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Strength Variability in Structural Materials

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Summary.

Strength variability in structural materials is discussed in its relation to the assessment of safe stresses for use in design, and to the loads to be prescribed for the approval of a structural design by test.

The structural penalty imposed by the latter method of approval is examined and means of reducing severity are discussed.

A list is given of variability values (coefficients of variation) obtained from tests on a wide variety of aeronautical structural materials, fastenings and components.

*Replaces R.A.E. Technical Report 69 015—A.R.C. 31 416.

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1. *General Considerations.*

Any constructional material of a given nominal type will vary in strength from piece to piece even when made by a controlled standardized process. This variation not only occurs between one batch and another, but also between different parts of the same batch.

To design a safe and efficient structure it is therefore necessary to know not only the average or typical strength of the material, but also the range of its variability, so that the risk of encountering in production a piece of material at any given low level of strength can be assessed.

A knowledge of strength variability is thus necessary in practice for three purposes:

(i) To fix a safe strength for design calculations on the basis that only a low proportion of the material or product will have a strength lower than this design strength.

(ii) To judge what margin of strength over the required design strength a part needs to show in an approval test in order to provide an assurance that the risk of using an under-strength part is acceptably small.

(iii) To enable a decision to be made on whether efforts to reduce the variability would be worth while.

Wide variability in a material does not make it inherently less safe, provided that the degree of variability is known and remains constant throughout the whole production range. A highly variable material is, however, at some disadvantage as far as design is concerned, in that in fixing a safe design stress from a set of coupon tests, an allowance must be made in case these coupon tests have been made on material from the high end of the strength distribution. This allowance is higher the greater the variability, but can be reduced by making a larger number of coupon tests. Also the strength of those parts that do fall below the design strength is likely to be lower the higher the variability.

When parts are to be approved by a test, high variability is a definite disadvantage, as the test factor required increases with the variability. This is discussed in detail in Section 3.

2. *Evaluation of a Safe Design Strength.*

If a number of strength test results are available for a product, and the strength variability and the shape of the strength distribution for the product are known, a safe design strength can be calculated, to comply with any required safety condition, to any required degree of confidence.

In evaluating safe strengths for the materials and fasteners used in aircraft, it is the practice to stipulate the following safety conditions, which are to be satisfied to a 97½* per cent degree of confidence:

(i) not more than one part in ten shall be below the required design strength, and

(ii) not more than one part in a thousand shall be below 0.9 × required design strength.

A detailed method of calculating design strengths under the above conditions is given in the Appendix.

3. *Strength Checking by Test.*

When a specimen is selected for test, it will not be known where the strength of the specimen lies, within the range of strength variability of the product. For safety it is therefore necessary to assume that its strength is in the upper portion of the strength range, so that on this assumption there would be specimens markedly weaker than the one selected for test. Therefore the acceptance procedure must allow for this by requiring the test specimen to achieve a strength K times higher than the required design strength,

$$\text{where } K = \frac{\text{Strength at an upper limit on the variability}}{\text{Required design strength}}$$

The upper limit is fixed so that the chance of specimens occurring outside the limit (and thus the risk that the approval procedure may not ensure a safe design) is acceptably small. In practice this degree of risk is taken the same as that stated in Section 2, i.e. 2½ per cent.

K is the variability test factor, and a method of analysis for deriving it is given in the Appendix. Other methods of analysis are also used, notably that proposed by Bullen¹.

*i.e. a 2½ per cent risk that the required safety conditions will *not* be satisfied.

Obviously K increases with increase in scatter, so that for a given required design strength, the strength to be achieved in the check test goes up with the scatter.

If the part had been designed so that a specimen at the lower limit of variability had the required design strength, i.e. if the design conditions of Section 2 were *just* satisfied, then, although in this case all the production parts would be acceptably safe, the product would nevertheless have a very poor chance of satisfying the approval test, since only the strongest specimens would satisfy the requirement of being as strong as K times the required design strength, and the chance of picking a sufficiently strong specimen in a random selection would thus be very low.

Because manufacturers cannot afford to take the risk of having expensive batches of parts rejected, some of them design so that the specimen at the lower limit of variability is as strong as K times the required design strength. In this way they ensure a high probability of passing the test, but the parts are heavy and this weight penalty is greater, the greater the variability. In fact, this practice of designing to K times the required design strength is often unnecessarily severe particularly where the variability is high, and in many cases it is possible to design to a lower load and still have a reasonable chance of satisfying the factored test.

Curves showing the test factors applicable for various numbers of tests are given in Figs. 1 to 4.

4. *Parameters that Affect the Test Factor.*

In view of what was said in the last section it is obviously desirable to have as low a test factor as possible, and it will therefore be useful to examine the parameters that fix its value.

4.1. *Number of Specimens Tested.*

If more than one specimen is tested, the test factor required is lower because of the extra information given by the tests on the additional specimens.

However it is not usually worth testing more than about six specimens, as the reduction of factor for numbers greater than this is small.

For example, even in a case where the variability (i.e. coefficient of variation) is as high as 20 per cent, the test factor is 3.3 for one test, 2.9 for three tests, 2.75 for six tests, 2.65 for nine tests and 2.45 for one hundred tests, (applied to the mean value of the test strengths).

4.2. *Magnitude of the Variability.*

Where the variability is high (coefficient of variation, V , from 10 per cent upwards) it may be worth while to try to reduce the variability of the product by closer quality control.

For example, a product with $V = 15$ per cent requires a test factor of 2.2 (test of one specimen) but this would become 1.56 if V could be reduced to 10 per cent.

Where the variability is initially lower the profit from reducing the variability is not so attractive; for instance, reducing V from 10 per cent to 6.7 per cent (the same proportional reductions as before) only reduces the test factor from 1.56 to 1.29, and also it will usually be more difficult to reduce variability from an initially low value.

4.3. *Accuracy of Estimation of the Variability.*

It will usually be necessary to estimate the value of the coefficient of variation V of the product from the results of comparatively few tests. Unfortunately the precision of such an evaluation is not high.

For example, to a 95 per cent degree of confidence*, the coefficient of variation of a population of items could lie between 0.85 times and 1.27 times the value calculated from the results of tests on 30 specimens taken at random from the population. This can represent quite a large difference in the relevant test factor, particularly if the variability is high.

*i.e. a $2\frac{1}{2}$ per cent chance that the coefficient of variation is less than the lower limit quoted and a $2\frac{1}{2}$ per cent chance that it is higher than the higher limit. It thus represents the same order of risk as that used in Section 2.

The precision of estimation of variability from various numbers of tests, (to a 95 per cent degree of confidence), is shown in Table 1.

It may be noted that strength approval by test is usually prescribed in cases where (a) it is thought that the nature of the part and/or complexity of loading are such as to make calculation insufficiently precise and/or (b) where the strength of the material is not sufficiently well known. However, factored test procedures are also subject to doubt, particularly where thermal stresses are critical, or if the load on the part is conditioned by a supporting structure which has not been designed to take the factored test load of the supported member. This doubt arises because it is difficult to ensure that the correct stress distribution is preserved in the test specimen at loads above the normal design load.

In some cases of this nature imprecise calculations may be no worse than unrepresentative tests, and it might be that a combination of calculations and instrumented test would be practicable solution. With calculations made reasonably precise, the test would serve primarily to check the assumptions made in the calculations concerning stress distribution and mode of failure, and secondarily as check that no gross error had been made in calculating the strength.

The development of such an approval procedure would be very well worth while as it could largely eliminate the excess weight which at present has inevitably to be built into a part that is to be approved by test.

5. Available Data on Variability.

A search has been made through the records of Structures Department and the available data on variability has been extracted and summarised in Table 2. All the data are for items approved for aeronautical use.

The values quoted for variability are in most cases those that are considered, on the basis of all the information available, to be reasonably conservative figures to use in the calculation of safe stresses or test factors, but in this context the notes given against various items in the table should be considered, and in general it should be understood that values based on less than one hundred results may refer to items for which the sampling has not been as wide as could be desired. These latter figures should therefore be regarded as 'values which have been obtained' rather than as generally representative figures.

As can be seen there is a large amount of data available on the variability found in coupon tests of materials and fastenings, but very little on the variability of structural parts.

This is unfortunate as it is the variability of the structural part that is needed in evaluating test factors.

6. Future Objectives.

It is obviously of prime importance for the future development of new materials, particularly fibrous materials, that the penalty imposed by the present factor approval test procedure should be minimised.

There are two general ways in which this can be approached:

(i) *Reduction of the variability of the product.* This would involve research into the causes of variability and the devising of control systems in manufacture to limit the variability arising from these sources. Tests on sets of nominally similar parts would then be required so that the variation could be evaluated fairly precisely in numerical terms, and thus full advantage be taken of any reduction in the variability.

As shown in a previous section, reduction of variability becomes less rewarding in terms of test factor reduction as the level of variability goes down, and there will be a level at which it is not economic to try to reduce the variability any further.

(ii) *More precise analytical determination of the stress behaviour and modes of failure of the material* so that parts could be approved mainly by calculation with an instrumented test to confirm that the stress distribution and the mode of failure are in fact those assumed in the calculations. This would need considerable mathematical research, supported by tests, and as the strength calculations would need a basis of materials data, a supporting research exercise would also be needed to establish the framework of a materials test routine that would supply the right kind of basic data.

The above two approaches should not be regarded as mutually exclusive, because even if it should be found possible to approve a structure by calculation, without a factored test, it would still be desirable to investigate ways of reducing variability by control in manufacture, as this would give a more consistent and dependable product.

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APPENDIX

Evaluation of Safe Design Strengths and of Test Factors

A.1. *Design strength.*

In 1948 Atkinson proposed² that acceptable conditions for the strength of a material or part should be that :

- (a) Not more than one tenth of the product should be weaker than the required design strength, and
- (b) not more than one thousandth of the product should be weaker than $0.9 \times$ the required design strength.

(The condition giving the most severe case to be used*.)

Since that date these conditions have been widely used in the calculation of design strengths and test factors.

Atkinson went on to apply these conditions to practical cases, making the further stipulation that the values should be determined to a confidence of $97\frac{1}{2}$ per cent (i.e. only one case in 40 would not be covered). On the basis that the strength distribution is Gaussian, and that the coefficient of variation, V , is known from background experience, the following relations can be derived :

$$f_{0.1} = \frac{\bar{X} (1 - 1.28V)}{\left(1 + \frac{2V}{\sqrt{N}}\right)}$$

$$f_{0.001} = \frac{1.11 \bar{X} (1 - 3.09V)}{\left(1 + \frac{2V}{\sqrt{N}}\right)}$$

where

$f_{0.1}$ = design stress complying with condition (a)

$f_{0.001}$ = design stress complying with condition (b)

N = number of test results available

\bar{X} = mean value of the N results

V = coefficient of variation of the whole population of strength.

A.2. *For test factors to be applied when testing to check that the product is safe for a given load, the corresponding relations are :*

$$F_1 = \frac{\left(1 + \frac{2V}{\sqrt{N}}\right)}{(1 - 1.28V)}$$

$$F_2 = \frac{0.9 \left(1 + \frac{2V}{\sqrt{N}}\right)}{(1 - 3.09V)}$$

*Up to values of $V = 0.052$, (a) is the overriding case. Above this, (b) is the more severe.

where $F_1 =$ the test factor for condition (a)

$F_2 =$ the test factor for condition (b)

$N =$ number of specimens tested.

The larger of F_1 or F_2 is to be used and it is to be applied to the mean value of the N results, that is to say that:

$$\frac{\text{Mean of } N \text{ test results}}{\text{Required design strength}}$$

must be greater than or equal to the test factor. Values of test factors calculated on this basis, are given in Figs. 1 and 2.

It is sometimes required to use the *lowest* of the N results as a criterion instead of the mean of N , while still complying with the conditions (a) and (b), and the 97½ per cent confidence requirement. In this case the calculation of the test factors is slightly more complicated, as follows:

$$F_1 = \frac{(1 + KV)}{(1 - 1.28V)}$$

$$F_2 = \frac{0.9(1 + KV)}{(1 - 3.09V)}$$

K is obtained as follows:

Calculate $p = \sqrt[N]{0.025}$. Then K is the number of standard deviations, away from the mean, corresponding with a proportion p , at the upper end of the normal Gaussian curve, and can be obtained from statistical tables.

For example, if $N = 3$ then $p = \sqrt[3]{0.025} = 0.292$. The value of K corresponding with 0.292 is 0.55 (from tables),

therefore

$$F_1 = \frac{1 + 0.55V}{(1 - 1.28V)}$$

$$F_2 = \frac{0.9(1 + 0.55V)}{(1 - 3.09V)}$$

Values of the higher of F_1 or F_2 for various values of N and V , calculated on this basis are given in Figs. 3 and 4.

For these factors the ratio:

$$\frac{\text{Strength of weakest specimen tested}}{\text{Required design strength}}$$

must be greater than or equal to the test factor.

TABLE 1

Limits of the Coefficient of Variation, V.

If V_N = coefficient of variation obtained from a sample of N results, then to 95 per cent confidence, the coefficient of variation of the whole population from which the sample was drawn will be between :

$$(M_U \times V_N) \quad \text{and} \quad (M_L \times V_N)$$

(Gaussian distribution of the population has been assumed.)

$N =$	10	15	20	30	40	50	100
$M_U =$	1.58	1.43	1.35	1.27	1.23	1.19	1.13
$M_L =$	0.73	0.77	0.79	0.82	0.84	0.86	0.9

TABLE 2

Representative Values of Coefficients of Variation.

Item	Representative value of coefficient of variation V	No. of results on which based*	Remarks	Reference Nos.		
ATTACHMENTS						
<i>Snaphead Solid Aluminium Alloy Rivets in Aluminium Alloy Sheet</i>						
Failing load, rivet shear range	0.06	About 2800	Some evidence that for small dia: ($\frac{3}{32}$ in) rivets, V as high as 0.09 may occur. Values of V as high as 0.16 have been found in some cases.	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13		
Failing load, sheet tearing range	0.09					
Proof load, rivet shear range	0.07	About 1900				
Proof load, sheet tearing range	0.10					
<i>Snaphead 'Chobert' Steel Rivets in Aluminium Alloy Sheet</i>						
Failing load, rivet shear range	0.06	About 1800				14, 15
Failing load, sheet tearing range	0.09					
Proof load, rivet shear range	0.07	About 300				
Proof load, sheet tearing range	0.09					
<i>120° Countersunk-Head Solid Aluminium Alloy Rivets in Aluminium Alloy Sheet</i>						
Failing load, rivet shear range	0.07	About 1300		3, 16, 17, 18, 19		
Failing load, sheet tearing range	0.12					
Proof load, rivet shear range	0.07	About 400				
Proof load, sheet tearing range	0.15					
<i>Countersunk-Head 'Chobert' Steel Rivets in Aluminium Alloy Sheet</i>						
Failing load, rivet shear range	0.06	About 170				20, 21
Failing load, sheet tearing range	0.12					
Proof load, rivet shear range	0.07	About 170				
Proof load, sheet tearing range	0.15					
<i>Welds (hand)</i>						
<i>Failing load</i>						
Aluminium low alloy sheet	0.08	240	Gas butt welds	22		
Magnesium alloy sheet	0.19	About 400	Gas butt welds	23, 24		
Steel sheet	0.10	About 270	Gas or arc butt welds. Isolated batches of 9 tests gave V as high as 0.18.			
Steel sheet	0.07	120	Gas or arc fillet welds			
Steel tube (complex part)	0.14	40	Gas welds			

*The values of V given are not, in general, overall values for the number of results quoted, but are values assessed from the V values of individual smaller batches, within the overall total of results.

Item	Representative value of coefficient of variation V	No. of results on which based*	Remarks	Reference Nos.
<i>Steel Bolts</i> In tension on nut In double shear of shank	0.02 to 0.07 0.02 to 0.07	130 40		24
WROUGHT MATERIAL IN COUPON TESTS IN TENSION <i>Aluminium Alloy Bars and Extrusions</i> Failing strength Proof strength	0.01 to 0.05 0.01 to 0.05	Over 5500 Over 5500	One batch of 75 specimens gave $V = 0.06$ One batch of 75 specimens gave $V = 0.07$	26, 27, 28, 29, 30
<i>Aluminium Alloy Sheet</i> Failing strength Proof strength	0.01 to 0.05 0.01 to 0.05	About 950 About 950	One batch of 60 gave $V = 0.08$	29, 31
<i>Aluminium Alloy Tube</i> Failing strength Proof strength	0.03 0.05	60 60	Limited sampling, one alloy only Limited sampling, one alloy only	29
<i>Magnesium Alloy Bar</i> Failing strength Proof strength	0.02 to 0.04 0.07	About 130 About 130	One alloy only One alloy only	29
<i>Steel Bar</i> Failing strength Proof strength	0.01 to 0.02 0.01 to 0.03	About 50 About 50		32
WROUGHT MATERIALS IN COUPON TESTS IN COMPRESSION <i>Aluminium Alloy Bars and Extrusions</i> Proof strength	0.01 to 0.04	About 140		30, 33, 34
<i>Aluminium Alloy Sheet</i> Proof strength	0.01 to 0.03	About 180		35
<i>Steel Bar</i> Proof strength	0.02 to 0.03	About 50		32
<i>Steel Sheet</i> Proof strength	0.02 to 0.05	About 120		36

*The values of V given are not, in general, overall values for the number of results quoted but are values assessed from the V values of individual smaller batches, within the overall total of results.

TABLE 2—continued

Item	Representative value of coefficient of variation V	No. of results on which based*	Remarks	Reference Nos.
CAST MATERIAL				
<i>Material from Aluminium Alloy Sand Castings</i>				
Tensile failing strength	0.05 to 0.17	About 680		37, 38, 39
Tensile proof strength	0.05 to 0.15	About 670		
<i>Material from Magnesium Alloy Sand Castings</i>				
Tensile Failing Strength	0.07 to 0.19	About 240		38, 39
Tensile Proof Strength	0.07 to 0.21	About 240		
<i>Material from Steel Sand Castings</i>				
Tensile failing strength	0.05	About 190	One batch of 41 tests gave $V = 0.095$	40, 41, 42
Tensile proof strength	0.07 to 0.08	About 60		
<i>Material from Precision Investment Steel Castings</i>				
Tensile failing strength	0.01 to 0.03	About 300		43
Tensile proof strength	0.01 to 0.05	About 500		
GLASS, ETC.				
<i>Soda-Lime Plate Glass—Annealed</i>				
Failing strength in bending	0.12 to 0.28	About 300		44, 45, 46, 47, 48
<i>Soda-Lime Plate Glass—Toughened</i>				
Failing strength in bending	0.09 to 0.19	About 500		48
<i>Quartz Plates ('Spectrosil')</i>				
Failing strength in bending	0.12 to 0.14	About 60		49, 50
<i>Alumino-Silicate Plate Glass—Annealed</i>				
Failing strength in bending	0.23	40	One batch only	51
<i>Alumino-Silicate Plate Glass—Toughened</i>				
Failing strength in bending	0.15	40	One batch only	52
<i>Glass-Fibre Laminate</i> (tension, compression and bearing tests)	0.07	Over 1250	P6 woven glass cloth with polyester resin. Laid up by four firms, by a prescribed, non-production process, from eight different rolls of cloth and four bulk lots of resin.	53

*The values of V given are not, in general, overall values for the number of results quoted, but are values assessed from the V values of individual smaller batches, within the overall total of results.

Item	Representative value of coefficient of variation V	No. of results on which based*	Remarks	Reference Nos.
'Perspex' Failing strength in tension	0.04 to 0.09	About 100 in eight batches	Specimens cut from 'Vampire' hoods after pressure test.	62, 63
'Perspex' notched specimens Failing strength in tension	0.12 0.25	24 15	Specimens tested at 20.5°C Specimens tested at -25°C	64
COMPLETE STRUCTURAL UNITS <i>Wooden Aircraft Structure ('Master' tailplane)</i> Failing load	0.073	60	Built-up structures in Grade A spruce, loads applied to front spar	54
<i>Metal aircraft structure ('Typhoon' tailplane)</i> Failing load	0.02 to 0.03	35	2-spar light alloy structure	55
'Perspex' hood from 'Vampire' Failing pressure	0.11 to 0.29	90	Double-shell hood	56, 57, 58
<i>Glass Panel Windcreens from 'Vanguard'</i> Failing pressure	0.07 to 0.17	About 150	Includes 4 different types of panel	60
<i>Toughened Glass Sheets for TSR 2</i> Failing pressure	0.10 to 0.16	80	Component glasses shaped and toughened ready for laminating. Tested with edges anchored between shaped wooden formers.	61
'Perspex' Astrodomes from 'Wellington'	0.24	30	Taken from scrapped aircraft	59

*The values of V given are not, in general, overall values for the number of results quoted, but are values assessed from the V values of individual smaller batches, within the overall total of results.

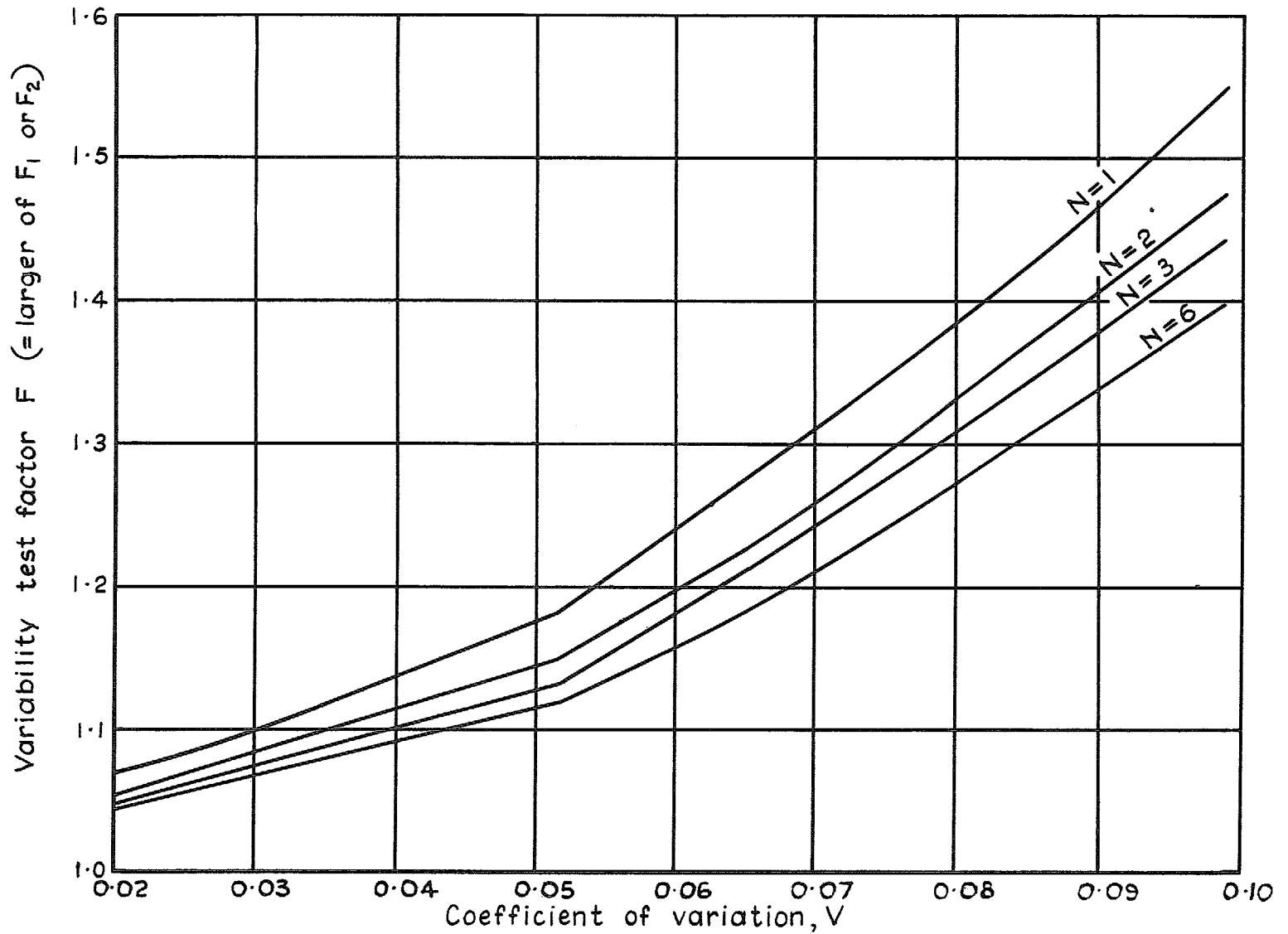


Fig. 1. Variability test factor required on the mean of N tests.

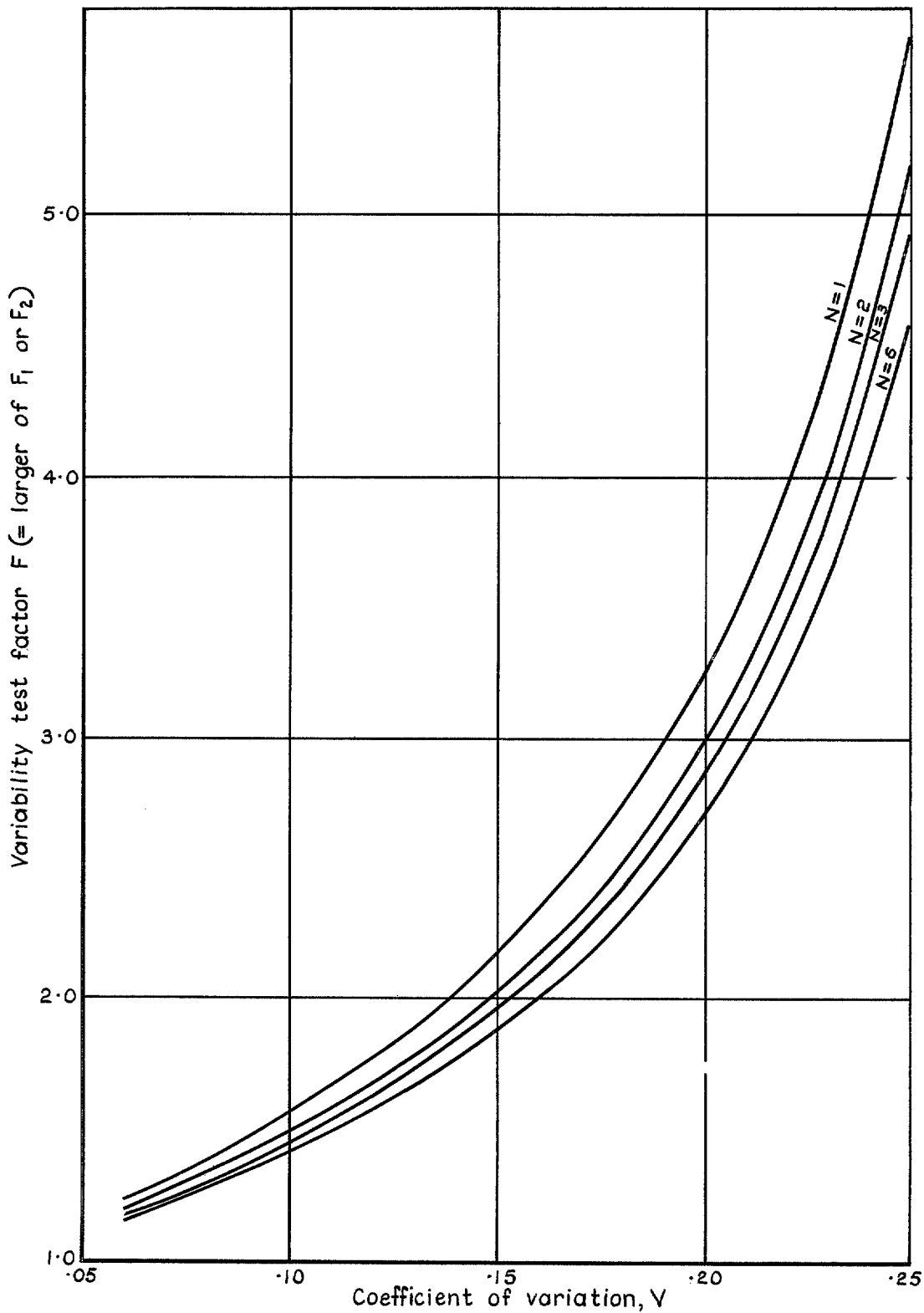


Fig. 2. Variability test factor required on the mean of N tests.

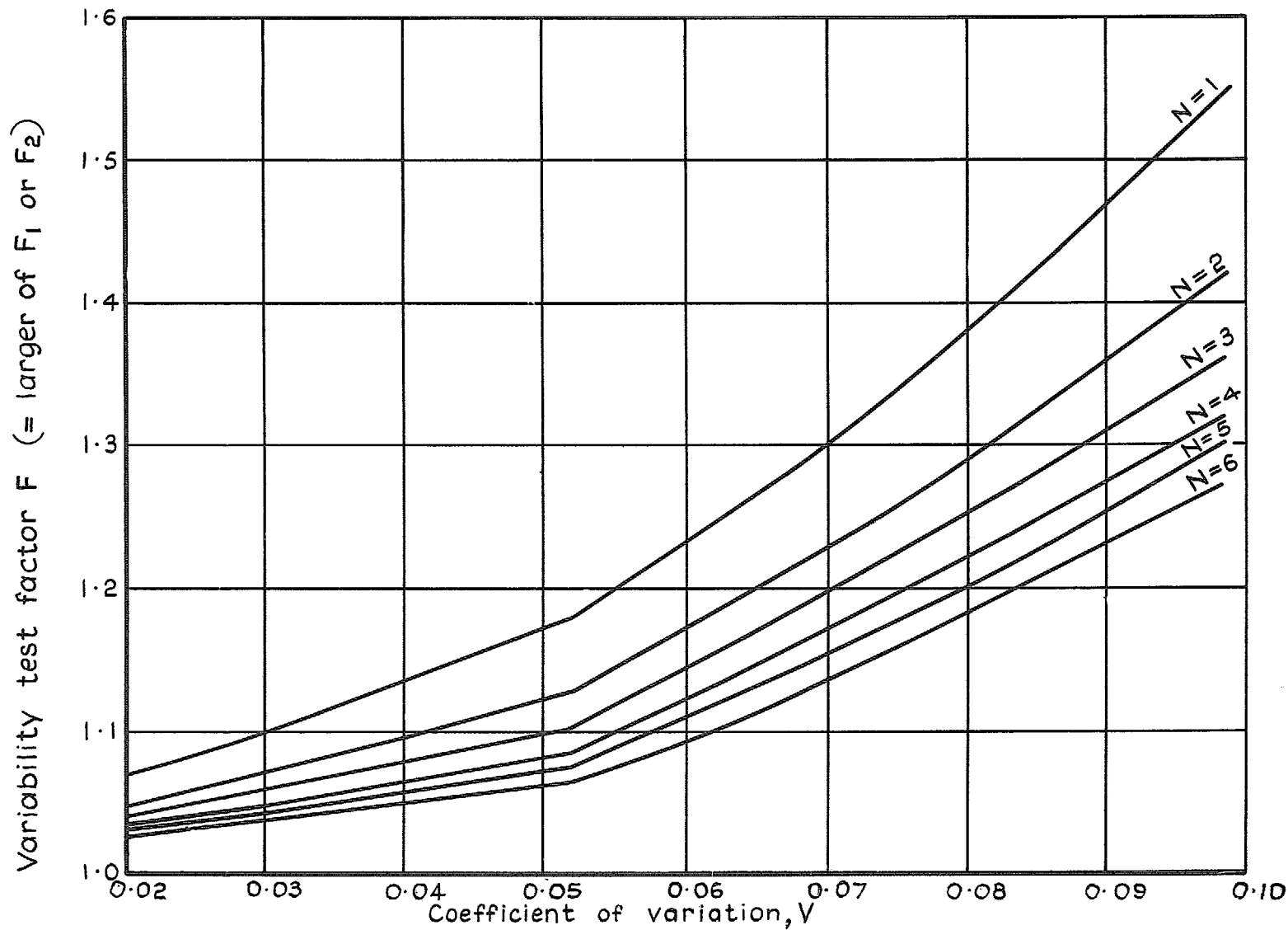


Fig. 3. Variability test factor required on the minimum of N tests.

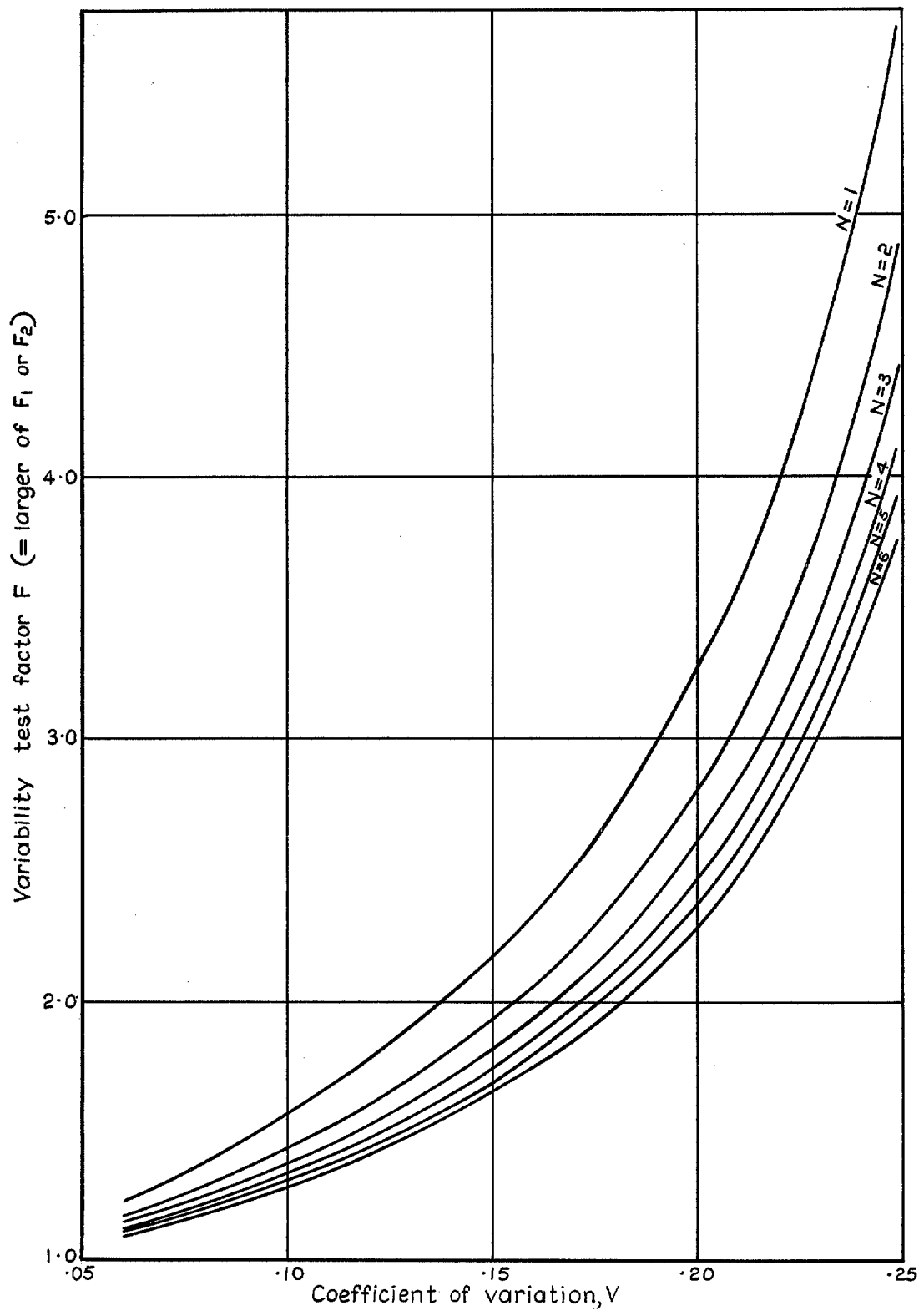


Fig. 4. Variability test factor required on the minimum of N tests.

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