R. & M. No. 2526
(4921)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Note on Differential Gearing as a Means of Aileron Balance

By

S. B. Gates, M.A.

LONDON: HIS MAJESTY'S STATIONERY OFFICE
1951
FOUR SHILLINGS NET

Note on Differential Gearing as a means of Aileron Balance

By
S. B. Gates, M.A.
Communicated by the Principal Director of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 2526*
December, 1940

Summary.—It has been suggested in some American investigations that differential gearing, combined with adjustment of the aileron floating angle by means of a tab, may be a powerful method of balancing ailerons. This report sets out the theory of this method of balance and analyses it in relation to the most pressing problem of aileron design, which is to obtain close balance at high speed without overbalance in any part of the range, or uncomfortable lightness at slow speed. It is shown that this result can be achieved more directly by differential balance than by any other method if the differential and the tab setting are nicely adjusted to the natural floating properties of the aileron. Thus if the aileron tends to float up as incidence increases, a differential giving more downward than upward movement must be used, and this must be combined with an upward-set tab; while if the aileron tends to float down as the incidence increases, a differential giving more upward than downward movement must be used, combined with a down-set tab. After examining the possible disadvantages of the downward differential, and the loads set up by the tab, it is concluded that there is a strong case for exploration in flight of differential gearing as a major means of aileron balance.

Some notes on the geometry of differential gearing are given in an Appendix.

1. *Introduction.*—In the past, when ailerons have been moved differentially, it has always been with the object rather of improving the rolling moments than of lightening the stick force. The potentiality of differential gearing as a major means of aileron balance has been broached in two American reports[1, 2], but the theory of this method is probably still unfamiliar to many designers in this country. It may be profitable, therefore, to make a rather more extended analysis of the subject, relating it particularly to the central problem of aileron design, which is to make the aileron light enough at very high speed, while avoiding overbalance in any part of the range and retaining sufficient feel at low speed. There seems to be much promise in the development of differential balance, but only if the designer realises its power and its limitations in relation to other balancing methods at his disposal, and designs his gear deliberately to fit the hinge-moment characteristics of the aileron he is seeking to balance.

2. *Definition of Differential Gearing.*—A sketch of a method of analysis which seems simpler and more illuminating than the American discussion has been given in Appendix I of Ref. 3, and will now be developed in more detail. Instead of working with ξ_u, ξ_d, defined respectively as the settings of the up-and-down aileron, each being regarded as positive, it seems clearer to base the analysis on two quantities, ξ, ε defined as

$$\xi = \frac{\xi_u + \xi_d}{2}, \qquad \varepsilon = \frac{\xi_u - \xi_d}{2}, \qquad \qquad (1)$$

so that $\xi_u = \xi + \varepsilon$ and $\xi_d = \xi - \varepsilon$.

* R.A.E. Report No. B.A. 1642—received 18th January, 1941.

With these definitions the ailerons can be regarded as moving through $\pm \xi$ from the common setting ε. ξ will be called the *displacement* of the system and ε the *eccentricity*. So long as the setting of either aileron does not exceed the angular range in which the rolling-moment coefficient C_l is proportional to aileron setting, the displacement ξ fixes C_l whatever the differential. Under this assumption ξ is therefore the basic variable for assessing the balancing power of differential, since it concentrates on the magnitude of the force to produce a given rolling moment.

A differential gear is specified when ξ_u and ξ_d, or alternatively ξ and ε, are given as functions of x, the movement of the pilot's hand from the central position. A typical sketch is given in Fig. 1. It should be realised at the outset that the essence of a differential gear, and the property by virtue of which it provides balance, is that *the eccentricity* ε *should vary as the stick goes over*. In an ordinary non-differential gear ε is zero, and when this is worked from a rigged up or down position ε is constant. It is only when ε varies with x or ξ that a true differential is obtained and the possibility of balance arises.

The amount of differential D is usually taken to be the ratio of ξ_u to ξ_d at full movement. The common usage will be retained here in spite of its failure to define the differential in any strict sense, and using the suffix max. to denote full movement, we have

$$D = \left(\frac{\xi + \varepsilon}{\xi - \varepsilon}\right)_{\text{max.}} .$$

Thus for an upward differential ($\xi_u > \xi_d$), D is greater than 1 and ε and $d\varepsilon/d\xi$ are positive, and for a downward differential D is less than 1 and ε and $d\varepsilon/d\xi$ are negative. The spurious case of non-differential from a non-central position (ε constant) should be excluded from this definition.

3. *Analysis of Pilot's Force.*—The equation of virtual work is

$$Pdx + \{ C_{H,d}\, d\xi_d + C_{H,u}\,(- d\,\xi_u) \}\, \tfrac{1}{2} S_\xi \,.\, c_\xi\, q = 0 \,,$$

where
P pilot's force in the direction of displacement,
S_ξ total aileron area,
c_ξ aileron chord,
$C_{H,u}$, $C_{H,d}$ respectively the hinge-moment coefficients of the up and downgoing ailerons, referred to the area of one aileron,

$$q = \tfrac{1}{2}\rho V^2 .$$

Rearranging in terms of ξ and ε we have

$$\frac{P}{S_\xi\, c_\xi\, q} = \frac{d\xi}{dx}\left(\frac{C_{H,u} - C_{H,d}}{2} + \frac{C_{H,u} + C_{H,d}}{2}\frac{d\varepsilon}{d\xi}\right) . \qquad (3)$$

Assume now that the hinge-moment coefficient of one aileron is linear in local incidence and aileron displacement, so that

$$C_H = b_0 + b_1 \text{ (mean aileron incidence)} + b_2 \text{ (aileron displacement)*}.$$

Assume also that response in roll due to the mean displacement ξ is proportional to ξ, so that the mean incidence is increased by $n\xi$ at the upgoing aileron and decreased by $n\xi$ at the downgoing aileron. It follows that if α is the incidence before the control is applied, the downgoing aileron has mean incidence $\alpha - n\xi$ and displacement $\xi - \varepsilon$, while the upgoing aileron has mean incidence $\alpha + n\xi$ and displacement $-(\xi + \varepsilon)$. We therefore have

$$C_{H,u} = b_0 + b_1 (\alpha + n\xi) - b_2 (\xi + \varepsilon) \,,$$
$$C_{H,d} = b_0 + b_1 (\alpha - n\xi) + b_2 (\xi - \varepsilon) \,,$$

* No attempt is made to discuss aileron types, such as the Frise whose hinge moments are far from linear.

so that $\dfrac{C_{H,u} - C_{H,d}}{2} = (nb_1 - b_2)\,\xi$

and $\dfrac{C_{H,u} + C_{H,d}}{2} = b_0 + b_1\alpha - b_2\varepsilon\,.$

The discussion of response in Ref. 3 leads to a response factor

$$K = 1 - n\,\frac{b_1}{b_2}\,,$$

and so we have

$$\frac{C_{H,u} - C_{H,d}}{2} = -\,Kb_2\xi\,. \qquad (4)$$

Again, if ξ_f is the floating angle of the aileron at incidence α, measured positive when upward, we have

$$b_0 + b_1\alpha = b_2\,\xi_f\,, \qquad (5)$$

and so $\dfrac{C_{H,u} + C_{H,d}}{2} = b_2\,(\xi_f - \varepsilon)\,. \qquad (6)$

Substituting in (3) from (4) and (5) we have finally

$$p = \frac{P}{S_\xi c_\xi q} = -\,b_2\,\frac{d\xi}{dx}\left\{K\xi - (\xi_f - \varepsilon)\,\frac{d\varepsilon}{d\xi}\right\}, \qquad (7)$$

and in order that the control may never be overbalanced we must have

$$\frac{dp}{dx} > 0. \qquad (8)$$

4. *Preliminary Discussion.*—Equation (7) is the fundamental expression for pilot's force. The aerodynamics of the control is represented by the quantities b_2, K and ξ_f, the geometry of the gearing by ξ, ε and x, and it is the designer's problem to adjust the geometry to the aerodynamics of the control so that the force to provide a given rolling-moment coefficient is small enough at high speed without being too small at low speed. An analysis of the geometry of a simple form of differential gearing is given in the Appendix. It is clear from (7) that differential provides balance in virtue of the term $(\xi_f - \varepsilon)\, d\varepsilon/d\xi$. Now in all practical cases the eccentricity ε will increase steadily, either upward or downward, as the control goes over, that is, ε and $d\varepsilon/d\xi$ will have the same sign. Hence to make the differential effective as a balance the floating angle ξ_f must be arranged to exceed ε numerically throughout the range. Thus with upward differential a large upfloating angle, and with downward differential a large downfloating angle, is required to produce an effective balance. The Americans[1,2] have pointed out that a fixed tab gives a simple and effective means of providing any floating angle that is required. It must be noted, however, that the floating angle ξ_f has in general an important variation with the incidence α, since

$$\xi_f = \frac{b_0}{b_2} + \frac{b_1}{b_2}\,\alpha\,.$$

The tab is available for the adjustment of b_0, but as will be seen later, the part of ξ_f represented by $(b_1/b_2)\alpha$ must be carefully considered in designing a differential for satisfactory operation over the whole of the incidence range.

One other general conclusion from equation (7) may be mentioned There will be a general tendency for a differential to give most balance for small movements of the control, since $\xi_f - \varepsilon$ decreases as the control goes over and will not be compensated by an equal increase in $d\varepsilon/d\xi$ unless the gear is specially designed to provide this.

5. *Basis of Comparison with no Differential.*—Before discussing the matter further, a basis of comparison with the case of no differential must be decided. Referring to Fig. 3 let us consider more closely what the designer's problem actually is. He makes the best use of cockpit space to get a large stick movement, so that $x_{max.}$ may be regarded as fixed. He then chooses a range of aileron displacement $\pm \xi_{max.}$ which gives what he considers to be a satisfactory maximum rolling-moment coefficient with a linear non-differential gear, represented by the straight line OP. The question at issue is, how much can he lighten the control by using a differential while retaining the same aileron power ? If the differential is restricted to the linear rolling-moment range, the condition is that $\xi_{max.}$ must remain the same, and so in Fig. 1 we compare the non-differential gear represented by the straight line OP with the differential gear represented by the curves OP and OQ. This seems the fairest basis of comparison, admitting of course that if the differential is extreme there will be some loss of power due to breakdown in linearity. Equation (7) can now be modified to express this line of argument. The non-differential gearing m or $\xi_{max.}/x_{max.}$ is the mean value of $d\xi/dx$, and writing

$$\frac{d\xi}{dx} = \mu m ,$$

equation (7) can be conveniently rearranged in the form

$$F = \frac{p}{mKb_2} = -\xi\mu\left(1 - \frac{\xi_f - \varepsilon}{K\xi}\frac{d\varepsilon}{d\xi}\right). \quad \ldots \quad (9)$$

F is a function of ξ and α which is equal to $-\xi$ when there is no differential. The factor μ, although introduced by the differential, is not a true differential effect since it would be produced by a non-differential gear represented by the curve OP. The characteristic differential effect is represented solely by the factor $1 - \dfrac{\xi_f - \varepsilon}{K\xi}\dfrac{d\varepsilon}{d\xi}$. The pilot's force P is obtained from F by multiplying it by $mKb_2S_\xi c_\xi q$.

There is another basis of comparison which may be made when the designer finds that he has some aileron power in hand and can sacrifice some of this to lighten his control. He therefore uses a differential which reduces the downward movement while retaining the maximum upward movement. This is illustrated in Fig. 2, where the no-differential (straight line OP) is to be compared with the differential curves OP′, OQ. Here, quite apart from the true effect of the differential, the loss of power represented by PP′ appears as a gain in lightness represented by the angle POP′; it could of course be produced by merely reducing the range and retaining equal up and down movement (straight line OP′). The equation for this comparison is

$$F = -\xi\mu\frac{\xi_{max.}}{\xi_{\mu\,max.}}\left(1 - \frac{\xi_f - \varepsilon}{K\xi}\frac{d\varepsilon}{d\xi}\right). \quad \ldots \quad (10)$$

6. *Form of Differential to give Constant Balance.*—In general the variation of force with displacement under a differential will not be linear, but the conditions for linearity are of some interest. If the differential is to multiply the force at any displacement ξ by the constant factor k, the general condition is from (9),

$$\mu\left(1 - \frac{\xi_f - \varepsilon}{K\xi}\frac{d\varepsilon}{d\xi}\right) = k ,$$

and if the displacement remains linear with stick movement ($\mu = 1$), the equation for the eccentricity becomes

$$1 - \frac{\xi_f - \varepsilon}{K\xi}\frac{d\xi}{d\xi} = k .$$

Integrating this under the condition that ε and ξ vanish together, we have

$$K(1-k)\left(\frac{\xi}{\xi_f}\right)^2 + \left(\frac{\varepsilon}{\xi_f} - 1\right)^2 = 1 . \quad \ldots \quad (11)$$

This elliptic variation of ε with ξ is plotted in Fig. 3 for various values of $K(1 - k)$. Complete balance is obtained when $k = 0$, and so the number attached to any curve is the value of the response factor K for which there is complete balance with that form of the eccentricity. For instance, if the response factor is 0·6, any curve to the right of 0·6 will give less than complete balance, and any curve to the left will give overbalance.

With $K = 1$ the eccentricity curve for complete balance is an arc of a circle. This has been further analysed in Fig. 4, where curves of $\varepsilon/\xi_{max.}$ against $\xi/\xi_{max.}$ are plotted for various values of $\xi_f/\xi_{max.}$, the condition being complete balance with $K = 1$ and $d\xi/dx$ is constant; all these curves are circular arcs. The point to notice here is that it is aerodynamically possible to get complete balance by differential without using an impossibly large tab. It appears from Fig. 20 (reproduced from Fig. 19 of Ref. 4) that a floating angle of at least 20 deg. can be obtained with a tab whose chord does not exceed 20 per cent. of the aileron chord and whose angle does not exceed 15 deg. Taking $\xi_f = 20$ deg. and a maximum displacement $\xi_{max.}$ of 16 deg. we have the curve labelled 1·25 of Fig. 4. This differential gearing has been drawn in more detail in Fig. 5, and is aerodynamically quite feasible. It represents about the lowest eccentricity which can be used in practice to give complete balance. Higher eccentricity, combined with a smaller tab, would give the same result, the limit occurring when the floating angle is equal to the maximum displacement.

It should be noticed that if ξ_f varies with incidence, complete balance can only be obtained at one point of the incidence range.

7. *Parabolic Differential.*—In several actual examples of differential gear which have been examined, the displacement ξ varies linearly with the stick movement x and the eccentricity ε varies as the square of the displacement ξ.* This typical parabolic gear is useful as a basis for further discussion. In this case we have

$$\mu = 1,\ \frac{d\varepsilon}{d\xi} = \lambda\xi,\ \varepsilon = \tfrac{1}{2}\lambda\xi^2,\ D = \frac{1 + \dfrac{\lambda}{2}\,\xi_{max.}}{1 - \dfrac{\lambda}{2}\,\xi_{max.}}\,,$$

and so equation (9) becomes

$$F = -\,\xi\left\{1 - \frac{\lambda}{K}\left(\xi_f - \frac{\lambda}{2}\,\xi^2\right)\right\}. \qquad (12)$$

Since
$$\frac{dF}{d\xi} = -\left\{1 - \frac{\lambda}{K}\,\xi_f + \frac{3}{2K}\,\lambda^2\xi^2\right\},$$

the balance decreases as the displacement increases, and the condition for stability is

$$1 - \frac{\lambda}{K}\,\xi_f > 0\,,$$

complete balance being obtained at $\xi = 0$ when

$$\xi_f = \frac{K}{\lambda}\,. \qquad (13)$$

As illustrations of the kind of force reduction which can be obtained with various amounts of differential combined with various floating angles, two cases have been worked out for parabolic gears, assuming that $K = 1$, *i.e.*, that the differential balance is independent of incidence :

(1) Parabolic gears with the same maximum displacement, $\xi_{max.} = 16$ deg.

(2) Parabolic gears with the same maximum upward movement $\xi_{u\,max.} = 20$ deg.

These illustrate respectively the two bases of comparison discussed in section 5.

* See for instance the gears tried in flight in Ref. 2. It is shown in the Appendix that this gearing is obtained when the stick crank is eccentric and the aileron crank central in the neutral position.

(1) *Parabolic Gears with Same Maximum Displacement,* $\xi_{\max.} = 16$ *deg.*—The gearings as developed from the black line representing no differential are shown for a range of D in Fig. 6, and the force function F is plotted against ξ for three floating angles (0, 10, and 20 deg.) in Fig. 7. In Fig. 7 the black line at 45 deg. represents no differential, and overbalance occurs as soon as the slope at the origin becomes positive. The diagram shows the marked increase of curvature at a given floating angle as the differential increases, and indicates clearly that with this form of gear it pays to use a moderate eccentricity combined with a large floating angle, rather than a large eccentricity combined with a moderate floating angle, if the object is to get a large reduction of force over the whole range of displacement. This is seen for example by comparing the curves for $D = 2$, $\xi_f = 20$, and $D = 6$, $\xi_f = 10$. These give roughly the same force up to $\xi = 5$ deg., but at maximum displacement the force with the small differential is still comparatively small, while with the large differential it is actually greater than with no differential, since the eccentricity has exceeded the floating angle.

Figs. 6 and 7 are drawn for upward differentials ($\xi_u > \xi_d$) but they apply unchanged for downward differential if ξ_u and ξ_d are interchanged, ε and ξ_f are changed in sign, and D is inverted. This remark applies also to Figs. 8 and 9 (*see* below).

(2) *Parabolic Gears with the Same Maximum Upward Movement,* $\xi_{u,\ \max.} = 20$ *deg.*—The gearings, as developed from the black line representing no differential, are shown for a range of D in Fig. 8, and the force function F is plotted against ξ for the floating angle in Fig. 9.

In Case (1) above all the systems had the same maximum power. In this case the decrease in $\xi_{\max.}$ as the differential increases represents a deliberate loss of power, which is reflected by a reduction in force quite independent of the true differential balance. The loss in power is shown in Fig. 9, and the corresponding reduction in force is represented by the broken line.

8. *Choice of Differential and Tab Setting when the Floating Angle Varies with Incidence.*—The preceding discussion has assumed that the floating angle is independent of incidence. We have now to discuss the crucial problem of differential balance, which is how best to make it effective over the incidence range between diving and landing when the floating angle varies with incidence. Let the upfloating angle in the dive (*i.e.* near $\alpha = 0$) be ξ_{f0} and let the increase in upfloating angle at landing (*i.e.* near $\alpha = 15$ deg.) be $\triangle$, so that the extreme values of the floating angle are $\xi_f = \xi_{f0}$ and $\xi_f = \xi_{f0} + \triangle$. The corresponding values of the force function F are, for a parabolic differential,

$$\left.\begin{aligned} F_0 &= -\,\xi\left\{1 - \frac{\lambda}{K}\left(\xi_{f0} - \frac{\lambda}{2}\xi^2\right)\right\} \text{ in the dive} \\ \text{and} \quad F_\triangle &= -\,\xi\left\{1 - \frac{\lambda}{K}\left(\xi_{f0} + \triangle - \frac{\lambda}{2}\xi^2\right)\right\} \text{ at landing} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (14)$$

The designer's problem may be put as follows. The sign and magnitude of $\triangle$ is fixed by the type of aileron he has chosen. $\triangle$ will be positive or negative according as the aileron is convergent or divergent (*i.e.* according as b_1/b_2 is positive or negative)*, and since the range of b_1/b_2 may well be $\pm$ 1 and the incidence range is of the order 15 deg., the range of $\triangle$ to be considered is of the order $\pm$ 15 deg. The designer has to arrange the sign of his differential (λ is + or − according as the differential is upward or downward) and the value of ξ_{f0} (controlled by tab size and setting) to give, with the $\triangle$ with which he is working, the best distribution of balance over the incidence range. The best balance distribution is governed by the following considerations:

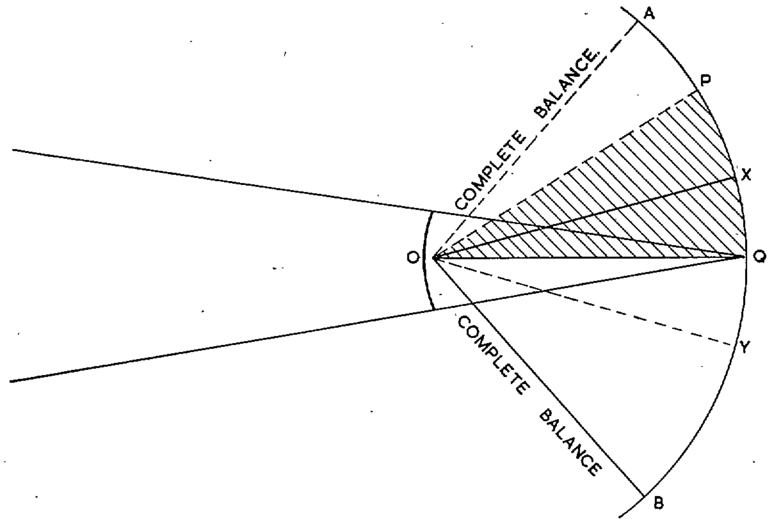
(1) F_0 should be as small as possible.

(2) $F_\triangle$ should be numerically considerably greater than F_0 if sufficient feel is to be retained at slow speed, for the speed-squared factor between diving and landing is at least 20.

(3) Overbalance must be avoided in any part of the range. With a parabolic differential this means that $dF_0/d\xi$ and $dF_\triangle/d\xi$ must both be < 0 when $\xi = 0$, whatever the incidence. That is, ξ_f must not exceed K/λ numerically (*see* equation (13)).

* An aileron is convergent or divergent according as it tends to float upwards or downwards when the incidence is increased. It is null when the floating angle is not affected by incidence.

We can now see in diagrammatic form the solution of the problem.



Convergent Aileron

$PQ = AX = YB = \triangle$

$AQ = QB = K/\lambda$

$AP = XQ = QY = \dfrac{K}{\lambda} - \triangle$

- - - - - - - low speed

———— high speed

Suppose that a convergent aileron ($\triangle$ positive) is to be balanced by a differential λ which may be of either sign. If the floating angle is zero at the high speed limit, the range of floating angles is represented by the sector OPQ, where $PQ = \triangle$. Now the floating angle for complete balance is K/λ; this is represented by OA when the differential is upward, and by OB when it is downward. Adjustment of the floating angle ξ_{f0} by means of a tab is represented by rotation of the sector OPQ *within the limiting sector* OAB. Thus if upward differential is used, the most that can be done through the adjustment is to rotate OPQ to the position OAX through the small upward floating angle AP, thus reaching complete balance at slow speed (OA) and XQ/AQ of complete balance at high speed (OX)*. This is the opposite of what is wanted. If, however, downward differential is used, tab adjustment can rotate OPQ to the position OYB through the large negative floating angle QB, thus reaching complete balance at high speed (OB), and QY/QB of complete balance at low speed (OY). This is what is wanted.

The condition for balancing a divergent aileron ($\triangle$ negative) being exactly the opposite of the above, we are led to the following general rule:

> To obtain good balance by differential, a downward differential, combined with an upward-set tab, must be used to balance a convergent aileron, and an upward differential, combined with a down-set tab, must be used to balance a divergent aileron. If the aileron is null ($\triangle = 0$), differential of either sign may be used, but in this case the balance is invariable over the speed range.

This discussion establishes one favourable factor which seems to be peculiar to differential balance, namely, that the convergence or divergence of the aileron can be used to make the hinge-moment coefficient progressively heavier as the speed falls, and so the speed-squared law can to some extent be defeated.

The above argument has been illustrated in Figs. 11 to 15 by working out the force functions F_0 and $F_\triangle$ for the parabolic gearing $\lambda = \pm 0{\cdot}05$ (Fig. 10) when it is used to balance

(*a*) a convergent aileron, $b_1/b_2 = 1$, $\triangle = 15$ deg. (Figs. 11 and 12),

and (*b*) a divergent aileron, $b_1/b_2 = -1$, $\triangle = -15$ deg. (Figs. 14 and 15).

The value of K is taken to be $1 - \dfrac{1}{5}\, b_1/b_2$.

To round off the argument, results for a null aileron ($\triangle = 0$) are shown in Fig. 13.

---

* The argument is of course limited to $\xi = 0$; balance decreases as ξ increases

Fig. 16 shows the force functions when these parabolic gears are combined with the various ailerons to give the best possible results, that is, when ξ_{f0} is arranged so that in each case there is complete balance, when $\xi = 0$, at one extreme of the incidence range.

9. *Design Conditions for Optimum Balance by Differential with Fixed Tab.*—It will now be clear that if differential gear is to be used as a major means of aileron balance, its design is bound to run counter to accepted practice, which is to balance mainly by other means and to use an upward differential to improve the aileron rolling and yawing moments at large displacements and high incidence; as no more than the natural floating angle is used, the differential balance at high speed is feeble and may be of either sign. Now it is highly probable that an unbalanced aileron will be markedly convergent, and can only be made divergent when it is aerodynamically in close balance. Hence if the designer decides to balance such an aileron by differential, he is virtually committed to a downgoing differential in combination with an upward-set tab. He will enter on the credit side of his design account:—

(1) A direct attack on his main objective—fine balance at small displacements at high speed, coarse balance at large displacements at low speed—which is very difficult to attain by any other means.

(2) Suppression of most of the serious troubles arising from lack of hinge-moment linearity in common types of nose balance, such as the Frise.

On the debit side of his account there will appear :—

(3) Some loss of rolling moment and increase in adverse yawing moment due to the sign of the differential.

(4) Increases in load in the control system, in pitching moment and in drag, due to the tab.

As regards (3) a very rough idea of the losses involved in a downward differential is given in Figs. 17 to 19, which show the variations in rolling and yawing moment coefficients (wind axes) when the differentials of Fig. 6 are applied to results for plain ailerons on rectangular wings given in Table 2 of Ref. 5. The loss in rolling moment is due to the earlier breakdown in flow over the downgoing aileron. This can be retrieved to some extent by the use of a slot. The increase in adverse yawing moment is due partly to increase in profile drag of the downgoing aileron and partly to increased induced drag moment; the former, but not the latter, is recoverable with a slotted aileron. Lacking full-scale evidence, it is impossible to say how serious the adverse yaw effect is likely to be, but the fact that opposing moments from rudder power and weathercock stability are both much greater now than formerly is a strongly mitigating factor. Certainly the ill effects of adverse yaw are much less noticeable now than they used to be, and the possibility of trouble from this source should not act as a deterrent to exploration of the full potentialities of differential balance.

As regards (4), the loads due to the tab, let $\delta\xi_f$ be the increment in floating angle which it provides. Then the neutral load in the control circuit is simply that due to a hinge-moment coefficient $b_2 \delta \xi_f$. Again it appears from the multi-flap theory of R. & M. 1171[6] that the corresponding increment in pitching moment coefficient about the quarter-chord point is roughly represented by

$$\delta C_{M0} = 0 \cdot 1 \; \delta\xi_f$$

over the part of the wing covered by the tab, the angle being measured in radians. Thus if the tab increases the downfloating angle by 20 deg., it produces a local increase of C_{M0} of about $0 \cdot 03$ and an average increase of C_{M0} over the whole wing of about $0 \cdot 01$.

10. *Upward Differential to Balance Convergent Ailerons.* The necessity of using downward differential for close balance of convergent ailerons arises only if the differential tab is fixed. Upward differential can be used in association with a tab which is arranged to move downwards as the incidence decreases. This could be achieved by way of the fore-and-aft stability by arranging a suitable gearing between the differential tab and the fore-and-aft movement of the stick.

Apart from a tendency to give less balance in a turn than in straight flight at the same incidence, this should be aerodynamically quite satisfactory. There is, however, no need to face the mechanical complication of this scheme until it has been proved in flight that downward differential has serious drawbacks.

11. *Conclusions.*—(1) The essence of balance by differential is that the eccentricity ε varies with the displacement ξ, and that the naturally small floating angle near no-lift is augmented by a tab so that it exceeds ε numerically over the range of ξ. It is theoretically possible to design the gear so that the hinge moment remains linear with displacement, but with the simple crank system in common use the heaviness increases with the displacement.

(2) The main object of current aileron design is to get the balance as close as possible for small displacements at low incidence while retaining coarser balance for large displacements at high incidence, thus defeating the speed-squared law as far as may be. If the aileron is convergent this can be arranged by using a downward differential with an upward-set tab; and if divergent, by an upward differential with a down-set tab. If the aileron is null either differential system may be used, but in this case the balance will be invariable with incidence.

(3) There is a strong case for exploring in flight the possibilities of balancing an initially unbalanced aileron by differential action alone. The downward differential may have some unfavourable effects on rolling and yawing moment at slow speed; this can only be determined by full-scale work. Close balance can probably be obtained by using a fairly small tab set at an angle not exceeding 20 deg.; the loads introduced by the tab are not prohibitive.

(4) If downward differential should prove to have serious effects on aileron control, there remains the possibility of balancing convergent ailerons by upward differential combined with a tab geared to the fore-and-aft movement of the stick.

Notation

ξ_u up aileron angle.

ξ_d down aileron angle.

$\xi = \dfrac{\xi_u + \xi_d}{2}$, displacement.

$\varepsilon = \dfrac{\xi_u - \xi_d}{2}$, eccentricity.

$D = \xi_u/\xi_d$ at maximum displacement.

P pilot's force in direction of movement of stick.

x movement of pilot's hand.

S_ξ total aileron area.

c_ξ aileron mean chord.

$C_{H,u}$, $C_{H,d}$ hinge-moment coefficients of the up and downgoing ailerons, referred to the area of one aileron.

$C_H = b_0 + b_1$ (mean aileron incidence) $+ b_2$ (aileron displacement).

K response factor $= 1 - n\dfrac{b_1}{b_2}\left(n \text{ is taken as } \dfrac{1}{5}\right)$.

$p = \dfrac{P}{S_\xi c_\xi q}$.

m gearing between stick and aileron in radians per foot when there is no differential, $= \xi_{max.}/x_{max.}$

$\mu = \dfrac{1}{m}\dfrac{d\xi}{dx}$.

F $\quad \dfrac{p}{mKb_2} = \dfrac{P}{mKb_2 S_\xi \, c_\xi \, q}$, force function.

k factor by which the force at any displacement is multiplied by the differential gear.

λ $\quad \dfrac{d\varepsilon}{d\xi} = \lambda\xi$ for parabolic gear.

ξ_f floating angle, positive when upward.

ξ_{f0} floating angle at minimum incidence.

$\triangle$ increase in floating angle between minimum and maximum incidence.

$\delta\xi_f$ increment in floating angle provided by tab.

F_0 force function at minimum incidence.

$F_\triangle$ force function at maximum incidence.

APPENDIX

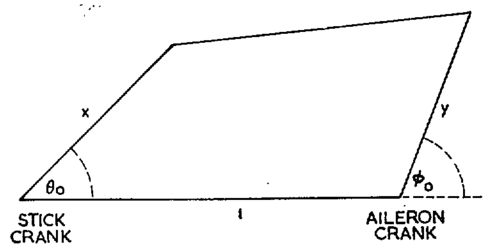
Geometry of Differential Gear

Differential gearing as used at present consists essentially of a stick crank and an aileron crank, which may be of different radii, one or both of which are set eccentrically in the neutral position. Actually of course the control system must make a right-angle turn between stick and aileron, but the general features of the system will be obtained by confining the geometry to two dimensions, and in the common case when the crank radii are small compared with the distance between their centres, the approximation is probably fairly close.

Taking the distance between the crank centres as the unit of length, the neutral position of the system is defined by

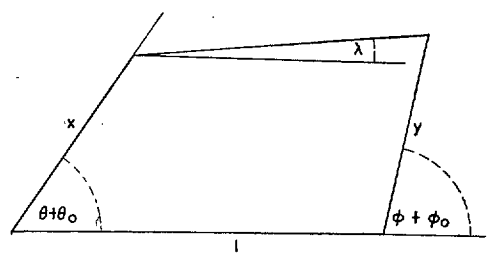
x, y the radii respectively of stick and aileron crank,

and θ_0, ϕ_0 the angular settings of the crank, as shown below.



Neutral Position.

When the stick is displaced through an angle θ, let the up-aileron movement be ϕ and the down-aileron movement ϕ'. Then the displaced position for the up-aileron movement will be as shown below:—



Displaced Position.

Considering a small change $d\theta$, $d\phi$ from this displaced position we have

$$x\,d\theta\,.\,\cos\left(\frac{\pi}{2} - (\theta + \theta_0) + \lambda\right) = y d\phi\,.\,\cos\left(\frac{\pi}{2} - (\phi + \phi_0) + \lambda\right),$$

or

$$\frac{d\phi}{d\theta} = \frac{x \sin(\theta + \theta_0 - \lambda)}{y \sin(\phi + \phi_0 - \lambda)}, \qquad \ldots \qquad (1)$$

where

$$\tan \lambda = \frac{y \sin(\phi + \phi_0) - x \sin(\theta + \theta_0)}{1 - x \cos(\theta + \theta_0) + y \cos(\phi + \phi_0)};$$

and so, eliminating λ, we have

$$\frac{d\phi}{d\theta} = \frac{x}{y} \cdot \frac{\sin(\theta + \theta_0) + y \sin(\theta_0 - \phi_0 + \theta - \phi)}{\sin(\phi + \phi_0) + x \sin(\theta_0 - \phi_0 + \theta - \phi)}. \qquad \ldots \qquad (2)$$

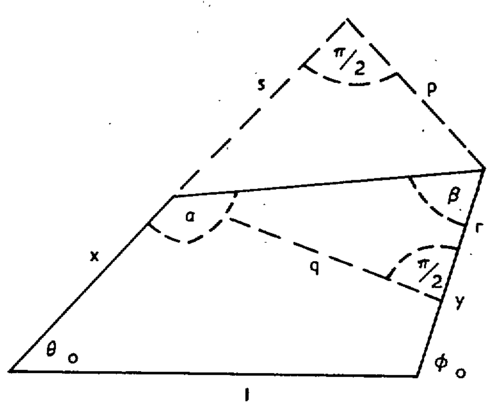
If θ and ϕ are small, this can be expanded in the form

$$\frac{d\phi}{d\theta} = a_0 + a_1\theta + a_2\phi\,, \qquad \ldots \qquad (3)$$

where

$$\left.\begin{aligned} a_0 &= \frac{\sin\theta_0 - y\sin(\phi_0 - \theta_0)}{\sin\phi_0 - x\sin(\phi_0 - \theta_0)} \cdot \frac{x}{y} = \frac{xp}{yq}, \\ \frac{a_1}{a_0} &= \frac{\sin\phi_0\{\cos\theta_0 - x + y\cos(\phi_0 - \theta_0)\}}{\{\sin\theta_0 - y\sin(\phi_0 - \theta_0)\}\{\sin\phi_0 - x\sin(\phi_0 - \theta_0)\}} = \frac{s\sin\phi_0}{pq}, \\ \frac{a_2}{a_0} &= \frac{-\sin\theta_0\{\cos\phi_0 + y - x\cos(\phi_0 - \theta_0)\}}{\{\sin\theta_0 - y\sin(\phi_0 - \theta_0)\}\{\sin\phi_0 - x\sin(\phi_0 - \theta_0)\}} = \frac{-r\sin\theta_0}{pq}, \end{aligned}\right\} \qquad (4)$$

and p, q, r, s are as shown below:—



Thus $a_1 = 0$ if $\alpha = \pi/2$, and $a_2 = 0$ if $\beta = \pi/2$.

If in addition x and y also are small we have from (2)

$$\frac{d\phi}{d\theta} = \frac{x}{y} \cdot \frac{\sin(\theta + \theta_0)}{\sin(\phi + \phi_0)}, \qquad \ldots \qquad (2')$$

$$\left.\begin{aligned} a_0 &= \frac{x \sin\theta_0}{y \sin\phi_0}, \\ \frac{a_1}{a_0} &= \cot\theta_0\,, \\ \frac{a_2}{a_0} &= -\cot\phi_0\,. \end{aligned}\right\} \qquad \ldots \qquad (4')$$

In this case $a_1 = 0$ if $\theta_0 = \pi/2$ and $a_2 = 0$ if $\phi_0 = \pi/2$.

Similarly, if ϕ' is the movement of the downward aileron corresponding to the stick movement θ we have

$$\frac{d\phi'}{d\theta} = a_0 - a_1\theta - a_2\phi' . \qquad \qquad (5)$$

It follows now from (3) and (5), by writing

$$\xi = \frac{\phi + \phi'}{2} , \quad \varepsilon = \frac{\phi - \phi'}{2} ,$$

(as in the main analysis), that

$$\left.\begin{aligned} \frac{d\xi}{d\theta} &= a_0 + a_2\varepsilon \\ \frac{d\varepsilon}{d\theta} &= a_1\theta + a_2\xi \end{aligned}\right\} \qquad \qquad (6)$$

These are equivalent to

$$\frac{d^2\xi}{d\theta^2} = a_2{}^2\xi = a_1a_2\theta ,$$

$$\frac{d^2\varepsilon}{d\theta^2} = a_2{}^2\varepsilon = a_1 + a_0a_2 ,$$

and so they may be integrated in the form

$$\left.\begin{aligned} \xi &= \left(\frac{a_0}{a_2} + \frac{a_1}{a_2{}^2}\right) \sinh a_2\theta - \frac{a_1}{a_2}\,\theta \\ \varepsilon &= \left(\frac{a_0}{a_2} + \frac{a_1}{a_2{}^2}\right) (\cosh a_2\theta - 1) \end{aligned}\right\} \qquad \qquad (7)$$

Special Cases.—(*a*) If $a_1 = 0$ we have

$$\xi = \frac{a_0}{a_2} \sinh a_2\theta ,$$

$$\varepsilon = \frac{a_0}{a_2} (\cosh a_2\theta - 1) ,$$

and so
$$\left(\frac{a_2}{a_0}\,\varepsilon + 1\right)^2 - \left(\frac{a_2}{a_0}\,\xi\right)^2 = 1 .$$

Thus the variation of ε with ξ is hyperbolic.

If x, y are small, we have in this case $\theta_0 = \pi/2$ and $a_2/a_0 = -\cot\phi_0$.

Hence the eccentricity ε is positive or negative (*i.e.* ϕ is greater or less than ϕ') according as ϕ_0 is greater or less than $\pi/2$. The differential depends only on ϕ_0 and is independent of x/y.

(*b*) If $a_2 = 0$ we have from (6)

$$\xi = a_0\theta ,$$
$$\varepsilon = \tfrac{1}{2}\, a_1\theta^2 .$$

Thus $\varepsilon = \dfrac{a_1}{a_0{}^2}\,\dfrac{\xi^2}{2}$ and the variation of ε with ξ is parabolic.

This is the case which has been used for illustration in the aerodynamic analysis.

If x and y are small we have $\phi_0 = \pi/2$

and $$\frac{a_1}{a_0^2} = \frac{y \cos \theta_0}{x \sin^2 \theta_0}.$$

Thus the eccentricity ε is positive or negative according as θ_0 is less than or greater than $\pi/2$, and is increased by making the aileron crank larger than the stick crank.

REFERENCES

No.	Author	Title, etc.
1	R. T. Jones and A. I. Nerken ..	The Reduction of Aileron Operating Force by Differential Linkage. N.A.C.A. Technical Note No. 586. (December, 1936.)
2	H. A. Soulé and J. A. Hootman ..	A Flight Investigation of the Reduction of Aileron Operating Force by Means of Fixed Tabs and Differential Linkage, with Notes on Linkage Design. N.A.C.A. Technical Note No. 653. (June, 1938.)
3	S. B. Gates and H. B. Irving ..	An Analysis of Aileron Performance. A.R.C. 4652. (July, 1940.)
4	F. E. Weick and R. T. Jones	Resumé and Analysis of N.A.C.A. Lateral Control Research. N.A.C.A. Report No. 605. (1937.)
5	F. E. Weick and C. J. Wenzinger ..	Wind Tunnel Research Comparing Lateral Control Devices Particularly at High Angles of Attack. I. Ordinary Ailerons on Rectangular Wings. N.A.C.A. Report No. 419. (1932.)
6	W. G. A. Perring	Theoretical Relationships for an Aerofoil with a Multiple-Hinged Flap System. R. & M. 1171. (April, 1928.)

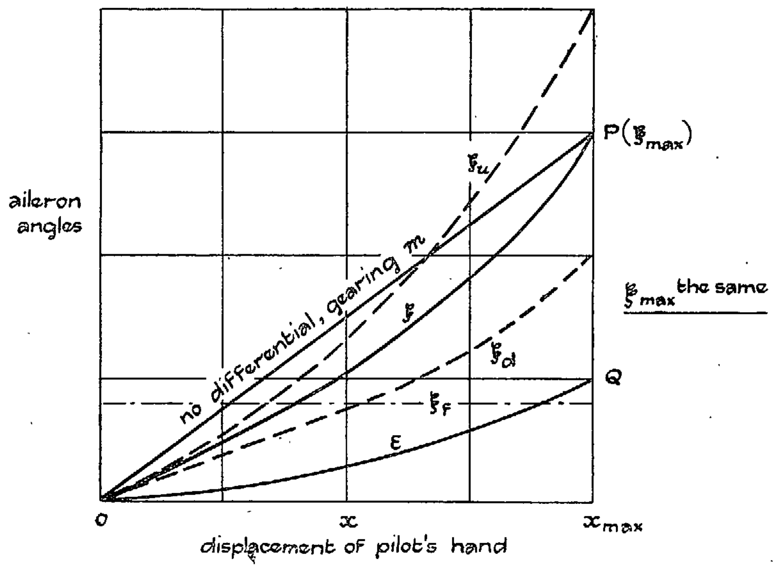


FIG. 1.

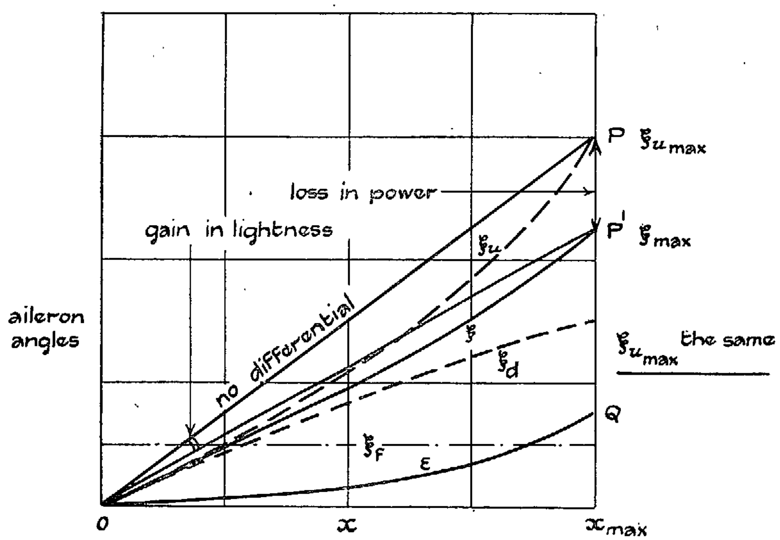


FIG. 2. Comparisons between Differential and Non-differential Systems.

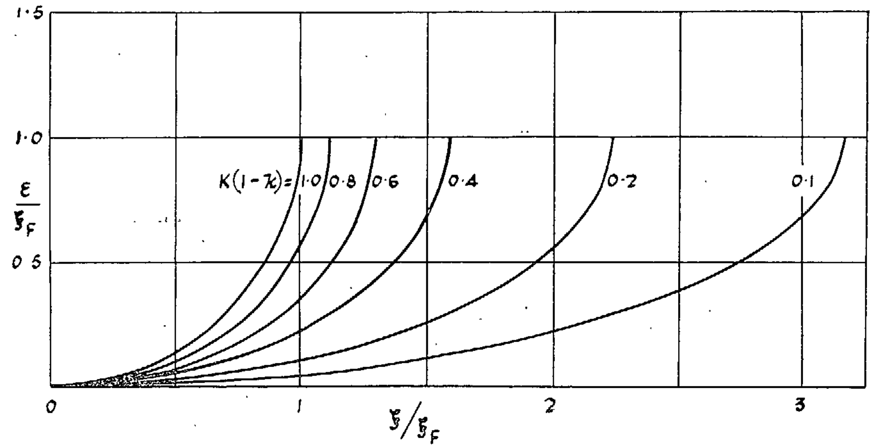


FIG. 3. Eccentricity for Constant Balance by Differential. Differential decreases force in the ratio k : 1, $d\xi/dx$ assumed constant.

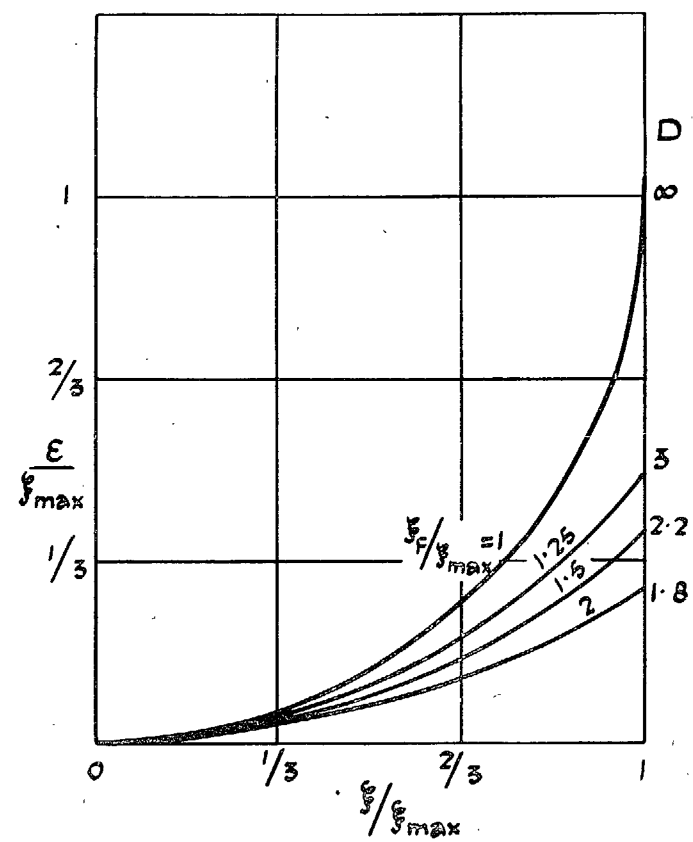


FIG. 4. Eccentricity for Complete Balance by Differential ($k = 0$) when $K = 1$.

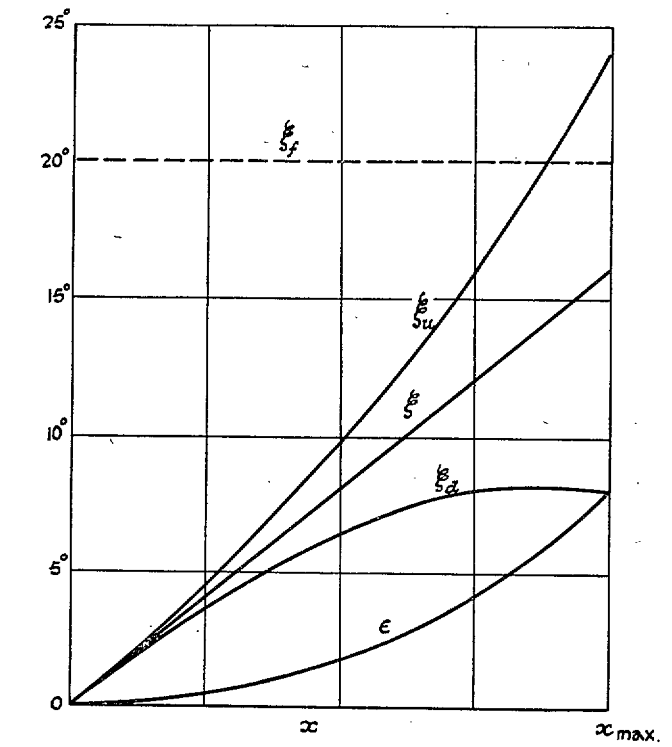


FIG. 5. Differential Gear which would give Zero Force throughout the Stick Movement when $K = 1$. Maximum displacement 16 deg. Up-floating angle 20 deg.

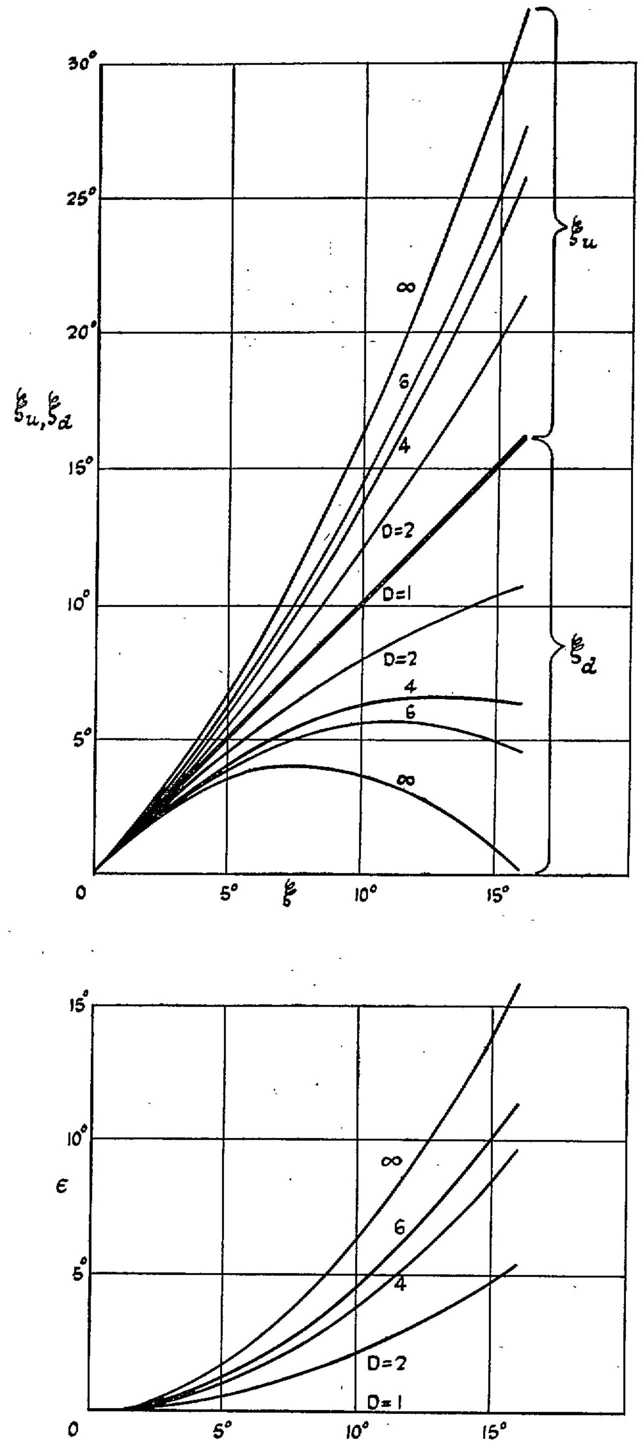


FIG. 6. Parabolic Differentials with same $\xi_{max.} = 16$ deg.

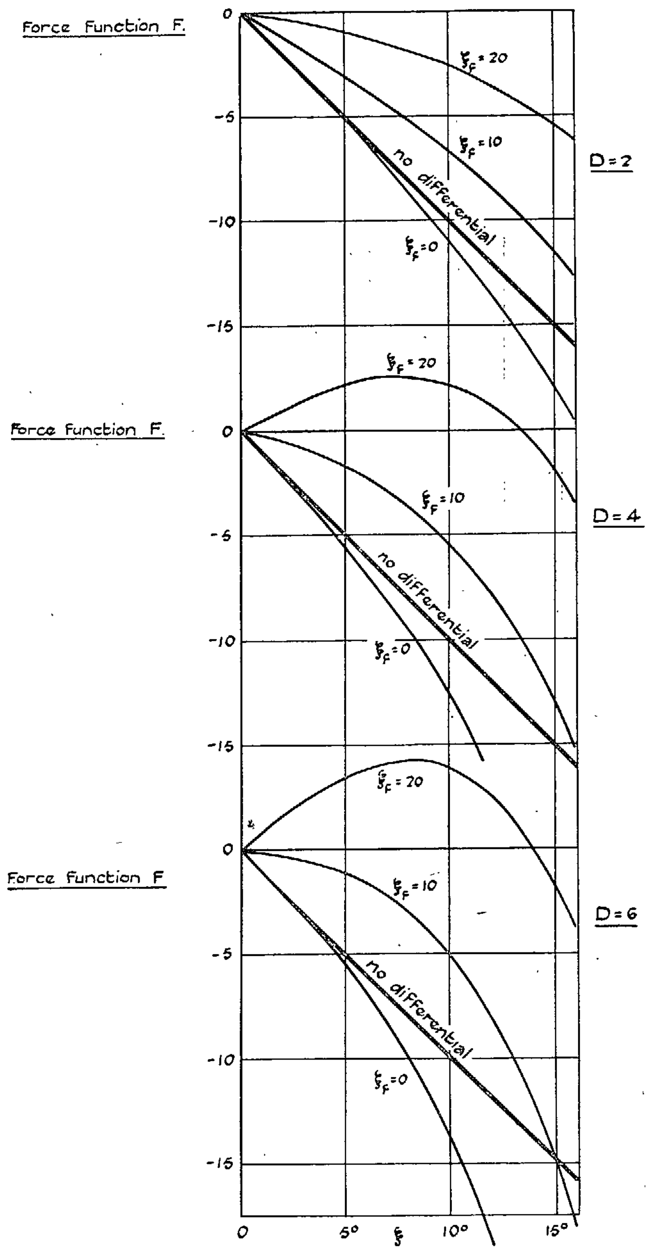


FIG. 7. Parabolic Differentials with same $\xi_{max.} = 16$ deg. ($b_1 = 0$).

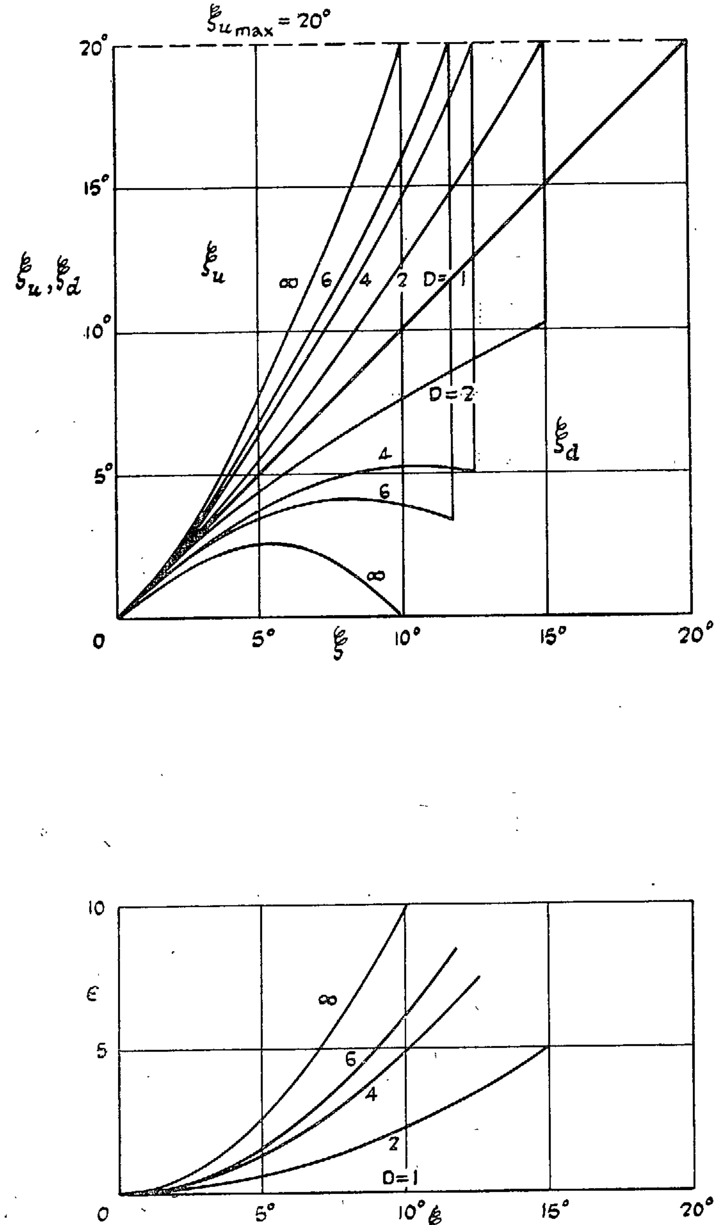


FIG. 8. Parabolic Differentials with the same $\xi_{u\,max.} = 20$ deg.

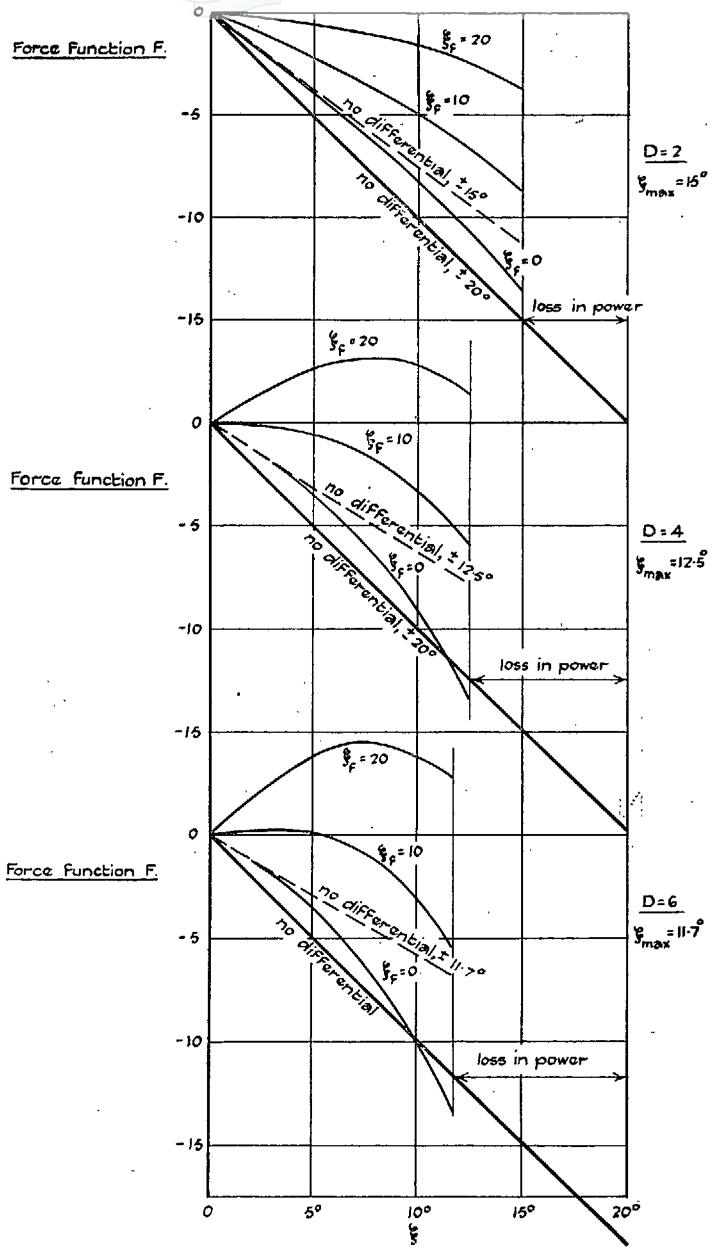


FIG. 9. Parabolic Differentials with the same $\xi_{u\,max.} = 20$ deg. ($b_1 = 0$).

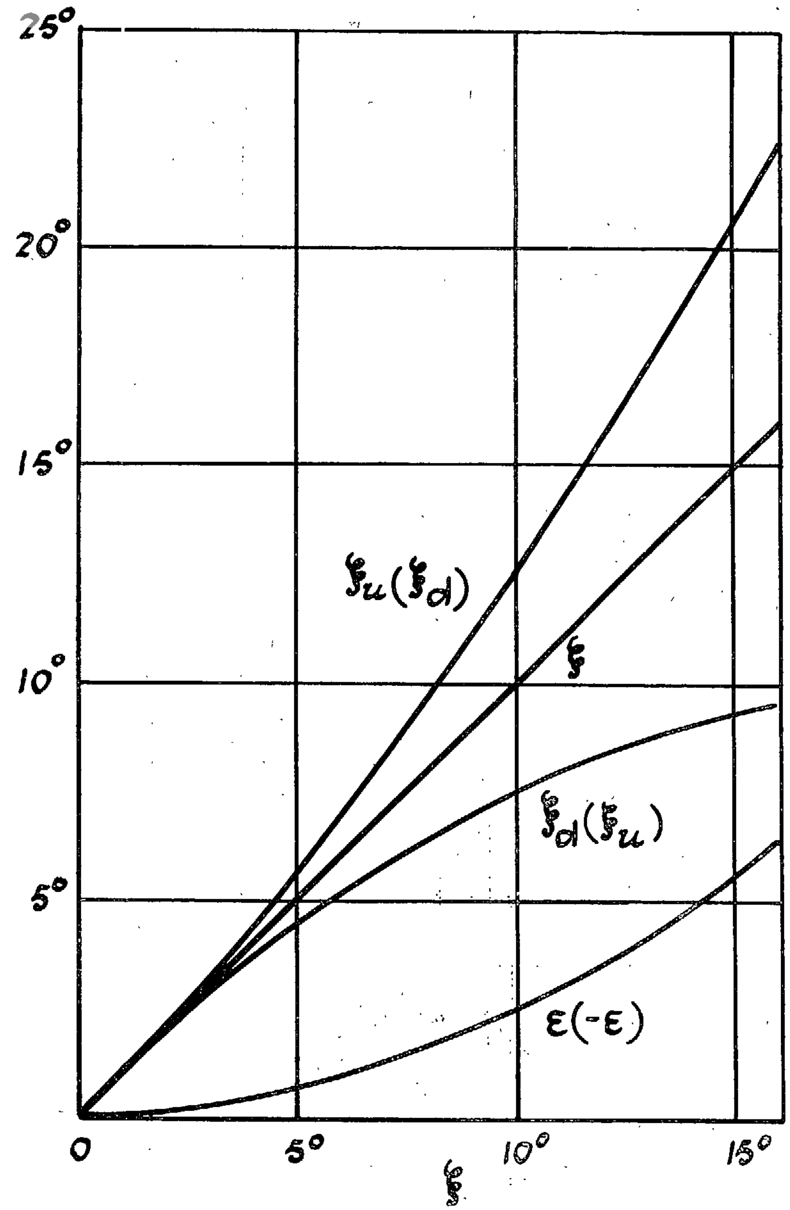


FIG. 10. Parabolic Differential ($\lambda = \pm 0{\cdot}05$) used as Illustration in Figs. 11–15.
Unbracketed symbols refer to upward differential ($\lambda = 0{\cdot}05$).
Bracketed symbols refer to downward differential ($\lambda = -0{\cdot}05$).

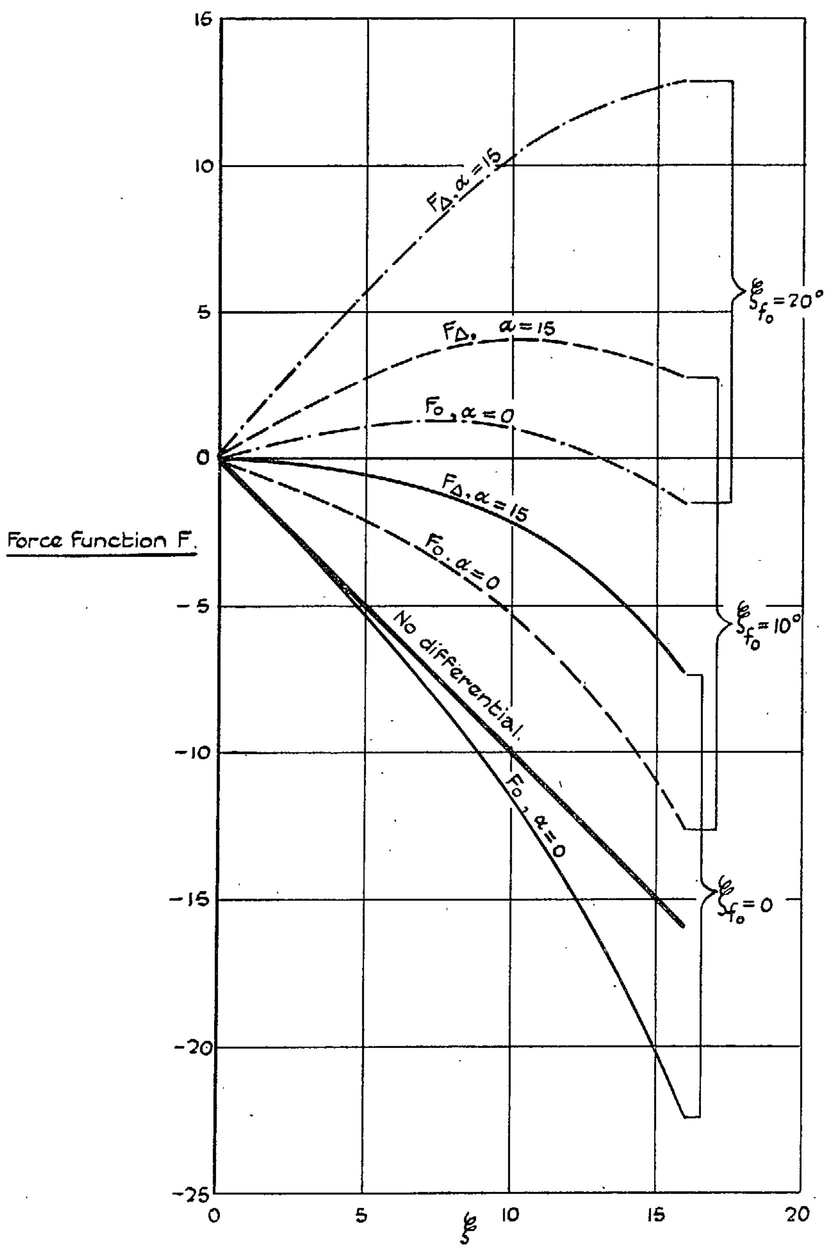


FIG. 11. Convergent Aileron ($b_1/b_2 = 1$, $\triangle = 15$ deg.) balanced by Upward Parabolic Differential ($\lambda = 0 \cdot 05$).

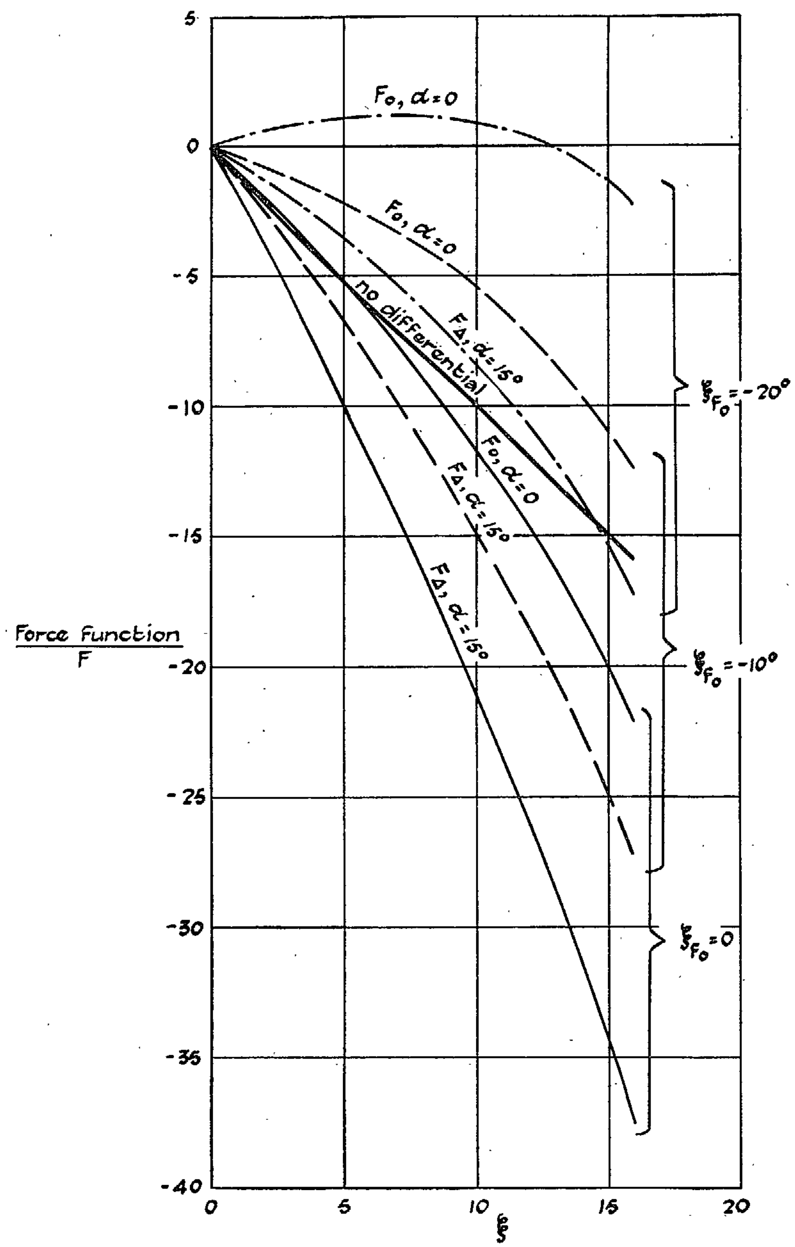


FIG. 12. Convergent Aileron ($b_1/b_2 = 1$, $\triangle = 15$ deg.) balanced by Downward Parabolic Differential ($\lambda = - 0 \cdot 05$).

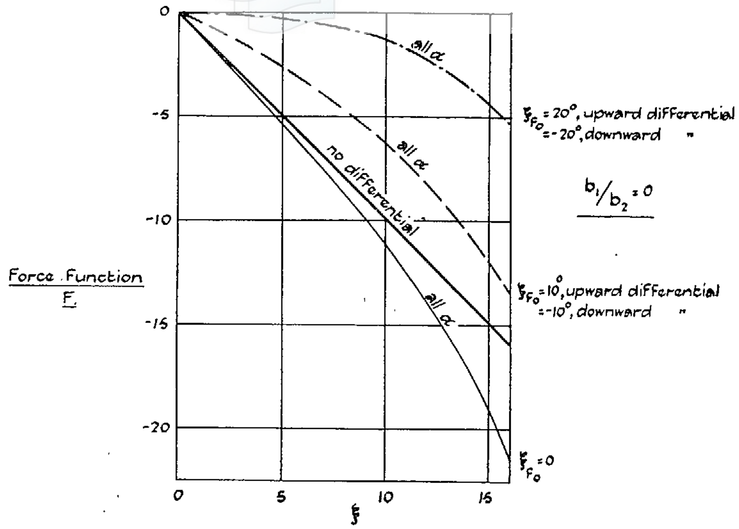


FIG. 13. Null Aileron ($b_1 = 0$, $\triangle = 0$) balanced by Parabolic Differential $\lambda = \pm 0 \cdot 05$).

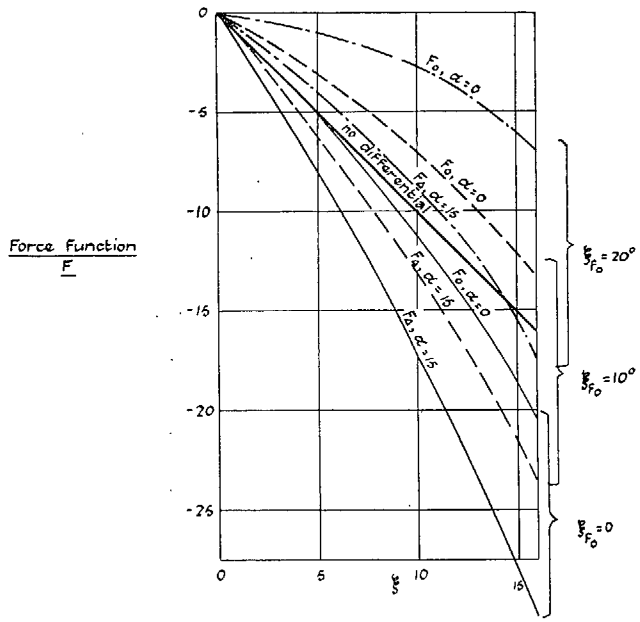


FIG. 14. Divergent Aileron ($b_1/b_2 = -1$, $\triangle = -15$ deg.) balanced by Upward Parabolic Differential ($\lambda = 0 \cdot 05$).

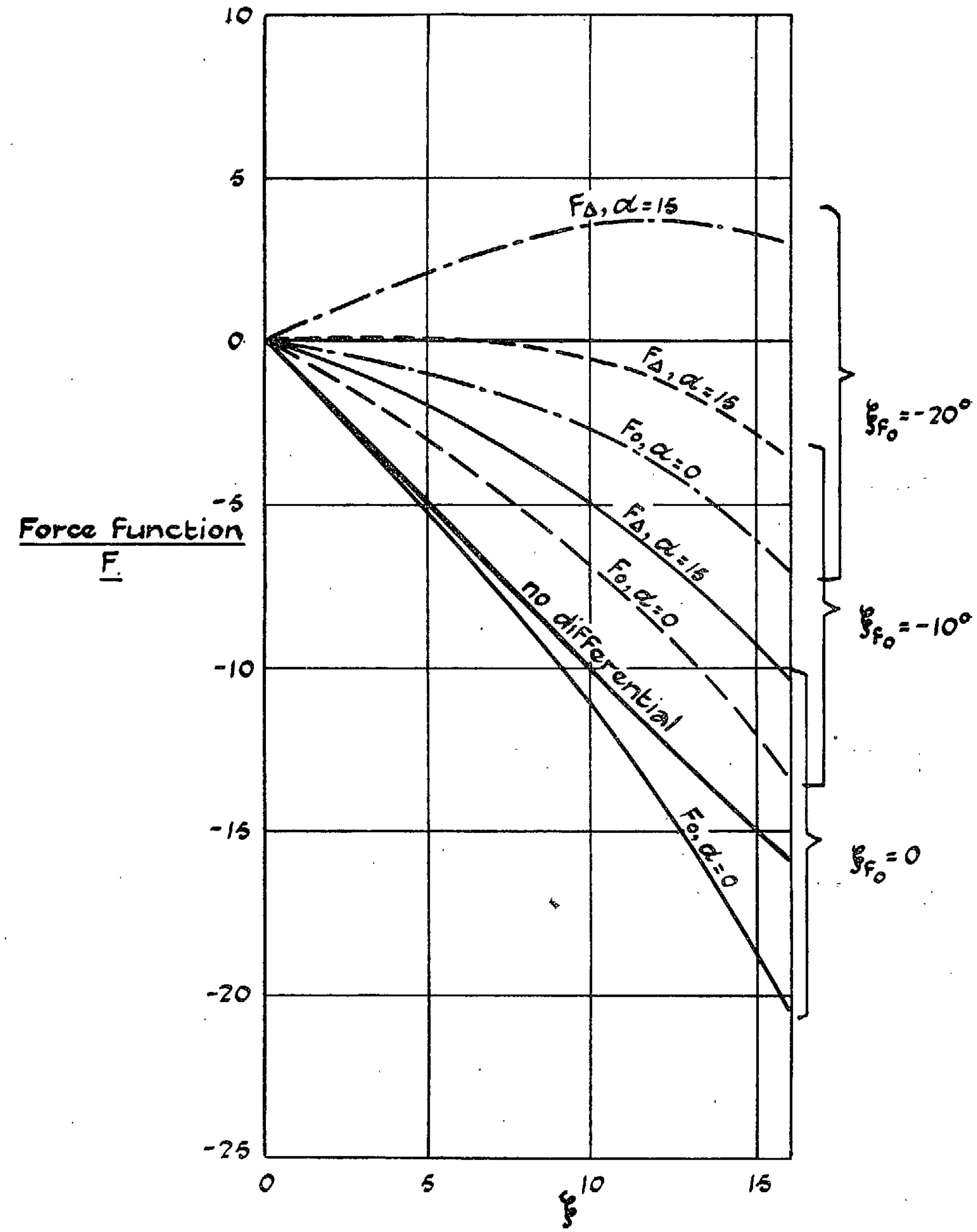


FIG. 15. Divergent Aileron ($b_1/b_2 = -1$, $\triangle = -15$ deg.) balanced by Downward Parabolic Differential ($\lambda = -0 \cdot 05$).

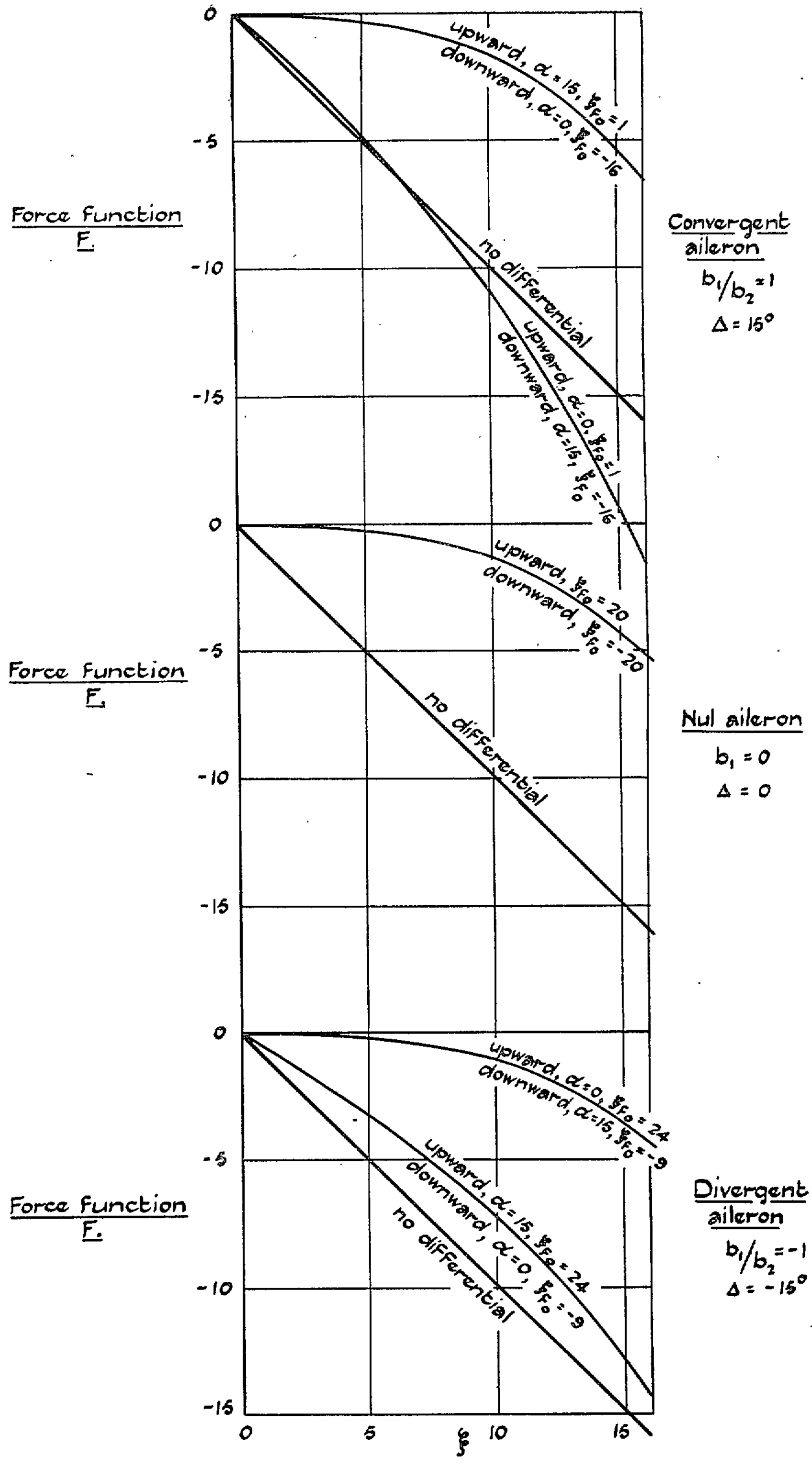


FIG. 16. Force Functions when Parabolic Gearing $\lambda = \pm 0{\cdot}05$ is Combined with Various Ailerons to give the Best Results.

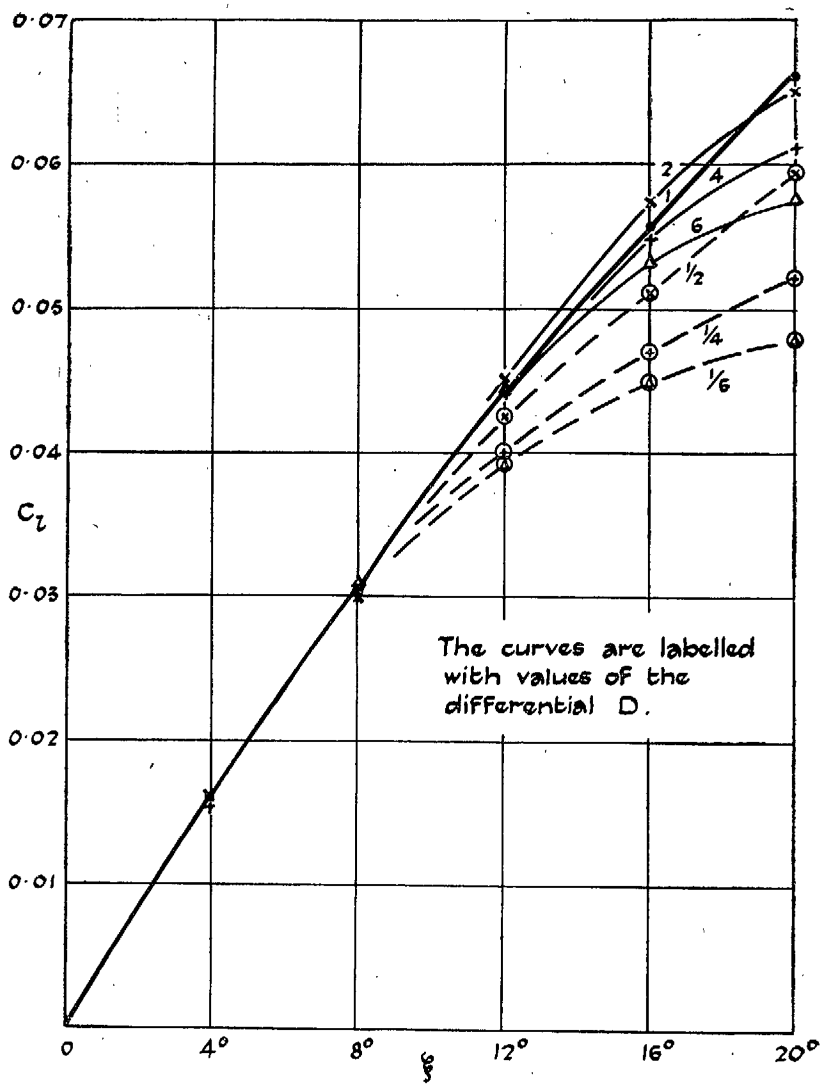


FIG. 17. Effect of Differential on Rolling Moment (Plain ailerons, rectangular wing). Low incidence, $\alpha = 0$ deg.

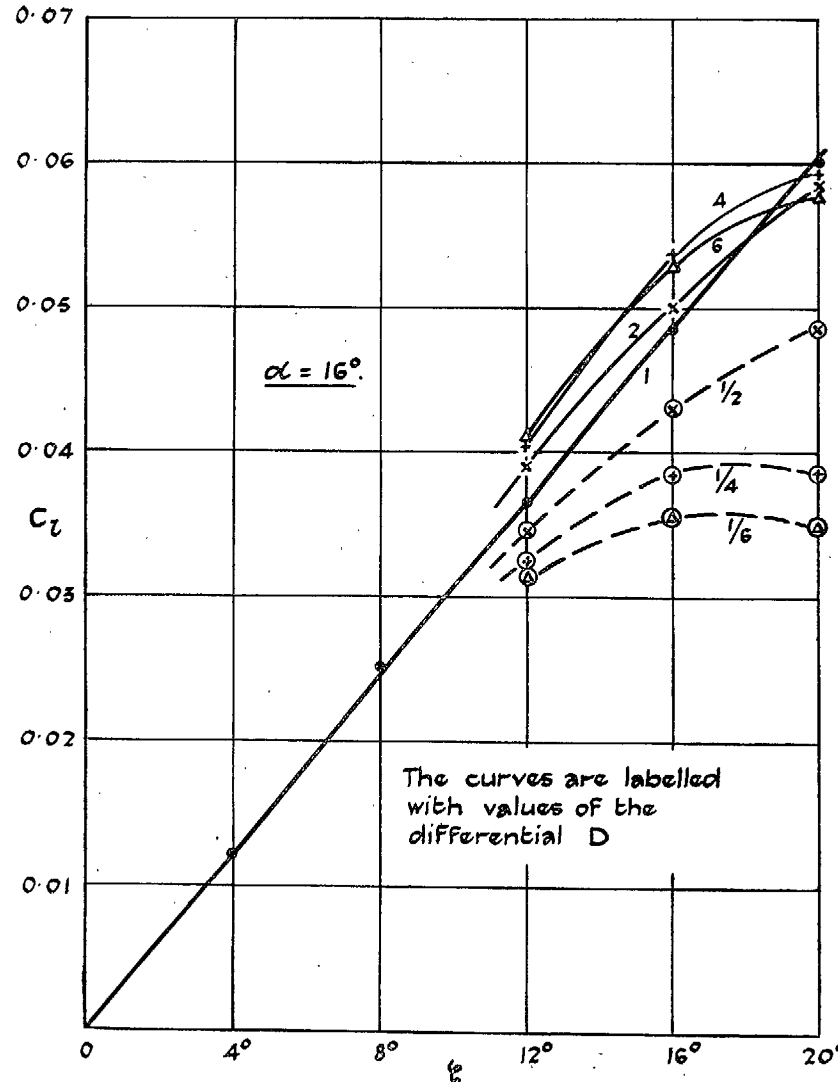


FIG. 18. Effect of Differential on Rolling Moment (Plain ailerons, rectangular wing). High incidence, $\alpha = 16$ deg.

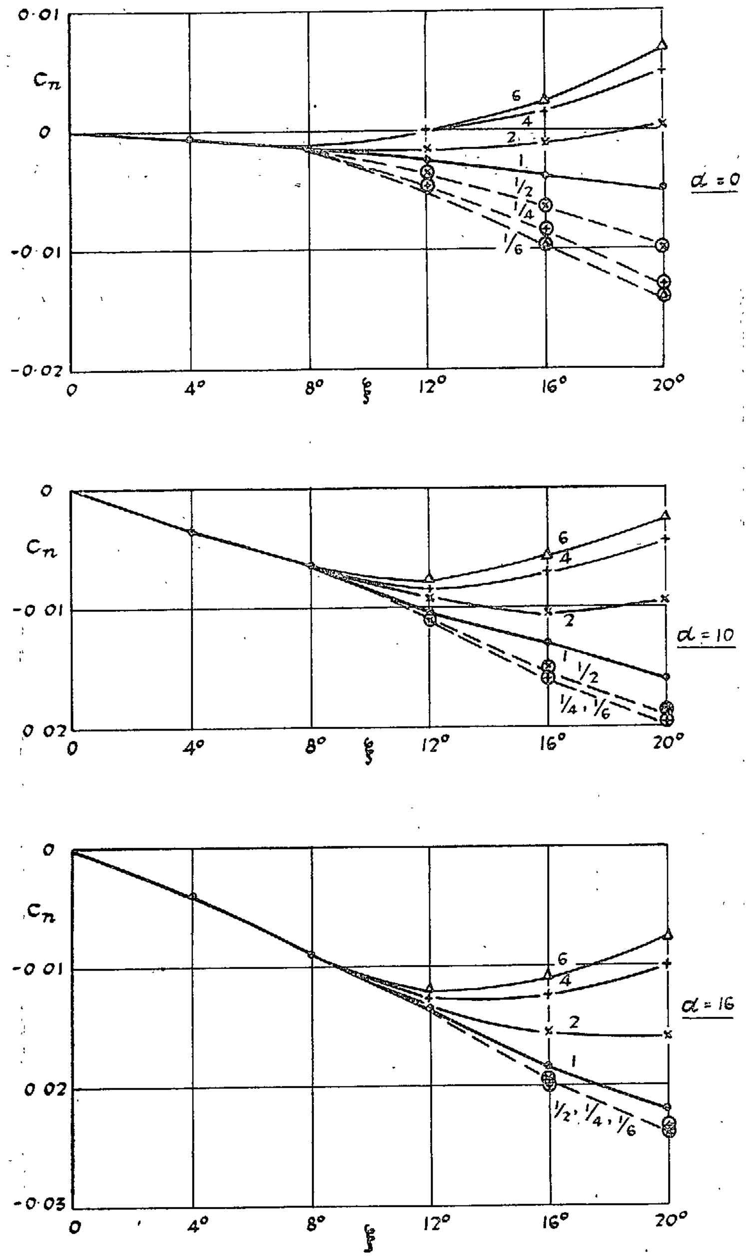


FIG. 19. Effect of Differential on the Yawing Moment (Wind axes) associated with a Positive Rolling Moment (Plain ailerons, rectangular wing).

Negative C_n indicates adverse yaw. The curves are labelled with values of the differential D.

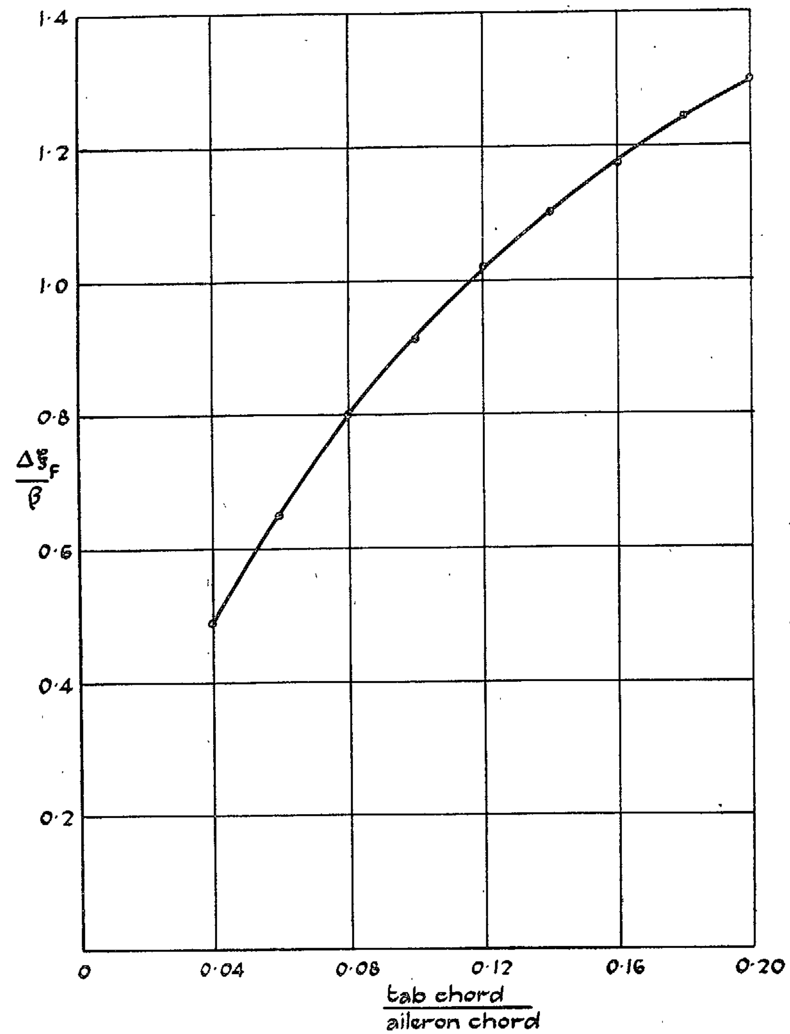


FIG. 20. Increase in Floating Angle ($\triangle\xi_F$) Produced by Tab Angle β—Plain Aileron.

(94980) Wt. 14/806 K. 5 4/51 Hw.

PRINTED IN GREAT BRITAIN

R. & M. No. 2526
(4921)
A.R.C. Technical Report

Publications of the Aeronautical Research Committee

TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE—

1934–35 Vol. I. Aerodynamics. 40s. (40s. 8d.)
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40s. (40s. 8d.)

1935–36 Vol. I. Aerodynamics. 30s. (30s. 7d.)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30s. (30s. 7d.)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 9d.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50s. (50s. 10d.)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 9d.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60s. (61s.)

1938 Vol. I. Aerodynamics General, Performance, Airscrews, 50s. (51s.)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30s. (30s. 9d.)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (50s. 11d.)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (64s. 2d.)

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COMMITTEE—

1933–34	1s. 6d. (1s. 8d.)
1934–35	1s. 6d. (1s. 8d.)
April 1, 1935 to December 31, 1936.	4s. (4s. 4d.)
1937	2s. (2s. 2d.)
1938	1s. 6d. (1s. 8d.)
1939-48	*In the press*

INDEXES TO THE TECHNICAL REPORTS OF THE ADVISORY COMMITTEE ON AERONAUTICS—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1s. 3d. (1s. 5d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1s. (1s. 2d.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1s. (1s. 1d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1s. 3d. (1s. 4d.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1s. 3d. (1s. 4d.)

Prices in brackets include postage.

Obtainable from

His Majesty's Stationery Office

York House, Kingsway, LONDON, W.C.2 429 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller.

S.O. Code No. 23-2526