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Supports for Vibration Isolation

By

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Supports for Vibration Isolation

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SUMMARY

Spring arrangements are described that provide flexible supports of very low stiffness for a limited range of movement. They are suitable as supports for vibration isolation provided the levels of vibration are not too large.

Some particular applications of the arrangements for vibration isolation and in other fields are discussed.

LIST OF CONTENTS

		Page
1	Introduction	3
2	Spring arrangements	3
	2.1 Inclined springs	3
	2.2 Inclined springs and axial spring	4
	2.3 General purpose support	6
	2.4 Alternative arrangements	6
3	Some applications to vibration isolation	7
4	Other applications	7
5	Conclusions	8
References		8

LIST OF ILLUSTRATIONS

	Figure
Spring arrangement - linear displacement	1
Load and stiffness functions for inclined spring arrangement	2
Load function - inclined springs with optimum axial spring	3
Stiffness function - " " " " "	4
Frequency function - " " " " "	5
General purpose support	6
Alternatives to inclined compression springs	7
Spring arrangement - angular displacement	8
Application to road vehicle suspension systems	9
Cantilever supported bucket seat	10a
Suspension for siesmic vibration transducer	10b
Other applications	11

1 Introduction

The problem of vibration isolation arises in all fields of engineering where rotating or vibrating machinery is used. Unbalanced forces in the machinery produce unwanted vibrations in the surrounding structure and the problem is either to isolate the machinery from the structure or, if this is impracticable, to isolate components mounted on the structure from the structural vibrations.

The usual practice in vibration isolation is to mount the component to be isolated on flexible supports so that the rigid body frequencies of the component on the supports are very low by comparison with the frequency of the troublesome vibrations[1,2]. In this circumstance the level of vibration transmitted through the supports is a function of the ratio of the rigid body frequencies to the vibration frequency[3]. Obviously the ideal requirement for such supports is that they should provide zero rigid body frequencies, i.e. their stiffness should be zero.

One support satisfying this ideal requirement is the sine spring[4], which provides a pure zero stiffness for a considerable range of movement. However, the latter feature can be an embarrassment since it implies that any slight change in the effective mass of the supported component leads to large excursions of the component on its supports, with no restoring force to return it to the datum position. Further, the sine spring is not well suited for supporting very large loads.

In principle it is adequate for vibration isolation if the support provides zero stiffness over a greater range of movement than the amplitude level of the troublesome vibration. Such levels are generally of the order of hundredths of an inch.

In what follows a spring arrangement with a non-linear stiffness characteristic is described* that provides zero, or very small stiffness for a limited range of movement. Though the stiffness characteristics of the arrangement are non-linear the components used are conventional linear springs. Some applications of the arrangement to vibration isolation and in other fields are discussed.

2 Spring arrangements

2.1 Inclined springs

Consider first the spring arrangement shown in Fig.1. Here there are two inclined compression springs AB and AC each of stiffness k freely hinged at A, B and C to form a triangle of base BC = a and vertex A. Let the unstressed spring lengths AB, AC equal ℓ, where

$$\frac{2\ell}{a} = \sec \alpha . \tag{1}$$

Now suppose a load W is applied at A so that A moves to A', the springs AB, AC being compressed, taking up an inclination θ. The location of A' on the line AA' is defined by

$$x = \frac{a}{2} \tan \theta . \tag{2}$$

Then the expressions for the load W and the stiffness $\frac{dW}{dx}$ along the axis AA' are:

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* Patent applications 10722/56 and 24268/56.

$$\frac{W}{ka} = (\sec\alpha - \sec\theta)\sin\theta \qquad (3)$$

$$-\frac{1}{k} \cdot \frac{dW}{dx} = 2(1 - \sec\alpha\cos^3\theta) \cdot \qquad (4)$$

A negative value for $\frac{dW}{dx}$ corresponds to positive stiffness, for x decreases as W increases.

It is apparent from equation (4) that the stiffness along AA' is zero when

$$\cos\theta = \cos^{\frac{1}{3}}\alpha \qquad (5)$$

and there is a maximum in the load-displacement curve for this condition.

Cos α defines the ratio of the ultimate compressed length of the springs to the unstressed length. In Fig.2 the load and stiffness functions (equations (3) and (4)) are plotted for various values of α.

It is apparent that this spring arrangement can provide zero stiffness, but it has little practical value as a support for vibration isolation, since the conditions for zero stiffness result in unstable equilibrium of the system. Any slight increase in load beyond the optimum value leads to a negative stiffness condition, and A then moves rapidly to the opposite side of the datum BC, ultimately coming to rest when the extension in AB and AC balances the applied load.

It is apparent that it is not sufficient merely to satisfy the condition that the support stiffness must be zero; a stability requirement must also be satisfied. The stability requirement is satisfied if the support provides zero stiffness for some optimum load, and positive stiffness if the load is increased or decreased from this optimum value, i.e. the zero stiffness point must be a minimum for the stiffness-displacement curve. This requirement also has the effect of limiting excursions of the load on the support since in any such excursion there will be a restoring force to return the load to the zero stiffness condition.

From a further differentiation of equation (4) we have

$$\frac{a}{k} \cdot \frac{d^2W}{dx^2} = -12\sec\alpha\cos^4\theta\sin\theta \qquad (6)$$

which indicates that the stiffness is a minimum when θ is zero (as can be seen from Fig.2b).

This minimum provides zero (rather than negative) stiffness only when α is also zero (equation (5)), i.e. when the unstressed spring length ℓ equals $\frac{a}{2}$, the springs then lying along the datum BC. From equation (3) this requires that there is zero applied load, so that the arrangement is useless as a load carrying flexible support.

2.2 Inclined springs and axial spring

Now consider the effect of adding a further spring DE of stiffness qk along the axis AA' (Fig.1). Prior to coupling E to A let E be at a distance s from the datum BC.

With E coupled to A the expressions for load and stiffness along the axis AA' are

$$\frac{W}{ka} = (\sec\alpha - \sec\theta)\sin\theta + \frac{q}{a}\left(s - \frac{a}{2}\tan\theta\right) \qquad (7)$$

$$-\frac{1}{k}\cdot\frac{dW}{dx} = 2\,(1 - \sec\alpha\cos^3\theta) + q \qquad (8)$$

and from a further differentiation of equation (8) we have

$$\frac{a}{k}\cdot\frac{d^2W}{dx^2} = -12\sec\alpha\cos^4\theta\sin\theta\,. \qquad (9)$$

From equations (8) and (9) the stiffness has a minimum value of zero when θ is zero and when

$$\sec\alpha = 1 + \frac{q}{2} \qquad (10)$$

and for these conditions there is a point of inflexion in the load displacement curve, the optimum load being given from equation (7) as

$$W = q\,k\,s\,. \qquad (11)$$

It is apparent from equations (1) and (10) that the ratio q of the axial spring stiffness to that of an inclined spring is determined solely by the ratio of the unstressed length to the ultimate compressed length of the inclined springs. Further, from equation (11), the optimum load is simply the load required to extend the axial spring to the datum BC, and the optimum load for the support can therefore be varied simply by moving the anchorage D of the axial spring so as to vary s.

In Fig.3 the load function of equation (7) is plotted against θ and $\tan\theta$ for two different initial conditions for the inclined springs ($\alpha = 30^\circ$ and $\alpha = 60^\circ$). In both cases the value of s for the axial spring is $\frac{a}{2}$. The point of inflexion when θ is zero can be seen. Also plotted against $\tan\theta$ is the corresponding load function for the axial springs alone, and it can be seen that for values of θ between α and zero the combined system carries more load than the axial spring alone, and between zero and $-\alpha$ it carries less load.

The stiffness function for the two conditions is plotted in Fig.4, together with the curve for the axial springs alone. The stiffness of the combined system is less than that of the appropriate axial spring for an appreciable range of $\tan\theta$, and in particular has a minimum zero stiffness when θ is zero.

The significant parameter for vibration isolation is the support frequency, and accordingly the frequency function for the support is plotted in Fig.5. This function is the square root of the ratio of stiffness function to load function, i.e. $\left(\frac{a}{W}\cdot\frac{dW}{dx}\right)^{\frac{1}{2}}$.

When α is 60° the frequency function for the combined system is less than that of the axial spring for a wide range of $\tan\theta$, and in particular is zero when θ is zero. When α is 30° the range of reduced frequency is more limited. However, since the frequency of the axial spring under applied load is inversely proportional to the square root of the extension of the spring[5], the range of reduced frequency for the combined springs over that of the axial spring alone will depend on the initial value of s for the axial spring. Whatever the value of s the frequency of the combined springs will be very low for a limited range of movement in the region of $\theta = 0$, thus satisfying the requirements for a support for vibration isolation.

2.3 General purpose support

In Fig.6 a general purpose support for vibration isolation is shown that is based upon the arrangement of inclined springs and an axial spring described above. A series of inclined compression springs are spaced around a central pillar which carries the component to be isolated. The springs are so spaced that there is no net side load on the pillar for axial movement. An axial compression spring is provided, and its stiffness is determined from the general form of equation (10). When there are n inclined springs each of stiffness k it is easily shown that the general expression for the ratio q of stiffness of the axial spring to that of an inclined spring is given by

$$\sec\alpha = 1 + \frac{q}{n} . \qquad (12)$$

With this condition satisfied it is then only necessary for the inclined springs to be at zero inclination under the applied load for the support to provide zero stiffness. If the applied load is too great the inclination becomes negative and the adjusting screw must be screwed in until the zero inclination requirement is satisfied, and conversely. The range of load that can be supported with zero stiffness on a particular support is therefore determined by the range of the adjusting screw and physical limitations on the axial spring.

A support of this type used for mounting equipment in an aircraft, would normally only provide zero stiffness for the straight and level flight condition. In a climb, dive or other manoeuvre the stiffness will be positive with consequent loss in effectiveness for vibration isolation. This problem has been considered by Shapiro[1] in relation to the sine spring, and he suggests a method for its alleviation by applying "g" sensitive bias to the system. The method could be used with the present device, the adjusting screw being geared to a "g" sensitive lever so that increased "g" increased the compression in the axial spring and vice versa, while maintaining the auxiliary springs at zero inclination.

To support large loads, such as a complete aircraft for ground resonance tests[5], very powerful springs can be used, or alternatively a series of supports with less powerful springs can be arranged along an axial shaft which carries the main load.

2.4 Alternative arrangements

In the arrangements discussed above the inclined springs are helical compression springs, but alternative arrangements using tension springs or cantilever springs can be devised, as shown in Fig.7. Similarly, different types of springs can be used for the axial spring.

Furthermore, the principles involved are not limited to providing a zero linear stiffness but can also provide a zero torsional stiffness for a system under torque load. Consider, for example the system shown in Fig.8. A linkage arm AB is clamped to the torsion member and has compression springs AC, BD of stiffness k and unstressed length ℓ at its ends. Following a similar procedure to that for the axial load system it can be shown that the torsional stiffness of the loaded shaft is a minimum when the shaft is twisted so that DBAC are in line. The springs are then under maximum compression with a compressed length of $\frac{a}{2}$. The stiffness minimum is zero (rather than negative or positive) when

$$\frac{2\ell}{a} = 1 + \frac{q}{2} \tag{13}$$

where the torsional stiffness of the shaft is defined as qkb^2.

This equation is identical in form with equation (10) since

$$\sec \alpha = \frac{2\ell}{a} .$$

Further the torsion load T in the shaft when the stiffness is zero is given by

$$T = q\,k\,b^2\psi \tag{14}$$

which, for a given spring system depends only on the initial angle ψ of the shaft when unstressed. This equation may be comparea with equation (11) of the axial load system.

3 Some applications to vibration isolation

A few special applications of the linear and torsional systems to vibration isolation are considered. Fig.9 shows their application to road vehicle suspension systems. Coil spring, cantilever spring and torsion bar suspensions are all in current use, but by the addition of an inclined spring the stiffness of the suspension can be markedly reduced, without increasing the deflection. By reducing suspension stiffness a greater degree of isolation of the body from road shocks is obtained.

Fig.10a shows how the stiffness of a cantilever bucket seat can be reduced, isolating the occupant from vibration and shocks and providing greater comfort. Such seats are in common use on agricultural machinery. Applications for other types of sprung seat can be visualised.

Fig.10b shows the suspension for a siesmic type of vibration transducer. This provides very small stiffness for a limited range of movement, which is a necessary requirement for instruments of this type.

4 Other applications

Applications of these spring arrangements in fields other than vibration isolation can be visualised. Three such applications are shown in Fig.11.

Fig.11a shows a mechanical tension meter, that provides a sensitive indication of small variations in tension about some predetermined level. It could be used, for example, as a weighing device to detect small variations in weight of nominally identical objects.

Fig.11b shows a non-lincar torque meter, that could be used as the indicator in a variety of instruments. This provides a sensitive indication of torque variations about some predetermined level.

Fig.11c shows a torque regulator, providing a sensitive means of detecting variations in the torque transmitted by a rotating shaft. It could be used in conjunction with an automatic gear changing device.

5 Conclusions

The inclined spring arrangements which have been described are capable of providing low stiffness for a limited range of movement. They can, with advantage, be used as support springs in vibration isolation mechanisms provided the levels of vibration are not too great and for moderate levels of applied acceleration. In aircraft applications the applied acceleration aspect is an important one, because of the accelerations encountered during manoeuvres. The inclined spring arrangement is, however, no worse off in catering for this case than the conventional type of spring mounting and has the advantage of providing improved isolation under normal conditions.

There are many other possible applications for spring arrangements based on this principle. Some of these are described in the paper.

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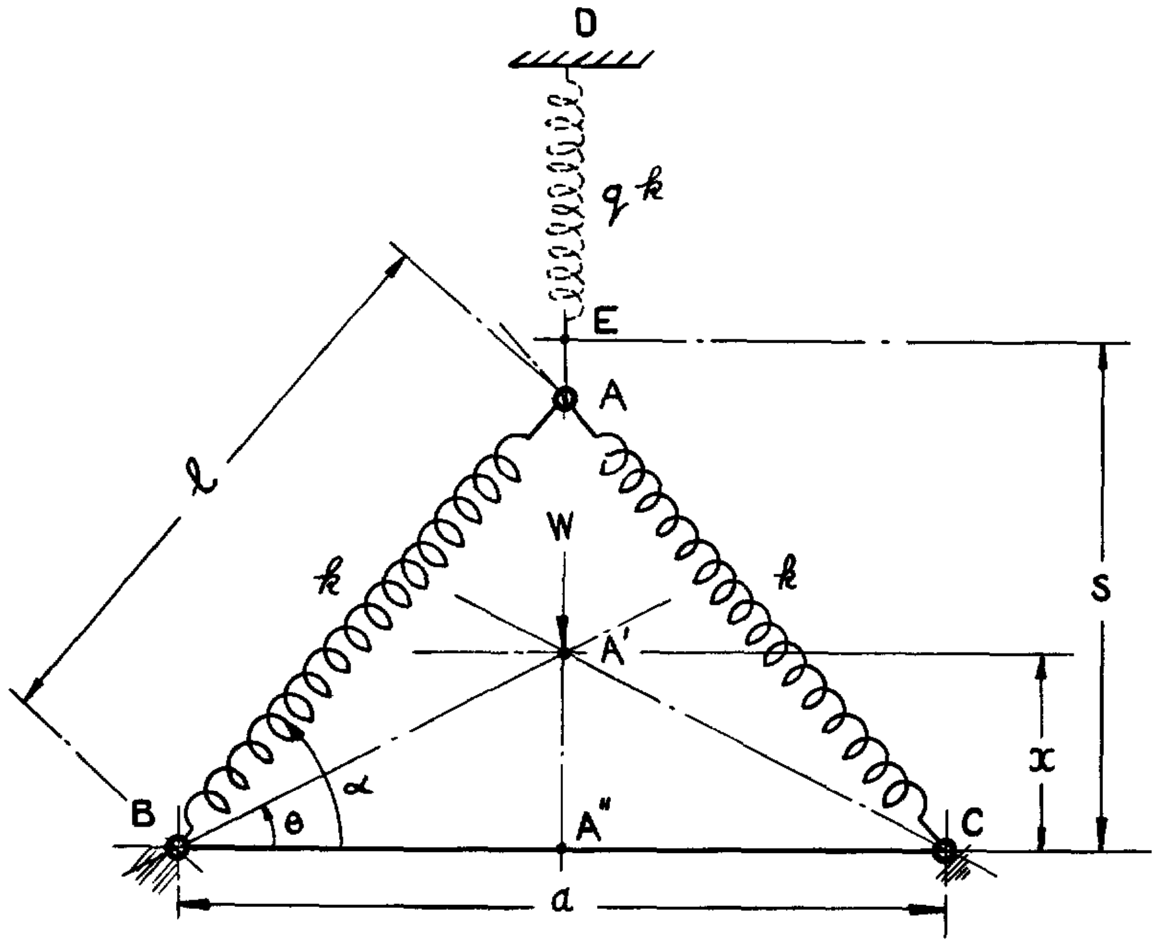

FIG.1. SPRING ARRANGEMENT.

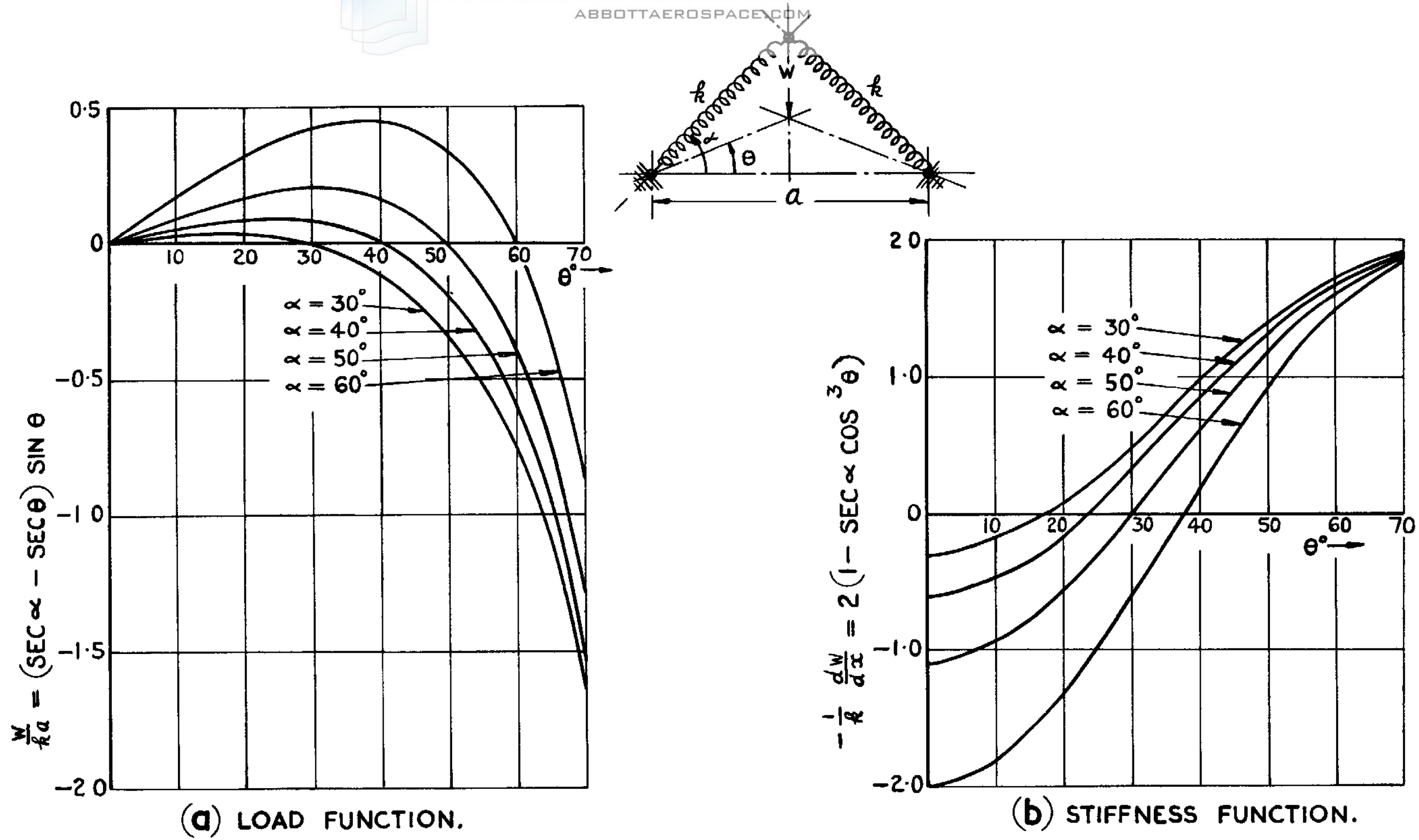

FIG. 2(a & b) LOAD & STIFFNESS FUNCTIONS FOR INCLINED SPRING ARRANGEMENT.

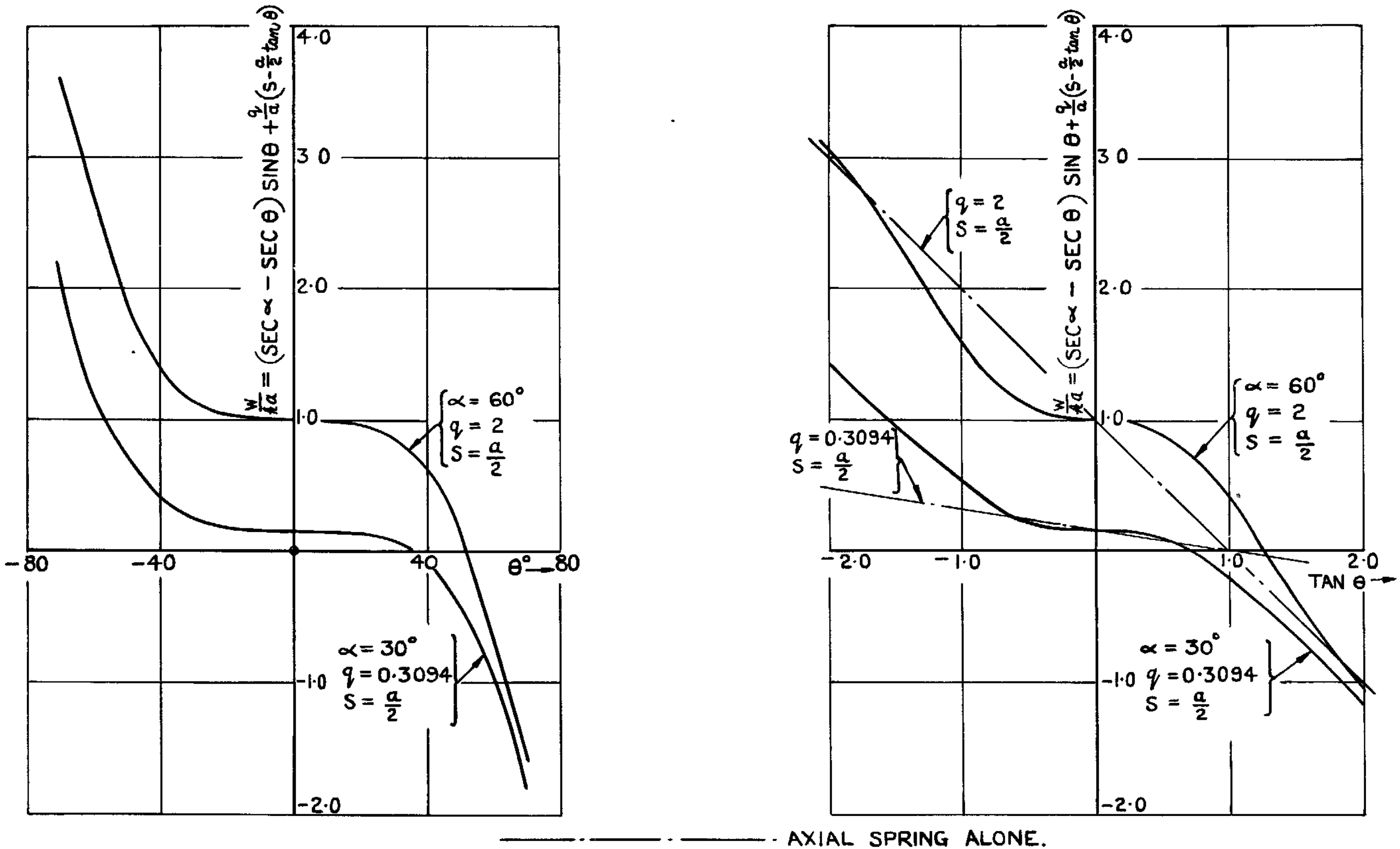

FIG. 3. LOAD FUNCTION—INCLINED SPRINGS WITH OPTIMUM AXIAL SPRING.

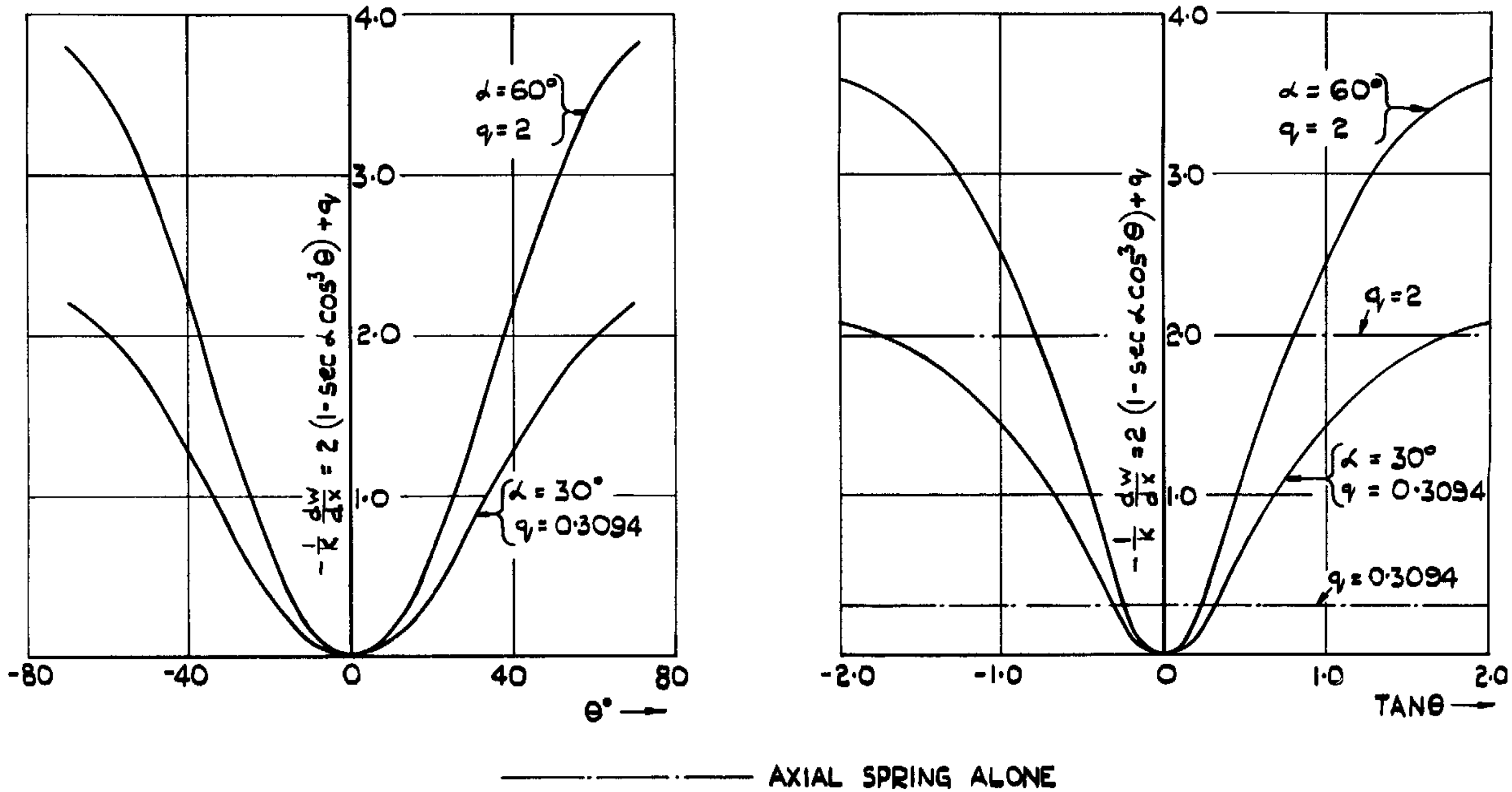

FIG. 4. STIFFNESS FUNCTION – INCLINED SPRINGS WITH OPTIMUM AXIAL SPRING

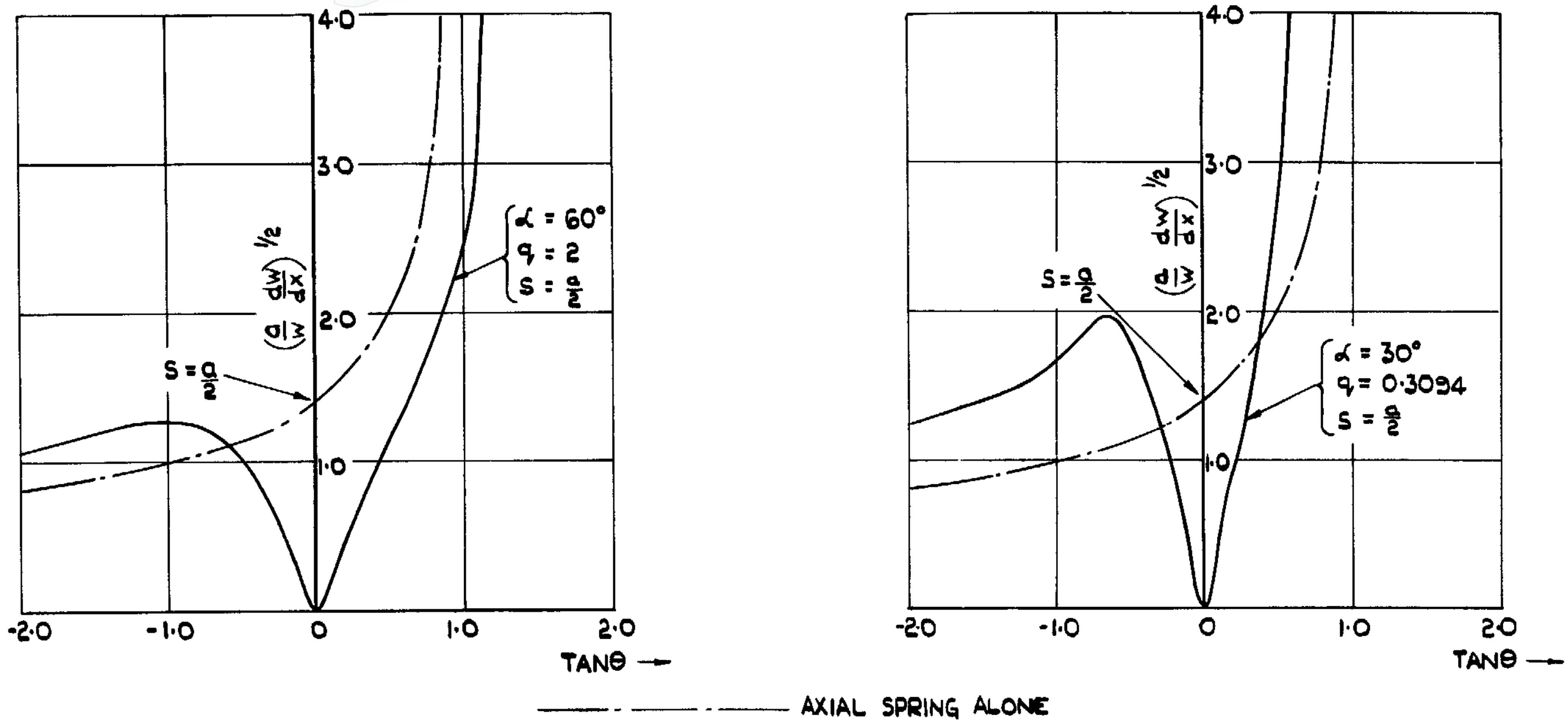

FIG. 5. FREQUENCY FUNCTION – INCLINED SPRINGS WITH OPTIMUM AXIAL SPRING.

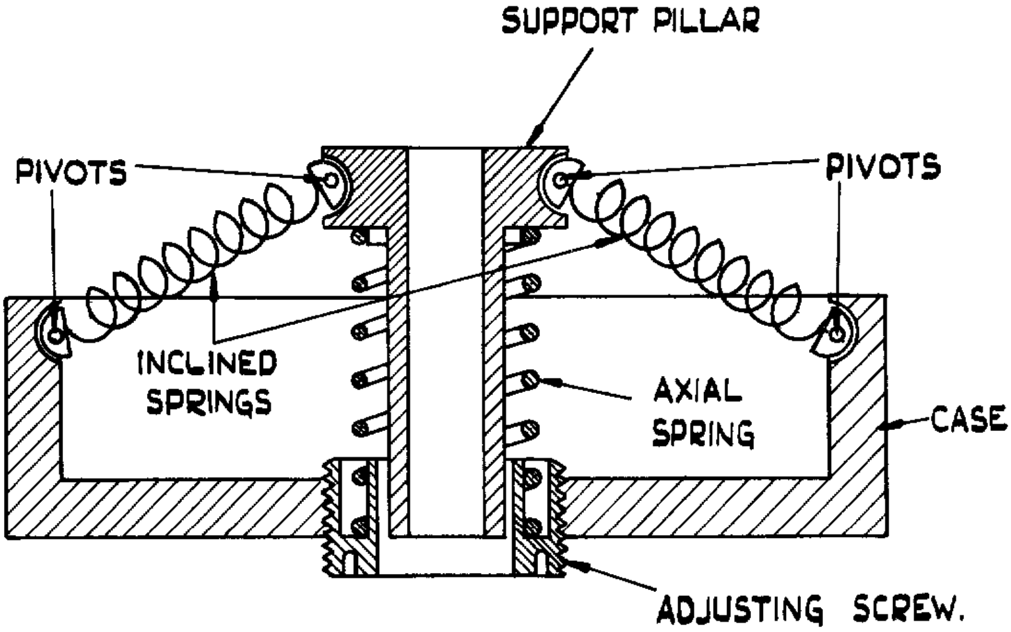

FIG. 6. GENERAL PURPOSE SUPPORT FOR VIBRATION ISOLATION.

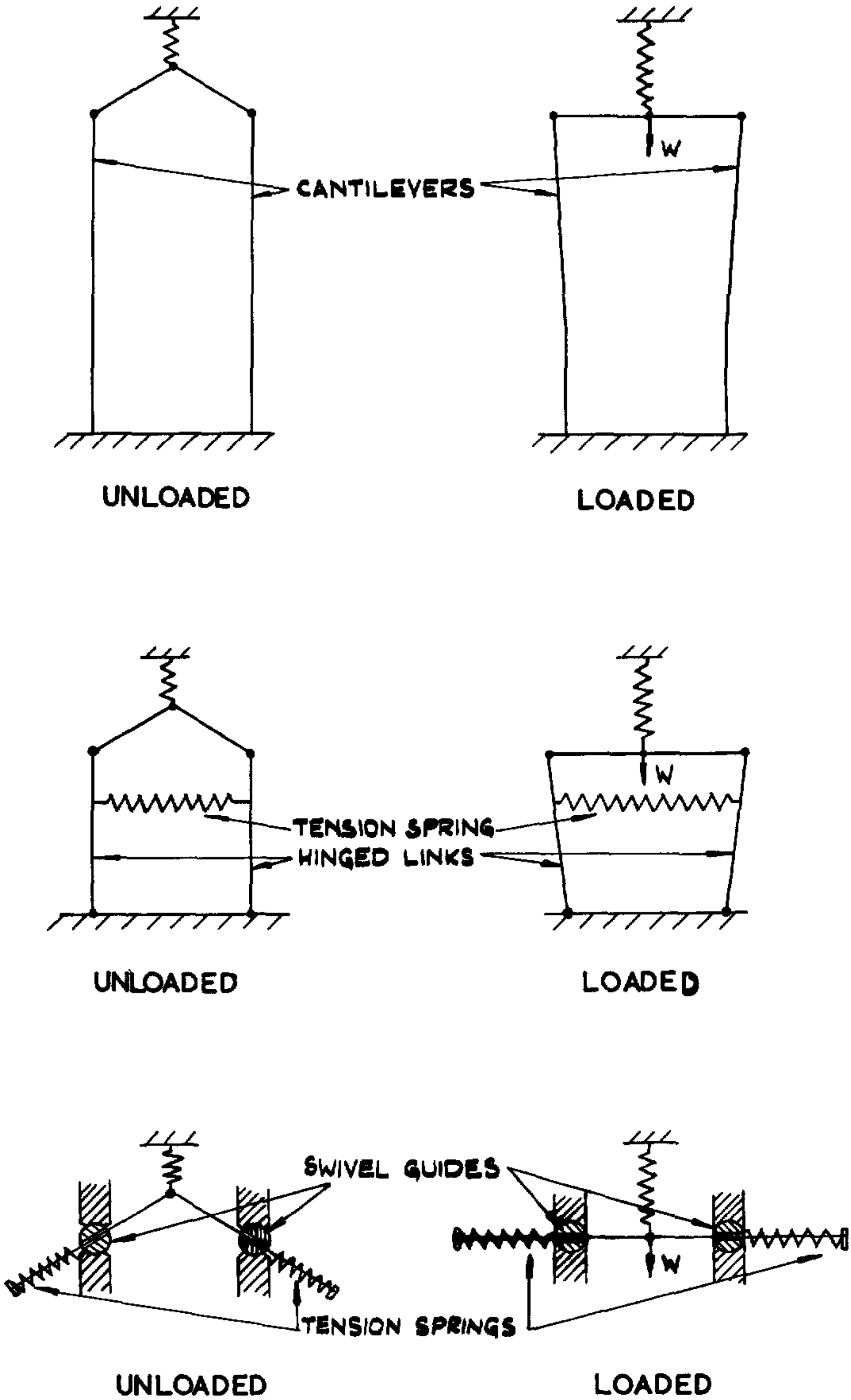

FIG.7. ALTERNATIVE ARRANGEMENTS.

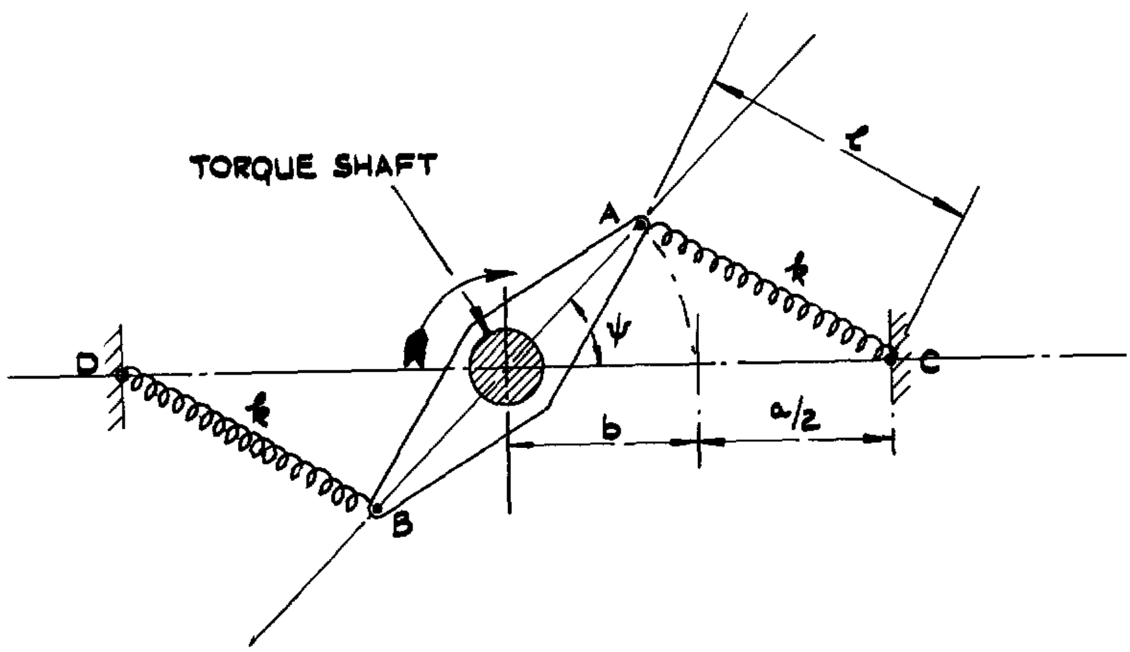

FIG. 8. SPRING ARRANGEMENT - ANGULAR DISPLACEMENT.

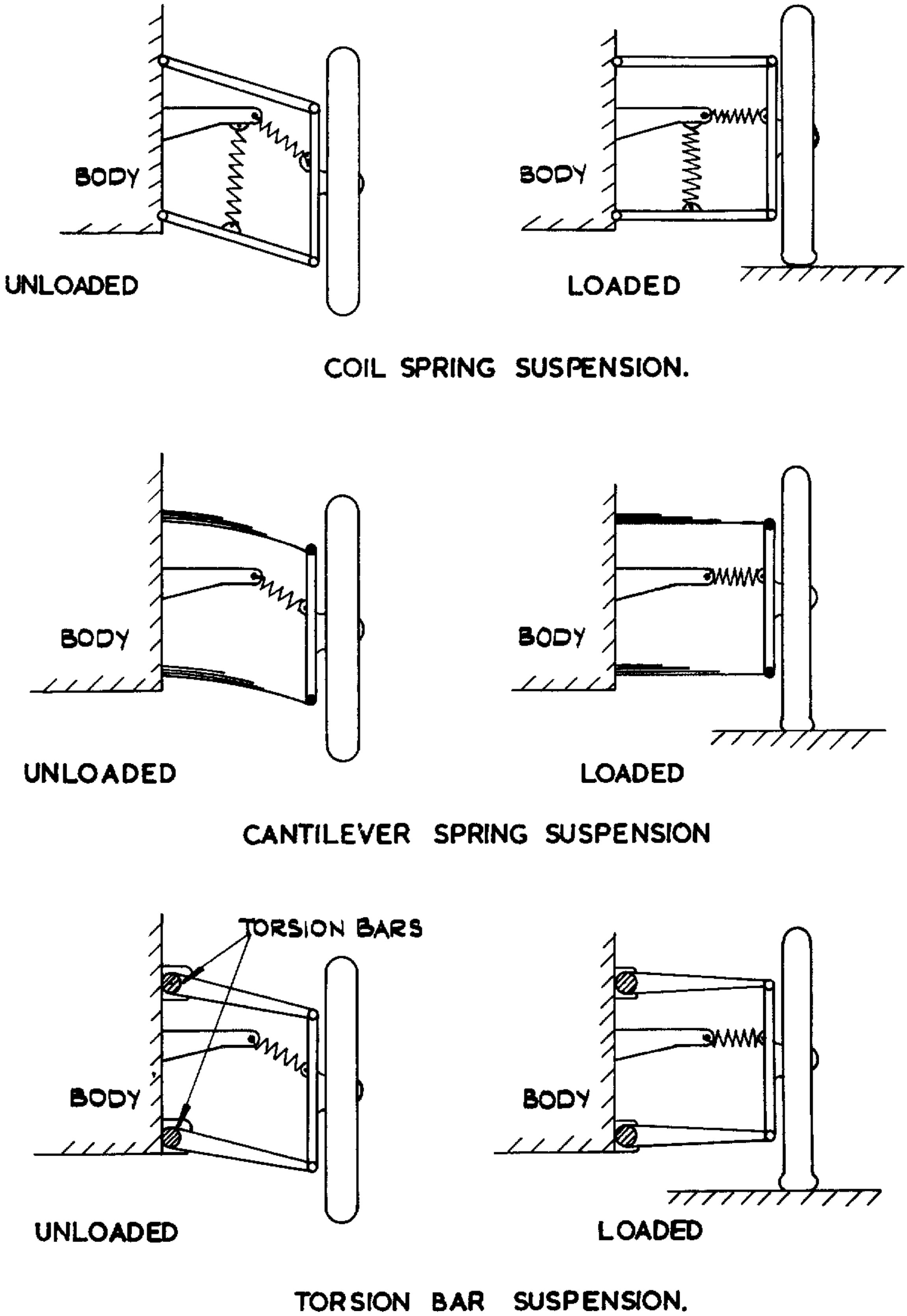

FIG. 9. APPLICATION TO ROAD VEHICLE SUSPENSION SYSTEMS.

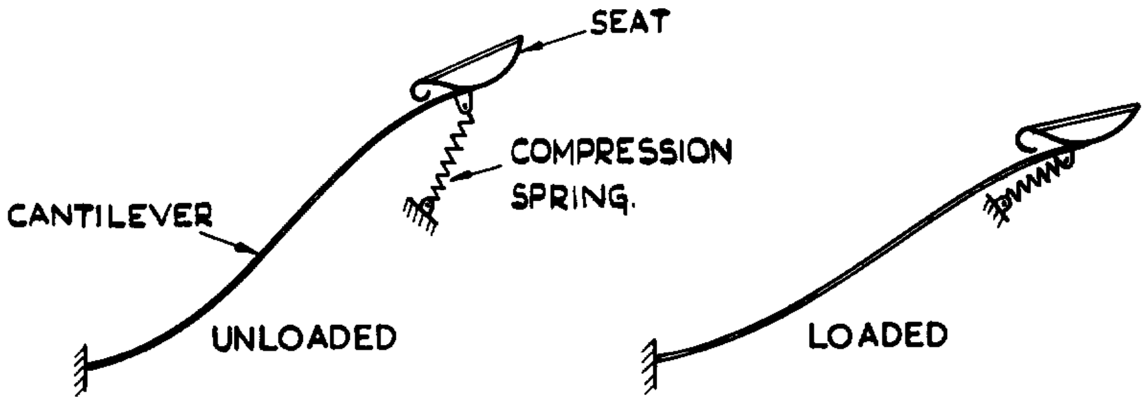

FIG. 10(a) CANTILEVER SUPPORTED BUCKET SEAT.

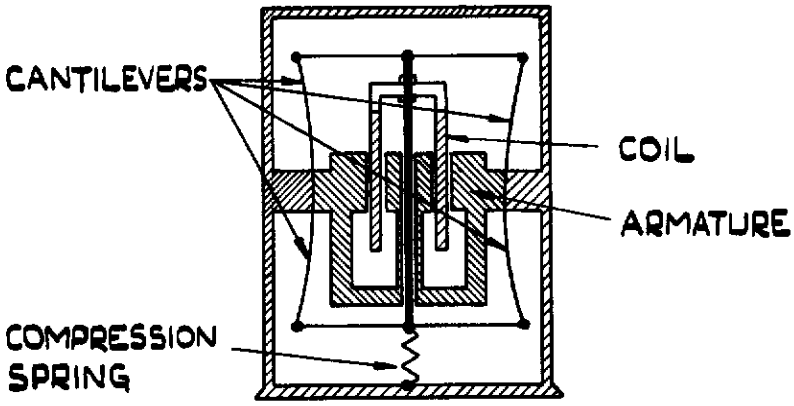

FIG. 10(b) SUSPENSION FOR SIESMIC VIBRATION TRANSDUCER

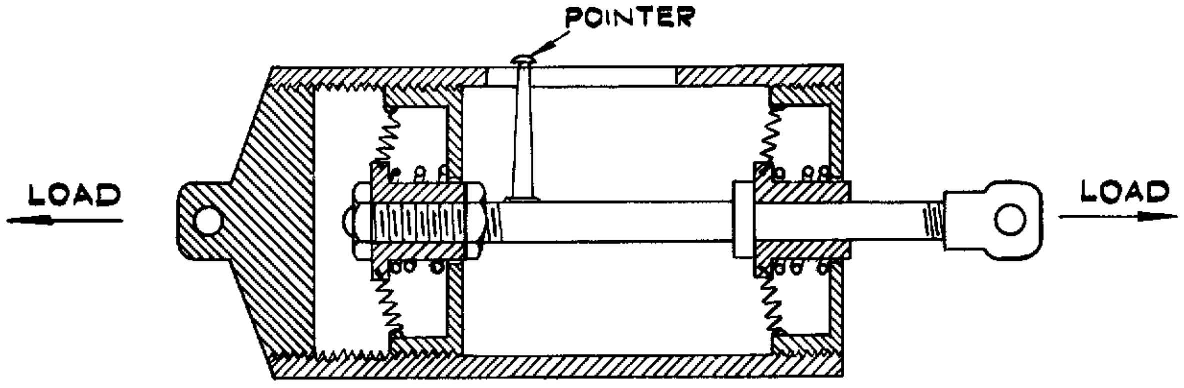

FIG. 11 (a) TENSION METER

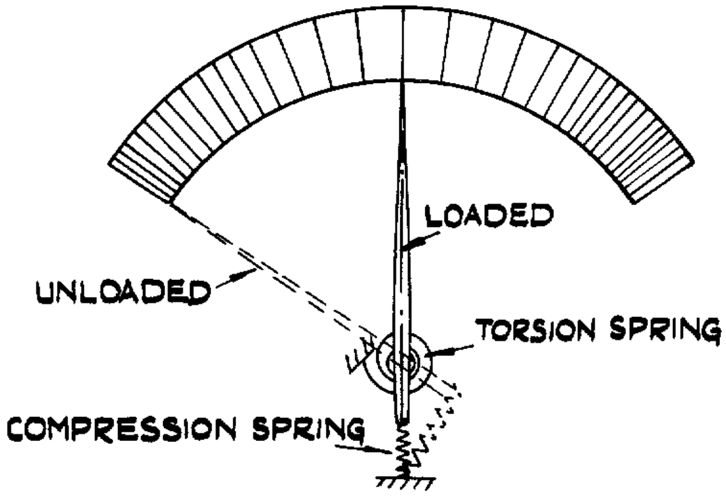

FIG. 11(b) NON-LINEAR TORQUE METER.

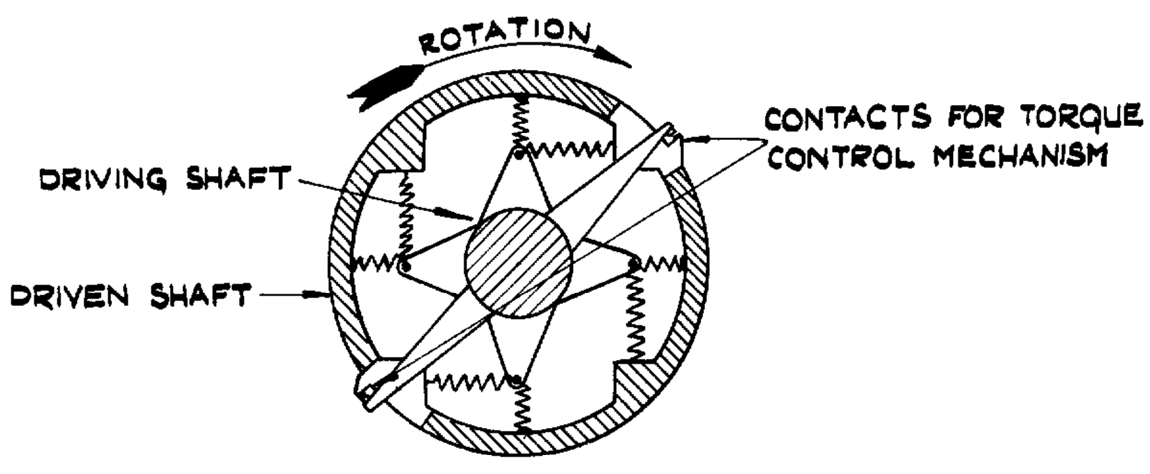

FIG. 11. (c) TORQUE REGULATOR COUPLING.

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