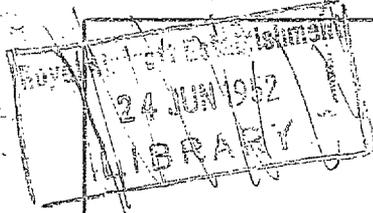


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Calculated Loadings due to  
Incidence of a Number of Straight  
and Swept-back Wings

By

V. M. FALKNER, B.Sc., A.M.I.MECH.E., with Appendix by  
DORIS LEHRMAN, B.Sc., of the Aerodynamics Division, N.P.L.

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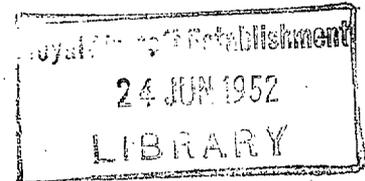
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# Calculated Loadings due to Incidence of a Number of Straight and Swept-back Wings

By

V. M. FALKNER, B.Sc., A.M.I.MECH.E.

with Appendix by DORIS LEHRMAN, B.Sc.  
of the Aerodynamics Division, N.P.L.



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*Summary.*—In this report are collected together the calculated aerodynamic loadings due to incidence of a number of straight and swept-back wings. The calculations follow in the main the routine described previously in another report<sup>1</sup>, but include additions concerned with induced camber and induced drag. An additional investigation is made of the effect of the N.A.C.A. camber on the properties at zero lift of a rectangular wing of aspect ratio 6.

A table is given of loading functions for use in auxiliary solutions when the wing plan has a discontinuity of direction at an arbitrary position along the span.

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1. *Introduction.*—In this report are collected together the calculated aerodynamic loadings due to incidence of a number of straight and swept-back wings. The calculations are based on the vortex-lattice method described in Ref. 1, but have additional interest in that the full effects of the discontinuity at the median section are now apparent, and calculations relating to induced camber and induced drag are included.

A further extension is that the auxiliary loading functions required to allow for the effect of the discontinuity in plan, when this occurs at the median section, have been established as dependent on the aspect ratio only.

2. *Range of Solutions.*—The wings described in this report include twelve wings of constant chord, of aspect ratios 6, 4, 2 and 1 and with angles of sweep-back 0, 30 and 45 deg, a triangular wing with 90 deg apex angle, aspect ratio 4; and two other wings derived from the latter by cropping the tips to reduce to aspect ratios 3 and 2·309 respectively. Results for other wings will be included in a later report.

The work includes calculation of the lift, local lift coefficient ( $C_{LL}/C_L$ ), loading coefficient ( $C_{LLC}/C_L\bar{c}$ ), local aerodynamic centre, induced drag and in some cases the induced camber.

Some wind-tunnel results which show the overall agreement between theoretical and experimental results are included.

3. *Loading Functions.*—All of the wings dealt with in this report have the discontinuity of leading edge at the wing centre, and it has been shown in Ref. 1 that solutions are obtained by first calculating the standard solution for control points at 0·2, 0·6 and 0·8 and modifying

this by a 'rounding off' solution involving the use of special circulation functions of which the induced downwash curve is polygonal. For cases with the discontinuity at the median section, the special functions for incidence solutions are built up from the two functions  $P_a$  and  $P_b$  which have double values and which, described previously in Ref. 1, are now given again in Table 1. Various combinations of the functions are given in Table 2, and in every case the 0.1 lattice values are to be used in the lattice for finding the expressions for the downwash at the control points, while the true values are to be used in the analysis subsequent to the establishment of the formula for the vortex sheet. Although such cases are not treated here, the presence of discontinuities along the span requires the use of further functions and their interpolates if necessary, and a range of these, again with double values, is given in Table 3. The distribution of induced downwash for each is symmetrical and of the roof-top type, with apices at different positions along the span, and the values are included in the Tables as well as being plotted for the same cases in Fig. 1.

The functions are derived by combining the Tables of Ref. 2 in certain proportions as follows:—

$$\begin{aligned}
 P_a &\equiv TF_{00} - 10P_{S00} + 9P_{S10} \\
 P_b &\equiv TF_{00} - 5P_{S00} + 4P_{S20} \\
 P_1 &\equiv 10P_{S00} - 18P_{S10} + 8P_{S20} \\
 P_2 &\equiv 9P_{S10} - 16P_{S20} + 7P_{S30} \\
 P_3 &\equiv 8P_{S20} - 14P_{S30} + 6P_{S40} \\
 P_4 &\equiv 7P_{S30} - 12P_{S40} + 5P_{S50} \\
 P_5 &\equiv 6P_{S40} - 10P_{S50} + 4P_{S60} \\
 P_6 &\equiv 5P_{S50} - 8P_{S60} + 3P_{S70} \\
 P_7 &\equiv 4P_{S60} - 6P_{S70} + 2P_{S80}
 \end{aligned}$$

For each case, the four constants  $T_{15}$ ,  $T_{16}$ ,  $T_{19}$  and  $T_{20}$  are given, where, for  $K/4sV = P$  (see Ref. 1),  $C_L = (8s^2/S) T_{15}$ ,  $T_{16} = (16/\pi) T_{15}$ , the position of the centre-of-pressure of the symmetrical half-wing loading in terms of the semispan is  $2T_{19}/T_{15}$ , and  $P(\eta \rightarrow 1) = T_{20}\sqrt{(1 - \eta^2)}$ .

3.1. By the use of auxiliary solutions in the region of the discontinuity, which involve control points placed at  $\eta = 0$  and 0.1 as well as at the standard positions, the proportions of the two functions  $P_a$  and  $P_b$  which must be used in the combined function have been estimated for several wings of varying taper, and aspect ratio. The results show that the proportions depend mainly on aspect ratio, and can be selected, without necessity for close accuracy, from the diagram of Fig. 2, which has been plotted from the following data.

A.R.6. Taper 1 to 1: 45 deg sweep.	0.68 $P_a$ + 0.32 $P_b$
A.R.4. Taper 1 to 1: 45 deg sweep.	0.66 $P_a$ + 0.34 $P_b$
A.R.1. Taper 1 to 1: 45 deg sweep.	0.51 $P_a$ + 0.49 $P_b$
Delta wing, equilateral triangle, A.R.2.31.	0.61 $P_a$ + 0.39 $P_b$
A.R.5.82. Taper 0.323: 45 deg sweep.	0.66 $P_a$ + 0.34 $P_b$

The function for all wings with the discontinuity at the median section can, therefore, be chosen by reference to this diagram, but it must be noted that they will be applicable only to incidence solutions, and that for wing twist different functions are required.

4. *Basis of solutions.*—The solutions are based generally on the following formula for the vortex sheet, as given in A.R.C. Report 10,895:—

$$\frac{kc}{8sV \sin \alpha} = F_0 \cot \frac{1}{2}\theta + F_1 \sin \theta + F_2 \sin 2\theta$$

where

$$\begin{aligned} F_0 &= \sqrt{(1 - \eta^2)} (a_0 + b_0\eta + c_0\eta^2 + \dots) + p_0P \\ F_1 &= \sqrt{(1 - \eta^2)} (a_1 + b_1\eta + c_1\eta^2 + \dots) + p_1P \\ F_2 &= \sqrt{(1 - \eta^2)} (a_2 + b_2\eta + c_2\eta^2 + \dots) + p_2P \end{aligned}$$

where  $P$  is the auxiliary function as described in the preceding paragraph. Unless  $p_0$ ,  $p_1$  and  $p_2$  are zero, the induced drag is not directly calculable, and a numerical integration will be required, based on the general expression for induced drag.

$$C_{DI} = \frac{8s^2}{S} \int_{-1}^1 \left(\frac{w}{V}\right) \left(\frac{K}{4sV}\right) d\eta$$

An example of this is given for solution 7 in Table 4. The downwash is made up of two parts, the first derived from the Fourier terms, given by

$$\frac{w}{V} = \frac{\sum n A_n \sin n\phi}{\sin \phi},$$

and the second from the polygonal distribution associated with the auxiliary functions. The values of  $\sin n\phi$  are tabulated in Ref. 3.

For

$$\frac{kc}{8sV \sin \alpha} = p_0P \cot \frac{1}{2}\theta + p_1P \sin \theta + p_2P \sin 2\theta,$$

$$\frac{K}{4sV \sin \alpha} = \pi (p_0 + \frac{1}{2}p_1) P,$$

and the downwash per radian is  $\pi (p_0 + \frac{1}{2}p_1)$  at  $\eta = 0$ , this being the factor to be applied to the standard values for  $w/V$  given in Tables 1, 2 and 3.

Instead of the circulation, values will be given of a loading coefficient, defined as  $C_{LLc}/C_L\bar{c}$ , the advantage being that the area under all curves is unity. For constant chord wings this coefficient is the same as  $C_{LL}/C_L$  and the general formula is

$$4\pi A (F_0 + \frac{1}{2}F_1)/(dc_L/d\alpha) \text{ where } A \text{ is aspect ratio.}$$

5. *Induced Camber.*—The induced camber is defined in the following way—if the downwash due to the chordwise distribution of vorticity be computed on the assumption that the flow is two-dimensional with a straight airstream, the mean camber line would require to be curved in order to satisfy the tangential flow condition. The displacement of this curved camber line from the straight line joining the leading and trailing edges is defined as the induced camber.

5.1. The induced camber is calculated as follows:—

The three-dimensional vorticity distribution on a wing is given by

$$\frac{kc}{8sV \sin \alpha} = F_0 \cot \frac{1}{2}\theta + F_1 \sin \theta + F_2 \sin 2\theta.$$

The two-dimensional equivalent is usually written

$$k = 2V (A_0 \cot \frac{1}{2}\theta + A_1 \sin \theta + A_2 \sin 2\theta)$$

and the two expressions are identical if

$$A_0 = \frac{4s}{c} F_0 \sin \alpha, \quad A_1 = \frac{4s}{c} F_1 \sin \alpha, \text{ etc.}$$

Now the downwash, positive downwards, is given by

$$\frac{w}{V} = A_0 - A_1 \cos \theta - A_2 \cos 2\theta.$$

The slope of the camber line is  $-w/V$ , and the ordinates ( $z$ ) are obtained by integrating  $-w/V$  from the leading edge to the point concerned. Hence,

$$\frac{z}{c} = \frac{1}{c} \int_{c/2}^x (A_0 - A_1 \cos \theta - A_2 \cos 2\theta) dx$$

or since  $x = \frac{1}{2}c \cos \theta$ ,

$$\frac{z}{c} = -\frac{1}{2} \int_0^\theta (A_0 - A_1 \cos \theta - A_2 \cos 2\theta) \sin \theta d\theta.$$

Therefore,

$$\frac{z}{c} = \frac{1}{2} \left[ A_0 (\cos \theta - 1) - \frac{1}{4}A_1 (\cos 2\theta - 1) - \frac{1}{8}A_2 (\cos 3\theta - 1) + \frac{1}{2}A_2 (\cos \theta - 1) \right].$$

Substituting  $\cos \theta = \xi$ ,

$$\frac{z}{c} = \frac{1}{2}A_0 (\xi - 1) - \frac{1}{4}A_1 (\xi^2 - 1) - \frac{1}{8}A_2 (2\xi^3 - 3\xi + 1).$$

At the leading edge

$$\xi = 1 \text{ and } z/c = 0,$$

while at the trailing edge,

$$\xi = -1 \text{ and } z/c = -A_0 - \frac{1}{3}A_2.$$

Hence, the induced camber is given by

$$\begin{aligned} \frac{\text{camber}}{\text{chord}} &= \frac{1}{2}A_0 (\xi - 1) - \frac{1}{4}A_1 (\xi^2 - 1) - \frac{1}{8}A_2 (2\xi^3 - 3\xi + 1) \\ &\quad + (A_0 + \frac{1}{3}A_2) \frac{1 - \xi}{2} \end{aligned}$$

where  $(1 - \xi)/2$  is the distance from the leading edge in terms of the chord.

Finally,

$$\frac{\text{camber}}{\text{chord}} = \frac{1}{4}A_1 (1 - \xi^2) + \frac{1}{3}A_2 (\xi - \xi^3) = \frac{s}{c} \sin \alpha (1 - \xi^2) (F_1 + \frac{4}{3}F_2\xi)$$

5.2. From the nature of the calculations by which the coefficients  $a_0$ ,  $a_1$ , etc., are computed, the results for induced camber will not have the same precision as the main properties of the wing. The occurrence of ill-conditioned equations might affect adversely the accuracy of  $F_1$  and  $F_2$ , and, when obtained from comparatively few control points, it should be noted that the values of induced camber may sometimes only represent the order of the effect, rather than exact values. Accuracy can, of course, be improved by the use of more control points and normalisation.

6. *Results.*—The results are given in Tables 4 to 40, and, as the calculations follow standard methods, a minimum of explanation will be needed. The induced cambers have all been calculated for  $C_L = 1$ .

*Rectangular wing: aspect ratio 6: 0 deg sweep-back: Tables 5 to 9.*—In this set are included standard solutions with 6 and 9 points using the 84-vortex lattice; a 6-point solution by Blenk<sup>4</sup> revised by Argyris; a lifting-line solution; and a recalculation of  $\alpha_0$  and  $C_{m_0}$  for the N.A.C.A. camber.

The lifting-plane solutions by the vortex-lattice method are in good general agreement. The solution given by Blenk required revision as it was based on surface integrals which suffered from inaccuracy towards the tips of the wing. The errors have been corrected as the result of an unpublished investigation by Argyris, and the solution now given, which can be taken as based on exact values of downwash integrals at the control points, agrees quite well with the remainder of the calculations, any slight discrepancies being accounted for by the variation in the disposition of the control points. The lifting-line solution has been included in order to show the error involved in this approximation. Solution 5 (*see R. & M. 1910<sup>b</sup>*) was calculated originally by using the actual slopes of the surface at the control points, and is of importance in that it demonstrates the little-known fact that  $\alpha_0$  and  $C_{m_0}$  are variable with aspect ratio for a uniform cambered wing. The revised calculations, which give  $\alpha_0$  and  $C_{m_0}$  for aspect ratios 6 and infinity with the position of maximum camber varying from 0.2 to 0.7, are based on equivalent slopes, the origin of which is described in an Appendix to this report.

The complete 9-point equations were used for this work, and, in order to show the effect of varying the number of control points, the case  $P = 0.2$  has also been calculated for a 6-point solution with two terms chordwise. This gave  $\alpha_0 = -1.597$  and  $C_{m_0} = -1.744$  instead of  $-1.633$  and  $-1.748$  respectively, showing that the important quantity  $C_{m_0}$  can be calculated quite accurately by the use of six points.

6.1. *Constant-chord wing: aspect ratio 6: 30 deg sweep-back: Tables 10 and 11.*—The two solutions, standard 6-point and standard 6-point modified, are based on the 126-vortex pattern. The auxiliary loading function used was  $0.65P_a + 0.35P_b$ , and the main effect of the modification to 'round off' the discontinuity is to reduce the loading and shift back the local aerodynamic centre at the wing centre, increase the induced drag, and alter appreciably the induced camber. The integration for evaluating the induced drag is given as a typical example in Table 4.

6.2. *Constant-chord wing: aspect ratio 6: 45 deg sweep-back: Tables 12 to 18.*—For this wing, a comprehensive range of solutions has been calculated. These start with solutions 8, 9 and 10, standard and modified solutions with the 21-vortex pattern, and continue with solution 11, a 126-vortex standard solution, solutions 12 and 14, two auxiliary solutions, and finally solution 13, a full 8-point solution. The first point to be noted is that solution 10, in which  $P_a$  and  $P_b$  were both used, gives proportions of  $P_a$  and  $P_b$  which agree with the ratio 0.65 to 0.35 used in the more elaborate solutions, and confirms that the use of the  $\frac{1}{4}/\frac{3}{4}$  approximation is exceedingly valuable in making a preliminary survey of any problem. It must be pointed out, however, that it is of limited use as it would not be possible to establish functions of which the main effect is to shift backwards the local aerodynamic centre. The effect of the rounding-off, as shown by a comparison of solutions 11 and 13, is to decrease the load coefficient at the median section

by about 14 per cent, to shift back the local aerodynamic centre at the median section by 0.083 chord and the overall aerodynamic centre by 0.04 $\bar{c}$ , and to increase the induced drag factor from 1.107 to 1.157. The two simplified auxiliary solutions give an excellent approximation to the full 8-point solution. It has hitherto been accepted as good evidence of accuracy if the loading coefficient is in agreement for  $\frac{1}{4}/\frac{3}{4}$  and the more complicated solutions. This criterion becomes less acceptable when the discontinuity is acute, as in the present case, and it is thought that differences are not necessarily due to inaccuracy, but may indicate that the  $\frac{1}{4}/\frac{3}{4}$  solution is inadequate to represent the flow in the median region. The differences in  $C_{LL}/C_L$  for  $\eta = 0$  between solutions 10 and 13, which are not serious and amount to a maximum of 4 per cent at  $\eta = 0$ , but do not exceed 2 per cent between  $\eta = 0.2$  and 0.8, do not, therefore, disprove the accuracy of the calculations.

6.3. *Rectangular wing: aspect ratio 4: 0, 30, 45 deg sweep-back. Tables 19 to 24.*

*Rectangular wing: aspect ratio 2: 0, 30, 45 deg sweep-back. Tables 25 to 29.*

*Rectangular wing: aspect ratio 1: 0, 30, 45 deg sweep-back. Tables 30 to 34.*

These wings have been calculated by the standard method, the 84-vortex pattern being used for the straight wings, and the 126 for those with sweep-back. The auxiliary loading functions used were as follows:—

$$\text{A.R.4. } 0.65P_a + 0.35P_b$$

$$\text{A.R.2. } 0.60P_a + 0.40P_b$$

$$\text{A.R.1. } 0.50P_a + 0.50P_b$$

6.4. *Triangular wing, 90 deg apex angle, aspect ratio 4 and two other wings obtained by cropping the tips to aspect ratios 3 and 2.309.*—These wings have been calculated by the standard 126-vortex lattice, and the results are given in Tables 35 to 40.

The auxiliary function used for solutions 32 and 36 was  $0.65P_a + 0.35P_b$ .

6.5. A correction on  $dc_L/d\alpha$  obtained by the 126-vortex lattice is sometimes required in order to bring it to the full potential value. It has not been possible to establish the factor for all cases, but figures derived from a delta and other wings suggest that the factor can be taken tentatively as independent of aspect ratio, and to vary as  $1 + 0.029$  (tangent of sweep-back of quarter-chord). All results given in the tables are uncorrected.

7. *Discussion of Results.*—A remarkable change in the solutions will be noted as the aspect ratio decreases. The loss in lift due to the centre-line correction is found to decrease with decreasing aspect ratio, while at the same time the induced drag decreases to the minimum of  $(1/\pi A) c_L^2$  and the spanwise distribution of circulation approaches the elliptic form. The change is shown graphically in Fig. 3, in which are plotted the loading coefficient and local aerodynamic centre for sweep-back 0, 30 and 45 deg for wings of A.R.6 and A.R.1. The effect is independent of wing plan, as the triangular wings show precisely the same tendency. It does not necessarily follow that there is a possible saving in effort by assuming the elliptic distribution, as trial calculations with a 4-point solution for aspect ratio 1 show that the lift coefficient will be fairly accurately assessed, but the locus of local aerodynamic centres may be considerably in error unless the full number of control points are used.

8. *Comparison with Experiments.*—Some experimental results at a high Reynolds number for the three delta wings of section 6.2 taken from a report<sup>8</sup> issued by the Royal Aircraft Establishment, are given in Table 41 with the theoretical values. The values of  $dc_L/d\alpha$  for the latter have

been multiplied by the correction factor 1.022, and it then appears that the wind-tunnel values for  $dc_L/d\alpha$  are about 0.95 of the theoretical, and the measured aerodynamic centre does not vary by more than 0.017 mean chords from the theoretical. In view of the fact that the section  $t/c$  is about 10 per cent, these results must be regarded as very reasonable, although they are not intended to provide conclusive evidence as to the accuracy of the calculations.

9. *Acknowledgements.*—The writer desires to acknowledge the valuable assistance given by his colleague Mr. H. L. Nixon in some of the earlier work, by Miss D. E. Lehrian in the organisation of the latter stages of the work and by Misses S. D. Brown, W. M. Tafe and B. M. Skelton who took considerable responsibility in carrying out successfully the laborious task of computing the solutions.

### REFERENCES

<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	V. M. Falkner .. .. .	The Solution of Lifting Plane Problems by Vortex Lattice Theory. Report A.R.C. 10,895. (To be published).
2	V. M. Falkner .. .. .	Appendix by E. J. Watson. Tables of Multhopp and Other Functions for Use in Lifting Line and Lifting Plane Theory. Report A.R.C. 11,234. (To be published).
3	V. M. Falkner .. .. .	The Solution by Lifting Line Theory of Problems Involving Discontinuities. Report A.R.C. 10,922. (To be published).
4	H. Blenk .. .. .	Der Eindecker als tragende Wirbelfläche. <i>Z.A.M.M.</i> Vol. 5. 1925.
5	V. M. Falkner .. .. .	The Calculation of Aerodynamic Loading on Surfaces of Any Shape. R. & M. 1910. August, 1943.
6	R. Hills, R. C. Lock, and J. G. Ross	Interim Note on Wind-tunnel Tests of a Model Delta Wing. Report A.R.C. 10,535. (To be published).
7	E. N. Jacobs, K. M. Ward, and R. M. Pinkerton.	The Characteristics of the 78 Related Aerofoil Sections from Tests in the V.D. Tunnel. N.A.C.A. Report 460.
8	V. M. Falkner .. .. .	The Use of Equivalent Slopes in Vortex Lattice Theory. R. & M. 2293. March, 1946.

### APPENDIX

*Calculation of Equivalent Slopes to represent the N.A.C.A. Camber-line*

*By*

DORIS LEHRIAN, B.Sc.

If  $X$  be the distance from the leading edge in terms of the chord, and  $Y$  the distance in the direction normal to the line joining leading and trailing edges, in terms of the chord, the equation of the camber line is

$$Y = \frac{M}{P^2} (2PX - X^2) \text{ for } 0 \leq X \leq P.$$

and 
$$Y = \frac{M}{(1-P)^2} (1 - 2P + 2PX - X^2) \text{ for } P \leq X \leq 1$$

where  $M =$  maximum value of  $Y$ , at  $X = P$ .

For the three-term formula  $k/2V = A_0 \cot \frac{1}{2}\theta + A_1 \sin \theta + A_2 \sin 2\theta$  the equivalent slope is  $A_0 - A_1 \cos \theta - A_2 \cos 2\theta$ . The chordwise analysis for wing camber gives the solution

$$A_0 = -I_0 + \frac{3}{8}I_1 + \frac{1}{8}I_2$$

$$A_1 = \frac{1}{4}(I_1 - I_2)$$

$$A_2 = \frac{3}{4}(I_2 - I_1)$$

where 
$$I_0 = \frac{1}{\pi} \int_0^\pi \frac{dY}{dX} \cdot d\theta, I_n = \frac{2}{\pi} \int_0^\pi \frac{dY}{dX} \cdot \cos n\theta \cdot d\theta$$

and 
$$X = \frac{1}{2}(1 - \cos \theta).$$

Applying this analysis to the N.A.C.A. camber line and writing  $\theta = \phi$  at  $X = P, Y = M$ , the integrals evaluate to

$$I_0 = \frac{16M \cos \phi (\sin \phi - \phi \cos \phi)}{\pi (1 - \cos^2 \phi)^2} - \frac{4M \cos \phi}{(1 + \cos \phi)^2}$$

$$I_1 = \frac{16M \cos \phi (\phi - \sin \phi \cos \phi)}{\pi (1 - \cos^2 \phi)^2} + \frac{4M}{(1 + \cos \phi)^2}$$

$$I_2 = \frac{32M \cos \phi}{3\pi \sin \phi}.$$

The values of  $A_0, A_1, A_2$  are obtained immediately and the values of the equivalent slopes at the positions  $S_1$  to  $S_9$  are calculated for values of  $P$  equal to 0.2 (0.1) 0.7. These values are set out in Table A and are given in terms of the maximum value  $M$ .

For the two term formula  $k/2V = A_0 \cot \frac{1}{2}\theta + A_1 \sin \theta$ , the equivalent slope is  $A_0 - A_1 \cos \theta$ . In this case the analysis for wing camber gives the solution

$$A_0 = -I_0 + \frac{1}{2}I_2$$

$$A_1 = I_1 - I_2$$

where the integrals  $I_0, I_1, I_2$  have the same values as above. The equivalent slopes are calculated for the positions  $S_1$  to  $S_9$  for values of  $P$  equal to 0.2(0.1)0.7 and these values are set out in Table B.

TABLE A

*Equivalent slopes to represent N.A.C.A. camber-line:—3-term formula*

Slope	Position on chord-X	$P = 0.2$	$P = 0.3$	$P = 0.4$	$P = 0.5$	$P = 0.6$	$P = 0.7$
$S_1$	0	2.4522M	2.7415M	3.0809M	3.5000M	4.0500M	4.8349M
$S_2$	0.16	0.6874	0.6058	0.5446	0.5000	0.4741	0.4765
$S_3$	0.25	0.0991	- 0.1060	- 0.3008	- 0.5000	- 0.7179	- 0.9763
$S_4$	0.33	- 0.2931	- 0.5806	- 0.8644	- 1.1667	- 1.5125	- 1.9448
$S_5$	0.50	- 0.4892	- 0.8179	- 1.1462	- 1.5000	- 1.9098	- 2.4291
$S_6$	0.66	0.0991	- 0.1060	- 0.3008	- 0.5000	- 0.7179	- 0.9763
$S_7$	0.75	0.6874	0.6058	0.5446	0.5000	0.4741	0.4765
$S_8$	0.83	1.4717	1.5550	1.6718	1.8333	2.0634	2.4136
$S_9$	1.00	3.6287M	4.1653M	4.7717M	5.5000M	6.4339M	7.7405M

TABLE B

*Equivalent slopes to represent N.A.C.A. camber line:—2-term formula*

Slope	Position on chord-X	$P = 0.2$	$P = 0.3$	$P = 0.4$	$P = 0.5$	$P = 0.6$	$P = 0.7$
$S_1$	0	- 1.9598M	- 2.5977M	- 3.2598M	- 4.0000M	- 4.8897M	- 6.0611M
$S_2$	0.16	- 1.1755	- 1.6485	- 2.1326	- 2.6667	- 3.3004	- 4.1240
$S_3$	0.25	- 0.7833	- 1.1739	- 1.5690	- 2.0000	- 2.5058	- 3.1555
$S_4$	0.33	- 0.3911	- 0.6993	- 1.0053	- 1.3333	- 1.7112	- 2.1870
$S_5$	0.50	0.3932	0.2499	0.1219	0	- 0.1219	- 0.2499
$S_6$	0.66	1.1776	1.1991	1.2491	1.3333	1.4674	1.6872
$S_7$	0.75	1.5698	1.6737	1.8127	2.0000	2.2620	2.6557
$S_8$	0.83	1.9619	2.1483	2.3764	2.6667	3.0567	3.6242
$S_9$	1.00	2.7436M	3.0975M	3.5036M	4.0000M	4.6459M	5.5613M

TABLE 1

*True and special values of P functions*

$\eta$	$P_a$			$P_b$		
	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$
0	0.14308	0.1578	1.0	0.24197	0.2518	1.00
0.05	0.12579		0.5	0.23047		0.75
0.10	0.09888	0.0940	0	0.20726	0.2092	0.50
0.15	0.08356		0	0.17952		0.25
0.20	0.07366	0.0726	0	0.15307	0.1493	0
0.25	0.06610		0	0.13523		0
0.30	0.05994	0.0594	0	0.12179	0.1206	0
0.35	0.05467		0	0.11067		0
0.40	0.05003	0.0497	0	0.10104	0.1004	0
0.45	0.04584		0	0.09244		0
0.50	0.04201	0.0418	0	0.08461	0.0841	0
0.55	0.03842		0	0.07731		0
0.60	0.03504	0.0349	0	0.07043	0.0701	0
0.65	0.03176		0	0.06380		0
0.70	0.02855	0.0284	0	0.05733	0.0571	0
0.75	0.02535		0	0.05089		0
0.80	0.02209	0.0220	0	0.04433	0.0442	0
0.85	0.01866		0	0.03744		0
0.90	0.01490	0.0149	0	0.02986	0.0299	0
0.95	0.01028		0	0.02064		0
0.9625	0.00887	0.0089	0	0.01778	0.0178	0
1.00	0		0	0		0
$T_{15}$	0.09992			0.19933		
$T_{16}$	0.50888			1.01517		
$T_{19}$	0.01599			0.03233		
$T_{20}$	0.03186			0.06388		

TABLE 2

Combinations of  $P$  functions

11

$\eta$	$P = 0.65P_a + 0.35P_b$			$P = 0.60P_a + 0.40P_b$			$P = 0.50P_a + 0.50P_b$		
	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$
0	0.17769	0.1907	1.0000	0.18264	0.1954	1.00	0.19252	0.2048	1.000
0.05	0.16243		0.5875	0.16766		0.60	0.17813		0.625
0.10	0.13681	0.1343	0.1750	0.14223	0.1401	0.20	0.15307	0.1516	0.250
0.15	0.11715		0.0875	0.12194		0.10	0.13154		0.125
0.20	0.10145	0.0994	0	0.10542	0.1033	0	0.11336	0.1109	0
0.25	0.09030		0	0.09375		0	0.10066		0
0.30	0.08159	0.0808	0	0.08468	0.0839	0	0.09086	0.0900	0
0.35	0.07427		0	0.07707		0	0.08267		0
0.40	0.06788	0.0674	0	0.07043	0.0700	0	0.07554	0.0750	0
0.45	0.06215		0	0.06448		0	0.06914		0
0.50	0.05692	0.0566	0	0.05905	0.0587	0	0.06331	0.0630	0
0.55	0.05203		0	0.05398		0	0.05786		0
0.60	0.04743	0.0472	0	0.04920	0.0490	0	0.05274	0.0525	0
0.65	0.04297		0	0.04458		0	0.04778		0
0.70	0.03862	0.0385	0	0.04006	0.0399	0	0.04294	0.0428	0
0.75	0.03429		0	0.03557		0	0.03812		0
0.80	0.02987	0.0298	0	0.03099	0.0309	0	0.03321	0.0331	0
0.85	0.02523		0	0.02617		0	0.02805		0
0.90	0.02014	0.0201	0	0.02088	0.0209	0	0.02238	0.0224	0
0.95	0.01391		0	0.01442		0	0.01546		0
0.9625	0.01199	0.0120	0	0.01243	0.0124	0	0.01332	0.0133	0
1.00	0		0	0		0	0		0
$T_{15}$	0.13471			0.13968			0.14962		
$T_{16}$	0.68608			0.71140			0.76202		
$T_{19}$	0.02171			0.02253			0.02416		
$T_{20}$	0.04307			0.04467			0.04787		

TABLE 3

*True and special values of P functions*

$\eta$	$P_1$			$P_2$			$P_3$		
	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$
0	0.19778	0.1881	0	0.14730	0.1451	0	0.11987	0.1188	0
0.05	0.20936		0.5	0.14966		0	0.12078		0
0.10	0.21675	0.2303	1.0	0.15883	0.1535	0	0.12372	0.1223	0
0.15	0.19191		0.5	0.18047		0.5	0.12949		0
0.20	0.15884	0.1535	0	0.19316	0.2075	1.0	0.14104	0.1360	0
0.25	0.13826		0	0.17172		0.5	0.16446		0.5
0.30	0.12371	0.1223	0	0.14103	0.1360	0	0.17847	0.1930	1.0
0.35	0.11200		0	0.12218		0	0.15806		0.5
0.40	0.10201	0.1013	0	0.10897	0.1077	0	0.12815	0.1232	0
0.45	0.09320		0	0.09824		0	0.10993		0
0.50	0.08519	0.0847	0	0.08900	0.0884	0	0.09717	0.0960	0
0.55	0.07778		0	0.08075		0	0.08678		0
0.60	0.07079	0.0705	0	0.07314	0.0727	0	0.07776	0.0772	0
0.65	0.06409		0	0.06598		0	0.06957		0
0.70	0.05757	0.0573	0	0.05906	0.0588	0	0.06191	0.0616	0
0.75	0.05109		0	0.05228		0	0.05452		0
0.80	0.04448	0.0444	0	0.04542	0.0453	0	0.04720	0.0470	0
0.85	0.03757		0	0.03829		0	0.03964		0
0.90	0.02992	0.0300	0	0.03048	0.0305	0	0.03147	0.0315	0
0.95	0.02071		0	0.02104		0	0.02167		0
0.9625	0.01783	0.0178	0	0.01811	0.0181	0	0.01865	0.0187	0
1.00	0		0	0		0	0		0
$T_{15}$	0.19882			0.19578			0.19059		
$T_{16}$	1.01260			0.99712			0.97068		
$T_{19}$	0.03268			0.03412			0.03572		
$T_{20}$	0.06405			0.06502			0.06683		

12

TABLE 3—continued

True and special values of  $P$  functions

13

$\eta$	$P_4$			$P_5$			$P_6$		
	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$	True	0.1 Lattice	$w/V$
0	0.10004	0.0994	0	0.08401	0.0836	0	0.07002	0.0698	0
0.05	0.10053		0	0.08428		0	0.07021		0
0.10	0.10200	0.1013	0	0.08516	0.0847	0	0.07076	0.0705	0
0.15	0.10468		0	0.08669		0	0.07171		0
0.20	0.10896	0.1077	0	0.08900	0.0884	0	0.07312	0.0728	0
0.25	0.11576		0	0.09235		0	0.07506		0
0.30	0.12814	0.1232	0	0.09715	0.0960	0	0.07772	0.0772	0
0.35	0.15220		0.5	0.10437		0	0.08130		0
0.40	0.16672	0.1813	1.0	0.11704	0.1122	0	0.08628	0.0852	0
0.45	0.14669		0.5	0.14132		0.5	0.09362		0
0.50	0.11705	0.1122	0	0.15598	0.1707	1.0	0.10632	0.1015	0
0.55	0.09900		0	0.13600		0.5	0.13056		0.5
0.60	0.08631	0.0852	0	0.10633	0.1015	0	0.14511	0.1599	1.0
0.65	0.07588		0	0.08815		0	0.12491		0.5
0.70	0.06668	0.0662	0	0.07522	0.0742	0	0.09491	0.0902	0
0.75	0.05819		0	0.06439		0	0.07623		0
0.80	0.05000	0.0498	0	0.05458	0.0542	0	0.06256	0.0617	0
0.85	0.04178		0	0.04512		0	0.05066		0
0.90	0.03298	0.0330	0	0.03537	0.0353	0	0.03911	0.0391	0
0.95	0.02263		0	0.02410		0	0.02635		0
0.9625	0.01945	0.0195	0	0.02069	0.0207	0	0.02257	0.0226	0
1.00	0		0	0		0	0		0
$T_{15}$	0.18309			0.17294			0.15967		
$T_{16}$	0.93248			0.88080			0.81320		
$T_{19}$	0.03711			0.03797			0.03794		
$T_{20}$	0.06957			0.07368			0.07985		

TABLE 3—continued

*True and special values of P functions*

$\eta$	$P_7$		
	True	0.1 Lattice	$w/V$
0	0.05701	0.0570	0
0.05	0.05713		0
0.10	0.05749	0.0575	0
0.15	0.05810		0
0.20	0.05900	0.0590	0
0.25	0.06022		0
0.30	0.06182	0.0617	0
0.35	0.06392		0
0.40	0.06663	0.0664	0
0.45	0.07022		0
0.50	0.07515	0.0744	0
0.55	0.08236		0
0.60	0.09486	0.0904	0
0.65	0.11879		0.5
0.70	0.13290	0.1481	1.0
0.75	0.11210		0.5
0.80	0.08127	0.0771	0
0.85	0.06134		0
0.90	0.04571	0.0457	0
0.95	0.03010		0
0.9625	0.02566	0.0257	0
1.00	0		0
$T_{15}$	0.14237		
$T_{16}$	0.72506		
$T_{19}$	0.03653		
$T_{20}$	0.08974		

TABLE 4

*Calculation of induced drag for Solution 7*

$$\left(\frac{w}{V}\right)_{\text{Fourier}} = \frac{\sum n A_n \sin n\phi}{\sin \phi}$$

where  $A_1 = \pi (a_0 + 0.5a_1 + 0.25c_0 + 0.125c_1 + 0.125e_0 + 0.0625e_1)$

$A_3 = \pi (0.25c_0 + 0.125c_1 + 0.1875e_0 + 0.09375e_1)$

$A_5 = \pi (0.0625e_0 + 0.03125e_1)$

$w/V$  for  $P$  function =  $\pi (-0.05183) (w/V)_P$  where  $(w/V)_P$  is the induced downwash due to  $K/4sV = P$ .

$\eta$	$K/4sV$	$w/V$ Fourier	$w/V$ $P$ function	Total $w/V$	$(K/4sV)$ $\times$ $(w/V)$	Factors
0	0.1623	0.1354	- 0.1628	- 0.0274	- 0.0044	1
0.05	0.1648	0.1362	- 0.0957	0.0405	0.0067	4
0.10	0.1690	0.1386	- 0.0285	0.1101	0.0186	2
0.15	0.1724	0.1425	- 0.0142	0.1283	0.0221	4
0.20	0.1751	0.1480	0	0.1480	0.0259	2
0.25	0.1771	0.1552	0	0.1552	0.0275	4
0.30	0.1785	0.1640	0	0.1640	0.0293	2
0.35	0.1796	0.1745	0	0.1745	0.0313	4
0.40	0.1804	0.1867	0	0.1867	0.0337	2
0.45	0.1807	0.2007	0	0.2007	0.0363	4
0.50	0.1805	0.2166	0	0.2166	0.0391	2
0.55	0.1795	0.2344	0	0.2344	0.0421	4
0.60	0.1776	0.2543	0	0.2543	0.0452	2
0.65	0.1746	0.2762	0	0.2762	0.0482	4
0.70	0.1701	0.3003	0	0.3003	0.0511	2
0.75	0.1635	0.3266	0	0.3266	0.0534	4
0.80	0.1541	0.3554	0	0.3554	0.0548	2
0.85	0.1408	0.3867	0	0.3867	0.0544	4
0.90	0.1213	0.4205	0	0.4205	0.0510	1.800
0.95	0.0906	0.4572	0	0.4572	0.0414	4.525
1.00	0	0.4967	0	0.4967	0	0.675

$$C_{DI} = \frac{16s^2}{S} \frac{\text{Integral}}{60} = 1.082 \frac{1}{\pi A} C_L^2.$$

TABLE 5

*Solution 1*

*Rectangular wing, aspect ratio 6, 84 vortex 9-point solution*

$a_0$	0.06811	$c_0$	0.01638	$e_0$	0.02716
$a_1$	- 0.00197	$c_1$	- 0.00155	$e_1$	- 0.03698
$a_2$	- 0.00057	$c_2$	0.00444	$e_2$	- 0.01369

$$dc_L/d\alpha = 4.270 \quad C_{DI} = 1.018 \frac{1}{\pi A} C_L^2$$

Aerodynamic centre 0.23997 behind leading edge.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.185	0.247	0.55	1.071	0.241
0.05	1.184	0.247	0.60	1.044	0.239
0.10	1.182	0.247	0.65	1.010	0.237
0.15	1.178	0.247	0.70	0.969	0.234
0.20	1.172	0.247	0.75	0.919	0.231
0.25	1.165	0.247	0.80	0.855	0.227
0.30	1.156	0.246	0.85	0.772	0.223
0.35	1.144	0.246	0.90	0.658	0.218
0.40	1.130	0.245	0.95	0.487	0.214
0.45	1.114	0.244	1.00	0	0.208
0.50	1.094	0.242			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	- 0.001	- 0.003	- 0.004	- 0.003
0.2	- 0.001	- 0.006	- 0.006	- 0.006
0.3	- 0.001	- 0.007	- 0.008	- 0.007
0.4	- 0.001	- 0.008	- 0.008	- 0.008
0.5	- 0.001	- 0.008	- 0.008	- 0.007
0.6	- 0.001	- 0.007	- 0.008	- 0.007
0.7	- 0.001	- 0.006	- 0.006	- 0.005
0.8	- 0.001	- 0.004	- 0.004	- 0.004
0.9	0	- 0.002	- 0.002	- 0.002
1.0	0	0	0	0

Local a.c. = Local aerodynamic centre.

TABLE 6

*Solution 2*

*Rectangular wing, aspect ratio 6, 84 vortex 6-point solution*

$$\begin{array}{llll}
 a_0 & 0.06759 & c_0 & 0.01933 & e_0 & 0.01738 \\
 a_1 & -0.00140 & c_1 & -0.00654 & e_1 & -0.02168
 \end{array}$$

$$dc_L/d\alpha = 4.247 \quad C_{DI} = 1.016 \frac{1}{\pi A} C_L^2$$

Aerodynamic centre  $0.2400\bar{7}$  behind leading edge.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.188	0.247	0.55	1.073	0.241
0.05	1.187	0.247	0.60	1.044	0.239
0.10	1.184	0.247	0.65	1.010	0.236
0.15	1.180	0.247	0.70	0.968	0.234
0.20	1.175	0.247	0.75	0.916	0.231
0.25	1.168	0.246	0.80	0.851	0.227
0.30	1.158	0.246	0.85	0.766	0.223
0.35	1.147	0.245	0.90	0.652	0.219
0.40	1.133	0.245	0.95	0.481	0.214
0.45	1.116	0.244	1.00	0	0.209
0.50	1.096	0.242			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	0	-0.002	-0.002	-0.002
0.2	-0.001	-0.004	-0.004	-0.004
0.3	-0.001	-0.005	-0.005	-0.005
0.4	-0.001	-0.006	-0.006	-0.005
0.5	-0.001	-0.006	-0.006	-0.006
0.6	-0.001	-0.006	-0.006	-0.005
0.7	-0.001	-0.005	-0.005	-0.005
0.8	-0.001	-0.004	-0.004	-0.004
0.9	0	-0.002	-0.002	-0.002
1.0	0	0	0	0

TABLE 7

*Solution 3*

*Rectangular wing, aspect ratio 6, Blenk's 6-point solution, with  $\eta = 0.2$  and  $0.8$ , revised by Argyris*

$$a_0 \quad 0.06713 \qquad c_0 \quad 0.03196$$

$$a_1 \quad -0.00156 \qquad c_1 \quad -0.02312$$

$$a_2 \quad 0.00009 \qquad c_2 \quad -0.00474$$

$$dc_L/d\alpha = 4.231 \qquad C_{DI} = 1.015 \frac{1}{\pi A} C_L^2$$

Aerodynamic centre  $0.2391\bar{c}$  behind leading edge.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.182	0.247	0.55	1.079	0.238
0.05	1.182	0.247	0.60	1.051	0.236
0.10	1.180	0.247	0.65	1.015	0.234
0.15	1.177	0.246	0.70	0.972	0.233
0.20	1.173	0.246	0.75	0.917	0.231
0.25	1.167	0.245	0.80	0.849	0.229
0.30	1.159	0.244	0.85	0.761	0.227
0.35	1.149	0.243	0.90	0.644	0.225
0.40	1.137	0.242	0.95	0.472	0.223
0.45	1.122	0.240	1.00	0	0.221
0.50	1.103	0.239			

TABLE 8

*Solution 4*

*Rectangular wing, aspect ratio 6, 4-term lifting-line solution*

$$K/4sV = 0.2402 \sin \theta + 0.0289 \sin 3\theta + 0.0057 \sin 5\theta + 0.0010 \sin 7\theta$$

$$dc_L/d\alpha = 4.527 \quad C_{DI} = 1.046 \frac{1}{\pi A} C_L^2$$

Local and overall aerodynamic centre 0.25 chord.

η	C <sub>LL</sub>	η	C <sub>LL</sub>
0	1.145	0.6	1.043
0.1	1.143	0.7	0.988
0.2	1.137	0.8	0.900
0.3	1.125	0.9	0.728
0.4	1.107	1.0	0
0.5	1.081		

TABLE 9

*Solution 5*

*Rectangular wing, aspect ratio 6, 84-vortex 9-point solutions for wing with N.A.C.A. camber, at zero lift, based on equivalent slopes*

$C_{m0}$  and  $\alpha_0$  are given in terms of the maximum camber  $M$ , which is located at a ratio  $P$  of the chord.

P	A.R.6.		A.R. infinity	
	α <sub>0</sub>	C <sub>m0</sub>	α <sub>0</sub>	C <sub>m0</sub>
0.2	− 1.633	− 1.748	− 1.570	− 1.848
0.3	− 1.750	− 2.115	− 1.674	− 2.236
0.4	− 1.904	− 2.512	− 1.813	− 2.656
0.5	− 2.108	− 2.972	− 2.000	− 3.142
0.6	− 2.390	− 3.542	− 2.262	− 3.745
0.7	− 2.812	− 4.317	− 2.656	− 4.564

TABLE 10

*Solution 6*

*Constant-chord wing, aspect ratio 6, 30 deg sweep-back, 126-vortex 6-point standard solution*

$a_0$ 0.05703	$c_0$ 0.03557	$e_0$ 0.03123
$a_1$ 0.00136	$c_1$ - 0.00172	$e_1$ - 0.05625

$$dc_L/d\alpha = 3.955 \quad C_{DI} = 1.058 \frac{1}{\pi A} C_L^2$$

Aerodynamic centre  $1.035\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.100	0.253	0.55	1.091	0.242
0.05	1.101	0.253	0.60	1.077	0.238
0.10	1.101	0.253	0.65	1.057	0.234
0.15	1.103	0.253	0.70	1.027	0.228
0.20	1.104	0.252	0.75	0.986	0.222
0.25	1.106	0.252	0.80	0.929	0.215
0.30	1.107	0.252	0.85	0.848	0.207
0.35	1.107	0.251	0.90	0.730	0.198
0.40	1.107	0.249	0.95	0.545	0.187
0.45	1.104	0.248	1.00	0	0.176
0.50	1.099	0.245			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0.000	0.000	0.000	0.000
0.1	0.000	- 0.004	- 0.004	- 0.004
0.2	0.001	- 0.007	- 0.008	- 0.007
0.3	0.001	- 0.009	- 0.010	- 0.009
0.4	0.001	- 0.010	- 0.012	- 0.010
0.5	0.001	- 0.010	- 0.012	- 0.011
0.6	0.001	- 0.010	- 0.012	- 0.010
0.7	0.001	- 0.009	- 0.010	- 0.009
0.8	0.001	- 0.007	- 0.008	- 0.007
0.9	0.000	- 0.004	- 0.004	- 0.004
1.0	0.000	0.000	0.000	0.000

TABLE 11

*Solution 7*

*Constant chord wing, aspect ratio 6, 30 deg sweep-back, 126-vortex 6-point standard solution modified.*

Auxiliary solution:—

$$\begin{aligned}
 a_0' &= 0.01583 & \phi_0 &= -0.18500 \\
 a_1' &= -0.02537 & \phi_1 &= 0.26634 \\
 dc_L/d\alpha &= 3.877 & C_{DI} &= 1.082 \frac{1}{\pi A} C_L^2
 \end{aligned}$$

Aerodynamic centre  $1.053\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.004	0.306	0.55	1.111	0.242
0.05	1.020	0.296	0.60	1.100	0.238
0.10	1.046	0.279	0.65	1.081	0.234
0.15	1.067	0.267	0.70	1.053	0.228
0.20	1.084	0.257	0.75	1.012	0.222
0.25	1.096	0.252	0.80	0.954	0.215
0.30	1.105	0.252	0.85	0.872	0.207
0.35	1.112	0.251	0.90	0.751	0.198
0.40	1.117	0.249	0.95	0.561	0.187
0.45	1.118	0.248	1.00	0	0.176
0.50	1.117	0.245			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	0.006	-0.006	-0.006	-0.005
0.2	0.012	-0.010	-0.011	-0.009
0.3	0.015	-0.014	-0.014	-0.012
0.4	0.017	-0.016	-0.016	-0.014
0.5	0.018	-0.016	-0.017	-0.014
0.6	0.017	-0.016	-0.016	-0.014
0.7	0.015	-0.014	-0.014	-0.012
0.8	0.012	-0.010	-0.011	-0.009
0.9	0.006	-0.006	-0.006	-0.005
1.0	0	0	0	0

TABLE 12

*Solution 8*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 21-vortex standard solution: 3 control points at  $\eta = 0.2, 0.6, 0.8$*

$a_0$  0.04686                       $dc_L/d\alpha = 3.323$   
 $c_0$  0.03035                       $C_{DI} = 1.098 \frac{1}{\pi A} C_L^2$   
 $e_0$  0.01329                      Local aerodynamic centre 0.25 chord.

$\eta$	$C_{LL}/C_L$	$\eta$	$C_{LL}/C_L$
0	1.063	0.6	1.080
0.1	1.065	0.7	1.052
0.2	1.069	0.8	0.977
0.3	1.076	0.9	0.793
0.4	1.083	0.95	0.602
0.5	1.086	1.0	0

TABLE 13

*Solution 9*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 21-vortex standard solution: 4 control points at  $\eta = 0.2, 0.4, 0.6, 0.8$*

$a_0$  0.04709                       $dc_L/d\alpha = 3.339$   
 $c_0$  0.03197                       $C_{DI} = 1.094 \frac{1}{\pi A} C_L^2$   
 $e_0$  0.01037                      Local aerodynamic centre 0.25 chord.

$\eta$	$C_{LL}/C_L$	$\eta$	$C_{LL}/C_L$
0	1.064	0.6	1.083
0.1	1.065	0.7	1.052
0.2	1.071	0.8	0.973
0.3	1.078	0.9	0.786
0.4	1.086	0.95	0.595
0.5	1.090	1.0	0

TABLE 14

*Solution 10*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 21-vortex solution: 6 control points at  $\eta = 0, 0.1, 0.2, 0.4, 0.6, 0.8$*

$a_0 = 0.05071$                        $p_{0a} = -0.03990$

$c_0 = 0.02797$                        $p_{0b} = -0.02105$

$e_0 = 0.01326$

$dc_L/d\alpha = 3.207$

$C_{DI} = 1.131 \frac{1}{\pi A} C_L^2$

Local aerodynamic centre 0.25 chord.

$\eta$	$C_{LL}/C_L$	$\eta$	$C_{LL}/C_L$
0	0.938	0.6	1.108
0.1	0.997	0.7	1.080
0.2	1.050	0.8	1.002
0.3	1.080	0.9	0.812
0.4	1.100	0.95	0.617
0.5	1.110	1.0	0

TABLE 15

*Solution 11*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 126-vortex 6-point standard solution:*  
 $\eta = 0.2, 0.6, 0.8$

$$\begin{array}{llll}
 a_0 & 0.04734 & c_1 & 0.01354 & dc_L/d\alpha = 3.464 \\
 a_1 & 0.00184 & e_0 & 0.04771 & C_{DI} = 1.107 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.02800 & e_1 & -0.07056 & 
 \end{array}$$

Aerodynamic centre  $1.657\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.050	0.255	0.55	1.089	0.249
0.05	1.051	0.255	0.60	1.086	0.245
0.10	1.052	0.255	0.65	1.078	0.240
0.15	1.055	0.255	0.70	1.061	0.234
0.20	1.059	0.256	0.75	1.033	0.228
0.25	1.064	0.256	0.80	0.987	0.220
0.30	1.069	0.256	0.85	0.916	0.211
0.35	1.074	0.256	0.90	0.802	0.200
0.40	1.080	0.255	0.95	0.610	0.190
0.45	1.085	0.254	1.00	0	0.178
0.50	1.088	0.252			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	0.001	-0.003	-0.004	-0.004
0.2	0.001	-0.006	-0.008	-0.008
0.3	0.001	-0.008	-0.011	-0.010
0.4	0.002	-0.009	-0.012	-0.011
0.5	0.002	-0.010	-0.013	-0.012
0.6	0.002	-0.009	-0.012	-0.011
0.7	0.001	-0.008	-0.011	-0.010
0.8	0.001	-0.006	-0.008	-0.008
0.9	0.001	-0.003	-0.004	-0.004
1.0	0	0	0	0

TABLE 16

*Solution 12*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 126-vortex standard solution modified by 6 variable 6-point auxiliary solution*

$$\begin{array}{lll}
 a_0' & 0.01512 & \phi_{0a} - 0.24688 & \phi_{0b} - 0.00216 \\
 a_1' & - 0.02176 & \phi_{1a} & 0.39712 & \phi_{1b} - 0.04012 \\
 & & dc_L/d\alpha = 3.367 & & 
 \end{array}$$

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	0.900	0.335	0.50	1.114	0.252
0.05	0.925	0.313	0.60	1.121	0.245
0.10	0.968	0.283	0.70	1.100	0.234
0.15	1.000	0.268	0.80	1.027	0.220
0.20	1.027	0.261	0.90	0.836	0.200
0.25	1.048	0.256	0.95	0.636	0.190
0.30	1.065	0.256	1.00	0	0.178
0.40	1.094	0.255			

TABLE 17

*Solution 13*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 126-vortex 8-point solution:*  
 $\eta = 0, 0.2, 0.6, 0.8$

$a_0$	0.06977	$e_0$	0.07542	$dc_L/d\alpha = 3.304$
$a_1$	-0.03426	$e_1$	-0.11816	$C_{DI} = 1.157 \frac{1}{\pi A} C_L^2$
$c_0$	-0.00719	$p_0$	-0.24857	
$c_1$	0.07404	$p_1$	0.34940	

Aerodynamic centre 1.697 $\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	0.902	0.338	0.55	1.116	0.248
0.05	0.928	0.320	0.60	1.116	0.246
0.10	0.971	0.292	0.65	1.109	0.242
0.15	1.005	0.274	0.70	1.095	0.236
0.20	1.033	0.263	0.75	1.068	0.229
0.25	1.053	0.257	0.80	1.023	0.220
0.30	1.069	0.253	0.85	0.952	0.209
0.35	1.083	0.252	0.90	0.836	0.197
0.40	1.095	0.251	0.95	0.638	0.183
0.45	1.105	0.250	1.00	0	0.167
0.50	1.112	0.250			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	0.009	-0.004	-0.005	-0.005
0.2	0.016	-0.006	-0.009	-0.009
0.3	0.021	-0.008	-0.012	-0.012
0.4	0.024	-0.009	-0.014	-0.013
0.5	0.025	-0.010	-0.014	-0.014
0.6	0.024	-0.009	-0.014	-0.013
0.7	0.021	-0.008	-0.012	-0.012
0.8	0.016	-0.006	-0.009	-0.009
0.9	0.009	-0.004	-0.005	-0.005
1.0	0	0	0	0

TABLE 18

*Solution 14*

*Constant chord wing, aspect ratio 6, 45 deg sweep-back, 126-vortex standard solution modified by 4 variable 4-point auxiliary solution*

$$\begin{array}{llll}
 a_0' & 0.02010 & p_0 & - 0.23686 & d_{C_L}/d\alpha & = 3.365 \\
 a_1' & - 0.03061 & p_1 & 0.32271 & & 
 \end{array}$$

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	0.888	0.340	0.50	1.117	0.252
0.05	0.914	0.323	0.60	1.124	0.245
0.10	0.959	0.296	0.70	1.104	0.234
0.15	0.994	0.277	0.80	1.030	0.220
0.20	1.024	0.264	0.90	0.839	0.200
0.25	1.046	0.256	0.95	0.638	0.190
0.30	1.065	0.256	1.00	0	0.178
0.40	1.095	0.255			

TABLE 19

*Solution 15*

*Rectangular wing, aspect ratio 4, 84-vortex, 6-point solution:  $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{llll}
 a_0 & 0.09213 & c_1 & - 0.00535 & d_{C_L}/d\alpha & = 3.639 \\
 a_1 & - 0.00799 & e_0 & 0.02116 & C_{DI} & = 1.006 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.01880 & e_1 & - 0.04195 & & 
 \end{array}$$

Aerodynamic centre  $0.2302\bar{7}$  behind leading edge.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.217	0.239	0.55	1.073	0.232
0.05	1.216	0.239	0.60	1.038	0.230
0.10	1.213	0.239	0.65	0.997	0.227
0.15	1.208	0.238	0.70	0.948	0.223
0.20	1.201	0.238	0.75	0.888	0.219
0.25	1.192	0.238	0.80	0.816	0.214
0.30	1.180	0.238	0.85	0.727	0.208
0.35	1.166	0.237	0.90	0.610	0.201
0.40	1.148	0.236	0.95	0.444	0.193
0.45	1.127	0.235	1.00	0	0.184
0.50	1.102	0.234			

TABLE 20

*Solution 16*

*Constant chord wing, aspect ratio 4, 30 deg sweep-back, 126-vortex 6-point standard solution:  
 $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{lll}
 a_0 & 0.07722 & c_1 - 0.03666 \quad dc_L/d\alpha = 3.467 \\
 a_1 & 0.00331 & e_0 \quad 0.01921 \quad C_{DI} = 1.027 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.05918 & e_1 - 0.05885
 \end{array}$$

Aerodynamic centre  $0.750\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.144	0.255	0.55	1.093	0.232
0.05	1.144	0.255	0.60	1.070	0.226
0.10	1.144	0.255	0.65	1.039	0.220
0.15	1.144	0.254	0.70	0.998	0.213
0.20	1.143	0.253	0.75	0.946	0.204
0.25	1.142	0.251	0.80	0.877	0.195
0.30	1.140	0.249	0.85	0.787	0.185
0.35	1.137	0.247	0.90	0.665	0.173
0.40	1.131	0.244	0.95	0.486	0.160
0.45	1.123	0.241	1.00	0	0.145
0.50	1.110	0.237			

TABLE 21

*Solution 17*

*Constant-chord wing, aspect ratio 4, 30 deg sweep-back, 126-vortex 6-point standard solution  
 modified by 4-point auxiliary solution*

$$\begin{array}{lll}
 a_0' & 0.01804 & p_0 - 0.21921 \quad dc_L/d\alpha = 3.400 \\
 a_1' & -0.03130 & p_1 \quad 0.34336 \quad C_{DI} = 1.040 \frac{1}{\pi A} C_L^2
 \end{array}$$

Aerodynamic centre  $0.762\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.076	0.307	0.55	1.108	0.232
0.05	1.087	0.297	0.60	1.086	0.226
0.10	1.105	0.281	0.65	1.056	0.220
0.15	1.119	0.269	0.70	1.016	0.213
0.20	1.129	0.260	0.75	0.964	0.204
0.25	1.136	0.252	0.80	0.894	0.195
0.30	1.139	0.249	0.85	0.804	0.185
0.35	1.140	0.247	0.90	0.679	0.173
0.40	1.138	0.244	0.95	0.497	0.160
0.45	1.133	0.241	1.00	0	0.145
0.50	1.123	0.237			

TABLE 22

Solution 18

*Constant-chord wing, aspect ratio 4, 45 deg sweep-back, 126-point vortex 6-point standard solution:  $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{lll}
 a_0 & 0.06463 & c_1 - 0.00344 \quad dc_L/d\alpha = 3.121 \\
 a_1 & 0.00530 & e_0 \quad 0.04759 \quad C_{DI} = 1.061 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.05306 & e_1 - 0.11200
 \end{array}$$

Aerodynamics centre 1.157 $\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.084	0.260	0.55	1.104	0.241
0.05	1.084	0.260	0.60	1.091	0.235
0.10	1.086	0.260	0.65	1.070	0.227
0.15	1.090	0.259	0.70	1.040	0.218
0.20	1.094	0.259	0.75	0.996	0.207
0.25	1.098	0.258	0.80	0.934	0.195
0.30	1.104	0.257	0.85	0.848	0.180
0.35	1.108	0.255	0.90	0.726	0.164
0.40	1.111	0.253	0.95	0.537	0.146
0.45	1.112	0.250	1.00	0	0.125
0.50	1.110	0.246			

TABLE 23

Solution 19

*Constant-chord wing, aspect ratio 4, 45 deg sweep-back, 126-vortex 8-point solution:  $\eta = 0, 0.2, 0.6, 0.8$*

$$\begin{array}{lll}
 a_0 & 0.09138 & e_0 \quad 0.07958 \quad dc_L/d\alpha = 2.986 \\
 a_1 & -0.04157 & e_1 - 0.16959 \quad C_{DI} = 1.092 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.01219 & p_0 - 0.30843 \\
 c_1 & 0.07073 & p_1 \quad 0.47236
 \end{array}$$

Aerodynamic centre 1.182 $\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	0.972	0.342	0.55	1.125	0.240
0.05	0.991	0.325	0.60	1.114	0.235
0.10	1.024	0.299	0.65	1.096	0.228
0.15	1.050	0.282	0.70	1.067	0.219
0.20	1.072	0.269	0.75	1.024	0.208
0.25	1.089	0.262	0.80	0.963	0.195
0.30	1.102	0.257	0.85	0.876	0.179
0.35	1.113	0.254	0.90	0.751	0.160
0.40	1.122	0.251	0.95	0.557	0.138
0.45	1.127	0.248	1.00	0	0.113
0.50	1.128	0.244			

TABLE 24

*Solution 20*

*Constant-chord wing, aspect ratio 4, 45 deg sweep-back, 126-vortex 6-point standard solution  
 modified: auxiliary solution*

$$\begin{array}{llll} a_0' & 0.02504 & p_0 & - 0.30052 & dc_L/d\alpha = 3.039 \\ a_1' & - 0.04137 & p_1 & 0.45074 & \end{array}$$

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	0.964	0.344	0.50	1.132	0.246
0.05	0.984	0.328	0.60	1.119	0.235
0.10	1.018	0.302	0.70	1.072	0.218
0.15	1.045	0.284	0.80	0.966	0.195
0.20	1.068	0.270	0.90	0.752	0.164
0.30	1.101	0.257	0.95	0.557	0.146
0.40	1.123	0.253	1.00	0	0.125

TABLE 25

*Solution 21*

*Rectangular wing, aspect ratio 2, 126-vortex 6-point standard solution:  $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{llll} a_0 & 0.14153 & c_1 & - 0.02988 & dc_L/d\alpha = 2.524 \\ a_1 & - 0.03065 & e_0 & 0.00754 & C_{DI} = 1.001 \frac{1}{\pi A} C_L^2 \\ c_0 & 0.02135 & e_1 & - 0.01406 & \end{array}$$

Aerodynamic centre 0.21107 behind leading edge.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.257	0.220	0.55	1.066	0.210
0.05	1.255	0.220	0.60	1.024	0.208
0.10	1.251	0.219	0.65	0.976	0.206
0.15	1.244	0.219	0.70	0.921	0.203
0.20	1.234	0.218	0.75	0.856	0.200
0.25	1.221	0.218	0.80	0.780	0.197
0.30	1.204	0.217	0.85	0.688	0.193
0.35	1.184	0.216	0.90	0.572	0.189
0.40	1.161	0.215	0.95	0.412	0.185
0.45	1.134	0.214	1.00	0	0.180
0.50	1.102	0.212			

TABLE 26

*Solution 22*

*Constant-chord wing, aspect ratio 2, 30 deg sweep-back, 126-vortex standard 6-point solution*

$$\begin{array}{lll}
 a_0 & 0.12334 & c_1 - 0.15199 \quad dc_L/d\alpha = 2.480 \\
 a_1 & -0.00471 & e_0 - 0.02948 \quad C_{DI} = 1.003 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.10075 & e_1 \quad 0.03426
 \end{array}$$

Aerodynamic centre  $0.462\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.226	0.245	0.55	1.078	0.203
0.05	1.225	0.245	0.60	1.040	0.196
0.10	1.222	0.244	0.65	0.995	0.189
0.15	1.218	0.242	0.70	0.942	0.182
0.20	1.211	0.239	0.75	0.878	0.174
0.25	1.202	0.236	0.80	0.801	0.167
0.30	1.190	0.232	0.85	0.707	0.159
0.35	1.176	0.227	0.90	0.587	0.151
0.40	1.158	0.222	0.95	0.422	0.143
0.45	1.136	0.216	1.00	0	0.135
0.50	1.109	0.210			

TABLE 27

*Solution 23*

*Constant-chord wing, aspect ratio 2, 30 deg sweep-back, 126-vortex standard 6-point solution modified by auxiliary solution*

$$\begin{array}{lll}
 a_0' & 0.01455 & p_0 - 0.20014 \quad dc_L/d\alpha = 2.441 \\
 a_1' & -0.02794 & p_1 \quad 0.34280 \quad C_{DI} = 1.005 \frac{1}{\pi A} C_L^2
 \end{array}$$

Aerodynamic centre  $0.469\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.198	0.282	0.55	1.084	0.203
0.05	1.201	0.276	0.60	1.047	0.196
0.10	1.206	0.266	0.65	1.003	0.189
0.15	1.207	0.256	0.70	0.949	0.182
0.20	1.205	0.248	0.75	0.886	0.174
0.25	1.199	0.241	0.80	0.808	0.167
0.30	1.190	0.234	0.85	0.714	0.159
0.35	1.177	0.227	0.90	0.593	0.151
0.40	1.161	0.222	0.95	0.426	0.143
0.45	1.140	0.216	1.00	0	0.135
0.50	1.115	0.210			

TABLE 28

*Solution 24*

*Constant-chord wing, aspect ratio 2, 45 deg sweep-back, 126-vortex 6-point standard solution*

$$\begin{array}{lll}
 a_0 & 0.10315 & c_1 - 0.18082 \quad dc_L/d\alpha = 2.359 \\
 a_1 & 0.01680 & e_0 - 0.03370 \quad C_{DI} = 1.009 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.13559 & e_1 \quad 0.01396
 \end{array}$$

Aerodynamic centre  $0.661\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.188	0.269	0.55	1.092	0.213
0.05	1.188	0.268	0.60	1.060	0.203
0.10	1.187	0.267	0.65	1.019	0.193
0.15	1.186	0.264	0.70	0.968	0.183
0.20	1.183	0.261	0.75	0.906	0.172
0.25	1.179	0.256	0.80	0.828	0.160
0.30	1.173	0.251	0.85	0.731	0.148
0.35	1.164	0.244	0.90	0.607	0.135
0.40	1.153	0.238	0.95	0.434	0.121
0.45	1.138	0.230	1.00	0	0.106
0.50	1.118	0.222			

TABLE 29

*Solution 25*

*Constant-chord wing, aspect ratio 2, 45 deg sweep-back, 126-vortex 6-point standard solution modified by auxiliary solution*

$$\begin{array}{lll}
 a_0' & 0.02633 & p_0 - 0.34507 \quad dc_L/d\alpha = 2.300 \\
 a_1' & -0.04655 & p_1 \quad 0.55418 \quad C_{DI} = 1.019 \frac{1}{\pi A} C_L^2
 \end{array}$$

Aerodynamic centre  $0.676\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.117	0.337	0.55	1.108	0.213
0.05	1.128	0.326	0.60	1.077	0.203
0.10	1.145	0.306	0.65	1.038	0.193
0.15	1.159	0.290	0.70	0.987	0.183
0.20	1.168	0.276	0.75	0.925	0.172
0.25	1.172	0.264	0.80	0.846	0.160
0.30	1.172	0.254	0.85	0.748	0.148
0.35	1.168	0.244	0.90	0.621	0.135
0.40	1.161	0.238	0.95	0.445	0.121
0.45	1.149	0.230	1.00	0	0.106
0.50	1.132	0.222			

TABLE 30

*Solution 26*

*Square wing, aspect ratio 1, 84-vortex 6-point standard solution:  $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{llll}
 a_0 & 0.20900 & c_1 & -0.01109 & dc_L/d\alpha = 1.489 \\
 a_1 & -0.11680 & e_0 & 0.00839 & C_{DI} = 1.000 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.00640 & e_1 & -0.01657 & 
 \end{array}$$

Aerodynamic centre  $0.1482\bar{c}$  behind leading edge.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.271	0.153	0.55	1.064	0.149
0.05	1.270	0.153	0.60	1.019	0.148
0.10	1.265	0.153	0.65	0.969	0.147
0.15	1.257	0.153	0.70	0.911	0.146
0.20	1.246	0.153	0.75	0.844	0.144
0.25	1.231	0.152	0.80	0.766	0.142
0.30	1.213	0.152	0.85	0.673	0.140
0.35	1.192	0.152	0.90	0.557	0.137
0.40	1.166	0.151	0.95	0.399	0.134
0.45	1.137	0.151	1.00	0	0.131
0.50	1.103	0.150			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	-0.014	-0.010	-0.007	-0.005
0.2	-0.025	-0.017	-0.013	-0.009
0.3	-0.033	-0.022	-0.017	-0.012
0.4	-0.038	-0.025	-0.019	-0.014
0.5	-0.039	-0.026	-0.020	-0.015
0.6	-0.038	-0.025	-0.019	-0.014
0.7	-0.033	-0.022	-0.017	-0.012
0.8	-0.025	-0.017	-0.013	-0.009
0.9	-0.014	-0.010	-0.007	-0.005
1.0	0	0	0	0

TABLE 31

*Solution 27*

*Constant-chord wing, aspect ratio 1, 30 deg sweep-back, 126-vortex 6-point standard solution:*  
 $\eta = 0.2, 0.6, 0.8$

$$\begin{array}{llll}
 a_0 & 0.19336 & c_1 & -0.13039 & dc_L/d\alpha = 1.493 \\
 a_1 & -0.08460 & e_0 & -0.02761 & C_{DI} = 1.000 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.06674 & e_1 & 0.05322 & 
 \end{array}$$

Aerodynamic centre  $0.281\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.271	0.180	0.55	1.064	0.152
0.05	1.270	0.180	0.60	1.020	0.147
0.10	1.265	0.179	0.65	0.969	0.143
0.15	1.257	0.178	0.70	0.911	0.138
0.20	1.246	0.176	0.75	0.844	0.134
0.25	1.232	0.174	0.80	0.766	0.129
0.30	1.214	0.171	0.85	0.672	0.126
0.35	1.192	0.168	0.90	0.556	0.122
0.40	1.167	0.164	0.95	0.398	0.119
0.45	1.137	0.160	1.00	0	0.117
0.50	1.103	0.156			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	-0.010	-0.011	-0.008	-0.006
0.2	-0.018	-0.019	-0.014	-0.011
0.3	-0.024	-0.025	-0.019	-0.014
0.4	-0.027	-0.028	-0.022	-0.016
0.5	-0.028	-0.029	-0.023	-0.017
0.6	-0.027	-0.028	-0.022	-0.016
0.7	-0.024	-0.025	-0.019	-0.014
0.8	-0.018	-0.019	-0.014	-0.011
0.9	-0.010	-0.011	-0.008	-0.006
1.0	0	0	0	0

TABLE 32

*Solution 28*

*Constant-chord wing, aspect ratio 1, 30 deg sweep-back, 126-vortex 6-point standard solution modified by auxiliary solution*

$$a_0' \quad 0.00519 \quad p_0 \quad -0.10036 \quad dc_L/d\alpha = 1.486$$

$$a_1' \quad -0.01125 \quad p_1 \quad 0.19381 \quad C_{DI} = 1.000 \frac{1}{\pi A} C_L^2$$

Aerodynamic centre  $0.287\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.268	0.201	0.55	1.065	0.153
0.05	1.267	0.199	0.60	1.020	0.148
0.10	1.263	0.194	0.65	0.970	0.143
0.15	1.256	0.189	0.70	0.912	0.138
0.20	1.245	0.185	0.75	0.844	0.134
0.25	1.231	0.180	0.80	0.766	0.129
0.30	1.214	0.176	0.85	0.673	0.126
0.35	1.192	0.172	0.90	0.557	0.122
0.40	1.167	0.168	0.95	0.399	0.119
0.45	1.138	0.163	1.00	0	0.117
0.50	1.104	0.158			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	-0.007	-0.011	-0.008	-0.006
0.2	-0.013	-0.019	-0.015	-0.011
0.3	-0.016	-0.025	-0.019	-0.014
0.4	-0.019	-0.028	-0.022	-0.016
0.5	-0.020	-0.030	-0.023	-0.017
0.6	-0.019	-0.028	-0.022	-0.016
0.7	-0.016	-0.025	-0.019	-0.014
0.8	-0.013	-0.019	-0.015	-0.011
0.9	-0.007	-0.011	-0.008	-0.006
1.0	0	0	0	0

TABLE 33

*Solution 29*

*Constant-chord wing, aspect ratio 1, 45 deg sweep-back, 126-vortex 6-point standard solution:*  
 $\eta = 0.2, 0.6, 0.8$

$$\begin{array}{llll}
 a_0 & 0.16730 & c_1 & -0.26228 & dc_L/d\alpha = 1.484 \\
 a_1 & -0.03574 & e_0 & -0.07894 & C_{DI} = 1.000 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.13721 & e_1 & 0.14867 & 
 \end{array}$$

Aerodynamic centre  $0.394\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.265	0.220	0.55	1.067	0.166
0.05	1.264	0.220	0.60	1.023	0.158
0.10	1.259	0.218	0.65	0.973	0.151
0.15	1.252	0.215	0.70	0.915	0.144
0.20	1.242	0.212	0.75	0.848	0.138
0.25	1.228	0.207	0.80	0.769	0.132
0.30	1.211	0.202	0.85	0.675	0.128
0.35	1.190	0.195	0.90	0.558	0.126
0.40	1.166	0.189	0.95	0.400	0.125
0.45	1.138	0.181	1.00	0	0.126
0.50	1.105	0.174			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	-0.004	-0.010	-0.008	-0.006
0.2	-0.008	-0.018	-0.014	-0.010
0.3	-0.010	-0.024	-0.019	-0.013
0.4	-0.012	-0.028	-0.021	-0.015
0.5	-0.012	-0.029	-0.022	-0.016
0.6	-0.012	-0.028	-0.021	-0.015
0.7	-0.010	-0.024	-0.019	-0.013
0.8	-0.008	-0.018	-0.014	-0.010
0.9	-0.004	-0.010	-0.008	-0.006
1.0	0	0	0	0

TABLE 34

*Solution 30*

*Constant-chord wing, aspect ratio 1, 45 deg sweep-back, 126-vortex 6-point standard solution  
 modified by auxiliary solution*

$$\begin{aligned}
 a_0' &= 0.01300 & p_{a1} &= 0.17130 & dc_L/d\alpha &= 1.469 \\
 a_1' &= -0.02620 & p_{b0} &= -0.11582 & C_{DI} &= 1.000 \frac{1}{\pi A} C_L^2 \\
 p_{a0} &= -0.09363 & p_{b1} &= 0.21654 & &
 \end{aligned}$$

Aerodynamic centre  $0.405\bar{c}$  behind apex.

*Loading*

$\eta$	$C_{LL}/C_L$	Local a.c.	$\eta$	$C_{LL}/C_L$	Local a.c.
0	1.252	0.263	0.55	1.070	0.167
0.05	1.253	0.258	0.60	1.026	0.158
0.10	1.252	0.248	0.65	0.976	0.151
0.15	1.247	0.238	0.70	0.918	0.144
0.20	1.239	0.228	0.75	0.851	0.138
0.25	1.227	0.220	0.80	0.772	0.132
0.30	1.211	0.211	0.85	0.678	0.128
0.35	1.191	0.203	0.90	0.561	0.126
0.40	1.168	0.194	0.95	0.402	0.125
0.45	1.140	0.185	1.00	0	0.126
0.50	1.107	0.176			

*Induced camber*

Position on chord	Spanwise location: value of $\eta$			
	0	0.8	0.9	0.95
0	0	0	0	0
0.1	0.002	-0.011	-0.008	-0.006
0.2	0.003	-0.019	-0.015	-0.011
0.3	0.004	-0.025	-0.020	-0.014
0.4	0.005	-0.029	-0.022	-0.016
0.5	0.005	-0.030	-0.023	-0.017
0.6	0.005	-0.029	-0.022	-0.016
0.7	0.004	-0.025	-0.020	-0.014
0.8	0.003	-0.019	-0.015	-0.011
0.9	0.002	-0.011	-0.008	-0.006
1.0	0	0	0	0

TABLE 35

Solution 31

*Triangular wing, 90 deg apex angle, aspect ratio 4, standard 126-vortex 6-point solution:*

$$\eta = 0.2, 0.6, 0.8$$

$$\begin{array}{lll} a_0 & 0.08851 & c_1 - 0.04994 \quad dc_L/d\alpha = 3.470 \\ a_1 & 0.01339 & e_0 - 0.02830 \quad C_{DI} = 1.026 \frac{1}{\pi A} C_L^2 \\ c_0 & 0.00187 & e_1 \quad 0.03216 \end{array}$$

Aerodynamic centre  $1.116\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.690	1.379	0.268	0.55	1.171	1.054	0.252
0.05	0.724	1.376	0.267	0.60	1.236	0.989	0.249
0.10	0.760	1.369	0.267	0.65	1.309	0.916	0.247
0.15	0.798	1.356	0.266	0.70	1.396	0.837	0.245
0.20	0.836	1.338	0.265	0.75	1.501	0.751	0.243
0.25	0.876	1.314	0.264	0.80	1.639	0.655	0.241
0.30	0.918	1.285	0.262	0.85	1.835	0.550	0.240
0.35	0.962	1.251	0.268	0.90	2.162	0.432	0.239
0.40	1.009	1.211	0.258	0.95	2.913	0.291	0.239
0.45	1.059	1.165	0.256	1.00		0	0.241
0.50	1.112	1.112	0.254				

TABLE 36

Solution 32

*Triangular wing, 90 deg apex angle, aspect ratio 4, standard 126-vortex 6-point solution modified by 4-point conditional auxiliary solution*

$$\begin{array}{lll} a_0' & 0.01304 & p_0' - 0.17388 \quad dc_L/d\alpha = 3.470 \\ a_1' & -0.02608 & p_1' \quad 0.34776 \quad C_{DI} = 1.026 \frac{1}{\pi A} C_L^2 \end{array}$$

Aerodynamic centre  $1.133\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.690	1.379	0.314	0.55	1.171	1.054	0.252
0.05	0.724	1.376	0.308	0.60	1.236	0.989	0.249
0.10	0.760	1.369	0.296	0.65	1.309	0.916	0.247
0.15	0.798	1.356	0.286	0.70	1.396	0.837	0.245
0.20	0.836	1.338	0.278	0.75	1.501	0.751	0.243
0.25	0.876	1.314	0.272	0.80	1.639	0.655	0.241
0.30	0.918	1.285	0.267	0.85	1.835	0.550	0.240
0.35	0.962	1.251	0.262	0.90	2.162	0.432	0.239
0.40	1.009	1.211	0.258	0.95	2.913	0.291	0.239
0.45	1.059	1.165	0.256	1.00		0	0.241
0.50	1.112	1.112	0.254				

TABLE 37

Solution 33

*Triangular wing, 90 deg angle, with tips cropped to aspect ratio 3, standard 126-vortex 6-point solution:  $\eta = 0.2, 0.6, 0.8$*

$$\begin{array}{lll} a_0 & 0.10118 & c_1 - 0.07024 & dc_L/d\alpha = 3.142 \\ a_1 & 0.01426 & e_0 - 0.00869 & C_{DI} = 1.001 \frac{1}{\pi A} C_L^2 \\ c_0 & 0.02309 & e_1 & 0.03020 \end{array}$$

Aerodynamic centre  $0.916\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.743	1.300	0.266	0.55	1.141	1.055	0.245
0.05	0.775	1.298	0.266	0.60	1.184	1.006	0.242
0.10	0.807	1.292	0.266	0.65	1.228	0.952	0.238
0.15	0.841	1.282	0.265	0.70	1.273	0.891	0.234
0.20	0.874	1.268	0.263	0.75	1.316	0.822	0.231
0.25	0.909	1.250	0.262	0.80	1.352	0.743	0.228
0.30	0.945	1.228	0.260	0.85	1.371	0.651	0.225
0.35	0.981	1.202	0.257	0.90	1.344	0.538	0.222
0.40	1.019	1.172	0.254	0.95	1.184	0.385	0.220
0.45	1.058	1.137	0.252	1.00	0	0	0.219
0.50	1.099	1.099	0.248				

TABLE 38

Solution 34

*Triangular wing, 90 deg apex angle, with tips cropped to aspect ratio 3, 8-point solution:  $\eta = 0, 0.2, 0.6, 0.8$*

$$\begin{array}{lll} a_0 & 0.11415 & e_0 & 0.00028 & dc_L/d\alpha = 3.123 \\ a_1 & -0.01434 & e_1 & 0.00967 & C_{DI} = 1.001 \frac{1}{\pi A} C_L^2 \\ c_0 & 0.00975 & p_0 & -0.17274 \\ c_1 & -0.03973 & p_1 & 0.35454 \end{array}$$

Aerodynamic centre  $0.929\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.744	1.301	0.306	0.55	1.141	1.055	0.246
0.05	0.775	1.298	0.300	0.60	1.185	1.007	0.242
0.10	0.807	1.291	0.290	0.65	1.230	0.953	0.239
0.15	0.840	1.280	0.281	0.70	1.275	0.892	0.235
0.20	0.873	1.266	0.274	0.75	1.318	0.824	0.232
0.25	0.908	1.248	0.269	0.80	1.354	0.745	0.228
0.30	0.943	1.226	0.265	0.85	1.373	0.652	0.224
0.35	0.980	1.201	0.261	0.90	1.347	0.539	0.221
0.40	1.018	1.171	0.257	0.95	1.186	0.385	0.218
0.45	1.058	1.137	0.253	1.00	0	0	0.214
0.50	1.098	1.098	0.250				

TABLE 39

Solution 35

*Triangular wing, 90 deg apex angle, with tips cropped to aspect ratio 2.309, standard 126-vortex  
 6-point solution*

$$\begin{array}{lll}
 a_0 & 0.11541 & c_1 - 0.08365 & dc_L/d\alpha = 2.761 \\
 a_1 & 0.01165 & e_0 & 0.01183 & C_{DI} = 1.000 \frac{1}{\pi A} C_L^2 \\
 c_0 & 0.03985 & e_1 & -0.01769 & \lambda = 0.212
 \end{array}$$

Aerodynamic centre 0.756 $\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.808	1.274	0.262	0.55	1.127	1.062	0.234
0.05	0.838	1.273	0.262	0.60	1.150	1.017	0.228
0.10	0.867	1.268	0.261	0.65	1.168	0.966	0.222
0.15	0.897	1.260	0.260	0.70	1.181	0.908	0.215
0.20	0.927	1.248	0.258	0.75	1.184	0.842	0.208
0.25	0.957	1.233	0.256	0.80	1.170	0.764	0.199
0.30	0.986	1.214	0.254	0.85	1.128	0.672	0.190
0.35	1.016	1.192	0.251	0.90	1.036	0.557	0.180
0.40	1.045	1.166	0.248	0.95	0.833	0.400	0.170
0.45	1.074	1.136	0.244	1.00	0	0	0.158
0.50	1.101	1.101	0.239				

TABLE 40

Solution 36

*Triangular wing, 90 deg apex angle, with tips cropped to aspect ratio 2.309, standard 126-vortex  
 6-point solution modified by conditional 4-point auxiliary solution*

$$\begin{array}{lll}
 a_0' & 0.00882 & p_0 - 0.14653 & dc_L/d\alpha = 2.761 \\
 a_1' & -0.01764 & p_1 & 0.29306 & C_{DI} = 1.000 \frac{1}{\pi A} C_L^2
 \end{array}$$

Aerodynamic centre 0.767 $\bar{c}$  behind apex.

$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.	$\eta$	$C_{LL}/C_L$	$C_{LLc}/C_L\bar{c}$	Local a.c.
0	0.808	1.274	0.298	0.55	1.127	1.062	0.235
0.05	0.838	1.273	0.293	0.60	1.150	1.017	0.228
0.10	0.867	1.268	0.284	0.65	1.168	0.966	0.222
0.15	0.897	1.260	0.278	0.70	1.181	0.908	0.215
0.20	0.927	1.248	0.272	0.75	1.184	0.842	0.208
0.25	0.957	1.233	0.266	0.80	1.170	0.764	0.199
0.30	0.986	1.214	0.262	0.85	1.128	0.672	0.190
0.35	1.016	1.192	0.257	0.90	1.036	0.557	0.180
0.40	1.045	1.166	0.252	0.95	0.833	0.400	0.170
0.45	1.074	1.136	0.247	1.00	0	0	0.158
0.50	1.101	1.101	0.241				

TABLE 41

*Experimental and theoretical lift and aerodynamic centre for a delta wing, apex angle 90 deg with pointed and cropped tips*

Aspect ratio	$dc_L/d\alpha$		Ratio
	Wind tunnel	Theoretical	
4	3.342	3.547	0.942
3	3.048	3.192	0.955
2.31	2.720	2.821	0.964

Aspect ratio	a.c. in terms of $\bar{c}$ behind apex		Difference
	Wind tunnel	Theoretical	
4	1.126	1.133	- 0.007
3	0.943	0.929	0.014
2.31	0.784	0.767	0.017

*Note:*—The theoretical values of  $dc_L/d\alpha$  have been multiplied by the correction factor 1.022.

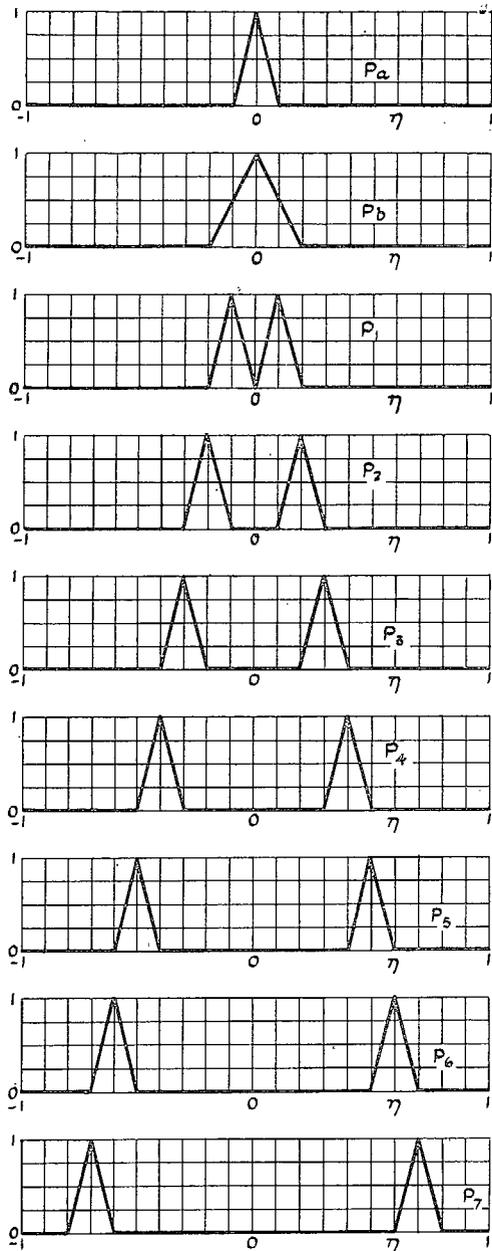


FIG. 1. Spanwise distribution of induced downwash for  $P$  functions.

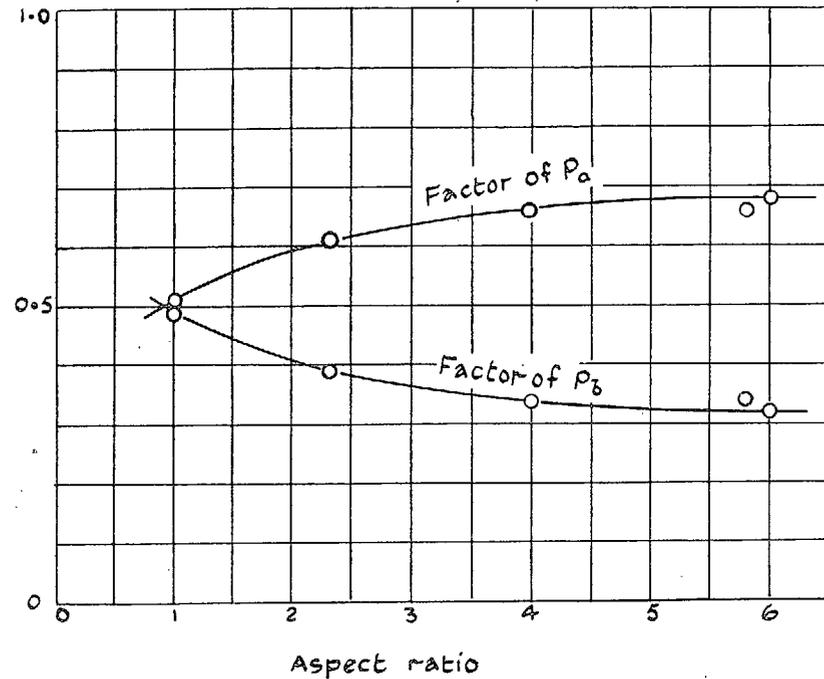


FIG. 2. Proportions of  $P_a$  and  $P_b$  for combined circulation function to allow for discontinuity at median section.

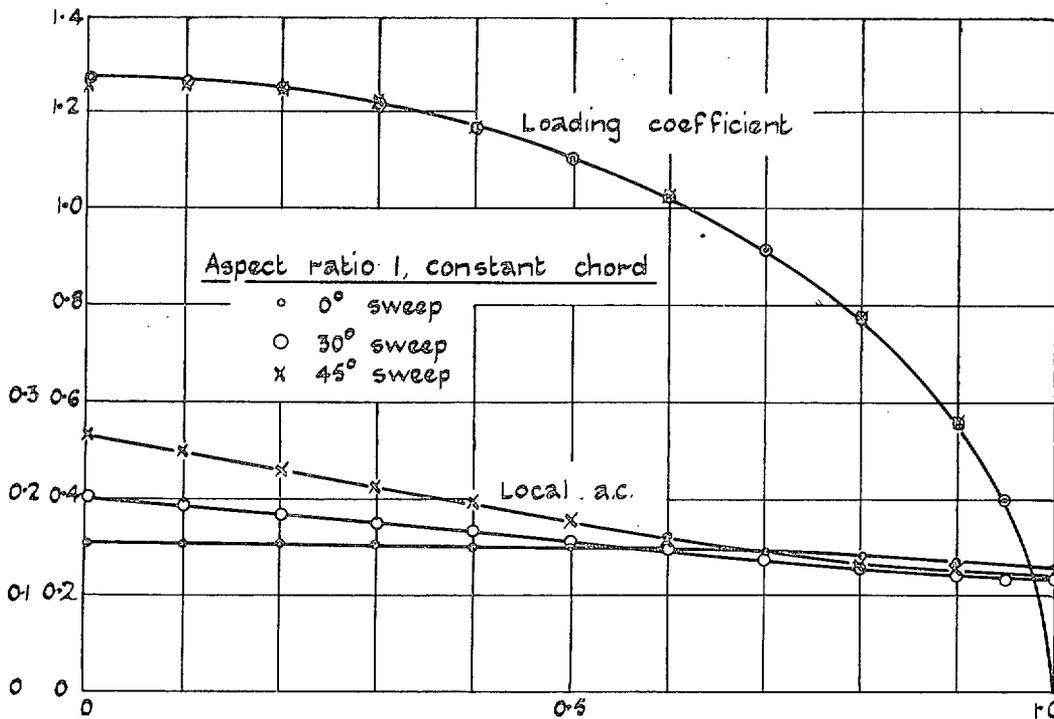
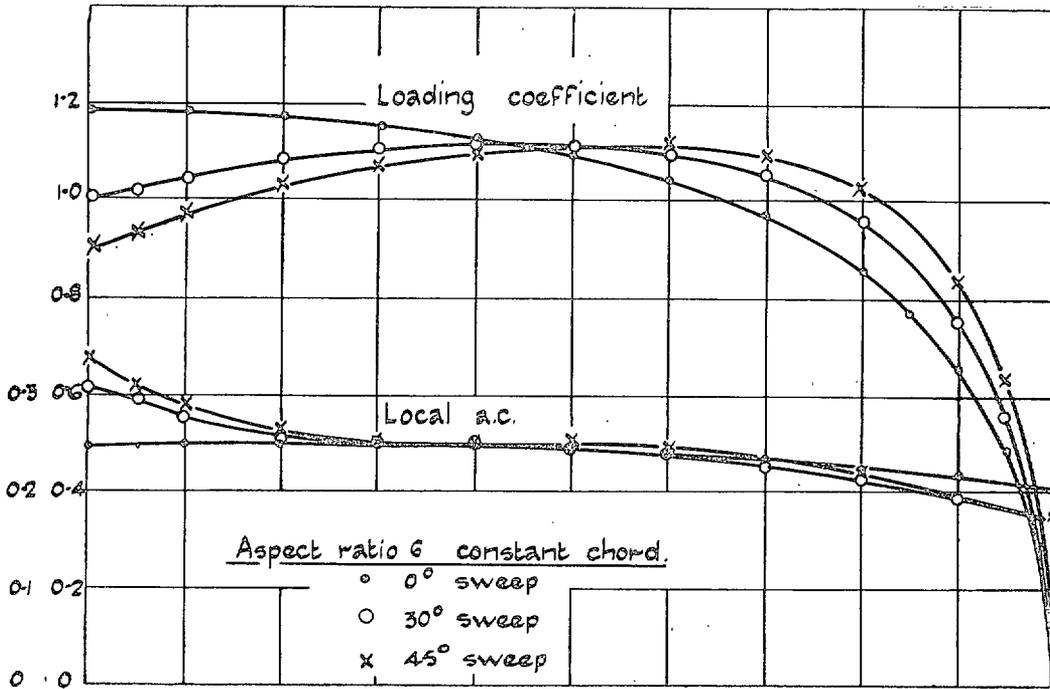


FIG. 3. Loading coefficient and local aerodynamic centre for two wings of constant chord.

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