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Summary.—An experimental study has been made of two-dimensional turbulent boundary-layer flow with zero pressure gradient. The investigation was made to determine the accuracy of a method proposed by Preston for measuring the local turbulent skin friction. The general momentum equation was used in conjunction with measurements of skin friction by Preston's method, to obtain increments in momentum thickness which could be compared with the measured values. The experiments were made on a smooth flat plate, 6 ft long, which spanned the working-section of a return-circuit wind tunnel. Transition was promoted by trip wires and glass-paper strips. A constant Reynolds number of $3 \cdot 9 \times 10^5$ per foot was maintained throughout the experiments.

The results, together with certain basic arguments, indicate that Preston's method will give the correct skin friction. For the range of Reynolds numbers covered in the experiments ($1500 < R_\theta < 4000$) the skin-friction coefficients obtained for different values of R_θ (the Reynolds number based on momentum thickness), were approximately 10 per cent and 6 per cent less than the corresponding Prandtl-Schlichting and Schoenherr values respectively.

1. *Introduction.*—1.1. *Introductory.*—Relf, Pankhurst and Walker[1] recently carried out experiments to determine the turbulent skin friction on a flat plate and they concluded that, in its existing form, Preston's[2] round pitot method of measuring local skin friction was not accurate. Their measured values of the mean skin friction were found to agree closely with the Prandtl-Schlichting law, but the local values of skin friction obtained by differentiating this law were 14 per cent greater than those found using Preston's method.

Preston's method depends on the assumption that the 'inner' law for the velocity distribution is the same in both turbulent pipe flow and boundary-layer flow and the results of Relf, Pankhurst and Walker's experiments suggest that this assumption is incorrect. In Ref. 3, Fage claims to have found further support for this conclusion. Using results obtained by Laufer[4] in fully developed turbulent pipe flow, and Klebanoff[5] in a turbulent boundary layer on a flat plate, he shows that on an 'inner law' basis the velocity distributions for the two types of flow differ by an amount which accounts almost exactly for the discrepancy noted by Relf. However, as shown by Preston in Ref. 6, the lack of an accurate method of determining the local skin friction on a flat plate has resulted in a large number of quite different 'inner laws' and for this reason Fage's contribution cannot be considered as very strong evidence in favour of Relf's conclusion.

In the present investigation the general momentum equation is used to make a completely independent check on the accuracy of the values of skin friction found using Preston's method.

The experimental results obtained, together with certain basic arguments, provide substantial confirmation of the accuracy of the method. On the basis of this, the distribution of skin friction on a flat plate is considered and the results of the present experiments are compared with some established skin-friction laws in the Reynolds-number range $3 \cdot 9 \times 10^5$ to 2×10^6.

1.2. *Notation.*

ρ		Density of air
μ		Viscosity of air
ν		Kinematic viscosity of air
τ_0		Local skin friction
P		Total pressure recorded by a pitot-tube
P_0		Static pressure at reference position in tunnel working-section
p		Mean static pressure at any height in the boundary layer
U_1		Free-stream velocity
q_1	=	$\frac{1}{2}\rho U_1^2$ Free-stream dynamic pressure
u		Mean velocity component parallel to wall at any point in the boundary layer
v		Mean velocity component normal to wall at any point in the boundary layer
u', v', w'		Instantaneous values of the fluctuating velocity components
u_τ	=	$\sqrt{(\tau_0/\rho)}$
x		Distance along flat plate from leading edge
y		Vertical distance above surface
δ		Boundary-layer thickness
θ		Boundary-layer momentum thickness
	=	$\int_0^\delta \frac{u}{U_1}\left(1 - \frac{u}{U_1}\right) dy$
h		Height of centre of pitot-tube
d		External diameter of mouth of pitot-tube
c_f	=	$\frac{\tau_0}{\frac{1}{2}\rho U_1^2}$ Local skin-friction coefficient
R_x	=	$\frac{U_1 \rho x}{\mu}$ Reynolds number based on x
R_θ	=	$\frac{U_1 \rho \theta}{\mu}$ Reynolds number based on θ

1.3. *Brief Outline of Preston's Method.*—Preston's method assumes, on the basis of Ludweig and Tillmann's[7] experiments and from consideration of local dynamical similarity, that in a region close to the surface in both fully developed turbulent pipe flow and in boundary-layer flow

$$\frac{u}{u_\tau} = f\left(\frac{u_\tau \rho y}{\mu}\right) \quad \ldots \quad (1)$$

and further that this function would be the same for both types of flow. If equation (1) is approximately correct it suggests that, in the region close to the surface, conditions depend only on the properties of the fluid, the surface friction, and a suitable length. Hence, by dimensional analysis

$$\frac{(P - p_0)d^2}{\rho \nu^2} = F\left(\frac{\tau_0 d^2}{\rho \nu^2}\right), \quad \ldots \quad (2)$$

where P is the total pressure read by a pitot-tube resting on the surface, p_0 is the static pressure there, and d is the diameter of the pitot-tube. With the above assumption the function F can be determined from measurements in a pipe, since τ_0 can then be found accurately from the pressure drop. Hence equation (2) provides the basis of a convenient method for obtaining the local skin friction.

1.4. *Basis of the Present Investigation*.—It is well known that the measurement of skin friction is exceedingly difficult. The existing methods available are open to question for the reasons given in Ref. 6 and because of this it was considered that they could not be used to check the accuracy of Preston's method. However, the boundary-layer momentum equation provides a convenient means of obtaining an independent assessment of the accuracy of the method. This equation was therefore used in the following way in the analysis of the present results.

The general momentum equation for two-dimensional turbulent boundary-layer flow can be written

$$\frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - \frac{1}{U_1^2}\int_0^\delta \left\{\frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{v'^2}}{\partial x}\right\} dy$$
$$+ \frac{1}{U_1^2}\left[\int_0^\delta \left\{\int_0^y \frac{\partial^2 \overline{u'v'}}{\partial x^2}\, dy - \delta\frac{\partial^2 \overline{u'v'}}{\partial x^2}\right\} dy\right]. \quad \ldots \quad (3)$$

The influence of the turbulence terms in the above equation has been examined by several authors (*see* Ref. 8) and in general it has been concluded that the effects are negligible, except where separation is approached. However, a re-analysis of this problem using experimental results obtained by Klebanoff and Diehl[9] and by Klebanoff[5] showed that even with approximately zero pressure gradient the term containing $\overline{u'^2}$ and $\overline{v'^2}$ can contribute as much as 2 to $2\frac{1}{2}$ per cent of the skin-friction coefficient. Hence for the purpose of the present investigation it was decided that these turbulence terms could not be neglected.

In Appendix I it is shown how the corrections for the turbulence terms can be estimated in a particularly simple way, provided it is assumed that the distributions of $\overline{u'^2}/U_1^2$ and $\overline{v'^2}/U_1^2$ exhibit longitudinal similarity in a turbulent boundary layer with zero pressure gradient. The argument supporting this assumption is given in the Appendix. The two terms containing $\overline{u'v'}$ can be shown to be negligible compared with the remaining term containing $\overline{u'^2}$ and $\overline{v'^2}$ and are neglected. Equation (3) can therefore be reduced to

$$\frac{c_f}{2} = \frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - K\frac{d\delta}{dx} + 2K\frac{\delta}{\rho U_1^2}\frac{dp}{dx}. \quad \ldots \quad (4)$$

where

$$K = \int_0^1 \left(\frac{\overline{u'^2} - \overline{v'^2}}{U_1^2}\right) d\left(\frac{y}{\delta}\right)$$

and, with the above assumption, is a constant. Equation (4) can be written

$$(\theta_2 - \theta_1) = \int_1^2 \frac{c_f}{2}\, dx + \int_1^2 (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx}\, dx + K[\delta_2 - \delta_1] - K\int_1^2 \frac{2\delta}{\rho U_1^2}\frac{dp}{dx}\, dx, \quad \ldots \quad (5)$$

where the limits 1 and 2 refer to any two successive values of x along the surface. Hence, if the distributions of skin friction and pressure along the centre-line of a flat plate are measured, together with velocity profiles at chosen stations also along this line, increments in the momentum thickness θ can be obtained from equation (5) and compared with the measured values. Also, if the measured value of θ at one of the stations is taken as a starting value, the development of θ along the plate can be calculated and again compared with the measured results. If the pressure gradient is made as small as possible and accurate values of θ are found, a reliable check on the method of determining the skin friction should be obtained. This was the method employed in the present investigation.

2. *Description of Apparatus*.—2.1. *General Arrangement of Apparatus*.—The boundary-layer measurements were carried out on a smooth flat plate which spanned the working-section of a return-circuit wind tunnel. The plate was a solid Duralumin sheet 6 ft long and $\frac{1}{4}$ in. thick and had an elliptical leading edge. Details of this leading edge, together with other important dimensions on the plate, are shown in Fig. 1.

A special effort was made to achieve a high degree of surface finish and over the centre 12 in. of the span the plate was, by normal standards, exceptionally free from waves.

The pressure gradient could be adjusted by slightly changing the incidence of the plate and by altering the inclination of the roof of the tunnel working-section. The pressure distribution along the plate could be found using the 0·020-in. diameter static holes which were drilled on a line one inch to the side of the plate centre-line and at the following distances from the leading edge: $\frac{3}{4}$, 2, 3, 5, 10, 15, 30, 45 and 60 in. (*see* Fig. 1).

Transition and artificial thickening of the boundary layer on the plate was accomplished in two ways: by using trip wires and by using a glass-paper strip. Two different trip wires were used. They were held in contact with the surface one inch from the leading edge and were 0·013 in. and 0·022 in. in diameter. The glass-paper strip was 2·5 in. wide and was cemented onto the plate 0·75 in. from the leading edge. It had a particle size of approximately 0·015 to 0·020 in.

2.2. *Pitot-tubes*.—All the boundary-layer traverses were made by means of a small flattened pitot-tube, its geometric centre being at a height $y = 0 \cdot 0028$ in. when it rested on the surface. The dimensions of the opening were: breadth 0·047 in. and height 0·002 in.

For the measurement of skin friction, using Preston's method, the original set of five geometrically similar pitot-tubes were used. The dimensions of the openings of these are given in the Table below and the method of construction is described in detail in Ref. 2. The pitot-tubes could be located on the plate at any one of the seven positions shown in Fig. 1.

Dimensions of the Geometrically Similar Round Pitot Tubes

Pitot number	External diameter (in.)	$\frac{\text{Internal diameter}}{\text{External diameter}}$
1	0·0239	0·5942
2	0·02915	0·602
3	0·0544	0·603
4	0·0907	0·599
5	0·1214	0·598

2.3. *Manometers*.—All pressure differences were measured on two sensitive null-reading manometers of the inclined tube type, the accuracy of each individual reading being within 1 per cent.

3. *Description of Experiments and Procedure*.—Preliminary experiments were made to ensure that the pressure distribution over the plate was as nearly uniform as possible. The final distribution obtained is shown in Fig. 2. Also, the two-dimensional nature of the flow was checked by measuring velocity profiles at three spanwise positions 45 in. from the leading edge.

Boundary-layer traverses and skin-friction measurements were then made at each of the five stations along the centre-line of the plate with each of the three transition devices. Some or all of the five geometrically similar round pitot-tubes were used for the skin-friction measurements, the number used at each station being determined by the condition that each one lay wholly in the region where the 'inner' law was applicable, *i.e.*, approximately the inner fifth of the boundary layer. The skin friction at each station was then taken as the mean of the values obtained with the appropriate number of pitot-tubes.

Considerable difficulty was experienced in repeating sets of results and over a period of several days the boundary-layer development and the distribution of skin friction were found to change to a marked degree for a given transition device. The reasons for these changes were eventually found and they are discussed later. However, by constant checking and repeating of measurements, five complete sets of results were obtained, two with a 0·013-in. diameter transition wire, one with a 0·022-in. diameter wire and two with the glass-paper strips. Results were also obtained from a third set of measurements with the 0·013-in. transition wire, when the boundary layer was turbulent only over the rear half of the plate.

All experiments were carried out with a constant Reynolds number per foot ($3 \cdot 9 \times 10^5$) in the working-section. Adjustments were made to the free-stream velocity, before each measurement of dynamic pressure, to take account of changing temperature and pressure.

4. *Reduction of Observations and Corrections etc*.—4.1. *Velocity Distribution**, δ *and* θ.—Corrections were made for the effect of the presence of the pitot-tube on the pressure at the static hole, before the values of u/U_1 were computed. It may be noted here that corrections due solely to viscous effects (*see* Ref. 10) were not applied because, for the pitot-tube Reynolds numbers of these experiments, the correction to the measured dynamic pressure would be constant and would therefore have no effect on u/U_1. Also, corrections for the effects of the turbulent fluctuating velocities were not necessary because the differences they produce in the values of u/U_1 have a negligible effect on the calculated values of the momentum thickness.

The measured values of y in all the pitot traverses were corrected to allow for the displacement of the effective centre of the flattened pitot-tube. This displacement was taken to be the Young and Maas[11] value of $0 \cdot 24h$ (where h is the height of the pitot-tube).

The boundary-layer thickness δ was obtained by plotting the outer part of each profile in the form $\log_{10} y$ vs. $\log_{10} u/U_1$. The curve through the points up to a value of $u/U_1 = 0 \cdot 997$ was then extrapolated to the y axis and the value of y there taken as the boundary-layer thickness. Thus consistent values of δ were found.

The values of momentum thickness were obtained in the usual way by plotting curves of y vs. $(u/U_1)\{1 - (u/U_1)\}$ and measuring the necessary areas.

* The velocity distributions are described in detail in a later report.

4.2. *The Skin Friction from the Round Pitot-Tubes on the Wall.*—The large-scale calibration curves described by Preston in Ref. 2 were used to obtain the skin friction from the measurements with round pitot-tubes. The probable error in the skin friction incurred in reading these curves was less than 1 per cent. The necessary calculations were made for the temperature and pressure conditions existing in the tunnel at the time of each measurement.

Again the observations were corrected to allow for the effect of the pitot-tube on the pressures at the static holes.

5. *Results and Discussion.*—5.1. *Skin Friction Measurements.*—In Figs. 3a and 4a the actual values of $c_f/2$ found with each round pitot-tube at the five stations along the plate, for two typical sets of results, are plotted against the corresponding values of d/δ. The reason for this method of plotting the results will be apparent later. The distributions of the mean values of $c_f/2$ along the plate, corresponding to the above two sets of results, are shown in Figs. 3b and 4b.

From these figures it will be evident that the accuracy obtained for the values of $c_f/2$ is very high ; in fact, at each station $c_f/2$ is found to be within $\pm\frac{1}{2}$ per cent of the mean.

5.2. *The Pressure Gradient and Turbulence Terms Appearing in the General Momentum Equation.*—Typical values of the pressure-gradient term $(H + 2)\{\theta/(\rho U_1^2)\}(dp/dx)$ and the turbulence term $K(d\delta/dx)$ which appear in the momentum equation (4) are shown in Fig. 5, where they are compared with the appropriate distribution of skin friction. It may be noted that for the other sets of results the only significant change in the pressure gradient term occurred between the 45 and 60 in. stations and the term $K(d\delta/dx)$ remained effectively the same. For all the results the term $\{2/(\rho U_1^2)\}\ \delta K\ (dp/dx)$ was small enough to be neglected.

Fig. 5 shows that over the greater part of the plate the pressure-gradient term lies between $\frac{1}{2}$ per cent and $1\frac{1}{2}$ per cent of $c_f/2$ but over the last 15 in. it increases to 25 per cent, even though the change in pressure in this region is less than 1 per cent q_0. From the same figure the value of $K(d\delta/dx)$ is seen to be approximately 2 per cent of $c_f/2$ all along the surface. Thus it is seen that while the pressure gradient and turbulence terms are small they are also quite significant, even for approximately zero pressure gradient, and they must therefore be included in the momentum equation if an accurate estimate of the momentum balance is required.

5.3. *Comparison Between the Measured and Computed Development of Momentum Thickness.*—It can be seen now that all the terms appearing in the integrated momentum equation (5) can be found. Hence, taking the measured values of θ at the 10-in. station as starting values, this equation can be used to calculate the growth of momentum thickness along the plate for each set of conditions. This calculated development is compared with the measured results in Fig. 6.

While very good agreement is obtained for three sets of results, discrepancies are observed in the comparison with the first set of results obtained with the 2·5-in. glass-paper strip and also with the results obtained with the 0·022-in. diameter transition wire. However, even here, if the two sides of equation (5) are compared, it is found that the right-hand side is only 6 per cent higher and 7 per cent lower respectively than the left-hand side when the limits 1 and 2 refer to the 10 and 60-in. stations on the plate. For the other three sets of results the corresponding differences are from $\frac{1}{2}$ per cent to $1\frac{1}{2}$ per cent too low and these very small variations must certainly be considered to lie within the limits of the experimental accuracy.

5.4. *Possible Reasons for the Discrepancies.*—A likely explanation of the larger discrepancies can be found by first considering the rather strange phenomenon experienced earlier in the tests. It was found that the boundary-layer development over the plate could change for no apparent reason and a change of this type, for instance, produced the two quite different sets of results obtained with the glass-paper-strip transition device. After some thought it was decided that the changes were due to very small alterations in the direction of the free stream at the leading edge of the plate. These would cause quite significant changes in the pressure distribution in the region of the leading edge but would have a negligible effect on the distribution over the part of the plate used for measurements, *i.e.*, 10 in. to 60 in. Although the actual origin of the changes could not be established this theory was substantially confirmed near the end of the tests when a turbulence screen in the settling chamber of the tunnel was cleaned ; this operation had a marked effect on the boundary-layer development and even on the pressure distribution over the length of the plate. After the first changes had been noted certain observations were repeated a number of times to check that the boundary-layer development remained constant for each particular experiment. Consequently it is unlikely that the above can account for the discrepancies between some of the measured and calculated results. However, it is likely that these discrepancies are due to the sensitivity of the boundary-layer development to the leading-edge conditions, and two alternative suggestions are put forward.

The first of these is that the boundary layer on the upper surface of the plate was systematically affected by the movement of the traverse gear below. At the beginning of the investigation it was observed that this had only a very small influence on the pressure gradient over the plate and it was therefore assumed that it would have a negligible effect on the boundary-layer development. This assumption was also checked by measuring the dynamic pressure at a fixed height in the boundary layer at the station 60 in. from the leading edge and with the traverse gear in each of its five positions below the plate. The dynamic pressure appeared to be independent of the traverse gear position. It was realised later, however, that this may only have been true for the general conditions existing at the time of the test. Already it has been seen that the sudden changes in the boundary-layer flow over the plate were associated with changes in the position of the stagnation point near the leading edge, and in view of this it appeared possible that certain stagnation point positions might be sensitive to small pressure variations caused by the movement of the traverse gear. If this did happen it would account for some of the results being quite poor while other sets were very good.

The other reason was also connected with the position of the stagnation 'line' along the leading edge of the plate. If, for one particular test, this was in a critical position, then it seems rather unlikely that it would occupy exactly the same position with respect to the leading edge all the way across the plate. If it did not, then the flow would be three-dimensional and although the effects might be expected to be small for approximately zero pressure gradient, they might be sufficient to make it unsatisfactory to use the two-dimensional momentum equation. Also, it is difficult to ensure that transition wires and strips have a uniform spanwise influence and the slow lateral spread of an initial non-uniform entry condition would only be felt towards the rear of the plate.

Unfortunately time did not permit further experiments which would have confirmed or disproved these ideas.

6. *Discussion of the Accuracy of Preston's Method in the Light of the Present Investigation.*— Although the present investigation has not provided conclusive proof that Preston's method (Ref. 2) may be used to obtain highly accurate values of skin friction, it does suggest very strongly that this is the case. The procedure adopted to check particular skin-friction measurements indicates that correct values were certainly obtained in three different sets of measurements, and a likely explanation, which does not involve the accuracy of the skin-friction values, can be found (*see* previous section) to account for the discrepancies occurring in the remaining two sets of results. No evidence has been found of any consistent systematic error in the skin-friction values and the experiments therefore provide no support whatever for the conclusion reached by Relf[1], namely, that the skin-friction values found by Preston's method will be a constant 14 per cent too low.

Consideration of Figs. 3a and 4a, where $c_f/2$ is plotted against d/δ, throws further doubt on the conclusion drawn by Relf. There it will be seen that as the boundary-layer thickness increases, d/δ becomes smaller, until at the 60-in. position the diameter of the smallest round pitot-tube is approximately 1/50th of the boundary-layer thickness, but in spite of this, in all the sets of results, the skin friction found by the smallest pitot-tube is the same as that obtained from the remaining larger pitot-tubes. This is very significant because it proves beyond doubt that there is a region of local dynamical similarity in the turbulent boundary layer and further that, for the same skin friction, the velocity profiles in pipe and boundary-layer flow are at least geometrically similar in the region covered by the round pitot-tubes, *i.e.*, from $0 \cdot 02\delta$ to $0 \cdot 2\delta$. Consequently the suggestion implied in Ref. 1, that for the same skin friction the turbulent velocity profiles in pipe and boundary-layer flow are different, can only be true if the transition from the position where they must be exactly the same, *i.e.*, in the immediate neighbourhood of the surface, to the point where they are geometrically similar, takes place within 1/50th of the boundary-layer thickness. This is certainly difficult to imagine because as the surface is approached the physical differences between the two types of flow must become less and less significant, and hence, if there were a transitional region of the type mentioned above, it would not be expected to occur so close to the surface. In fact, the effect of curvature of the surface would be expected to become more important as the distance from the surface increased, so that constant values of skin friction would not be obtained with the larger pitots.

The hypothesis that for the same skin friction two different profiles (and hence two different 'inner laws') exist, one for turbulent pipe flow and one for flat-plate turbulent boundary-layer flow, may be argued on the grounds that the two types of flow differ in the following ways:

(*a*) A pressure gradient always exists in the pipe flow

(*b*) The surface of a pipe is curved

(*c*) Turbulent flow in pipes is nowhere intermittent, whereas patches of non-turbulent fluid occur in the outer regions of turbulent boundary layers

(*d*) The turbulent 'boundary layer' in a pipe always has a constant thickness equal to the pipe radius.

However, previous experimental evidence indicates that at least (*a*) and (*c*) should not be put forward in support of this hypothesis. It is well established (*see* Ref. 8) that the inner law is independent of pressure gradient. Secondly it has been shown by Schubauer[12] that in spite of the intermittent character of the outer part of the flat-plate boundary-layer, the actual distribution of turbulent energy in pipe flow is the same as in the fully turbulent part of the boundary-layer flow ; also, patches of non-turbulent fluid are not found in the region $y/\delta < 0 \cdot 4$. Hence, it appears that if the inner law is different for pipe and boundary-layer flow, this must be due to either the effects of surface curvature or the fact that the flat-plate boundary layer is continually growing, while the equivalent thickness in the pipe remains constant. If this is so it implies that one (or both), of the parameters describing (*b*) and (*d*), is important and must therefore be included in the derivation of the corresponding inner law. Under these circumstances the existing form of the inner law would not be universal for either pipe or boundary-layer flow and this, of course, is contrary to a large amount of experimental evidence collected in the past. In particular it is contrary to the present results. As previously mentioned, these show that within the accuracy of the measurements, $(P - p_0)d^2/\rho\nu^2$ is a function only of $\tau_0 d^2/\rho\nu^2$ (and from section $1 \cdot 3$ this is equivalent to saying that u/u_τ is a function only of $u_\tau \rho_y/\mu$) in the region $0 \cdot 02\delta$ to $0 \cdot 2\delta$.

The foregoing arguments, while possibly not conclusive, suggest strongly that the existence of two different (but universal for each type of flow) relations of the form $(u/u_\tau) = f(u_\tau \rho_y/\mu)$ is extremely unlikely. Moreover, since the present experiments have provided no evidence of systematic errors in the skin-friction values, the necessity for postulating two such different relations does not appear. Consequently it is concluded that the values of skin friction found by Preston's method are correct.

7. *Distribution of Local Turbulent Skin Friction on a Smooth Flat Plate*.—The conclusion that Preston's method is correct permits a comparison between the present measurements of skin friction and the results given by various empirical skin-friction laws for smooth flat plates.

In most of the early investigations the skin-friction coefficient was expressed as a function of R_x, the Reynolds number based on the distance from the leading edge of the plate, and this has been the likely cause of some of the considerable discrepancies between the results of various experimenters. This is illustrated clearly in Figs. 7 and 8 which show the variation of R_θ and $c_f/2$ respectively with R_x. The values of $c_f/2$ and R_θ at a particular R_x are seen to depend appreciably on both the transition device and the position of the stagnation point relative to the leading edge (*see* section 5.4) and hence neither $c_f/2$ nor R_θ can be expressed as a unique function of R_x. It is essential, therefore, that $c_f/2$ should be correlated with some quantity other than R_x, which is independent of entry conditions.

Both Coles[13] and Landweber[14] have shown that for fully developed turbulent flow

$$c_f/2 = f(R_\theta) , \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

where the function is universal and independent of entry conditions provided the inner law and the outer velocity defect law are assumed to hold and further, that there is a common region in which they are both applicable. Hamma[15] also shows that the relation can take the form

$$\sqrt{(2/c_f)} = \frac{U_1}{u_\tau} = A \log R_\theta + B \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

and the results of various experimenters, together with those of the present investigation, are plotted in this way in Fig. 9. The large discrepancies between the established results are clearly shown in this diagram and they must be entirely due to the difficulty of measuring skin friction in the past. The results of the present experiments show only a small amount of scatter about a straight line for $1500 < R_\theta < 4000$, and are in good agreement with the result predicted by Coles.

In Fig. 10 some of the results shown in Fig. 9 are replotted in the form $c_f/2$ vs. R_θ. The values of $c_f/2$ found from the Prandtl-Schlichting and Schoenherr laws are estimated to be respectively 10 per cent and 6 per cent higher than the corresponding values obtained in the present investigation.

8. *General Conclusions*.—The existence of a region of local dynamical similarity in the turbulent boundary layer, where conditions are dependent only on the properties of the fluid, the wall friction, and a particular length, has been firmly established, for zero pressure gradient. This supports Preston's experiments in a pipe and on the walls of a wind tunnel where this similarity was deduced from the readings of similar pitot-tubes. The region is found to extend certainly from $y = 0$ to $y = 0 \cdot 2\delta$. This fact therefore shows that under the above conditions $(u/u_\tau) = f(u_\tau \rho_y/\mu)$, just as for fully developed turbulent pipe flow but, of course, it does not prove that the functions are precisely the same for both types of flow. However, certain basic arguments can be put forward (*see* section 6), which indicate that the function is the same for both types of flow and thus the basis of Preston's method would appear to be perfectly sound. This is quite firmly supported by the checks made on the measured skin friction using the method based on the general momentum equation. It was therefore concluded that, contrary to the results obtained by Relf[1], the values of skin friction found by Preston's method were correct.

It is evident from the results obtained in the investigation that an accurate estimate of the momentum balance in the boundary layer can only be obtained using the general momentum equation. Very small pressure gradients, and the turbulent fluctuating velocities occurring under these conditions, are each found to contribute significant amounts to this momentum balance and can not therefore be neglected. A method is suggested by which the magnitude of the significant turbulence terms appearing in the general momentum equation can be easily calculated. For the present results this ranges from 2 to $2\frac{1}{2}$ per cent of the local skin-friction coefficient.

It is apparent that the leading-edge conditions have an appreciable influence on the distribution of skin friction and the boundary-layer development over the plate, and it is clear that large errors can arise from a relation which expresses the local skin-friction coefficient solely in terms of R_x, the Reynolds number based on the distance from the leading edge of the plate. However, it is found that, again as suggested by Coles and Landweber, $c_f/2$ can be expressed as a universal function of R_θ. The present results disagree appreciably with those of Prandtl and Schlichting and of Schoenherr, the values of $c_f/2$ obtained from these laws being respectively 10 per cent and 6 per cent greater than the corresponding values obtained in the present investigation. However, the present results are substantiated by the results obtained by Coles from an analysis of a large amount of experimental data, and it is believed that they give an accurate picture of the variation of $c_f/2$ with R_θ in the range $1500 < R_\theta < 4000$.

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APPENDIX I

The Influence of the Turbulent Fluctuating Velocities on the Von Kármán Momentum Equation for Two-Dimensional Flow

The general momentum equation for turbulent boundary-layer flow containing the terms due to pressure gradients and turbulent fluctuating velocities, can be written as

$$\frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - \frac{1}{U_1^2}\left[\int_0^\delta \frac{\partial \overline{u'^2}}{\partial x}\,dy - \int_0^\delta \frac{\partial \overline{v'^2}}{\partial x}\,dy\right]$$

$$+ \frac{1}{U_1^2}\int_0^\delta\left[\int_0^y \frac{\partial^2 \overline{u'v'}}{\partial x^2}\,dy - \delta\,\frac{\partial^2 \overline{u'v'}}{\partial x^2}\right]dy\,. \quad .. \quad .. \quad (1)$$

The influence of the turbulence terms in this equation has been examined by several authors (*see*, for example, Ref. 8) and in general it has been concluded that the effects are negligible except when separation is approached. A re-analysis of this problem, however, showed that the Reynolds normal stress term, could contribute as much as 5 per cent of the skin-friction coefficient in regions of small pressure gradient and where the boundary layer is a long way from separation. Moreover, it was found that the influence of the term containing $\overline{v'^2}$ (in equation (1)) must also be considered; this reduces the effect of the Reynolds normal stress term ($\overline{u'^2}$) but the resultant term still appears to be significant when compared with the skin-friction coefficient. A similar analysis of results obtained in zero pressure-gradient conditions shows that even there the two terms mentioned above can contribute as much as 2 per cent of the skin-friction coefficient. Thus it appears that if an accurate estimate of the momentum balance is required the turbulence terms cannot be omitted from equation (1). A simple method is given below which will enable a sufficiently accurate estimate of the magnitude of these terms to be made.

The experimental data at present available is, unfortunately, not sufficient to permit the calculation of the terms containing $\partial^2\overline{u'v'}/\partial x^2$, but a reasonable estimate of their magnitude can be made and as this indicates that they are both less than 1 per cent of the Reynolds normal stress term, they are neglected in the following. Equation (1) therefore reduces to

$$\frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - \frac{1}{U_1^2}\left[\int_0^\delta \frac{\partial \overline{u'^2}}{\partial x}\, dy - \int_0^\delta \frac{\partial \overline{v'^2}}{\partial x}\, dy\right], \quad \ldots \quad \ldots \quad (2)$$

which may be expanded in the form

$$\frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - \left[\frac{\partial}{\partial x}\,\delta\int_0^1 \frac{\overline{u'^2}}{U_1^2}\, d\!\left(\frac{y}{\delta}\right) + \frac{2}{U_1}\,\delta\,\frac{dU_1}{dx}\int_0^1 \frac{\overline{u'^2}}{U_1^2}\, d\!\left(\frac{y}{\delta}\right)\right]$$

$$+ \left[\frac{\partial}{\partial x}\,\delta\int_0^1 \frac{\overline{v'^2}}{U_1^2}\, d\!\left(\frac{y}{\delta}\right) + \frac{2}{U_1}\,\delta\,\frac{dU_1}{dx}\int_0^1 \frac{\overline{v'^2}}{U_1^2}\, d\!\left(\frac{y}{\delta}\right)\right]. \quad \ldots \quad (3)$$

In this form equation (3) is quite intractable but it may be considerably simplified if it is assumed that the distribution of $\overline{u'^2}/U_1^2$ and $\overline{v'^2}/U_1^2$ exhibit longitudinal similarity in a fully developed turbulent boundary layer on a flat plate. With this assumption equation (3) reduces to

$$\frac{\tau_0}{\rho U_1^2} = \frac{d\theta}{dx} - (H + 2)\frac{\theta}{\rho U_1^2}\frac{dp}{dx} - K\frac{d\delta}{dx} + 2K\frac{\delta}{\rho U_1^2}\frac{dp}{dx}, \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where

$$K = \left[\int_0^1 \frac{\overline{u'^2} - \overline{v'^2}}{U_1^2}\, d\!\left(\frac{y}{\delta}\right)\right]$$

and is a constant.

The above assumption is supported by the limited amount of experimental evidence available. The experiments of Klebanoff and Diehl[9], for instance, show that the distribution of u'/U_1 through a fully developed turbulent boundary layer remains constant (within the accuracy of the measurements) over the Reynolds-number range $3 \cdot 5 \times 10^6$ to 7×10^6.

Using data obtained recently by Klebanoff[5], K is found to equal $0 \cdot 00174$, a value which is substantiated by the experimental results quoted in Ref. 9. As the terms involving K are likely to be small, *i.e.*, they are only 'corrective' terms in equation (4), they should be obtained with sufficient accuracy using this value and it was therefore used in the analysis of the results of the present investigation.

It might also be noted that if a 1/7th power law, for instance, is used to express δ in terms of θ, equation (4) can be written

$$\frac{\tau_0}{\rho U_1^2} = A\frac{d\theta}{dx} - [(H + 2) - B] \times \frac{\theta}{\rho U_1^2}\frac{dp}{dx}, \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where A and B are $0 \cdot 983$ and $0 \cdot 035$ respectively, taking $K = 0 \cdot 00174$.

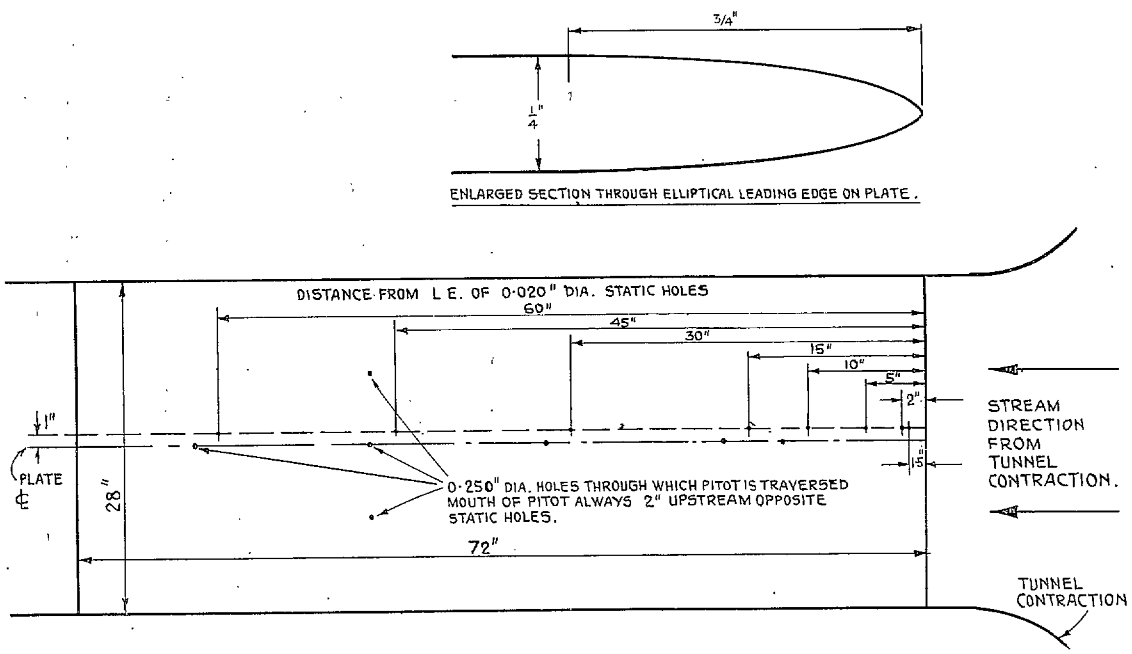


FIG. 1. Plan of flat plate showing principal dimensions.

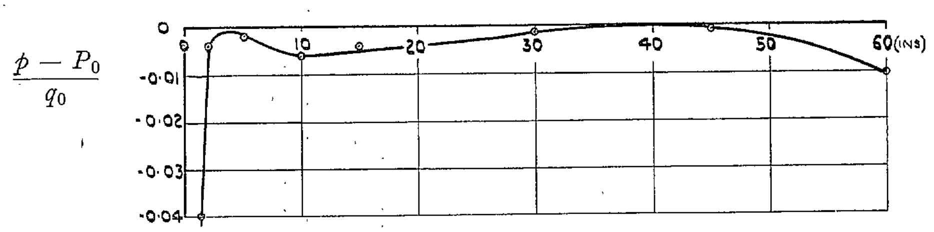


FIG. 2. Pressure distribution over the flat plate.

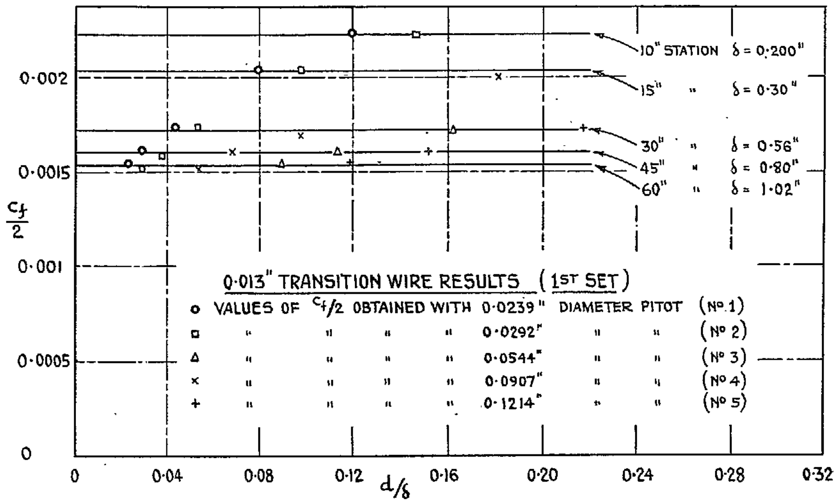


FIG. 3a. The values of the skin friction coefficient at each station along plate found by Preston's method using a set of geometrically similar pitot-tubes.

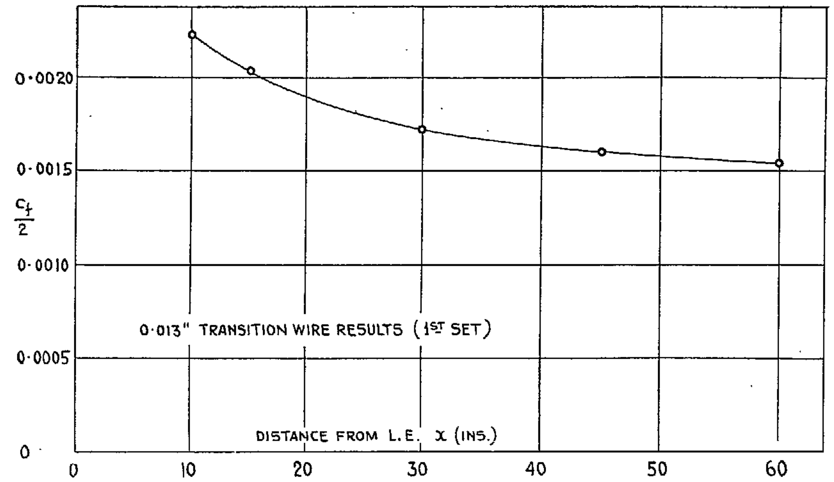


FIG. 3b. Distribution of skin friction along flat plate found by Preston's method.

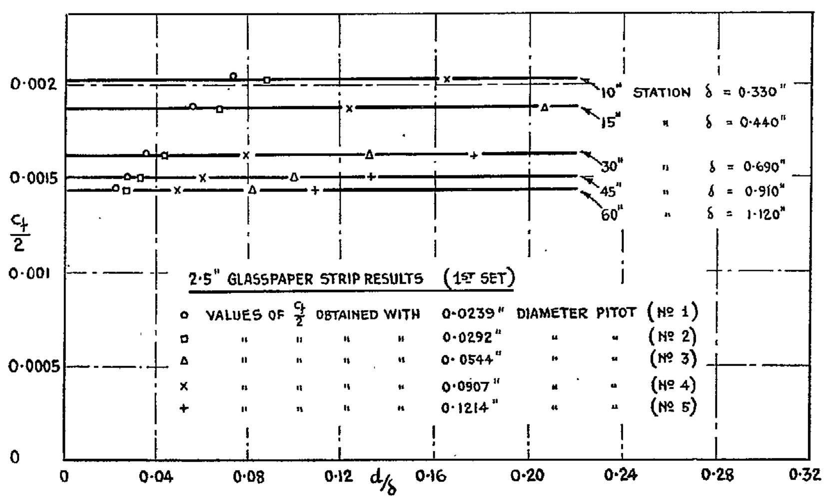


FIG. 4a. The values of skin-friction coefficient at each station along flat plate found by Preston's method using a set of geometrically similar pitot-tubes.

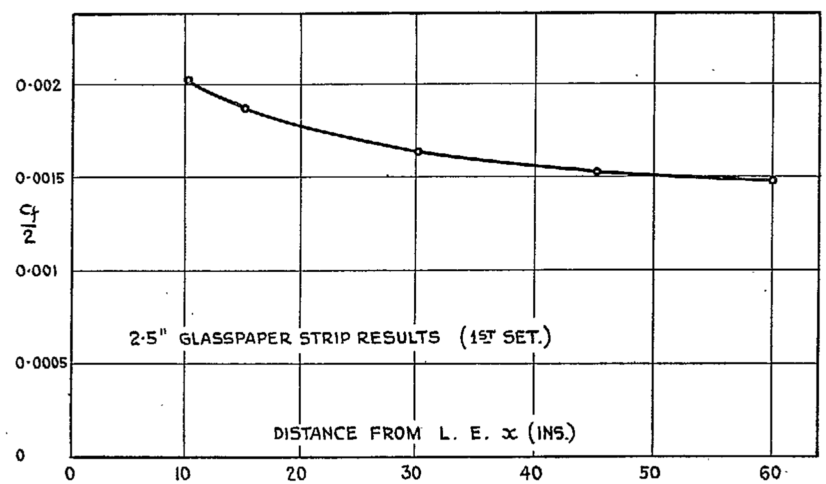


FIG. 4b. Distribution of skin friction along flat plate found by Preston's method.

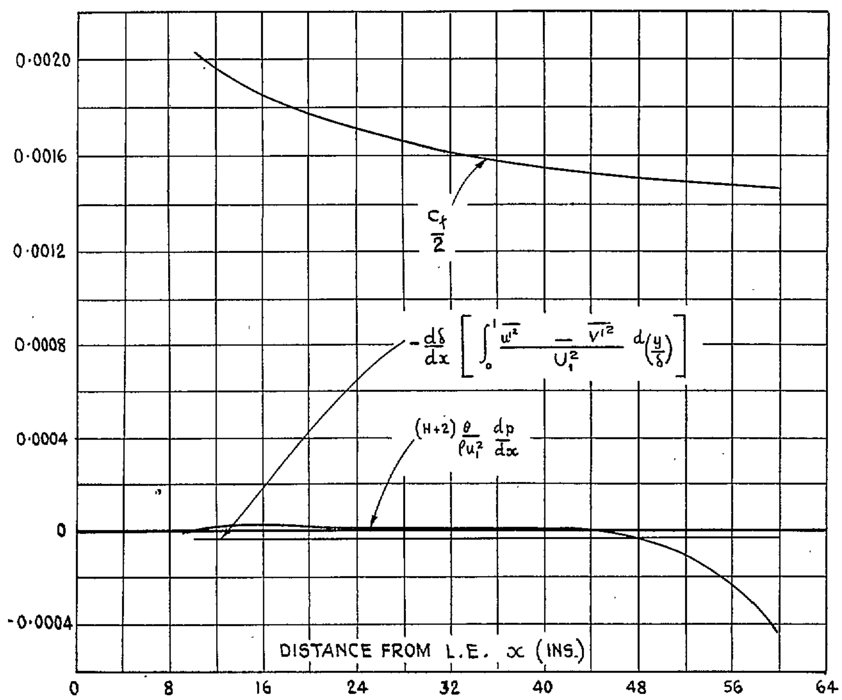
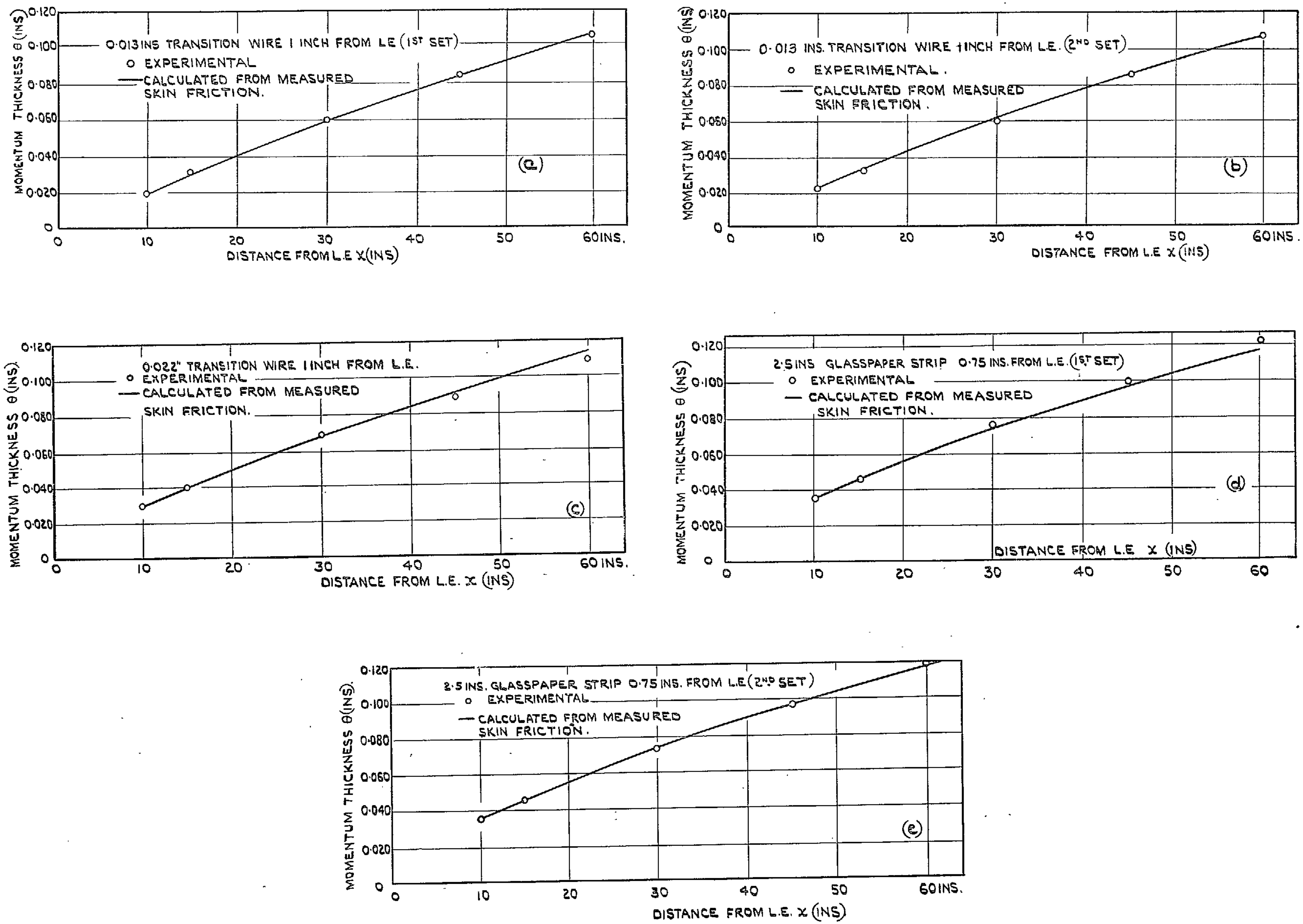


FIG. 5. Typical comparison between the terms appearing in the boundary-layer momentum equation.



FIGS. 6a to 6e. The comparison between the measured and predicted development of θ along the plate. Calculated values of θ found from the momentum equation using values of skin friction found by Preston's method and measured value of θ at 10 in. as a starting value.

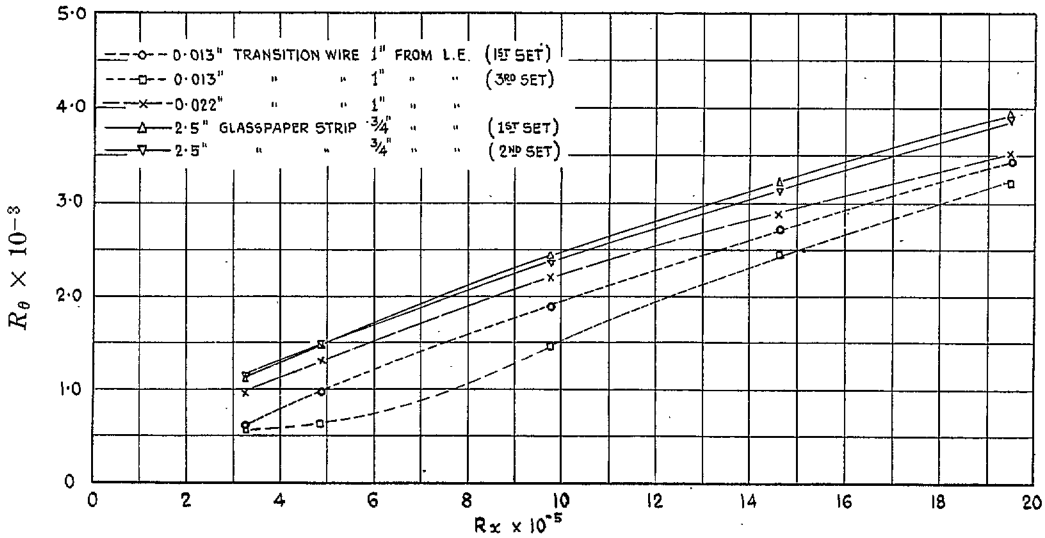


FIG. 7. The influence of leading-edge conditions on the development of the turbulent boundary layer along a flat plate.

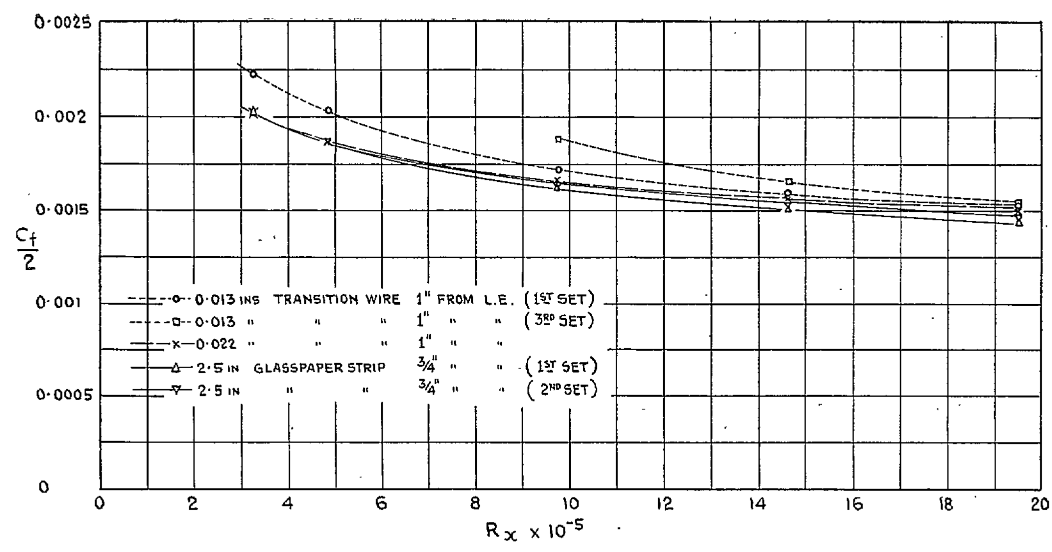


FIG. 8. Distribution of skin friction obtained by Preston's method on the flat plate with different transition devices and leading-edge conditions.

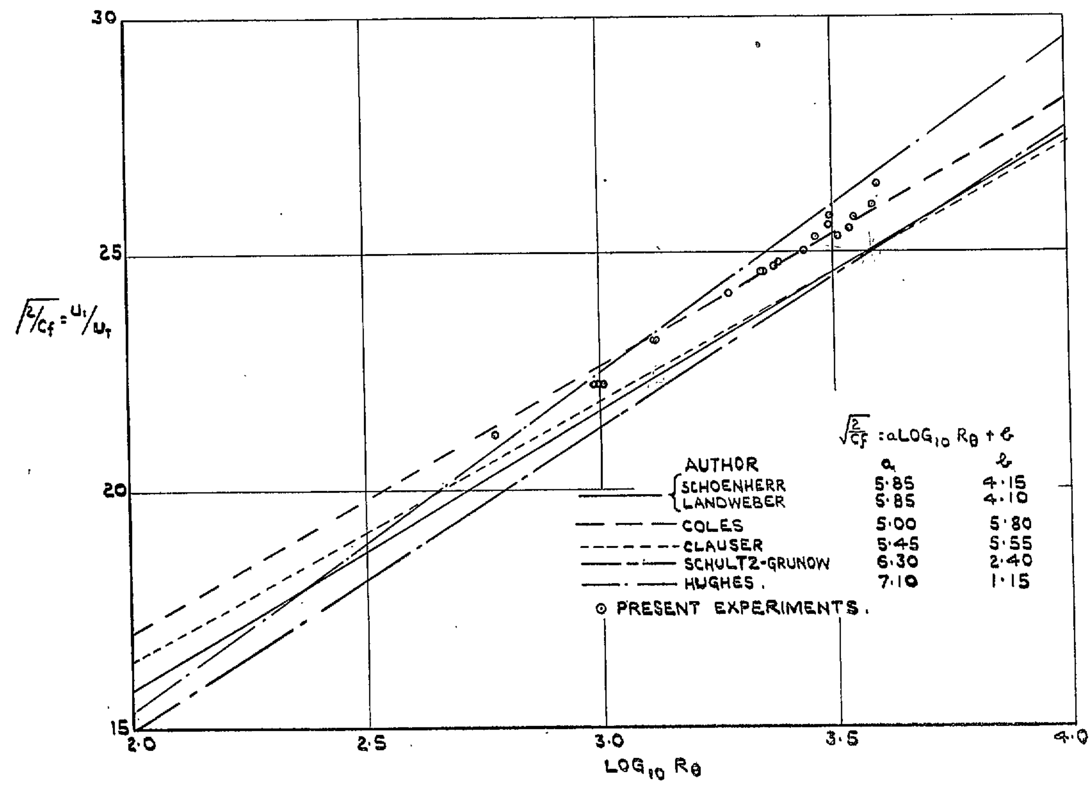


FIG. 9. Comparison between present results and various local skin-friction laws.

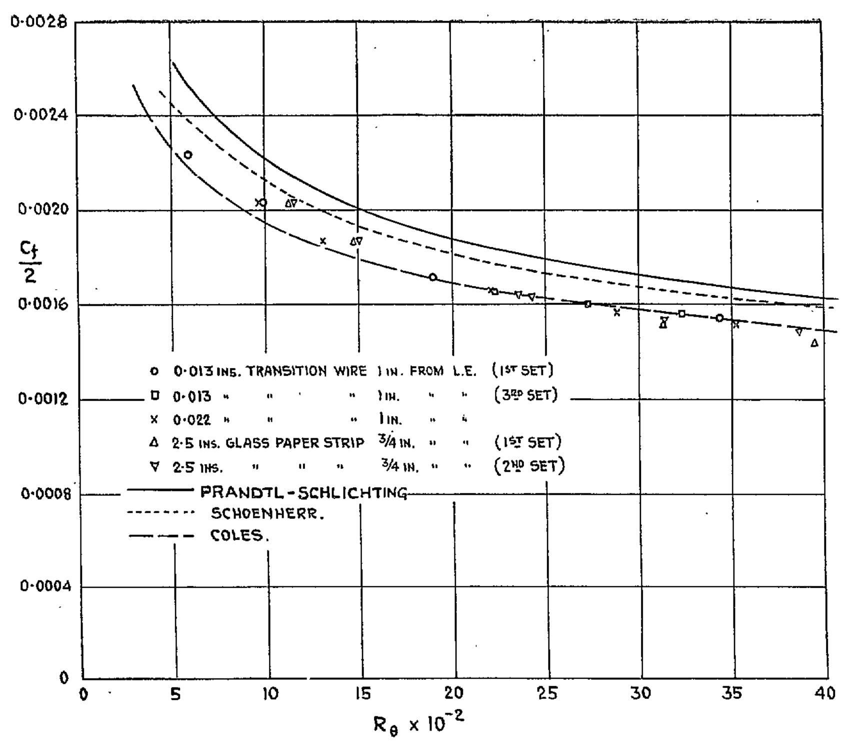


FIG. 10. Local skin friction on a flat plate.

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