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Theoretical Analysis of a Gust Alleviator Used on a *Lancaster* Aircraft and Comparison with Experiment

By J. K. ZBROŻEK

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Theoretical Analysis of a Gust Alleviator Used on a *Lancaster* Aircraft and Comparison with Experiment

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COMMUNICATED BY THE DEPUTY CONTROLLER AIRCRAFT (RESEARCH AND DEVELOPMENT), MINISTRY OF AVIATION

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Summary.

A theoretical analysis of a gust alleviator using the spectral technique is made and the results compared with experiment. The comparison is made in terms of the frequency of occurrence of normal accelerations at the aircraft c.g. The estimated loss of alleviator effectiveness with increasing magnitude of the normal acceleration and with increasing static alleviation is in close agreement with the observed phenomena, which the single-gust approach had failed to explain.

The importance of turbulence of wavelengths longer than 1000 ft is emphasised.

1. *Introduction.*

The object of this report is two-fold. First, it is to attempt to explain theoretically the measured behaviour of a gust alleviator as fitted to a *Lancaster* aircraft[1], and second to demonstrate the application of the power-spectral technique to the estimation of gust-induced accelerations. The single-gust approach, which cannot account correctly for aircraft and air turbulence dynamics, failed utterly to explain the measured behaviour of the gust alleviator. The present spectral calculations explain at least qualitatively, if not entirely quantitatively, all the peculiarities observed experimentally.

2. *Description of the Gust Alleviator.*

For the sake of completeness, a brief description of the gust alleviator is given; a more comprehensive description can be found in Ref. 1. The gust alleviator was basically a wing-lift-slope reducing device. Incidence was measured some distance ahead of the aircraft by a nose-boom-mounted pitchmeter. The signal from the pitchmeter was fed to an electronic computer, called the 'electronic link', which, *via* a hydraulic servo, deflected both ailerons in the same sense (both up or down). The 'electronic link' incorporated phase advance and filter circuits.

For this type of system, the reduction in lift slope due to the alleviator only is a_2k, where a_2 is the aileron lift slope and k is the amount of upward aileron deflection produced by unit positive change in aircraft incidence. The proportion of alleviation in a gust load, neglecting the effect of aircraft response, is a_2k/a, where a is the wing lift slope, and this quantity was termed the static alleviation.

* Replaces R.A.E. Report No. Aero. 2645—A.R.C. 22,856.

Unfortunately the deflection of the ailerons produced not only a reduction in lift, but also a pitching moment. At the time when the gust alleviator was designed, it was thought that the main aircraft gust loads were produced by relatively sudden, sharp gusts, the 'official' gust of 100 ft length being then the accepted standard. The pitching motion of the aircraft was erroneously considered to be of secondary importance.

3. *Experimental Results.*

The assessment of the gust alleviator was based on the measured normal acceleration at the aircraft c.g. A counting accelerometer was used with reading intervals of $0 \cdot 1g$, and the aircraft was flown with the gust alleviator switched ON and OFF automatically at 30 second intervals. The length of each sortie was from 1 to 2 hours and the total flying time was about 40 hours. Results were presented as the number of times the acceleration exceeded a given level plotted against the acceleration level. The alleviation was defined as:

$$\frac{\text{(number of accelerations with alleviator OFF)—(number with alleviator ON)}}{\text{(number with alleviator OFF)}}$$

and it was plotted also against the level of acceleration. A typical result is shown in Fig. 1 which is reproduced from Ref. 1, Fig. 8. It can be seen that the alleviator did not produce the expected alleviation by decreasing the number of accelerations exceeding a given level, but actually increased this number. What was interesting and typical for every record analysed was that the alleviation, as defined above, decreased with increasing magnitude of acceleration and was mainly negative. There was also an indication that the alleviator efficiency in terms of the mean alleviation decreased with increasing 'static alleviation'. This suspected trend is shown in Fig. 2 which is Fig. 13 of Ref. 1.

To explain in terms of discrete gusts, the decrease of alleviation with increasing magnitude of the gust loads, one has to assume that gust length increases in proportion to gust magnitude, the bigger gust being of the order 400 to 500 ft long (Fig. 26 of Ref. 1). To explain the loss of measured alleviation with increasing 'static alleviation' by the concept of discrete gusts is even more difficult. It can only be said that increasing 'static alleviation' decreases the static and manoeuvre margins, making them negative for 'static alleviation' greater than 33% and 47% respectively (Fig. 23 of Ref. 1). Thus one is led to conclude that the length of the 'important gusts' is of the order of the short-period wavelength.

It was felt that a better understanding of the gust-alleviator behaviour could be obtained using the spectral technique, and accordingly some calculations have been made. The results are presented in the following paragraphs.

4. *Theoretical Calculations.*

4.1. *Assumptions.*

(i) The calculations have been made for one flight condition, *viz.*, 150 knots T.A.S. and air density $\rho = 0 \cdot 0023$ slug/ft^3; this roughly corresponds to the average flight conditions during the tests.

(ii) The aerodynamic derivatives were obtained from Ref. 1.

(iii) The aircraft was assumed rigid. The first fundamental structural frequency was of the order of 3 c/s and it was known that this flexibility had some effect on the accelerations at the c.g. In addition there was a coupling between the gust alleviator and the first fundamental mode. However, the dynamic behaviour of the gust alleviator at this frequency was rather erratic and no reliable transfer functions were available, so that this elastic mode could not be included in the calculations. As a result, the calculated accelerations at the aircraft c.g. are probably slightly under-estimated, but for the purpose of comparison between different settings of the gust alleviator (including zero, i.e., no gust alleviator), the assumption of a rigid aircraft is probably valid.

(iv) The unsteady lift due to a gust was included using data of Ref. 2 interpolated to an aspect ratio of 8.

(v) The power spectrum of atmospheric turbulence was assumed to be[3]:

$$G(\Omega) = {\sigma_w}^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1+\Omega^2 L^2)^2}$$

where L, the turbulence scale, was assumed to be equal to 1000 ft.

The assumed turbulence spectrum shape and scale are open to criticism, especially remembering that the *Lancaster* flight tests were made in Africa and at low altitude. However it was felt that the numerical calculations are not too sensitive to the assumed turbulence scale, due to the rather short wavelength of the aircraft short-period oscillation (of the order of 500 ft). For this particular aircraft and speed, the gust loads are relatively insensitive to the turbulence spectrum shape at very low frequencies (long wavelength).

4.2. *Calculations.*

The numerical calculations were made using the theory of Ref. 2 for five settings of the gust alleviator:

(1) basic aircraft	0% static alleviation
(2)	10% static alleviation
(3)	20% static alleviation
(4)	+40% static alleviation
(5) alleviator reversed	−25% static alleviation

The modulus squared of the loading transfer function is shown in Fig. 3. It can be seen that for frequencies above about 0·3 c/s the dynamic behaviour of the alleviator is proportional to its static alleviation and the alleviator decreases the normal accelerations. Below a frequency of about 0·2 c/s the action of the alleviator is reversed; increase of static alleviation increases, instead of decreases, the gust loads. This is mainly an effect of loss of static stability with increasing static alleviation.

The corresponding non-dimensional power spectra of the c.g. acceleration are shown in Fig. 4. The spectral 'gust-alleviation factor', is defined as:

$$F_{sp} = \frac{1}{\sigma_w} \left[\int_0^\infty |K(\Omega)|^2 G(\Omega) d\Omega \right]^{1/2}, \tag{1}$$

and the corresponding relationship between the R.M.S.'s of the c.g. normal acceleration and of the gust velocity is given by:

$$\sigma_n = \frac{\rho V a}{2W/S} F_{sp} \sigma_w . \tag{2}$$

The characteristic frequency of the aircraft in turbulent air is computed from the relationship:

$$\Omega_0 = \frac{\frac{1}{\sigma_w}\left[\int_0^\infty \Omega^2 \,|K(\Omega)|^2 G(\Omega) d\Omega\right]^{1/2}}{F_{sp}}, \tag{3}$$

and this gives the number of zero crossings per second,

$$N_0 = \frac{V}{2\pi}\Omega_0. \tag{4}$$

The value of V in the case considered here was 253 ft/sec.

The upper limit of the integral {equation (3)} was chosen to be 7 c/s, as it was assumed that the counting accelerometer, on which the experimental data are based, would have insignificant response at higher frequencies.

The results of calculations in terms of the spectral gust-alleviation factor, F_{sp}, and the number of zero crossings per second, N_0, are tabulated below and shown in Fig. 5.

TABLE 1

Static alleviation	F_{sp}	N_0 per second $V = 253$ ft/sec
−25%	0·511	0·805
0%	0·484	0·700
+10%	0·487	0·640
20%	0·508	0·575
40%	0·60	0·440

It can be seen that increasing the static alleviation increases the overall level of gust loads (F_{sp}), but decreases the frequency of zero crossing (N_0). The first effect is due to the considerable increase in gust loadings at very low frequencies, and the second, due to a decrease of loading at high frequencies, Fig. 4. The decrease in the value of N_0 and the increase in the value of F_{sp} have opposite effects on the distribution of the frequency of occurrence of gust loads, and thus this distribution has to be studied in more detail.

In order to compare the computed responses with the measured counts of the normal accelerometer, the non-dimensional alleviation factor, F_{sp}, and a frequency, N_0, had to be converted to the frequency distribution of normal accelerations. For that purpose a very simple model of atmospheric turbulence was assumed.

It was assumed that there is only one component of atmospheric turbulence, with Gaussian probability distribution and of intensity $\sigma_w = 7$ ft/sec, occurring for half the time, $P = 0 \cdot 5$ (*see* Ref. 3). It follows that the overall mean intensity of turbulence is $\sigma_w \simeq 5$ ft/sec, which is probably a not unreasonable value for African desert conditions during the month of October. It should be mentioned that the final results of the present analysis would be negligibly changed if an entirely different model of atmospheric turbulence was assumed.

5. *Comparison with Experiment.*

In order to compare the theoretical values with the experimental results of Ref. 1, the computed frequencies of occurrence of accelerations were converted to relative alleviation at discrete increments of normal acceleration according to the definition used in Ref. 1, which is

$$\frac{\text{(number of accelerations with alleviator OFF)—(number with alleviator ON)}}{\text{(number with alleviator OFF)}}$$

at each discrete level of acceleration increment.

The results are plotted in Figs. 6a, 6b and 6c for static alleviations of 10%, 20% and −25% respectively. On the same figures the experimental results corresponding approximately to the assumed alleviator settings and flight conditions are replotted from Ref. 1.

The agreement between the predicted and measured behaviour of the gust alleviator is rather good. The main features observed experimentally are faithfully reproduced by the theory. First, the alleviation decreases with an increasing level of acceleration encountered, being slightly positive only for small incremental accelerations. Second, the computed and measured alleviations decrease with increasing values of static alleviation. To illustrate this point further the computed alleviation at a given increment of c.g. normal acceleration is plotted against the static alleviation in Fig. 7. This figure should be compared with Fig. 2 of the present report (or Fig. 13 of Ref. 1). It can be seen that the predicted trends correspond closely to the measured results.

6. *Conclusions.*

The performance of a simple gust alleviator as measured and reported in Ref. 1 has been estimated accurately using the power-spectral technique, in spite of a lack of precise knowledge of the turbulence above the North African desert.

The estimated and observed loss of alleviator effectiveness shows that, at least in this particular case, the long waves of turbulence, longer than say 1000 ft, contribute more to the overall loads on the aircraft than the shorter waves. This conclusion is in agreement with a tentative conclusion of Ref. 1, which already suggested that the important gusts must be much longer than the standard 100 ft.

It should be pointed out, that for current and even more so for future large aircraft, the very low frequency range of turbulence spectrum (long waves) is becoming more and more important.

It might be concluded, that in cases where the aircraft dynamics are different from the 'standard, well-behaved aircraft' the discrete gust approach based on one 100 ft long gust and corresponding 'gust-alleviation factor' is useless. With the present state of knowledge, the power-spectrum approach cannot as yet entirely replace the single-gust approach, but it appears to be much more promising. The success of the power-spectrum technique depends on two fundamental questions, and these can only be answered experimentally:

(i) Can all types of atmospheric turbulence be defined satisfactorily by their power spectrum (e.g., thunderstorm turbulence)?

(ii) Is it possible for purposes of practical application to standardise the turbulence spectra (in analogy with the 100 ft ramp length standardised for the single gust)?

LIST OF SYMBOLS

a		Wing lift slope
a_2		Aileron lift slope
f		Frequency, cycles per second
F_{sp}		Spectral gust-alleviation factor
g		Acceleration of gravity, ft/sec^2
$G(\Omega)$		One-dimensional spectral density of the vertical component of atmospheric turbulence, ft^3/sec^2
k		Aileron gearing, aileron deflection per unit change of wing incidence
$K(\Omega)$		Frequency response of loading function
L		Turbulence scale, ft
Δn		Increment of normal acceleration due to gust, in 'g'
N_0		Average number of zero crossings, per sec
P		Proportion of time flown in turbulence
S		Wing area, ft^2
V		Aircraft speed, ft/sec
W		Aircraft weight, lb
λ	=	$\frac{2\pi}{\Omega}$, wavelength of turbulence, ft
ξ		Mean angle of ailerons
ρ		Air density, slug/ft^3
σ_n		R.M.S. (root mean square) of normal acceleration, in 'g'
σ_w		R.M.S. of the vertical component of atmospheric turbulence, ft/sec
Ω	=	$\frac{2\pi}{\lambda}$, space frequency of turbulence, per ft

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No.	Author(s)	Title, etc.
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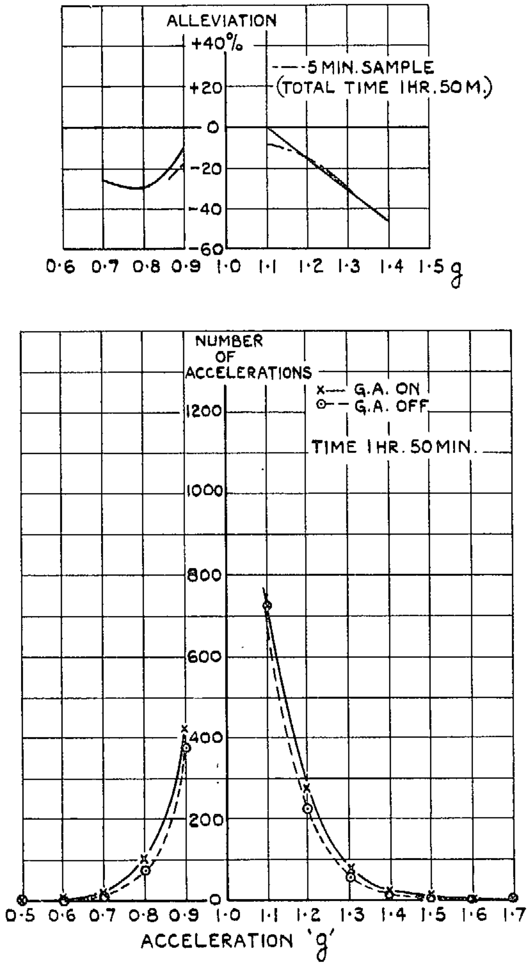
2	J. K. Zbrożek	Longitudinal response of aircraft to oscillatory vertical gusts. A.R.C. 18,940. November, 1955.
3	J. K. Zbrożek	The relationship between the discrete gust and power spectra presentations of atmospheric turbulence, with a suggested model of low-altitude turbulence. A.R.C. R. & M. 3216. March, 1960.



SETTING

FILTER	IN
STIFFNESS	-15 DB
PHASE ADVANCE	SINGLE STAGE
GAIN	110°
STATIC ALLEVIATION	19%
A.S.I.	150 KNOTS
MEAN MEASURED ALLEV.	-9%

FIG. 1. A typical result of gust-alleviator effect on normal acceleration distribution. (Fig. 8 of Ref. 1.)

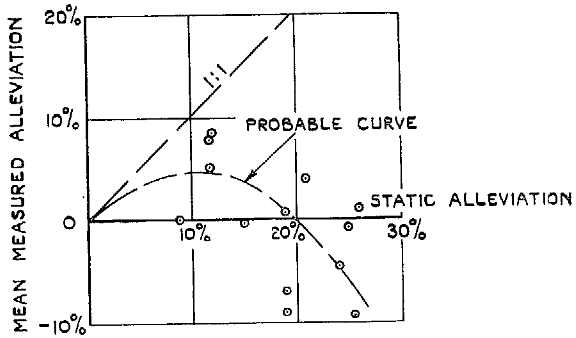


FIG. 2. The effect of static alleviation on the measured alleviation. (Fig. 13 of Ref. 1.)

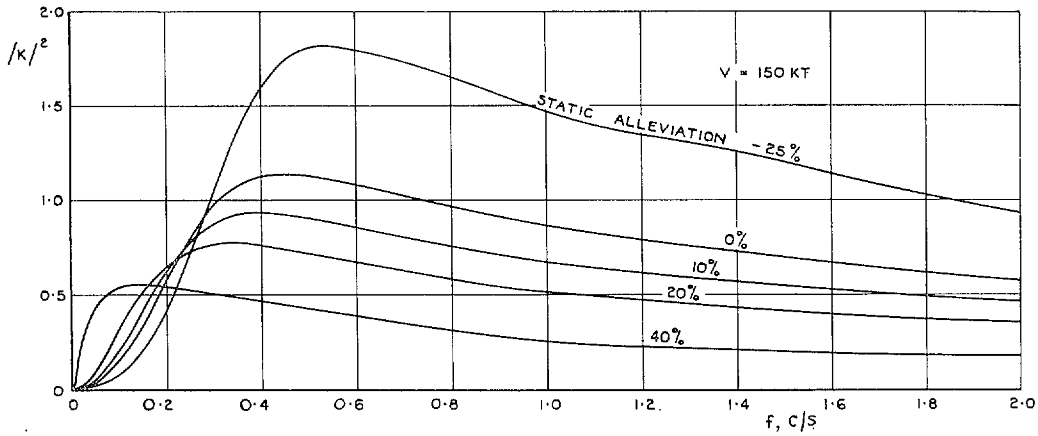


Fig. 3. Modulus squared of loading transfer function.

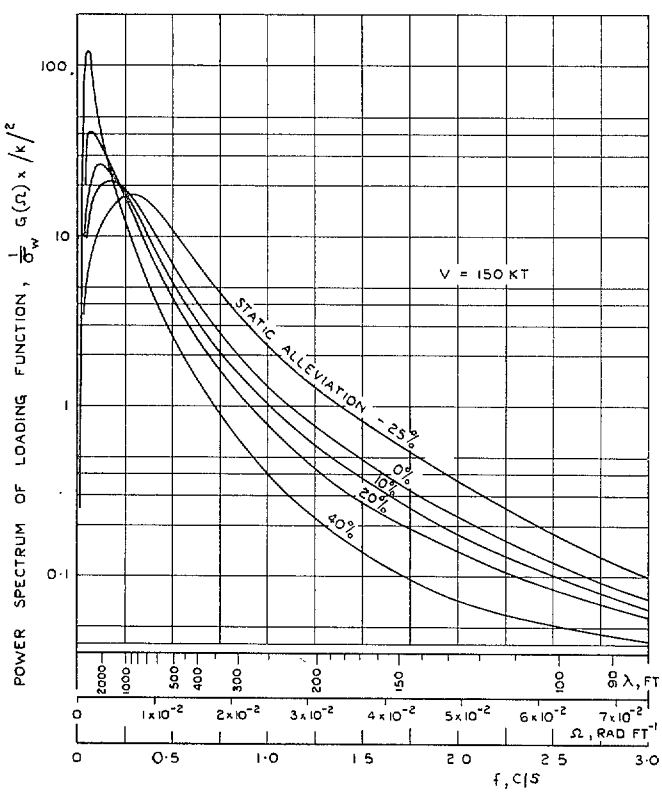


Fig. 4. Non-dimensional power spectra of c.g. acceleration.

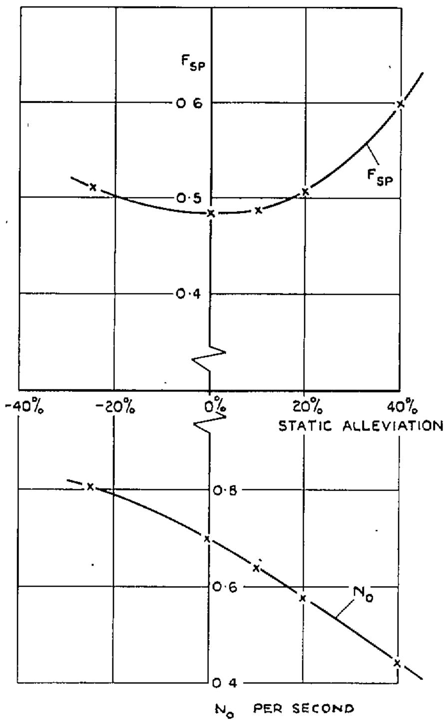
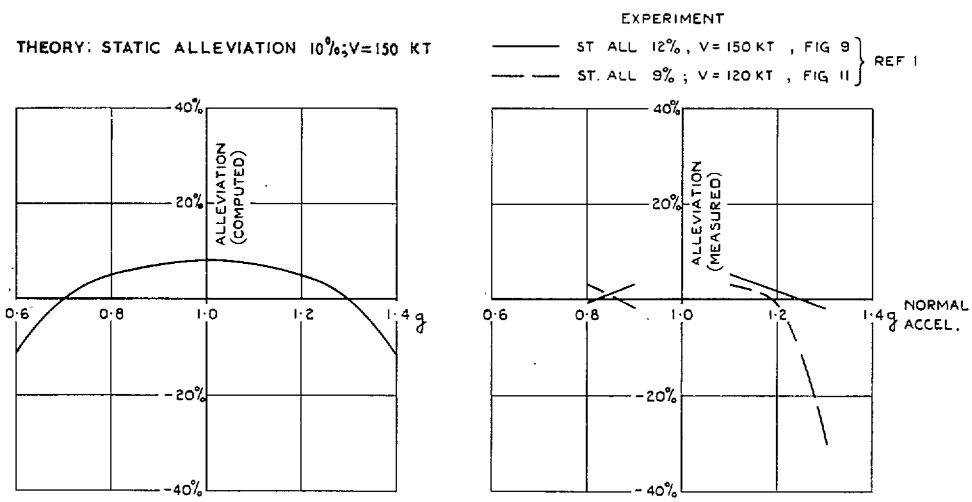
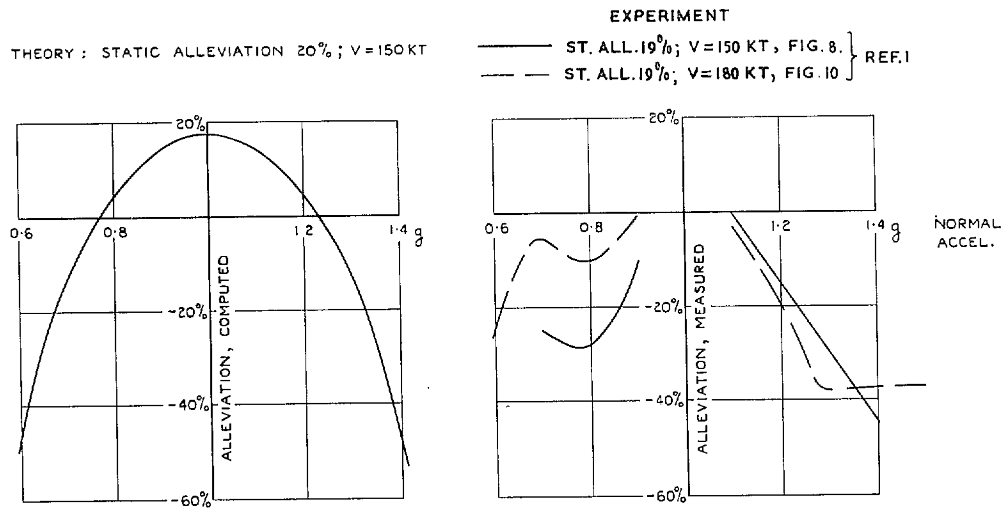


FIG. 5. The effect of alleviator setting on the spectral alleviation factor, F_{sp}, and on the frequency of zero crossing, N_0, per second ($V = 253$ ft/sec).

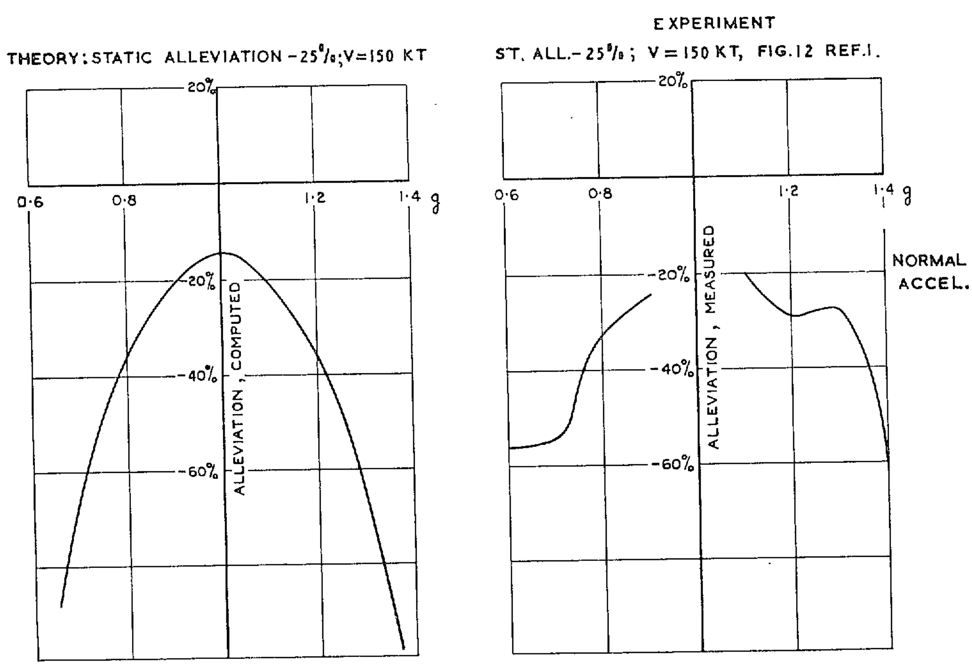


(*a*) Static alleviation +10%, nominal.

FIG. 6. Comparison between the theoretical and experimental values of gust alleviation plotted against the magnitude of normal acceleration due to gusts.



(b) Static alleviation +20%, nominal.



(c) Static alleviation −25%, nominal.

FIG. 6. Comparison between the theoretical and experimental values of gust alleviation plotted against the magnitude of normal acceleration due to gusts.

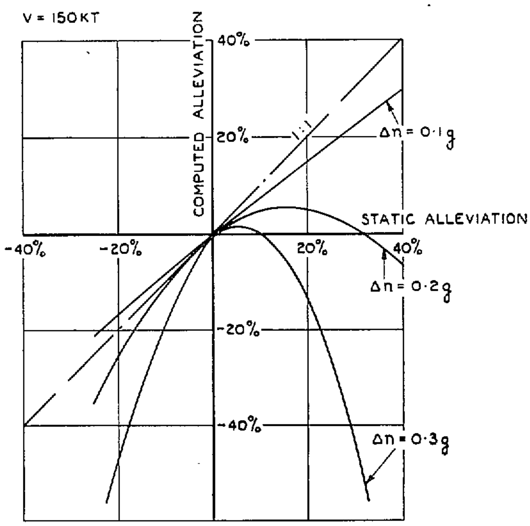


FIG. 7. The effect of static alleviation on the actual (computed) alleviation for different levels of increments of normal acceleration due to gusts (σ_w = 7 ft/sec, P = 0·5).

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