

A GENERAL THEORY OF THE AUTOGYRO.

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Summary.—(a) *Introductory.*—An autogyro obtains remarkably high lift forces from a system of freely rotating blades and it is important to develop a theory which will explain the behaviour of an autogyro and will provide a method of estimating the effect of changes in the fundamental parameters of the system.

(b) *Range of Investigation.*—A theory is developed depending on the assumptions that the angles of incidence of the blade elements are small, that the interference flow is similar to that caused by an ordinary aerofoil, and that only first order harmonics of periodic terms need be retained in the equations. An alternative method of analysis by considering the energy losses of an autogyro is developed in an appendix to the main report.

(c) *Conclusions.*—The maximum lift coefficient of an autogyro, using the disc area as fundamental area and the forward speed as fundamental speed, lies between 0.5 and 0.6 in general, and the best lift-drag ratio is of the order of 6 or 8 at most. Also, owing to the necessity of maintaining a sufficient ratio of tip speed to forward speed, the stalling speed of an autogyro must rise with the maximum speed of level flight, and so the principal merit of the autogyro system, the low landing speed, would disappear in the case of high speed aircraft.

(d) *Further developments.*—The analysis is confined to the case of blades of constant chord and angle of pitch, but there would be no difficulty in extending the theory to tapered and twisted blades, provided these variations can be expressed in a simple mathematical form. It is not anticipated that an improvement of more than a few per cent. could be achieved by any such modifications.

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1. *Introduction.*—The lifting system of an autogyro or gyroplane consists essentially of a windmill of large radius R with three or more identical blades, whose angular rotation Ω is maintained by the forward speed V of the aircraft. Each blade is also free to rotate about a hinge at its root which is normal to the shaft of the autogyro. In the simplest case, the chord c of the blades is constant from root to tip, and the blade is attached to the shaft at a small positive angle of pitch θ , while the shape of the blade is concave downwards when viewed from front or rear. The shape of the blades will be assumed to be of this simple form in the subsequent analysis, although in practice the corners of the blades are rounded off at the tips and the chord tapers to the dimensions of the spar at the root. Variations of the chord and angle of pitch along the blade would not necessitate any fundamental changes in the method of analysis, but would involve greater complexity at all stages.

When the shaft of the autogyro is inclined backwards at angle i (fig. 1) to the normal to the direction of motion, the autogyro will be said to be at angle of incidence i . The resultant force acting on the autogyro can then be resolved conveniently into the following components :—

T , the thrust along the shaft.

H , the longitudinal force at right angles to the shaft in the plane of the shaft and of the direction of motion.

Y , the lateral force, normal to the previous components and positive to the side on which the blades are advancing in the direction of motion.

The lift Z and the drag X of the autogyro are expressed simply in terms of the thrust and longitudinal force by the equations :—

$$\left. \begin{aligned} Z &= T \cos i - H \sin i \\ X &= T \sin i + H \cos i \end{aligned} \right\} \quad \dots \quad (1)$$

Owing to the method of attachment of the blades to the shaft, the only couple which can be transmitted to the autogyro is a torque Q about the shaft. The torque will be regarded as positive when it opposes the rotation of the autogyro.

In order to express these force components in the form of non-dimensional coefficients, it is convenient, when considering the aerodynamics of the rotating system, to use the disc area πR^2 of the windmill as the fundamental area and the tip speed ΩR as the fundamental speed. Accordingly, non-dimensional coefficients are defined by the equations

$$\left. \begin{aligned} T &= T_c \pi R^2 \rho \Omega^2 R^2 \\ H &= H_c \pi R^2 \rho \Omega^2 R^2 \\ Y &= Y_c \pi R^2 \rho \Omega^2 R^2 \\ Q &= Q_c \pi R^2 \rho \Omega^2 R^3 \end{aligned} \right\} \dots \dots (2)$$

On the other hand, when considering the motion of the aircraft as a whole, it is necessary to use the forward speed V as fundamental speed, and to define the non-dimensional coefficients of drag, lateral force, and lift by the equations

$$\left. \begin{aligned} X &= k_x \pi R^2 \rho V^2 \\ Y &= k_y \pi R^2 \rho V^2 \\ Z &= k_z \pi R^2 \rho V^2 \end{aligned} \right\} \dots \dots (3)$$

The relationships between the two sets of coefficients involves a single parameter λ , which is the ratio of the forward speed to the tip speed

$$\lambda = \frac{V}{\Omega R} \dots \dots (4)$$

and in particular the equations (1) become

$$\left. \begin{aligned} \lambda^2 k_z &= T_c \cos i - H_c \sin i \\ \lambda^2 k_x &= T_c \sin i + H_c \cos i \end{aligned} \right\} \dots \dots (5)$$

It may be noted that the angle of incidence i and the speed ratio λ define the state of working of the windmill.

2. *Motion of the blades.*—Each blade is hinged at its root about an axis normal to the shaft of the autogyro. The plan form of the blades is approximately rectangular, but the blades are curved so as to be concave downwards. Take the line joining the root to the tip as base line (fig. 2), let h be the ordinate at radial distance r , and let χ be the slope of the tangent at this point, so that

$$\chi = \frac{dh}{dr}$$

In the subsequent analysis the values of the following integrals are required, depending on the curvature of the blades :—

$$\left. \begin{aligned} \int_0^R \chi \, dr &= 0 \\ \int_0^R \chi r \, dr &= - \int_0^R h \, dr = - \eta_1 R^2 \\ \int_0^R \chi r^2 \, dr &= - 2 \int_0^R h r \, dr = - 2 \eta_2 R^3 \\ \int_0^R \chi^2 r \, dr &= \xi R^2 \end{aligned} \right\} \quad (6)$$

and for the purposes of the analysis the curvature of the blades is completely represented by the values of the three coefficients, η_1 , η_2 , and ξ .

The flapping of the blades also depends on the following three integrals, involving the line density m of the blade and the total weight W_1 of one blade :—

$$\left. \begin{aligned} G_1 &= \int_0^R m g r \, dr = \mu_1 W_1 R \\ I_1 &= \int_0^R m r^2 \, dr = \mu_2 \frac{W_1}{g} R^2 \\ J_1 &= \int_0^R m h r \, dr = \varepsilon I_1 \end{aligned} \right\} \quad (7)$$

The general analysis will be developed in terms of the six coefficients defined by equations (6) and (7), but in numerical applications it will be assumed that the blade has the shape of a circular arc and that the line density m is constant along the blade. In this special case ε is the camber of the circular arc and the other coefficients have the values

$$\left. \begin{aligned} \mu_1 &= \frac{1}{2}, & \mu_2 &= \frac{1}{3} \\ \eta_1 &= \frac{2}{3} \varepsilon, & \eta_2 &= \frac{1}{3} \varepsilon, & \xi &= \frac{8}{3} \varepsilon^2 \end{aligned} \right\} \quad (8)$$

A typical numerical value for ε is 0.03 and then

$$\eta_1 = 0.02, \quad \eta_2 = 0.01, \quad \xi = 0.0024$$

The position of a blade at any moment can be defined by the angle ψ through which it has rotated about the shaft from the downwind position and by the angle β which is its upward inclination above the plane normal to the shaft. Then, provided that the

angles β and χ are small, the equation of motion for the flapping of the blade is

$$\int_0^R m r^2 \ddot{\beta} dr = \int_0^R \frac{dT_1}{dr} r dr - \int_0^R mg r dr - \int_0^R m \Omega^2 r (r \beta + h) dr$$

or

$$I_1 (\ddot{\beta} + \Omega^2 \beta) = (TM)_1 - G_1 - \Omega^2 J_1$$

where the suffix (1) denotes that the values refer to a single blade and where $(TM)_1$ is the moment of the thrust about the hinge of the blade.

The angle β can be expressed quite generally in the form of the Fourier series

$$\beta = \beta_0 - \beta_1 \cos(\psi - \psi_1) - \beta_2 \cos 2(\psi - \psi_2)$$

where β_1, β_2 etc. may be assumed to be positive. For the present it is proposed to retain only the first harmonic term, so that the flapping is equivalent to a tilt of the plane of rotation through an angle β_1 with the lowest point in the angular position ψ_1 , combined with a general upward tilt of all the blades through the coning angle β_0 . On this basis

$$\beta = \beta_0 - \beta_1 \cos(\psi - \psi_1) \quad \dots \quad (9)$$

and, by virtue of equations (7), the equation of motion for the flapping of the blades reduces to

$$\frac{(TM)_1}{RW_1} = \mu_1 + \mu_2 (\beta_0 + \epsilon) \frac{\Omega^2 R}{g} \quad \dots \quad (10)$$

Hence, to this order of approximation, the moment of the thrust on a blade about its hinge is independent of the angular position ψ , and the evaluation of the angles ψ_1, β_0 , and β_1 follows from the consideration of the aerodynamic expression for the thrust moment.

The subsequent analysis is based entirely on the assumption that it is sufficiently accurate to retain only the principal oscillation of the flapping, and to neglect all higher harmonics involving $\cos 2\psi, \cos 3\psi$, etc.

3. Interference flow.—An autogyro at angle of incidence i is essentially a windmill descending with the axial velocity $V \sin i$ and with the velocity of sideslip $V \cos i$, but owing to the fact that the velocity of sideslip is considerably greater than the axial velocity, the induced velocity due to the system of trailing vortices will correspond more closely to the induced velocity of an aerofoil than to that usually associated with an airscrew and its slipstream. The principal force component is the thrust T and so the induced velocity v will be assumed to be parallel to the shaft of the autogyro (fig. 3).

In the first place also, this induced velocity will be assumed to have a constant value over the whole disc of the autogyro, and the consideration of the effect of variation of the induced velocity will be postponed to a later stage (Para. 10).

The resultant velocity V' experienced by the autogyro is the resultant of the forward speed V and the axial induced velocity v , and may be written in the alternative forms

$$\begin{aligned} V'^2 &= (V - v \sin i)^2 + v^2 \cos^2 i \\ &= (V \sin i - v)^2 + V^2 \cos^2 i \end{aligned}$$

The formula proposed for the axial induced velocity is

$$v = \frac{T}{2 \pi R^2 \rho V'} \quad \dots \quad (11)$$

which is a logical generalisation of the ordinary aerofoil formula. When i and T are small the formula gives approximately

$$v = \frac{T}{2 \pi R^2 \rho V}$$

which is the standard formula for the normal induced velocity of an aerofoil of semi-span R giving the lift T , and when i is nearly 90° it gives

$$T = 2 \pi R^2 \rho (V - v) v$$

which is the ordinary momentum formula for an airscrew. It is anticipated therefore that the formula (11) will be valid over a wide range of angle of incidence.

The axial velocity through the disc of the autogyro is

$$u = V \sin i - v$$

and it is convenient to write

$$V \sin i - v = u = \Omega R x \quad \dots \quad (12)$$

The equation (11) for the induced velocity may then be expressed in the form

$$\lambda \sin i = x + \frac{\frac{1}{2} T_c}{\sqrt{\lambda^2 \cos^2 i + x^2}} \quad \dots \quad (13)$$

and for small angles of incidence a good approximation can be obtained by neglecting x in comparison with $\lambda \cos i$.

4. *Flow at blade element.*—Consider the element dr at radial distance r on the blade which is at angular position ψ to the downwind position. Due to the flapping of the blade and to its curvature the blade element is inclined at the small angle $(\beta + \chi)$ to the

normal to the shaft. Now the velocity of the air relative to the autogyro has the components u along the shaft and $V \cos i$ normal to the shaft, while the blade element is moving with the angular velocities Ω about the shaft of the autogyro and β about the hinge of the blade. The velocity of the air relative to the blade element has therefore the following components (see fig. 4) :—

(1) normal to the shaft and to the element dr

$$U \cos \phi = \Omega r + V \cos i \sin \psi$$

(2) normal to this first component and to the element dr

$$U \sin \phi = u - r \dot{\beta} - (\beta + \chi) V \cos i \cos \psi$$

(3) radial along the element dr

$$u (\beta + \chi) + V \cos i \cos \psi$$

The radial velocity component will be ignored. Retaining only first harmonics of the angle ψ and regarding the angle ϕ as small, the other two components become

$$\left. \begin{aligned} U &= \Omega r + V \cos i \sin \psi \\ \phi U &= \Omega R x - \Omega r \beta_1 \sin (\psi - \psi_1) \\ &\quad - (\beta_0 + \chi) V \cos i \cos \psi \end{aligned} \right\} (14)$$

and to this same order of approximation

$$\left. \begin{aligned} U^2 &= \Omega^2 r^2 + 2\Omega r V \cos i \sin \psi \\ \phi U^2 &= \Omega^2 R x^2 + 2\Omega R x V \cos i \sin \psi \\ &\quad - \Omega^2 r^2 \beta_1 \sin (\psi - \psi_1) - \Omega r \\ &\quad (\beta_0 + \chi) V \cos i \cos \psi \\ \phi^2 U^2 &= \Omega^2 r^2 \chi^2 - 2\Omega^2 R x \beta_1 \sin (\psi - \psi_1) \\ &\quad - 2\Omega R x (\beta_0 + \chi) V \cos i \cos \psi \end{aligned} \right\} (15)$$

The assumptions on which these formulae are based clearly break down towards the root of all of the blades and over a wider range of the retreating blades, since the angle ϕ ceases to be small, while on the inner portion of the retreating blade the air flow will actually strike the rear of the aerofoil section. The failure of the approximation towards the root of the blade is not of any practical importance, but the method of analysis will cease to be valid when the approximations break down over a large part of the retreating blades. It is not possible to assign an exact limit to the validity of the approximations but it is proposed to take the condition that the velocity component $U \cos \phi$ must be positive over the outer half of the retreating blade. On this basis the limit of validity is that

$$\frac{V \cos i}{\Omega R} < \frac{1}{2} \quad \dots \quad (16)$$

The lift and drag coefficients of the aerofoil section correspond to two dimensional motion at the angle of incidence

$$\alpha = \theta + \phi$$

For small angles of incidence the lift coefficient is simply proportional to the angle of incidence, since the aerofoil sections are of symmetrical shape. Also, the drag contributes only a small correction to the force components due to the lift, and it is therefore legitimate to replace the actual drag coefficients by a mean value δ . This value will, however, be greater than the profile drag coefficient of the aerofoil section at small angles of incidence, since it must take account of the increased drag coefficients which occur on the re-treating blade where the angle of incidence is large. The analysis will be developed on the assumption that

$$\left. \begin{aligned} k_L &= 3 (\theta + \phi) \\ k_D &= \delta \end{aligned} \right\} \dots \dots (17)$$

The assumption that the lift coefficient is simply proportional to the angle of incidence will cease to be valid if the angle of incidence rises to the neighbourhood of the critical angle. Now the symmetrical aerofoil sections Göttingen 429 and R.A.F. 30 stall at an angle of incidence of 9° or 0.16 radian in two dimensional motion, and hence the limit of validity may be taken to be

$$\theta + \phi < 0.15$$

Ignoring periodic terms, equations (14) give $\phi = xR/r$ and if the blade elements are to operate below the critical angle over the outer halves of the blades, it is necessary that

$$\theta + 2x < 0.15 \dots \dots (18)$$

This condition imposes an upper limit to the angle of pitch θ for which the method of analysis is valid, and on inserting the values of x determined at a larger stage (Para. 7), the following limits are obtained.

$\delta = 0.004$	0.006	0.008	0.010
$\theta = 7.8$	7.4	7.0	6.6 degrees.

5. *Thrust.*—For one blade of the autogyro

$$\frac{dT_1}{dr} = 3 (\theta + \phi) c \rho U^2$$

and by virtue of equations (15)

$$3 (\theta + \phi) U^2 = 3 \Omega^2 (\theta r^2 + x R r) + \sin \psi \left\{ \begin{aligned} &3 (2 \theta r + x R) \Omega V \cos i \\ &- 3 \Omega^2 r^2 \beta_1 \cos \psi_1 \end{aligned} \right\} + \cos \psi \left\{ \begin{aligned} &3 \Omega^2 r^2 \beta_1 \sin \psi_1 \\ &- 3 \Omega r (\beta_0 + \chi) V \cos i \end{aligned} \right\} \quad (19)$$

The periodic terms disappear on summing over the B blades of the autogyro and hence the total thrust is

$$T = B c \rho \Omega^2 R^3 \left(\theta + \frac{3}{2} x \right)$$

Now $B c R$ is the total blade area and it is convenient to write σ for the solidity or the ratio of the blade to the disc area :—

$$\sigma = \frac{Bc}{\pi R} \quad \dots \quad \dots \quad \dots \quad (20)$$

and then the thrust coefficient is

$$T_c = \sigma \left(\theta + \frac{3}{2} x \right) \quad \dots \quad \dots \quad \dots \quad (21)$$

The periodic part of the thrust on one blade is obtained by integration as

$$c \rho \Omega^2 R^3 \left[\sin \psi \left\{ 3 \left(\theta + x \right) \frac{V \cos i}{\Omega R} - \beta_1 \cos \psi_1 \right\} + \cos \psi \left\{ \beta_1 \sin \psi_1 - \left(\frac{3}{2} \beta_0 - 3 \eta_1 \right) \frac{V \cos i}{\Omega R} \right\} \right]$$

and on inserting the values from equations (23) below, this expression becomes

$$c \rho \Omega^2 R^3 \left\{ \left(\frac{1}{3} \theta + x \right) \sin \psi - \left(\frac{1}{6} \beta_0 - 3 \eta_1 + 8 \eta_2 \right) \cos \psi \right\} \frac{V \cos i}{\Omega R}$$

so that the thrust on one blade is

$$\frac{T_1}{c \rho \Omega^2 R^3} = \left(\theta + \frac{3}{2} x \right) + \left\{ \left(\frac{1}{3} \theta + x \right) \sin \psi - \left(\frac{1}{6} \beta_0 - 3 \eta_1 + 8 \eta_2 \right) \cos \psi \right\} \frac{V \cos i}{\Omega R} \quad (22)$$

Inserting typical numerical values* this expression gives

$$\frac{T_1}{c \rho \Omega^2 R^3} = 0.068 + \left\{ 0.034 \sin \psi - 0.039 \cos \psi \right\} \frac{V \cos i}{\Omega R}$$

and when $V \cos i = \frac{1}{2} \Omega R$, its largest legitimate value, the thrust on one blade oscillates 38 per cent. on each side of its mean value.

* The typical values inserted throughout the report refer to an autogyro defined by the values

$B = 4, \theta = 2^\circ, \sigma = 0.2, \delta = 0.006, \epsilon = 0.03, \frac{W_1}{W} = 0.03, W/\pi R^2 = 2, R = 17.5.$

6. *Thrust moment and flapping.*—For one blade of the autogyro

$$r \frac{dT_1}{dr} = 3 (\theta + \phi) c \rho U^2 r$$

and according to the analysis of Para. 2 the thrust moment must be independent of the angle ψ and have the value given by equation (10). On integration, the coefficients of $\sin \psi$ and $\cos \psi$ give respectively

$$(2\theta + \frac{3}{2}x) \Omega R^3 V \cos i - \frac{3}{4} \Omega^2 R^4 \beta_1 \cos \psi_1 = 0$$

$$\frac{3}{4} \Omega^2 R^4 \beta_1 \sin \psi_1 - (\beta_0 - 6\eta_2) \Omega R^3 V \cos i = 0$$

or

$$\left. \begin{aligned} \beta_1 \sin \psi_1 &= \frac{4}{3} (\beta_0 - 6\eta_2) \frac{V \cos i}{\Omega R} \\ \beta_1 \cos \psi_1 &= \frac{8}{3} (\theta + \frac{3}{4}x) \frac{V \cos i}{\Omega R} \end{aligned} \right\} \dots (23)$$

and hence

$$\tan \psi_1 = \frac{\frac{1}{2} \beta_0 - 3\eta_2}{\theta + \frac{3}{4}x} \dots \dots (24)$$

The thrust moment on each blade is

$$(TM)_1 = c \rho \Omega^2 R^4 (\frac{3}{4}\theta + x)$$

and by means of equation (10) it is possible to determine the value of the coning angle β_0 . The thrust T is sensibly equal to the total weight W of the aircraft and hence the angular velocity is given by the equation

$$W = T = B c \rho \Omega^2 R^3 (\theta + \frac{3}{2}x) \dots \dots (25)$$

Equation (10) now gives

$$\beta_0 + \varepsilon = \frac{g \rho \sigma \pi R^3}{\mu_2} \left\{ \frac{(\frac{3}{4}\theta + x)}{B W_1} - \frac{\mu_1 (\theta + \frac{3}{2}x)}{W} \right\} \dots (26)$$

or which typical numerical values are

$$\beta_0 + 0.030 = 0.160 - 0.014 = 0.146$$

$$\beta_0 = 0.116, \text{ or } 6\frac{1}{2} \text{ degrees.}$$

Also

$$\tan \psi_1 = 0.54, \psi_1 = 28\frac{1}{2} \text{ degrees.}$$

and

$$\beta_1 = 0.157 \frac{V \cos i}{\Omega R} \text{ or } 9^{\circ}.0 \frac{V \cos i}{\Omega R}$$

It should be noted that the values of ψ_1 , β_0 , and β_1 are very sensitive to the weight and curvature of the blades. The values are also modified considerably if the axial flow is regarded as periodic (see para. 10 below).

7. *Torque.*—For one blade of the autogyro

$$\begin{aligned} \frac{dQ_1}{dr} &= (k_D - \phi k_L) c \rho U^2 r \\ &= (\delta - 3\theta\phi - 3\phi^2) c \rho U^2 r \end{aligned}$$

and by virtue of equations (15)

$$\begin{aligned} (\delta - 3\theta\phi - 3\phi^2) U^2 &= \Omega^2 (\delta r^2 - 3\theta x R r - 3x^2 R^2) \\ &+ \sin \psi \left\{ \begin{aligned} &(2\delta r - 3\theta x R) \Omega V \cos i \\ &+ 3\Omega^2 (\theta r^2 + 2x R r) \beta_1 \cos \psi_1 \end{aligned} \right\} \\ &+ \cos \psi \left\{ \begin{aligned} &3(\theta r + 2x R) (\beta_0 + \chi) \Omega V \cos i \\ &- 3\Omega^2 (\theta r^2 + 2x R r) \beta_1 \sin \psi_1 \end{aligned} \right\} \end{aligned} \quad (27)$$

The periodic terms disappear on summing over the B blades of the autogyro and hence the total torque is

$$Q = B c \rho \Omega^2 R^4 \left(\frac{1}{4} \delta - \theta x - \frac{3}{2} x^2 \right)$$

and the torque coefficient is

$$\begin{aligned} Q_c &= \frac{1}{4} \sigma \left\{ \delta - 4x \left(\theta + \frac{3}{2} x \right) \right\} \\ &= \frac{1}{4} \sigma \delta - x T_c \quad \dots \quad \dots \quad \dots \quad (28) \end{aligned}$$

In steady motion the torque must be zero and hence the state of operation of the autogyro is determined by the equation

$$\delta = 4x \left(\theta + \frac{3}{2} x \right) \quad \dots \quad \dots \quad \dots \quad (29)$$

This equation determines the parameter x in terms of the angle of pitch θ and of the mean profile drag coefficient δ . Now x is the ratio of the axial velocity u to the tip speed ΩR , and equations (25) and (29) taken in combination show that a given autogyro operates with definite values of the angular velocity and of the axial velocity u , which are independent of the angle of incidence.

The value of x is determined in any particular case by rewriting equation (29) in the form

$$x = \frac{1}{3} \left\{ \sqrt{\theta^2 + \frac{3}{2} \delta - \theta} \right\}$$

and the following table gives the value of x for a suitable range of values of θ and δ .

$\theta =$	<i>Values of x.</i>			
	0°	2°	4°	6°
$\delta = 0$	0	0	0	0
0.003	0.0224	0.0136	0.0090	0.0065
0.006	0.0316	0.0220	0.0160	0.0121
0.010	0.0408	0.0308	0.0237	0.0188
0.015	0.0500	0.0397	0.0318	0.0260

The torque on the individual blades is due solely to the periodic terms which give

$$Q_1 = c \rho \Omega^2 R^4 \left[\sin \psi \left\{ \left(\frac{2}{3} \delta - \frac{3}{2} \theta x \right) \frac{V \cos i}{\Omega R} + \left(\frac{3}{2} \theta + 2x \right) \beta_1 \cos \psi_1 \right\} + \cos \psi \left\{ \left(\theta \beta_0 + 3x \beta_0 - 6\theta \eta_2 - 6x \eta_1 \right) \frac{V \cos i}{\Omega R} - \left(\frac{3}{2} \theta + 2x \right) \beta_1 \sin \psi_1 \right\} \right]$$

and on substituting from equations (23)

$$\frac{Q_1}{c \rho \Omega^2 R^4} = \left\{ \left(\frac{2}{3} \delta + 2\theta^2 + \frac{16}{3} \theta x + 4x^2 \right) \sin \psi + x \left(\frac{1}{3} \beta_0 - 6\eta_1 + 16\eta_2 \right) \cos \psi \right\} \frac{V \cos i}{\Omega R} \dots \quad (30)$$

for which typical numerical values are

$$\frac{Q_1}{c \rho \Omega^2 R^4} = (0.0125 \sin \psi + 0.0017 \cos \psi) \frac{V \cos i}{\Omega R}$$

Thus, to a close approximation, the torque is retarding on the advancing blades and accelerating on the retreating blades.

8. *Longitudinal force.*—The longitudinal force on one blade is calculated from the equation

$$\frac{dH_1}{dr} = \frac{1}{r} \frac{dQ_1}{dr} \sin \psi - \frac{dT_1}{dr} (\beta + \chi) \cos \psi$$

where $\beta = \beta_0 - \beta_1 \cos(\psi - \psi_1)$.

To obtain the sum over all the blades it is sufficient to neglect odd powers of $\sin \psi$ and $\cos \psi$ in the expansion of this expression and to

replace $\sin^2 \psi$ or $\cos^2 \psi$ by $\frac{1}{2} B$. Proceeding by this method and using the expansions given in equations (19) and (27), the longitudinal force is obtained in the form

$$\begin{aligned} \frac{2}{B C \rho} \frac{d H}{d r} &= (2 \delta r - 3 \theta x R) \Omega V \cos i \\ &+ 3 \Omega^2 (\theta r^2 + 2 x R r) \beta_1 \cos \psi_1 \\ &- 3 \Omega^2 r^2 (\beta_0 + \chi) \beta_1 \sin \psi_1 \\ &\quad + 3 \Omega r (\beta_0 + \chi)^2 V \cos i \\ &+ 3 \Omega^2 (\theta r^2 + x R r) \beta_1 \cos \psi_1 \end{aligned}$$

On integrating

$$\begin{aligned} \frac{2 H}{B c \rho \Omega^2 R^3} &= (\delta - 3 \theta x + \frac{3}{2} \beta_0^2 - 6 \beta_0 \eta_1 + 3 \xi) \frac{V \cos i}{\Omega R} \\ &+ (2 \theta + \frac{9}{2} x) \beta_1 \cos \psi_1 - (\beta_0 - 6 \eta_2) \beta_1 \sin \psi_1 \end{aligned}$$

and then substituting from equations (23)

$$\begin{aligned} \frac{H}{B c \rho \Omega^2 R^3} &= \left\{ \frac{1}{2} \delta + \frac{8}{3} \theta^2 + \frac{13}{2} \theta x + \frac{9}{2} x^2 + \frac{1}{12} \beta_0^2 \right. \\ &\quad \left. + (8 \eta_2 - 3 \eta_1) \beta_0 - 24 \eta_2^2 + \frac{1}{2} \xi \right\} \frac{V \cos i}{\Omega R} \quad \dots \quad (31) \end{aligned}$$

Inserting typical numerical values

$$\frac{H}{T} = 0.264 \frac{V \cos i}{\Omega R}$$

and if only the first four terms of the expression for H are retained, the numerical factor falls to 0.198. The difference is 25 per cent. of the full value, but the longitudinal force usually contributes less than the thrust to the drag, and there is therefore some justification in retaining only the first four terms as an approximate expression for the longitudinal force. Moreover, the later terms depend on the weight and curvature of the blades, so that it is not possible to assess their value until the full details of the blades are known, whereas the earlier terms depend only on the angle of pitch and on the mean profile drag coefficient. It is therefore proposed to adopt the approximate expression

$$\begin{aligned} H_c &= \sigma \left(\frac{1}{2} \delta + \frac{8}{3} \theta^2 + \frac{13}{2} \theta x + \frac{9}{2} x^2 \right) \frac{V \cos i}{\Omega R} \\ &= \sigma \left(\frac{8}{3} \theta^2 + \frac{17}{2} \theta x + \frac{15}{2} x^2 \right) \frac{V \cos i}{\Omega R} \quad \dots \quad (32) \end{aligned}$$

In any case of special importance, however, it would be desirable to use the full expression (31), particularly at small angles of incidence.

9. *Lateral force.*—The lateral force on one blade is calculated from the equation

$$\frac{d Y_1}{d r} = - \frac{1}{r} \frac{d Q_1}{d r} \cos \psi - \frac{d T_1}{d r} (\beta + \chi) \sin \psi$$

and proceeding as in the case of the longitudinal force

$$\begin{aligned} \frac{2}{B c \rho} \frac{d Y}{d r} &= 3 \Omega^2 (\theta r^2 + 2 x R r) \beta_1 \sin \psi_1 \\ &\quad - 3 (\theta r + 2 x R) (\beta_0 + \chi) \Omega V \cos i \\ &\quad - 3 (2 \theta r + x R) (\beta_0 + \chi) \Omega V \cos i \\ &\quad + 3 \Omega^2 r^2 (\beta_0 + \chi) \beta_1 \cos \psi_1 \\ &\quad + 3 \Omega^2 (\theta r^2 + x R r) \beta_1 \sin \psi_1 \end{aligned}$$

On integrating

$$\begin{aligned} \frac{2 Y}{B c \rho \Omega^2 R^3} &= (9 \theta \eta_1 - \frac{9}{2} \theta \beta_0 - 9 x \beta_0) \frac{V \cos i}{\Omega R} \\ &\quad + (\beta_0 - 6 \eta^2) \beta_1 \cos \psi_1 + (2 \theta + \frac{9}{2} x) \beta_1 \sin \psi_1 \end{aligned}$$

and then substituting from equations (23)

$$\begin{aligned} \frac{Y}{B c \rho \Omega^2 R^3} &= \left\{ \theta \left(\frac{5}{12} \beta_0 + \frac{9}{2} \eta_1 - 16 \eta_2 \right) \right. \\ &\quad \left. - x \left(\frac{1}{2} \beta_0 + 24 \eta^2 \right) \right\} \frac{V \cos i}{\Omega R} \quad \dots \quad (33) \end{aligned}$$

Inserting typical numerical values

$$\frac{Y}{T} = - 0.108 \frac{V \cos i}{\Omega R}$$

indicating a lateral force to port, where the blades are retreating, and a magnitude proportional to the forward speed of the autogyro.

Experimentally the lateral force appears to be to port at high speed and to starboard at low speed. Thus the sense of the variation of the lateral force with speed has been obtained correctly, but there is a discrepancy in the value at low speeds. To explain this divergence it is necessary to abandon the assumption that the axial velocity u is constant over the whole disc and to consider the effect of a varying induced velocity.

10. *Periodic induced velocity.*—Hitherto the normal induced velocity has been assumed to have a constant value v over the whole disc of the autogyro, but it is evident on physical grounds that the induced velocity will in fact be greater to the rear and less to the

front of the disc. If the increment of the induced velocity is proportional to the distance behind the centre of the disc, then at small angles of incidence it will be of the form

$$v + v_1 \frac{r}{R} \cos \psi$$

It is not possible to assign an exact value for v_1 , but it is probably of the same order as v , and in numerical estimates it is perhaps legitimate to assume that $v_1 = v$.

Now from equation (11) the mean induced velocity at small angles of incidence is

$$v = \frac{T}{2 \pi R^2 \rho V}$$

or

$$\frac{v}{\Omega R} = \frac{T_c}{2 \lambda}$$

and, when allowance is made for the varying induced velocity, the flow through the disc becomes

$$\Omega R (x - x_1 \cos \psi)$$

where

$$x_1 = \frac{v_1}{\Omega R} \cdot \frac{r}{R} = \frac{v_1}{v} \frac{T_c}{2 \lambda} \frac{r}{R}$$

The following corrections are then necessary to the expressions (19) and (27)

$$\begin{aligned} \Delta \left\{ 3 (\theta + \phi) U^2 \right\} &= - 3 \Omega^2 R r x_1 \cos \psi \\ &= - 3 \Omega^2 r^2 \frac{v_1}{v} \frac{T_c}{2 \lambda} \cos \psi \end{aligned}$$

$$\begin{aligned} \Delta \left\{ (\delta - 3 \theta \phi - 3 \phi^2) U^2 \right\} &= \Omega^2 (3 \theta R r + 6 x R^2) x_1 \cos \psi \\ &= 3 \Omega^2 (\theta r^2 + 2 x R r) \frac{v_1}{v} \frac{T_c}{2 \lambda} \cos \psi \end{aligned}$$

The total thrust and torque of the autogyro are unaltered, though there will be some modification to their periodic parts. There is a correction to the coefficient of $\cos \psi$ in the expression for the thrust moment, which gives

$$\Delta (\beta_1 \sin \psi_1) = \frac{v_1}{v} \frac{T_c}{2 \lambda}$$

and so the phase angle ψ_1 must be determined from the equation

$$\tan \psi_1 = \frac{\frac{1}{2} \beta_0 - 3\eta^2 + \frac{3}{16} \frac{v_1}{v} \frac{T_c}{\lambda^2}}{\theta + \frac{3}{4} x} \quad \dots \quad (34)$$

which has the typical numerical value

$$\tan \psi_1 = 0.54 + \frac{0.050}{\lambda^2}$$

The increment of the longitudinal force H is obtained as

$$\Delta \left(\frac{2}{B c \rho} \frac{dH}{dr} \right) = -3 \Omega^2 r^2 (\beta_0 + \chi) \Delta (\beta_1 \sin \psi_1) + 3 \Omega^2 R r x_1 (\beta_0 + \chi)$$

which vanishes identically, so that the lift and drag of the autogyro are not altered.

The increment of the lateral force Y is obtained as

$$\begin{aligned} \Delta \left(\frac{2}{B c \rho} \frac{dY}{dr} \right) &= 3 \Omega^2 (2 \theta r^2 + 3 x R r) \Delta (\beta_1 \sin \psi_1) \\ &\quad - 3 \Omega^2 (\theta R r + 2 x R^2) x_1 \\ &= 3 \Omega^2 (\theta r^2 + x R r) \frac{v_1}{v} \frac{T_c}{2 \lambda} \end{aligned}$$

and hence

$$\Delta \left(\frac{Y}{B c \rho \Omega^2 R^3} \right) = \frac{1}{4} \left(\theta + \frac{3}{2} x \right) \frac{v_1}{v} \frac{T_c}{\lambda}$$

or

$$\frac{\Delta Y}{T} = \frac{1}{4} \frac{v_1}{v} \frac{T_c}{\lambda} \quad \dots \quad (35)$$

This increment indicates a force to starboard, where the blades are advancing, and a magnitude increasing as the forward speed decreases. The correction is therefore of the type required to explain the discrepancy mentioned in the previous paragraph. Numerically, however, the correction is not sufficiently large, for with typical values the lateral force becomes

$$\frac{Y}{T} = \frac{0.0034}{\lambda} - 0.108 \lambda$$

which vanishes when $\lambda = 0.178$. This value corresponds to an angle of incidence in the neighbourhood of 30° , and the evidence available from full scale flight appears to indicate that the lateral force should be to starboard at a moderate angle such as 15° . The source of this residual discrepancy may possibly lie in the numerical values which have been used to illustrate the general results.

11. *Lift and drag.*—The aerodynamic characteristics of an autogyro depend on the values of three fundamental parameters:—

- θ = the angle of pitch of the blades,
- σ = the solidity of the blades,
- δ = the mean profile drag coefficient,

and when these values are known, the lift and drag of the autogyro are calculated by means of the following equations:—

$$\left. \begin{aligned}
 \delta &= 4x \left(\theta + \frac{3}{2}x \right) \\
 T_c &= \sigma \left(\theta + \frac{3}{2}x \right) = \frac{\sigma \delta}{4x} \\
 \zeta &= \frac{8}{3} \theta^2 + \frac{17}{2} \theta x + \frac{15}{2} x^2 \\
 H_c &= \sigma \zeta \lambda \cos i \\
 \lambda \sin i &= x + \frac{\frac{1}{2} T_c}{\sqrt{\lambda^2 \cos^2 i + x^2}} \\
 \lambda^2 k_z &= T_c \cos i - H_c \sin i \\
 \lambda^2 k_x &= T_c \sin i + H_c \cos i
 \end{aligned} \right\} \dots (36)$$

The method of using these equations is to calculate, from the known values of the three fundamental parameters, the values of x , T_c , and ζ . Then, starting with a suitable series of values of $\lambda \cos i$, it is possible to calculate in turn the values of $\lambda \sin i$, i , λ , H_c , k_z and k_x .

No ambiguity exists as to the value of θ , but if the aerofoil sections are not of symmetrical shape this angle should be measured from the no lift line of the section and not from the chord.

The solidity σ has been defined as the ratio of the total blade area to the disc area, but the analysis has been developed on the assumption of blades of constant chord. Thus in the application of the equations to the case of blades which are rounded at the tips and tapered slightly at the root, the value of should be taken to be

$$\sigma = \frac{B c}{\pi R}$$

where c is the chord length over the greater part of the blade, rather than the actual ratio of blade area to disc area.

The value of δ is less certain, since it represents the effective mean value of the profile drag coefficient. As an aid to the choice

of a suitable value of δ , it is useful to calculate the mean lift coefficient \bar{k}_L given by the aerofoil sections. Approximately

$$T = \int_0^R k_L B c \rho \Omega^2 r^2 dr$$

$$= \frac{1}{3} \bar{k}_L B c \rho \Omega^2 R^3$$

or $T_c = \frac{1}{3} \sigma \bar{k}_L$

so that

$$\bar{k}_L = 3 \left(\theta + \frac{3}{2} x \right) = \frac{3 \delta}{4 x}$$

Numerical values are given in the following table.

Mean lift coefficient.

$\theta =$	0°	2°	4°	6°
$\delta = 0$	0	0.105	0.210	0.315
0.003	0.101	0.166	0.250	0.345
0.006	0.142	0.204	0.282	0.370
0.010	0.184	0.244	0.317	0.400
0.015	0.225	0.284	0.353	0.432

The mean lift coefficient increases with the angle of pitch and the corresponding value of δ may also be expected to increase also. As a rough guide to a suitable value of δ it is suggested to take the profile drag coefficient of the aerofoil section at the mean lift coefficient and to increase it 50 per cent. Thus, in the case of a good symmetrical section the profile drag coefficient at small lift coefficients is of the order of 0.004, and so a typical value of δ is 0.006. The question is complicated by the scale effect on the profile drag, both at small angles and near the critical angle, which is attained on the retreating blade, and a final decision as to suitable values for δ can be attained only by comparison with experimental results. It is also probable that the value of δ which occurs in equations (31) and (32) for the longitudinal force should be greater than the value in equations (28) and (29) for the torque, since the roots of the blades, where the drag coefficient is high, exert a relatively greater influence on the longitudinal force than on the torque.

12. *The ideal autogyro*—Before discussing the fundamental equations (36) in general, it is of interest to examine the limiting condition when δ tends to zero, which is the case of the ideal autogyro. It appears that x also tends to zero, indicating that there is no axial

flow through the disc, and so the ideal autogyro cannot be regarded as a physical reality but only as a limit towards which a good autogyro may tend.

After a little reduction the lift and drag coefficients of the ideal autogyro can be obtained in the form

$$k_z = \sin 2i (\cos i - \frac{4}{3} \theta \sqrt{\sigma \theta \sin 2i})$$

$$k_x = \sin 2i \sin i + \frac{4}{3} \theta (1 + \cos 2i) \sqrt{\sigma \theta \sin 2i} \dots (37)$$

and the maximum lift coefficient is practically independent of the values of θ and σ . Approximately

$$k_z (\text{max}) = 0.77, \quad \text{at } i = 35\frac{1}{2} \text{ degrees.}$$

Also at small angles of incidence

$$\left. \begin{aligned} k_z &= 2i \\ k_x &= 2i^2 + \frac{8}{3} \theta \sqrt{2\sigma\theta i} \\ \frac{X}{Z} &= i + \frac{8}{3} \theta \sqrt{\frac{\sigma\theta}{2i}} \end{aligned} \right\} (38)$$

and the minimum value of the drag-lift ratio is $2\theta(3\sigma)^{1/3}$ at the angle $\frac{2}{3}\theta(3\sigma)^{1/3}$. Numerically, when $\theta = 2^\circ$ and $\sigma = 0.2$ the maximum lift-drag ratio is 17 and occurs where $i = 1^\circ.12$, $k_z = 0.039$. Full numerical values in this case are given in Table 3.

It is of interest to note that an ideal aerofoil of semi span R and aspect ratio A with elliptic loading gives the values

$$k_z = \frac{Z}{\pi R^2 \rho V^2} = \frac{4}{2+A} i$$

$$k_x = \frac{X}{\pi R^2 \rho V^2} = \frac{1}{2} k_z^2$$

Thus, as regards lift, the ideal autogyro is equivalent to an aerofoil of zero aspect ratio or infinite chord. Also the drag coefficient of the autogyro can be written in the form

$$k_x = \frac{1}{2} k_z^2 + \frac{8}{3} \theta \sqrt{\sigma \theta k_z}$$

and so the drag of the ideal autogyro is higher than that of the ideal aerofoil.

13. *Maximum lift coefficient.*—A good approximation to the maximum lift coefficient of any autogyro can be derived from the following approximations to the fundamental equations (36) :—

$$\left. \begin{aligned} \delta &= 4x \left(\theta + \frac{3}{2} x \right) \\ T_c &= \frac{\sigma \delta}{4x} \\ \lambda \sin i &= x + \frac{T_c}{2\lambda \cos i} \\ \lambda^2 k_z &= T_c \cos i \end{aligned} \right\} \dots (39)$$

Hence

$$\begin{aligned} \lambda^2 \sin i \cos i - x \lambda \cos i &= \frac{\sigma \delta}{8x} \\ \lambda^2 k_z &= \frac{\sigma \delta}{4x} \cos i \end{aligned}$$

and by differentiation with respect to λ and i , the maximum lift coefficient is found to occur when

$$2\lambda(3\sin^2 i - 1) = 3x \sin i$$

Substituting back in the previous equations

$$\left. \begin{aligned} k_z(\text{max}) &= \frac{2}{3} \frac{(2 - 3\sin^2 i) \cos^2 i}{\sin i} \\ \text{where} \quad \frac{(3\sin^2 i - 1)^2}{(2 - 3\sin^2 i) \sin i \cos i} &= \frac{6x^3}{\sigma \delta} \end{aligned} \right\} \dots (40)$$

which give the following numerical results :—

i	35½	36	37	38	39	40
$\frac{6x^3}{\sigma \delta}$	0	·003	·017	·045	·089	·153
$k_z(\text{max.})$	·770	·715	·645	·580	·520	·463

By means of this table the maximum lift coefficient of any autogyro, defined by the values of θ , σ and δ , can be determined, and Table 1 gives the values for a suitable range of values of the three fundamental parameters. The maximum lift coefficient increases as θ or σ increases and as δ decreases.

14. *Maximum lift drag ratio.*—At small angles of incidence the fundamental equations (36) can be replaced by the approximations

$$\left. \begin{aligned} \delta &= 4x \left(\theta + \frac{3}{2} x \right) \\ T_c &= \frac{\sigma \delta}{4x} \\ \zeta &= \frac{8}{3} \theta^2 + \frac{17}{2} \theta x + \frac{15}{2} x^2 \\ H_c &= \sigma \zeta \lambda \\ \lambda(\lambda i - x) &= \frac{\sigma \delta}{8x} \\ \frac{X}{Z} &= i + \frac{4x \zeta \lambda}{\delta} \end{aligned} \right\} \dots \quad (41)$$

By differentiation with respect to λ and i , the minimum value of the drag-lift ratio is found to occur where

$$2 \lambda i - x = \frac{4x \zeta \lambda^2}{\delta}$$

and then writing

$$\lambda' = 2 \lambda \sqrt{\frac{\zeta}{\delta}}$$

the results can be expressed in the form

$$\left. \begin{aligned} \lambda' (\lambda'^2 - 1) &= \frac{\sigma \sqrt{\zeta \delta}}{2x^2} \\ i &= x \sqrt{\frac{\zeta}{\delta}} \left(\lambda' + \frac{1}{\lambda'} \right) \\ \frac{X}{Z} \text{ (min.)} &= x \sqrt{\frac{\zeta}{\delta}} \left(3 \lambda' + \frac{1}{\lambda'} \right) \end{aligned} \right\} \dots \quad (42)$$

The general numerical solution of these equations is as follows:—

$\frac{\sigma \sqrt{\zeta \delta}}{2x^2}$	0.5	1.0	1.5	2	3	4
λ'	1.192	1.325	1.432	1.522	1.672	1.797
$\frac{i}{x} \sqrt{\frac{\delta}{\zeta}}$	2.03	2.08	2.13	2.18	2.27	2.35
$\left(\frac{X}{Z}\right) \frac{1}{x} \sqrt{\frac{\delta}{\zeta}}$	4.41	4.73	4.99	5.22	5.61	5.95

The maximum lift-drag ratio can be determined from this table for any autogyro when the values of the three fundamental parameters, θ , σ and δ are known. Typical values are given in Table 2, and it appears that the lift-drag ratio increases as θ , σ , or δ decreases.

15. *General discussion.*—Within the limits of validity of the preceding analysis, as discussed in para. 4, the characteristics of an autogyro depend essentially on the values of the three fundamental parameters θ , σ , and δ , but the flapping of the blades and the lateral force Y depend also on the curvature, weight, and size of the blades. The longitudinal force and drag are also influenced to a smaller extent by these latter characteristics.

The analysis of the torque led to the equation (29) which shows that the axial velocity u bears a constant ratio to the tip velocity ΩR for a given autogyro. Also, according to equation (21), the thrust coefficient T_c has a constant value, and since in level flight the thrust is sensibly equal to the weight of the aircraft, it appears that the tip velocity must have the constant value

$$\Omega R = \sqrt{\frac{W}{T_c \rho \pi R^2}} = 2 \sqrt{\frac{x w}{\rho \sigma \delta}} \quad \dots \quad (43)$$

and if $\theta = 2^\circ$, $\delta = 0.006$.

$$\Omega R = 78.8 \sqrt{\frac{w}{\sigma}}$$

The maximum lift coefficient of an autogyro is of the order of 0.5 to 0.6, occurring at an angle of incidence in the neighbourhood of 38° , and if the autogyro is loaded 2 lb. per sq. ft. of disc area, the stalling speed will be of the order of 25 to 28 m.p.h.

The maximum lift-drag ratio of the rotating wings is poor compared with that of ordinary fixed wings: its ordinary value is approximately 6, and it is unlikely to exceed 8 in any practical case. It occurs at a small value of the lift coefficient, in the neighbourhood of 0.05, and so at a speed approximately three times the stalling speed.

The lateral force on an autogyro whose starboard blades are advancing, is to port at high speeds and to starboard at low speeds. If the shaft of the autogyro is inclined sideways so as to maintain the aircraft on a level keel at a mean angle of incidence, it is to be anticipated that the aircraft will fly inclined downwards to port at high speed and downwards to starboard at low speeds.

Numerical solutions, illustrating the general equations (36), are given in Tables 4 to 7. Table 4 gives the solution in the typical case $\theta = 2^\circ$, $\sigma = 0.2$, $\delta = 0.006$, and this solution is also shown in fig. 5. It is of interest to notice the curved shape of the lift curve, which first increases very slowly with incidence, then at uniform rate, and finally falls off to a very gradual stall. Figs. 6 to 8 show

respectively the effect of variation of the profile drag, solidity and angle of pitch. It is desirable that the profile drag should be as low as possible, a solidity of 0.2 represents a good mean condition but a lower solidity is advantageous for high speed, and an angle of pitch of 2° is probably the best for the ordinary range of flying speeds.

A discussion of the best conditions for maximum speed and of the possibility of vertical descent is given in the appendices. The important conclusion is reached that as the maximum speed of a gyroplane is increased, the loading must also be increased in order to maintain a sufficient ratio of tip speed to forward speed, and there is a corresponding increase of the stalling speed. In the typical case $\theta = 2^\circ$, $\sigma = 0.02$, $\delta = 0.006$, the values are :—

Maximum speed	...	85	150	200	m.p.h.
Loading ($W/\pi R^2$)	...	2.0	6.2	11.0	lb./sq. ft.
Stalling speed	$26\frac{1}{4}$	$26\frac{1}{2}$	62	m.p.h.

Thus the principal merit of a gyroplane, its low landing speed, inevitably disappears when high speed of level flight is required, and there remains only the absence of a sudden stall to counter-balance the very poor efficiency as compared with an aeroplane.

APPENDIX I.

THE ENERGY LOSSES OF AN AUTOGYRO.

The theory of the autogyro, as given in the main body of the report is developed by considering the aerodynamic forces on the rotating blades. The theory necessarily involves certain assumptions and approximations, which unfortunately become less accurate at small angles of incidence and introduce some uncertainty in the determination of the maximum lift-drag ratio. An attempt has, therefore, been made to analyse the energy account of an autogyro in order to provide an independent check on the previous results. This analysis gives an upper limit only to the possible lift-drag ratio since it is not possible to evaluate fully every possible source of loss of energy. For simplicity also the analysis has been confined to small angles of incidence, *i.e.* to the region where the results of the previous theory are most likely to be in error.

Two main sources of loss of energy are considered, due respectively to the induced velocity caused by the thrust and to the profile drag of the blades. An additional source of loss of energy is the periodic distribution of thrust over the disc of the windmill but no simple method has been found of estimating its magnitude.

The analysis assumes the angle of incidence i to be small, and hence it is legitimate to replace $\cos i$ by unity and to regard the lift Z as identical with the thrust T . If E is the loss of energy in unit time, the drag X of the windmill is determined by the equation

$$X V = E$$

and the drag-lift ratio of the windmill is obtained as

$$\frac{X}{Z} = \frac{E}{V T}$$

The thrust of the windmill causes an induced velocity v and a corresponding loss of energy Tv . The induced velocity will be assumed to have a constant value over the disc of the windmill and to be given by the equation

$$V = \frac{T}{2 \pi R^2 \rho V'} \quad \dots \quad (a)$$

as in the previous analysis. The velocity V' in this equation is the resultant of V and v , and for small angles of incidence it is sufficiently accurate to take $V' = V$. The element of drag corresponding to the induced velocity v is then calculated simply as

$$\frac{X_1}{Z} = \frac{v}{V} = \frac{T}{2 \pi R^2 \rho V^2} = \frac{T_c}{2 \lambda^2} \quad \dots \quad (b)$$

The energy loss due to the drag of the blades will be calculated on the assumption of a mean profile drag coefficient δ for the whole of the blades. As a first approximation also, the velocity of the air relative to a typical element of the blade at angle ψ from the downwind position is $(\Omega r + V \sin \psi)$ and the corresponding loss of energy is

$$E = \Sigma \int_0^R \delta c \rho (\Omega r + V \sin \psi)^3 dr$$

where the summation extends over all the blades. Thus

$$E = B \int_0^R \delta c \rho (\Omega^2 r^3 + \frac{3}{2} V^2 \Omega r) dr$$

$$= \frac{1}{4} \delta B c \rho \Omega^3 R^4 (1 + 3 \lambda^2)$$

and the corresponding element of drag is

$$\frac{X_2}{Z} = \frac{E}{VT} = \frac{\sigma \delta}{4 \lambda T_c} (1 + 3 \lambda^2) \quad \dots \quad (c)$$

This first estimate of the energy absorbed by the drag of the blades ignores the axial velocity u and the radial velocity $V \cos \psi$. At small angles of incidence the axial velocity u is small compared with V and may be neglected, but the effect of the radial velocity is quite important. It is necessary therefore to calculate the energy absorbed by a blade evaluating the integral

$$\int_0^R \delta c \rho \left\{ (\Omega r + V \sin \psi)^2 + V^2 \cos^2 \psi \right\}^{\frac{3}{2}} dr$$

and the mean energy loss is then the mean value of this integral for all values of ψ , multiplied by the number of blades. In the absence of a simple algebraic evaluation of the mean value of this integral, the method has been adopted of evaluating the integral for four suitable values of ψ and of accepting the sum of the four values so obtained as the value of E . It is convenient, by comparison with equation (c), to express the result in the form

$$\frac{X_2}{Z} = \frac{E}{VT} = \frac{\sigma \delta}{4 \lambda T_c} (1 + n \lambda^2) \quad \dots \quad (d)$$

where n is now a function of λ , determined by the equation

$$1 + n \lambda^2 = \Sigma \int_0^1 (\xi^2 + 2 \xi \lambda \sin \psi + \lambda^2)^{\frac{3}{2}} d \xi$$

where

$$\xi = \frac{r}{R}$$

and

$$\lambda = \frac{V}{\Omega R} < 1$$

The integral has been evaluated for the angles $\psi = 0, 90, 180,$ and 270 degrees in the first place.

With $\psi = 90^\circ$, the integral is

$$I(90) = \int_0^1 (\xi + \lambda)^3 d \xi = \frac{1}{4} (1 + \lambda)^4 - \frac{1}{4} \lambda^4$$

With $\psi = 270^\circ$, the integrand must be taken to be $(\lambda - \xi)^3$ or $(\xi - \lambda)^3$ according as ξ is less than or greater than λ , and hence

$$I(270) = \int_\lambda^1 (\xi - \lambda)^3 d \xi + \int_0^\lambda (\lambda - \xi)^3 d \xi$$

$$= \frac{1}{4} (1 - \lambda)^4 + \frac{1}{4} \lambda^4$$

With $\psi = 0$ or 180° , the integral is

$$\begin{aligned} I(0) = I(180) &= \int_0^1 (\xi^2 + \lambda^2)^{3/2} d\xi \\ &= \frac{1}{8} (2 + 5\lambda^2) \sqrt{1 + \lambda^2} \\ &\quad + \frac{3}{16} \lambda^4 \log \frac{\sqrt{1 + \lambda^2} + 1}{\sqrt{1 + \lambda^2} - 1} \end{aligned}$$

Finally, by adding the values of the four integrals, we obtain

$$\begin{aligned} 1 + n\lambda^2 &= \frac{1}{2} (1 + 6\lambda^2 + \lambda^4) + \frac{1}{4} (2 + 5\lambda^2) \sqrt{1 + \lambda^2} \\ &\quad + \frac{3}{8} \lambda^4 \log \frac{\sqrt{1 + \lambda^2} + 1}{\sqrt{1 + \lambda^2} - 1} \quad \dots \quad (e) \end{aligned}$$

The numerical values of $(1 + n\lambda^2)$ and n are given below in Table A. It appears that n increases with λ instead of having the constant value 3 as given by the approximate equation (c). Moreover, its value is of the order of 5 in the region where the maximum lift-drag ratio is likely to occur, and hence it is important to retain the radial velocity in the calculation of the energy losses.

As a check on the accuracy of the method of calculating n , the integrals have also been evaluated for the angles $\psi = 45, 135, 225,$ and 315 degrees for the special case $\lambda = 1$. The value derived for n was 6.09, instead of 6.13, so the agreement is satisfactory.

TABLE A.

Values of n .

λ	$1 + n\lambda^2$	n
1.00	7.13	6.13
0.75	4.11	5.53
0.60	2.88	5.22
0.50	2.26	5.03
0.40	1.78	4.87
0.30	1.43	4.73
0	1.00	4.50

By considering the loss of energy of an autogyro the drag-lift ratio has been obtained in the form

$$\frac{X}{Z} = \frac{T_c}{2\lambda^2} + \frac{\sigma\delta}{4\lambda T_c} (1 + n\lambda^2)$$

whereas the result given in the main body of the report may be expressed in the form

$$\frac{X}{Z} = \frac{T_c}{2\lambda^2} + \frac{\sigma\delta}{4\lambda T_c} + \frac{\sigma\lambda\zeta}{T_c}$$

where
$$\delta = 4x\left(\theta + \frac{3}{2}x\right)$$

$$\zeta = \frac{8}{3}\theta^2 + \frac{17}{2}\theta x + \frac{15}{2}x^2$$

The comparison of the two results therefore depends on the terms

$$\frac{\sigma\lambda}{T_c}, \frac{n\delta}{4} \text{ and } \frac{\sigma\lambda}{T_c}\zeta$$

The first point to notice is that n depends only on λ , while ζ depends on θ and δ so that there is an essential difference between the two results, depending on the fact that the earlier analysis assumes λ to be small and the present analysis neglects the effect of the periodic distribution of thrust over the disc of the windmill. The earlier analysis gives the higher drag, the difference being

$$\frac{\Delta X}{Z} = \frac{\sigma\lambda}{T_c}\left(\zeta - \frac{n\delta}{4}\right)$$

Now at high speed λ is of the order 0.5 and so we can take approximately $n=5$. Then

$$\begin{aligned} \zeta - \frac{1}{4}n\delta &= \left(\frac{8}{3}\theta^2 + \frac{17}{2}\theta x + \frac{15}{2}x^2\right) - 5x\left(\theta + \frac{3}{2}x\right) \\ &= \frac{8}{3}\theta^2 + \frac{7}{2}\theta x \end{aligned}$$

which lies between

$$\frac{7}{3}\theta\left(\theta + \frac{3}{2}x\right) \text{ and } \frac{8}{3}\theta\left(\theta + \frac{3}{2}x\right).$$

Hence we have approximately

$$\frac{\Delta X}{Z} = \frac{\sigma\lambda}{T_c}\left\{\frac{5}{2}\theta\left(\theta + \frac{3}{2}x\right)\right\} = \frac{5}{2}\theta\lambda$$

It appears therefore that the two theories agree approximately when the blade angle is zero, but that there is an increasing divergence as the blade angle increases. A crucial test of the two alternative results would therefore be the experimental determination of the effect of varying the blade angle of an autogyro.

With the usual blade angle of 2° the difference in the values of X/Z is 0.087λ . According to the earlier analysis an autogyro defined by the values $\sigma = 0.20$ and $\delta = 0.006$ had the minimum drag-lift ratio of 0.170 when λ was approximately 0.5. According to the present analysis this value would be reduced to 0.126, and the lift-drag ratio would rise from 5.9 to 7.9. The

earlier analysis possibly underestimates the merit of the auto-gyro and the present theory certainly overestimates it. The truth must lie somewhere between the two values given.

The profile drag coefficient of the aerofoil section used on the autogyro is determined by model experiments as

$$k_D = 0.0040 + 0.025 k_L^2$$

This value will be reduced by a factor 0.8 to allow for scale effect, and the mean profile drag coefficient δ will be assumed to be 50 per cent. greater than the simple aerofoil value. Then

$$\delta = 0.0048 + 0.030 k_L^2$$

which gives the value $\delta = 0.0060$ at $k_L = 0.2$ as used in the earlier work.

Now

$$\delta = 4x \left(\theta + \frac{3}{2} x \right)$$

and the mean lift coefficient is

$$k_L = \frac{3}{\sigma} T_c = 3 \left(\theta + \frac{3}{2} x \right)$$

and hence we derive the equations

$$x = 0.0225 k_L + \frac{0.0036}{k_L}$$

$$\theta = 0.299 k_L - \frac{0.0054}{k_L}$$

The relationship between θ , k_L , x , and δ is shown in the following table.

TABLE B.

θ	k_L	x	δ
0°	0.134	0.0299	0.0054
2°	0.205	0.0222	0.0061
4°	0.299	0.0188	0.0074
6°	0.397	0.0180	0.0095

The lift-drag ratio can now be expressed in the form

$$\begin{aligned} \frac{X}{Z} &= \frac{T_c}{2 \lambda^2} + \frac{\sigma \delta}{4 \lambda T_c} (1 + n \lambda^2) \\ &= \frac{\sigma k_L}{6 \lambda^2} + \frac{x}{\lambda} (1 + n \lambda^2) \end{aligned}$$

and Table C shows the variation of the lift-drag ratio with the parameter λ and the blade angle θ for an autogyro of solidity $\sigma = 0.20$.

TABLE C.
 Lift-drag ratio ($\sigma = 0.20$).

	$\theta=0$	2°	4°	6°
$\lambda = 1.00$	4.61	6.07	6.95	6.90
0.75	5.84	7.46	8.15	8.19
0.60	6.44	7.96	8.47	8.12
0.50	6.56	7.84	8.00	7.45
0.40	6.24	7.07	6.85	6.15
0.30	5.24	5.50	5.00	4.30

According to these calculations the best blade angle for high speed would be approximately 4°, but unless the ratio of forward speed to tip speed exceeds 1/2 there is little advantage over the blade angle 2°, and the latter is superior at lower forward speeds.

The analysis of the energy losses of an autogyro has led to a rather more favourable estimate of the efficiency of an autogyro than was suggested by the previous analysis, the maximum lift-drag ratio for a typical autogyro rising from 6 to 8. It must be remembered, however, that the energy calculations represent an optimum condition which will not be realised, since some sources of loss of energy have been neglected.

The chief difference from the previous results is the effect of the blade angle. According to the earlier analysis the blade angle should be reduced below 2° for high speed, while according to the present calculations it should be increased to 4° or even higher. A useful check on the two alternative methods of calculation would be provided by the test of a model autogyro with blade angles 0°, 2°, and 4°.

APPENDIX 2.

Conditions for maximum speed.

The aerodynamic characteristics of an autogyro are determined by the analysis of the main report, and in particular, equations (41) represent the conditions at small angles of incidence. It is possible therefore to determine the thrust horse power ηP which is required to overcome the drag of the rotating blades, and it is found that for any speed of horizontal flight there is an optimum loading w which requires minimum power $\eta P/W$, or alternatively the curve of power against speed can be drawn for any given loading.

In horizontal flight the tip velocity of the windmill is, from equation (43),

$$\Omega R = 2\sqrt{\frac{xw}{\rho\sigma\delta}}$$

and the forward speed is

$$V = \lambda \Omega R$$

Also by virtue of equations (41), the power taken by the windmill is

$$550 \eta P = V X$$

or

$$\begin{aligned} 550 \frac{\eta P}{W} &= \frac{V X}{Z} = V \left\{ i + \frac{4x\zeta\lambda}{\delta} \right\} \\ &= 2x \sqrt{\frac{xw}{\rho\sigma\delta}} + \frac{w}{2\rho V} + 2\zeta V^2 \sqrt{\frac{\rho\sigma x}{\delta w}} \dots \quad (a) \end{aligned}$$

The loading w which requires minimum power at a given forward speed, or which gives the maximum speed for given power, is determined by the equation

$$x \sqrt{\frac{x w}{\rho \sigma \delta}} + \frac{w}{2 \rho V} = \zeta V^2 \sqrt{\frac{\rho \sigma x}{\delta w}}$$

and since

$$w = k_z \rho V^2$$

this condition is equivalent to

$$k_z^{3/2} + 2 x \sqrt{\frac{x}{\sigma \delta}} k_z = 2 \zeta \sqrt{\frac{\sigma x}{\delta}} \quad \dots \quad (b)$$

In this case also

$$\left. \begin{aligned} 550 \frac{\eta P}{W} &= V \left(\frac{3}{2} k_z + 4 x \sqrt{\frac{x k_z}{\sigma \delta}} \right) \\ \frac{\Omega R}{V} &= 2 \sqrt{\frac{x k_z}{\sigma \delta}} \end{aligned} \right\} \quad \dots \quad (c)$$

Equation (a) can be used to determine the power taken by the windmill at any speed V with any loading w , while equations (b) and (c) determine the best conditions for horizontal flight. The numerical solution for an autogyro $\theta = 2^\circ$, $\delta = 0.006$ is given in the following table, which represents the optimum conditions :—

σ	0.10	0.15	0.20	0.25	0.30
k_z	0.036	0.046	0.054	0.061	0.068
$\frac{\Omega R}{V}$	2.28	2.12	2.00	1.90	1.82
$10^4 \frac{w}{V^2}$	0.84	1.09	1.28	1.45	1.61
$10^4 \frac{\eta P}{W V}$	2.79	2.95	3.08	3.19	3.30

It has been suggested in the main report that, for efficient working, the ratio $\Omega R/V$ must not fall below the value 2. The optimum conditions for solidities equal to or less than 0.2 satisfy this condition, but in the case of the higher solidities it would be necessary to use a slightly heavier loading than the optimum value.

For the solidity 0.2 the optimum loading is the lightest that can be used and this loading varies with the speed as follows :—

$V =$	85	150	200	m.p.h.
$w =$	2.0	6.2	11.0	lb./sq. ft.

A reduction of the solidity leads to improved speed of horizontal flight since the power taken by the windmill is reduced. Also the best loading falls more rapidly than the maximum lift coefficient and hence the higher top speed is accompanied by a lower stalling speed. The limiting condition for this method of improvement is clearly the impossibility of making very thin blades of large radius and is a matter of structural strength. In any case, however, it appears to be inevitable that the loading and stalling speed must rise with the top speed of the gyroplane.

APPENDIX 3.

Vertical descent.

The problem of the vertical descent of a gyroplane is essentially that of determining the drag of a windmill subject to an axial velocity V . The equations of the main report remain valid with the exception of those which define or involve the axial induced velocity. The velocity of flow through the disc u is unaltered and the tip speed of the windmill is given by the equation :—

$$\Omega R = 2 \sqrt{\frac{x w}{\rho \sigma \delta}}$$

In order to determine the velocity of descent it is necessary to turn to the ordinary airscrew theory. It is customary to write

$$\begin{aligned} T &= 2 \pi R^2 \rho V^2 f \\ &= 2 \pi R^2 \rho u^2 F \end{aligned}$$

and in the régime in which the windmill is operating the coefficients f and F are connected by a purely empirical relationship. The value of F is obtained at once as

$$F = \frac{T_c}{2 x^2} = \frac{\sigma \delta}{8 x^3}$$

and if the corresponding empirical value of f is known, the speed V is obtained finally from the equation

$$V = \sqrt{\frac{w}{2 \rho f}}$$

If the windmill behaved as a parachute of the same disc area, f would have the value 0.3, but experimental evidence appears to indicate that the value of f may be rather higher and may possibly, as an extreme limit, attain the value 0.5. There is no evidence to indicate a value higher than 0.5.

Taking $f = 0.5$ as the highest possible value, the speed of descent becomes

$$V = 20.5 \sqrt{w}$$

which is of the order of 30 f.p.s. or more for ordinary loadings.

A typical value of F , when $\theta = 2^\circ$, $\sigma = 0.2$, and $\delta = 0.006$ is 14 and for values of F in this neighbourhood the empirical value of f is given by the equation

$$\frac{1}{f} = 2 + \sqrt{\frac{3}{F}}$$

In this case, therefore, $f = 0.4$, and the velocity of descent becomes

$$V = 23.0 \sqrt{w}$$

It is possible that there may be some cushioning effect on approaching the ground, but in free descent with a loading of 2 it is improbable that the velocity is less than 30 f.p.s. and 35 f.p.s. is a more probable figure. It is doubtful, however, whether the controls of a gyroplane are adequate to hold the aircraft in a steady vertical descent.

APPENDIX 4.

Notation.

Dimensions of blades.

- B = number of blades.
- θ = angle of pitch.
- R = extreme radius.
- r = radius to blade element.
- c = chord of blade element.
- h = ordinate from base line (Fig. 2).
- λ = slope of blade element.
- W_1 = weight of one blade.
- G_1 = weight moment about hinge.
- I_1, J_1 = moment and product of inertia.

Motion of blades

- Ω = angular velocity about shaft.
- ψ = angular position of blade.
- β = angular rotation of blade about its hinge
- $\beta = \beta_0 - \beta_1 \cos (\psi - \psi_1).$

General motion.

- i = angle of incidence of autogyro.
- V = forward speed.
- v = axial induced velocity.
- V' = resultant of V and v.
- u = axial velocity through disc.
- U = resultant velocity relative to blade element.
- ϕ = inclination of velocity U (Fig. 4).

Forces.

- W = total weight.
- $w = \frac{W}{\pi R^2}$
- T = thrust.
- H = longitudinal force.
- X = drag.
- Y = lateral force.
- Z = lift.
- Q = torque.

Coefficients.

- $T_c = T/\pi R^2 \rho \Omega^2 R^2$, etc.
- $k_x = X/\pi R^2 \rho V^2$, etc.
- k_L, k_D = lift and drag coefficients of blade element.
- δ = mean profile drag coefficient.
- $\sigma = \frac{B c}{\pi R}$ (the solidity).
- $\lambda = \frac{V}{\Omega R}$
- $x = \frac{u}{\Omega R}$
- $\zeta = \frac{8}{3} \theta^2 + \frac{17}{2} \theta x + \frac{15}{2} x^2$
- ξ, η_1, η_2 = coefficients of the blade curvature, equations (6).
- ϵ, μ_1, μ_2 = coefficients of the blade density, equations (7).

TABLE I.

Maximum lift coefficient.

θ	σ	δ	i (degrees).	k_z (max.).
2°	0.2	0	35.25	0.77
—	—	0.003	37.4	0.62
—	—	0.006	38.3	0.56
—	—	0.010	39.0	0.52
—	—	0.015	39.6	0.485
—	—	0.020	40.1	0.46
2°	0.1	0.006	39.3	0.50
—	0.2	—	38.3	0.56
—	0.3	—	37.7	0.595
0°	0.2	0.006	40.1	0.46
2°	—	—	38.3	0.56
4°	—	—	37.2	0.63
6°	—	—	36.5	0.68

TABLE 2.

Maximum lift-drag ratio.

θ	σ	δ	i (degrees).	$\frac{Z}{X}$ (max.).
2°	0.2	0	1.1	17.0
—	—	0.003	3.0	7.8
—	—	0.006	4.1	5.9
—	—	0.010	5.2	4.7
—	—	0.015	6.4	3.8
—	—	0.020	7.4	3.4
2°	0.1	0.006	3.9	6.5
—	0.2	—	4.1	5.9
—	0.3	—	4.2	5.5
0°	0.2	0.006	4.1	6.2
2°	—	—	4.1	5.9
4°	—	—	4.5	5.0
6°	—	—	5.0	4.2

TABLE 3.

Ideal autogyro.

$\theta = 2^\circ, \sigma = 0.20.$

<i>i</i> (degrees).	k_z	k_x	k_x/k_z
0.5	0.0175	0.00118	0.068
1	0.0349	0.00207	0.059
2	0.0696	0.00450	0.065
5	0.173	0.0183	0.106
10	0.336	0.0637	0.190
15	0.482	0.1346	0.279
20	0.602	0.226	0.375
25	0.691	0.330	0.478
30	0.747	0.438	0.587
35	0.766	0.544	0.710
40	0.751	0.638	0.850

TABLE 4.

Standard autogyro.

$\theta = 2^\circ, \sigma = 0.20, \delta = 0.006.$

<i>i</i> (degrees).	k_z	k_x	k_x/k_z
1.65	0.0135	0.00308	0.228
3.2	0.0373	0.00654	0.175
4.7	0.0658	0.0114	0.173
8.5	0.145	0.0302	0.208
11.1	0.203	0.0502	0.247
15.6	0.300	0.0967	0.322
20.0	0.385	0.154	0.400
24.0	0.456	0.220	0.483
29.5	0.524	0.316	0.604
37.0	0.561	0.442	0.788
41.5	0.560	0.519	0.928

TABLE 5.

Effect of profile drag.

$\theta = 2^\circ, \quad \sigma = 0.20.$

$\delta = 0.003$			$\delta = 0.010$			$\delta = 0.015$		
<i>i</i> degrees.	k_x	k_x/k_z	<i>i</i> degrees.	k_x	k_x/k_z	<i>i</i> degrees.	k_x	k_x/k_z
1.1	0.011	0.177	2.2	0.016	0.282	2.8	0.019	0.339
2.2	0.030	0.133	4.2	0.044	0.221	5.3	0.051	0.268
3.3	0.054	0.129	6.2	0.078	0.218	7.7	0.089	0.266
6.1	0.120	0.154	10.9	0.168	0.267	13.3	0.190	0.328
8.1	0.171	0.182	14.1	0.232	0.316	17.1	0.257	0.389
11.6	0.257	0.238	19.5	0.334	0.408	23.3	0.355	0.505
16.6	0.375	0.327	24.5	0.412	0.508	28.9	0.428	0.618
26.2	0.544	0.515	29.1	0.468	0.603	33.8	0.471	0.733
34.3	0.613	0.708	35.1	0.510	0.753	40.0	0.484	0.908
39.5	0.613	0.847	38.7	0.520	0.849	43.7	0.479	1.025
			42.8	0.516	0.973			

TABLE 6.

Effect of solidity.

$\theta = 2^\circ, \quad \delta = 0.006.$

$\sigma = 0.10$			$\sigma = 0.30$		
<i>i</i> degrees.	k_x	k_x/k_z	<i>i</i> degrees.	k_x	k_x/k_z
1.5	0.007	0.222	1.9	0.020	0.231
2.7	0.019	0.166	3.7	0.056	0.185
3.8	0.033	0.155	5.7	0.099	0.190
6.3	0.074	0.171	10.5	0.214	0.246
8.1	0.104	0.194	14.1	0.293	0.306
11.0	0.160	0.235	20.0	0.416	0.411
16.5	0.265	0.329	25.6	0.509	0.519
22.5	0.368	0.443	30.7	0.562	0.638
28.9	0.454	0.577	37.4	0.590	0.805
38.2	0.507	0.813	45.8	0.556	1.074
44.3	0.499	1.004			

R. & M. III.

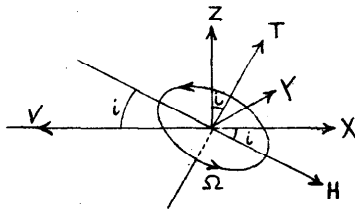


FIG. 1.

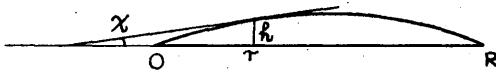


FIG. 2.

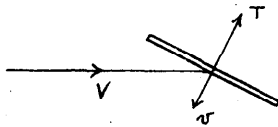


FIG. 3.

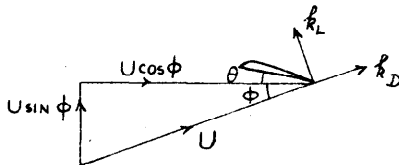


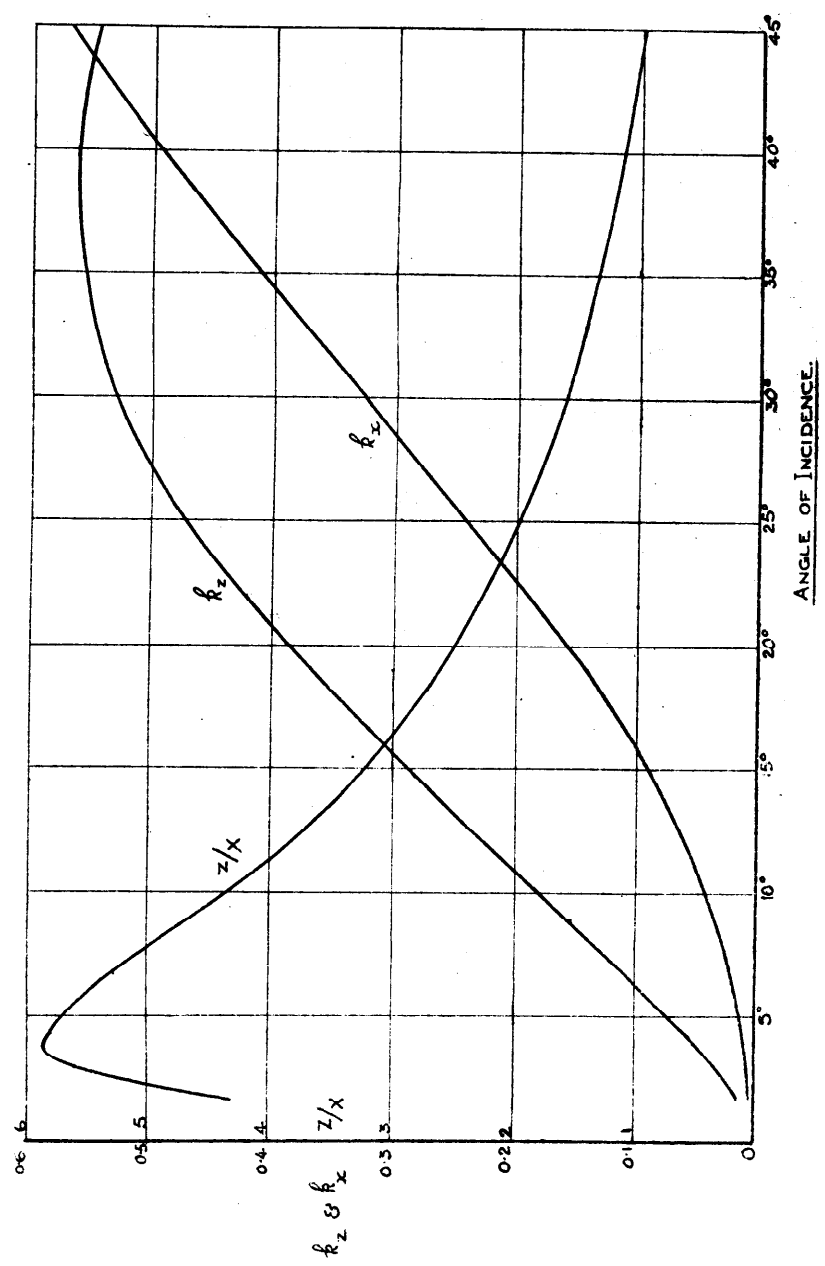
FIG. 4.

R. & M. 1111.

FIG. 5.

LIFT AND DRAG.

$$\left\{ \begin{array}{l} \theta = 2^\circ. \\ \sigma = 0.20. \\ \delta = 0.006. \end{array} \right.$$

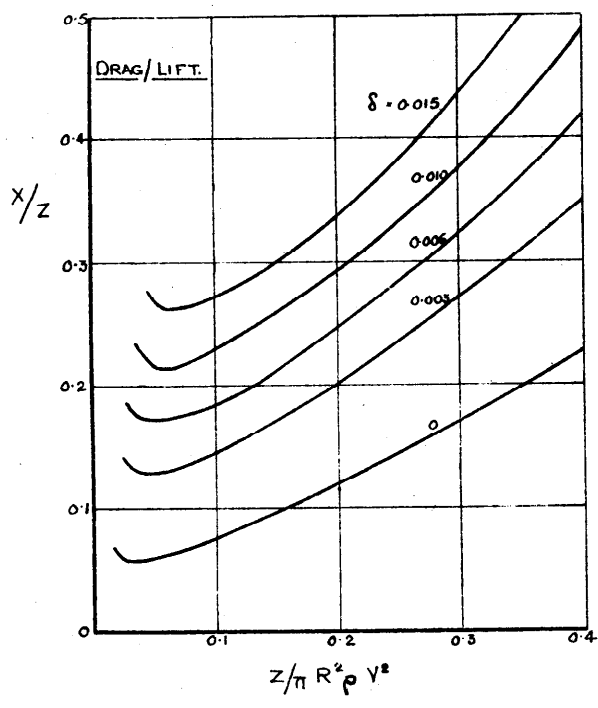
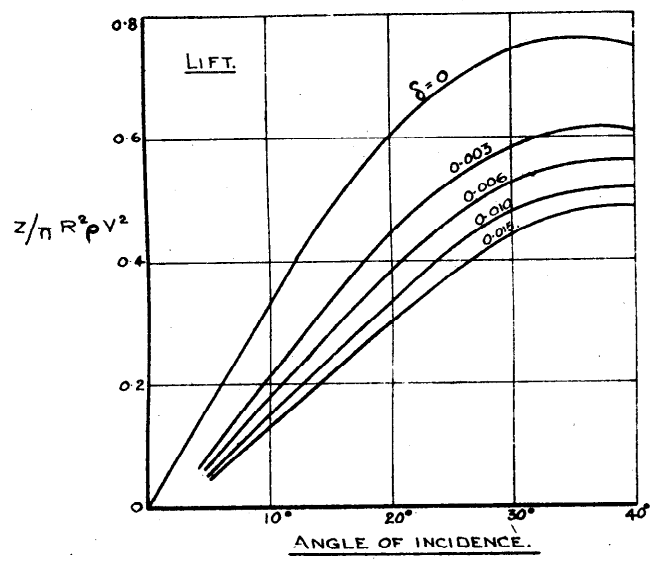


R&M. IIII.

FIG. 6.

EFFECT OF PROFILE DRAG

$$\begin{cases} \theta = 2^\circ. \\ \sigma = 0.20 \end{cases}$$

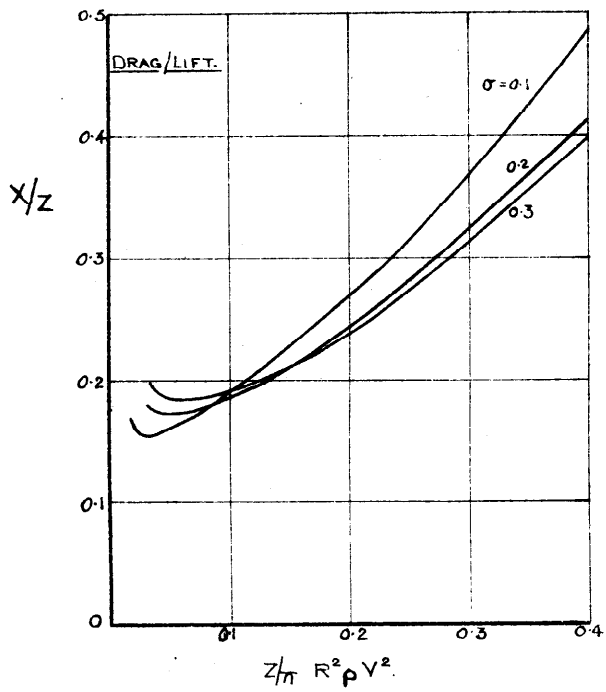
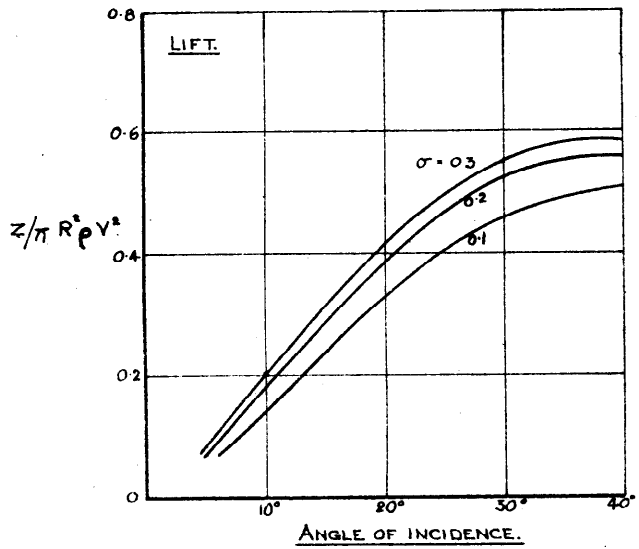


R. & M. IIII.

FIG. 7.

EFFECT OF SOLIDITY.

$$\left\{ \begin{array}{l} \theta = 2^\circ \\ \delta = 0.006. \end{array} \right.$$



R.&M. IIII.

FIG. 8.

EFFECT OF ANGLE OF PITCH.

$$\begin{cases} \sigma = 0.20. \\ \delta = 0.006. \end{cases}$$

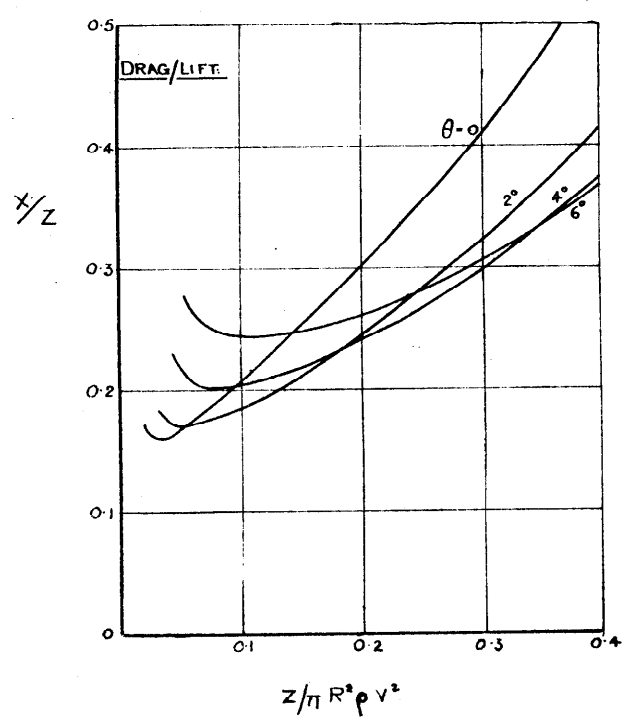
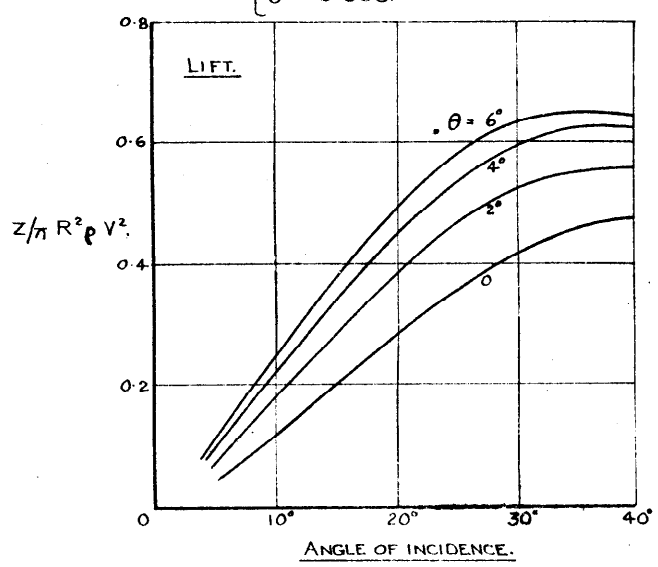


TABLE 7.

Effect of angle of pitch.

$\sigma = 0.20, \quad \sigma = 0.006.$

$\theta = 0.$			$\theta = 4^\circ.$			$\theta = 6^\circ.$		
<i>i</i> degrees.	k_z	k_x/k_z	<i>i</i> degrees.	k_z	k_x/k_z	<i>i</i> degrees.	k_z	k_x/k_z
2.1	0.010	0.195	1.5	0.019	0.287	1.4	0.025	0.362
3.8	0.026	0.161	3.0	0.051	0.212	3.1	0.068	0.258
5.3	0.046	0.165	4.7	0.091	0.201	5.0	0.119	0.242
9.0	0.100	0.206	9.0	0.199	0.238	10.1	0.258	0.284
11.4	0.143	0.241	12.1	0.277	0.283	13.9	0.355	0.337
15.4	0.209	0.310	17.5	0.399	0.373	20.3	0.498	0.450
22.6	0.327	0.444	27.6	0.569	0.575	29.2	0.629	0.628
30.2	0.420	0.610	32.3	0.606	0.682	35.7	0.648	0.795
37.6	0.469	0.793	38.0	0.624	0.834	43.8	0.625	1.034
42.2	0.474	0.936	42.5	0.608	0.970			
47.3	0.454	1.115						