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Summary.

The paper gives a theory of vortex flow in turbo-machines in which the basic assumption is made that the axial-velocity profiles always belong to a family governed by a single parameter, λ. This parameter is a measure of the radial displacement of the streamlines. The flow equations then reduce to a single linear differential equation for λ and a corresponding simplification of ideas about the way in which these flows behave is achieved. Comparison with experimental results by Horlock and more-accurate calculations by Wu shows that the theory is sufficiently accurate for design purposes.

The theory is applied to compressors with a large number of identical stages, and it is shown how a rapid solution may be obtained by matrix methods. Examples of flow in a ten-stage compressor are given, and the effects of variation of mass flow and of modifications to the inlet guide vanes are demonstrated.

1. *Introduction.*

In the design of an axial-flow compressor or turbine it is usual to proceed in three steps. First the overall conditions for each stage are determined by specifying the axial and radial velocities of the flow at some design radius. Then the radial variations of these velocities are considered, and finally blading is specified which will realize the desired angles of flow. This paper is concerned with the second step of this process. The flow is therefore considered to be axi-symmetric, and as is usual the effects of viscosity and turbulence will be neglected. Consequently the theory will be in error near the inside and outside walls of the machine, but may be expected to hold over a substantial proportion of the blade height.

A large number of methods has been proposed for calculating these flows, so that it may be useful to enumerate the criteria by which it is suggested that these methods should be judged. Firstly the method must be sufficiently accurate. The accuracy required is not high, since the changes in axial velocity resulting from the three-dimensional effects which are to be calculated correspond to changes in the flow angle of a few degrees only. Discrepancies of 10% in the theory will not therefore give rise to measurable effects. The second criterion is that results should be easily obtainable. The advent of digital computers means that volume of arithmetic is no longer the primary consideration, and ease of programming becomes more important. Thirdly, the theory should be conceptually simple, so that the qualitative effects of changes in the design can be easily visualized.

The simplest type of theory assumes that the flow is in 'radial equilibrium'. It is now recognized that except for certain special designs this is very inaccurate.

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The most accurate type of solution is based on a numerical solution of the full equations of motion through the blade rows. The general theory has been given by Wu[23], and solutions for compressible flow in a single-stage free-vortex turbine have been obtained (Wu[24]). This method has also been used for a single-stage and for a seven-stage compressor (Wu[25]). These solutions are extremely valuable as yardsticks against which more-approximate theories may be measured, but the method appears to be far too complicated for general use.

Approximate solutions using semi-graphical methods have been given by Smith, Traugott, and Wislicenus[18] and by Holmquist and Rannie[5]. These methods do not appear to be suitable for digital computers.

Another method of solution is to assume that the streamlines in a plane containing the axis of the machine are sinusoidal, with a wavelength equal to the axial length of a stage. This approach has been used by Traupel[20]. Wu and Wolfenstein[22] and Schnittger[17] have shown how this method can be extended for the case where the flow has not yet become completely periodic. While these methods may be expected to work well in the later stages of a compressor, where the flow has settled down to a repeating pattern, they seem likely to be inaccurate in the first few stages.

Another approximate type of theory which has received considerable attention is the actuator-disc theory. This theory involves replacing a blade row by an infinitely thin disc which causes a sudden discontinuity in whirl velocity and static pressure. The results of this type of theory have been summarized by Marble[13] and a number of papers are included in the Bibliography. Recent developments include the application of actuator-disc theories to machines with conical walls by Lewis[10], and to flows with compressibility effects by Horlock[8] and by Hawthorne and Ringrose[4]. Although there is some difficulty in deciding exactly where to place the actuator discs, it appears that the accuracy attained by the actuator-disc theories is very satisfactory. The design problem can be solved very readily by actuator-disc methods (Horlock and Carmichael[2]) but the problem of the analysis of the flow in a given multi-stage machine is much more complicated and there appears to be some difficulty in programming this for digital computers. Also, since the problem remains one with two independent variables (radius and axial distance) the effects of design modifications are hard to visualize. In addition, since in the simplest theory the blade is reduced to a disc of infinitely small axial length, this theory does not yield any information about the velocity field within the blade passage itself. This difficulty can be overcome by replacing the blade row by more than one actuator disc.

A theory is now being developed by the Authors which can be used to solve both the direct and inverse problems of the flow of a compressible fluid through a multi-stage compressor with varying hub and tip radii. This paper is concerned with the fundamental part of the theory, which is the analysis of the flow of a perfect fluid through a given compressor of constant hub and tip radii. In this theory as well as making most of the usual approximations, an approximation to the form of the axial-velocity profile will be made. This makes the problem simple to consider, yet retains the general overall features of the fluid flow. It will be assumed that the distribution of axial velocity with radius can be represented as a single-parameter family of profiles. It is thought that this new approximation is as good as is warranted by the assumption of non-turbulent and inviscid flow. Using this assumption, the hydrodynamical equations are satisfied at the design radius only, where the design radius is defined as the root mean square of the hub and tip radii. The equations are solved in terms of a non-dimensional slope of the axial-velocity profile, which can be assumed to give an approximate picture of the radial displacement of the streamline at the design radius.

2. *Approximations.*

In this paper the following approximations will be made:

(*a*) The air is assumed to be incompressible and inviscid, and the flow is non-turbulent.

(*b*) The flow is assumed to be rotationally symmetric. This neglects all blade grid effects such as wakes and secondary flows.

(*c*) It is assumed that the air outlet angle from each blade row is constant. This will be very nearly true as long as the blades are not stalled.

The two latter approximations are equivalent to replacing the actual blades by a cascade of infinitely thin blades, very close together, but having the same axial chord as the actual blades. The outlet angle of these blades is the air outlet angle, but the inlet angle is the air inlet angle determined by the conditions at exit from the previous row. This inlet angle will vary with mass flow, and, except when the blades operate at zero incidence, bears no relation to the actual designed blade inlet angle.

(*d*) It is assumed that the total pressure loss in each cascade relative to that cascade is independent of radius.

(*e*) It is assumed that the radial displacement of the streamlines is small.

(*f*) Variation of axial velocity with radius is assumed to be of the form of a family of curves governed by a single parameter.

(*g*) The equations of motion will be satisfied only at a design radius.

Away from the design radius the equations of motion will not be satisfied exactly, but since the axial-velocity profile and its radial derivative are correctly determined at the design radius it is possible to draw the complete axial-velocity profile with little error.

(*h*) It is assumed that the blades exert no radial force on the air.

Since a twisted blade can only be radial at one axial station this will only be exactly true at one axial station. Wu[25] found that the effect of radial force was negligible in his solutions for compressors, but that there was a significant effect in a free-vortex turbine (Wu[24]), where the blades are highly twisted. This is therefore a small effect which could be incorporated into the present theory, but will be neglected in this paper.

(*i*) It is assumed that the air flows through each row of blades on a path such that the tangent of the relative air angle varies linearly from the leading to the trailing edge of the blades. This assumption is exact for a row of infinitely thin blades designed on a parabolic centre-line.

This last assumption is not essential to the theory, which can readily be modified to allow for any given variation of relative air angle through a blade row.

In addition to the above approximations, the theory in this paper is established for the special case of a compressor with constant hub and tip radii.

3. *Basic Theory.*

3.1. *Assumed Axial-Velocity Profiles, and Continuity.*

In accordance with the fundamental approximation that the distribution of axial velocity with radius in a machine can be represented as a single-parameter family of profiles, it will be assumed that the axial velocity v is given by

$$\frac{v}{V_R} = 1 + \frac{\lambda}{2}\left\{\left(\frac{r}{R}\right)^2 - 1\right\}. \qquad (1)$$

Profiles for various values of the parameter λ are shown in Fig. 5. At the design radius, R, the axial velocity is always V_R, and the shape of the curve at this point is controlled by the parameter λ. This of course is far from being the only family of profiles which satisfies the required conditions, but it has been chosen both for algebraical simplicity and since it gives profiles which are similar to those used in practice.

The design radius R is chosen as the root mean square of the hub and tip radii,

$$R^2 = \tfrac{1}{2}(t^2+h^2). \tag{2}$$

The mass flow through the machine is then given by

$$\int_h^t 2\pi r\rho v\,dr = 2\pi\rho V_R \int_h^t \left\{r + \tfrac{1}{2}\lambda r\left(\frac{r^2}{R^2} - 1\right)\right\} dr$$
$$= \pi\rho V_R(t^2-h^2)$$

from equations (1) and (2). This is independent of the parameter λ {the choice of design radius in equation (2) has been made to this end}. Since it is being assumed that ρ, t, and h are constants, this shows that V_R is a constant throughout the length of the machine, and is determined by the mass flow. The problem of finding the flow in the machine is therefore reduced to finding the way in which λ varies along the length of the machine.

The continuity equation allows a stream function ψ to be defined so that

$$rv = -\frac{\partial\psi}{\partial r}, \tag{3}$$

and

$$ru = \frac{\partial\psi}{\partial x}. \tag{4}$$

Substituting for v from equation (1) in equation (3) gives

$$\frac{\partial\psi}{\partial r} = -rV_R - \tfrac{1}{2}\lambda r V_R\left\{\left(\frac{r}{R}\right)^2 - 1\right\}.$$

Integrating this with respect to r, and choosing the constant of integration so that ψ is zero at the hub gives

$$\psi = -\tfrac{1}{2}V_R(r^2-h^2) + \frac{\lambda V_R}{8R^2}(t^2-r^2)(r^2-h^2). \tag{5}$$

If this is now substituted into equation (4) an expression for the radial velocity is obtained as

$$u = \frac{V_R}{8rR^2}(t^2-r^2)(r^2-h^2)\frac{d\lambda}{dx}. \tag{6}$$

The stream function may also be used to compute the radial displacement of the streamlines. It will be assumed in the theory that this radial displacement is small. Consider in particular a streamline which lies at the design radius at an axial station where the axial-velocity profile is specified by λ (point A in Fig. 4). Farther downstream the streamline lies at B where the radius is $(R+\delta R)$, and λ has increased to $(\lambda+\delta\lambda)$. For the point D, which is the point on the design radius at the second axial station, the stream function is given by equation (5) as

$$\psi_D = \psi_A + \frac{V_R}{8R^2}(t^2-R^2)(R^2-h^2)\delta\lambda.$$

Also from equation (3) for the second station

$$\psi_B = \psi_D + \frac{\partial\psi}{\partial r}\,\delta R = \psi_D - RV_R\delta R.$$

But since A and B are on the same streamline $\psi_A = \psi_B$, and eliminating the stream functions gives

$$\delta R = \frac{1}{8R^3}(t^2 - R^2)(R^2 - h^2)\delta\lambda.$$

This may be written

$$\delta R = \frac{a^2k^2}{R}\,\delta\lambda, \tag{7}$$

where k is a constant which depends mainly on the aspect ratio of the blades and is given by

$$k = \frac{(t^2 - h^2)}{4\sqrt{2}\,Ra}. \tag{8}$$

Equation (7) shows that, for small displacements, the radial displacement of a streamline near the design radius is directly in proportion to the change of λ, and this is probably the easiest way to visualize λ. λ is also simply related to the slope of the axial-velocity profile at the design radius, since differentiation of equation (1) gives at the design radius

$$\lambda = \frac{R}{V_R}\left(\frac{\partial v}{\partial r}\right)_{r=R}. \tag{9}$$

3.2. *The Basic Differential Equation for λ.*

In order to find the way in which λ varies along the machine an equation of motion is required. Assuming that the blades exert no radial force on the fluid, the radial equation of motion for axi-symmetric flow is

$$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial x} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}. \tag{10}$$

Owing to the restricted range of axial-velocity profiles which are available it is only possible to satisfy this equation exactly at one radius, and this will be chosen as the design radius. From equation (6) at the design radius,

$$\left(\frac{\partial u}{\partial x}\right)_R = \frac{V_R(t^2 - R^2)(R^2 - h^2)}{8R^3}\frac{d^2\lambda}{dx^2}$$

Putting this into equation (10) for the design radius gives

$$\frac{V_R{}^2(t^2 - R^2)(R^2 - h^2)}{8R^3}\frac{d^2\lambda}{dx^2} - \frac{W_R{}^2}{R} = -\left\{\frac{\partial}{\partial r}\left(\frac{p}{\rho} + \tfrac{1}{2}u^2\right)\right\}_R.$$

This may be written

$$k^2a^2\frac{d^2\lambda}{dx^2} = \left(\frac{W_R}{V_R}\right)^2 - g, \tag{11}$$

where

$$g = \frac{R}{V_R{}^2}\left\{\frac{\partial}{\partial r}\left(\frac{p}{\rho} + \tfrac{1}{2}u^2\right)\right\}_R. \tag{12}$$

Since the radial velocities will be small the $\frac{1}{2}u^2$ term in equation (12) is negligible. g may therefore be thought of as the static-pressure gradient at the design radius. Equation (11) then shows how the curvature of the streamlines is determined by the centrifugal effect of the whirl velocity and by the static-pressure gradient. This is the equation governing the variation of λ along the machine. However, as the static-pressure gradient g will vary along the machine, it is useful to replace it by the gradient of stagnation pressure.

The stagnation pressure of the fluid relative to the blade row through which it is flowing is given by

$$\frac{P}{\rho} = \frac{p}{\rho} + \tfrac{1}{2}u^2 + \tfrac{1}{2}v^2 + \tfrac{1}{2}(\beta v)^2,$$

where β is the tangent of the flow angle in the circumferential plane, so that βv is the relative whirl velocity. Differentiation with respect to r gives

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{\partial}{\partial r}\left(\frac{p}{\rho} + \tfrac{1}{2}u^2\right) + v\,(1+\beta^2)\frac{\partial v}{\partial r} + v^2\beta\frac{\partial \beta}{\partial r}. \tag{13}$$

At the design radius this gives

$$G = g + \lambda(1+\beta_R{}^2) + \left(R\beta\frac{\partial \beta}{\partial r}\right)_R, \tag{14}$$

where

$$G = \frac{R}{\rho V_R{}^2}\left(\frac{\partial P}{\partial r}\right)_R. \tag{15}$$

G is the gradient of relative stagnation pressure at the design radius.

Eliminating g from equations (11) and (14) gives

$$k^2a^2\frac{d^2\lambda}{dx^2} - \lambda(1+\beta_R{}^2) = \left(\frac{W_R}{V_R}\right)^2 + \left(R\beta\frac{\partial \beta}{\partial r}\right)_R - G. \tag{16}$$

This is the basic differential equation for the variation of λ within a blade passage. The quantities β_R, $(\partial\beta/\partial r)_R$, and W_R/V_R are presumed to be known functions of x, since they are determined by the blade geometry.

It will also be shown that provided the displacement of the streamlines is small, G is a constant in any one blade passage. For, considering the two streamlines AB and CD in Fig. 4 which are always close to the design radius, the change in stream function $\delta\psi$ from one line to the other is constant and, at the design radius, the axial velocity V_R is constant, so that from equation (3) the distance between the two streamlines δR is also constant. For flow without loss through stators P is constant along a streamline. The corresponding condition for rotors is that $(P+\frac{1}{2}\rho\Omega^2r^2)$ is constant along a streamline. Now since the streamlines AB and CD are deflected by the same distance, the term $\frac{1}{2}\rho\Omega^2r^2$ will change by the same amount for each of them, and so δP, the difference in stagnation pressure between the two lines, is constant. Therefore, since δP and δR are both constant, G must also be constant.

If there are losses in the flow, the assumption that G is constant will still hold, provided that the losses are the same at all radii.

Before considering methods of solving equation (16) within the blade rows, the flow in the annular ducts which form the inlet and outlet to the machine, and also in the gaps between the blade rows, will be considered.

4. *Flow in an Annular Duct.*

For flow in an annular duct the angular momentum, wr, and the stagnation pressure are constant along streamlines. At the design radius the separation of the streamlines is constant, and therefore

$$\frac{\partial}{\partial r}(wr)_R = \text{constant} = \omega V_R \tag{17}$$

and

$$G = \text{constant}. \tag{18}$$

ω is a constant which determines the amount by which the flow departs from free-vortex conditions.

Referring to Fig. 4 in which points A and D are at the design radius, the angular momentum at B is given by

$$(wr)_{\mathrm{B}} = (wr)_{\mathrm{D}} + \frac{\partial}{\partial r}(wr)\delta R.$$

But since points A and B are on the same streamline, $(wr)_{\mathrm{B}} = (wr)_{\mathrm{A}}$, and using equation (7) for the displacement of the streamlines, and also equation (17),

$$(wr)_{\mathrm{A}} = (wr)_{\mathrm{D}} + \frac{\omega V_R a^2 k^2}{R}\delta\lambda.$$

This may be written

$$\left(wr + \frac{\omega V_R a^2 k^2}{R}\lambda\right)_{\mathrm{A}} = \left(wr + \frac{\omega V_R a^2 k^2}{R}\lambda\right)_{\mathrm{D}},$$

so that at the design radius and providing the displacement of the streamlines remains small

$$wR^2 + \omega V_R a^2 k^2 \lambda = \text{constant}. \tag{19}$$

On substituting $w = \beta v$, equation (17) gives

$$\beta_R V_R + \beta_R R\left(\frac{\partial v}{\partial r}\right)_R + V_R R\left(\frac{\partial \beta}{\partial r}\right)_R = \omega V_R$$

or, using equation (9)

$$\beta_R(1+\lambda) + R\left(\frac{\partial \beta}{\partial r}\right)_R = \omega. \tag{20}$$

Similarly equation (19) gives

$$\beta_R + \frac{\omega a^2 k^2}{R^2}\lambda = \text{constant}. \tag{21}$$

Eliminating β_R, $(\partial\beta/\partial r)_R$, and G from equations (16), (18), (20) and (21) then gives

$$\frac{a^2}{\dfrac{1}{k^2} - \left(\dfrac{\omega a}{R}\right)^2}\frac{d^2\lambda}{dx^2} - \lambda = \text{constant} = -\lambda_\infty.$$

This may be written

$$k'^2 a^2 \frac{d^2\lambda}{dx^2} - \lambda = -\lambda_\infty, \tag{22}$$

where

$$\frac{1}{k'^2} = \frac{1}{k^2} - \left(\frac{\omega a}{R}\right)^2. \tag{23}$$

The solution of equation (22) is

$$\lambda = \lambda_\infty + Ae^{+x/k'a} + Be^{-x/k'a} \tag{24}$$

where A and B are arbitrary constants.

For an inlet duct the value of λ must not become infinite far upstream where x tends to $-\infty$. This means that B is zero. Similarly, for an outlet duct the solution must remain finite when x tends to $+\infty$, and then A is zero. Hence, if λ_0 is the value of λ when $x = 0$,

$$(\lambda - \lambda_\infty) = (\lambda_0 - \lambda_\infty)e^{\pm x/k'a}, \tag{25}$$

where the upper sign refers to the inlet duct and the lower sign to an outlet duct. It is seen that λ_∞ is the value of λ far away from the machine.

Differentiating equation (25) gives

$$k'a\frac{d\lambda}{dx} = \pm(\lambda_0 - \lambda_\infty)e^{\pm x/k'a}. \tag{26}$$

Differentiating again and using equations (11) and (23) gives for the static-pressure gradient

$$g = -\left(1 - \frac{k^2a^2\omega^2}{R^2}\right)(\lambda_0 - \lambda_\infty)e^{\pm x/k'a} + \beta_R^{\,2}. \tag{27}$$

The result obtained in this section may be compared with the exact solution for flow in an annular duct when $\lambda_\infty = 0$ and $\omega = 0$ (Bragg and Hawthorne[1]). This exact solution has the form of a series in which each term varies exponentially with x in the same way as in equation (24) but with a different value of k for each term. These values of k are the roots of

$$\frac{J_1\left(\dfrac{t}{ka}\right)}{Y_1\left(\dfrac{t}{ka}\right)} = \frac{J_1\left(\dfrac{h}{ka}\right)}{Y_1\left(\dfrac{h}{ka}\right)} \tag{28}$$

where J and Y are the usual Bessel functions. The axial-velocity profile corresponding to the lowest root has no maximum or minimum for $h < r < t$ (although the slope is zero at $r = h$ and $r = t$) and is therefore comparable to the profiles assumed in this paper. The values of ka/t calculated from equation (8) and for the lowest root of equation (28) are compared in Fig. 6. The agreement is seen to be very close.

When ω is not zero the exact linearized solution for flow in an annular duct has been given by Marble[13]. The effect on the rate of decay of disturbances agrees exactly with equation (23).

5. *Flow in an Isolated Blade Passage.*

It has already been demonstrated that for a compressor with constant hub and tip radii the values of R and V_R are constant through the compressor, and in particular through each individual blade row. In addition G has been assumed to be constant in a blade passage. Thus in order to solve equation (16) it is necessary to know β_R, $(\partial\beta/\partial r)_R$ and w_R as functions of x. However, from Fig. 3 it is seen that w_R for both a rotor row and a stator row depends only on β_R, as given below.

For a rotor row

$$\left(\frac{w_R}{V_R}\right)^2 = \left(\frac{1}{m} - \beta_R\right)^2 \tag{29a}$$

and, for a stator row

$$\left(\frac{w_R}{V_R}\right)^2 = \beta_R^{\,2} \tag{29b}$$

where m is a mass-flow parameter defined by $V_R/\Omega R$ which is constant in any blade passage. Thus any analysis applied to equation (16) for the blade passage of a rotor row will apply for the blade passage of a stator row, if $1/m$ is put equal to zero. The remainder of this section will therefore be devoted to the flow through an isolated rotor row of blades.

The values of β_R and $(\partial\beta/\partial r)_R$ at entry to and exit from a blade passage are determined by the flow conditions at inlet and exit. However, we can assume any reasonable variation of β_R and $(\partial\beta/\partial r)_R$ within the blade passage itself. In this analysis it has been assumed that both β_R and $(\partial\beta/\partial r)_R$ vary linearly with x through the blade passage, so that β_R and $(\partial\beta/\partial r)_R$ at any point within the passage are given by

$$\left.\begin{aligned}\beta_R &= \beta_{R1} + (\beta_{R2}-\beta_{R1})\frac{x}{a}\\ \left(\frac{\partial\beta}{\partial r}\right)_R &= \left(\frac{\partial\beta}{\partial r}\right)_{R1} + \left[\left(\frac{\partial\beta}{\partial r}\right)_{R2} - \left(\frac{\partial\beta}{\partial r}\right)_{R1}\right]\frac{x}{a}.\end{aligned}\right\} \tag{30}$$

This assumption is very close for the central streamlines in a blade passage, and is exact for a row of infinitely thin blades designed with a parabolic centre-line, working at zero incidence. A more accurate variation could be obtained by analysing the pressure distribution round the cascade blade and putting in the correct variation of mean flow angle.

Substituting equation (30) in equation (16) yields

$$\frac{d^2\lambda}{dx^2} - (L+Px+Qx^2)\lambda = W + Ux + Vx^2, \qquad 0 \leqslant x \leqslant a \tag{31}$$

where

$$\begin{aligned}
k^2a^2L &= 1 + \beta_{R1}{}^2\\
k^2a^3P &= 2\beta_{R1}(\beta_{R2}-\beta_{R1})\\
k^2a^4Q &= (\beta_{R2}-\beta_{R1})^2\\
k^2a^2W &= \left(\frac{1}{m} - \beta_{R1}\right)^2 - G + \left(R\beta\frac{\partial\beta}{\partial r}\right)_{R1}\\
k^2a^3U &= R\beta_{R1}\left[\left(\frac{\partial\beta}{\partial r}\right)_{R2} - \left(\frac{\partial\beta}{\partial r}\right)_{R1}\right] + R\left(\frac{\partial\beta}{\partial r}\right)_{R1}(\beta_{R2}-\beta_{R1}) - 2\left(\frac{1}{m} - \beta_{R1}\right)(\beta_{R2}-\beta_{R1})\\
k^2a^4V &= (\beta_{R2}-\beta_{R1})^2 + R(\beta_{R2}-\beta_{R1})\left[\left(\frac{\partial\beta}{\partial r}\right)_{R2} - \left(\frac{\partial\beta}{\partial r}\right)_{R1}\right].
\end{aligned}$$

This is a linear differential equation for λ, the solution of which will contain two arbitrary constants, one of which is determined by the conditions at entry to the passage and the other by the conditions at exit from the passage.

If we assume that the isolated blade row is preceded by an inlet duct and followed by an outlet duct, it follows from equations (25) and (26) that the solution of equation (31) at inlet to the blade passage must satisfy the equation,

$$k'a\frac{d\lambda}{dx} = (\lambda - \lambda_{\infty u}). \tag{32}$$

Similarly at outlet from the blade row, the solution must satisfy

$$k'a\frac{d\lambda}{dx} = -(\lambda - \lambda_{\infty d}) \tag{33}$$

where $\lambda_{\infty u}$ and $\lambda_{\infty d}$ are calculated from equations (16), (20), (22) and (23). Since, in general, it will not be possible to obtain an exact analytical solution of equation (31) for a blade passage, an approximate solution (e.g. a power-series solution) must be found, or a numerical method of solution may be used. In each case the method of attack is the same. An initial guess for the value of λ at inlet to the blade passage is made, and $(d\lambda/dx)$ is calculated from equation (32). Solving equation (31) then yields the values of λ and $(d\lambda/dx)$ at exit from the blade passage. These values should satisfy equations (23) and (33), which are used to provide a closer initial guess for λ. Further iterations are performed until a solution is found which satisfied the conditions at exit.

The solution for the flow through an isolated blade row followed closely by a row of outlet guide vanes can readily be obtained. In this case the value of ω in equation (23) for the outlet duct becomes zero, and assuming the gap between the two blade rows to be small the problem assumes a linear form. To solve this two initial guesses for the value of λ at inlet are made, and the respective values of $(d\lambda/dx)$ are calculated from equation (32). The two solutions at exit are scaled by a factor b and substituted in equation (33), producing a linear equation for b. Knowing the scaling factor b the true values of λ at every point on the mean radius within the blade passage can at once be obtained from the two initial solutions.

Some solutions for the flow in a blade passage have also been obtained using an alternative assumption about the variation of β_R and $(\partial\beta/\partial r)_R$ through the blade passage. By analogy with equation (22) for the flow in an annular duct, equation (16) for a blade passage may be written

$$k^2a^2\frac{d^2\lambda}{dx^2} - \lambda = -\lambda_\infty \tag{34}$$

where

$$\lambda_\infty = -\lambda\beta_R{}^2 - \left(\frac{W_R}{V_R}\right)^2 - \left(R\beta\frac{\partial\beta}{\partial r}\right)_R + G \tag{35}$$

and λ_∞ is assumed to vary linearly through the blade passage so that

$$\lambda_\infty = \lambda_{\infty u} + (\lambda_{\infty d} - \lambda_{\infty u})\frac{x}{a}, \tag{36}$$

then the solution for λ is

$$\lambda = C_1\sinh\left(\frac{x}{ka}\right) + C_2\cosh\left(\frac{x}{ka}\right) + \lambda_{\infty u} + (\lambda_{\infty d} - \lambda_{\infty u})\frac{x}{a} \tag{37}$$

where C_1 and C_2 are constants. Calculating these in terms of the inlet values of λ and $(d\lambda/dx)$ gives

$$C_1 = ka\left(\frac{d\lambda}{dx}\right)_1 - k(\lambda_{\infty d} - \lambda_{\infty u}),$$

$$C_2 = \lambda_1 - \lambda_{\infty u}.$$

Thus the value of λ at exit from the blade row is given by

$$\lambda_2 = \left\{ka\left(\frac{d\lambda}{dx}\right)_1 - k(\lambda_{\infty d} - \lambda_{\infty u})\right\}\sinh\left(\frac{1}{k}\right) + (\lambda_1 - \lambda_{\infty u})\cosh\left(\frac{1}{k}\right) + \lambda_{\infty d}. \tag{38}$$

The values of $\lambda_{\infty u}$ and $\lambda_{\infty d}$ are calculated from equations (16) and (34) and the same method of solving the problem is used as was described in the previous paragraph. It is interesting to note from Fig. 7 that (for a multi-stage compressor) the values of λ obtained using this assumption agree very closely with the values obtained using the assumption of a linear variation of tan α. Since this assumption leads to a vast saving of arithmetic and seems to be quite accurate it is recommended for use when a digital computer is not available. However, since equation (35) contains λ, which is the unknown which we are trying to find, the physical meaning of the assumption in equation (36) must always be dubious, and when a computer is available the assumption of linear variation of β_R and $(\partial\beta/\partial r)_R$ is preferred.

6. *Flow in a Multi-Stage Compressor—General Case.*

A compressor consists of alternate rotating and stationary rows of blades, in each of which equation (16) is true. Thus the solution of this equation for a complete compressor consists primarily of solving the equation for each individual blade row in turn. On passing from one blade passage into the next continuity of axial, radial and whirl velocities must exist, and in addition the static-pressure gradient (g) must be the same. Therefore, if for the present the axial gaps between the blade rows are neglected, the values of β_R and $(\partial\beta/\partial r)_R$ at entry to a blade passage can at once be written in terms of the given values at exit from the preceding passage.

For a stator, the whirl velocity is given by $w = \beta v$. For a rotor, the whirl velocity is $w = -\beta v + \Omega r$. Therefore from Fig. 3, at exit from a rotor row,

$$-\beta_2 v + \Omega r = \beta_3 v$$

so that

$$\beta_3 = -\beta_2 + \frac{\Omega r}{v},$$

and

$$\frac{\partial\beta_3}{\partial r} = -\frac{\partial\beta_2}{\partial r} + \frac{\Omega}{v} - \frac{\Omega r}{v^2}\frac{\partial v}{\partial r}.$$

Thus at the design radius, putting $r = R$ and $m = V_R/\Omega R$,

$$\beta_3 = -\beta_2 + \frac{1}{m} \tag{39}$$

and

$$R\left(\frac{\partial\beta_3}{\partial r}\right)_R = -R\left(\frac{\partial\beta_2}{\partial r}\right)_R + \frac{1}{m} - \frac{\lambda_2}{m}. \tag{40}$$

Similar expressions can be obtained for a stator exit-rotor entry interface.

Since g is constant at the interface between two blade rows the value of the stagnation-pressure gradient G in a blade passage can be found in terms of the value of G in the previous blade passage by writing equation (14) for both blade passages at the interface. Finally λ and $(d\lambda/dx)$ must be continuous from one blade passage to the next. Thus if a solution of equation (16) has been found for one blade passage of a multi-stage compressor, a solution for a blade passage in the next row can readily be obtained.

It is not necessary to neglect the axial gaps between the blade rows when considering a multi-stage machine. Having obtained a solution in a blade passage, the theory of Section 4 can be used to find the solution in the gap after the passage. This will give values of λ and $(d\lambda/dx)$ at entry to the next blade passage, and these values together with equations (14), (39) and (40) will enable the solution to be continued through this passage.

Equation (16) is linear within each blade passage and in the axial gaps between the blade rows and at exit from the compressor, but the boundary conditions are non-linear, since the value of the constant k' for each of these sections depends on the value of λ at entry to the section. However, the axial length of the gaps between the blade rows is usually very small compared with the axial length of the blades, so that they are usually neglected. If they are not neglected the solution within each gap is given by equation (24) as

$$\lambda = \lambda_\infty + Ae^{x/k'a} + Be^{-x/k'a},$$

where

$$\left(\frac{1}{k'a}\right)^2 = \left(\frac{1}{ka}\right)^2 - \left(\frac{\omega}{R}\right)^2$$

Now (ω/R), given by equation (20), will in general be small compared with $(1/ka)$ which is itself small. Since x is also small, $(x/k'a)$ will be much less than unity so that within the gap, equation (24) may be written

$$\lambda = \lambda_\infty + A\left(1 + \frac{x}{k'a}\right) + B\left(1 - \frac{x}{k'a}\right)$$

and the effect on λ of neglecting the modification to $1/ka$ will not be significant. Thus within the gap between blade rows it will be assumed that k' is equal to k.

At exit from the compressor the air will normally flow through a set of outlet guide vanes before entering the outlet duct. These guide vanes will remove the swirl component of velocity so that ω becomes zero in the outlet duct and k' assumes the constant value k. Consequently, the problem is linear within the whole compressor and a method of solution similar to that for a single blade row followed by a row of outlet guide vanes can be used. Two initial guesses are made for the value of λ at inlet to the first row of blades, and two complete solutions using these initial guesses are then obtained. The solutions at entry to the outlet duct are scaled linearly to fit the boundary conditions at that point expressed by equation (26) in the form

$$k'a\frac{d\lambda}{dx} = -(\lambda - \lambda_\infty).$$

The scaling factor b thus derived is used to calculate the correct solution at that point and thus at every point within the compressor.

The special case when there are no outlet guide vanes (and thus k' is not equal to k in the outlet duct) can still be treated in the same manner, since k' only occurs in the solution when the scaling factor b is being calculated at exit from the final blade row. This is given by

$$b = \cfrac{1}{1 - \cfrac{\left(\frac{d\lambda}{dx}\right)_1 + \frac{1}{k'a}(\lambda_1 - \lambda_{\infty 1})}{\left(\frac{d\lambda}{dx}\right)_2 + \frac{1}{k'a}(\lambda_2 - \lambda_{\infty 2})}}$$

where the subscripts 1 and 2 refer to the two initial solutions. Since $(d\lambda/dx)$ and $(\lambda - \lambda_\infty)$ will be very large at this point for any initial guess which is not the exact initial value of λ, a small change in the small number $1/k'a$ will not significantly affect b, and so k' may be assumed to have the value k.

Since the solution for a multi-stage compressor is basically a series of solutions for individual blade rows, the problem of the flow through a multi-stage machine lends itself very readily to an efficient digital-computer solution. A programme for the solution of equation (16) in any compressor has been written for use on the EDSAC 2 computer of the Cambridge University Mathematical Laboratory. Details of this programme are given in Appendix I. This programme has been used to obtain the flow patterns through several compressor designs, the results of which are shown in Figs. 7 to 14.

7. *Flow in a Multi-Stage Compressor—Identical Repeating Stages.*

In the previous section the flow through a general multi-stage compressor has been considered, and it has been shown how the problem can be solved numerically. A numerical solution, of necessity, involves dividing each section into small intervals, and solving the equation step by step. However, for the case of a set of identical repeating stages a method of solution has been evolved which will provide values of λ at the leading and trailing edges of all the blade rows, without having to solve the differential equation at any intermediate points within the blade rows. It will be assumed that the compressor under analysis consists of a number of identical stages which may be preceded by several stages which do not form part of the repeating group. The compressor will be terminated by a row of outlet guide vanes followed by a duct. The axial gaps between the blade rows will be neglected.

Consider a row of rotor blades. The linearity of equation (16) permits the solution at the trailing edge (station 2) to be written in terms of that at the leading edge (station 1) in the form

$$\left.\begin{aligned} \lambda_2 &= a_1\lambda_1 + a_2\left(a\frac{d\lambda}{dx}\right)_1 + a_3 g_1 + a_4 \\ \left(a\frac{d\lambda}{dx}\right)_2 &= a_5\lambda_1 + a_6\left(a\frac{d\lambda}{dx}\right)_1 + a_7 g_1 + a_8 \\ g_2 &= a_9\lambda_1 + a_{10}\left(a\frac{d\lambda}{dx}\right)_1 + a_{11} g_1 + a_{12} \end{aligned}\right\} \qquad (41)$$

where a_1 to a_{12} are constants and a is the axial-chord length of the blade row.

Similarly for a stator row the solution can be written

$$\left.\begin{aligned} \lambda_4 &= b_1\lambda_3 + b_2\left(a\frac{d\lambda}{dx}\right)_3 + b_3 g_3 + b_4 \\ \left(a\frac{d\lambda}{dx}\right)_4 &= b_5\lambda_3 + b_6\left(a\frac{d\lambda}{dx}\right)_3 + b_7 g_3 + b_8 \\ g_4 &= b_9\lambda_3 + b_{10}\left(a\frac{d\lambda}{dx}\right)_3 + b_{11} g_3 + b_{12} \end{aligned}\right\}. \qquad (42)$$

Now a column matrix, X_n, is defined such that

$$X_n = \begin{bmatrix} \lambda_n \\ (ad\lambda/dx)_n \\ g_n \end{bmatrix} \qquad (43)$$

and in addition define matrices A, D, B, E such that

$$A \equiv \begin{bmatrix} a_1 & a_2 & a_3 \\ a_5 & a_6 & a_7 \\ a_9 & a_{10} & a_{11} \end{bmatrix} \qquad D \equiv \begin{bmatrix} a_4 \\ a_8 \\ a_{12} \end{bmatrix}$$

$$B \equiv \begin{bmatrix} b_1 & b_2 & b_3 \\ b_5 & b_6 & b_7 \\ b_9 & b_{10} & b_{11} \end{bmatrix} \qquad E \equiv \begin{bmatrix} b_4 \\ b_8 \\ b_{12} \end{bmatrix}.$$

Then equations (31) and (32) may be written in the form

$$X_2 = AX_1 + D \tag{44}$$

$$X_4 = BX_3 + E. \tag{45}$$

If now, in the usual way for a compressor, a 'stage' is defined to be a rotor row followed by a stator row, then by continuity of λ, $(d\lambda/dx)$ and g at the rotor-stator interface, and neglecting the axial gap between the blade rows,

$$X_2 \equiv X_3.$$

Thus by combination of equations (44) and (45), there follows for a stage

$$X_4 = CX_1 + H \tag{46}$$

where

$$\left.\begin{aligned} C &= BA \\ H &= BD + E. \end{aligned}\right\} \tag{47}$$

and

Therefore, writing ${}_{(n)}X$ as the value of X at entry to the nth repeating stage,

$${}_{(n+1)}X = C\,{}_{(n)}X + H. \tag{48}$$

This can be written in the form

$${}_{(n+1)}X' = C\,{}_{(n)}X' \tag{49}$$

where

$${}_{(n)}X' = {}_{(n)}X - M$$

and

$$M = (1-C)^{-1}H.$$

Hence it follows that

$${}_{(n)}X' = C^n\,{}_{(1)}X'. \tag{50}$$

Let the eigenvalues of C be μ_1, μ_2 and μ_3 with corresponding eigenvectors Ξ_1, Ξ_2, Ξ_3. Thus ${}_{(1)}X'$ can be written as the sum of the three eigenvectors in the form

$${}_{(1)}X' = \gamma_1\Xi_1 + \gamma_2\Xi_2 + \gamma_3\Xi_3$$

where γ_1, γ_2 and γ_3 are constants.

And since, by definition, $C\Xi_1 = \mu_1\Xi_1$ it follows that

$${}_{(n)}X' = \gamma_1\mu_1^{n-1}\Xi_1 + \gamma_2\mu_2^{n-1}\Xi_2 + \gamma_3\mu_3^{n-1}\Xi_3 \tag{51}$$

and therefore

$${}_{(n)}X = \gamma_1\mu_1^{n-1}\Xi_1 + \gamma_2\mu_2^{n-1}\Xi_2 + \gamma_3\mu_3^{n-1}\Xi_3 + M. \tag{51a}$$

There are now three conditions available to determine the three constants γ_1, γ_2, γ_3. Two of these conditions are at entry to the compressor. One is the relation between λ and $(d\lambda/dx)$ given by equation (32), and the other is the condition for the correct stagnation-pressure gradient at inlet given by equation (14). The third condition which must be satisfied is the relation between λ and $(d\lambda/dx)$ given by equation (33).

The three terms on the right-hand side of equation (51) may now be regarded as disturbance terms to the regular flow pattern, these being significant at the ends of the compressor only. Thus for a compressor with a large number of repeating stages, two of these terms will be significant at the front and become small towards the middle, and the third will be significant at the back and likewise become small towards the middle. If all the eigenvalues are real then

$$-1 < \mu_1 < 1$$

$$-1 < \mu_2 < 1$$

and

$$\mu_3 < -1 \text{ or } \mu_3 > +1 .$$

However, in practice it has usually been found that μ_1 and μ_2 are complex conjugates and will be denoted by $\phi \pm i\psi$. In this case the modulus of the complex roots must be less than unity. For complex conjugate eigenvalues the eigenvectors will also be complex conjugates, which will be denoted by $\Lambda \pm i\Pi$.

Using these definitions, the solution at entry to the first repeating stage can be written as

$$_{(1)}X' = \gamma_1(\Lambda + i\Pi) + \gamma_2(\Lambda - i\Pi) \tag{52}$$

and the solution at exit from a compressor of n repeating stages is

$$_{(n+1)}X' = \gamma_3 \mu_2{}^n \Xi_3 . \tag{53}$$

Since $_{(1)}X'$ must necessarily be real, it follows at once from equation (52) that γ_1 and γ_2 are complex conjugates, which will be represented by $\rho \pm i\sigma$.

Now, if λ_0 is the value of λ at entry to the compressor,

then

$$X_0 = \lambda_0 \begin{bmatrix} 1 \\ 1/k \\ -1 \end{bmatrix},$$

and it can be seen that the value of X after all the stages which precede the repeating stages (i.e. the value of X at entry to the first repeating stage) may be expressed in the form

$$_{(1)}X = \lambda_0 P \begin{bmatrix} 1 \\ 1/k \\ -1 \end{bmatrix} + Q \tag{54}$$

where P is a 3 by 3 matrix and Q is a 3 by 1 matrix devised from the matrices which define the blade rows preceding the repeating stages.

Therefore, equating equations (52) and (54) the matrix equation

$$(\rho + i\sigma)(\Lambda + i\Pi) + (\rho - i\sigma)(\Lambda - i\Pi) = \lambda_0 P \begin{bmatrix} 1 \\ 1/k \\ -1 \end{bmatrix} + Q - M \tag{55}$$

is obtained.

This can be written in the form

$$\left[2\Lambda, -2\Pi, P\begin{bmatrix}1\\1/k\\-1\end{bmatrix}\right]\begin{bmatrix}\rho\\\sigma\\-\lambda_0\end{bmatrix} = \left[Q - M\right]$$

or

$$\begin{bmatrix}\rho\\\sigma\\-\lambda_0\end{bmatrix} = \left[2\Lambda, -2\Pi, P\begin{bmatrix}1\\1/k\\-1\end{bmatrix}\right]^{-1}\left[Q - M\right]. \tag{56}$$

This equation determines λ_0, ρ and σ. Hence, the solution at the front of the compressor and the constants γ_1 and γ_2 have been found, so that there remains only γ_3 undetermined.

To find γ_3, equation (33) expressed at exit from the outlet guide vanes is used. Since ω is zero in the outlet duct, k' has the constant value k and equation (33) can be expressed in the form

$$a\frac{d\lambda}{dx} = \frac{1}{k}(g - \beta^2). \tag{57}$$

If J and K are respectively the 3×3 and 1×3 matrices governing the flow through the outlet guide vanes, the matrix X at exit from these blades is given by X_e, where

$$X_e = J(\gamma_3\mu_3{}^n\Xi_3 + M) + K$$

which can be written as

$$X_e = \gamma_3\mu_3{}^n(J\Xi_3) + (JM + K). \tag{58}$$

If equation (58) is now written in terms of the matrix components in the form

$$X_e = \gamma_3\mu_3{}^n\begin{bmatrix}\zeta_1\\\zeta_2\\\zeta_3\end{bmatrix} + \begin{bmatrix}\xi_1\\\xi_2\\\xi_3\end{bmatrix}$$

equation (57) can be used to obtain the equation

$$\gamma_3\mu_3{}^n[k\zeta_2 - \zeta_3] = [\xi_3 - k\xi_2] - \beta^2. \tag{59}$$

Equation (59) yields γ_3 directly.

This matrix analysis furnishes a quick method for obtaining the solution of equation (16) at the trailing edges of all blade rows in a multi-stage compressor of identical stages. First the coefficients of the matrices A, B, D, E are obtained for an isolated rotor row and an isolated stator row using a numerical or approximate analytical method. The eigenvalues and eigenvectors of the three by three matrix C are next found, and then the constants γ_1, γ_2 and γ_3 are evaluated. Equation (51a) gives the value of X after every repeating stage, and equation (44) gives the values of X after the rotor rows.

For the special case of 50% reaction stages, the values of X' after the rotor rows can be found by taking n as the corresponding half-integer {e.g. the value of n to give the solution after the nth rotor is $(2n+1)/2$}. This follows because the matrices A and B are identical, and thus the eigenvalues of A are the square roots of the eigenvalues of C, the eigenvectors remaining the same. However, instead of adding the matrix M to ${}_{(n)}X'$ to get ${}_{(n)}X$ in this case we must add the matrix $[AM + D]$.

This matrix method of solution is considerably quicker than the numerical method, and is particularly advantageous for a compressor having a large number of identical repeating stages. The main disadvantage of the method is that it is only possible to obtain solutions at the leading and trailing edges of each blade row, and thus no indication is given about the form of the solution within the blade passages themselves. The method of solving equation (16) described in this section has also been programmed for use on the EDSAC computer, and details of this programme are given in Appendix II. Results obtained by this method have been found to agree exactly with results obtained by the longer numerical method of solution.

Simple calculation of the regular flow pattern.

The regular flow pattern which occurs in the middle of a multi-stage compressor with many stages may be obtained by evaluating the matrix M, but the result may be obtained much more conveniently by starting from the basic equations for flow in a blade row. It will be assumed in this section that the aspect ratio of the blades is high, so that the change in λ across either the rotor or the stator is small, and

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda .$$

Since the flow is periodic it will also be assumed that

$$\left(\frac{d^2\lambda}{dx^2}\right)_1 = -\left(\frac{d^2\lambda}{dx^2}\right)_2 = -\left(\frac{d^2\lambda}{dx^2}\right)_3 = \left(\frac{d^2\lambda}{dx^2}\right)_4 .$$

Also, since there is no change of stagnation-pressure gradient across each blade row,

$$G_1 = G_2, \qquad G_3 = G_4 .$$

Equations (39) and (40) relate the flow angles at stator inlet to the conditions at rotor outlet. Since the pattern is periodic, similar equations relate the conditions at rotor inlet to conditions at stator outlet, namely

$$\beta_1 = \frac{1}{m} - \beta_4 ,$$

and

$$R\left(\frac{\partial\beta_1}{\partial r}\right)_R = -R\left(\frac{\partial\beta_4}{\partial r}\right)_R + \frac{1}{m} - \frac{\lambda_2}{m} .$$

If equation (16) is written for each of stations 1, 2, 3 and 4, there are then sufficient equations for the unknowns to be found, and the result for λ is

$$\lambda = \frac{\dfrac{2}{m} - (\beta_2+\beta_4) - \left(R\dfrac{\partial\beta_2}{\partial r} + R\dfrac{\partial\beta_4}{\partial r}\right)}{\beta_2 + \beta_4} . \qquad (60)$$

The corresponding result for the gradient of stagnation pressure relative to stators is

$$G_3 = \frac{1}{2(\beta_2+\beta_4)}\left[\frac{4}{m} - 2\beta_2 - 2\beta_4 + \frac{2}{m^2}\beta_4 + \frac{2}{m}\beta_4{}^2 - \frac{2}{m}\beta_2\beta_4 + \right.$$
$$+ R\frac{\partial\beta_2}{\partial r}\left(-2 - \beta_4{}^2 + \beta_2\beta_4 - \frac{1}{m}\beta_4\right) +$$
$$\left. + R\frac{\partial\beta_4}{\partial r}\left(-2 - \beta_2{}^2 + \beta_2\beta_4 + \frac{1}{m}\beta_2\right)\right] . \qquad (61)$$

For the special case of 50% reaction at all radii

$$\beta_2 = \beta_4 = \beta$$

$$\left(\frac{\partial\beta}{\partial r}\right)_2 = \left(\frac{\partial\beta}{\partial r}\right)_4 = \left(\frac{\partial\beta}{\partial r}\right).$$

Then

$$\lambda = \frac{1}{m\beta} - 1 - \frac{R}{\beta}\frac{\partial\beta}{\partial r} \tag{62}$$

and

$$G_3 = \frac{1}{m\beta} - 1 - \frac{R}{\beta}\frac{\partial\beta}{\partial r} + \frac{1}{2m^2}. \tag{63}$$

These results are particularly useful in the preliminary design calculations for a compressor.

8. *Discussion of Results.*

8.1. *Model Compressor.*

To illustrate the method of using the theory presented in the foregoing sections, it has been applied to calculate the flow through a hypothetical ten-stage compressor both in the 'as-designed' condition and for conditions when various modifications are made to the compressor geometry and to the mass-flow rate through the compressor.

The machine consists of a row of inlet guide vanes followed by ten identical stages, the axial gaps between the blade rows being ignored. It is presumed that the blading has been designed by assuming constant axial velocity throughout (i.e. $\lambda = 0$), equal work done at all radii, and 50% reaction at all radii. The rotor blades are therefore mirror images of the stator blades.

The information relevant to the specification of the compressor is summarized below.

Constant tip radius	t	12 in.
Constant hub radius	h	6 in.
Angular velocity of rotor	Ω	1208 r.p.m.
Axial-chord length of all blades	a	1·6086 in.
Exit air angle relative to blade	α	30°
$\dfrac{\text{Stage work}}{(\text{Blade speed at design radius})^2}$	$\dfrac{\Delta H}{(\Omega R)^2}$	0·4
50% reaction at all radii.		

Hence,

Design radius	R	9·4868 in.
Blade speed at design radius	ΩR	100 ft/sec
Mass-flow parameter	m	0·5196
Aspect ratio		3·73
Blade twist at exit	$\partial\beta/\partial r$	0·1420.

This value of $\partial\beta/\partial r$ is obtained by putting $\lambda = 0$ in equation (62).

As-designed conditions.—The vortex flow for the hypothetical compressor under as-designed conditions is shown in Fig. 7. It is seen that the flow pattern can be divided into three distinct phases. First there is a basic periodic variation which persists right through the compressor; then superimposed on this basic flow are the other two phases. These consist of disturbances at inlet and exit caused by the conditions existing in the inlet and exit ducts respectively. Each of these phases of the flow will be considered in turn.

After the first four stages the variation in λ consists entirely of the basic periodic variation, called the regular flow. This regular flow has a wavelength equal to the axial length of a single stage and is of very small amplitude. This result verifies the assumption used by Traupel[20], Wu and Wolfenstein[22], and Schnittger[17], that the streamlines in a plane containing the axis of the machine vary in a periodic manner.

The regular flow comprises a radially inward deflection of the air through the second half of a rotor row and the leading half of a stator row, and a radially outward deflection through the latter half of a stator row and the leading half of a rotor row. These flow variations are small, and can normally be neglected for design purposes, since the overall effect is to increase by approximately $\frac{1}{4}°$ the air angles on to the leading edges of the blades at the stator tip and the rotor root, and to decrease by the same amount the air angles on to the leading edges at the rotor tip and stator root.

Now consider the disturbances at inlet to and exit from the compressor. As would be expected, the effect at inlet propagates downstream much farther than the effect at exit propagates upstream. As the air enters the compressor it is deflected radially inwards through the inlet guide vanes and the first-stage rotor, the maximum deflection occurring just before the trailing edge of these rotor blades. The radial deflection then gets progressively smaller, and through the third-stage rotor there is almost no variation of the flow in the radial direction. Through the third-stage stator there is a small outward radial deflection as the disturbance at inlet dies away and the flow settles down to the regular flow. The inward deflection of the air causes a maximum increase of $7\frac{1}{2}°$ in the air angle on to the tip and a decrease of $4\frac{1}{2}°$ on to the root at the leading edge of the first row of stator blades, compared with the design condition of zero deflection. As the air leaves the compressor it flows into an annular duct, the effect of which is to cause an outward flow of air in the last stator row, but it has negligible influence on the flow farther than one blade chord upstream.

Also shown in Fig. 7 are values obtained by the matrix method of solution described in Section 7, using the assumption of a linear variation of λ_∞ mentioned in Section 5. The agreement between the two solutions is close.

Effect of varying the mass flow.—Fig. 8 shows the effect of varying the mass-flow rate through the hypothetical compressor. All the solutions exhibit the same three characteristics, namely, an initial disturbance which settles down after the fourth stage to a regular periodic flow, followed by an outward deflection of the flow in the stator row of the final stage.

As the mass-flow rate is decreased from the design flow rate the maximum inward deflection of the air in the inlet guide vanes and first rotor row is decreased, but a large outward deflection is induced in the second and third stages. After the fourth stage the periodic flow is such that more air is going through the tip section than the root section. This will tend to stall the root section more than the tip. It will be noticed that as the mass flow is decreased so the amplitude of the periodic variation increases. This is caused by the air having greater difficulty in trying to attain radial equilibrium conditions for the smaller mass-flow rates.

For mass-flow rates greater than the designed flow rate the maximum inward radial deflection is increased, and the flow eventually settles down with more going through the root section than the tip. It appears from Fig. 8 that as the mass-flow rate is increased the amplitude of the periodic variation decreases, so that the flow through a stage becomes more uniform although radially along a blade row there is more air going through the root than the tip.

Modifications to the inlet guide vanes.—The considerable inward flow set up in the first few stages of the hypothetical compressor running under as-designed conditions is a departure from the design assumptions and is therefore very undesirable. Consequently a simple method of reducing this deviation from the zero-slope design condition was sought. Two methods were tried, the first being a complete removal of the inlet guide vanes and the second a modification to the exit air angles of these blades. The results of these investigations are shown in Fig. 9.

It is seen that removing the inlet guide vanes has the effect of replacing the inward radial deflection by an almost equal outward deflection. In this case the maximum deflection occurs just before the trailing edge of the first row of stator blades. As in the as-designed case, the flow settles down to a periodic variation after the large deflection, and becomes identical to the original flow pattern after the fifth stage.

The reason for the inward deflection of the flow is that a non-constant stagnation-pressure profile is required in the later stages in order that equilibrium may be attained. This stagnation-pressure gradient is given by equation (63). Since the stages are designed for constant work, the first stages must depart from their design conditions in order to produce the required stagnation-pressure profile. If therefore the inlet guide vanes are designed so that the required stagnation-pressure gradient is produced in the first stage of the compressor, the departure from the design axial-velocity profile will be reduced. Consequently the air angles out of the inlet guide vanes were modified to produce this effect. This was done by changing the twist on the trailing edges of the guide vanes, keeping the exit angle of 30° at the design radius.

The rise of stagnation pressure in the first stage is

$$\frac{\Delta P}{\rho} = \Omega r(w_2 - w_1)$$

or

$$\frac{\Delta P}{\rho} = \Omega r(\Omega r - \beta_2 v - \beta_e v)\,, \tag{64}$$

where the subscript e denotes conditions at the trailing edge of the inlet guide vanes. Since

$$G = \frac{R}{\rho V_R^2}\left(\frac{\partial P}{\partial r}\right)_R,$$

equation (64) can be differentiated to give (after putting $\lambda = 0$)

$$G = \frac{2}{m^2} - \frac{1}{m}\left(\beta_2 + \beta_e + R\frac{\partial\beta_2}{\partial r} + R\frac{\partial\beta_e}{\partial r}\right). \tag{65}$$

The hypothetical compressor under analysis has been designed for 50% reaction at all radii. Consequently,

$$\beta_2 = \beta_e = \beta_4$$

and

$$\frac{\partial\beta_2}{\partial r} = \frac{\partial\beta_4}{\partial r}$$

and the value of $(\partial\beta_e/\partial r)$ can be found using equations (62), (63) and (65). In this case the value is $0\cdot04057$, compared with the as-designed value of $0\cdot14200$, and the twist on the inlet guide vanes is considerably reduced.

Using the modified inlet guide vanes, the resulting flow shown in Fig. 9 is seen to possess a much smaller inward deflection in the first two blade rows. As in the two previous cases the flow assumes a periodic form after the first four stages, and again becomes identical with the original design flow after the fifth stage.

Alternative design.—Another possible basis for the design of a compressor is to make the stagnation-pressure gradient zero in the regular flow. This would have the advantage of giving nearly constant stagnation pressure at outlet from the compressor. The twist on the blades has therefore been altered to achieve this condition. The required twist is obtained by putting $G = 0$ in equation (63) giving

$$\frac{\partial\beta}{\partial r} = \frac{\beta}{r}\left(\frac{1}{m\beta} - 1 + \frac{1}{2m^2}\right) = 0\cdot25470\,.$$

Comparing this with the value of $0\cdot14200$ for the constant-axial-velocity design, it is seen that the constant stagnation-pressure design requires blades with a much higher degree of twist.

The design value of λ is given from equation (62) as

$$\lambda = -\frac{1}{2m^2} = -1\cdot852\,.$$

This design axial-velocity profile is in fact the mean of the radial equilibrium profiles before and after a rotor row, assuming constant stagnation pressure with radius.

Fig. 10 shows the computed flow for this compressor. It is seen that the design condition is reached after about 3 stages.

8.2. *Comparison with Existing Data.*

An attempt has been made to compare the theory with experimental results obtained by Horlock[6] and with the theoretical results of Wu[25] for single-stage compressors of constant hub and tip radii. The results of these investigations are presented in Figs. 11 to 14.

The compressor analysed by Horlock had a hub-to-tip ratio of $0\cdot4$, and consisted of a row of inlet guide vanes followed by the single stage. This was followed by an annular duct about nine axial-chords long, which was terminated by a row of outlet guide vanes. Calculations were performed for the compressor both with and without the outlet guide vanes, and, as would be expected, it was found that these guide vanes had no effect on the slope of the axial-velocity profile within the three blade rows constituting the compressor.

Two sets of calculations have been performed, one using the air angles predicted by Horlock for the actuator-disc theory, and the other using air angles observed by that author during the experiments. The variations in λ through the compressor for each of these cases are shown in Fig. 11 together with approximate values of λ calculated from Horlock's experimental axial-velocity profiles. Fig. 12 shows the axial velocities within the compressor at values of the hub-to-tip ratio of $0\cdot6$, $0\cdot7616$ (the design value) and $0\cdot9$. At each of these values four axial-velocity variations are presented. These consist of two curves obtained from the calculations using the two sets of air angles mentioned above, the experimental values obtained by Horlock and finally values predicted by that author using the actuator-disc theory with the discs placed at the centres of pressure of the blades. It is seen that the agreement between experiment and the vortex-flow theory is, in this case, comparable with the agreement between experiment and the actuator-disc theory.

Wu[25] obtained the solution of the incompressible flow through a single-stage compressor. He considered a compressor consisting of inlet guide vanes, a rotor row and a stator row, each blade having a hub-to-tip ratio of 0·6 and an aspect ratio (based on axial-blade length) of 2·67. The ratio of inlet velocity to rotor tip speed was assumed to be 0·7378. In order to proceed with the solution Wu had first to specify the axial rate of change of wr on the mean stream surface. Having done this he obtained solutions for the axial velocity at all points within the compressor.

The air angles obtained by Wu have been used by the Authors to obtain a solution for the flow through the specified compressor by means of the present theory. The slope of the axial-velocity profile is shown in Fig. 13 and the axial velocities at three radii derived from this solution for λ are shown in Fig. 14. In this case the radius and axial velocities have been rendered dimensionless by dividing by the tip radius and the rotor tip speed respectively. Also plotted in this figure are the axial velocities obtained by Wu.

For this compressor the value of r/t at the design radius is 0·82462. In the neighbourhood of this design radius the axial velocities derived by the two methods agree very closely, while at the hub and tip the values again agree closely, differing only in the stator row where the greatest discrepancy is 5% at the hub. These results verify the fundamental assumption that the axial-velocity profile can be represented by a single-parameter family of curves. The agreement is particularly good, since the vortex-flow theory assumes that the chosen axial-velocity profile is true only in the vicinity of the design radius, where the equations of motion are satisfied. Hence the close agreement attained in the neighbourhood of the design radius would be expected. The fairly close agreement between the two axial velocities at points on the blade well away from the design radius shows that the assumed axial-velocity profile is acceptable over a much greater proportion of the blade length.

9. *Conclusions.*

The comparison of the results of Wu's[25] calculation and Horlock's[6] experiment with the present theory shows good agreement, and it is suggested that the present theory is quite sufficiently accurate for design and performance calculations on axial-flow turbo-machines. It is also suggested that greater accuracy of the calculations is not warranted unless an adequate means of calculating the effects of the boundary layers on the inside and outside walls of the machine is included. The limitation to a one-parameter family of axial-velocity profiles leads to a great simplification of ideas about how these flows behave. An illustration of this is the example showing how the first stage of a compressor may be designed to do varying work with radius, in order to achieve a more nearly constant axial velocity in the first few stages.

The calculation may very easily be programmed for digital computers. EDSAC 2 takes about five minutes to calculate a ten-stage compressor. An alternative assumption, which is recommended if no computer is available, has also been given, and this method appears to give equally accurate results, although the physical basis for the assumption is not entirely satisfactory.

The calculations for a model compressor with identical stages show how the flow consists of a regular pattern which repeats from stage to stage, together with perturbations at the front and the back of the compressor. The flow fluctuation in the regular repeating pattern is small and can probably be neglected in design, and the outlet perturbation is also small, but the inlet perturbation should certainly be included in design calculations. The effect of varying mass flow has been studied, and also the effect of modifications to the design of the inlet guide vanes, and to the basis of design of the vortex flow in the compressor. It is found that if a compressor is designed to have any constant axial-velocity profile, and is designed to do constant work with radius, then the design profile will be achieved after the first few stages. There will however be substantial deviations from design in the first few stages, and the stagnation pressure will vary with radius at exit.

Further work now being undertaken is to include the effects of taper of the inside and outside walls of the machine, and also the effect of compressibility. These are known to be effects of importance in turbines and in the first stages of axial compressors. Other effects which could be taken into account but which are probably of minor importance are the effects of the radial force exerted by the blades, the effect of taper of blade thickness, and the accurate representation of the blade loading working from the pressure distribution round the blades, which could be obtained from cascade tests or by calculation.

10. *Acknowledgment.*

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NOTATION

The notation used is illustrated in Figs. 1, 2 and 3.

General Notation

r		Radial co-ordinate
x		Axial co-ordinate
θ		Tangential co-ordinate
u		Radial component of velocity
v		Axial component of velocity
w		Tangential component of velocity
a		Axial chord of blade
g		Static-pressure gradient
G		Stagnation-pressure gradient relative to the blade
h		Hub radius
k		Constant defined by equation (8)
m	=	$V_R/\Omega R$, mass-flow parameter
p		Static pressure
P		Stagnation pressure
R	=	$\{\frac{1}{2}(t^2+h^2)\}^{1/2}$, design radius
t		Tip radius
α		Air angle relative to the blade
β	=	$\tan\alpha$
λ	=	$\frac{R}{V_R}\left(\frac{\partial v}{\partial r}\right)_{r=R}$, slope of axial-velocity profile at the design radius
ρ		Air density
ω		A measure of rotation in the flow
Ω		Angular velocity of rotor

Matrix Notation

A	3×3	matrices for flow through rotor row
D	3×1	
B	3×3	matrices for flow through stator row
E	3×1	

Matrix Notation—continued

C	$3 \times 3 = B \times A$
H	$3 \times 1 = B \times D + E$
M	$3 \times 1 = (1-C)^{-1}H$
P	3×3 } matrices defining flow through all stages preceding repeating stages
Q	3×1 }
X	$3 \times 1 = \{\lambda, a\, d\lambda/dx, g\}$
μ_1, μ_2, μ_3	Eigenvalues of C
Ξ_1, Ξ_2, Ξ_3	Associated eigenvectors of C
Λ	Real part of Ξ_1 and Ξ_2
Π	Imaginary part of Ξ_1 and Ξ_2
ϕ	Real part of μ_1 and μ_2
ψ	Imaginary part of μ_1 and μ_2, and also used for the stream function
$\gamma_1, \gamma_2, \gamma_3$	Constants used in defining flow
ρ	Real part of γ_1 and γ_2
σ	Imaginary part of γ_1 and γ_2
$\zeta_1, \zeta_2, \zeta_3$ } ξ_1, ξ_2, ξ_3 }	Components of 3×1 matrices used in calculating γ_3

Suffices

R	Conditions at the design radius
0	Conditions at entry to the compressor
1	Conditions at entry to a rotor row
2	Conditions at exit from a rotor row
3	Conditions at entry to a stator row
4	Conditions at exit from a stator row

Prefixed subscript

(n)	Conditions at entry to the nth repeating stage in a multi-stage compressor

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APPENDIX I

Numerical Solution on the EDSAC Computer

A programme for the EDSAC computer has been written which will solve the basic differential equation {equation (16)} for any compressor consisting of rotor rows, stator rows and small lengths of duct. A Runge-Kutta-Gill numerical method has been used to furnish the solution, which consists of the values of λ and $(d\lambda/dx)$ everywhere within the compressor under consideration. Two initial guesses are made for the value of λ at inlet to the compressor, and using these values two complete solutions are evaluated simultaneously throughout the whole compressor. These solutions are then scaled to satisfy the exit condition given by equation (26), and the scaling factor so obtained is used to provide the exact solution everywhere within the compressor.

To enable the solution to proceed, certain data about the compressor under consideration must be provided, and this must be done in a definite format. This consists of a block of general data about the compressor, followed by small data blocks for each of the compressor components, arranged in the order the components appear in the machine.

The general data block must supply the following quantities: hub radius, tip radius, axial velocity at entry, angular velocity of rotor, Runge-Kutta-Gill interval.

Each component data block must consist of—the axial-chord length, β_R at exit, $(\partial\beta/\partial r)_R$ at exit and the number of values per section required in the output.

APPENDIX II

Computer Solution of the Flow in a Multi-Stage Machine with Identical Repeating Stages

This solution consists of two phases. The first of these is the calculation of the matrix coefficients for the individual blade rows, which coefficients are then used in the second phase to obtain the values of λ, $(d\lambda/dx)$ and g at the leading and trailing edges of all the blade rows comprising the compressor.

Twelve matrix coefficients have to be evaluated for every blade row which does not constitute part of a repeating stage (as defined in Section 7), and also for an isolated rotor row and isolated stator row which go to form one of the repeating stages. These coefficients are obtained simply by solving equation (16) in the isolated row for the three different cases obtained when two of the variables λ, $(d\lambda/dx)$ and g at inlet are put equal to 0 and the third is made equal to unity, and for the fourth case when all three variables at inlet are put equal to zero. The solution through the blade row is obtained using the Runge-Kutta-Gill numerical integration mentioned in Appendix I.

The matrices evaluated in the first phase of this solution are now used in the second phase to evaluate λ, $(d\lambda/dx)$ and g at all the blade interfaces. The algebraic method based on the Caley-Hamilton theorem is used to set up the characteristic equation of the matrix C, the dominant eigenvalue of which is then extracted by Newton's method. To give an idea of the magnitudes of the errors in the method the imaginary parts of all the results (which should be zero) are printed in the output in addition to the real parts. In all the compressors thus far tested, the largest imaginary part recorded has been of the order 10^{-11}.

In order to evaluate the matrix coefficients the following information about each blade row must be supplied—the axial chord, the axial velocity at the design radius at the leading edge of the blade, the hub and tip radii, β_R and $(\partial\beta/\partial r)_R$ at exit from the blade and also at exit from the blades of the preceding row, and the angular velocity of the compressor rotor. Additional data required for the second phase of the solution are the number of blade rows before the repeating stages and the number of repeating stages in the compressor.

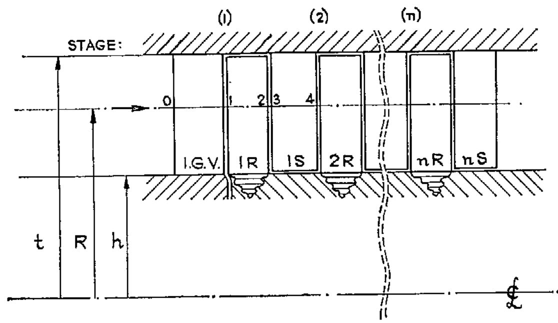

Fig. 1. System under analysis.

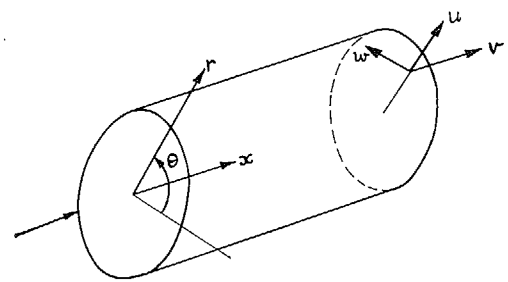

Fig. 2. Co-ordinative system.

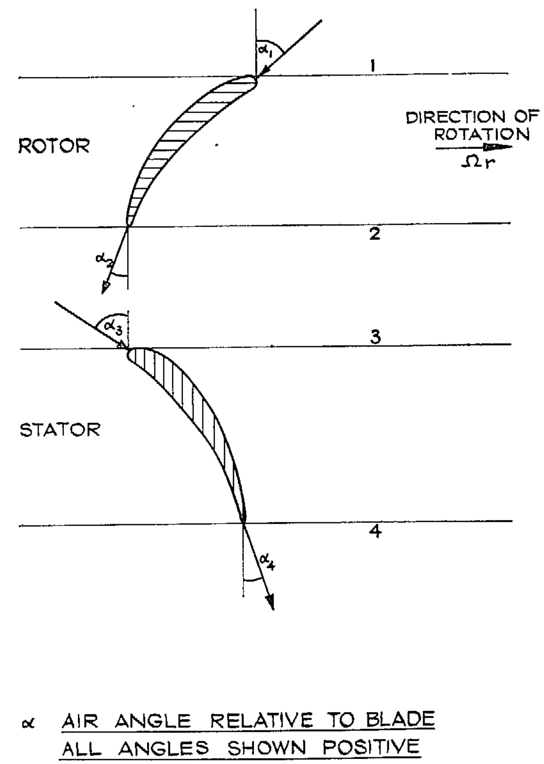

Fig. 3. Notation for blade angles.

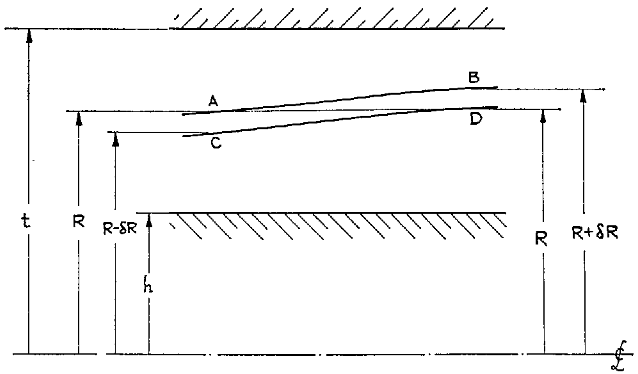

FIG. 4. Displacement of streamlines near design radius.

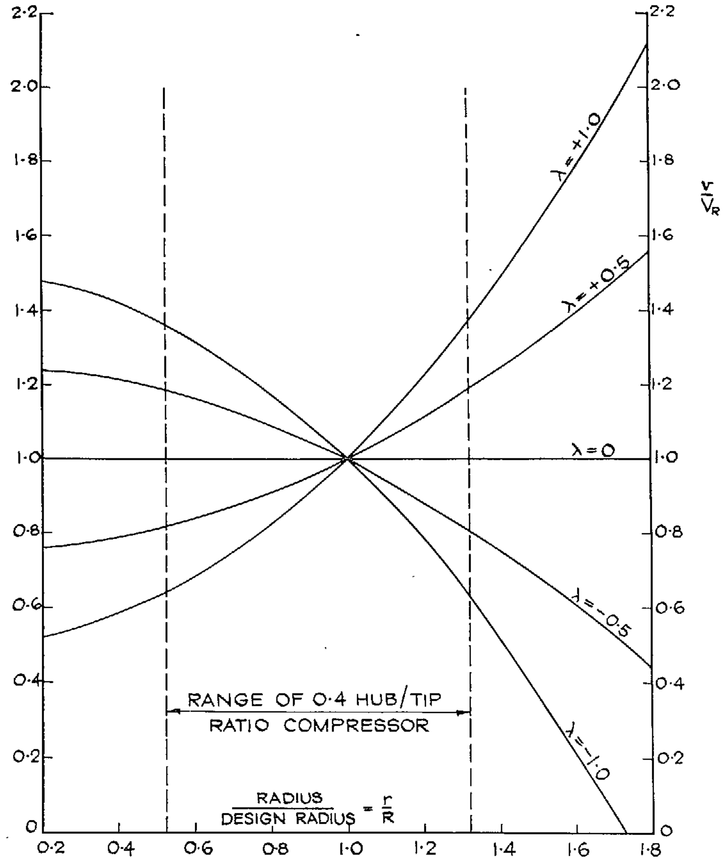

FIG. 5. Axial-velocity profiles.

$$\frac{v}{V_R} = 1 + \frac{\lambda}{2}\left\{\left(\frac{r}{R}\right)^2 - 1\right\}$$

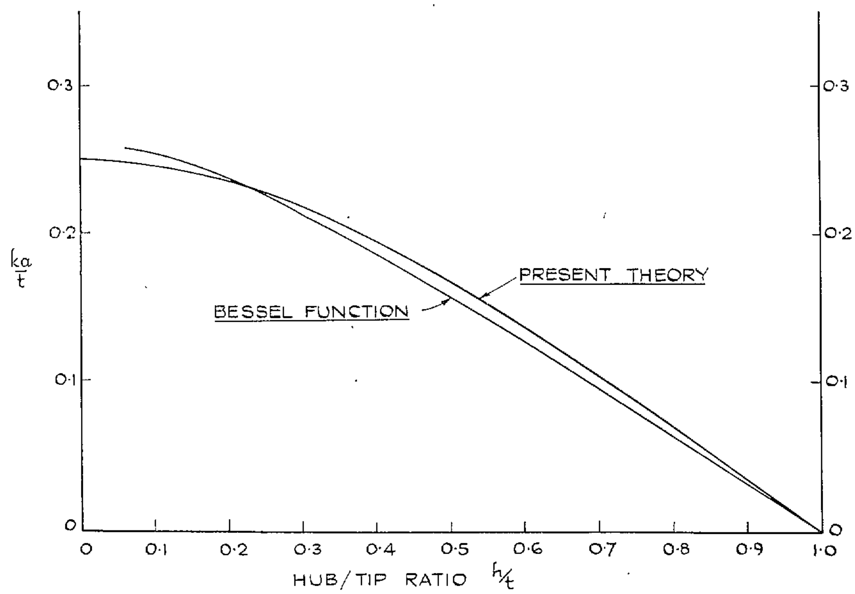

FIG. 6. Aspect-ratio parameter ka/t.

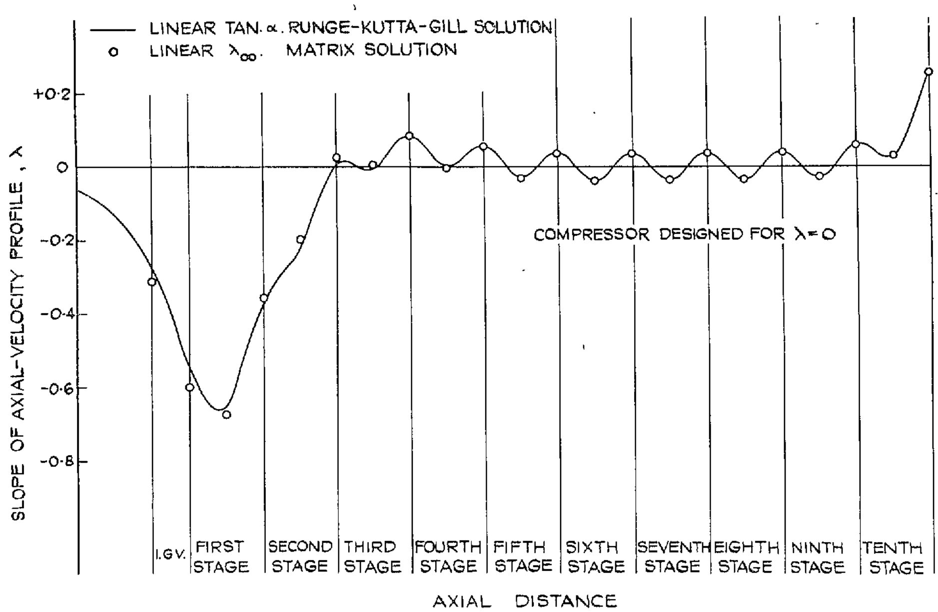

FIG. 7. Vortex flow in axial compressor. Constant-axial-velocity design.

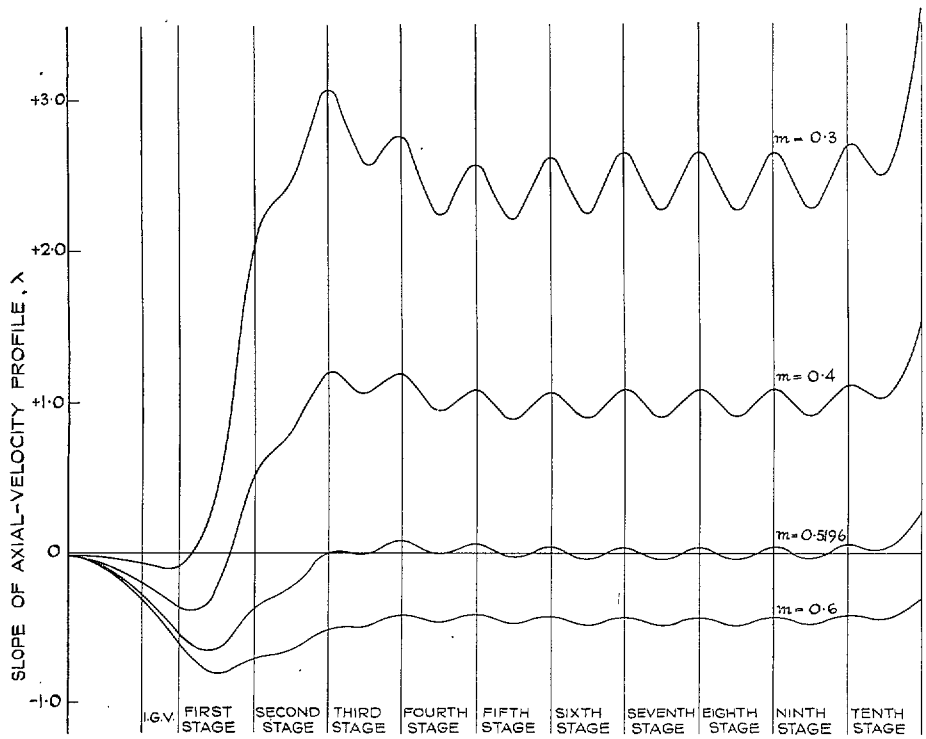

FIG. 8. Vortex flow in ten-stage compressor. Effect of varying the mass flow.

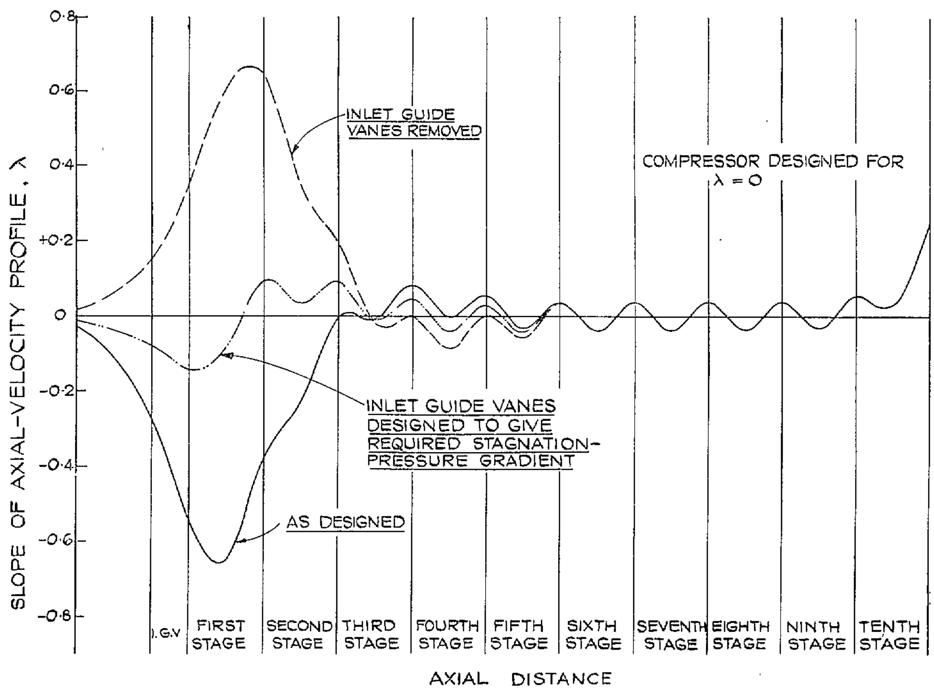

FIG. 9. Vortex flow in ten-stage compressor. Effect of modifications to inlet guide vanes.

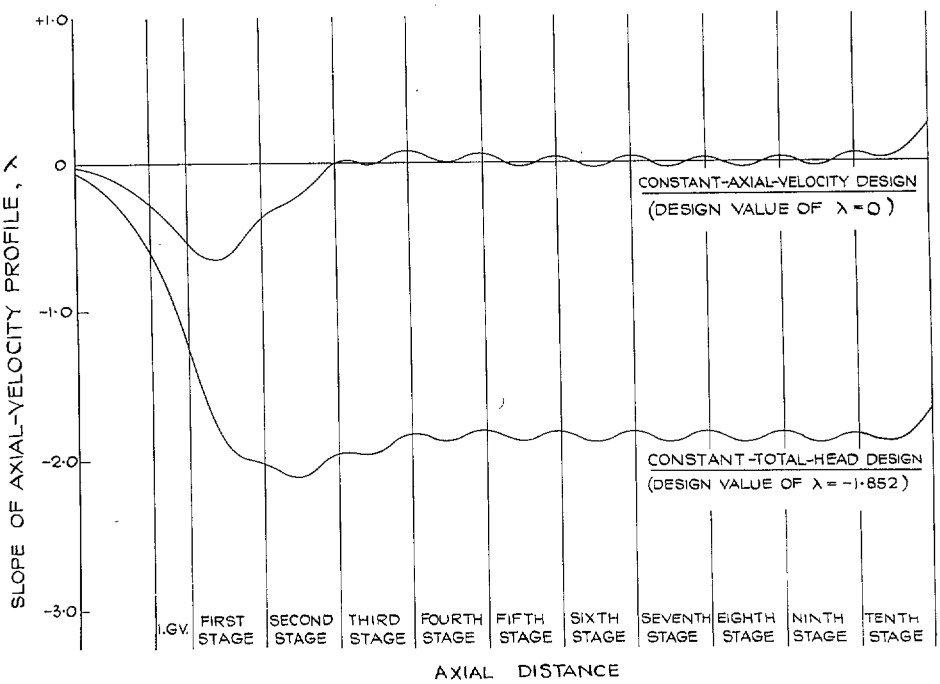

FIG. 10. Vortex flow in ten-stage compressor. Comparison of two designs.

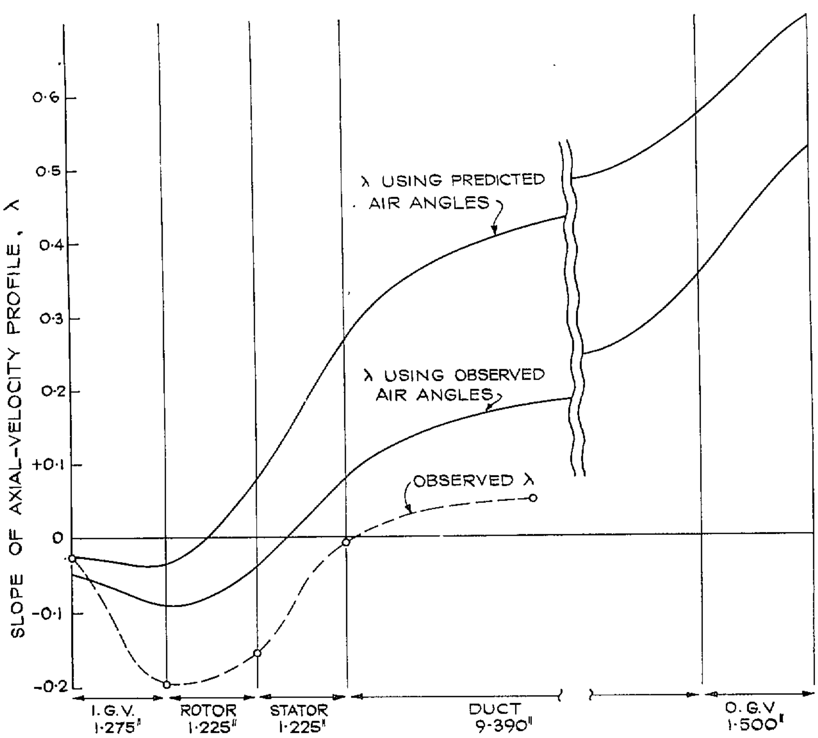

FIG. 11. Comparison of theoretical and experimental values of λ for a single-stage compressor (Horlock).

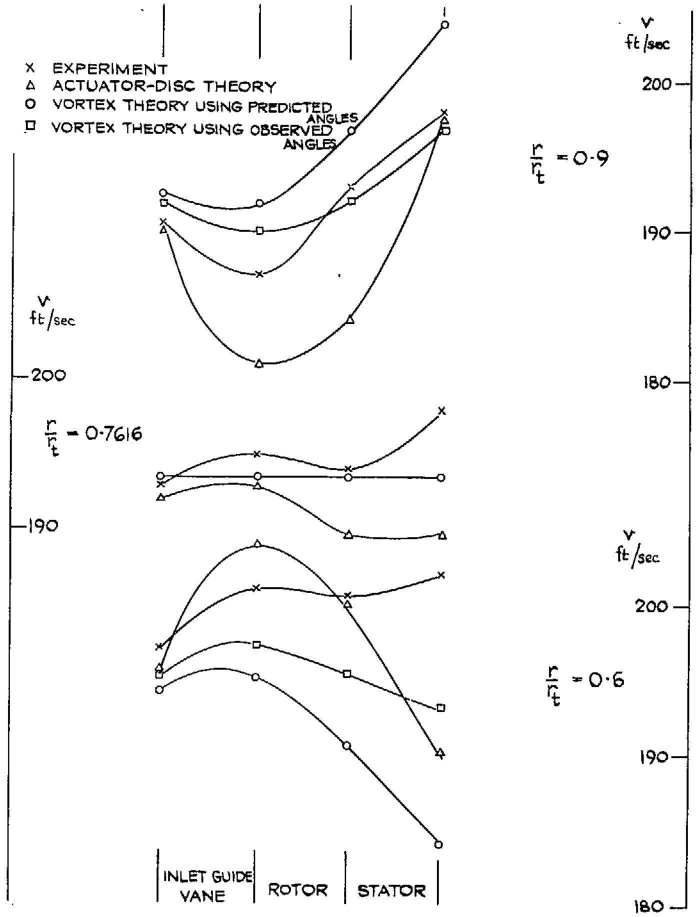

FIG. 12. Axial-velocity profiles for single-stage compressor (Horlock).

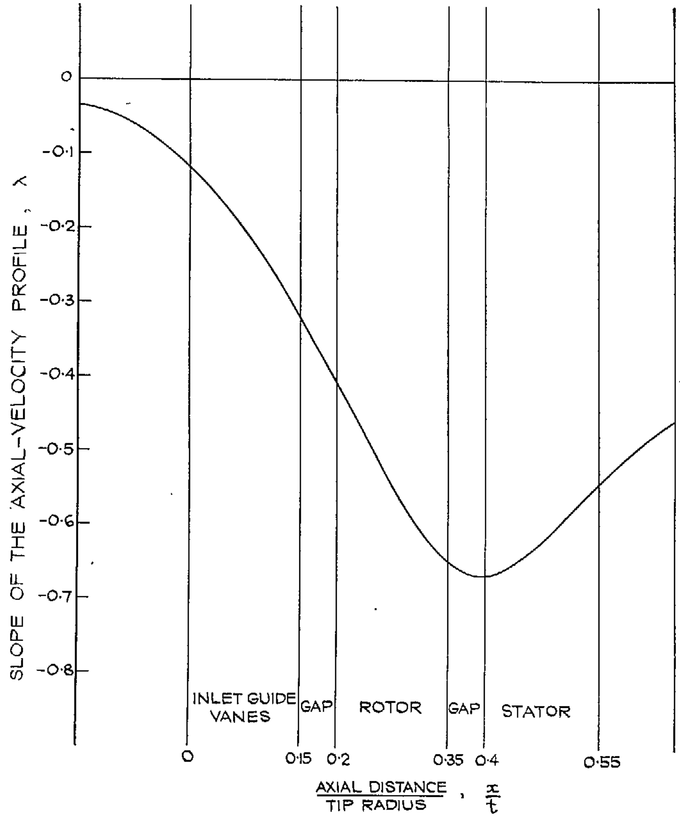

FIG. 13. Slope of axial-velocity profile for a single-stage compressor (Wu).

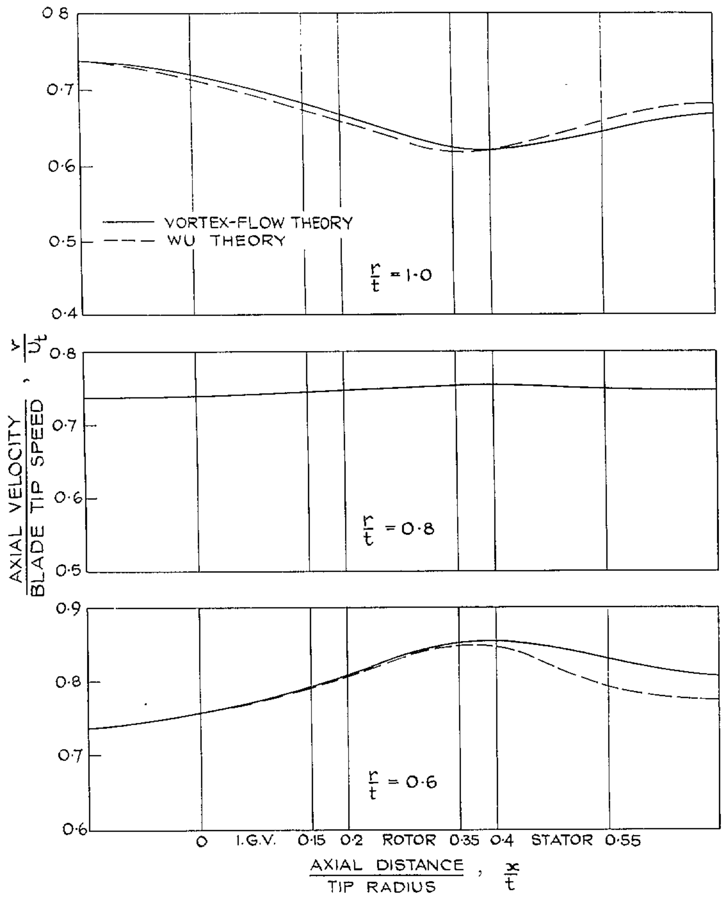

FIG. 14. Axial-velocity variations through a single-stage compressor (Wu).

(87503) Wt. 64/1857 K.5 8/63 Hw.

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