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(With Particular Reference to the Chemical Method of Indicating Boundary-layer Transition)

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

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Summary.—The problem of predicting the rate of transport of a gas from or into the surface of a two-dimensional body in an airstream is discussed. The principal object of the investigation is to provide a means of estimating the time required to obtain an experimental record of boundary-layer transition when a chemical technique is used. The methods evolved should, however, find an application to other forced diffusion phenomena.

The general approach is based on the analogy between mass transfer, heat transfer and skin friction, and the analysis is applied to both a laminar and a turbulent boundary-layer on the surface of the body; it also includes the problem of diffusion commencing in an established boundary-layer. For this problem, an approximate, alternative solution to that of O. G. Sutton, for a turbulent boundary layer, is given.

Particular attention is paid to a description of the boundary condition at the surface of the body, and it is concluded that, for evaporation, the usual assumption that the air is saturated with the diffusing substance is, in general, satisfactory.

The influence of molecular diffusion on the transfer of a gas through a turbulent boundary layer is considered; it is demonstrated that the effects of molecular diffusion in the laminar sub-layer may be important. A simple, approximate method of estimating the molecular diffusion coefficient for a pair of gases is derived.

A description is given of a wind-tunnel experiment in which measurements were made of the rates of sublimation of some chemicals from a small part of a flat plate with a turbulent boundary layer: they were found to agree fairly well with the corresponding theoretical estimates.

Finally, as an example of the application of the methods, a calculation of the effect of altitude on the rate of sublimation of a chemical from the surface of an aircraft is made; this agrees with flight tests, which had established that the rate of sublimation decreases very rapidly with increase of altitude.

1. *Introduction.*—The chemical method of indicating transition from laminar to turbulent boundary-layer flow has been used for some time in both flight and wind-tunnel experiments. Following the introduction of the method by W. E. Gray (1944), considerable experience has been gained in deciding which particular technique is the most suitable in given circumstances, and how best to apply it[2,3,4], but no calculations of the influence on the rate of chemical or physical reaction of variables such as surface temperature, skin friction and wind speed, appear to have been made. Until recently, the need for such quantitative information was not great, because, in general, each set of boundary-layer observations was made under a fairly

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narrow range of conditions, so that, having selected a certain chemical indicator and a corresponding technique, the changes in reaction time accompanying small changes in, say, wind-speed were of little significance. However, the increasing application of the method to aircraft which may operate over a wide range of altitude and forward speed makes it desirable to have available some method of calculating the approximate rates of reaction, for flight testing experience has shown that evaporative solids are much more sensitive to temperature effects than had previously been realized[3,4].

Two methods of distinguishing between regions of laminar and turbulent boundary-layer flow commonly in use are the 'sublimation' method[2] and the 'contamination' method[1]. In the sublimation method, the surface under observation is coated with a feebly volatile substance; during exposure to an airstream, sublimation occurs along the surface at varying rates depending mainly upon the magnitude of the local shear stress within the boundary layer. The greatest rates of sublimation are usually achieved in regions of high shear stress or skin friction. It follows that, for a surface such as that of an aerofoil on which there may be distinct areas of laminar and turbulent flow, sublimation will first be complete where the flow is turbulent, except possibly within a small region near the stagnation point. The physical processes which occur in the contamination method are similar. The surface is coated with a chemical, and an active gas is introduced into the airstream; the transport of the gas through the boundary layer, and consequently reaction between the gas and the surface, proceeds at a greater rate in regions where the flow is turbulent than in those where it is laminar, again with the possible exception of the neighbourhood of the stagnation point.

Clearly, the problem of predicting the rate of sublimation, or of chemical reaction, reduces to a determination of the rate of transport of a foreign gas through the boundary layer, given the concentrations of the gas at the surface and in the main stream: stated in this way, the solution of the problem will have an application to a wider class of diffusive phenomena than simply that of the chemical indication of boundary-layer transition. The method given in this note is essentially an approximate one based on the analogy between diffusion, heat transfer and skin friction, and, although certain of the results have appeared in other papers (*e.g.*, Ref. 5), the analysis is developed from first principles for the sake of completeness. The treatment of this purely aerodynamic aspect of the problem is independent of whether the diffusion is directed towards or away from the surface; on the other hand, the absolute rate of mass transfer depends on the surface concentration of the diffusing gas, which can only be specified in general for an evaporation or sublimation process, where it is proportional to the vapour pressure of the diffusing substance. In a contamination process, involving the absorption of an active gas from the main stream, the surface concentration will depend on the nature of the chemical reaction occurring between the gas and the material with which the surface of the body is coated and, except in special circumstances—for instance, when the temperature of the surface is high—can only be determined by experiment. Accordingly, in the discussion of practical examples of mass transfer, emphasis is placed on cases of evaporation or sublimation.

The accuracy of the method should be adequate for most practical purposes. For example, it is sufficient to indicate the effects of a change in altitude on the time required to obtain a record of transition in a flight experiment when a 'sublimation' technique is used. Generally speaking, the limit to the degree of refinement worth attaining in any theory of forced evaporation or sublimation is set by the accuracy with which the vapour pressure of the diffusing substance is known. This may be high for evaporating liquids at ordinary temperatures, but, for solids of small volatility, vapour pressure data from various sources may be found to differ by as much as 100 per cent. With this reservation as to the order of accuracy to be expected from the theory, the calculated rates of sublimation of some solid substances were found to agree satisfactorily with measurements made in a wind tunnel.

2. *The Two-dimensional Diffusion Equation.*—We shall consider the diffusion of a gas from, or into, a surface $y = 0$ in a two-dimensional airflow. Let U be the velocity in the x-direction at the outer edge of the boundary layer and u,v the mean velocity components inside the layer.

The concentration of the gas will be assumed so small that variations in the density of the mixture may be neglected and only the diffusion of the contaminating gas into the air need be considered. Variations in the density of the mixture due to variations in temperature across the boundary layer are also neglected, thus the analysis is restricted to values of U well below the speed of sound. Other properties of the mixture, such as ν, the kinematic viscosity, and k, the thermal conductivity, will be assumed independent of the presence of a small quantity of the contaminating gas and their values will be taken to be the same as those in an uncontaminated airstream.

The assumption of a low concentration also implies that the rate of transfer of mass across the surface is small and it is, therefore, permissible to retain the boundary condition $v = 0$ appropriate to an impervious surface ; the effect of a finite velocity component normal to the surface, due to large rates of mass transfer, has been calculated by Schuh[6] (1944) for laminar flow*.

Throughout the analysis it will be supposed that the concentrations in the main stream and at the surface, or part of it, are known and are constant.

2.1. *Laminar Boundary-layer Flow.*—If ρ is the density of the contaminating gas, the mass of this gas diffusing across an elementary area δS in unit time is

$$-j\frac{\partial \rho}{\partial n}\delta S \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1)$$

where n is the normal to the area δS, drawn in the direction of diffusion, and j is the molecular diffusion coefficient[14].

Consider the accumulation of gas within an elementary rectangle having sides δx, δy. The mass carried into the element in unit time by diffusion is

$$j\left(\frac{\partial^2 \rho}{\partial x^2}+\frac{\partial^2 \rho}{\partial y^2}\right)\delta x\,\delta y$$

and by convection

$$-\left(u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}\right)\delta x\,\delta y \ .$$

The net rate of change of mass per unit volume is, therefore, given by

$$\frac{\partial \rho}{\partial t}=-u\frac{\partial \rho}{\partial x}-v\frac{\partial \rho}{\partial y}+j\left(\frac{\partial^2 \rho}{\partial x^2}+\frac{\partial^2 \rho}{\partial y^2}\right) . \quad .. \quad .. \quad .. \quad .. \quad .. \quad (2)$$

When conditions are steady, equation (2) reduces to

$$u\frac{\partial \psi}{\partial x}+v\frac{\partial \psi}{\partial y}=j\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right) \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3)$$

with

$$\psi=\frac{\rho}{\rho_0} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4)$$

where ρ_0 is the density of the mixture, assumed equal to that of uncontaminated air. ψ is called the concentration.

A more exact form of equation (3), allowing for variations in the density of both constituent gases, is derived by Squire in Ref. 5.

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* A further inference from the assumption of small concentration is that the boundary-layer velocity-profile is unaffected by the mass transfer. Large rates of mass transfer, such as might be encountered in sweat cooling, lead to a considerable distortion of the velocity profile ; in the case of laminar flow, the distortion is such that the boundary-layer stability is reduced when mass is transferred from the surface to the main stream and increased when the transfer is into the surface.

The boundary conditions are

$$\left.\begin{aligned} \psi &= \psi_1, \quad y = 0 \\ \psi &\to \psi_0, \quad y \to \infty \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The former condition may not always be satisfied for all values of x along the surface (*cf.* section 3.3).

The diffusion equation, (3), resembles the equation of motion in a laminar boundary layer with zero pressure gradient :

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

As will be discussed later (section 7), j and ν are of the same order of magnitude, and this suggests that the diffusion is confined to a layer near the surface, of dimensions comparable with those of the viscous boundary layer. If we suppose a diffusion boundary layer of thickness δ_1 and a viscous boundary layer of thickness δ, and take U and $(\psi_1 - \psi_0)$ as quantities of standard order, where ψ_0 may now be identified with the concentration at the outer edge of the diffusion boundary layer, the magnitudes of the terms in equation (3) are as follows :

$$u \text{ is } O(1), \qquad u\frac{\partial \psi}{\partial x} \text{ is } O(1), \qquad \frac{\partial^2 \psi}{\partial x^2} \text{ is } O(1) ;$$

$$v \text{ is } O(\delta), \qquad v\frac{\partial \psi}{\partial y} \text{ is } O\left(\frac{\delta}{\delta_1}\right), \qquad \frac{\partial^2 \psi}{\partial y^2} \text{ is } O\left(\frac{1}{{\delta_1}^2}\right).$$

Hence, $\partial^2\psi/\partial x^2$ may be neglected in comparison with $\partial^2\psi/\partial y^2$, and, if this latter term is to be of the same order as the convection terms, $j/{\delta_1}^2$ must be $O(1)$; but ν/δ^2 is $O(1)$, so that

$$\frac{\delta_1}{\delta} \text{ is } O\left(\sqrt{\frac{j}{\nu}}\right).$$

It follows that $v\dfrac{\partial \psi}{\partial y}$ is of the same order of magnitude as $u\dfrac{\partial \psi}{\partial x}$ and the diffusion equation in a two-dimensional laminar flow thus becomes

$$u\frac{\partial \psi}{\partial x} + v\frac{\partial \psi}{\partial y} = j\frac{\partial^2 \psi}{\partial y^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

2.2. *Turbulent Boundary-layer Flow.*—When the flow in the boundary layer is turbulent, the effect of the velocity fluctuations is to increase the mean rate of transfer of the contaminating gas by an amount

$$\overline{v'\rho'}$$

per unit area, where dashes denote fluctuating quantities and the bar signifies an average with respect to time. The resultant rate at which the mass of the contaminating gas is transferred across a unit area parallel to the surface is then

$$-j\frac{\partial \rho}{\partial y} + \overline{v'\rho'} .$$

According to the turbulent mixing length theory (Ref. 7, Chapter v)

$$\overline{v'\rho'} = -\overline{lv}\frac{\partial \rho}{\partial y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where ρ is the mean density of the gas at any point and $\overline{lv}$ is the turbulent exchange coefficient or eddy diffusivity. The diffusion equation in a two-dimensional turbulent flow may, therefore, be written,

$$u\frac{\partial\psi}{\partial x}+v\frac{\partial\psi}{\partial y}=\frac{\partial}{\partial y}\left((j+\overline{lv})\frac{\partial\psi}{\partial y}\right) \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (9)$$

with the boundary conditions

$$\left.\begin{aligned}\psi&=\psi_1\ , \quad y=0\\ \psi&\to\psi_0\ , \quad y\to\infty\end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (10)$$

As in (5) the former of these conditions may not necessarily hold for all values of x along the surface, (*cf.* section 4.3).

3. *Diffusion in a Laminar Flow.*—3.1. *Diffusion from a Flat Plate.*—The problem of diffusion in the laminar boundary layer on a flat plate lying parallel to the airstream, when the area from which diffusion occurs extends to the leading edge of the plate, is identical with the heat transfer problem solved by Pohlhausen (Ref. 8, Chap. xiv). In the case of heat transfer we have the energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho_0 C_p}\frac{\partial^2 T}{\partial y^2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11)$$

together with the boundary conditions

$$\left.\begin{aligned}T&=T_1\ , \quad y=0\\ T&\to T_0\ , \quad y\to\infty\end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (12)$$

in which T is the temperature, k the thermal conductivity, ρ_0 the density and C_p the specific heat at constant pressure of the air.

The corresponding equation for diffusion is

$$u\frac{\partial\psi}{\partial x}+v\frac{\partial\psi}{\partial y}=j\frac{\partial^2\psi}{\partial y^2},$$

with

$$\left.\begin{aligned}\psi&=\psi_1\ , \quad y=0\\ \psi&\to\psi_0\ , \quad y\to\infty\end{aligned}\right\}.$$

Thus, following from the Pohlhausen solution of (11) and (12)[8],

$$\left(\frac{\partial T}{\partial y}\right)_{y=0}=-0{\cdot}332\left(\frac{\mu C_p}{k}\right)^{1/3}(T_1-T_0)\left(\frac{U_0}{\nu x}\right)^{1/2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (13)$$

we have

$$\left(\frac{\partial\psi}{\partial y}\right)_{y=0}=-0{\cdot}332\left(\frac{\nu}{j}\right)^{1/3}(\psi_1-\psi_0)\left(\frac{U_0}{\nu x}\right)^{1/2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (14)$$

U_0 is the velocity of the undisturbed flow; x is measured from the leading edge of the plate.

Introducing a local mass-transfer coefficient, K_g, defined by

$$K_g=\frac{g_s}{\rho_0 U_0(\psi_1-\psi_0)} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (15)$$

where g_s is the rate at which mass is being transferred across a unit area of the surface, *i.e.*, $-\rho_0 j\left(\frac{\partial\psi}{\partial y}\right)_{y=0}$ equation (14) may be written

$$K_g=0{\cdot}332\left(\frac{j}{\nu}\right)^{2/3}\left(\frac{U_0 x}{\nu}\right)^{-1/2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (16)$$

Since the local skin friction coefficient, C_f, is given by (Ref. 7, Chap. iv)

$$C_f = 0{\cdot}664\left(\frac{U_0 x}{\nu}\right)^{-1/2} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (17)$$

it follows that

$$K_g = \frac{1}{2}\left(\frac{j}{\nu}\right)^{2/3} C_f \; . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (18)$$

Typical distributions of K_g as given by (16) or (18) are shown in Fig. 1.

3.2. *Diffusion from the Surface of an Aerofoil.*—When the velocity U outside the boundary layer varies along the surface, the mass-transfer rates may be estimated by the method for calculating the heat transfer, suggested by Squire (1942)[9].

As applied to the diffusion problem, the procedure is to solve the continuity equation†

$$\left(\frac{\partial}{\partial x}\right)\int_0^{\delta_1} u(\psi - \psi_0)\, dy = -j\left(\frac{\partial \psi}{\partial y}\right)_{y=0} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (19)$$

by assuming the distributions of u and ψ across their respective boundary layers to be equal to the Blasius velocity distribution[7] across the laminar boundary layer on a flat plate.

Defining displacement thicknesses δ^* and δ_1^* for the viscous boundary layer and diffusion boundary layer, respectively, by

$$\delta^* = \int_0^{\delta}\left(1 - \frac{u}{U}\right) dy \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (20)$$

and

$$\delta_1^* = \int_0^{\delta_1}\left(\frac{\psi - \psi_0}{\psi_1 - \psi_0}\right) dy \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (21)$$

the solution may be obtained from the following equations:

$$F\left(\frac{\delta_1^*}{\delta^*}\right) = 0{\cdot}3861\left(\frac{j}{\nu}\right) U^4 \frac{\int_0^x U\, dx}{\int_0^x U^5\, dx} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (22)$$

where F is a known function and U is given in terms of x, the distance along the surface from the front stagnation point;

$$(\delta^*)^2 = \frac{2{\cdot}96\nu}{U^6}\int_0^x U^5\, dx \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (23)$$

and

$$\left(\frac{\partial \psi}{\partial y}\right)_{y=0} = -0{\cdot}5715\frac{(\psi - \psi_0)}{\delta_1^*} \; . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (24)$$

† The continuity equation (19), which is analogous to the momentum equation for viscous flow and the integral energy equation for heat flow, may be obtained by integrating (7) with respect to y or, more simply, by considering the balance of mass within an elementary strip of the diffusion boundary layer.

The ratio δ_1^*/δ^* is found from (22); thence, using (23) we deduce δ_1^* which can be inserted in (24) to give $\left(\frac{\partial \psi}{\partial y}\right)_{y=0}^{\dagger}$.

The function $F(\delta_1^*/\delta^*)$ appearing in (22) is given in the Table below, which is reproduced from Ref. 9.

TABLE 1

δ_1^*/δ^*	0·500	0·625	0·667	0·833	1·000	1·250	1·429	1·667	1·818	2·000
$F(\delta_1^*/\delta^*)$	0·052	0·100	0·121	0·230	0·386	0·713	1·018	1·522	1·901	2·398

Near the front stagnation point, the velocity may be represented by

$$U = U_0 \frac{x}{s} \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \qquad (25)$$

where s is a typical length. Equations (22) and (23) then reduce to

$$F\left(\frac{\delta_1^*}{\delta^*}\right) = 1{\cdot}158\left(\frac{j}{\nu}\right) \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \qquad (26)$$

$$(\delta^*)^2 = 0{\cdot}493\,\frac{\nu s}{U_0} \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \qquad (27)$$

By way of example, the distribution of K_g over the forward part of the upper surface of the NACA 2409 aerofoil is shown in Fig. 2; the surface is assumed to be coated with naphthalene for which (j/ν) is 0·39 (*cf.* section 7).

As an alternative to the above method, when the skin friction at the surface of the aerofoil is known, an approximation to the mass-transfer rate can be obtained by assuming a relation equivalent to (18) to hold locally. This leads to

$$K_g = \frac{1}{2}\left(\frac{j}{\nu}\right)^{2/3} \frac{U_0}{U} C_f \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \qquad (28)$$

where both K_g and C_f are expressed in terms of the undisturbed velocity, U_0. The distribution of K_g found from equation (28) is compared with the more exact values for the NACA aerofoil in Fig. 2; except near the stagnation point, the agreement is fairly good.

3.3. *Diffusion from an Isolated Region of a Flat Plate.*—In some circumstances the region from which diffusion takes place may not extend so far upstream as the leading edge of the plate; accordingly, diffusion will commence in an established boundary layer and the Pohlhausen solution given in section 3.1 will not apply, since the boundary conditions (5) do not hold over the entire surface. However, if the dimension of the region in the direction of flow is small compared with the length of plate upstream, a situation likely to exist when diffusion occurs from a small area of the wall of a wind tunnel or duct, the problem may yet be compared with a similar one in heat transfer to which a solution has been given by Leveque[5].

† The right-hand side of equation (22) is a first approximation to a more complicated expression; a method of deriving successive approximations is described in Ref. 9. Alternatively, the ratio δ_1^*/δ^* can be found directly by solving the following equation step-by-step:

$$\frac{\delta_1^*}{\delta^*}\frac{d}{dx}\left[\frac{U\delta^*}{\nu}\cdot\frac{\delta^*}{\delta_1^*}, F\left(\frac{\delta_1^*}{\delta^*}\right)\right] = \frac{0{\cdot}5715}{\delta^*}\left(\frac{j}{\nu}\right).$$

Initial conditions are provided by (26) and (27).

The velocity profile in the boundary layer is assumed to be given by

$$\left.\begin{aligned} u &= \frac{y\tau_0}{\mu} \\ v &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

Equations (29) are equivalent to postulating the thickness of the diffusion boundary layer to be small in comparison with that of the viscous layer. τ_0 is the shear stress at the surface and is assumed to be constant.

The diffusion equation, (7), reduces to

$$\frac{\tau_0}{\mu}\frac{\partial \psi}{\partial x} = \frac{j}{y}\frac{\partial^2 \psi}{\partial y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

with the conditions

$$\left.\begin{aligned} &\psi = \psi_1 , \quad y = 0 , \quad x > 0 \\ &\psi = \psi_0 , \quad x \leqslant 0 \\ &\psi \to \psi_0 , \quad y \to \infty \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (31)$$

where x is measured from the upstream edge of the diffusing area.

The rate of mass transfer per unit area, as deduced from the corresponding Leveque solution of the thermal problem, is

$$-\rho_0 j\left(\frac{\partial \psi}{\partial y}\right)_{y=0} = 1\cdot 12\,\rho_0 j\,(\psi_1 - \psi_0)\left(\frac{\tau_0}{9\mu j x}\right)^{1/3} \quad \ldots\ldots\ldots\ldots \quad (32)$$

The local mass-transfer coefficient is given by

$$K_g = 0\cdot 428\left(\frac{j}{\nu}\right)^{2/3} C_f^{1/3}\left(\frac{\nu}{U_0 x}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

in which $C_f = \dfrac{\tau_0}{\frac{1}{2}\rho_0 U_0^2}$.

The above solution is compared in Fig. 1 with that given by (18) for diffusion from the entire length of the plate.

A relation very similar to (33) can be derived directly from the continuity equation, (19), by assuming a distribution of ψ across the diffusion boundary layer in the form of a polynomial in y. With a quartic, the conditions which may be satisfied are

$$\left.\begin{aligned} &y = \delta_1 ; \ \psi = \psi_0, && y = 0 ; \psi = \psi_1 \\ &\frac{\partial \psi}{\partial y} = 0 ; && \frac{\partial^2 \psi}{\partial y^2} = 0 \\ &\frac{\partial^2 \psi}{\partial y^2} = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (34)$$

The last of these conditions follows from the diffusion equation, (7).

The appropriate distribution of ψ is then given by

$$\frac{\psi_1 - \psi}{\psi_1 - \psi_0} = \left(\frac{y}{\delta_1}\right)\left[2 - 2\left(\frac{y}{\delta_1}\right)^2 + \left(\frac{y}{\delta_1}\right)^3\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (35)$$

With this expression for ψ, and the velocity profile given by (29), the continuity equation (19) reduces to

$$\delta_1^2\frac{d\delta_1}{dx} = \frac{15\mu j}{\tau_0}$$

so that

$$\delta_1 = \left(\frac{45\mu j x}{\tau_0}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

since $\delta_1 = 0$ when $x = 0$.

For the local mass-transfer coefficient, we have

$$K_g = \frac{-\rho_0 j \left(\frac{\partial \psi}{\partial y}\right)_{y=0}}{\rho_0 U_0(\psi_1 - \psi_0)} = \frac{2j}{U_0 \delta_1} . \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

Finally, from (36) and (37),

$$K_g = 0{\cdot}446 \left(\frac{j}{\nu}\right)^{2/3} C_f^{1/3} \left(\frac{\nu}{U_0 x}\right)^{1/3} . \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

Equations (33) and (38) are identical in form : they differ only by 4 per cent in the value of the constant.

The advantage of the approximate method over the exact Leveque solution is that it may be extended to the case where the diffusion and viscous boundary layers are of comparable thickness, simply by replacing (29) by a polynomial of the Pohlhausen type, *i.e.*,

$$\frac{u}{U_0} = \frac{y}{\delta}\left[2 - 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3\right]$$

where δ is the thickness of the viscous boundary layer.

4. *Diffusion in a Turbulent Flow.*—4.1. *The analogy Between Mass Transfer, Skin Friction and Heat Transfer : Diffusion from a Flat Plate.*—The expressions corresponding to (8) for the turbulent transfer of momentum and heat are

$$\left.\begin{aligned} \rho_0 \overline{v'u'} &= -\rho_0 \overline{l_1 v} \frac{\partial u}{\partial y} \\ \text{and} \qquad \rho_0 C_p \overline{v'T'} &= -\rho_0 C_p \overline{l_2 v} \frac{\partial T}{\partial y} \end{aligned}\right\} . \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

Since the transfer of heat and of mass by the turbulent velocities may be supposed to occur in the same way, it is admissible to write

$$\overline{l_2 v} = \overline{lv} .$$

Moreover, in the Reynolds analogy between heat transfer and skin friction (Ref. 8, Chap. xv) it is assumed that

$$\overline{l_2 v} = \overline{l_1 v} .$$

The momentum, energy and diffusion equations in a turbulent flow with zero pressure gradient may, therefore, be written

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[(\nu + \overline{lv})\frac{\partial u}{\partial y}\right] \qquad \ldots\ldots \quad (40a)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{k}{\rho_0 C_p} + \overline{lv}\right)\frac{\partial T}{\partial y}\right] \qquad \ldots\ldots \quad (40b)$$

$$u\frac{\partial \psi}{\partial x} + v\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left[(j + \overline{lv})\frac{\partial \psi}{\partial y}\right] \qquad \ldots\ldots \quad (40c)$$

with the respective boundary conditions

$$u = 0 , \quad y = 0 ; \quad u \to U_0, \quad y \to \infty \qquad \ldots\ldots \quad (41a)$$

$$T = T_1, \quad y = 0 ; \quad T \to T_0, \quad y \to \infty \qquad \ldots\ldots \quad (41b)$$

$$\psi = \psi_1, \quad y = 0 ; \quad \psi \to \psi_0, \quad y \to \infty \qquad \ldots\ldots \quad (41c)$$

provided that the viscous, thermal and diffusion boundary layers have a common origin on the surface.

In a fully developed turbulent flow the molecular transfer coefficients j, ν and $k/\rho_0 C_p$ occurring in equations (40) may be neglected in comparison with the turbulent transfer coefficient, $\overline{lv}$, and, since the boundary conditions (41) are similar, (40) will have solutions related by

$$\frac{\psi - \psi_1}{\psi_0 - \psi_1} = \frac{T - T_1}{T_0 - T_1} = \frac{u}{U_0} \quad \ldots \quad (42)$$

together with

$$K_g = K_h = \tfrac{1}{2} C_f \ . \quad \ldots \quad (43)$$

K_h is the local heat-transfer coefficient defined by

$$K_h = \frac{h}{\rho_0 U_0 C_p (T_1 - T_0)} \quad \ldots \quad (44)$$

where h is the rate at which heat is transferred across a unit area of the surface.

The relation (42) would not, in general, be expected to apply accurately throughout the boundary layer, owing to the existence of a laminar sub-layer adjacent to the surface in which the molecular transport terms dominate, and a transition layer where the molecular and turbulent transport terms are of equal importance. The analogies, (42) and (43) would be strictly valid over the entire boundary layer only in the case of ideal gases for which

$$\nu = \frac{k}{\rho_0 C_p} = j \ .$$

The extension of the analogy between heat transfer and skin friction in a real gas, to include the effects of the laminar sub-layer and transition layer, was made by von Kármán (Ref. 8, Chap. xv), who deduced an expression for the local heat-transfer coefficient

$$\frac{1}{K_h} = \sqrt{\left[\frac{2}{C_f}\right]}\left[\sqrt{\left(\frac{2}{C_f}\right)} + \Sigma(\sigma)\right] \quad \ldots \quad (45)$$

where

$$\Sigma(\sigma) = 5\left[\sigma - 1 + \log_e\left(\frac{5\sigma + 1}{6}\right)\right]$$

and σ is the Prandtl number, $\mu C_p/k$.

Von Kármán's argument may also be applied to the analogy between mass transfer and skin friction ; the relation between K_g and C_f is obtained by replacing σ in (45) by ν/j, thus :

$$\frac{1}{K_g} = \sqrt{\left[\frac{2}{C_f}\right]}\left[\sqrt{\left(\frac{2}{C_f}\right)} + J\left(\frac{j}{\nu}\right)\right] \quad \ldots \quad (46)$$

where,

$$J\left(\frac{j}{\nu}\right) = \Sigma\left(\frac{\nu}{j}\right) \ .$$

The function J is displayed in Fig. 3.

The distributions of K_g, as given by (46), along a flat plate parallel to the airstream are shown in Fig. 4 for surface coatings of naphthalene and of water. The respective values of j/ν for these two substances are 0·39 and 1·65 and are, in turn, representative of the magnitudes of j/ν for heavy organic vapours and elemental gases, or light vapours, diffusing into air. The skin-friction law adopted for the calculation was (Ref. 8, Chap. viii)

$$C_f = 0{\cdot}0592\left(\frac{U_0 x}{\nu}\right)^{-1/5} \ .$$

For comparison, the distribution of K_g as given by the simple relation (43), appropriate to j/ν equal to unity, is also shown in Fig. 4.

4.2. *Diffusion from the Surface of an Aerofoil.*—The analogy between mass transfer, skin friction and heat transfer discussed in the preceeding section is strictly true only for flows with zero pressure gradient. However, in the absence of a more refined theory, the variation in velocity outside the turbulent boundary layer of an aerofoil may be taken into account, as in heat-transfer calculations[9], by relating the distribution of ψ across the boundary layer to the local velocity. This leads to the expression, *cf.* equation (28),

$$\frac{1}{K_g} = \sqrt{\left[\frac{2}{C_f}\right]}\left[\frac{U}{U_0}\sqrt{\left(\frac{2}{C_f}\right)} + J\left(\frac{j}{\nu}\right)\right] \quad \ldots \quad (47)$$

where U is the local velocity at the outer edge of the boundary layer and U_0 is the velocity of the undisturbed stream. C_f and K_g are defined, as before, in terms of the velocity U_0.

The variation of K_g on the upper surface of an NACA 2409 aerofoil has been calculated from (47) and is shown in Fig. 2; the distribution of K_g is given for two assumed positions of transition*.

4.3. *Diffusion from an Isolated Region of a Flat Plate.*—The diffusion in a turbulent boundary layer from a part of a flat plate has been solved by O. G. Sutton (1934)[12] on the assumption that the variation of velocity with distance from the surface obeys a power law. Sutton's analysis does not, therefore, include the effects of molecular diffusion in the laminar sub-layer and, on this account, is applicable rigorously only to diffusion processes where j/ν is nearly unity. If j/ν departs appreciably from unity, as it does for example with complex organic vapours, like naphthalene, diffusing into air, the influence of molecular diffusion very close to the surface may be important; in fact, for the limiting case of a length of surface small compared with the thickness of the viscous boundary layer, the diffusion will be confined entirely within the laminar sub-layer (*see* Figs. 7 and 8) and the relevant solution is then given by equation (33). The following analysis provides an approximate method of allowing for the molecular diffusion.

The continuity equation for the diffusion boundary layer is, *cf.* section 3.2,

$$\frac{\partial}{\partial x}\int_0^{\delta_1} u(\psi - \psi_0)dy = -j\left(\frac{\partial \psi}{\partial y}\right)_{y=0} \quad \ldots \quad (48)$$

The boundary conditions are

$$\left.\begin{aligned} &\psi = \psi_1\ , \quad y = 0\ , \quad x > 0 \\ &\psi = \psi_0\ , \quad x \leqslant 0 \\ &\psi = \psi_0\ , \quad y \geqslant \delta_1 \end{aligned}\right\} \quad \ldots \quad (49)$$

The local mass-transfer coefficient is given by

$$K_g = \frac{-j\left(\frac{\partial \psi}{\partial y}\right)_{y=0}}{U_0(\psi_1 - \psi_0)} \quad \ldots \quad (50)$$

In order to evaluate the integral on the left of equation (48), some assumption is required concerning the variation of ψ across the boundary layer. The assumption we shall make is that the distribution of ψ through the diffusion boundary layer is similar to the distribution of temperature through a corresponding part of the boundary layer on a uniformly heated flat plate. Such a distribution has a certain plausibility: it is correct in the limit $x \to \infty$, and will be shown to lead to the correct form of solution for the mass transfer as $x \to 0$.

* The development of the boundary layer on the aerofoil was calculated by using Thwaites' simplified method (1949)[10] for the laminar layer, and the method of Squire and Young (1937)[11] for the turbulent layer.

Writing u_τ for the friction velocity, $\sqrt{(\tau_0/\rho_0)}$, and

$$y_\tau = yu_\tau/\nu$$
$$Y_\tau = \delta_1 u_\tau/\nu$$

the concentration at any point may be expressed in the form

$$\frac{\psi_1 - \psi}{\psi_1 - \psi_0} = \frac{g(y_\tau)}{g(Y_\tau)} \quad \ldots \quad (51)$$

$g(y_\tau)$ is identical with the function describing the variation of temperature across a heated turbulent boundary layer[5,8] except that the Prandtl number, σ, which occurs in the case of heat transfer is now replaced by ν/j. $g(y_\tau)$ is, therefore, defined by

$$\left.\begin{aligned} g(y_\tau) &= \frac{y_\tau \nu}{j}, && 0 < y_\tau < 5 \\ &= 5 \log_e \left(\frac{y_\tau \nu}{5j} + 1 - \frac{\nu}{j}\right) + \frac{5\nu}{j}, && 5 < y_\tau < 30 \\ &= \frac{u}{u_\tau} + J\left(\frac{j}{\nu}\right), && y_\tau > 30 \end{aligned}\right\} \quad \ldots \quad (52)$$

$J(j/\nu)$ is defined in section 4.1. The boundaries of the laminar sub-layer and transition layer are respectively given by $y_\tau = 5$ and $y_\tau = 30$.

With the expressions for $g(y_\tau)$ given by (52) the assumed distribution for ψ, equation (51), does not satisfy the condition $\partial\psi/\partial y = 0$ when $y = \delta_1$, characteristic of the edge of a boundary layer. However, since the method is used only for determining the mass-transfer rate, which depends on the integral of the concentration over the diffusion boundary layer, the error thus caused will be small. Equation (51) satisfies the more important boundary conditions, $\psi = \psi_1$, $y = 0$; $\psi = \psi_0$, $y = \delta_1$; $\partial^2\psi/\partial y^2 = 0$, $y = 0$.

The velocity distribution across the turbulent boundary layer on a flat plate is given by von Kármán (Ref. 8, Chap. xv) as

$$\frac{u}{u_\tau} = f(y_\tau)$$

where

$$\left.\begin{aligned} f(y_\tau) &= y_\tau && 0 < y_\tau < 5 \\ &= -3{\cdot}05 + 5 \log_e y_\tau, && 5 < y_\tau < 30 \\ &= 5{\cdot}5 + 2{\cdot}5 \log_e y_\tau, && y_\tau > 30 \end{aligned}\right\} \quad \ldots \quad (53)$$

The continuity equation, (48), may now be written

$$\left[\frac{g'(Y_\tau)}{g^2(Y_\tau)} \int_0^{Y_\tau} f(y_\tau)\, g(y_\tau)\, dy_\tau\right] \frac{dY_\tau}{dx} = \frac{u_\tau}{\nu} \frac{1}{g(Y_\tau)} \quad \ldots \quad (54)$$

the dash denoting differentiation with respect to Y_τ.

If we write

$$\frac{x}{\delta_0} = \chi$$

where δ_0 is the turbulent boundary-layer thickness at $x = 0$, equation (54) reduces to

$$\frac{dY_\tau}{d\chi} = \left(\frac{u_\tau \delta_0}{\nu}\right) \frac{g(Y_\tau)}{g'(Y_\tau)} \frac{1}{\int_0^{Y_\tau} f(y_\tau)\, g(y_\tau)\, dy_\tau} \quad \ldots \quad (55)$$

The expression for the local mass transfer coefficient, (50), becomes

$$K_g = \frac{u_\tau}{U_0}\frac{1}{g(Y_\tau)} \quad . \qquad (56)$$

The functions occurring on the right of equation (55) are easily calculated from (52) and (53); collecting them together, we may write

$$\frac{dY_\tau}{d\chi} = \left(\frac{u_\tau \delta_0}{\nu}\right) H(Y_\tau) \quad . \qquad (57)$$

$H(Y_\tau)$ and $g(Y_\tau)$ are given in Table 2 for a number of values of (j/ν); full algebraic expressions are contained in Appendix II.

Equation (57) can conveniently be integrated step-by-step to give Y_τ; K_g then follows from (56). The initial value of Y_τ is derived quite simply since, near the origin of the diffusion boundary layer, we have

$$\chi \to 0\,; \qquad Y_\tau \to 0, \qquad f(Y_\tau) \to Y_\tau, \qquad g(Y_\tau) \to \frac{\nu}{j} Y_\tau \quad .$$

Consequently,

$$\frac{dY_\tau}{d\chi} = 3\left(\frac{j}{\nu}\right)\left(\frac{u_\tau \delta_0}{\nu}\right)\frac{1}{Y_\tau^2} \quad .$$

Neglecting the variation of u_τ with χ, the preceding equation may be integrated to give

$$Y_\tau^3 = 9\left(\frac{j}{\nu}\right)\left(\frac{u_\tau \delta_0}{\nu}\right)\chi \quad . \qquad (58)$$

Equation (58) holds so long as the thickness of the diffusion boundary layer remains smaller than that of the laminar sub-layer and the integration of (57) may, therefore, be started at the value of χ corresponding to the edge of the laminar sub-layer. This leads to the initial condition

$$Y_\tau = 5\,; \qquad \chi = 13{\cdot}89\left(\frac{\nu}{j}\right)\left(\frac{\nu}{u_\tau \delta_0}\right) \quad .$$

It also follows from (58) that, as $\chi \to 0$,

$$K_g = \left(\frac{j}{\nu}\right)^{2/3}\left(\frac{u_\tau}{U_0}\right)\left(\frac{\nu}{9u_\tau \delta_0 \chi}\right)^{1/3}$$

or, expressed in terms of x and the local skin friction coefficient,

$$K_g = 0{\cdot}382\left(\frac{j}{\nu}\right)^{2/3} C_f^{1/3}\left(\frac{\nu}{U_0 x}\right)^{1/3} \quad . \qquad (59)$$

This equation differs from the Leveque solution, given by (33), only in the value of the constant; its exact value is roughly 10 per cent larger than the one given above*.

In contrast, as $\chi \to \infty$, the thickness of the diffusion boundary layer will approach that of the turbulent boundary layer and, from (52),

$$g(Y_\tau) \to \frac{U_0}{u_\tau} + J\left(\frac{j}{\nu}\right) \quad .$$

---

* In deducing equation (59), a linear distribution was, in effect, assumed for ψ through the laminar sub-layer, *cf.* equation (52). Equation (38), section 3.3, was obtained by the same method, but the distribution of ψ was assumed to be given by a quartic in y. Comparing the values of the constant, we find,

exact solution, equation (33)	:	0·428
linear distribution, equation (59)	:	0·382
quartic distribution, equation (38)	:	0·446.

Thus

$$K_g = \frac{u_\tau}{U_0} \frac{1}{\left[\frac{U_0}{u_\tau} + J\left(\frac{j}{\nu}\right)\right]}$$

or

$$\frac{1}{K_g} = \sqrt{\left[\frac{2}{C_f}\right]}\left[\sqrt{\left(\frac{2}{C_f}\right)} + J\left(\frac{j}{\nu}\right)\right]$$

which is the expression given in (46) for the mass-transfer coefficient when diffusion commences at the leading edge of a flat plate.

Comparison with Sutton's solution provides a further check on the present method when $j/\nu = 1$. The local mass-transfer coefficient deduced from Sutton's theory[13] is

$$K_g = 0{\cdot}0247\left(\frac{U_0 x}{\nu}\right)^{-1/9}\left(\frac{U_0 \delta}{\nu}\right)^{-1/9} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (60)$$

when the boundary-layer profile obeys a $\frac{1}{7}$th power law*. In the derivation of equation (60), the boundary-layer thickness, δ, is assumed constant. Some examples of variation of the K_g with x, as given by equation (60) and by equations (56) and (57), are shown in Figs. 5 and 6. Except near the origin, where the present solution is more nearly correct than Sutton's, the two results agree satisfactorily; the small differences could be explained by the different assumptions concerning the turbulent boundary-layer velocity-profile.

The significance of molecular diffusion in the laminar sub-layer, when j/ν differs from unity, can be seen from Figs. 5 and 6 which include curves of the mass-transfer coefficient for $j/\nu = 0{\cdot}333$.

Examples of the growth of the diffusion boundary layer are given in Figs. 7 and 8.

5. *The Concentration at the Boundary.*—In deriving the expressions for the mass-transfer rates it has been assumed that ψ_0 and ψ_1 are known quantities. The value of ψ_0 presents no difficulty: in an evaporation or sublimation process it may normally be taken as zero—except, of course, for the evaporation of water into a humid airstream—whereas in a contamination process it is prescribed by the concentration of the active gas present in the airstream. The value to be assigned to ψ_1, on the other hand, is less obvious. It is usually assumed for the evaporation of water[5,12] that ψ_1 is equal to the concentration of the saturated vapour at the temperature of the surface. Although it will be shown that this assumption is justified, it is not immediately apparent physically, particularly as saturation implies an equilibrium state, and therefore deserves a more detailed discussion.

5.1. *ψ_1 for Evaporation or Sublimation.*—The evaporation or sublimation of a substance into a stream of air may, for convenience, be imagined to occur in two stages. The first is a purely molecular process which takes place in an extremely thin layer adjacent to the surface and involves a continuous evaporation and recondensation of gas molecules. The second stage, represented by the diffusion through the boundary layer, can be regarded simply as a mechanism for conveying into the airstream those molecules which escape from the surface layer. The number of molecules which so escape is determined, amongst other things, by the difference between the partial pressure of the vapour at the surface and the saturation pressure; when this is zero no net evaporation can occur. Although this latter statement apparently contradicts the assumption that the concentration at the surface corresponds to saturation, it will be demonstrated that, in practice, a very small departure from the saturated condition may

* An expression similar to (60) can be deduced from some measurements of the temperature field due to a heat source located in the surface of a flat plate, made by Wieghardt (1948)[27]. This solution is given in Appendix III.

TABLE 2

Y_τ	$g(Y_\tau)$					$H(Y_\tau)$				
	$\frac{j}{\nu} = \frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{j}{\nu} = \frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$
5	15·00	7·50	5·00	3·75	3·00	0·0400	0·0800	0·1200	0·1600	0·2000
6	17·35	8·81	5·91	4·45	3·57	0·0424	0·0711	0·0990	0·1269	0·1551
7·5	19·58	10·30	7·03	5·34	4·31	0·0410	0·0598	0·0781	0·0964	0·1147
10	21·93	12·08	8·47	6·55	5·35	0·0365	0·0479	0·0591	0·0701	0·0808
15	24·73	14·43	10·49	8·33	6·94	0·0300	0·0363	0·0423	0·0481	0·0538
20	26·51	16·02	11·93	9·64	8·15	0·0262	0·0306	0·0347	0·0385	0·0423
25	27·82	17·23	13·05	10·68	9·12	0·0238	0·0272	0·0302	0·0331	0·0359
30	28·86	18·20	13·96	11·54	9·93	0·0220	0·0248	0·0272	0·0295	0·0317
30	28·86	18·20	13·96	11·54	9·93	0·0496	0·0525	0·0545	0·0559	0·0571
35	29·29	18·63	14·39	11·97	10·36	0·0367	0·0464	0·0493	0·0500	0·0512
40	29·63	18·96	14·72	12·30	10·70	0·0306	0·0425	0·0458	0·0459	0·0472
45	29·92	19·26	15·02	12·60	10·99	0·0270	0·0397	0·0432	0·0432	0·0443
50	30·18	19·52	15·28	12·86	11·25	0·0247	0·0377	0·0413	0·0411	0·0422
60	30·64	19·98	15·74	13·32	11·71	0·0218	0·0349	0·0385	0·0381	0·0392
70	31·03	20·36	16·12	13·70	12·09	0·0200	0·0329	0·0366	0·0361	0·0371
80	31·36	20·70	16·46	14·04	12·43	0·0188	0·0316	0·0351	0·0346	0·0355
100	31·92	21·26	17·01	14·59	12·99	0·0173	0·0296	0·0330	0·0324	0·0334
150	32·93	22·27	18·03	15·61	14·00	0·0154	0·0269	0·0300	0·0294	0·0302
200	33·65	22·99	18·75	16·33	14·72	0·0144	0·0254	0·0283	0·0283	0·0285
300	34·66	24·00	19·76	17·34	15·73	0·0134	0·0236	0·0263	0·0258	0·0264
400	35·38	24·72	20·48	18·06	16·45	0·0128	0·0226	0·0251	0·0245	0·0251
500	35·94	25·28	21·04	18·62	17·01	0·0125	0·0218	0·0242	0·0237	0·0243
1000	37·67	27·01	22·77	20·35	18·74	0·0115	0·0199	0·0219	0·0215	0·0219
2000	39·41	28·74	24·50	22·08	20·48	0·0107	0·0183	0·0200	0·0196	0·0200
3000	40·42	29·76	25·52	23·09	21·49	0·0104	0·0175	0·0191	0·0187	0·0190
4000	41·14	30·48	26·24	23·82	22·21	0·0101	0·0169	0·0184	0·0181	0·0184
5000	41·70	31·04	26·79	24·37	22·77	0·0099	0·0165	0·0180	0·0177	0·0179

suffice to account for the comparatively low rates of evaporation which are achieved. In other words, the number of molecules carried away from the surface by diffusion is small compared with the numbers evaporating and condensing in the surface layer. The condition at the surface is, therefore, almost one of dynamic equilibrium.

According to the kinetic theory of gases[14,19], the rate of evaporation from a substance at temperature T in contact with its vapour at the same temperature and at pressure p_1 is given by

$$G = \frac{\alpha(p_s - p_1)S}{\sqrt{(2\pi\bar{R}T/m)}} \qquad (61)$$

where p_s is the vapour pressure at temperature T, S is the area of the surface, m is the molecular weight of the vapour and $\bar{R}$ is the gas constant ($83{\cdot}15 \times 10^6$ ergs/°C/gm). α is called the coefficient of evaporation[14]: thus, of the total mass of vapour molecules striking the surface S in unit time, $(p_1 S/\sqrt{(2\pi\bar{R}T/m)}$, α represents the fraction which condenses to re-form a liquid or solid, the rest being reflected back into the vapour.

In the problem of evaporation from a surface into an atmosphere of air and contaminating vapour, p_1 may be taken to be the partial pressure of the vapour at the surface and is given by

$$p_1 = \frac{\bar{R}}{m}\rho_0\psi_1 T. \qquad (62)$$

Similarly,

$$p_s = \frac{\bar{R}}{m}\rho_0\psi_s T \qquad (63)$$

where ψ_s is the concentration of the saturated vapour.

Since the rate of evaporation given by (61) must be equal to the rate of transport of the vapour through the boundary layer,

$$K_g U_0(\psi_1 - \psi_0) = \alpha(\psi_s - \psi_1)\sqrt{\left(\frac{\bar{R}T}{2\pi m}\right)}.$$

Hence,

$$\psi_1 = \psi_s\left[\frac{1 + \frac{K_g U_0}{\alpha}\frac{\psi_0}{\psi_s}\sqrt{\left(\frac{2\pi m}{\bar{R}T}\right)}}{1 + \frac{K_g U_0}{\alpha}\sqrt{\left(\frac{2\pi m}{\bar{R}T}\right)}}\right] \qquad (64)$$

Clearly, the difference between ψ_1 and ψ_s will depend on the magnitude of the term $(K_g U_0/\alpha)\sqrt{(2\pi m/\bar{R}T)}$. The expression $K_g U_0\sqrt{(2\pi m/RT)}$ is, in general, very small. For example, in the case of water with $m = 18$, $T = 288$ deg K, $U_0 = 20$ m/sec and $K_g = 0{\cdot}001$, it is of the order of 10^{-4}; for a heavy organic vapour with $m = 150$, at a temperature and speed corresponding to the flight of an aircraft at high altitude, say $T = 230$ deg K and $U_0 = 150$ m/sec, it is of the order of 10^{-3}. It follows that $(K_g U_0/\alpha)\sqrt{(2\pi m/\bar{R}T)}$ can be comparable with unity only if α is small.

Measurements of α appear to have been made only in relatively few cases; a number of results are shown in Table 3 below.

TABLE 3

Substance	State	α	Ref.
Mercury	liquid	1·0	15
Mercury	solid	0·85	15
Cadmium	solid	0·98	15
Tantalum	solid	1·0	15
Tungsten	solid	1·0	15
Titanium	solid	1·0	17

TABLE 3—*continued*

Substance	State	α	Ref.
Rhombic Sulphur ..	solid	0·7	18
Thorium	liquid	1·0	15
Benzophenone	liquid	0·2 to 0·5	15
Water	liquid	0·004	16
Copper	liquid	1·0	28
Beryllium	liquid	1·0	28
Platinum	liquid	1·0	28
Silver	liquid	0·92	28
Iron	liquid	1·0	28

For evaporating liquids, α has generally been found to lie between 0·1 and 1·0. Consequently, the terms $(K_g U_0/\alpha)\sqrt{(2\pi m/\bar{R}T)}$ will be negligibly small in comparison with unity and the concentration at the surface will approximate closely to saturation. The only anomaly is water for which α is 0·004; with this value of α and with $T = 288$ deg K, $U_0 = 20$ m/sec and $K_g = 0{\cdot}001$, the value of $(K_g U_0/\alpha)\sqrt{(2\pi m/\bar{R}T)}$ becomes $3{\cdot}5 \times 10^{-2}$. This implies that, even in this extreme case, the concentration at the surface will only differ by about $3\frac{1}{2}$ per cent from the saturation value.

Available evidence on the magnitude of α for sublimation from the solid state suggests that it will also be of the order of unity, *cf.* Table 3. However, in a recent paper, Stranski and Wolff (1951)[19] maintain that this conclusion may not be generally valid. According to their argument, the molecular structure in the vapour may differ from that in the solid; thus, the solid state may be characterised by a crystalline arrangement of molecules which has associated with it a certain 'excitation energy.' It follows that, of the total number of vapour molecules striking the solid surface, only those possessing an energy in excess of the excitation energy will have the chance of condensing to reform crystals. If the excitation energy per gramme of substance is ε_c, the proportion of the total number of molecules striking the surface which have a kinetic energy in excess of ε_c is,

$$\exp\left(-\frac{\varepsilon_c m}{\bar{R}T}\right)$$

on the assumption that the molecular velocities in the vapour are distributed according to the Maxwell law. α will then be given by,

$$\alpha = \alpha_0 \exp\left(-\frac{\varepsilon_c m}{\bar{R}T}\right) \qquad (65)$$

where α_0 is a constant, probably of the order of unity*. Stranski and Wolff cite carbon as an example of a case in which large differences in the magnitude of α between evaporation and sublimation have been observed; they attribute this difference to the term $\exp(-\varepsilon_c m/\bar{R}T)$. On the other hand, the substances used for boundary-layer transition indication all have low melting points (< 200 deg C) and this suggests that, at ordinary temperatures, the intermolecular forces in the solid state will be comparatively small. It is, therefore, reasonable to assume that ε_c for such substances will also be small and consequently $\exp(-\varepsilon_c m/\bar{R}T)$ and α will not differ much from unity. There is also some evidence produced by Hartshorne (1949)[18] to show that, even when ε_c is appreciable, (65) may give too low a value for α.

* α_0 may differ from unity because, in addition to the energy condition, there may be a geometrical condition for condensation, requiring a certain orientation of the molecules striking the surface.

As a result of these arguments it may be inferred that, for the sublimation of substances used for boundary-layer observation and certainly for evaporating liquids, the boundary condition at the surface may, to a close approximation, be written

$$\left.\begin{aligned} &\psi_1 = \psi_s \\ \text{or} \quad &\psi_1 = \frac{mp_s}{\rho_0 \bar{R}_0 T} = \frac{mp_s}{m_0 p_0} \end{aligned}\right\} \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (66)$$

where m_0 is the molecular weight of air, and p_1 is the static pressure in the airstream.

When the approaching airstream is uncontaminated by any vapour, (66) implies that the rate of evaporation of a substance should be proportional to its vapour pressure. This was confirmed experimentally for Toluene, Chlorobenzene, m-Xylene and Nitrobenzene by Hine (1924)[20].

It is of interest to observe that the condition (66) has a similarity to the no-slip condition for the viscous flow of a dense gas past a body, and the condition for heat flow that the temperature varies continuously across the surface of the body. This may be seen by considering the diffusion from a flat plate in a laminar flow, for which it may easily be shown that

$$\frac{K_g U_0}{\alpha}\sqrt{\left(\frac{2\pi m}{\bar{R}T}\right)} \text{ is } O\left(\frac{M}{R^{1/2}}\right)$$

in which α is assumed to be $O(1)$; M is the Mach number of the flow and R the Reynolds number based on the length of the plate. When the Reynolds number is large, $M/\sqrt{R}$ is of the same order of magnitude as $\bar{l}/\delta$, where $\bar{l}$ is the molecular mean free path in the air and δ is the thickness of the boundary layer. Consequently, the condition $\psi_1 = \psi_s$ is valid provided that $\bar{l}/\delta$ or $M/\sqrt{R}$ is small. It is, however, in precisely these circumstances that the conditions apply for zero slip and a continuous variation of temperature at the surface*.

5.2. *ψ_1 for Contamination.*—The rate of reaction between an active gas and a chemically coated surface may be deduced by an argument similar to that described in the previous section for crystal formation. Only a fraction of the gas molecules striking the surface may be expected to undergo a chemical combination : the remainder are re-emitted from the surface. The property required of the chemically active molecules is that they possess a kinetic energy in excess of a certain ' activation energy '[21]. If this energy is ε_A per gramme, the rate of transfer of the gas into the surface is given by,

$$G = \beta \exp\left(-\frac{\varepsilon_A m}{\bar{R}T}\right)\frac{p_1 S}{\sqrt{(2\pi \bar{R}T/m)}} \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (67)$$

where β is a constant. Since, in the steady state, this must be equal to the rate of transport of the gas through the boundary layer

$$K_g U_0(\psi_0 - \psi_1) = \beta\psi_1 \exp\left(-\frac{\varepsilon_A m}{\bar{R}T}\right)\sqrt{\left(\frac{\bar{R}T}{2\pi m}\right)}.$$

Thus,

$$\psi_1 = \frac{\psi_0}{1 + \dfrac{\beta}{K_g U_0}\exp\left(-\dfrac{\varepsilon_A m}{\bar{R}T}\right)\sqrt{\left(\dfrac{\bar{R}T}{2\pi m}\right)}} \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (68)$$

No values of ε_A are available for the reactions which occur with transition indicators ; in any case, ε_A may depend markedly on the amount of water vapour present. In the absence of definite information concerning the magnitude of ε_A it is, therefore, impossible to specify the correct value of ψ_1. All that can be deduced from equation (68) is that, as the surface temperature falls, ψ_1 will tend to the value ψ_0 so that, when the contamination method of boundary-layer observation is used in flight, the concentration of the active gas at the surface of the aircraft may be expected to increase with increasing altitude, thus reducing the rate of transfer of the gas through the boundary layer and, hence, the rate of chemical reaction.

* The exact value of $\bar{l}/\delta$ for which slip at the surface may be supposed to influence significantly the skin friction or the heat transfer may be only arbitrarily defined. It is usually accepted that slip flow occurs within the range $0{\cdot}01 < \bar{l}/\delta < 1$[22]. When $\bar{l}/\delta$ exceeds 1, the flow enters the ' free molecular ' régime.

6. *Vapour Pressure and Surface Temperature.*—The variation with temperature of the vapour pressures of most liquids and solids is found to obey approximately the Clausius-Clapeyron law over a limited temperature range, and is usually expressed in the form

$$\log_{10} p_s' = -0{\cdot}05223\,\frac{a}{T} + b \qquad (69)$$

where p_s' is the vapour pressure in mm Hg at temperature T and a, b are constants for a particular substance. a and b will not necessarily have the same values for evaporation from the liquid and from the solid states.

a is related to the heat of evaporation or sublimation, L per gramme, by

$$a = mL \times 10^{-7} \text{ ergs/gm.} \qquad (70)$$

Approximate values of a and b for substances suitable as boundary-layer transition indicators may be deduced from Fig. 1 of Ref. 4*; more reliable values are given in the Table below for a few of the solid substances, including Iodine which is toxic and therefore unsuitable, and for water.

TABLE 4

Substance	a	b	p_s' at 15°C	Ref.
Camphor	53,560	8·80	0·12	23
Hexachlorethane	50,450	8·64	0·21	24
Iodine	67,300	11·33	0·13	23
Napthalene	71,400	11·45	0·032	23
Thymol	91,900	14·32	0·049	25
Anthracene	70,390	8·71	$\simeq 10^{-4}$	24
Water	41,600	6·90	12·79	23

As remarked in section 1, the values of p_s for a certain substance obtained from different sources are not always consistent, even when a common method of measurement is used. This may be a result of the difficulty of determining very low vapour pressures, or, possibly, of slight differences in the surface texture of the chemicals employed.

The surface temperature T occurring in equation (69) may, in general, be taken to equal the boundary-layer stagnation temperature T_1 given by

$$T_1 = T_0 + \frac{U^2}{2JC_p}\sigma^{1/n} \qquad (71)$$

where for laminar layers n is equal to 2, and for turbulent layers it is given by Squire (1942)[9] as equal to 3. T_0 is the ambient temperature and U the velocity outside the boundary layer: J is the mechanical equivalent of heat. In identifying the surface temperature with the boundary-layer stagnation temperature, radiation from the surface to the airstream is neglected—this is permissible in subsonic flows under ordinary atmospheric conditions—and heat transfer across the surface is assumed to be zero. For the present purpose, T_1 is given with sufficient accuracy by

$$T_1 = T_0 + \frac{0{\cdot}9\,U^2}{2JC_p} \qquad (72)$$

for a surface on which the boundary layer is partly laminar and partly turbulent.

* The figures quoted by Main-Smith (1950) in Ref. 4 are accompanied by the cautionary note that they were extrapolated from measurements made at high temperatures, possibly above the melting points of the substances concerned. The values of p_s' deduced from Table 4 above, at a temperature of 20 deg C, for example, are in some cases only $\frac{1}{5}$th of the corresponding values given in Ref. 4.

In practice, a small quantity of heat must be transferred from the airstream to the surface in order to provide the heat of evaporation of the chemical; this implies that the surface temperature will be somewhat lower than the boundary-layer stagnation temperature. The difference, ΔT, is given by

$$K_g(\psi_1 - \psi_0)L = K_h J C_p \Delta T$$

or

$$\Delta T = \frac{K_g}{K_h}\frac{L}{JC_p}(\psi_1 - \psi_0) \quad \ldots \quad (73)$$

Since K_g and K_h are of the same order of magnitude, we obtain from (70) and (73) for evaporation into a vapour-free airstream,

$$\Delta T \simeq 0{\cdot}01 p_s a/p_0 \quad \ldots \quad (74)$$

where p_0 is the static pressure in the airstream. For the solids used as transition indicators, ΔT is insignificant at temperatures less than 20 deg C, being of the order of 0·1 deg C when p_0 is equal to ground level atmospheric pressure. On the other hand, it is appreciable in the evaporation of water: with a boundary-layer stagnation temperature of 15 deg C and a static pressure of one ground level atmosphere, ΔT is given by,

$$\Delta T \simeq 7(1 - r)$$

where r is the relative humidity in the approaching airstream. If, for example, the relative humidity is 0·7, the temperature drop is roughly 2 deg C and its effect on the vapour pressure is such as to reduce the rate of evaporation by some 10 per cent.

7. *The Molecular Diffusion Coefficient, j,*—The coefficients of molecular diffusion into air of many of the substances used for observing boundary-layer transition are not recorded in any of the standard works of reference[15,23,24,25]. Since the rate of mass transfer does not depend critically on the value of the diffusion coefficient, an approximate method of estimating it may suffice; a particularly simple one is suggested below.

The theory of Chapman and Langevin described by Kennard in his book on the *Kinetic Theory of Gases* (1938)[14], leads to the following expression for the coefficient of interdiffusion between gases 1 and 2,

$$j_{12} = A_{12}'\frac{1}{p}\left(\frac{m_1 + m_2}{m_1 m_2}\right)^{1/2} T^{3/2} \quad \ldots \quad (75)$$

where m_1 and m_2 are the respective molecular weights of the gases, T is the temperature and p the pressure of the mixture, and A_{12}' is a quantity depending on the effective collision diameter of the diffusing molecules; the collision diameter will, in turn, depend on the particular gases concerned and on the temperature. It may be noted that, according to (75), j_{12} is independent of the proportions in which the gases are mixed; this is confirmed closely by experiment.

It is found experimentally that, for a given pair of gases,

$$j_{12} \propto T^q$$

where q is usually either 1·75 or 2·0, *cf.* Ref. 15, from which it may be presumed that A_{12}' varies either as the square root or fourth root of the temperature. On this basis, a general expression for j_{12} is,

$$j_{12} = A_{12}\left(\frac{p_c}{p}\right)\left(\frac{m_1 + m_2}{m_1 m_2}\right)^{1/2}\left(\frac{T}{T_c}\right)^q \quad \ldots \quad (76)$$

where A_{12} is a constant for a given pair of gases and p_c, T_c are some standard pressure and temperature which we shall take to be 760 mm Hg and 0 deg C.

The collision diameter may be expected to increase, and A_{12} to decrease, with increasing complexity of the gas molecules. For the organic compounds primarily considered in this report, a very crude guide to the molecular complexity is presented by the molecular weight, and it is of interest, therefore, to discover whether, in fact, existing data suggest any distinct relation between A_{12} and the molecular weights of the interdiffusing gases.

Fig. 9 shows the coefficient A_{12} plotted against $(m_1 + m_2)$ for the gases listed in the Appendix ; the basic data for j_{12} were taken from Ref. 15. It will be seen that, with only two remarkable exceptions, the points lie within a well-defined band about a curve which approximates to a rectangular hyperbola. It may be significant that the two mixtures which deviate widely from the curve each contains a halogen or its radical. Apart from these exceptions, the generality of the result is surprising because, in addition to numerous organic gases and vapours, Fig. 9 includes water vapour as well as such elements as H_2, He, O_2, N_2 and Ar*.

It is suggested that, for the purpose of calculating rates of mass transfer for organic vapours, the mean curve in Fig. 9 may be used to estimate the molecular diffusion coefficients in the absence of reliable measurements.

In the analysis of sections 3 and 4 the diffusion coefficient appears only in the parameter (j/ν), where ν is taken to be the viscosity of the uncontaminated air. Within each of a series of temperature ranges, ν can be approximated by expressions of the type,

$$\frac{\nu}{\nu_0} = \left(\frac{p_c}{p}\right)\left(\frac{T}{T_c}\right)^s \qquad (77)$$

the values of s being found to lie between the two values quoted previously for q. The variation of (j/ν) with temperature may evidently be expected to be small. For example, Cope[26] has shown that a value of 1·89 for s represents adequately the temperature variation of ν for air over the range 90 deg K $\leqslant T \leqslant$ 300 deg K, so that the variation of (j/ν) between 300 deg K and 215 deg K, representative of the surface temperatures on a subsonic aircraft operating between ground level and the stratosphere, will only amount in the extreme case to some 5 per cent.

Some typical values of (j/ν) for substances diffusing into air are shown in Table 5.

TABLE 5

$\nu = 0{\cdot}132$ cm²/sec for air at 0°C and 760 mm Hg pressure
j/ν independent of temperature

Substance	j/ν	
Camphor	0·31‡	
Hexachlorethane	0·20‡	
Naphthalene	0·375‡	(0·39)§
Thymol	0·32‡	
Water vapour	1·54‡	(1·65)§

‡ j estimated from Fig. 9
§ j obtained from Ref. 15.

8. *Wind-tunnel Experiment.*—The rates of sublimation of Camphor, Naphthalene and Thymol from a flat surface were measured in the R.A.E. 4-ft × 3-ft Wind Tunnel in order to provide a rough comparison with the results of the preceding analysis†.

A solution in either Petroleum Ether or Acetone of each chemical was sprayed on to a rectangular metal strip 12·5 cm wide by 7·5 cm long ; a border, 1·25 cm wide, around the edges of the strip was kept free from the chemical coating. The strip was weighed and then, by means of cellophane tape, was fixed with its shorter edge parallel to the direction of flow to the surface of a large flat plate spanning the tunnel. The metal strip was 0·1 mm thick and the cellophane tape, which was attached to the border of the strip, was 0·075 mm thick. After running the tunnel at a known speed and for a given time, and having first carefully removed the cellophane tape and all traces of the adhesive, the strip was weighed again.

* In spite of this apparent generality, it is considered that the relation between A_{12} and molecular weight, suggested by the curve of Fig. 9, is valid only for organic substances containing combinations of a few elements, *i.e.*, Carbon, Hydrogen, Oxygen and Nitrogen, diffusing into relatively simple gases, such as air, carbon dioxide, hydrogen.

† The authors are indebted to Mr. Mikolajewski of Royal Aircraft Establishment, for supplying the solutions used in this experiment.

Boundary-layer transition was fixed near the leading edge of the large plate by a wire 0·7 mm diameter. The turbulent boundary-layer thickness in the neighbourhood of the coated strip, located approximately 2·1 m from the leading edge of the plate, was estimated to be of the order of 40 mm and so the protuberance formed by the combination of the strip and adhesive tape should have had a negligible effect on the flow.

No attempt was made to achieve a high degree of accuracy in the experiment, and the tunnel speeds were chosen so as to give a change in weight of the order of 0·05 gm during a run of 10-min duration. Since a certain time, of the order of 45 sec, is required at the beginning of a run for the air in the tunnel to accelerate to a steady speed and at the end of a run for it to return to rest, it is considered that the effective length of run was measured to an accuracy not much better than 5 per cent. If greater accuracy were required, the duration of the run could be increased, possibly with a corresponding decrease in tunnel speed. The change in weight was measured to within about 5 per cent.

The substances used in the experiment were selected from among the chemicals suitable for indicating boundary-layer transition, since detailed measurements of their vapour pressures were available ; even so, a disparity was noted between the vapour pressures quoted by different authorities (*see* for example, Fig. 11).

For comparison with the measurements, the mass-transfer coefficients were calculated by the method described in section 4.3 and converted into rates of sublimation by means of the vapour pressure data reproduced in Figs. 10, 11 and 12. The boundary-layer thickness and shear stress were estimated from the expressions (Ref. 8, Chap. viii)

$$\delta = 0{\cdot}37\, l \left(\frac{U_0 l}{\nu}\right)^{-1/5}$$

and

$$\frac{\tau_0}{\rho_0 U_0^2} = 0{\cdot}0296 \left(\frac{U_0 l}{\nu}\right)^{-1/5}$$

in which l is the distance of the strip from the leading edge of the plate*.

The results are shown in Table 6 below, and are presented in the form of non-dimensional rates of sublimation, $G/\rho_0 U_0 S$; G is the rate at which mass is transferred from the surface of area S into the airstream which has a velocity U_0 and density ρ_0.

TABLE 6

Substance	U_0 m/sec	T_0 °C	p_s' mm Hg	$\frac{G}{\rho_0 U_0 S} \times 10^6$	
				Measured	Estimated
Camphor	13·6	12·5	0·10	1·03	1·08
	19·8	12·5	0·10	0·98	0·96
Naphthalene	19·7	12·8	0·026	0·22	0·23
	26·4	12·0	0·023	0·22	0·20
	32·9	12·3	0·024	0·17	0·19
	39·5	13·0	0·026	0·18	0·19
	39·5	13·0	0·026	0·17	0·19
Thymol	57·9	14·3	0·0042	0·045	0·034
	58·0	15·5	0·0053	0·050	0·041
	49·9	15·5	0·0053	0·055	0·043

* The distributions of K_y over the strip for $j/\nu = 0{\cdot}333$ and 1·0 are shown in Figs. 5a, 5b, 6a and 6b. These Figures correspond respectively to tunnel wind speeds of 65·6, 49·2, 32·8 and 16·4 m/sec.

A similar comparison between the theoretical and experimental results, in terms of the mass-transfer coefficients, is made in Fig. 13.

Bearing in mind the questionable accuracy of the vapour pressure data on which the estimates were based, the agreement with the measured rates of sublimation in Table 6, and in Fig. 13, is as good as might reasonably be expected. It is, perhaps, significant that the divergence of the theoretical from the experimental values of $G/\rho_0 U_0 S$ and K_g increases as the volatility of the substance decreases; this trend might be accounted for by a progressively decreasing accuracy in the measurement of vapour pressure. It may be noted that the accuracy of the wind-tunnel measurements, although not claimed to be high, remained approximately the same for all the substances, since the windspeed was adjusted in each case to give roughly the same length of run and change in weight*.

9. *Application to the Observation of Boundary-layer Transition in Flight.*—The ' sublimation ' method of transition indication has been used successfully in flight by Dando (1949)[3] and others; this success has, however, depended on the fact that the range of altitudes to which the experiments were confined was generally fairly narrow. Recent flight experience has shown that the extension of this range to what might be representative of a modern jet aircraft introduces very serious difficulties in obtaining records of transition at 20,000 ft, and almost insuperable difficulties at greater altitudes. Other troubles which arise are due to seasonal variations in the temperature of the atmosphere.

The origin of these difficulties is clear from the preceding analysis. The rate of sublimation, g_s, into an unadulterated airstream can, in general, be expressed in the form

$$g_s = f\left(R, \frac{j}{\nu}\right)\frac{U_0 m p_s}{T}$$

where R is the Reynolds number and p_s, the vapour pressure of the diffusing substance, varies as $\exp(-0{\cdot}1203\ a/T)$; some values of a are given in Table 4, section 6. With a change in altitude, the values of R, U_0, T and p_s will alter, but the dominating effect will be that of the change in p_s. Thus, p_s at an altitude of 20,000 ft may be only 1 per cent, or less, of its value at ground level, owing to the difference in surface temperature†.

In order to illustrate the magnitude of these altitude effects, calculations were made of the rates of sublimation from the surfaces of two conjectural aircraft, which were assumed to climb to altitude at a constant forward speed and rate of ascent, followed by a level flight at constant Mach number. The individual performances were :—

(a) ' Fast ' aircraft :	Rate of climb	3000 ft/min
	Forward speed in the climb	300 ft/sec E.A.S.
	Mach number in level flight	0·65
(b) ' Slow ' aircraft :	Rate of climb	1500 ft/min
	Forward speed in the climb	200 ft/sec E.A.S.
	Mach number in level flight	0·35

* It seems quite feasible to adapt the technique used in this experiment to the measurement of the vapour pressure of solid volatile substances at ordinary temperatures, particularly if a laminar boundary layer were maintained on the plate. The methods described in section 3.3 could then be used to relate the concentration at the surface to the rate of sublimation. With a laminar boundary layer there is no uncertainty about the mechanism of the mass transfer process, and the theoretical solutions for the rate of transfer can be assumed correct.

† The extreme sensitivity of p_s to temperature makes it important to ensure that, in a flight experiment, the surface under observation is far removed from heat sources, such as engine exhausts and jet pipes, which may produce a non-uniform temperature distribution over the surface. Very roughly, for Naphthalene, an increase of 1 deg C in temperature increases the rate of evaporation by 15 per cent; it is, therefore, clear that chordwise variations in temperature of a few degrees could affect considerably the evaporation pattern.

The surface under observation was supposed to be a flat plate of 15-ft chord with a completely turbulent boundary layer, and the calculations were restricted to a unit area of the plate 0·6-chord behind the leading edge. The assumed surface coatings were Naphthalene for the slower aircraft, and both Thymol and Naphthalene for the faster one.

The results are shown in Figs. 14, 15 and 16, and are presented in the form of total mass evaporated as a function of altitude and the horizontal distance travelled from take-off*.

In practice, the surface densities of chemical corresponding to a very heavy coating, a moderate coating and a very light coating would be roughly 1·5, 0·75 and 0·4 gm/sq ft respectively.

Figs. 14 and 15 suggest that difficulty in obtaining transition records at high altitude within an acceptable flight range is likely to be encountered on the fast aircraft, if a chemical of low volatility is used (Fig. 14); if, however, a more highly volatile chemical were chosen, and it were heavily applied, it should be possible to obtain results at an altitude as high as 20,000 ft (Fig. 15). This is more clearly exemplified by Fig. 17 which shows how the extent of evaporation from the surface varies along the chord at different stages of a flight at 20,000 ft; to construct this Figure, it was assumed that the surface was coated with Naphthalene and that boundary-layer transition occurred at the leading edge during the climb, subsequently retreating to 0·4-chord in level flight. The curves imply that a satisfactory contrast between the regions of laminar and turbulent flow could be obtained in this particular case by coating the surface initially to a density of roughly 1·25 gm/sq ft, the observations being completed in a flight of 100 miles.

On the other hand, Fig. 16 indicates that, on the slower aircraft, the reduction in the rate of evaporation with increasing altitude is such as to make transition records unobtainable within a practicable flight range at heights greater than approximately 15,000 ft. The situation could not be improved by using a more volatile chemical, since it would require an initial coating of prohibitively high surface density.

In the interpretation of Figs. 14, 15 and 16, given above, it is supposed that the evaporation pattern can be recorded at altitude as soon as the position of transition becomes apparent. This may not always be feasible and, if records are obtained after the aircraft has landed, allowance must be made for the evaporation which occurs during the descent. For a given contrast between regions of laminar and turbulent flow, a reduction in the maximum altitude at which the observations can be made is required to counterbalance the effect of the additional evaporation; for the two aircraft considered here, the necessary reduction in altitude may amount to 5,000 ft, or more.

The conclusions given in this section confirm those previously reached by Mr. W. E. Gray from flight experiments.

10. *Conclusions.*—The transfer of a dilute gas from or into the surface of a body in a two-dimensional airstream has been studied by means of an analogy between forced diffusion, heat transfer and skin friction. Three particular surfaces were considered: a flat plate parallel to the flow, an aerofoil, and a part of a flat plate far removed from the leading edge. Expressions have been deduced for the rates of mass transfer when the boundary layers on the surfaces are either laminar or turbulent.

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* Figs. 14, 15 and 16 are intended to illustrate broadly the effects of altitude on the sublimation rates and, for this purpose, a surface with a completely turbulent boundary layer has been considered. In practice, higher rates of sublimation than those shown in the Figures may occur locally near transition, *cf.* Figs. 2 and 17. Figs. 14, 15 and 16 should not, therefore, be interpreted as giving an accurate measure of the distance which must be travelled at any altitude for the position of transition first to become apparent.

For laminar boundary layers, the parameter j/ν controls the mass transfer in a manner similar to that of the Prandtl number in the case of heat transfer ; j is the molecular diffusion coefficient and ν the kinematic viscosity of the mixture, assumed equal to that of the uncontaminated air. In particular, the mass transfer from a flat plate is proportional to $(j/\nu)^{2/3}$; this relation also holds approximately for the mass transfer from aerofoil surfaces.

For turbulent boundary layers, the molecular diffusion through the laminar sub-layer and transition layer has a significant effect on the mass-transfer rate, and for this reason an approximate, alternative solution to that of Sutton is given to the problem of evaporation from an isolated region of a flat plate. This solution is in fair agreement with Sutton's when j/ν is equal to unity ; it also agrees reasonably well with measurements of the rates of sublimation of Camphor, Thymol and Naphthalene, made in a wind tunnel, j/ν for these substances lying in the neighbourhood of $0{\cdot}35$.

An elementary examination of the boundary condition at an evaporating surface reveals that the usual assumption—namely, that the air adjacent to the surface is saturated with the diffusing vapour—is, in general, justified and bears a resemblance to the condition for no-slip in the flow of a viscous fluid, and the condition in heat transfer that the temperature varies continuously across the surface. The corresponding condition for the transfer of a chemically active gas into a surface is derived, but, without further experiments, no quantitative values can be assigned to the concentration at the surface.

An analysis of available measurements of the molecular diffusion coefficient for a pair of gases suggests an approximate relationship between the collision diameter of the molecules and the sum of the molecular weights of the gases. Combining this relation with the Chapman-Langevin formula, a rough value of the diffusion coefficient for a large class of gases can be easily obtained. When one of the gases is air, j/ν can be assumed independent of temperature, and may vary from approximately $0{\cdot}3$ for the diffusion of heavy organic vapours, like Camphor, to a value greater than unity for elementary vapours, such as water vapour.

Applying the theoretical methods to the problem of indicating boundary-layer transition by a chemical sublimation technique, it is shown that the time required to obtain a record of transition in a flight experiment increases rapidly with altitude, owing mainly to the reduction in surface temperature and consequent decrease in the vapour pressure of the diffusing chemical. This confirms the conclusion that had been reached by Mr. W. E. Gray from flight experiments at 20,000 ft, that the sublimation technique can be used satisfactorily only at low or moderate altitudes.

LIST OF SYMBOLS

a, b	Constants determining the vapour pressure of a substance (eqn. 69)
A_{12}	A constant for a pair of gases relating the diffusion coefficient to their molecular weights (eqn. 76)
C_p	Specific heat of air at constant pressure
C_f	Local skin friction coefficient : $\tau_0/\frac{1}{2}\rho_0 U_0^2$
g_s	Rate of transfer of mass from unit area of a surface
G	Rate of transfer of mass from a surface of area S
J	Mechanical equivalent of heat
$J(j/\nu)$	A function appearing in the expression for the turbulent mass transfer from a flat plate (eqn. 46)
j	Molecular diffusion coefficient (eqn. 1)

LIST OF SYMBOLS—*continued*

k	Thermal conductivity of air
K_h	Local heat-transfer coefficient (eqn. 44)
K_g	Local mass-transfer coefficient (eqn. 15)
$\overline{lv}$	Turbulent exchange coefficients (eqn. 8)
L	Latent heat of evaporation or sublimation per unit mass
m	Molecular weight
M	Mach number
p_0	Static pressure in the airstream
p_1	Partial pressure of contaminating vapour
p_c	A standard pressure (760 mm Hg)
p_s	Absolute vapour pressure
p_s'	Vapour pressure expressed in mm Hg
R	Reynolds number
$\overline{R}$	Universal gas constant
S	Area
t	Time
T	Temperature
T_0	Ambient temperature in the airstream
T_1	Surface temperature
T_c	A standard temperature (0 deg C)
U	Velocity just outside the boundary layer
U_0	Free-stream velocity
u, v	Velocity components in the boundary layer
u_τ	Friction velocity : $\sqrt{(\tau_0/\rho_0)}$
x	Distance measured along the surface from the origin of the diffusion boundary layer or from the front stagnation point on an aerofoil
y	Distance measured normal to the surface
y_τ	$y\ u_\tau/\nu$
Y_τ	The value of y_τ at the edge of the diffusion boundary layer
α	Coefficient of molecular evaporation (eqn. 61)
δ	Thickness of viscous boundary layer
δ_1	Thickness of diffusion boundary layer
δ^*	Displacement thickness of viscous boundary layer (eqn. 20)
δ_1^*	Displacement thickness of diffusion boundary layer (eqn. 21)
ε_A	Activation energy per unit mass (eqn. 67)
ε_c	Crystal excitation energy per unit mass (section 5.1)
μ	Coefficient of viscosity of air
ν	Coefficient of kinematic viscosity of air
ρ_0	Air density
ρ	Density of diffusing vapour
σ	Prandtl number : $\mu C_p/k$

LIST OF SYMBOLS—*continued*

Σ	Von Kármán function occurring in the expression for turbulent heat transfer from a flat plate (eqn. 45)
τ_0	Surface frictional stress
χ	x/δ
ψ	Concentration of diffusing vapour : ρ/ρ_0 (eqn. 4)
ψ_0	Concentration in the free stream
ψ_1	Concentration at the surface
ψ_s	Concentration corresponding to saturation (eqn. 63)

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APPENDIX I

The Coefficient of Interdiffusion Between a Pair of Gases

The values of A_{12} shown in Fig. 9 were deduced from the data contained in the *International Critical Tables*[15]; the gases to which they apply are tabulated below.

A_{12} is defined by the equation, *cf.* section 7,

$$j_{12} = A_{12}\left(\frac{m_1 + m_2}{m_1 m_2}\right)^{1/2}\left(\frac{p_c}{p}\right)\left(\frac{T}{T_c}\right)^q$$

TABLE 1

Gas 1	Gas 2	A_{12}		
		Air	CO_2	H_2
Benzene		0·352	0·280	0·411
Ethyl propionate		0·311	0·250	0·331
Isobutyl formate		0·336	—	—
Diethylamine		0·389	—	—
Ethyl ether		0·346	0·291	0·414
Methyl propionate		0·345	0·286	0·414
Ethyl acetate		0·336	0·254	0·382
Propyl formate		0·335	0·266	0·392
n-propyl alcohol		0·376	0·292	0·439
Isopropyl bromide		0·439	—	—
Methyl acetate		0·373	0·298	0·463
Carbon disulphide		0·408	0·333	0·515
Formic acid		0·550	0·415	0·705
Methyl alcohol		0·515	0·378	0·695
Acetic acid		0·471	0·361	0·366
Methyl formate		0·386	—	—

TABLE 1—*continued*

Gas 1	Gas 2	A_{12}		
		Air	CO_2	H_2
Propionic acid		0·378	0·308	0·460
Ethyl formate		0·383	0·301	0·471
Methyl acetate		0·383	0·298	0·467
n-propyl bromide		0·411	—	—
Ethyl alcohol		0·420	0·324	0·519
Diphenyl		0·299	—	—
Naphthalene		0·248	—	—
Amyl butyrate		0·198	—	—
n-octane		0·241	—	—
Amyl propionate		0·226	0·200	0·266
Ethyl benzene		0·314	—	—
Toluene		0·332	—	—
Propyl propionate		0·274	0·222	0·297
Anthracene		0·210	—	—
Benzidrin		0·149	—	—
Eugenol		0·187	—	—
Safrol		0·214	—	—
Isosafrol		0·226	—	—
Isobutyl valerate		0·210	0·180	0·242
Amyl isobuterate		0·207	—	0·240
n-propylbenzene		0·232	—	—
Mesitylene		0·270	—	—
Isopropylbenzene		0·236	—	—
Propyl valerate		0·229	0·197	0·266
Isobutyl isobutyrate		0·224	0·205	0·268
Isobutyl butyrate		0·230	0·190	0·260
Propyl isobutyrate		0·267	0·222	0·300
Propyl butyrate		0·258	0·208	0·289
Isopropyl isobutyrate		0·286	—	—
Isobutyl propionate		0·257	0·210	0·285
Ethyl valerate		0·249	0·210	0·288
Benzyl chloride		0·320	—	—
m-chlorotoluene		0·262	—	—
o-chlorotoluene		0·287	—	—
p-chlorotoluene		0·248	—	—
Hexyl alcohol		0·237	0·195	0·279
Ethyl benzene		0·314	—	—
m-xylene		0·281	—	—
o-xylene		0·296	—	—
p-xylene		0·268	—	—
Caproic acid		0·241	—	—
Isocaproic acid		0·247	—	—
Amyl formate		0·262	—	—
n-butyl acetate		0·280	—	—
Ethyl n-butyrate		0·279	0·230	0·313
Ethyl isobutyrate		0·285	0·232	0·322
Isoamyl formate		0·280	—	—
Isobutyl acetate		0·295	0·240	0·331
Methyl valerate		0·273	—	—
Propyl propionate		0·275	0·223	0·297
Aniline		0·327	—	—
n-amyl alcohol		0·275	0·228	0·329
amyl alcohol (fermented)		0·273	0·227	0·326
Isovaleric acid		0·259	0·208	0·298
n-valeric acid		0·238	—	—
Methyl butyrate		0·301	0·247	0·339
Methyl isobutyrate		0·304	0·250	0·360
Propyl acetate		0·318	—	—

TABLE 2

Gases	A_{12}
Helium—Argon	1·220
Hydrogen—Oxygen	0·954
Oxygen—Nitrogen	0·700
Oxygen—Carbon monoxide	0·714
Oxygen—Carbon dioxide	0·599
Oxygen—Air	0·695
Hydrogen—Sulphur dioxide	0·671
Hydrogen—Nitrogen	0·916
Hydrogen—Nitrous oxide	0·741
Hydrogen—Carbon monoxide	0·970
Hydrogen—Carbon dioxide	0·764
Hydrogen—Methane	0·834
Hydrogen—Ethylene	0·666
Hydrogen—Ethane	0·629
Hydrogen—Air	0·834
Nitrous oxide—Carbon dioxide	0·450
Carbon monoxide—Carbon dioxide	0·569
Carbon monoxide—Ethylene	0·435
Carbon dioxide—Methane	0·526
Carbon dioxide—Air	0·678
Iodine—Air	0·492
Water vapour—Air	0·735

APPENDIX II

The Functions g and H, cf. Section 4.3

$g(z)$ is defined by,

$$g(z) = \frac{zv}{j}, \qquad 0 < z < 5$$

$$= 5 \log_e \left(\frac{zv}{5j} + 1 - \frac{v}{j}\right) + \frac{5v}{j}, \qquad 5 < z < 30$$

$$= 5{\cdot}5 + 2{\cdot}5 \log_e z + J(j/v), \qquad z > 30 .$$

Then,

$$g'(z) = \frac{v}{j}, \qquad 0 < z < 5$$

$$= \frac{v}{j(zv/5j + 1 - v/j)}, \qquad 5 < z \leqslant 30$$

$$= \frac{2{\cdot}5}{z}, \qquad z \geqslant 30.$$

$H(z)$ is defined by :

$$H(z) = \frac{g(z)}{g'(z)} \frac{1}{\int_0^z f(\zeta)\, g(\zeta)\, d\zeta}$$

where $f(z)$ is given by equation (53), section 4.3.

Writing

$$I = \int_0^z f(\zeta)\, g(\zeta)\, d\zeta \ ,$$

$0 < z < 5$

$$I = \nu z^3/3j.$$

$5 < z < 30$

$$I = \log_e\left(\frac{z\nu}{5j} - 1 - \frac{\nu}{j}\right)\left\{-40{\cdot}25z - 201{\cdot}25\,\frac{j}{\nu} + 201{\cdot}25\right.$$

$$\left. + \left[25z + 125\left(\frac{j}{\nu} - 1\right)\right]\log_e z\right\} + 25\left(\frac{\nu}{j} - 1\right)\left(z\log_e z - 8{\cdot}05\right)$$

$$- 62{\cdot}5\left(\frac{j}{\nu} - 1\right)\left[\left(\log_e \frac{3\nu}{5j}\right)^2 - \left(\log_e \frac{\nu}{j}\right)^2\right]$$

$$+ 125\left(\frac{j}{\nu} - 1\right)\left\{\left[\frac{5\left(\frac{j}{\nu} - 1\right)}{z}\right] - \frac{1}{4}\left[\frac{5\left(\frac{j}{\nu} - 1\right)}{z}\right]^2\right.$$

$$+ \frac{1}{9}\left[\frac{5\left(\frac{j}{\nu} - 1\right)}{z}\right]^3 - \frac{1}{16}\left[\frac{5\left(\frac{j}{\nu} - 1\right)}{z}\right]^4 \cdots$$

$$\left. -\left(\frac{j}{\nu} - 1\right) + \frac{1}{4}\left(\frac{j}{\nu} - 1\right)^2 - \frac{1}{9}\left(\frac{j}{\nu} - 1\right)^3 + \frac{1}{16}\left(\frac{j}{\nu} - 1\right)^4 \cdots\right\}$$

$$+ 242{\cdot}9\,\frac{\nu}{j} - 40{\cdot}25z\,\frac{\nu}{j} + 65{\cdot}25z - 326{\cdot}25$$

$$5\left|\frac{j}{\nu} - 1\right| < z$$

$$= \log_e\left(\frac{z\nu}{5j} - 1 - \frac{\nu}{j}\right)\left\{-40{\cdot}25z - 201{\cdot}25\,\frac{j}{\nu} + 201{\cdot}25\right.$$

$$\left. + \left[25z + 125\left(\frac{j}{\nu} - 1\right)\right]\log_e z\right\}$$

$$+ 25\left(\frac{\nu}{j} - 1\right)\left(z\log_e z - 8{\cdot}05\right)$$

$$- 125\left(\frac{j}{\nu} - 1\right)\log_e\left(1 - \frac{\nu}{j}\right)\log_e \frac{z}{5}$$

$$- 125\left(\frac{j}{\nu} - 1\right)\left\{\left[\frac{z}{5\left(\frac{j}{\nu} - 1\right)}\right] - \frac{1}{4}\left[\frac{z}{5\left(\frac{j}{\nu} - 1\right)}\right]^2\right.$$

$$+ \frac{1}{9}\left[\frac{z}{5\left(\frac{j}{\nu} - 1\right)}\right]^3 - \frac{1}{16}\left[\frac{z}{5\left(\frac{j}{\nu} - 1\right)}\right]^4 \cdots$$

$$\left. - \left[\frac{1}{\left(\frac{j}{\nu} - 1\right)}\right] + \frac{1}{4}\left[\frac{1}{\left(\frac{j}{\nu} - 1\right)}\right]^2 - \frac{1}{9}\left[\frac{1}{\left(\frac{j}{\nu} - 1\right)}\right]^3 + \frac{1}{16}\left[\frac{1}{\left(\frac{j}{\nu} - 1\right)}\right]^4 \cdots \right\}$$

$$+ 242{\cdot}9\,\frac{\nu}{j} - 40{\cdot}25z\,\frac{\nu}{j} + 65{\cdot}25z - 326{\cdot}25$$

$$5\left|\frac{j}{\nu} - 1\right| > z$$

$\underline{z > 30}$

$$I = I(30) + \left[15{\cdot}25 + 3J\left(\frac{j}{\nu}\right)\right] + 15 + 2{\cdot}5J\left(\frac{j}{\nu}\right)\Big]z \log_e z$$

$$+ 6{\cdot}25z(\log_e z)^2 - \left[4157 + 345{\cdot}1J\left(\frac{j}{\nu}\right)\right]$$

APPENDIX III

Diffusion in a Turbulent Boundary Layer from an Isolated Region of a Flat Plate

An expression for the mass-transfer coefficient, similar in form to Sutton's, may be deduced very simply from some measurements, made by Wieghardt (1948)[27], of the temperature field due to a heat source placed in the surface of a flat plate. The plate was exposed to an air-stream of velocity U_0.

Wieghardt's investigation included both point and line sources; for a line source normal to the direction of flow, he obtained the empirical relation between the intensity of the source, q, and the temperature, T, at a point on the surface distant z from the source,

$$\rho_0 C_p(T - T_0) = 2{\cdot}52\left(\frac{U_0\delta}{\nu}\right)^{1/7}\left(\frac{U_0 z}{\nu}\right)^{-0{\cdot}91}\frac{q}{\nu} \quad \ldots \quad (1)$$

where δ is the boundary-layer thickness at the position of the source.

On the analogy between the diffusion of heat and of matter, the corresponding relation for the diffusion of a gas from a source of strength g_s per unit length normal to the flow may be written

$$\rho_0(\psi - \psi_0) = 2{\cdot}52\left(\frac{U_0\delta}{\nu}\right)^{1/7}\left(\frac{U_0 z}{\nu}\right)^{-0{\cdot}91}\frac{g_s}{\nu} \quad \ldots \quad (2)$$

The transfer of mass across a finite area extending downstream from $x = 0$, and forming part of the surface of a flat plate, may be represented by a distribution of sources of strength $g_s(x)$. The concentration, ψ_1, at a point x on the surface is then given by,

$$\rho_0(\psi_1 - \psi_0) = 2{\cdot}52\left(\frac{U_0}{\nu}\right)^{-0{\cdot}767} \int_0^x \frac{\delta^{1/7} g_s(\xi)\, d\xi}{(x - \xi)^{0{\cdot}91}} \quad \ldots \quad (3)$$

With the condition that ψ_1 is constant throughout the region occupied by the sources, the solution of (3) is,

$$g_s(x)\delta^{1/7} = 0{\cdot}0333\rho_0\nu x^{-0{\cdot}09}\left(\frac{U_0}{\nu}\right)^{0{\cdot}767}(\psi_1 - \psi_0).$$

The resulting expression for the mass-transfer coefficient is,

$$K_g = 0{\cdot}0333\left(\frac{U_0 x}{\nu}\right)^{-0{\cdot}09}\left(\frac{U_0\delta}{\nu}\right)^{-1/7} \quad \ldots \quad (4)$$

δ in the above equation relates to the boundary-layer thickness at the position x.

Values of K_g given by equation (4) are compared in the Table below with those obtained from Sutton's solution (equation (60), section 4.3) for $U_0\delta/\nu = 12{\cdot}52 \times 10^4$ and $4{\cdot}12 \times 10^4$.

x/δ	0·4	0·8	1·2	1·6	2·0
$U_0\delta/\nu = 12{\cdot}52 \times 10^4$					
K_g eqn. (4)	0·0023	0·0022	0·0021	0·0021	0·0020
K_g Sutton	0·0020	0·0019	0·0018	0·0018	0·0017
$U_0\delta/\nu = 4{\cdot}12 \times 10^4$					
K_g eqn. (4)	0·0031	0·0029	0·0028	0·0027	0·0027
K_g Sutton	0·0026	0·0024	0·0023	0·0022	0·0022

C

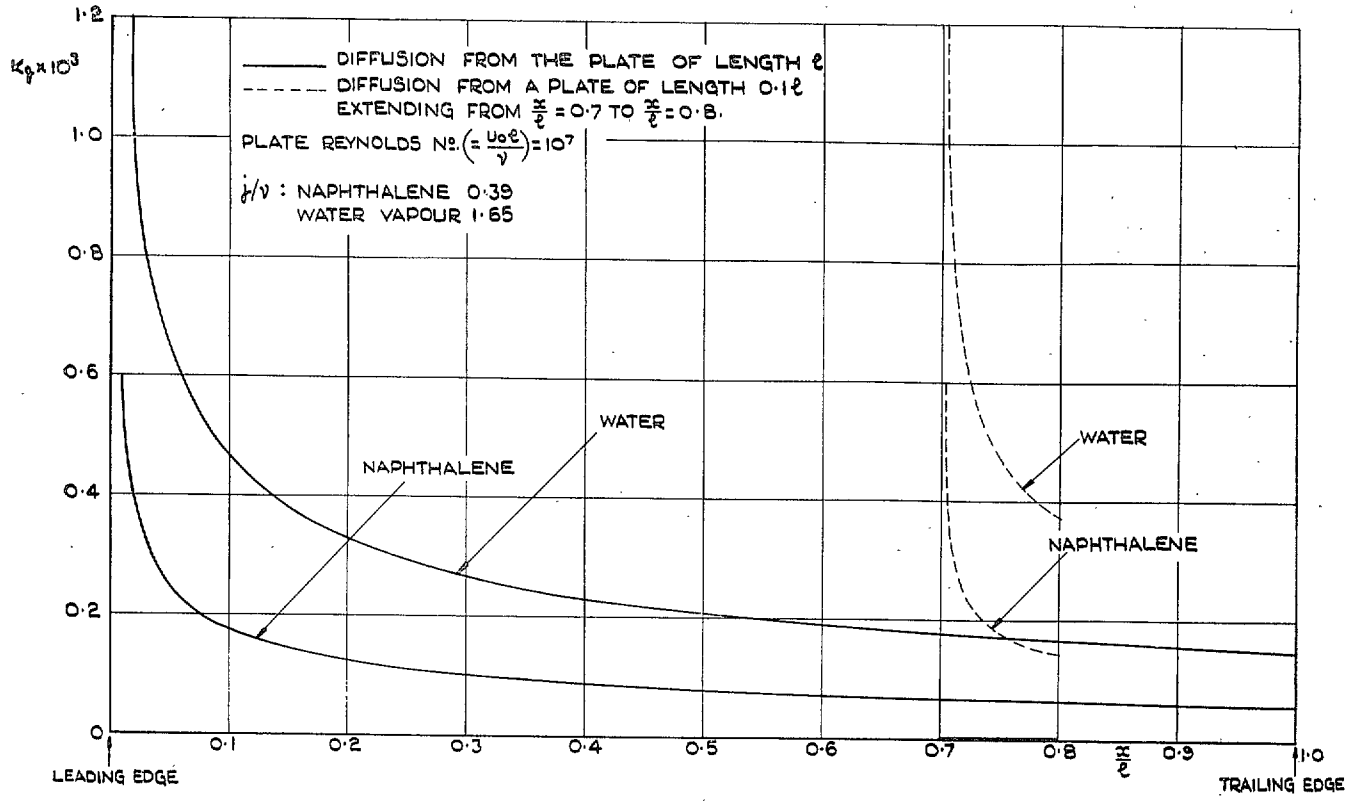
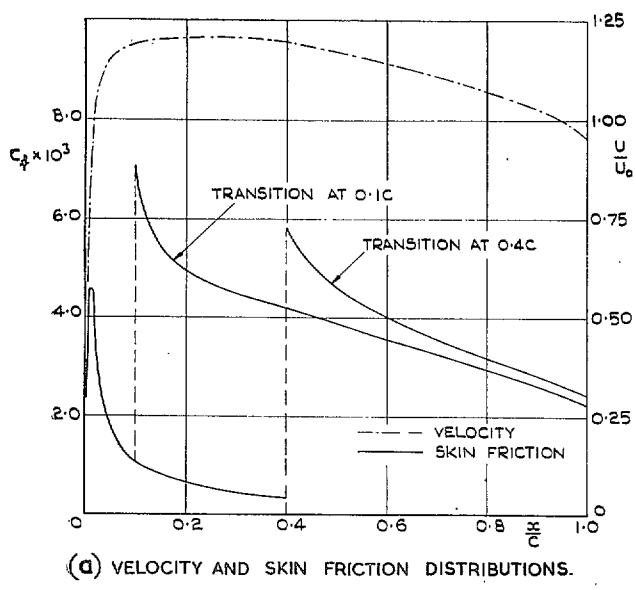
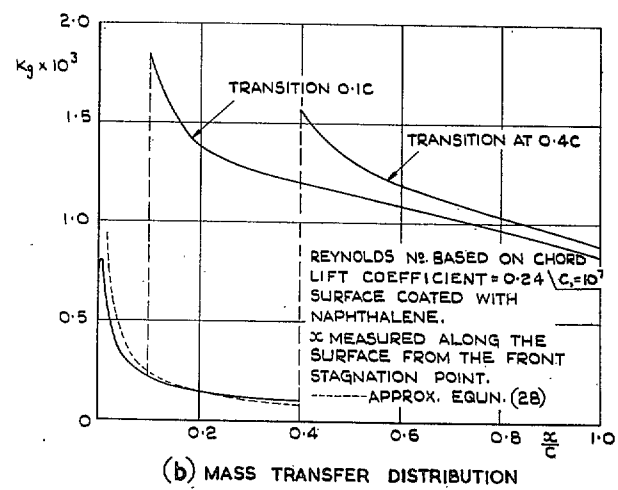


FIG. 1. Diffusion in a laminar boundary layer on a flat plate parallel to the airstream.



(a) VELOCITY AND SKIN FRICTION DISTRIBUTIONS.



(b) MASS TRANSFER DISTRIBUTION

FIGS. 2a and 2b. Diffusion from the upper surface of the NACA 2409 Aerofoil.

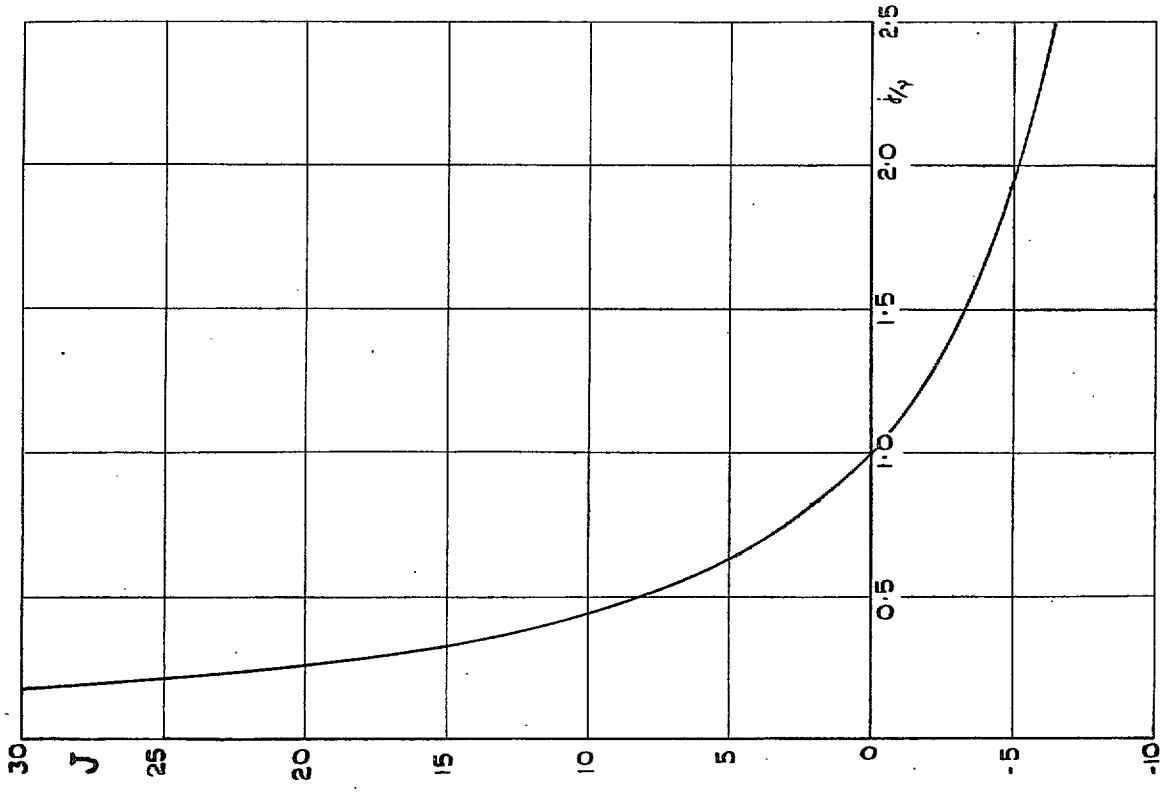


FIG. 3. Diffusion in a turbulent boundary layer : the function $J(j/\nu)$. (*cf.* equation (46).)

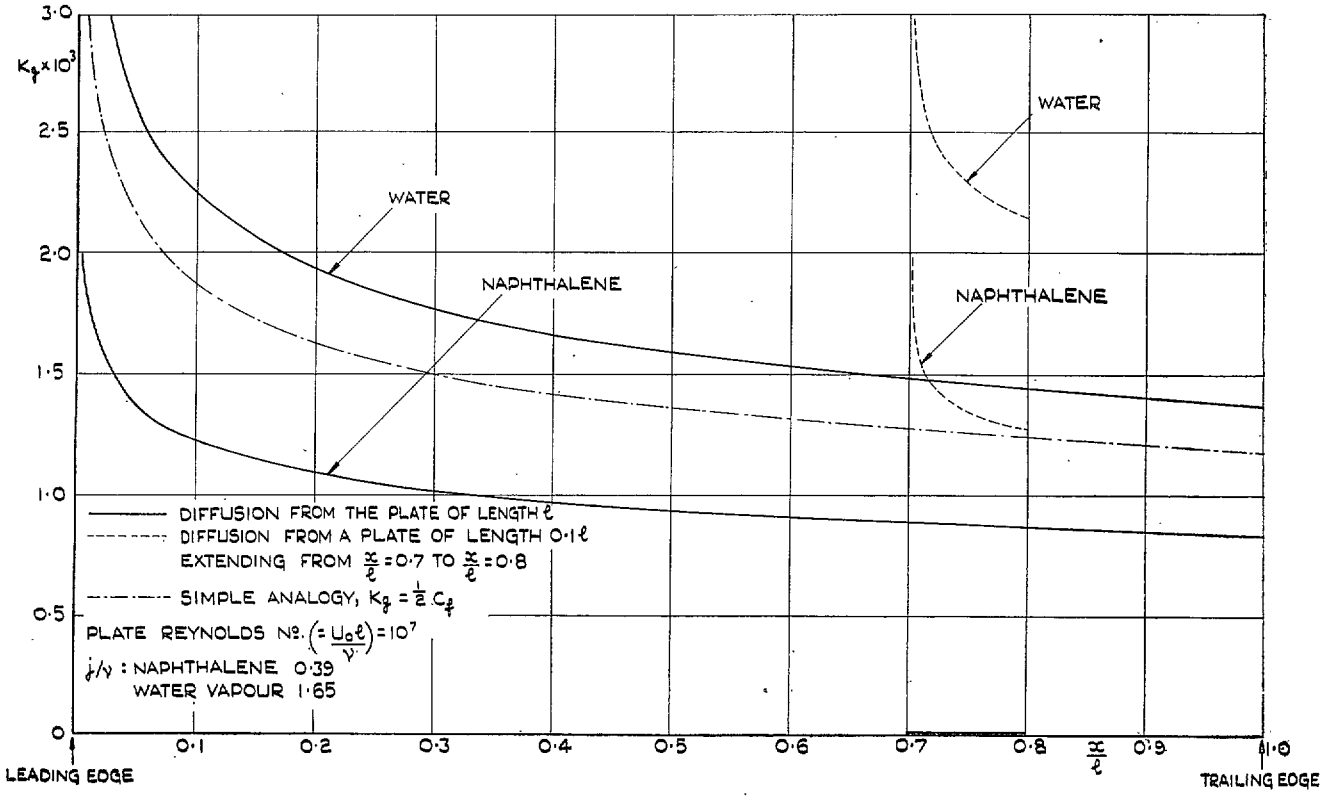
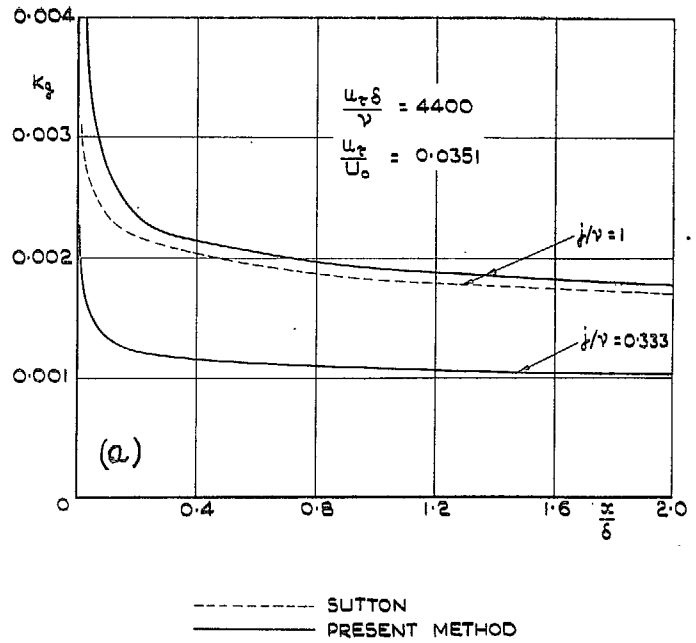
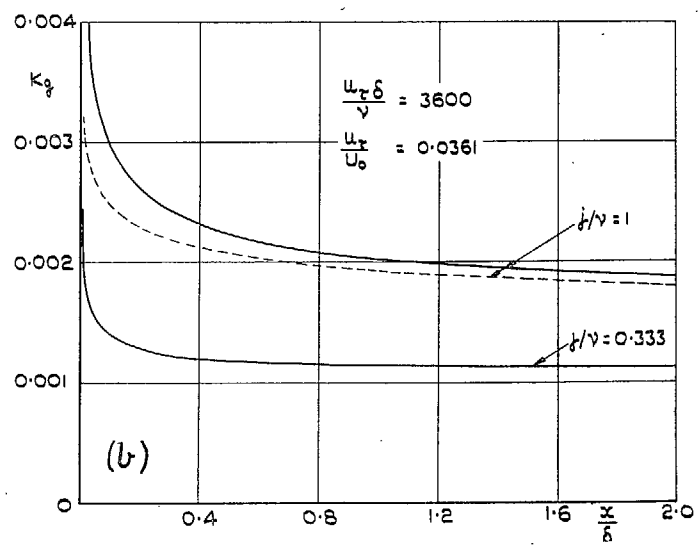
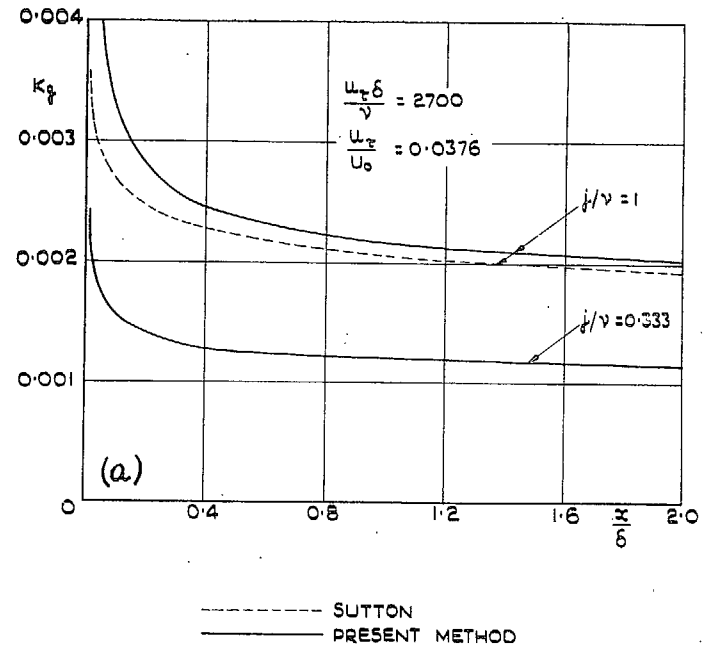


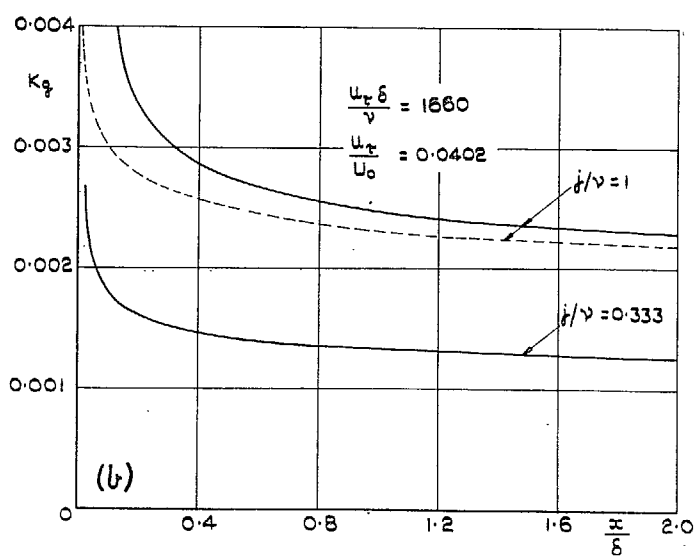
FIG. 4. Diffusion in a turbulent boundary layer on a flat plate parallel to the airstream.





FIGS. 5a and 5b. Diffusion in a turbulent boundary layer : comparison with Sutton's solution.





FIGS. 6a and 6b. Diffusion in a turbulent boundary layer : comparison with Sutton's solution.

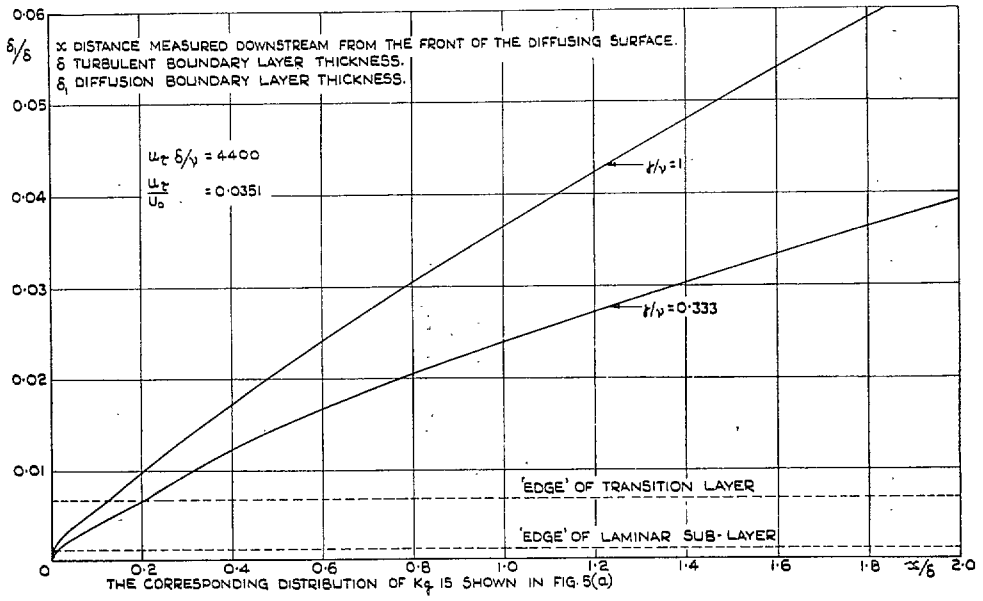


FIG. 7. Development of the diffusion boundary layer on a part of a flat plate.

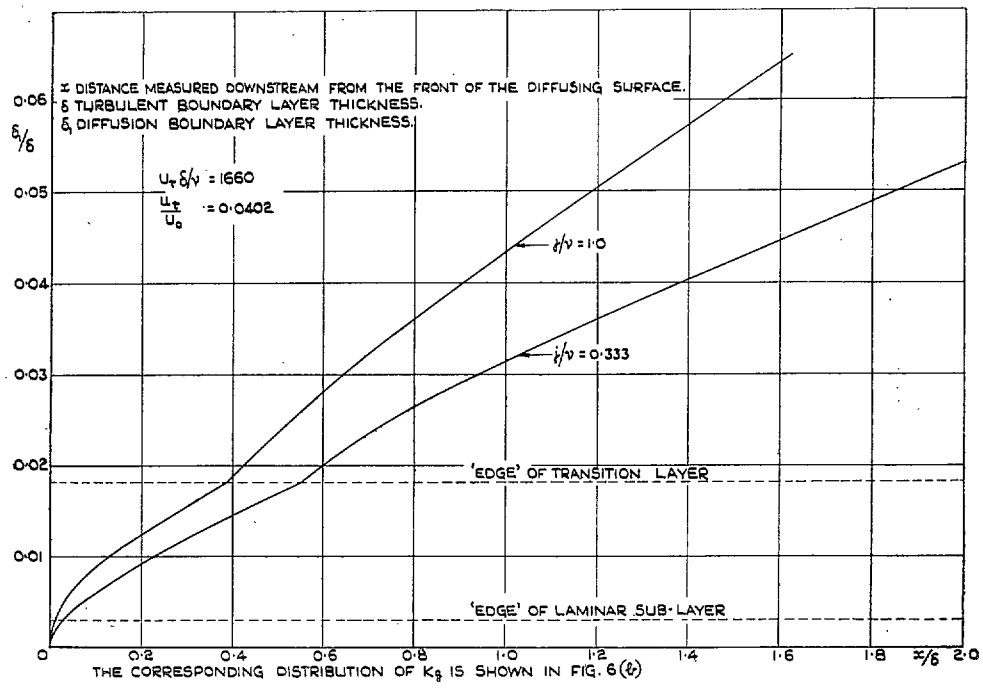
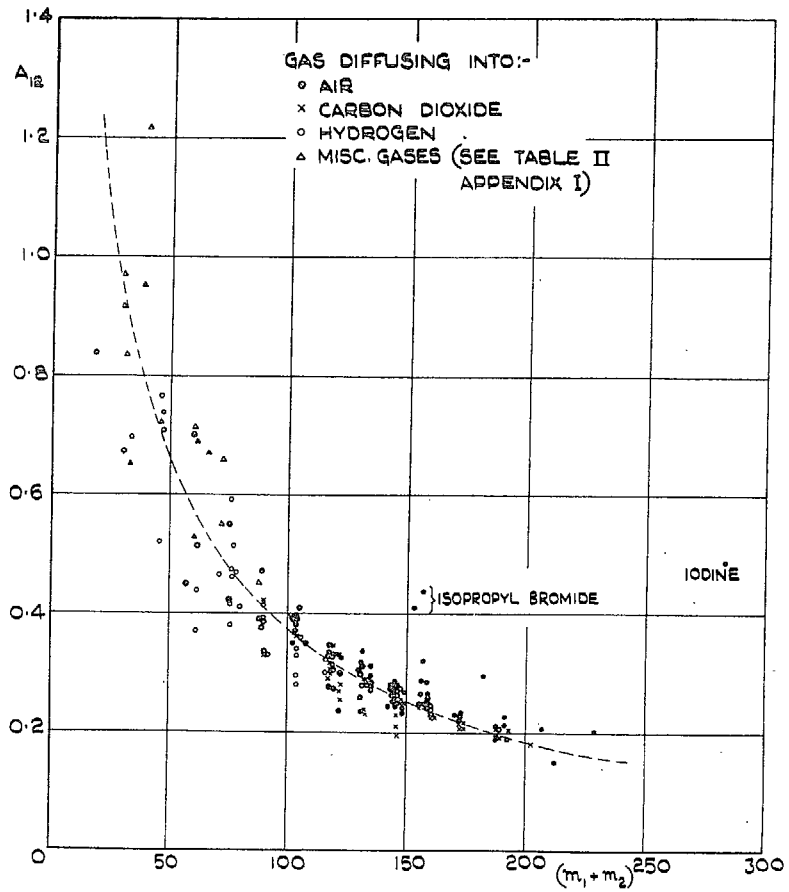
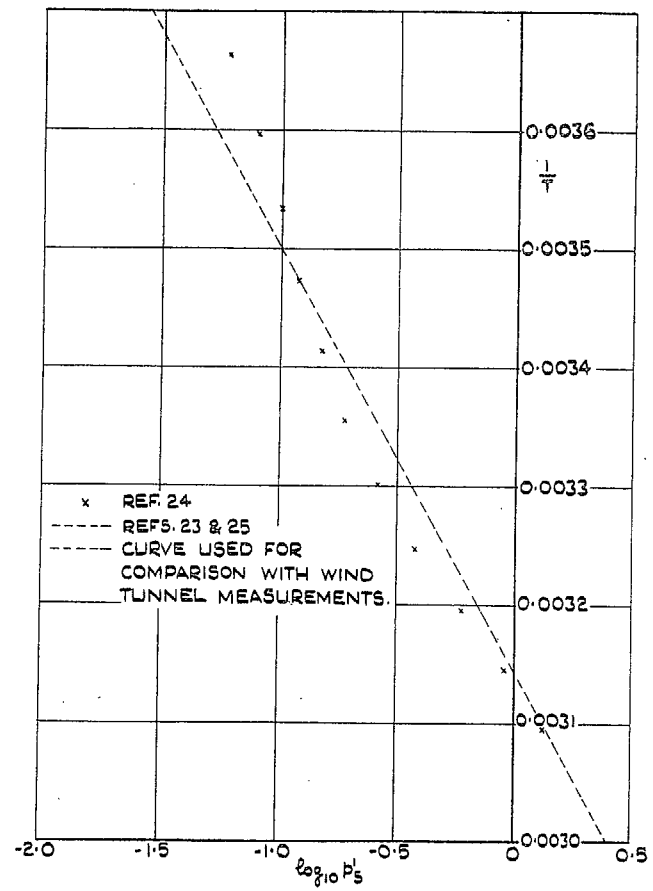


FIG. 8. Development of the diffusion boundary layer on a part of a flat plate.



m_1 & m_2 ARE THE MOLECULAR WEIGHTS OF THE DIFFUSING GASES
THE VALUES OF A_{12} PLOTTED IN THIS FIG. ARE TABULATED IN APPENDIX I.

FIG. 9. The coefficient of molecular diffusion between a pair of gases. The coefficient A_{12} appearing in equation (76).



p'_s VAPOUR PRESSURE IN mm. H_g AT TEMPERATURE T.

FIG. 10. Vapour pressures of Camphor.

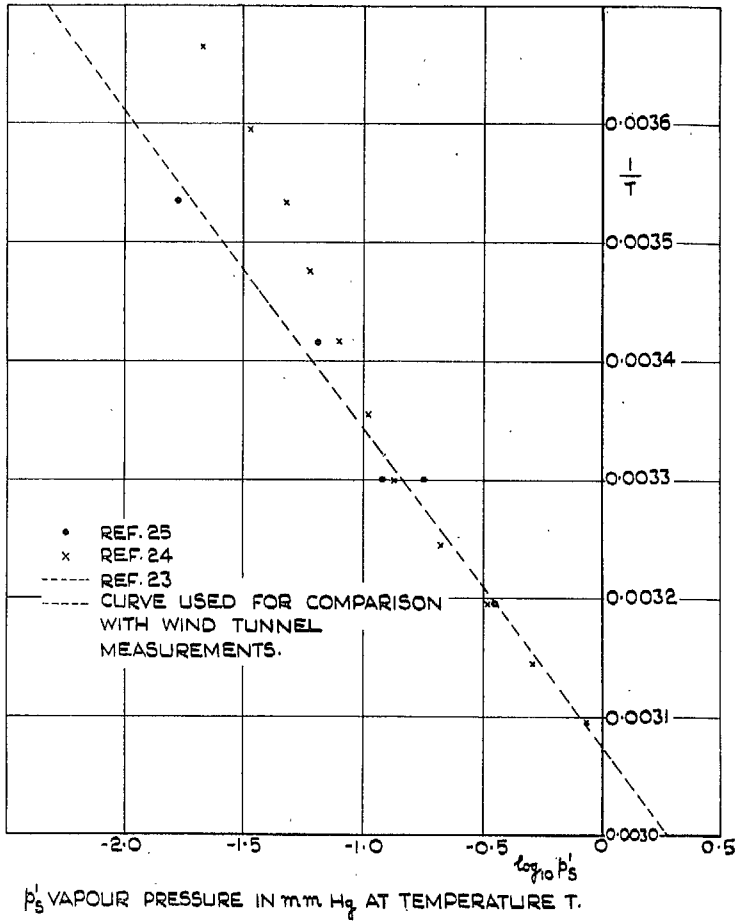


FIG. 11. Vapour pressures of Naphthalene.

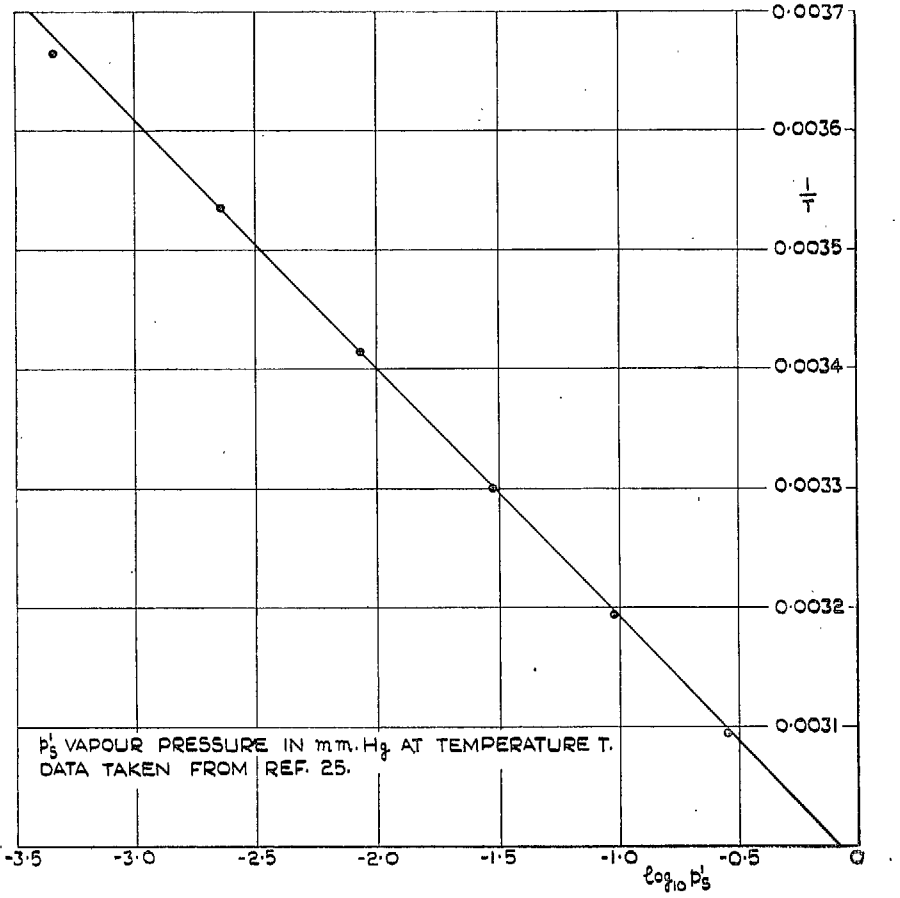


FIG. 12. Vapour pressures of Thymol.

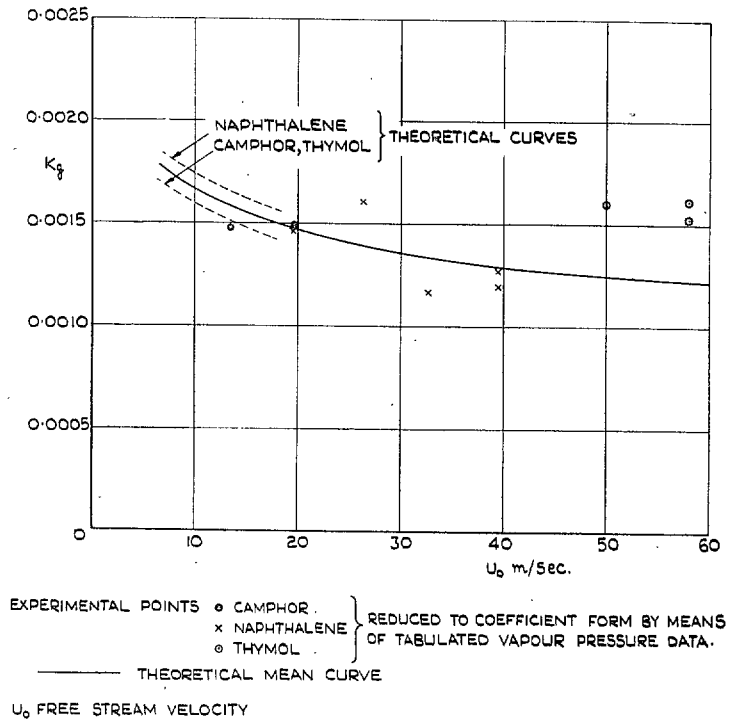


FIG. 13. Sublimation from part of a flat plate. Comparison with wind-tunnel measurements.

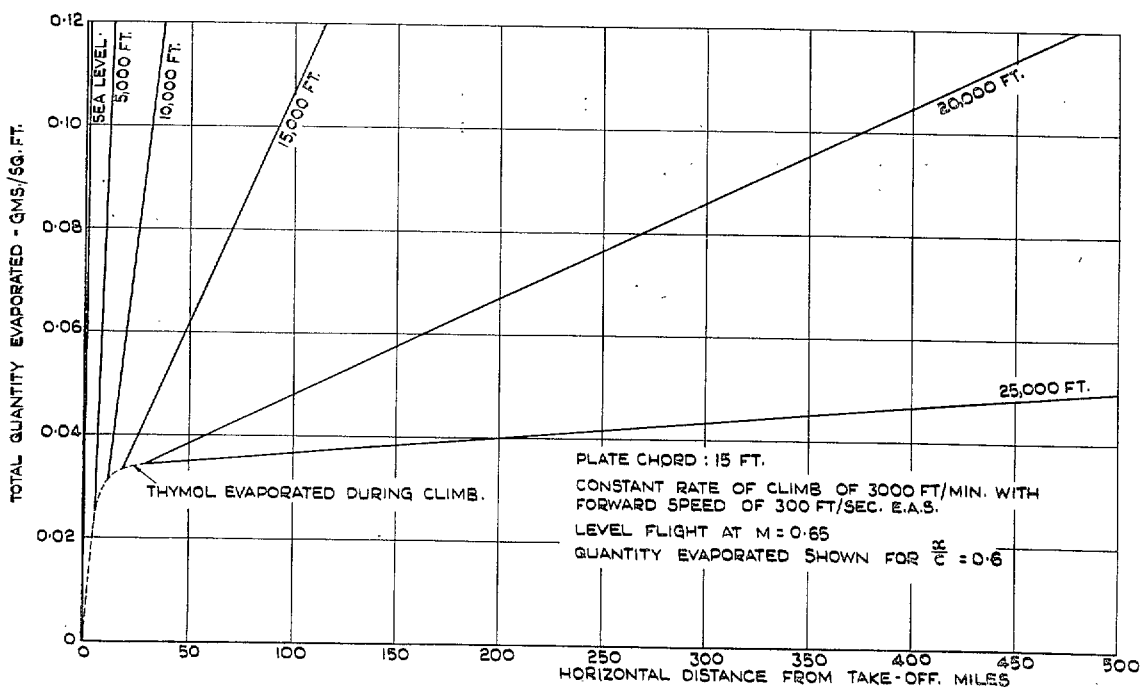


FIG. 14. Evaporation of Thymol from the surface of the faster aircraft. (*cf.* section 9.)

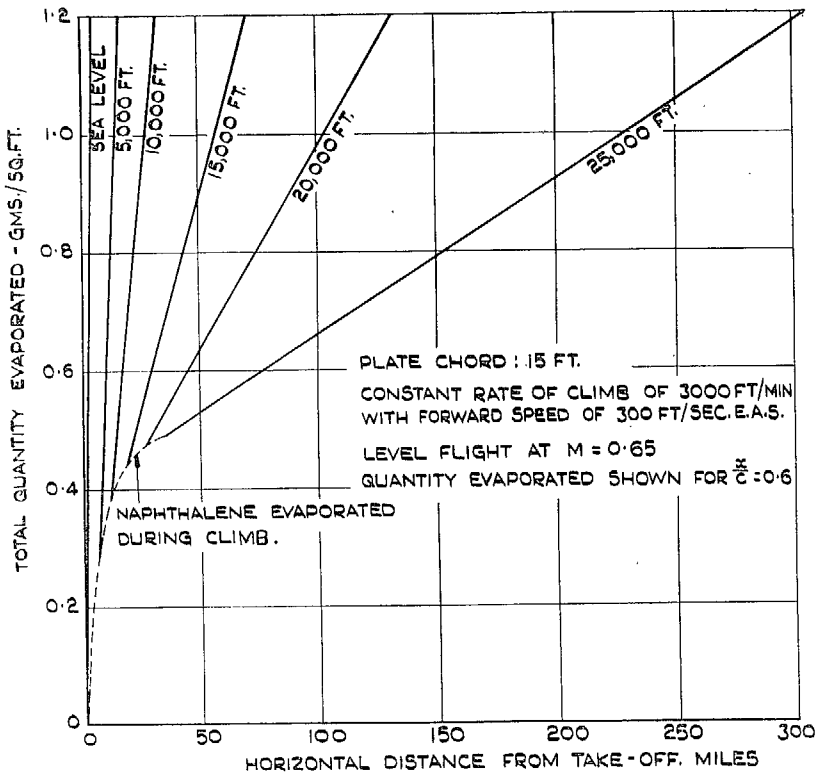


FIG. 15. Evaporation of Naphthalene from the surface of the faster aircraft. (*cf.* section 9.)

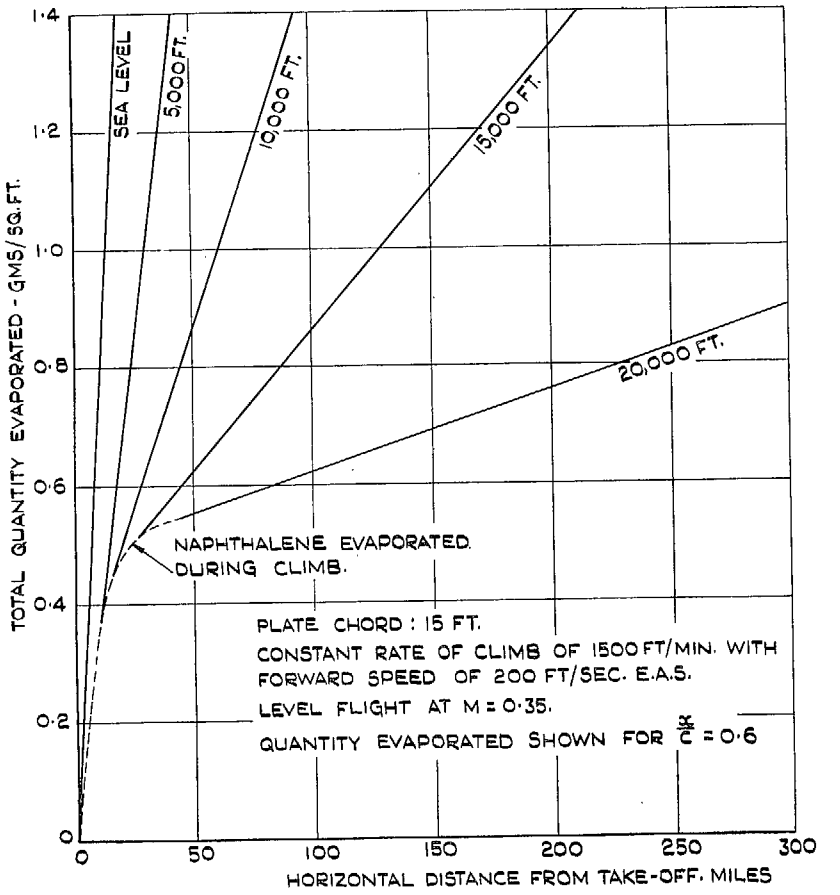
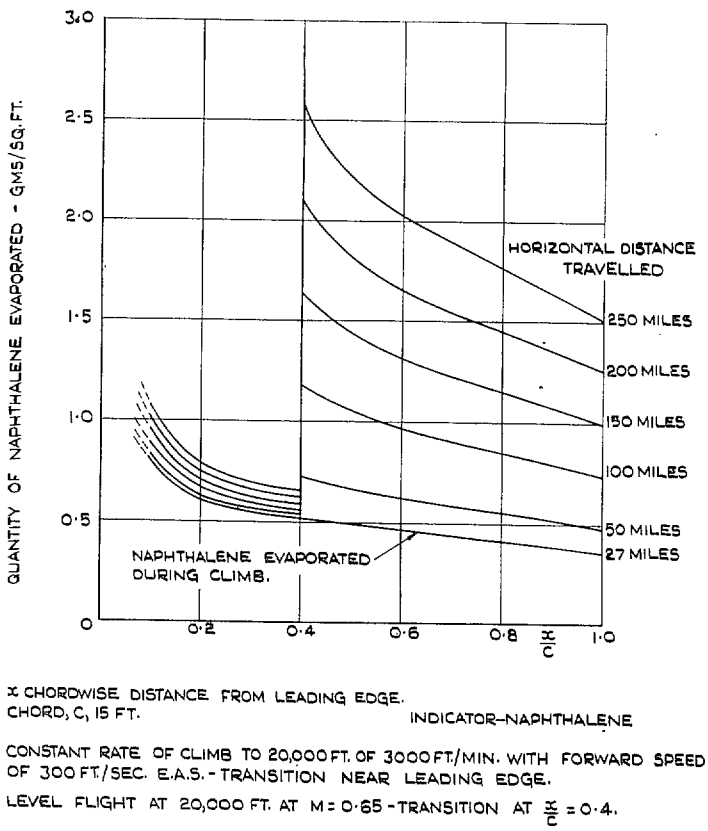


FIG. 16. Evaporation of Naphthalene from the surface of the slower aircraft. (*cf.* section 9.)



x CHORDWISE DISTANCE FROM LEADING EDGE.
CHORD, C, 15 FT.

INDICATOR–NAPHTHALENE

CONSTANT RATE OF CLIMB TO 20,000 FT. OF 3000 FT./MIN. WITH FORWARD SPEED OF 300 FT./SEC. E.A.S. - TRANSITION NEAR LEADING EDGE.

LEVEL FLIGHT AT 20,000 FT. AT M = 0·65 - TRANSITION AT $\frac{x}{c}$ = 0·4.

FIG. 17. Chordwise variation of the quantity of Naphthalene evaporated during a transition test at an altitude of 20,000 ft. (Faster aircraft.)

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