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COMMUNICATED BY THE DIRECTOR-GENERAL OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

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Summary.—The report gives the results of tests to investigate the aerodynamic effects of aspect ratio on the flutter of delta wings that are virtually rigid but are flexibly supported at the root. A function representing the aerodynamic effects of aspect ratio on wing flutter speeds is derived from the experiments.

Flutter calculations are made using this function as a correction to the derivatives obtained from two-dimensional theory, and there is good agreement between measured and calculated results. Calculations are also made using derivatives obtained from three-dimensional theory, but the agreement between measured and calculated results is poor.

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1. *Introduction.*—Wind-tunnel tests have been made on a series of semi-rigid delta wings to investigate the effects of aspect ratio on the flutter characteristics. The technique of these experiments was the same as that employed in previous work[1,2], in which the aerodynamic effects of aspect ratio on wing flutter are segregated from all the other effects by using wings that are virtually rigid but are flexibly supported at the root.

A function representing the aerodynamic effects of aspect ratio on wing flutter speed is obtained from the experiments, and by use of this function a close estimate of flutter speeds and frequencies of the wings has been obtained on the basis of two dimensional theory.

A parallel investigation to predict the wing flutter characteristics has been made using aerodynamic coefficients calculated for each wing on a three-dimensional basis[3]. The theoretical and experimental values are compared and the agreement is poor.

2. *Experimental Details.*—2.1. *Description of the Rig.*—The rig allowed wing freedoms in modes of linear flexure (roll) and uniform pitch. The wing root was at $0{\cdot}075s$ above the roll axis, and the pitching axis at 50 per cent of the root chord. Torsion bars of adjustable length on these axes provided the required stiffnesses, and sliding weights enabled the roll and pitch inertias to be adjusted. The lay-out of the rig is shown in Figs. 1 and 2. In Fig. 1, which is taken from above the rig, a typical torsion bar governing the rolling motion can be clearly seen. The sliding weights giving additional rolling inertia are more easily seen in Fig. 2, at the lower end of the vertical sideplates which hold the pitching body. The roll and pitch bodies of the rig were free to rotate on ball-races and, in general, friction damping was small.

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* R.A.E. Report Struct. 180, received 6th February, 1956.

The wing mounting was designed so that its product of inertia was zero, but the mountings contributed to the direct inertias of the wings so that means of adjusting them were required. The moments of inertia of the rig (wing plus mounting) about the axes of roll and pitch were adjusted by means of the sliding weights to vary as s^3 in roll and s in pitch, where s is the distance from roll axis to wing tip. The values are given in Table 1.

2.2. *Wind-Tunnel Measurements.*—The tests were conducted in the Royal Aircraft Establishment 5-ft Diameter Open-Jet Tunnel. All wings were mounted vertically above a reflector plate. The wing aspect ratios ranged from 1·87 to 5·62, being defined as $2s/c_m$ where c_m is the mean chord of each wing and was constant for the whole series.

To set up the wings, the torsion bars were adjusted so that for all the wings the frequencies of the corresponding modes were the same. Flutter tests were made for two frequency-ratio conditions, the pitching frequency being varied and the rolling frequency kept constant.

Flutter speeds and frequencies were measured for each wing, the speed being that at which the oscillation first damped out as speed was reduced. Measurements were also made of amplitude ratios and phase angles at flutter for one frequency ratio. For these measurements two potentiometers were attached to the roll and pitch bodies respectively and slide wire tappings were attached to fixed points on the rig. Voltage was applied to the potentiometers, and the outputs from the tappings, porportional to the displacements in roll and pitch, were displayed on a twin-beam oscilloscope and photographed as a continuous trace. Tests made with and without the slide wires showed that slide-wire friction had a negligible effect on flutter speed and frequency.

Measurements of the wing rolling moment in steady flow due to wing incidence were also made on all the wings. The purpose of these measurements is discussed in section 4.1.1.

3. *Experimental Results.*—The results of the wind-tunnel tests are plotted in Figs. 3 and 4 together with the results of the theoretical investigations. The results of Fig. 3 show that as aspect ratio decreases, the flutter speed increases linearly whilst the flutter frequency remains constant. The amplitude ratio, angle of roll to angle of pitch, also increases with decreasing aspect ratio whilst the phase angle is practically constant (Fig. 4). The rate of change of flutter speed with aspect ratio can be expressed in the form $V = V_0 f(A)$, $f(A)$ seems to be dependent on the wing modal frequencies, but there is too much scatter of results for this to be established positively. For the two frequency ratio conditions considered the appropriate results are given approximately by

(*a*) $N_R = 4\cdot 1$ c.p.s. , $N_P = 5\cdot 6$ c.p.s. $\quad f(A) = 1 + (2\cdot 3/A)$

(*b*) $N_R = 4\cdot 1$ c.p.s. , $N_P = 7\cdot 1$ c.p.s. $\quad f(A) = 1 + (2\cdot 5/A)$

where V is the critical flutter speed, V_0 is the calculated flutter speed using two-dimensional derivatives, and A is the aspect ratio.

The flutter frequency remains approximately constant over the range of aspect ratios considered in the tests, in the first instance at 4·85 c.p.s. and in the second at 5·05 c.p.s. However, the wing of smallest aspect ratio has a somewhat higher flutter frequency than the others of the series for both values of the pitching frequency.

4. *Theoretical Investigations and Comparison with Experiment.*—4.1 *Calculations Based on Two-dimensional Theory.*—The procedure here is to determine from the experiments factors to be applied to the theoretical aerodynamic derivatives for two-dimensional flow that will enable an accurate prediction of the wing flutter characteristics to be made. It can be seen that a relation of the form $V = V_0 f(A)$, which has been obtained experimentally, can be reproduced theoretically by multiplying the aerodynamic damping coefficients in the flutter equations for the infinite wing by $1/f(A)$ and the aerodynamic stiffness coefficients by $1/f(A)^2$. For the present

series of wings, to minimise the calculations a mean value (of the experimental results) for $f(A)$ of the form $f(A) = 1 + (2 \cdot 4/A)$ was used as the factor to be applied to the two-dimensional coefficients for both frequency ratios. The results are plotted in Figs. 3 and 4, and it can be seen that a close agreement of measured and calculated flutter characteristics is obtained, particularly so far as flutter speeds are concerned.

It should be noted (*see* diagram with Table 1) that reduction of aspect ratio for a delta wing leads to an increase of wing sweepback, so that the factor $f(A)$ above includes effects of both aspect ratio and sweepback on flutter. For swept wings it has been found that the effects of aspect ratio and sweepback can be separated for the purpose of calculation[2], the former being represented by the factors $1/F(A)$ and $1/\{F(A)\}^2$ applied to the damping and stiffness derivatives respectively, and the latter by the factor $\cos \Lambda$ applied to all the derivatives (where Λ is the leading-edge sweepback). The particular value assigned to the factor $F(A)$ as a result of the previous tests on unswept untapered, unswept tapered and swept untapered wings was $\{1 + (0 \cdot 8/A)\}$. It was decided in the first instance to see if this particular value had more general significance by applying it to this series of deltas, still assuming that the factor $\cos \Lambda$ represented the sweepback effect. The latter assumption involved a possible error as the factor $\cos \Lambda$ was derived for wings having both leading-edge and trailing-edge sweepback.

These calculations gave a close agreement between calculated and measured flutter frequencies, amplitude ratios and phase angles, but the flutter speeds were some 14 per cent less than the measured values. Accordingly a new value of $F(A)$ was derived of the form $1 + [(1/A)\{2 \cdot 0 - (1 \cdot 5/A)\}]$ and using this factor, in conjunction with the factor $\cos \Lambda$, a close agreement of all flutter characteristics was obtained.

4.1.1. *Correction factor derived from steady flow measurements.*—The suggestion has been put forward[2] that it might be possible by measuring in steady flow the variation of the slope of the lift curve (or some related parameter such as rolling moment) between the wings of a given set, to obtain the value of the aspect-ratio correction to be used in the flutter equations. The rolling moments of this series of wings, due to a given incidence were accordingly measured to investigate this possibility. The value of the function $F(A)$ found from these rolling-moment measurements gave calculated results having the same order of agreement with measured values as those obtained using the linear aspect-ratio factor $\{1 + (0 \cdot 8/A)\}$. A comparison of the measured flutter speeds with the calculated speeds using these two aspect-ratio factors is made in Fig. 5. Both factors are plotted in the figure.

4.2. *Calculations Using Three-dimensional Derivatives.*—The aerodynamic derivatives used in the calculation were obtained by interpolation from three other sets of derivatives:

(*a*) Those due to Miles[4] for very low aspect ratio (up to $AR = 1$).

(*b*) Woodcock's derivatives for a delta wing of aspect ratio three oscillating in elastic modes[3], with no chordwise distortion.

(*c*) The two-dimensional values.

In Woodcock's report derivatives are given only for the two frequency parameters of $0 \cdot 2\dot{6}$ and $0 \cdot 8$, and it was only possible to derive the coefficients here correspondingly. Hence it was not possible to obtain a balance between the actual and assumed frequency parameters in the flutter calculations. In one case, that of the infinite-aspect-ratio wing and for the first frequency ratio, the derivatives corresponding to both values of the frequency parameter were used in the calculations. The corresponding results were as follows:

Assumed frequency parameter	$0 \cdot 2\dot{6}$	$0 \cdot 8$
V_0 (ft/sec)	31·05	33·40
Calculated frequency parameter	0·479	0·466
Tip amplitude in roll/pitch angle (ft)	0·764	0·533
Phase angle—Roll leading pitch	45° 56′	49° 45′

The only parameter affected greatly by the change in assumed frequency parameter is the ratio, tip amplitude in roll to pitch angle. Amplitude ratio is obtained from this on division by the span s. However, the infinite-aspect-ratio wing for the first frequency ratio gives the greatest discrepancy between the calculated and assumed frequency parameters, and the difference will be less for the finite-aspect-ratio wings where the balance of frequency parameter is initially better. Hence, the change in amplitude ratio on balancing frequency parameter in the case of the finite span wing will be smaller than might appear from the above results. The slope of the line plotted in Fig. 4 is unlikely to be reduced appreciably.

The results of the calculations are plotted in Figs. 3 and 4 and it can be seen that, whereas a close agreement of measured and calculated flutter frequencies, amplitude ratios and phase angles is obtained, the agreement of flutter speeds is poor. The agreement of flutter speeds is, in fact, worse than is obtained using any of the modified two-dimensional approaches mentioned above.

5. *Conclusions*.—Tests on a series of delta wings having the same taper ratio and mean chord and whose aspect ratios are altered by changing the span show that if these wings have freedoms in roll and uniform pitch, then the flutter speeds are given quite accurately by a relation of the form $V = V_0 f(A)$, where V is the critical flutter speed, V_0 is the calculated flutter speed using two-dimensional derivatives, and A is the aspect ratio.

For these wings the mean value of the function $f(A)$ found from consideration of the experimental results was $f(A) = 1 + (2 \cdot 4/A)$. This function may be dependent to some extent on the frequency ratio between the degrees of freedom of the system.

By using this factor, which combines the effects of aspect ratio and sweepback on flutter as a correction to the aerodynamic derivatives for two-dimensional flow, quite good agreement between the measured and calculated flutter values is obtained.

As an alternative to the above procedure, aspect ratio and sweepback effects on flutter have been separated, sweepback being represented by a factor $\cos \Lambda$ based on leading-edge sweepback and aspect ratio being represented by the factor $F(A) = 1 + [(1/A)\{2 \cdot 0 - (1 \cdot 5/A)\}]$ applied as corrections to the derivatives for two-dimensional flow. Quite good agreement is obtained between measured and predicted flutter characteristics using this method.

A comparison of the measured results with those calculated using three-dimensional theory shows poor agreement for flutter speed and relatively good agreement for the other flutter characteristics.

Acknowledgement.—The authors' thanks are due to Miss J. M. Howarth formerly of Structures Department for the determination of the aerodynamic coefficients for these wings from three-dimensional theory.

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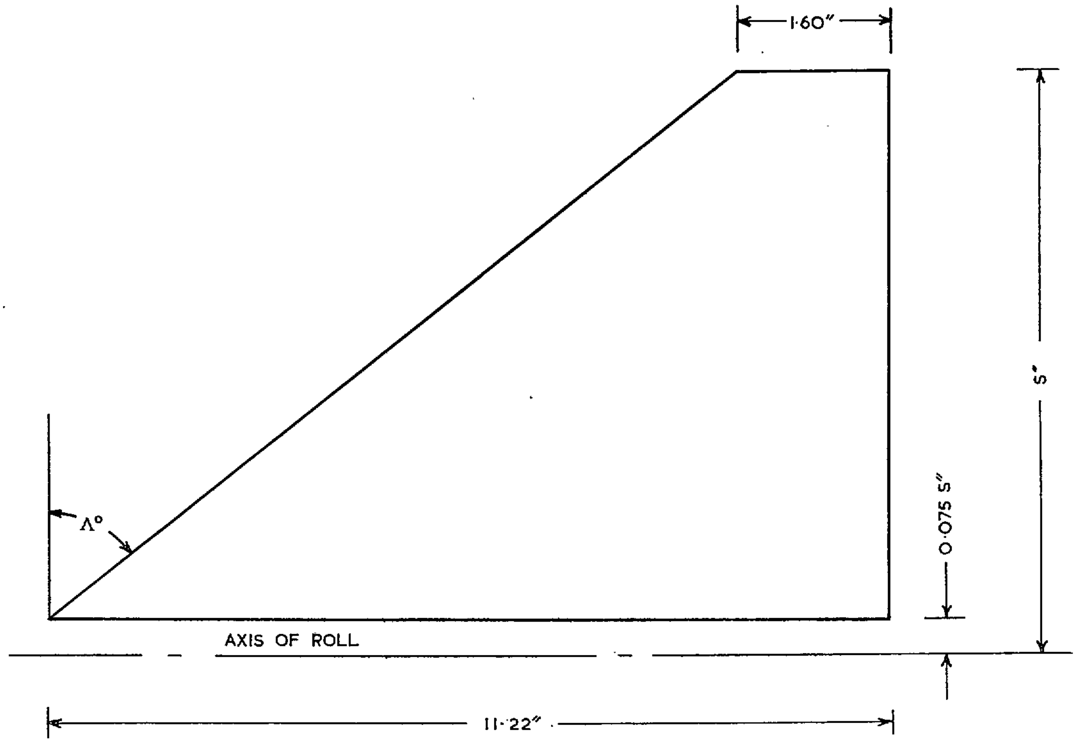
2	W. G. Molyneux and H. Hall ..	The aerodynamic effects of aspect ratio and sweepback on wing flutter. R. & M. 3011. February, 1955.
3	D. L. Woodcock	Aerodynamic derivatives for a delta wing oscillating in elastic modes. C.P. 170. July, 1952.
4	J. W. Miles	*Aero. Quart.* Vol. II. Part III. November, 1950.

TABLE 1

Geometrical and Structural Details of Wings



Wing mean chord = 0·534 ft
Wing thickness/chord = 0·08
Wing section = RAE 101

Wing number	Wing span S (in.)	Inertias Wing + Mounting (lb in.²)			Aspect ratio A	Sweepback Λ (deg)
		Roll inertia	Roll—Pitch cross inertias	Pitch inertia		
1	6	4·74	0·775	1·570	1·87	60
2	7	7·51	1·008	1·802	2·18	56
3	8	11·38	1·294	2·073	2·50	53
4	9	16·20	1·679	2·414	2·81	49
5	10	23·31	2·126	2·671	3·12	46
6	12	39·03	2·946	3·043	3·75	41
7	15	76·21	4·627	3·865	4·68	35
8	18	131·57	6·592	4·801	5·62	30

Wing frequencies { Roll = 4·1 c.p.s.
Pitch = 5·6 and 7·1 c.p.s.

(71645) B

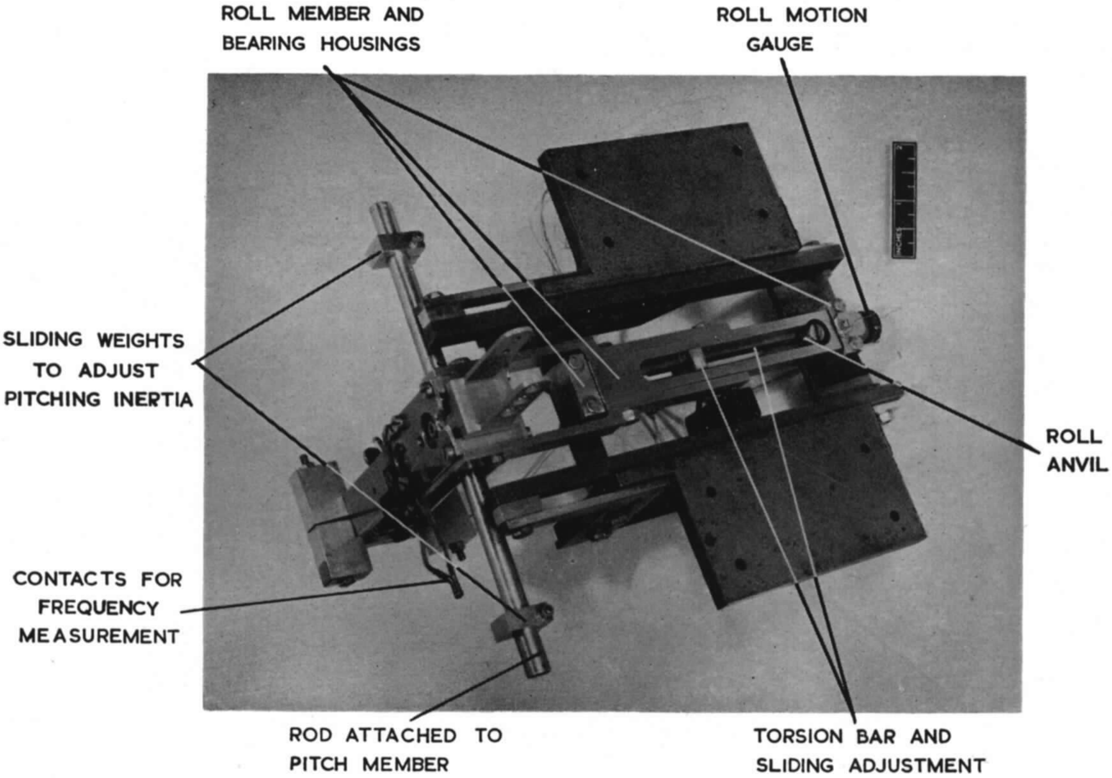


FIG. 1. Plan view.

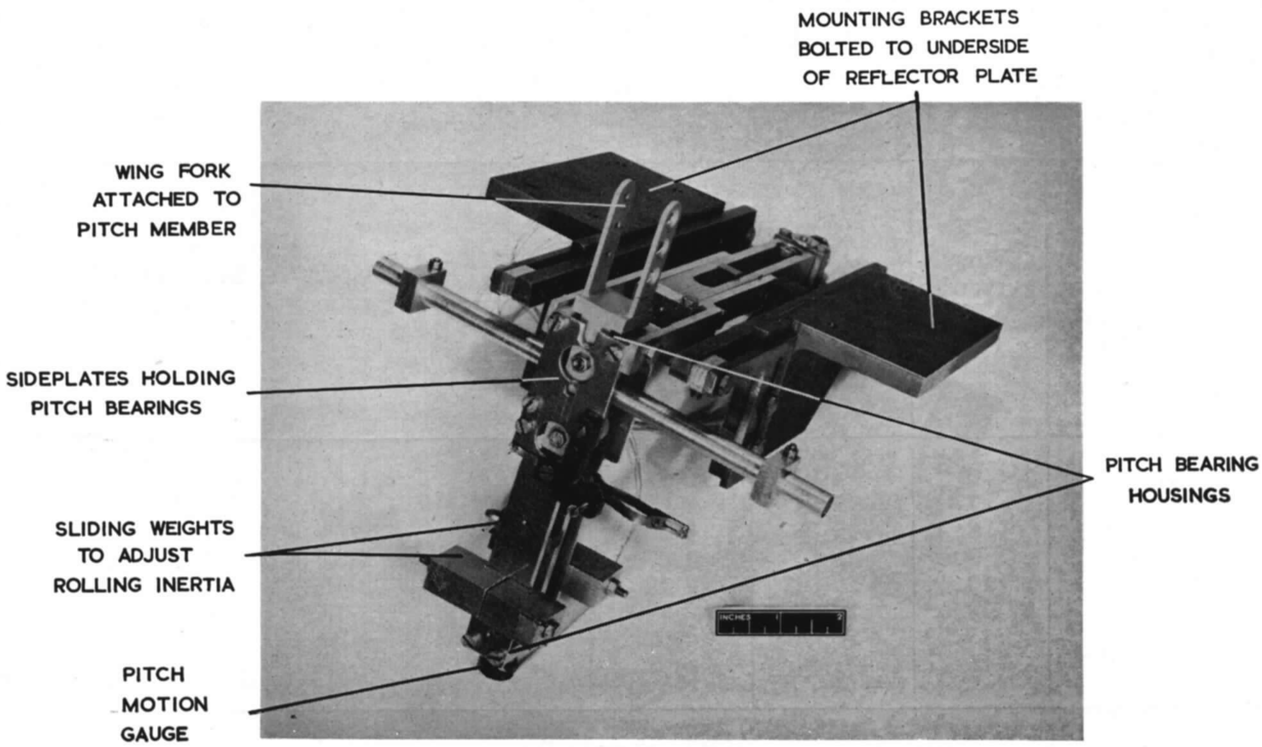


FIG. 2. Front elevation.

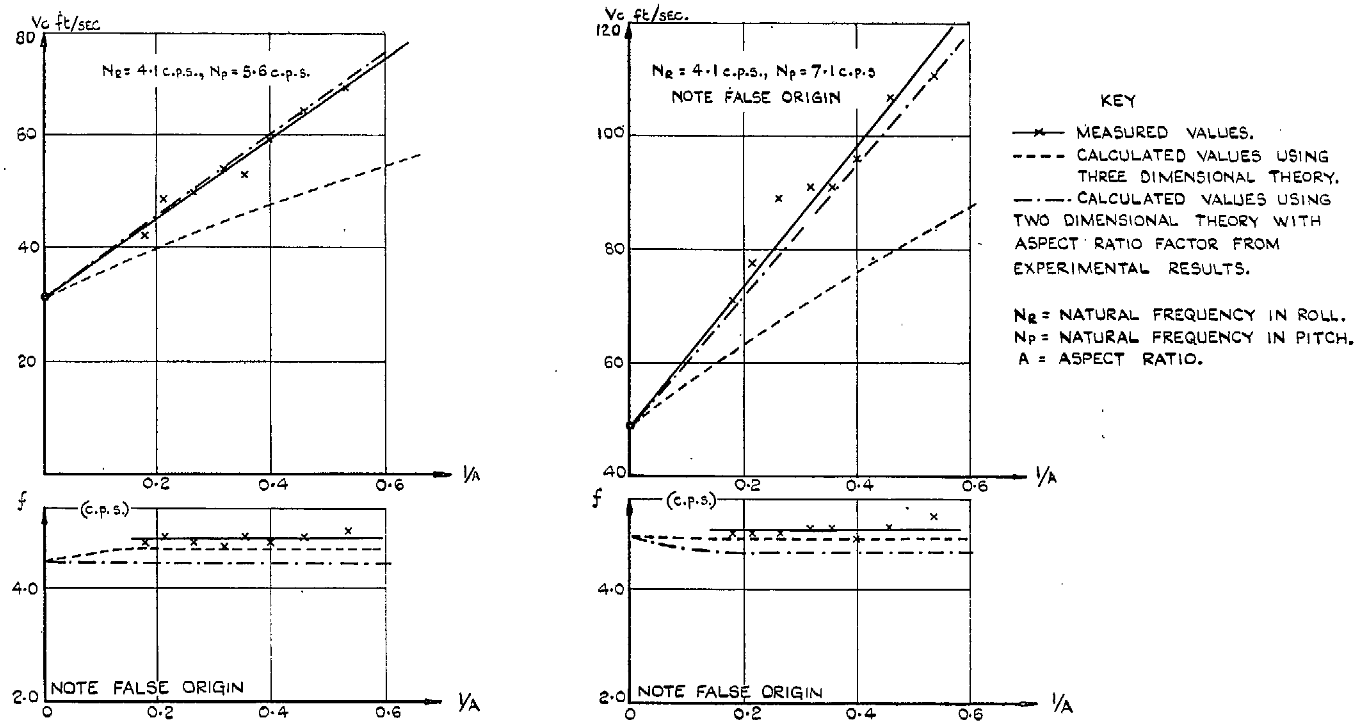


FIG. 3. The effect of aspect ratio on flutter speed and frequency.

(71645) Wt. 52 6943 K.8 2/58 Hw.

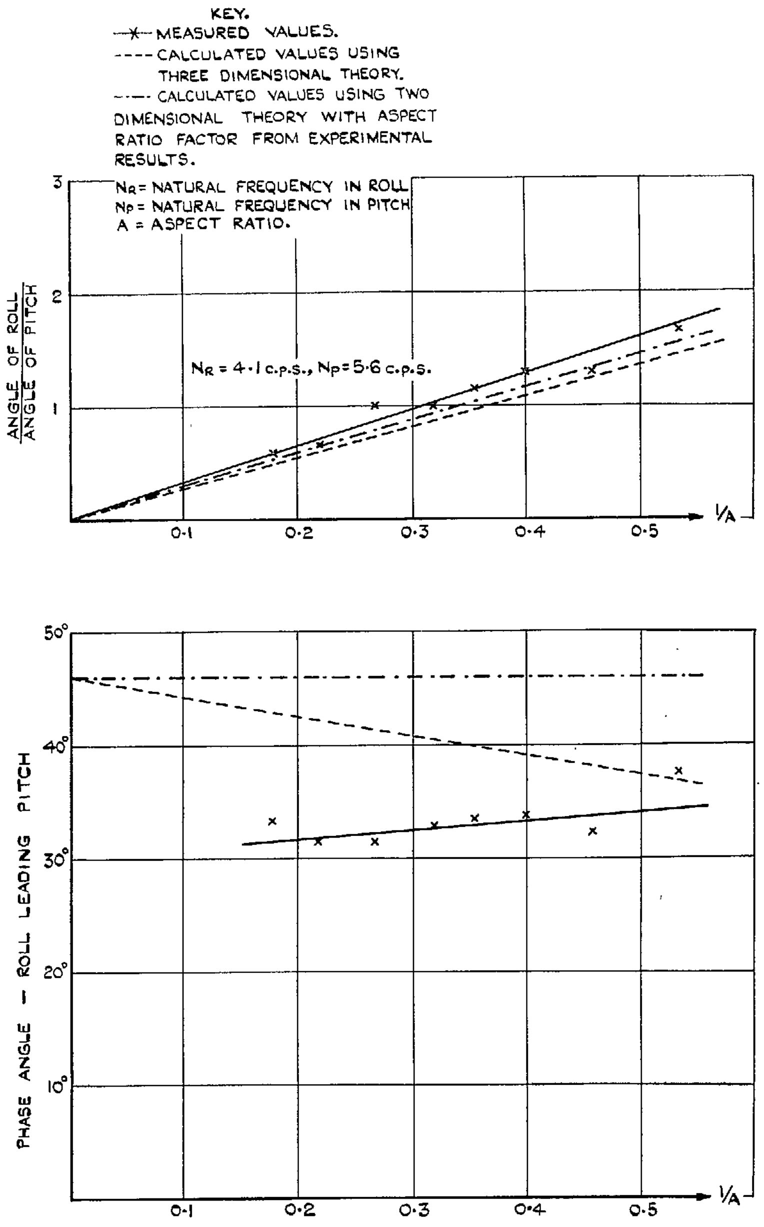


FIG. 4. The effect of aspect ratio on amplitude ratio and phase angle.

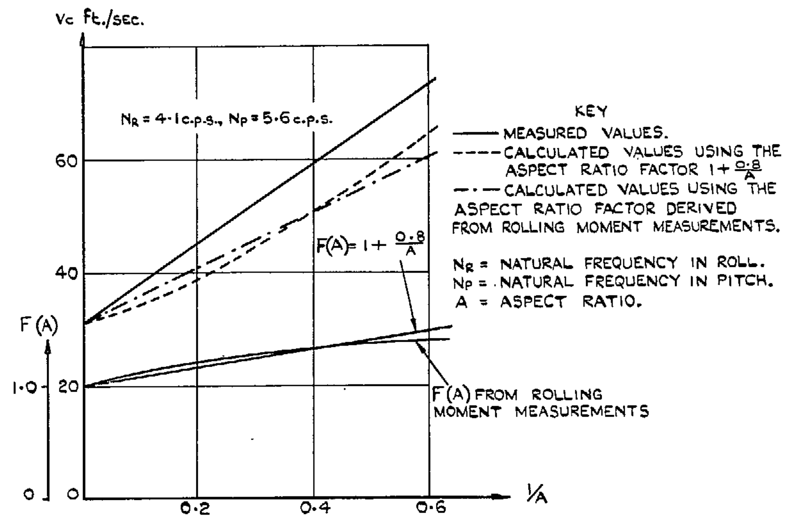


FIG. 5. A comparison of the predicted flutter speeds using two different factors to represent the aerodynamic effects of aspect ratio.

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