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Summary.—A method of calculation of the rate of heat transfer from the surface of an aerofoil maintained at a temperature above that of the stream was required, including allowance for the effect of dissipation of energy in the boundary layer. A convenient method of calculation is developed for laminar boundary layers, and the best method of applying Reynolds's analogy to the turbulent layer is discussed. The methods are applied to calculate the heat transfer from the aerofoils N.A.C.A. 2409 and 2415 at $C_L = 0 \cdot 24$ and $C_L = 0 \cdot 8$.

Simple formulae for the rise in surface temperature due to dissipation are derived.

CONTENTS

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1. *Introduction.*—The use of exhaust heat for the prevention of ice formation on wings and propellers makes it desirable to be able to calculate the rate of heat transfer from aerofoils maintained at a temperature above that of the stream. It is also desirable to be able to estimate the heating effect of dissipation in the boundary layer. This report deals with these aspects both for laminar and for turbulent boundary layers.

The laminar layer case is straightforward in the sense that the governing equations are known and the problem is to determine their solution in a convenient form. For the turbulent layer, it is, however, necessary to rely on the extended form of Reynolds's analogy, which is certainly not strictly valid in regions where the boundary layer is subjected to pressure gradients. Proper clarification of this matter will have to await more detailed knowledge of flow in turbulent boundary layers.

2. *Notation.*—The following notation will be used :—

Suffix 0 denotes values in the undisturbed stream.

Suffix 1 denotes values at the surface of the body.

T	Temperature at a point in the fluid.
T'	Temperature at edge of boundary layer.
T_0	Temperature in free stream.
T_1	Temperature at surface.
ΔT_1	Kinetic temperature rise at surface.
J	Mechanical equivalent of heat.
c_p	Specific heat at constant pressure.
c_v	Specific heat at constant volume.
x	Distance measured along the surface from the stagnation point.
x'	Distance measured along the surface from the leading edge.
y	Distance measured normal to the surface.
δ_1	Displacement thickness of boundary layer.
θ	Momentum thickness of boundary layer.
δ_2	Displacement thickness of thermal boundary layer.
u	Velocity in boundary layer parallel to surface.
U	Velocity at edge of boundary layer.
U_0	Stream velocity.
k	Thermal conductivity.
ρ	Density.
$\varkappa = k/\rho c_p$	Thermometric conductivity.
$\sigma = \nu/\varkappa$	Prandtl number.
ν	Kinematic viscosity.
d	Diameter of circular cylinder.
c	Aerofoil chord.
Q	Rate of heat transfer per unit area.
R	Reynolds number.
Nu	Nusselt heat transfer coefficient $\dfrac{lQ}{k(T_1 - T_0)}$ where l is a representative length.
k_H	Heat transfer coefficient $\dfrac{Q}{\rho c_p U_0(T_1 - T_0)}$ based on stream velocity.
k_H'	Heat transfer coefficient $\dfrac{Q}{\rho c_p U(T_1 - T_0)}$ based on velocity at edge of boundary layer.
ε	Total energy increment per unit mass defined as $(Jc_pT + \frac{1}{2}u^2) - (Jc_pT_0 + \frac{1}{2}U_0^2)$

3. *Laminar Boundary Layers.*

3.1. *Heat Transfer for Laminar Boundary Layers.*—The method of analysis for calculating heat transfer in laminar flow is analogous to that put forward in Ref. 1 for the calculation of laminar skin friction and consists in adopting a standard shape of velocity and temperature distribution across the boundary layer, the thicknesses of the momentum and thermal boundary layers then being determined by the momentum and energy equations. The selected standard shape for the velocity and temperature distributions is the Blasius velocity distribution for the flat plate.

Considering first the frictional boundary layer, the solution of the momentum equation may be taken to be[1]

$$\theta^2 = \frac{0 \cdot 441 \nu}{U^6} \int_0^x U^5 dx,$$

where x is measured along the surface from the forward stagnation point, θ is the momentum thickness and U is the velocity outside the boundary layer. For the Blasius velocity distribution $\delta_1/\theta = 2 \cdot 59$, where δ_1 is the displacement thickness, so that

$$\delta_1{}^2 = \frac{2 \cdot 960 \nu}{U^6} \int_0^x U^5 dx. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Let $\frac{u}{U} = f\left(\frac{0 \cdot 8604 y}{\delta_1}\right)$ be the Blasius velocity distribution, which is tabulated on p.136 of Ref. 2, where u is the velocity at a point distant y from the surface. The factor $0 \cdot 8604$ is introduced so that we shall have, according to the definition of δ_1,

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \frac{\delta_1}{0 \cdot 8604} \int_0^\infty [1 - f(\eta_1)] d\eta_1,$$

where

$$\eta_1 = \frac{0 \cdot 8604 y}{\delta_1} \; ;$$

this is satisfied, since

$$\int_0^\infty [1 - f(\eta_1)] d\eta_1 = 0 \cdot 8604.$$

For the thermal boundary layer we define a thermal displacement thickness δ_2 by the equation

$$\delta_2 = \int_0^\infty \left[\frac{T - T_0}{T_1 - T_0}\right] dy,$$

where T is the temperature at a point distant y from the surface, T_1 is the surface temperature and T_0 is the temperature in the free stream. It is assumed that the temperature distribution is similar to the velocity distribution, so that

$$\frac{T - T_0}{T_1 - T_0} = 1 - f\left(\frac{0 \cdot 8604 y}{\delta_2}\right) . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

If the velocity is low enough to permit the neglect of dissipation of energy due to friction*, and if the temperature differences are small enough to permit the neglect of the density variations, the energy equation for the thermal layer (Ref. 2, p. 615) becomes

$$\frac{d}{dx}\left[\int_0^\infty u(T - T_0) dy\right] = \varkappa \left(\frac{\delta T}{\delta y}\right)_1 , \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where $\varkappa$ is the thermometric conductivity $k/\rho c_p$ and the suffix 1 denotes the value at the surface $y = 0$.

* The principal effect of the dissipation is to reduce the effective temperature, if the surface is heated, by an amount which is calculated in §5.

From (2) we obtain

$$\frac{-1}{(T_1 - T_0)} \left(\frac{\partial T}{\partial y}\right)_1 = \frac{0 \cdot 8604}{\delta_2} f'(0) = \frac{0 \cdot 5715}{\delta_2}, \quad \ldots \quad (4)$$

since [2] $f'(0) = 0 \cdot 6641.$

We require to express the integral on the left-hand side of (3) in terms of δ_1 and δ_2. Substitution for u and T gives

$$\int_0^\infty \frac{u(T - T_0)}{U(T_1 - T_0)} dy = \int_0^\infty f\left(\frac{0 \cdot 8604y}{\delta_1}\right) \left[1 - f\left(\frac{0 \cdot 8604y}{\delta_2}\right)\right] dy$$

$$= \frac{\delta_2}{0 \cdot 8604} \int_0^\infty f\left(\frac{\delta_2 \eta_2}{\delta_1}\right) [1 - f(\eta_2)]\, d\eta_2,$$

on putting $\eta_2 = 0 \cdot 8604y/\delta_2$. Hence

$$\int_0^\infty u\,(T - T_0)\, dy = U(T_1 - T_0)\, \delta_2\, \phi\left(\frac{\delta_2}{\delta_1}\right), \quad \ldots \quad (5)$$

where

$$\phi\left(\frac{\delta_2}{\delta_1}\right) = \frac{1}{0 \cdot 8604} \int_0^\infty f\left(\frac{\delta_2 \eta_2}{\delta_1}\right) [1 - f(\eta_2)]\, d\eta_2 . \quad \ldots \quad (6)$$

From the known values of $f(\eta)$ the function ϕ has been tabulated for values of δ_2/δ_1 between $0 \cdot 5$ and $2 \cdot 0$ and is given in Table 1. Substituting from (4) and (5), equation (3) becomes

$$\frac{d}{dx}\left[U \delta_2\, \phi\left(\frac{\delta_2}{\delta_1}\right)\right] = \frac{0 \cdot 5715\varkappa}{\delta_2},$$

for which the integral is

$$\left[U\, \delta_2\, \phi\left(\frac{\delta_2}{\delta_1}\right)\right]^2 = 1 \cdot 143\varkappa \int_0^x U\phi dx.$$

Dividing by (1) we obtain

$$\frac{\delta_2^{\,2}}{\delta_1^{\,2}}\, \phi\left(\frac{\delta_2}{\delta_1}\right) = \frac{0 \cdot 3861}{\sigma} \cdot \frac{U^4 \int_0^x U\phi dx}{\phi \int_0^x U^5\, dx}, \quad \ldots \quad (7)$$

where $\sigma = \nu/\varkappa$ is the Prandtl number for the fluid.

TABLE 1

δ_2/δ_1	0·5	0·625	0·667	0·833	1·0	1·25	1·429	1·667	1·818	2·0
$\phi\,(\delta_2/\delta_1)$	0·2075	0·257	0·272	0·332	0·3861	0·4563	0·4988	0·5478	0·5750	0·5994
$\frac{\delta_2^{\,2}}{\delta_1^{\,2}}\, \phi\left(\frac{\delta_2}{\delta_1}\right)$ ..	0·052	0·100	0·121	0·230	0·386	0·713	1·018	1·522	1·901	2·398

Since ϕ is a known function of δ_2/δ_1 and the velocity distribution outside the boundary layer is also known, equation (7) determines δ_2/δ_1. The evaluation of ϕ from (7) is best effected by the method of successive approximation, as a first approximation omitting ϕ from the right-hand side, so that

$$\frac{\delta_2^2}{\delta_1^2}\phi\left(\frac{\delta_2}{\delta_1}\right) = \frac{0\cdot 3861}{\sigma}\cdot\frac{U^4\int_0^x U\,dx.}{\int_0^x U^5\,dx}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

The value of ϕ obtained from (8) is substituted in the right-hand side of (7) to give a second approximation, but, since ϕ is a slowly varying function of δ_2/δ_1, it will generally be found sufficient to stop at the first approximation. The left-hand side of (8) is tabulated in Table 1 and shown in Fig. 1.

The method of calculating the heat transfer from the surface for any particular case is now as follows :—

(*a*) Tabulate the right-hand side of (8).

(*b*) Determine the values of δ_2/δ_1 from Table 1 or Fig. 1 and equation (8).

(*c*) Determine δ_1 from (1).

(*d*) Determine δ_2 from (*b*) and (*c*).

(*e*) Determine the surface temperature gradient from (4). The local heat transfer per unit area is $-k(\partial T/\partial y)_2$ and the Nusselt number is given by $-d(\partial T/\partial y)_1/(T_1 - T_0)$, where d is a representative length.

3.2. *Comparison with More Accurate Solutions.*—To estimate the errors due to the approximations introduced to derive the above simple solution, comparison with two accurate solutions will be made.

(*a*) *Flat Plate.*—For the flat plate U is constant and equal to U_0 so that (1) and (8) give

$$\delta_1^2 = \frac{2\cdot 960\nu x}{U_0},$$

$$\frac{\delta_2^2}{\delta_1^2}\phi\left(\frac{\delta_2}{\delta_1}\right) = \frac{0\cdot 3861}{\sigma}.$$

From Fig. 1 or Table 1 values of δ_2/δ_1 can be derived for any value of σ and the surface temperature gradient is then given by (4). The results obtained are compared with Pohlhausen's exact solution (Ref. 2, p. 623) in Table 2 and good agreement is obtained.

TABLE 2

Values of $-\sqrt{\frac{\nu x}{U_0}}\,\frac{(\partial T/\partial y)_1}{(T_1 - T_0)}$ *for the flat plate.*

σ	0·6	0·7	0·8	0·9	1·0	1·1
Approximate solution	0·552	0·585	0·612	0·638	0·664	0·686
Exact solution	0·552	0·585	0·614	0·640	0·664	0·687

(*b*) *Stagnation Point.*—Near the stagnation point of a round-nosed body the velocity outside the boundary layer increases linearly with distance from the nose so that we may put $U = U_0 x/l$, where l is a representative length. Equations (1) and (8) then give

$$\delta_1{}^2 = 0 \cdot 493 \frac{\nu l}{U_0},$$

$$\frac{\delta_2{}^2}{\delta_1{}^2} \phi \left(\frac{\delta_2}{\delta_1}\right) = \frac{1 \cdot 158}{\sigma}.$$

The surface temperature gradient is derived as before from (4) and is compared with the exact solution (Ref. 2, p. 631) in Table 3.

TABLE 3

Values of $-\sqrt{\frac{\nu l}{U_0}} \cdot \frac{(\partial T/\partial y)_1}{T_1 - T_0}$ *for the stagnation point.*

σ	0·6	0·7	0·8	0·9	1·0	1·1
Approximate solution	0·445	0·474	0·499	0·521	0·543	0·562
Exact solution	0·466	0·495	0·521	0·546	0·570	0·592

The agreement is still quite good in this case, the approximate solution giving results about 5 per cent. too low : this is due to the use of the Blasius velocity distribution, which is far from exact near a stagnation point.

3.3. *Calculations for Laminar Flow.*—The method developed above for calculating heat transfer for laminar boundary layer has been applied to the circular cylinder and to the aerofoil sections N.A.C.A. 2409 and N.A.C.A. 2415 at $C_L = 0 \cdot 24$ and $C_L = 0 \cdot 8$, in each case taking $\sigma = 0 \cdot 715$.

The velocity distribution chosen for the circular cylinder is given in Fig. 2 and corresponds to the case of sub-critical flow with boundary layer separation at an angular position of about 80 deg. measured from the forward stagnation point. The results for the circular cylinder are given in Table 4 and Fig. 2, which gives values of $Nu/\sqrt{R}$ for angles (denoted by α) up to 80 deg. from the forward stagnation point. The Nusselt number Nu is defined as $- d\,(\partial T/\partial y)_1/(T_1 - T_0)$, where d is the diameter of the cylinder.

The velocity distributions for the aerofoils were taken from calculations made by Theodorsen's method, which were already available.[3] These velocity distributions and the calculated values of $Nu/\sqrt{R}$, for distances up to 26 per cent. of the chord measured along the surface, are given in Tables 5 and 6 and Figs. 3 and 4. For the aerofoils the Nusselt number is defined by taking the chord as the representative length. The magnitude of the peak heat transfer at the nose is very sensitive to the actual shape of the velocity distribution there, which is difficult to determine accurately, and consequently the heat transfer peak may be in error by as much as 20 per cent., but the error should be considerably less a short distance on either side of the peak.

3.4. *Comparison with Experiment.*—A comparison between theory and experiment for the circular cylinder is shown in Fig. 2, the experimental curve being the mean of a wide range of experimental data given by Schmidt and Wenner[4] and reported in Ref. 5. The agreement is quite satisfactory.

4. *Turbulent Boundary Layers.*

4.1. *Heat Transfer for Turbulent Boundary Layers.*—It is well established that Reynolds's analogy between heat transfer and skin friction, as modified by Kármán, gives satisfactory results for heat transfer to fluids flowing through pipes (Ref. 2, p. 658). There is no reason to doubt that this will also be true for heat transfer from flat plates at zero incidence, though the experimental data are less reliable. For aerofoils, however, it is not obvious how the analogy should be presented and the effect of the pressure gradients may destroy its validity.

With regard to the presentation of the analogy, it is usually stated, for the simple case $\sigma = 1$, in the form :—

$$k_H' = \frac{Q_1}{\rho c_p U(T_1 - T_0)} = \frac{c_f'}{2} = \frac{\tau_0}{\rho U^2}, \quad \ldots \quad \ldots \quad (9)$$

where the symbols have their usual significance. But it is not specified whether the velocity U refers to the stream velocity or to the velocity outside the boundary layer. To settle this, consider a cylinder with flat sides in a stream constricted by walls ; it is clear that, if the cylinder is long enough, the velocity to be used in applying Reynolds's analogy is the velocity outside the boundary layer on the flat sides of the cylinder. This illustration is sufficient to show that the velocity U in (9) should refer to the local velocity outside the boundary layer and not to the stream velocity.

The use of (9) to determine the heat transfer, with the velocity specified as above, involves the neglect of the effect of the pressure gradients along the boundary layer, which will certainly have some influence. An alternative analogy based on an assumed similarity between the velocity and temperature distributions across the boundary layer is considered in detail in Appendix I ; this leads to very different values for the heat transfer for the rear half of an aerofoil, where the effect of the pressure gradients is most important. However, in the absence of definite experimental data it is recommended that the analogy in the form (9) should be provisionally adopted. For σ not equal to unity the formula (9) is replaced by Kármán's generalisation (Ref. 2, p. 654)

$$\frac{1}{k_H'} = \frac{\rho c_p U(T_1 - T_0)}{Q_1} = \frac{\rho U^2}{\tau_0} + 5\sqrt{\frac{\rho U^2}{\tau_0}}\left[(\sigma - 1) + \log_e\left(1 + \frac{5}{6}(\sigma - 1)\right)\right]. \quad \ldots \quad (10)$$

If k_H is the standard heat transfer coefficient based on the stream velocity U_0 then

$$k_H = \frac{U}{U_0}\, k_H' . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

There are no experimental data reliable enough to check equations (10) and (11) as applied to aerofoils, or to decide definitely that this standard form of Reynolds's analogy is more nearly correct than the alternative analogy.

4.2. *Calculations for Heat Transfer in Turbulent Flow.*—Calculations of heat transfer have been made for the aerofoils N.A.C.A. 2409 and N.A.C.A. 2415, for $R = 10^7$, for $C_L = 0{\cdot}24$ and $C_L = 0{\cdot}8$, using equations (10) and (11) and taking $\sigma = 0{\cdot}715$. Transition to turbulent flow was assumed to occur in each case at 0·05c. and 0·10c. from the stagnation point measured along the arc. The results of the calculations are given in Tables 7–10 and in Figs. 5 and 6. For other values of R not differing from 10^7 by a factor of not more than 2 it is sufficiently accurate to assume that k_H is proportional to $R^{-0\cdot 2}$.

5. *Kinetic Temperature.*

5.1. *Kinetic Temperature. Laminar Flow.*—The kinetic temperature at a point of a body immersed in a stream of velocity U_0 at infinity is the temperature which is taken up when the body is non-conducting. For two-dimensional flow it is convenient to take the equation for the temperature in the form (Ref. 2, p. 612)

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(Jc_pT + \tfrac{1}{2}u^2\right) = \varkappa\frac{\partial^2}{\partial y^2}\left(Jc_pT + \tfrac{1}{2}\sigma u^2\right),$$

which becomes

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} - \varkappa\frac{\partial^2 \varepsilon}{\partial y^2} = \varkappa(\sigma - 1)\frac{\partial^2}{\partial y^2}\left(\frac{u^2}{2}\right), \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

on putting $\varepsilon = (Jc_pT + \frac{1}{2}u^2) - (Jc_pT_0 + \frac{1}{2}U_0{}^2)$. The quantity ε represents the increment in total energy per unit mass at a point in the fluid and vanishes outside the boundary layer. At the surface the normal temperature gradient vanishes when the body is non-conducting, so that $\partial\varepsilon/\partial_y = 0$ for $y = 0$, since $(\partial/\partial_y)(u^2/2) = 0$ for $y = 0$. The known solution $\varepsilon = 0$ for $\sigma = 1$ follows immediately from inspection of (12).

Consider the energy balance in a strip of the boundary layer starting at the forward stagnation point and extending to any station x along the surface. The energy inflow from the body vanishes and so does the energy outflow across the outer edge of the boundary layer. In addition the energy inflow at the stagnation point vanishes since the velocity is zero there and hence the energy outflow across any section of the boundary layer normal to the surface vanishes. Expressed mathematically this requires that

$$\int_0^\infty \rho u \varepsilon dy = 0 \quad \ldots \quad (13)$$

for all values of x; this result can also be obtained by integration of (12). It shows that the conditions at any station are independent of the flow upstream except in so far as this flow affects the velocity distribution across the boundary layer at the station. In addition (12) gives, on putting $y = 0$,

$$\left(\frac{\partial^2\varepsilon}{\partial y^2}\right)_1 = (1 - \sigma)\ \frac{\partial^2}{\partial y^2}\left(\frac{u^2}{2}\right)_1 = (1 - \sigma)\left(\frac{\partial u}{\partial y}\right)_1^2 . \quad \ldots \quad (14)$$

Equations (13) and (14) together with the equation of state suffice to determine the energy distribution across the boundary layer when the velocity distribution is known. For the flat plate the solution has been obtained in a different way by Pohlhausen (Ref. 2, p. 627), who derived for the energy at the surface the formula

$$\varepsilon_1 = Jc_p\,\Delta T_1 - \frac{U_0{}^2}{2} = \frac{U_0{}^2}{2}\,(\sigma^{1/2} - 1),$$

where ΔT_1 is the rise in temperature at the surface above the stream temperature. Now it follows from (13) and (14) that the only difference between a cylinder and a flat plate is that the velocity at the edge of the boundary layer at a station x is U instead of U_0 (provided that the velocity distributions across the boundary layers are the same), so that the surface energy for any cylinder is given by

$$\varepsilon_1 = Jc_p\Delta T_1 - \frac{U_0{}^2}{2} = \frac{U^2}{2}\,(\sigma^{1/2} - 1) \quad \ldots \quad (15)$$

The effect of various practicable velocity distributions across the boundary layer has been tested and found to be relatively unimportant* so that (15) may be assumed to be valid for all laminar layers. Thus we obtain finally for the kinetic temperature rise at a point on the wall

$$\Delta T_1 = \frac{U_0{}^2}{2Jc_p}\left[1 + \frac{U^2}{U_0{}^2}\,(\sigma^{1/2} - 1)\right]. \quad \ldots \quad (16)$$

5.2. *Comparison with Experiment.*—Measurements of the pressure distribution and surface temperature over a non-conducting circular cylinder of rubber have been made by Eckert and Drewitz[5] at a Mach number of 0·685. The measured temperature distribution is compared with the results given by (16) for values of σ equal to 0·7 and 0·75 in Fig. 7. It will be seen that the agreement is good.

Hilton[6] and Eckert[7] have shown that Pohlhausen's solution for the flat plate gives results in good agreement with experiment.

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* This is confirmed by calculations given in Reference 5.

5.3. *Kinetic Temperature. Turbulent Flow.*—The argument given in §5.1 about the energy flow across a section of the boundary layer applies equally to turbulent flow, provided that ρu represents the mean mass flow and ε the mean total energy per unit mass, including the energy associated with the turbulent velocity components: hence (13) is still valid. Also, since the flow near the wall is laminar the wall condition (14) is unchanged. It follows that the principal difference between laminar and turbulent flow is due to the different velocity distributions across the two kinds of boundary layer. It is true that the distribution of energy dissipation across the turbulent boundary layer is different from the distribution for the laminar layer, but this is not represented properly by the approximate solution for either case, appearing only through the wall condition (14), which is correct for both kinds of flow. The energy dissipation is normally largest at the wall for both layers, but is more concentrated near the wall for turbulent layers.

Since (13) and (14) hold for both laminar and turbulent layers we may expect that the wall temperature for the latter will be given by a formula similar to (16) with σ raised to a different power. This is confirmed in Appendix II, where it is concluded that the kinetic temperature for turbulent boundary layers is given by the formula

$$\Delta T_1 = \frac{U_0^2}{2Jc_p}\left[1 + \frac{U^2}{U_0^2}(\sigma^{1/3} - 1)\right] . \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Hilton's experiments[6] on plates of various thicknesses gave an extrapolated value for the kinetic temperature of a thin flat plate with a turbulent boundary layer in good agreement with (17). A really satisfactory experimental check can only be provided by kinetic temperature measurements on a body for which the velocity outside the boundary layer is considerably greater than the stream velocity and these measurements are not at present available.

6. *Application.*—The procedure for determining the rate of heat transfer from the surface of one of the aerofoils for which calculations have been made is as follows:—

(1) Determine the kinetic temperature distribution, for the conditions specified, from equations (16) and (17), using the correct velocity distribution outside the boundary layer, and estimate a suitable mean value. Assume for the subsequent calculations that the surface temperature is decreased by this mean kinetic temperature.

(2) With the reduced surface temperature given by (1) determine the rate of heat transfer for the laminar layer by the method of §3.1 or from the tables. The effect of change of scale is covered by the proportionality of Nu to $\sqrt{R}$ or of k_H to $1/\sqrt{R}$.

(3) With the reduced surface temperature determine the rate of heat transfer for the turbulent layer by the method of §4.1 or from the tables, bearing in mind that Reynolds's analogy is of doubtful application in regions where the pressure gradient is large. For values of R not differing from 10^7 by a factor of more than two k_H may be assumed to be proportional to $R^{-0 \cdot 2}$.

APPENDIX I

Alternative Form of Reynolds's Analogy for $\sigma = 1$.

The form of Reynolds's analogy represented by (9) is an analogy between the surface conditions controlling heat transfer and skin friction. Where there is no pressure gradient along the stream this form corresponds to similarity of the velocity and temperature distributions across the boundary layer, provided that $\sigma = 1$ and the validity of the momentum transfer theory is accepted, but this is not so when a pressure gradient is present (Ref. 2, p. 649). Some insight into the probable effect of the pressure gradients may be obtained by assuming that the temperature and velocity distributions are similar in the presence of pressure gradients, as this should give values for the heat transfer at the opposite extreme to those given by the standard formula (9). We shall therefore derive an expression for the heat transfer on this basis, limiting consideration to the simple case $\sigma = 1$.

The energy equation for the boundary layer, corresponding to (3), neglecting the heating due to friction and the variation of density in the field, is

$$\frac{d}{dx}\left[\int_0^\infty u(T - T_0)\,dy\right] = \frac{Q}{\rho c_p} \qquad (18)$$

Assuming that the temperature and velocity distributions across the boundary layer are similar, i.e. that

$$\frac{T - T_0}{T_1 - T_0} = 1 - \frac{u}{U},$$

equation (18) gives

$$k_H'' = \frac{Q_1}{\rho c_p U(T_1 - T_0)} = \frac{1}{U}\frac{d}{dx}\left[U \int \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right] = \frac{d\theta}{dx} + \frac{U'}{U}\,\theta,$$

where k_H'' is defined by this equation and θ is the momentum thickness of the boundary layer defined by

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy.$$

The value of U is known and θ may be calculated by the method of Ref. 8.

Since θ satisfies the equation[8]

$$\frac{d\theta}{dx} + \frac{U'}{U}(H + 2)\,\theta = \frac{\tau_0}{\rho U^2},$$

it follows that

$$k_H'' = \frac{\tau_0}{\rho U^2} - \frac{U'}{U}(H + 1)\,\theta.$$

The two different expressions for the heat transfer coefficient become identical when $U' = 0$, so that there is no pressure gradient. For $U' \neq 0$ there may be quite large differences between the values of k_H given by the two methods : this is illustrated by Fig. 8 which gives the different values of k_H for the upper surface of N.A.C.A. 2415 for $C_L = 0{\cdot}24$ and $C_L = 0{\cdot}8$, for $R = 10_L$ and $\sigma = 1{\cdot}0$.

The physical aspect of the increased rates of heat transfer given by the use of the alternative form of Reynolds's analogy in regions of rising pressure is that the increasing pressure causes rapid thickening of the boundary layer, which must therefore contain more heat than a thinner layer, if the temperature and velocity distributions are similar. This increased heat content in the boundary layer must be provided by an increased rate of heat transfer from the surface.

APPENDIX II

Kinetic Temperature for Turbulent Boundary Layers

For speeds less than half the speed of sound it is permissible to neglect the density variations across the boundary layer and take (13) in the form

$$\int_0^\infty u\varepsilon dy = 0. \qquad (19)$$

This equation, together with (14), can be used to determine the surface temperature. Let the velocity distribution in the boundary layer be given by $u/U = g(y)$, and the temperature distribution be similar to the square of the velocity distribution so that

$$Jc_p(T - T') = \lambda \frac{U^2}{2}\left[1 - g^2(y/\beta)\right];$$

in this equation λ and β are constants, and T' is the temperature at the edge of the layer, which is different from T_0 since the velocity U at the edge is different from U_0. This form satisfies the condition for zero temperature gradient at the surface and has a certain plausibility. Outside the boundary layer the total energy is constant so that

$$Jc_p\, T' + \tfrac{1}{2}U^2 = Jc_p\, T_0 + \tfrac{1}{2}U_0^2.$$

With these expressions for the velocity and temperature the increment in total energy takes the form

$$\varepsilon = \frac{U^2}{2}\left[\lambda\,\{1 - g^2(y/\beta)\} - \{1 - g^2(y)\}\right].$$

For $\sigma = 1$ we shall have $\lambda = \beta = 1$, $\varepsilon = 0$, as desired.

Equation (14) gives the relation

$$\lambda = \sigma\beta^2. \qquad (20)$$

Substituting the above expression for ε and u in (9) gives

$$\int_0^\infty g(y)\left[\lambda\,\{1 - g^2(y/\beta)\} - \{1 - g^2(y)\}\right] dy = 0. \qquad (21)$$

For turbulent boundary layers the velocity distribution can be fairly well represented by a power law of the form $g(y) \propto y^n$ where n is about 1/7. This fails to represent the conditions near the wall, but this will not seriously affect the value of the integrals across the layer appearing in (21). We therefore put

$$g(y) = (y/\delta)^n \quad \text{for} \quad 0 < y < \delta,$$
$$g(y) = 1 \quad \text{for} \quad y > \delta,$$

where δ is the boundary layer thickness. For values of β less than unity, equation (21) then leads after some reduction to the formula

$$\lambda\beta^{n+1} = 1.$$

Combining this with (20) gives

$$\lambda = \sigma^{\frac{n+1}{n+3}}, \quad \beta = \sigma^{-\frac{1}{n+3}}.$$

For $n \leqslant 1/7$ and values of σ not very different from unity it is sufficiently accurate to take

$$\lambda = \sigma^{1/3}, \ \beta = \sigma^{1/3}.$$

These give

$$\varepsilon_1 = \frac{U^2}{2}(\lambda - 1) = \frac{U^2}{2}(\sigma^{1/3} - 1),$$

for the value of the total energy increment at the surface, which leads to the expression

$$\Delta T_1 = \frac{U_0^2}{2}\left[1 + \frac{U^2}{U_0^2}(\sigma^{1/3} - 1)\right] \quad .. \quad .. \quad .. \quad (17)$$

for the kinetic temperature rise at the surface.

The above analysis has been carried through for $\beta < \delta$ which requires that $\sigma > 1$. For values of σ less than unity, which correspond to $\beta > \delta$, the evaluation of the left-hand side of (21) is complicated by the change of form of $g(y)$ as y passes through δ. It is found, however, that this does not affect to an appreciable degree the value of the surface temperature provided that σ is greater than 0·5.

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TABLE 4

Heat transfer for circular cylinder. Laminar boundary layer. $\sigma = 0{\cdot}715$.
α is the angle measured from the stagnation point

α	Rad. ..	0	0·2	0·4	0·6	0·8	1·0	1·2	1·4
	Deg. ..	0	11·4	22·9	34·2	45·7	57·3	68·7	80·1
U/U_0		0	0·361	0·708	1·026	1·296	1·497	1·592	1·543
$Nu/\sqrt{R}$		0·911	0·907	0·889	0·865	0·825	0·768	0·691	0·560

TABLE 5

Heat transfer for N.A.C.A. 2409. Laminar boundary layer. $\sigma = 0{\cdot}715$.

x denotes arc measured from the stagnation point.
c denotes aerofoil chord.
Positive values of x/c refer to the upper surface and negative values to the lower surface.
At $C_L = 0{\cdot}24$ the stagnation point is $0{\cdot}001c$ above the leading edge.
At $C_L = 0{\cdot}8$ the stagnation point is $0{\cdot}013c$ below the leading edge.

$$Nu = -c\left(\frac{\partial T}{\partial y}\right)_1 \Big/ (T_1 - T_0). \qquad k_H = \frac{Nu}{\sigma R}.$$

x/c	$C_L = 0{\cdot}24$		$C_L = 0{\cdot}8$	
	U/U_0	$Nu/\sqrt{R}$	U/U_0	$Nu/\sqrt{R}$
−0·26	−1·05	0·58	−0·89	0·61
−0·24	−1·055	0·60	−0·88	0·64
−0·22	−1·06	0·64	−0·87	0·66
−0·20	−1·065	0·67	−0·87	0·70
−0·18	−1·07	0·71	−0·86	0·74
−0·16	−1·075	0·76	−0·85	0·78
−0·14	−1·08	0·81	−0·83	0·84
−0·12	−1·09	0·89	−0·82	0·91
−0·10	−1·10	0·99	−0·80	0·99
−0·09	−1·11	1·07	−0·79	1·06
−0·08	−1·12	1·15	−0·78	1·14
−0·07	−1·125	1·25	−0·76	1·22
−0·06	−1·13	1·37	−0·73	1·29
−0·05	−1·135	1·54	−0·70	1·43
−0·04	−1·14	1·79	−0·65	1·57
−0·03	−1·145	2·19	−0·58	1·73
−0·02	−1·12	2·96	−0·50	1·89
−0·01	−0·81	3·93	−0·36	2·47
0	0	4·78	0	4·75
+0·01	+0·72	3·76	+1·25	5·29
0·02	0·99	2·81	1·82	3·90
0·03	1·08	2·24	1·84	2·76
0·04	1·12	1·88	1·77	2·18
0·05	1·15	1·66	1·70	1·79
0·06	1·16	1·48	1·66	1·56
0·07	1·17	1·36	1·625	1·39
0·08	1·18	1·26	1·59	1·27
0·09	1·185	1·19	1·565	1·17
0·10	1·19	1·12	1·545	1·10
0·12	1·195	1·01	1·51	0·98
0·14	1·20	0·93	1·485	0·89
0·16	1·20	0·86	1·46	0·82
0·18	1·20	0·81	1·44	0·76
0·20	1·20	0·76	1·425	0·72
0·22	1·205	0·72	1·41	0·68
0·24	1·205	0·69	1·40	0·64
0·26	1·205	0·66	1·39	0·61

TABLE 6

Heat transfer for N.A.C.A. 2415. Laminar boundary layer. $\sigma = 0{\cdot}715$.

At $C_L = 0{\cdot}24$ the stagnation point is $0{\cdot}001c$ above the leading edge.
At $C_L = 0{\cdot}8$ the stagnation point is $0{\cdot}018c$ below the leading edge.

x/c	$C_L = 0{\cdot}24$		$C_L = 0{\cdot}8$	
	U/U_0	$Nu/\sqrt{R}$	U/U_0	$Nu/\sqrt{R}$
−0·26	−1·135	0·62	−0·96	0·63
−0·24	−1·14	0·65	−0·96	0·67
−0·22	−1·15	0·68	−0·95	0·70
−0·20	−1·155	0·72	−0·945	0·74
−0·18	−1·16	0·77	−0·935	0·78
−0·16	−1·17	0·84	−0·925	0·83
−0·14	−1·175	0·91	−0·91	0·89
−0·12	−1·18	1·00	−0·89	0·97
−0·10	−1·19	1·14	−0·87	1·08
−0·09	−1·185	1·22	−0·85	1·14
−0·08	−1·18	1·29	−0·83	1·21
−0·07	−1·18	1·38	−0·80	1·29
−0·06	−1·17	1·52	−0·77	1·41
−0·05	−1·15	1·71	−0·73	1·52
−0·04	−1·12	2·00	−0·67	1·67
−0·03	−1·04	2·30	−0·58	1·84
−0·02	−0·94	2·69	−0·47	2·12
−0·01	−0·68	3·69	−0·29	2·43
0	0	4·79	0	3·04
+0·01	+0·62	3·43	+0·52	3·51
0·02	0·84	2·56	1·02	3·38
0·03	0·94	2·18	1·39	3·08
0·04	1·00	1·89	1·59	2·66
0·05	1·06	1·68	1·66	2·27
0·06	1·11	1·56	1·665	1·95
0·07	1·15	1·47	1·66	1·72
0·08	1·18	1·37	1·65	1·58
0·09	1·21	1·30	1·64	1·43
0·10	1·23	1·23	1·62	1·33
0·12	1·255	1·11	1·61	1·16
0·14	1·27	1·02	1·59	1·05
0·16	1·28	0·94	1·57	0·95
0·18	1·28	0·88	1·55	0·88
0·20	1·28	0·82	1·53	0·81
0·22	1·28	0·77	1·51	0·76
0·24	1·28	0·73	1·49	0·71
0·26	1·275	0·70	1·47	0·67

TABLE 7

Heat transfer for N.A.C.A. 2409. Turbulent boundary layer.

$R = 10^7$, $\sigma = 0{\cdot}715$.

Transition at $x/c = 0{\cdot}10$.

x/c	$C_L = 0{\cdot}24$		$C_L = 0{\cdot}8$	
	U/U_0	$10^3\,k_H$	U/U_0	$10^3\,k_H$
Upper Surface				
0·10	1·19	3·47	1·545	3·92
0·12	1·195	2·96	1·512	3·34
0·15	1·200	2·63	1·472	2·97
0·20	1·203	2·38	1·426	2·61
0·25	1·205	2·24	1·394	2·38
0·3	1·204	2·12	1·368	2·23
0·4	1·195	1·95	1·312	2·00
0·5	1·170	1·82	1·257	1·81
0·6	1·140	1·69	1·202	1·66
0·7	1·107	1·58	1·147	1·54
0·8	1·071	1·48	1·093	1·41
0·9	1·027	1·37	1·038	1·30
1·0	0·958	1·24	0·980	1·19
Lower Surface				
0·10	1·100	3·10	0·800	2·55
0·12	1·088	2·64	0·818	2·21
0·15	1·078	2·34	0·842	2·01
0·20	1·065	2·10	0·867	1·84
0·25	1·054	1·96	0·886	1·78
0·30	1·045	1·84	0·898	1·72
0·4	1·026	1·68	0·913	1·62
0·5	1·005	1·58	0·920	1·55
0·6	0·99	1·48	0·925	1·49
0·7	0·978	1·43	0·931	1·46
0·8	0·966	1·37	0·938	1·43
0·9	0·955	1·33	0·948	1·41
1·0	0·941	1·29	0·960	1·41

TABLE 8

Heat transfer for N.A.C.A. 2409. Turbulent boundary layer.

$R = 10^7$. $\sigma = 0 \cdot 715$.

Transition at $x/c = 0 \cdot 05$.

x/c	$C_L = 0 \cdot 24$		$C_L = 0 \cdot 8$	
	U/U_0	$10^3 k_H$	U/U_0	$10^3 k_H$
Upper Surface				
0·05	1·15	3·83	1·70	4·93
0·07	1·17	3·04	1·62	3·74
0·10	1·19	2·70	1·545	3·14
0·15	1·20	2·41	1·47	2·66
0·20	1·20	2·26	1·425	2·41
Lower Surface				
0·05	1·135	3·62	0·70	2·70
0·07	1·125	2·86	0·76	2·25
0·10	1·10	2·43	0·80	2·02
0·15	1·08	2·14	0·84	1·87
0·20	1·065	1·98	0·87	1·75

TABLE 9

Heat transfer for N.A.C.A. 2415. Turbulent boundary layer.

$R = 10^7$ $\sigma = 0{\cdot}715$.

Transition at $x/c = 0{\cdot}10$.

x/c	$C_L = 0{\cdot}24$		$C_L = 0{\cdot}8$	
	U/U_0	$10^3 k_H$	U/U_0	$10^3 k_H$
		Upper Surface		
0·10	1·234	3·76	1·633	4·41
0·12	1·254	3·13	1·611	3·69
0·15	1·273	2·80	1·580	3·22
0·20	1·280	2·51	1·528	2·80
0·25	1·276	2·35	1·481	2·52
0·3	1·271	2·21	1·440	2·33
0·4	1·254	2·03	1·384	2·09
0·5	1·229	1·88	1·327	1·90
0·6	1·190	1·74	1·252	1·72
0·7	1·142	1·60	1·180	1·56
0·8	1·090	1·47	1·111	1·40
0·9	1·027	1·33	1·042	1·27
1·0	0·920	1·13	0·974	1·15
		Lower Surface		
0·10	1·188	3·40	0·867	2·82
0·12	1·182	2·87	0·893	2·40
0·15	1·173	2·54	0·918	2·17
0·20	1·155	2·25	0·944	1·95
0·25	1·137	2·07	0·959	1·90
0·30	1·119	1·94	0·969	1·83
0·4	1·092	1·77	0·978	1·69
0·5	1·067	1·64	0·980	1·61
0·6	1·042	1·53	0·978	1·55
0·7	1·017	1·45	0·970	1·48
0·8	0·987	1·36	0·959	1·42
0·9	0·945	1·27	0·946	1·36
1·0	0·878	1·13	0·932	1·30

TABLE 10

Heat transfer for N.A.C.A. 2415. Turbulent boundary layer.

$R = 10^7$ $\sigma = 0{\cdot}715.$

Transition at $x/c = 0{\cdot}05$.

x/c	$C_L = 0{\cdot}24$		$C_L = 0{\cdot}8$	
	U/U_0	$10^3 k_H$	U/U_0	$10^3 k_H$
Upper Surface				
0·05	1·06	3·70	1·66	5·58
0·07	1·15	3·10	1·66	4·07
0·10	1·23	2·87	1·63	3·43
0·15	1·27	2·61	1·58	2·99
0·20	1·28	2·43	1·53	2·61
Lower Surface				
0·05	1·15	3·91	0·73	2·83
0·07	1·18	3·07	0·80	2·37
0·10	1·185	2·67	0·87	2·18
0·15	1·17	2·35	0·92	2·03
0·20	1·155	2·15	0·945	1·93

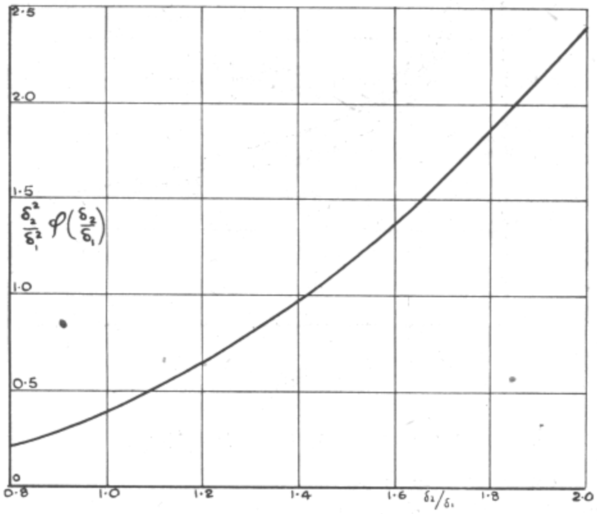

FIG. 1. Graph of the Function $(\delta_2/\delta_1)^2\ \phi\ (\delta_2/\delta_1)$.

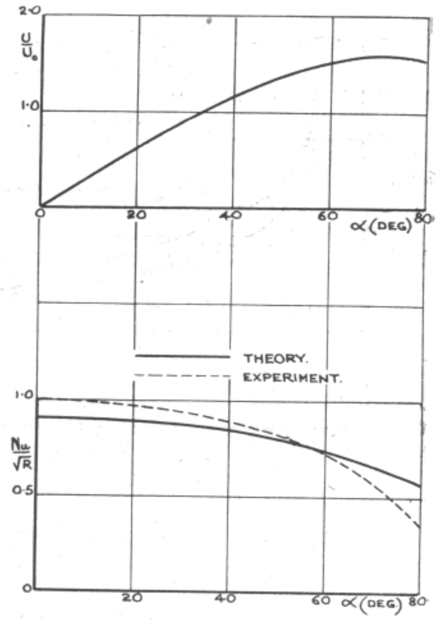

FIG. 2. Velocity and Heat Transfer Distributions for Circular Cylinder. $\sigma = 0{\cdot}715$.

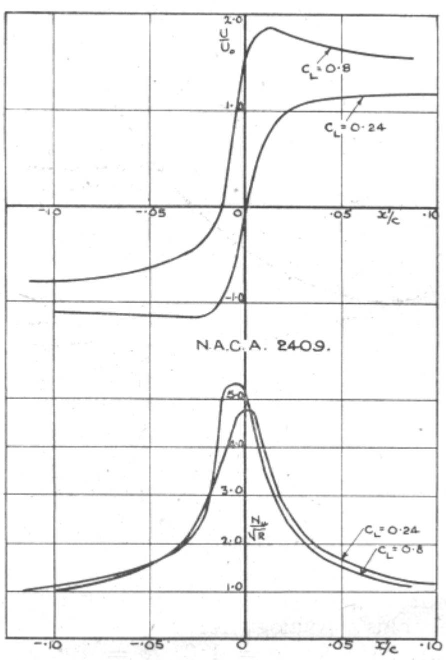

FIG. 3.

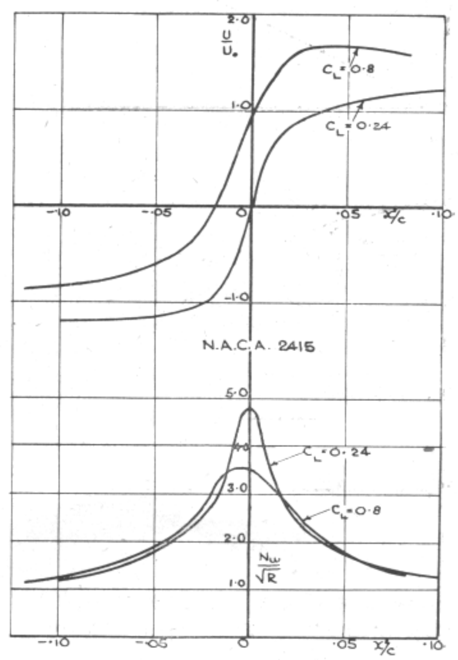

FIG. 4.

Heat Transfer for N.A.C.A. 2409 and N.A.C.A. 2415. Laminar Boundary Layers. $\sigma = 0{\cdot}715$.
(x' is distance along the surface measured from the leading edge.)

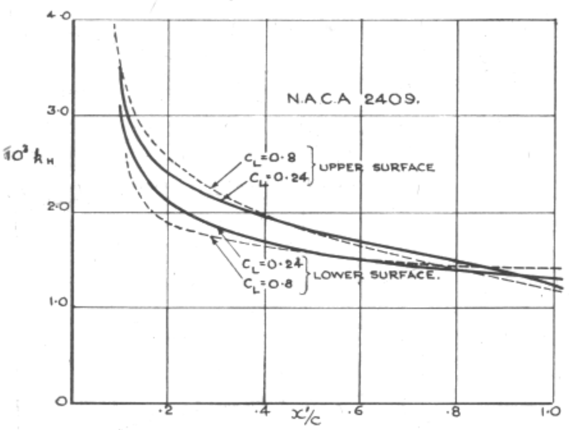

FIG. 5

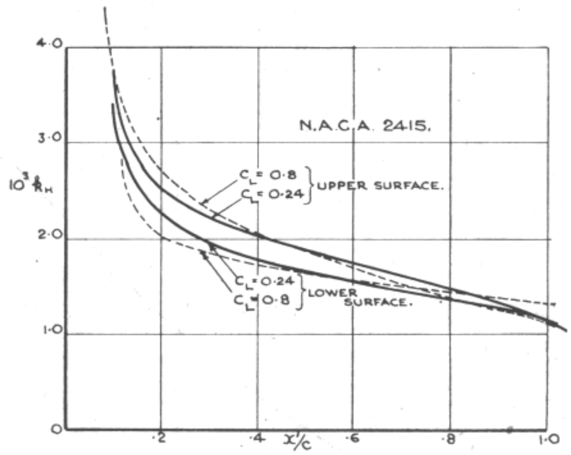

FIG. 6

Heat Transfer from N.A.C.A. 2409 and N.A.C.A. 2415. Turbulent Boundary Layers.

$\sigma = 0{\cdot}715$. $R = 10^7$. Transition at $x/c = 0{\cdot}10$.

(x' is distance along the surface measured from the leading edge).

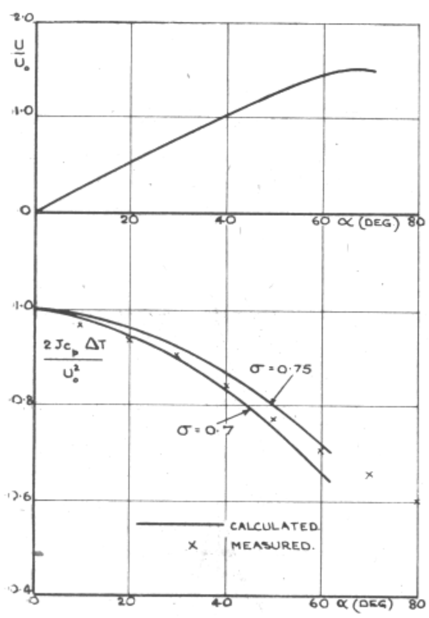

FIG 7

Comparison between Theory and Experiment for Kinetic Temperature on Circular Cylinder.

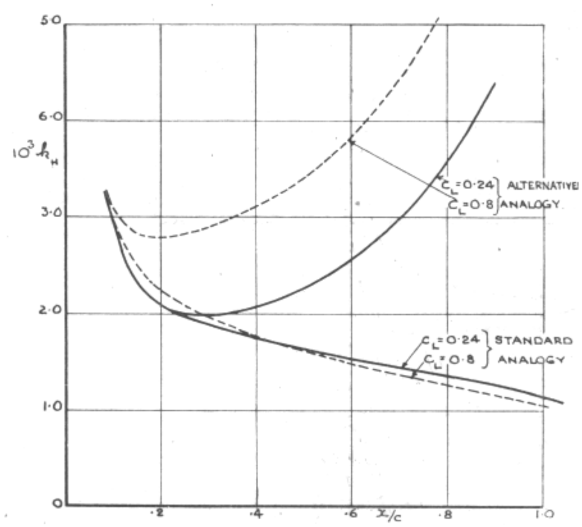

FIG 8

Comparison between Standard and Alternative Forms of Reynolds's Analogy for Upper Surface of N.A.C.A. 2415 for $\sigma = 1{\cdot}0$, $R = 10^7$.

(72173) Wt. 9/7116 12/45 Hw. G.377/1.