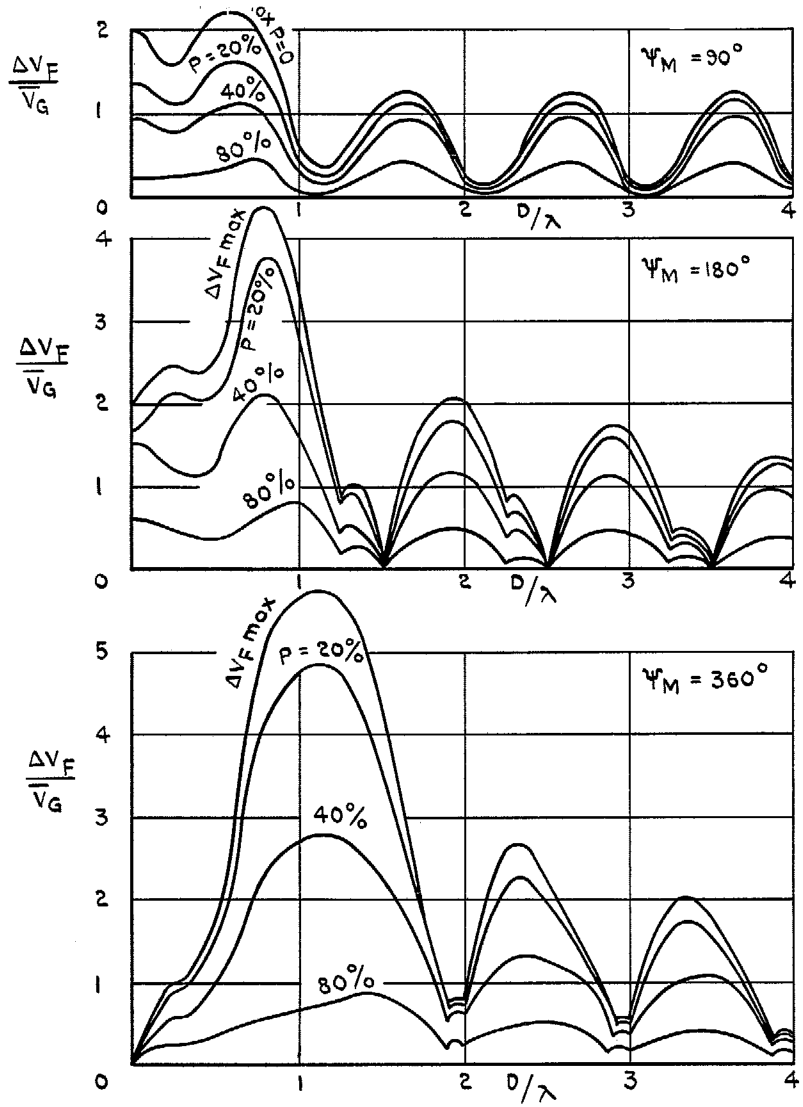


FIG. 13. Speed loss or gain which will be exceeded with probability P by aircraft turning through ψ_M in sinusoidal gust with amplitude $\overline{V}_G$ and wavelength λ.

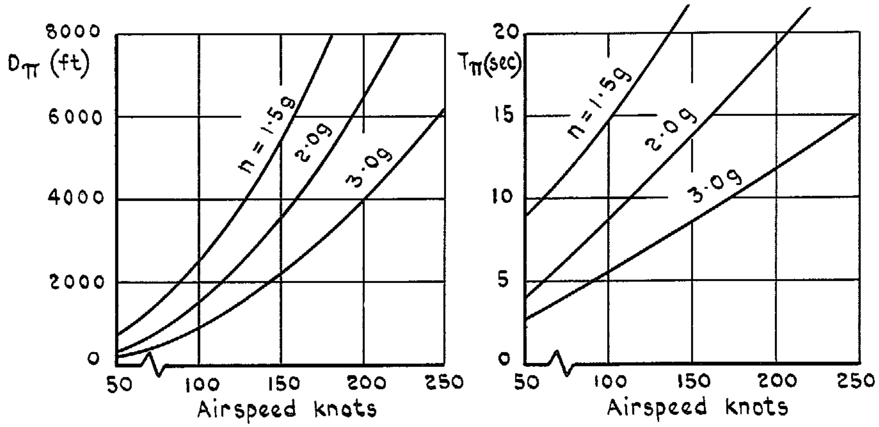


FIG. 14. Airdistance D_π and time T_π required to cover 180° turn hs a function of airspeed V_a and normal acceleration n.

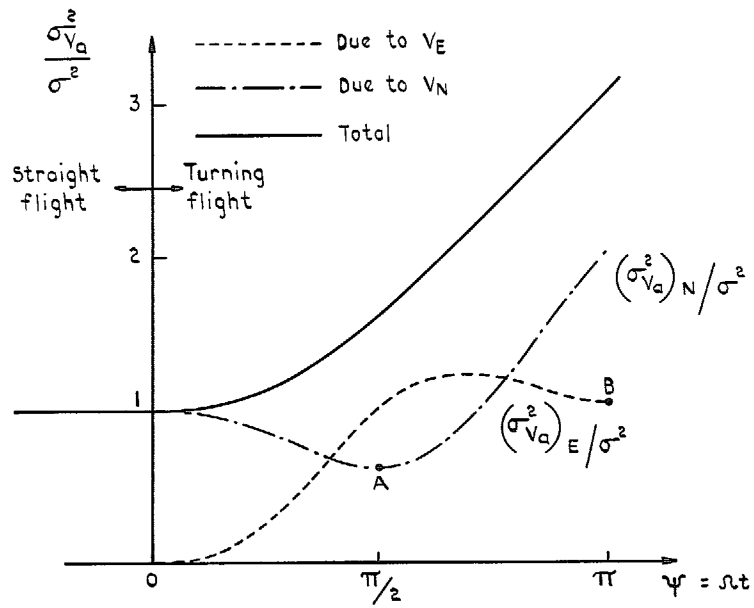


FIG. 16. Variance of airspeed changes due to random turbulence components V_N and V_E.

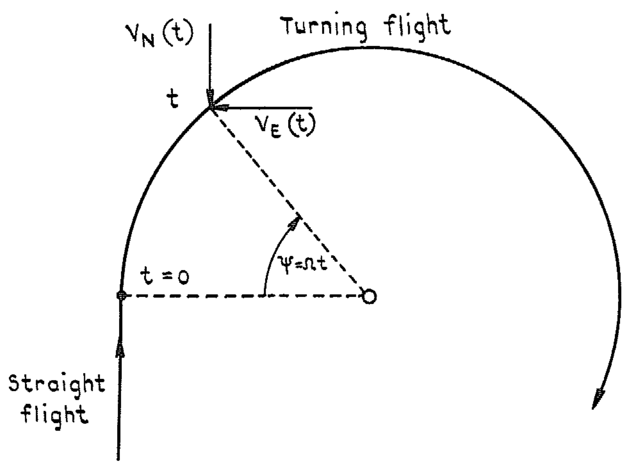


FIG. 15. The turning manoeevre (relative to mean wind) considered in app. C.

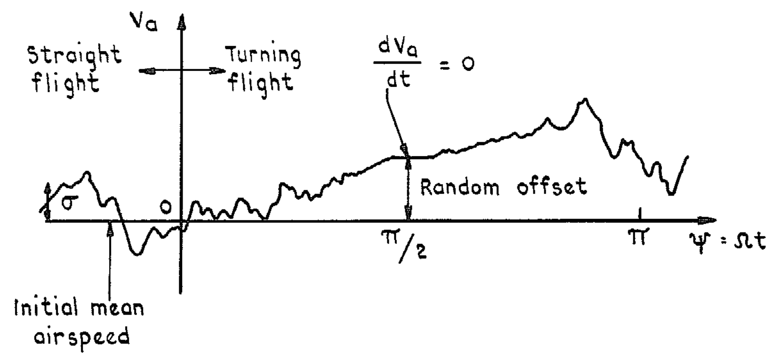


FIG. 17. Typical time variation of V_a due to V_N.

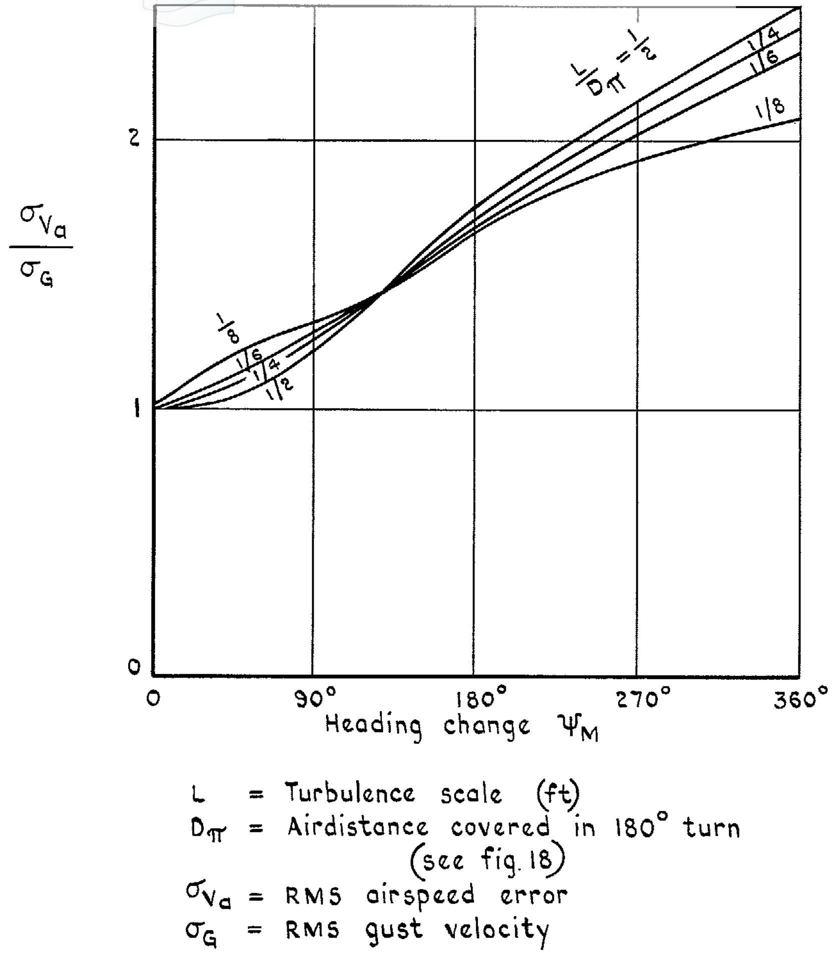


FIG. 18. Growth of RMS airspeed error in steady turning manoevre through random turbulence.

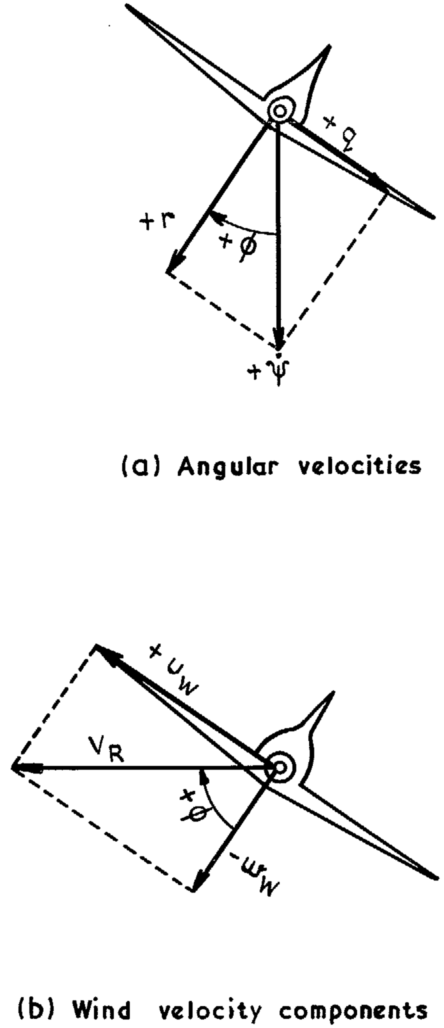


FIG. 19a & b. Definitions and resolution of angular velocities and windspeeds components in co-ordinated turning flight. (These sketches are viewed in the flight direction).

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