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Report of the Definitions Panel on the Definitions of the Thrust of a Jet Engine and of the Internal Drag of a Ducted Body

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Report of the Definitions Panel on the Definitions of the Thrust of a Jet Engine and of the Internal Drag of a Ducted Body.

20th May, 1954

Summary

The problems which occur in defining the thrust of a jet engine and the internal drag of a ducted body are considered, and formal definitions and names are given for the concepts considered to be of importance.

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1. Introduction

1.1 Recent measurements, in flight, of the thrusts of turbojet engines[1,2] have emphasized the need for a precise definition of jet-engine thrust, and also of the corresponding drag of the body housing the engine. Such definitions would be of interest to the thermodynamicist (who is concerned with the internal performance of the engine) and to the aerodynamicist (who deals with the airflow external to the engine, in connection with either aircraft performance estimation or thrust measurement in flight). Each of these considers certain forces to be important, and it is clearly necessary that the various terms employed by either should have a precise, unambiguous and mutually-agreed meaning.

The problem of defining the thrust of a jet engine and the drag of the body housing it is, of course, only a special case of the general problem of the flow through and round ducted bodies; it is fundamentally the same therefore as the problem of defining the internal and external drag, say, of a piston-engine cooling system, or a ducted heat exchanger. In all such problems it is usually convenient to consider separately the internal and external flows, and the thrusts and drags associated with them. The definitions of these forces must be consistent, however; that is, the definitions must be such that the overall longitudinal force on the body is given by the difference between the total thrust and the total drag. As the manner in which the longitudinal force can be divided into thrust and drag is to some extent arbitrary, it is not surprising that different definitions have been used in different contexts.

1.2 Attempts to resolve this problem were made independently by Jakobsson[3] and in an unpublished paper by Warren, who considered what concepts were important and suggested a number of names and precise definitions for the various terms involved. Warren's paper was discussed at a meeting of the Performance Sub-Committee of the Aeronautical Research Council in January 1953, when it was agreed that the subject required further consideration. At the Sub-Committee's request, the Council allowed a temporary Panel to be set up to consider the problem of defining thrust and drag, and also the related problem of the definitions to be used in the description and analysis of the drag of an aircraft. The Panel, which met first on 1st May, 1953, was composed of the following members, who were nominated either by the Performance or by the Engine Aerodynamics Sub-Committees.

Nominated by the Performance Sub-Committee

Prof. W. A. Mair (Cambridge University), Chairman
Mr. D. W. Bottle (A.& A.E.E.)
Dr. R. C. Pankhurst (N.P.L.)
Mr. T. V. Somerville (R.A.E.)
Mr. C. O. Vernon (S.B.A.C.)
Mr. C. H. E. Warren (R.A.E.)
Mr. E. W. E. Rogers (N.P.L.), Secretary.

Nominated by the Engine Aerodynamics Sub-Committee

Mr. S. Gray (Eng., R(a) M.O.S.)
Mr. H. Pearson (S.B.A.C.)
Dr. J. Seddon (R.A.E.)

Mr. A. R. Howell (N.G.T.E.) attended some of the meetings.

The present Report deals with the Panel's investigation of the problem of defining jet engine thrusts and concepts related to this; a Report dealing with the analysis of drag will be issued separately. The present investigation is limited to the case where the thrust acts in the direction of the undisturbed stream (i.e., along the line of flight), the jet exit plane being normal to this direction. The more

complicated/

complicated cases of an inclined thrust axis, an oblique jet exit plane*, or a jet exit plane upstream of the engine cowling exit with internal mixing between cooling and jet flows have been borne in mind throughout the Panel's discussions, but it was felt that it would be better to defer consideration of these more complicated problems until agreement had been reached for the simpler case.

1.3 In the course of the Panel's investigation it became clear that it might be convenient to introduce names for certain quantities having the dimensions of pressure. The Panel found, when reviewing these quantities, that some of the definitions relating to pressure, as laid down in the British Standard Glossary of Aeronautical Terms[4], were wholly unacceptable for compressible flow. The Panel would have liked to have suggested more suitable definitions, but was unable to agree on the names for certain quantities, or even on the need for names in certain cases**.

2. The Problem

2.1 The resultant force on a stationary ducted body in a steady stream of infinite extent can be obtained by considering the change of momentum of the fluid contained within an arbitrary surface completely surrounding the body. It is desirable, however, to divide the resultant force into internal "thrust" and external "drag" forces, the general case being considered as one in which the internal flow contributes a thrust rather than a drag, in view of the importance of jet engines. The results obtained will, of course, be immediately applicable to the related problem of internal drag, simply by interpreting thrust as negative drag. One could define the internal thrust and external drag as the sums of the components in the appropriate directions of the aerodynamic forces on the internal and external surfaces of the body, assuming that these surfaces could be precisely defined. Such an analysis, though quite valid, would be of interest mainly to the structural worker, who is concerned with the forces on the various parts of the body itself. It would be of less value, as will be seen later, to the aerodynamicist and thermodynamicist, who are concerned mainly with the fluid and the changes in its state, and who require definitions of the forces in terms of the rate of change of momentum of the fluid that flows through the body (internal flow) and of the fluid that flows round it (external flow).

2.2 In considering changes of momentum in the fluid flowing past a ducted body it has sometimes not been clear whether the relevant static pressures should be measured absolutely or relative to some datum pressure. For a closed (unducted) body this difficulty does not arise, since the subtraction of a datum pressure is equivalent to integrating a constant pressure over a closed surface and does not contribute to the overall force. But in considering parts of the surface and of the flow, as is done with ducted bodies, the derived forces depend on the datum pressure used. For a body at rest relative to the surrounding fluid the undisturbed fluid pressure acts on all surfaces, and the resultant force is zero. When there is relative motion between the fluid and the body this equilibrium is disturbed and the surface forces which are of significance are those due to the changes in surface pressure arising from the motion. Thus in the derivation of these forces all pressures must be referred to the undisturbed static pressure as a datum. This practice is adopted in the present Report.

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* If the nozzle is not square-cut and there is still some convergence in the final part of the nozzle it is clear that some deflection of the issuing jet from the free-stream direction will occur, entailing certain fundamental difficulties. Even if there is no convergence, some deflection will probably occur when the jet is choked.

** In any case, a revision of the names for various concepts relating to pressure is outside the terms of reference of the Panel, and is doubtless a matter for a more widely representative body than one set up primarily to consider the definitions of thrust and drag.

2.3 The question of the division of the resultant force into internal "thrust" and external "drag" components, resolves into a choice of the upstream and downstream stations between which the momentum relations are applied.

If the reference stations are taken at the entry and exit, the calculated values of these components will in general vary with any change in entry or exit area, though there would not necessarily be any change in the overall flow pattern. Thus if the thrust were defined in this way, the engine manufacturer would not be able to express the performance of his engine by a compact series of curves, applicable to all installations. Moreover, he would not be able to measure the thrust without traversing the flow at both the entry and exit stations. There would also be difficulty in defining the location of the entry station precisely. The use of the entry and exit stations is therefore particularly unsuitable for the definition of thrust, and some more generally applicable definition is required.

If a new reference station is considered, upstream of the entry station, then the calculated force associated with the internal flow will differ from that actually exerted on the internal surfaces of the body. It is shown in Appendix II that this difference is equal to the longitudinal component of the pressure forces acting on the boundaries of the "pre-entry streamtube" (Fig.1). A corresponding treatment of the external flow would yield a force differing from that actually exerted on the external surfaces by an equal and opposite amount. Physically it can be said that the flow in the pre-entry streamtube, induced by the engine, influences the external flow and thereby the pressure distribution on the external surfaces. Thus a force, equal in magnitude to the longitudinal component of the pressure forces on the boundaries of the pre-entry streamtube, is rightly attributed to the engine thrust, though usually acting on the external surfaces. This is in fact the basis on which engine thrusts have usually been derived in the past. Treating the external forces as positive in the drag sense, then a corresponding external drag is obtained which is equal to the force actually exerted on the external surfaces (as would be derived from the actual pressure distribution) increased by a quantity which has sometimes been referred to as the "pre-entry drag".

A similar argument can be applied to the flow downstream of the exit station by introducing the concept of a "post-exit streamtube" separating the internal and external flows. It is concluded that the fundamental definition of the thrust of a jet engine should be framed in terms of the stations at infinity upstream and infinity downstream.

2.4 Consideration of the pre-entry flow presents no difficulties, but there is in fact no physical post-exit streamtube, the flow downstream of the exit being complicated by the transfer of momentum and energy between the internal and external flows in the wake of the body. It has been customary in the past to assume that the local static pressure in the external flow in the region of the jet exit is equal to the undisturbed static pressure. In general, these pressures are not equal and it is necessary to take account of this in framing definitions of thrust. Moreover, the pressure of the internal fluid at the exit station may differ from that in the adjacent external fluid, and shock waves may be present in either the internal or external flows, or both. A major problem is, therefore, to devise practical definitions of thrust which take account of the complex flow conditions downstream of the exit, and which enable the thrust to be determined from practicable measurements. This is considered in the following Sections.

Some cautionary remarks about central bodies and side intakes are given in Appendix IV.

3. The Thrust and Related Quantities

3.1 The thrust obtained as a result of choosing reference stations at infinity both upstream and downstream of the body, and by assuming the existence of an effective post-exit streamtube, is given by the equation (derived in Appendix II)

$$\Theta = \int \rho_w V_w^2 dA_w - \rho_\infty V_\infty^2 A_\infty , \qquad \ldots(3.1)$$

where A is the area of the station (normal to the free-stream direction), ρ is the density of the fluid, and V is the local velocity, parallel to the free stream at both stations. Suffix w refers to the station at infinity downstream (where the static pressure is uniform and equal to that of the undisturbed stream), suffix ∞ refers to conditions at infinity upstream, and the integration is made over the mass of fluid flowing through the body.

The thrust given by equation (3.1) may be divided into three components:

$$\Theta = \Theta_{pre} + \Theta_{int} + \Theta_{post} , \qquad \ldots(3.2)$$

where Θ_{pre} and Θ_{post} are the pre-entry and post-exit thrusts, associated with the pressure exerted by the internal flow upon the external flow ahead of and behind the body, and Θ_{int} is the intrinsic thrust*, associated with the force on the internal surfaces of the body.

The components of Θ may be written as

$$\Theta_{pre} = \int (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty) dA_i - \rho_\infty V_\infty^2 A_\infty , \qquad \ldots(3.3)$$

$$\Theta_{int} = \int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e - \int (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty) dA_i , \qquad \ldots(3.4)$$

$$\Theta_{post} = \int \rho_w V_w^2 dA_w - \int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e , \qquad \ldots(3.5)$$

where suffices i and e denote the intake and exit stations respectively, and ϕ is the local inclination of a streamline to the free-stream direction.

The term

$$\rho_\infty V_\infty^2 A_\infty$$

in equation (3.3) is the "ram drag"**; by continuity this is equal to $m_i V_\infty$, where m_i is the internal rate of mass-flow at the duct entry.

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* The term "intrinsic thrust" is preferred to "internal thrust" because throughout this Report "internal" means "relating to the fluid that flows through the body", which embraces the fluid ahead of and behind the body as well as that actually within the body. The word "intrinsic" is thought to be etymologically suitable in this context.

** The term "ram drag" is due to Jakobssen[3] and is to be preferred to the term "intake drag", which is also used to denote the aerodynamic drag of the actual intake. Ram drag is the same as the "sink drag" which occurs with suction aerofoils.

The pre-entry thrust depends on the conditions at the entry and in the undisturbed stream, which are assumed to be known. The intrinsic thrust depends in addition on the conditions at exit, which in principle are assumed to be measurable or calculable. It follows from equation (3.2) that the use of θ depends on the successful determination of the post-exit thrust, and thus in any practical definitions of thrust, an attempt must be made to relate the conditions at infinity downstream to those at the jet exit; this approach is considered in Section 4 below.

3.2 The presence of a post-exit thrust depends on the existence, in the region surrounding the jet exit, of pressures differing from that of the undisturbed stream, and the value of the post-exit thrust depends on the magnitudes of these pressures and the subsequent behaviour of the combined internal and external flows. Hence any assessment of the post-exit thrust must rely on a measured or assumed distribution of pressure in the region surrounding the jet at exit. (The presence of a post-exit thrust implies that the internal flow from the exit affects the pressure distribution on the external surfaces.)

3.3 Some consideration must also be given to the conditions of the issuing jet at exit. When measurements are made at this position in flight or on a test-bed, the magnitude and direction of the resultant velocity must be found, at all points of the cross-section, in order to obtain the velocity component $V_e \cos\phi_e$. The engine manufacturer, in preparing brochures from which estimates can subsequently be made of the engine thrust in given conditions, envisages a somewhat idealized picture of the jet flow at the exit and assumes that the gas velocity is entirely in the direction of the undisturbed stream. An 'effective' exit area A_f (at some station f) is then quoted by the manufacturer, which differs from the real exit area A_e, to compensate for this simplification. These two viewpoints will be borne in mind when deriving expressions for thrust to be used in practice. A further discussion of the relation between quantities at stations e and f is given in Appendix III.

4. Practical Definitions of Thrust*

4.1 The Standard Thrust.- If the post-exit thrust can be neglected, the thrust is given by the sum of the pre-entry and intrinsic thrusts. This is called the net standard thrust and is the thrust normally specified in engine brochures. In terms of the conditions at the duct exit station the net standard thrust is defined by

$$\Theta_{sn} = \int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty)\, dA_e - m_i V_\infty . \qquad \ldots(4.1)$$

When an effective area is given (at station f), the net standard thrust is

$$\Theta_{sn} = \int (\rho_f V_f^2 + P_f - P_\infty)\, dA_f - m_i V_\infty . \qquad \ldots(4.2)$$

In the latter equation, $\cos^2\phi$ does not appear because at station f the flow is assumed to be in the direction of the undisturbed stream. The omission of the second term (the ram drag) in either of the above equations gives the gross standard thrust (Θ_{sg}).

4.2 The Jones Thrust.- If the jet is subsonic at exit, and exhausts into a region where the pressure differs from P_∞, the simplest assumption that can be made for calculating the post-exit thrust is that there is no transfer of momentum or energy, during

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*See Appendix II for a more detailed development of the equations given in this Section.

the subsequent flow to infinity where the pressure is P_∞. It is recommended that the thrust defined by this model of the flow, in which the fluid downstream of the exit is assumed to be expanded or compressed adiabatically and isentropically to the undisturbed pressure P_∞, should be called the net Jones thrust, since it represents an extension of B. M. Jones' original method[5] of obtaining the drag of a body in incompressible flow, to the problem of determining thrust.

Using the condition of continuity of mass-flow in the wake and the energy equation for the compressible flow of a perfect gas, it may be shown (Appendix II) that the net Jones thrust is given, in terms of conditions at the exit plane, by

$$\Theta_{Jn} = \int \rho_e V_e^2 \left[1 + \frac{2}{(\gamma_e - 1) M_e^2}\left\{1 - \left[\frac{P_\infty}{P_e}\right]^{\frac{\gamma_e - 1}{\gamma_e}}\right\}\right]^{\frac{1}{2}} \cos\phi_e \, dA_e - m_i V_\infty . \quad \ldots (4.3)$$

The ratio of the specific heats of the fluid at exit (γ_e) is assumed to remain constant in the subsequent flow to infinity, but may differ from the value of γ appropriate to conditions ahead of the body.

Substituting suffix f for suffix e and putting $\cos\phi$ equal to unity in equation (4.3) would give the net Jones thrust in terms of conditions at the idealized flow plane f, where the flow is assumed to be everywhere parallel to the undisturbed stream.

As before, the omission of the term $m_i V_\infty$ leads to the gross Jones thrust (Θ_{JG}).

4.3 The Pearson Thrust.- When there is a difference of pressure across the solid boundary between the internal and external flows at an infinitesimal distance upstream of the exit station, as there may be when the jet is supersonic, a more realistic assumption is that the issuing fluid either expands or is compressed until it attains the local static pressure of the surrounding fluid P_b (assumed uniform), by an adiabatic process in which the quantity

$$(\rho V^2 \cos^2\phi + P - P_b)\, dA \qquad \ldots (4.4)$$

remains constant along each streamtube. It is assumed that when the pressures in the internal and external flows have become equal, the flow is parallel to the direction of the undisturbed stream (station g) and there is then no transfer of momentum or energy during the subsequent flow to infinity where the pressure is P_∞.

Applying the energy equation to both parts of the flow (between the jet exit and station g, and between station g and infinity downstream) and using the condition that the quantity given by (4.4) remains constant between the exit and station g, the thrust obtained is (see Appendix II)

$$\Theta_{Pn} = \int \rho_e V_e^2 \left[\left[\frac{\rho_e V_e^2 \cos^2\phi_e + P_e - P_b}{\rho_e V_e^2}\right]^2 \left(\frac{P_\infty}{P_b}\right)^{\frac{\gamma_e - 1}{\gamma_e}} + \left\{1 - \left(\frac{P_\infty}{P_b}\right)^{\frac{\gamma_e - 1}{\gamma_e}}\right\}\left\{\frac{2}{(\gamma_e - 1) M_e^2} + 1\right\} \cos^2\phi_e\right]^{\frac{1}{2}} dA_e - m_i V_\infty . \quad \ldots (4.5a)$$

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Alternatively, in terms of conditions at the idealized flow station f, this can be written

$$\Theta_{Pn} = \int \rho_f V_f^2 \left[\left[\frac{\rho_f V_f^2 + P_f - P_b}{\rho_f V_f^2}\right]^2 \left(\frac{P_\infty}{P_b}\right)^{\frac{\gamma_f - 1}{\gamma_f}} + \left\{1 - \left(\frac{P_\infty}{P_b}\right)^{\frac{\gamma_f-1}{\gamma_f}}\right\} \left[\frac{2}{(\gamma_f - 1)M_f^2} + 1\right]\right]^{\frac{1}{2}} dA_f - m_1 V_\infty . \quad \ldots(4.5b)$$

It is recommended that the thrust Θ_{Pn} be called the <u>net Pearson thrust</u>, after the member of the Panel who derived the equations; the <u>gross Pearson thrust</u> Θ_{Pg} results from the exclusion of the ram-drag component $m_1 V_\infty$.

5. Comparison of the Three Practical Definitions of Thrust

If the jet discharges into a region where the static pressure P_b is equal to the undisturbed pressure P_∞, the post-exit thrust is zero and the standard thrust is appropriate for all exit velocities and pressures; the Pearson and standard thrusts are then identical. The former may therefore be regarded as a generalization of the standard thrust, to be used when the jet exhausts into a region whose local static pressure differs from that of the free stream.

When P_b is not equal to P_∞, there may be a post-exit thrust. For a subsonic jet, there can be no difference of pressure across the boundary between the internal and external flows at the exit, and if the flow inclination at the exit is small, so that $\cos\phi_e \approx 1$, the Pearson and Jones thrusts are then equal. Hence the Pearson thrust can be regarded as a generalization of the Jones thrust to the case where a difference of static pressure exists between the internal and external flow at exit, as in the case of a choked or supersonic jet.

Finally, a comparison may be made of the Jones and standard thrusts. For this purpose the flow inclination at exit ϕ_e will again be assumed to be small, so that $\cos\phi_e = 1$. Expanding the radical in equation (1.3) in powers of $(P_e - P_\infty)/\rho_e V_e^2$ then gives*

$$\Theta_{Jg} = \int \rho_e V_e^2 \left[1 + \frac{P_e - P_\infty}{\rho_e V_e^2} + \frac{M_e^2 - 1}{2}\left(\frac{P_e - P_\infty}{\rho_e V_e^2}\right)^2 + \ldots\right] dA_e$$

whereas, with the same assumption that $\cos\phi_e = 1$,

$$\Theta_{sg} = \int \rho_e V_e^2 \left[1 + \frac{P_e - P_\infty}{\rho_e V_e^2}\right] dA_e .$$

Thus (if ϕ_e is small) the standard thrust is equal to the first two terms only of the expanded formula for the Jones thrust. The difference between the two thrusts is of course equal to the post-exit thrust, and is shown in Fig. 2.

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* The subsequent equations could equally well be written in terms of conditions at station f.

The condition of zero momentum or energy transfer in the post-exit flow, assumed in deriving the Jones thrust, will not be obtained in practice, though with a subsonic jet the errors involved are unlikely to be large. In certain cases, however, the use of the Jones thrust is clearly wrong. For example, when the total pressure of the jet at exit is less than the static pressure of the undisturbed stream, the fluid flowing through the body cannot attain the pressure of the undisturbed stream again unless there is some transfer of momentum or energy*. Alternatively, if the velocity at exit is sonic or supersonic, there can be a difference of static pressure at exit between the internal and external flows. With an ideal fluid and no mixing, the wake could exhibit a periodic flow pattern extending to infinity and there would be no station at which the internal static pressure was at once uniform and equal to that of the undisturbed stream.

The latter case, which is the more important in thrust problems, can be dealt with by using the Pearson thrust. The assumptions made in deriving equations (4.5(a) and (b)) probably represent the best simple model of the flow that it is possible to obtain, though this model may not always correspond with reality. If, after the initial non-isentropic process, there is substantial mixing of the internal and external flows before the jet has reached the pressure P_∞, the actual thrust obtained will be affected and the assumption made by Pearson violated. In addition the external flow itself may not be isentropic; for example shock waves may be present. Moreover the Pearson thrust depends on the value of P_b; in practice this is unlikely to be uniform and some mean pressure must be used in the calculations. It is felt, however, that points such as these do not seriously affect the usefulness of the flow model and resulting thrust given by Pearson.

6. Internal Drag and Propulsive Thrust

In problems of internal drag similar concepts to the three types of thrust given in Section 4 are required. It is therefore recommended that the quantities defined by the negative of the three types of net thrust should be called the standard internal drag, the Jones internal drag and the Pearson internal drag, if such quantities are required. The adjective net is unnecessary because the concept of gross internal drag has no significance. Some formulae for internal drag are given in Appendix V.

The overall longitudinal force on a nacelle housing an engine, treated as a whole, is felt to be worthy of a name and it is suggested that this be called the propulsive thrust, as it is the thrust available for propelling the rest of the aircraft.

7. Concluding Remarks and Recommendations

The foregoing investigation has shown that the thrust can be divided into pre-entry thrust, intrinsic thrust and post-exit thrust. In the absence of the last component, the standard thrust is obtained. Two idealized models of the flow downstream from the exit were put forward and enabled an estimate to be made of the post-exit thrust in certain cases. This led to the concepts of Jones thrust and Pearson thrust, which can be used to assess the amount of overall longitudinal force that it is fair to attribute to the internal flow. The flow models are not claimed to be exact representations of the actual flow but it is felt that they are sufficiently realistic to be worth serious consideration. At this stage it is not possible to estimate with any accuracy their possible usefulness. There are related internal drag forces corresponding to each of the three types of thrust.

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* This type of flow may occur for example in the flow through a piston-engine cooling duct as the exit flap is opened (see also Appendix V).

It is suggested that, of the thrusts considered here, the Pearson thrust represents the best measure of the thrust that it is fair to ascribe to a jet engine in the general case. When the internal static pressure at the exit station is equal to the local static pressure surrounding the jet exit (subsonic, or unchoked, jet), the Pearson thrust reduces to the Jones thrust. When the local static pressure surrounding the jet exit is equal to the undisturbed static pressure, the Pearson thrust reduces to the standard thrust; in this case the post-exit thrust is zero and the standard thrust is the correct measure of the thrust.

For ease of reference the various thrusts are collated and briefly defined in Appendix I.

APPENDIX I/

APPENDIX I

Definitions of Some Quantities used in this Report

1. Internal Flow.- The flow of fluid which passes through the duct.

2. External Flow.- The flow of fluid which passes around the outside of the duct.

3. Pre-entry Streamtube.- The streamtube, extending from infinity upstream of the body to the duct entry, whose boundaries separate the internal and external flows.

4. Equivalent Post-exit Streamtube.- The streamtube, extending from the duct exit to infinity downstream, whose boundaries are assumed to separate the internal and external flows in the absence of mixing.

5. Ram Drag.- The force, acting in the downstream direction, which would arise if the momentum of the internal flow at infinity ahead of the body were destroyed. (It is equal to the product of the rate of mass flow at the duct entry and the velocity of the undisturbed relative airstream, and is the effective sink drag of the air intake.)

6. Thrust (associated with the internal flow).- The force corresponding to the rate of increase of momentum, in a direction parallel to the undisturbed relative airstream, of the internal flow through the duct, this increase being calculated between stations at infinite distances upstream and downstream of the body. The thrust can be divided conveniently into three parts, the pre-entry thrust, the intrinsic thrust, and the post-exit thrust.

7. Pre-entry Thrust.- The force, in a direction parallel to the undisturbed relative airstream, arising from the pressure forces acting on the internal surface of the pre-entry streamtube.

8. Intrinsic Thrust.- The force, in a direction parallel to the undisturbed relative airstream, arising from the pressure and friction forces acting on the internal surface of the duct.

9. Post-exit Thrust.- The force, in a direction parallel to the undisturbed relative airstream, arising from the pressure forces acting on the internal surface of the equivalent post-exit streamtube.

10. Standard Thrust.- The net standard thrust is the sum of the pre-entry thrust and the intrinsic thrust. The gross standard thrust is the arithmetic sum of the net standard thrust and the ram drag.

11. Jones Thrust.- The net Jones thrust is the sum of the pre-entry thrust, the intrinsic thrust and the post-exit thrust, the last being calculated by assuming that the fluid that has passed through the duct is compressed or expanded adiabatically and isentropically downstream of the duct exit until the undisturbed static pressure of the relative airstream is attained. The gross Jones thrust is the arithmetic sum of the net Jones thrust and the ram drag.

12. Pearson Thrust.- The net Pearson thrust is the sum of the pre-entry thrust, the intrinsic thrust and the post-exit thrust, the last being calculated by assuming that the flow downstream of the

duct/

duct exit has two stages. In the first, the fluid is compressed or expanded adiabatically and irreversibly until the local static pressure of the surrounding fluid (assumed uniform) is attained, the quantity $(\rho V^2 \cos^2 \phi + P - P_b)dA$ remaining constant along each streamline; the flow is then assumed to be parallel to the direction of the undisturbed relative airstream. In the second stage the fluid is compressed or expanded adiabatically and isentropically until the undisturbed static pressure of the relative airstream is attained. The gross Pearson thrust is the arithmetic sum of the net Pearson thrust and the ram drag.

13. Propulsive Thrust.- The overall force exerted on a nacelle housing an engine, in a direction parallel to the undisturbed relative airstream; it is considered positive when acting in the upstream direction.

14. Standard Internal Drag.- The negative of the net standard thrust.

15. Jones Internal Drag.- The negative of the net Jones thrust.

16. Pearson Internal Drag.- The negative of the net Pearson thrust.

APPENDIX II/

APPENDIX II

Derivation of the Thrust and Related Quantities

Fig. 1 represents the "internal" flow ahead of, through and to the rear of a stationary ducted body. Consider the control volume bounded by the intake and exit stations i and e (taken for convenience as normal to the free-stream direction) and the internal surfaces of the body. Momentum considerations show that the sum of the pressure and friction forces acting on the fluid at the boundaries is equal to the rate of change of momentum of the fluid flowing through the control volume. Resolving in the direction of the free stream gives

$$\int_i P_i \, dA_i - \int_e P_e \, dA_e + \int_{int} (P \sin\phi - F \cos\phi) \, dS$$

$$= \int_e \rho_e V_e^2 \cos^2\phi_e \, dA_e - \int_i \rho_i V_i^2 \cos^2\phi_i \, dA_i ,$$

where F is the local friction force per unit area, dS is an element of area of the internal surface, and ϕ is considered to be positive when the duct is diverging. (The third integral on the left-hand side represents the force in the free-stream direction exerted on the fluid by the internal surfaces of the duct.)

Hence $$\int_{int} (P\sin\phi - F\cos\phi) dS = \int_e (\rho_e V_e^2 \cos^2\phi_e + P_e) dA_e - \int_i (\rho_i V_i^2 \cos^2\phi_i + P_i) dA_i .$$

The integration of a constant pressure P_∞ (the undisturbed static pressure) round the closed boundary must give zero force,

i.e., $$\int_i P_\infty dA_i - \int_e P_\infty dA_e + \int_{int} P_\infty \sin\phi \, dS = 0.$$

Hence by subtraction

$$\int_{int} \left[(P-P_\infty)\sin\phi - F\cos\phi \right] dS = \int_e (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e - \int_i (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty) dA_i .$$

The forces that are of significance are those associated with pressures measured relative to the undisturbed static pressure. Thus the left-hand side of the above equation gives the force in the free-stream direction exerted on the fluid by the internal surfaces of the duct, measured positively in the downstream direction. This is clearly equal to the effective force in the direction of the free-stream exerted by the fluid on the internal surfaces, measured positively in the <u>upstream</u> (<u>thrust</u>) direction, and this quantity will be defined as the "<u>Intrinsic Thrust</u>", Θ_{int}.

Thus $$\Theta_{int} = \int_e (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e - \int_i (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty) dA_i \qquad \text{...(II.1)}$$

Now/

Now consider the flow between stations at infinity upstream (∞) and the exit (e); in similar manner a "thrust" can be derived which is given by

$$\int_e (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e - \int_\infty (\rho_\infty V_\infty^2 + P_\infty - P_\infty) dA_\infty, \qquad \ldots(\text{II}.2)$$

and is equal to the sum of the longitudinal forces on the boundaries of the pre-entry streamtube and the internal surfaces.

By subtraction, then:-

("Thrust" given by II.2) - Θ_{int} = axial force on pre-entry streamtube

$$= \int_i (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty)\, dA_i - \int_\infty \rho_\infty V_\infty^2\, dA_\infty.$$

This will be defined as the "Pre-entry Thrust", Θ_{pre}.

$$\text{Thus} \quad \Theta_{pre} = \int_i (\rho_i V_i^2 \cos^2\phi_i + P_i - P_\infty)\, dA_i - m_i V_\infty, \qquad \ldots(\text{II}.3)$$

since V_∞ is constant over the whole area A_∞ and $m_i = \rho_\infty V_\infty A_\infty$.

Similarly, if the flow is considered between stations at the exit and at infinity downstream (w), a thrust is introduced which is defined as the "Post-exit Thrust" and given by

$$\Theta_{post} = \int_w \rho_w V_w^2\, dA_w - \int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty)\, dA_e, \qquad \ldots(\text{II}.4)$$

since $P_w = P_\infty$ and $\phi_w = 0$, though V_w is not uniform.

Since both the pre-entry and post-exit thrusts are communicated to the body, the total thrust is given by

$$\Theta = \Theta_{pre} + \Theta_{int} + \Theta_{post}$$

$$= \int \rho_w V_w^2\, dA_w - m_i V_\infty. \qquad \ldots(\text{II}.5)$$

The net standard thrust is defined as the sum of the pre-entry and intrinsic thrusts and is given by

$$\Theta_{sn} = \int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty)\, dA_e - m_i V_\infty, \qquad \ldots(\text{II}.6)$$

or a similar equation in terms of station f.

In the Jones model of the flow downstream from station e or f, the expansion or compression from the local pressure P_b to the pressure of the undisturbed stream P_∞ at station w is adiabatic and isentropic, no mixing occurring between the external and internal flows.

The/

The energy equation for the compressible flow of a perfect gas can be written

$$\frac{\gamma_e}{\gamma_e - 1}\,\frac{P_e}{\rho_e} + \tfrac{1}{2} V_e^2 = \frac{\gamma_e}{\gamma_e - 1}\,\frac{P_w}{\rho_w} + \tfrac{1}{2} V_w^2 \,. \qquad \ldots(\text{II}.7)$$

Using the condition of isentropic flow, this equation becomes

$$V_w = V_e\left[1 + \frac{2}{(\gamma_e - 1)M_e^2}\left\{1 - \left(\frac{P_\infty}{P_e}\right)^{\frac{\gamma_e - 1}{\gamma_e}}\right\}\right]^{\frac{1}{2}} .$$

Substituting this value of V_w into equation (II.5) and using the condition of continuity,

$$\rho_w V_w dA_w = \rho_e V_e \cos\phi_e dA_e$$

gives the <u>net Jones thrust</u>

$$\Theta_{Jn} = \int \rho_e V_e^2 \left[1 + \frac{2}{(\gamma_e - 1)M_e^2}\left\{1 - \left(\frac{P_\infty}{P_e}\right)^{\frac{\gamma_e - 1}{\gamma_e}}\right\}\right]^{\frac{1}{2}} \cos\phi_e dA_e - m_i V_\infty \,. \qquad \ldots(\text{II}.8\text{a})$$

If the station f is used, ϕ_f is zero and

$$\Theta_{Jn} = \int \rho_f V_f^2 \left[1 + \frac{2}{(\gamma_f - 1)M_f^2}\left\{1 - \left(\frac{P_\infty}{P_f}\right)^{\frac{\gamma_f - 1}{\gamma_f}}\right\}\right]^{\frac{1}{2}} dA_f - m_i V_\infty \,. \qquad \ldots(\text{II}.8\text{b})$$

The Pearson model of the flow has for its initial stage a process in which the fluid flowing internally expands or contracts adiabatically and irreversibly from the internal static pressure at the exit to the local external pressure P_b, the process being complete at station g where the flow is assumed to be parallel to the undisturbed stream. During this process, the pressure at the dividing surface between the internal and external flows is assumed to have the uniform value P_b, and for each streamtube the quantity $(\rho V^2 \cos^2\phi + P - P_b)\, dA$ is assumed to remain constant between the exit and station g.

The gross Pearson thrust Θ_{Pg} is given by the general equation

$$\Theta = \int_w \rho_w V_w^2 dA_w$$

with conditions at station w related to those at the exit by means of the flow model suggested by Pearson. By continuity,

$$\rho_w V_w dA_w = \rho_e V_e \cos\phi_o dA_e \,.$$

Thus/

Thus

$$\theta_{Pg} = \int \rho_e V_e V_w \cos\phi_e dA_e . \qquad \ldots(II.9)$$

For the isentropic and adiabatic flow between stations g and w, the energy equation for a compressible fluid can be used, as in considering the Jones thrust, to give

$$V_w = V_g \left[\frac{2}{(\gamma_e - 1)M_g^2} \left\{ 1 - \left(\frac{P_\infty}{P_b} \right)^{\frac{\gamma_e - 1}{\gamma_e}} \right\} + 1 \right]^{\frac{1}{2}} . \qquad \ldots(II.10)$$

(It is assumed that γ does not change downstream of the exit.) Between stations e and g,

$$\rho_g V_g^2 \, dA_g - (\rho_e V_e^2 \cos^2\phi_e + P_e - P_b) \, dA_e = 0 .$$

But by continuity

$$\rho_g V_g dA_g = \rho_e V_e \cos\phi_e dA_e ,$$

and hence

$$V_g = \frac{\rho_e V_e^2 \cos^2\phi_e + P_e - P_b}{\rho_e V_e \cos\phi_e} . \qquad \ldots(II.11)$$

It remains now to eliminate M_g from equation (II.10). This can be done by applying the energy equation to the adiabatic flow between e and g, giving the following equation

$$\frac{2}{(\gamma_e - 1)M_g^2} = \left(\frac{V_e}{V_g} \right)^2 \left\{ \frac{2}{(\gamma_e - 1)M_e^2} + 1 \right\} - 1 . \qquad \ldots(II.12)$$

Substituting (II.11) and (II.12) in (II.10) and simplifying the result, leads to the following expression for the net Pearson thrust when the ram drag term is included.

$$\theta_{Pn} = \int \rho_e V_e^2 \left[\left[\frac{\rho_e V_e^2 \cos^2\phi_e + P_e - P_b}{\rho_e V_e^2} \right]^2 \left(\frac{P_\infty}{P_b} \right)^{\frac{\gamma_e - 1}{\gamma_e}} + \left[1 - \left(\frac{P_\infty}{P_b} \right)^{\frac{\gamma_e - 1}{\gamma_e}} \right] \left[\frac{2}{(\gamma_e - 1)M_e^2} + 1 \right] \cos^2\phi_e \right]^{\frac{1}{2}} dA_e - m_i V_\infty . \qquad \ldots(II.13a)$$

In/

In terms of conditions at station f, where the flow is assumed to be parallel to the direction of the free stream (i.e., $\phi_f = 0$) the net Pearson thrust is

$$\Theta_{Pn} = \int \rho_f V_f^2 \left[\left[\frac{\rho_f V_f^2 + P_f - P_b}{\rho_f V_f^2} \right]^2 \left(\frac{P_\infty}{P_b} \right)^{\frac{\gamma_f - 1}{\gamma_f}} + \left[1 - \left(\frac{P_\infty}{P_b} \right)^{\frac{\gamma_f - 1}{\gamma_f}} \right] \left[\frac{2}{(\gamma_f - 1) M_f^2} + 1 \right] \right]^{\frac{1}{2}} dA_f - m_i V_\infty . \quad \ldots (II.13b)$$

APPENDIX III/

APPENDIX III

Some Remarks on Real Jet Engines and the Use of Stations e and f

In general, the discharge coefficients of engine exhaust nozzles are less than unity. The engine manufacturers assume that the engine behaves as if the flow leaving the final nozzle converges and passes through some area (known as the effective area and denoted by A_f in the present Report) in which the streamlines are all parallel to the undisturbed stream. This area is calculated from a knowledge of the thrust, mass-flow and exhaust temperature, measured on a test-bed, or better still in an altitude test-cell. For estimating flight performance, either the effective area is assumed to be independent of altitude, or an assumed variation is used, or the results obtained from an altitude test-cell are employed. In the present context this variation and the method of obtaining it are immaterial; it is only necessary to allow for the existence of an effective area when constructing the expressions for the various thrusts, whether this area remains constant or not.

In the case of a real nozzle from which the flow contracts and passes through an "effective area", the contracting jet (which is assumed not to mix with the surrounding flow) can support a pressure from the external flow and thus contribute to the total thrust. The problem is unaltered if the boundary of the jet is replaced by a solid surface, so that for the purpose of defining the thrust an idealized picture of the jet may be used, in which the new exit area, at which the flow is parallel to the free-stream, is the effective exit area A_f. As was said earlier, the exit area given by the engine maker is this effective area A_f; expressions for the various thrusts in terms of conditions at station f are therefore required.

The important case of flight measurements of thrust must now be considered. These measurements are made in the jet, near the engine outlet, and a method is required for calculating the thrust from them. Now it may be readily shown that it is impossible to separate the internal thrust from the external drag unless the wake surveys are carried out in the plane of the final nozzle (station e), and a complete picture obtained of the flow at this position. It is therefore of great importance to have alternative expressions for the thrust in terms of quantities which can be determined in the actual exit plane.

One method that can be adopted is to use the results of the measurements at station e to calculate the conditions as station f, and this method will be used where a knowledge of the effective area is required. But in the case that is probably of the greatest importance, a choked jet exhausting into a region whose pressure is equal to the undisturbed pressure, a simpler approach can be used.* In the absence of any mixing with the external flow the pressure and velocity must be constant along the bounding streamtube of the jet, the pressure being equal to p_∞. Between stations e and f, the

following/

---

* The external pressure surrounding a jet exit under static conditions is not quite equal to the pressure of the undisturbed stream. This is because the jet induces an external flow around the nacelle which changes the external local pressure, particularly at the jet exit. The thrust obtained from engine brochures is ultimately based on test-bed measurements obtained under such conditions, and thus includes any post-exit thrust due to p_b not being equal to p_∞. The importance of this effect is not known.

following relation holds

$$\int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_\infty) dA_e - \int (\rho_f V_f^2 + P_f - P_\infty) dA_f = 0.$$

The difference between the two integrals given above is equal to the resolved component of the force due to the pressure difference $(P - P_\infty)$ acting on the boundary of the jet between the two stations. This difference must be zero, since on the boundary streamtube P is equal to P_∞. Thus the thrust, given in this case by the standard thrust, is obtainable from the first integral in the equation entirely in terms of quantities in the exit plane, where measurements of static and total pressure, temperature and angle of flow are required.

If the external pressure is not equal to the free-stream value, the relation given above must be modified. Assuming that the external pressure has the constant value P_b between stations e and f,

$$\int (\rho_e V_e^2 \cos^2\phi_e + P_e - P_b) dA_e = \int (\rho_f V_f^2 + P_f - P_b) dA_f,$$

so that the velocity V_g in the intermediate plane g can be related to conditions at station e.

Thus, for the purposes of the definition and calculation of thrust, when conditions at station e are known, it is often not necessary to calculate the conditions at station f. The choice between stations e and f is mainly one of convenience. In general, in the absence of measurements, it is almost impossible to estimate conditions at the exit plane, but it is relatively easy for the engine manufacturer to estimate the conditions at station f.

In many cases, too, the velocity and pressure can be assumed to be uniform across station f when estimating thrust from data supplied by the manufacturer, and there is then no need to integrate over the jet area. To define the thrusts in terms of uniform flow at station f would be unnecessarily restrictive, however, since the exact correspondence between points in stations e and f connected by a common streamtube would be lost.

APPENDIX IV/

APPENDIX IV

Some Cautionary Remarks Regarding Central Bodies and Side Intakes

All the thrusts (or internal drags) defined in Section 4 are based on the rate of change of momentum of the fluid that flows through the body between a station at infinity upstream and some downstream station. If, therefore, the ducted body has a central body protruding ahead of or behind the main body (see Fig.3), the drag of the portion of the central body protruding ahead of the entry is automatically included in the internal thrust or drag, just as are portions entirely within the main body. The drag of any portion of the central body protruding <u>behind</u> the exit is not necessarily included, however; the method to be used in allowing for this drag depends on the definition of thrust that is adopted.

When the central body protrudes a long way ahead of the entry, it is often more convenient to treat the central body as the main body, and to treat the ducted part as a 'side intake'. This is particularly necessary when the intake does not entirely surround the central body. In such cases the internal drag of the duct, under both definitions, will include the drag of the portions of the main body ahead of the entry that are wetted by the fluid that flows through the duct. However, it is usual to include the drag of the portions of the main body wetted by the internal flow in the external drag, and to correct for this by reducing the pre-entry drag of the intake by the same amount, to yield, say, a 'modified pre-entry drag'. That such a procedure is rational is apparent when the ducted parts of the body become so detached from the main body that they can be treated as separate nacelles.

APPENDIX V/

APPENDIX V

Formulae for Internal Drag

One of the earliest attempts to define the internal drag of a ducted body was made by Meredith[6], but he considered only the case when the exit static pressure was equal to the undisturbed pressure. In this case all the definitions compared in Section 5 are equivalent to one another and to Meredith's definition.

Other definitions (unpublished, 1936) covered the case in which the exit static pressure was different from the undisturbed pressure, and were equivalent to equation (3.1) of this report, but in order to apply the definition the velocity V_W was related to the exit velocity V_e and the exit static pressure P_e by the assumption of no transfer of momentum or energy in the wake. This method is now commonly called Jones's method, because Jones made the same assumptions in obtaining the profile drag of an unducted body from measurements in the wake[5]. Taylor[7] has pointed out, however, that the effect of turbulent mixing is not always negligible, particularly when the exit static pressure is appreciably different from the undisturbed pressure. Jones's formula, for constant mass flow through the duct, neglecting compressibility, is

$$D_J = \int \rho V_e \left[V_\infty - V_e \sqrt{1 + \frac{P_e - P_\infty}{\frac{1}{2}\rho V_e^2}} \right] dA_e \, . \qquad \ldots (V.1)$$

After it had been observed that the definition of internal drag given by equation (3.1) could be expressed in terms of measurable quantities by the use of what was, in effect, Jones's formula, it was natural that Betz's formula[8] should also have been considered, for it had been shown that the two formulae were equivalent to the first order when used to estimate the profile drag of unducted bodies[9]. Betz's formula, which applies to incompressible flow, is

$$D_B = \int \rho V_e \left[V_\infty - V_e \left\{ 1 + 2\,\frac{P_e - P_\infty}{\rho V_e^2} - \frac{V_\infty^2}{V_e^2}\left(1 - \sqrt{1 - \frac{P_e - P_\infty}{\frac{1}{2}\rho V_\infty^2}}\right)\right\}\right] dA_e, \qquad \ldots (V.2)$$

and on expanding the radical in powers of $(P_e - P_\infty)/\frac{1}{2}\rho V^2$ this becomes

$$D_B = \int \rho V_e \left[V_\infty - V_e \left\{ 1 + \frac{P_e - P_\infty}{\rho V_e^2} - \frac{1}{2}\left(\frac{P_e - P_\infty}{\rho V_e V_\infty}\right)^2 + \ldots \right\}\right] dA_e . \qquad \ldots (V.3)$$

By expanding the radical in equation (V.1) in powers of $(P_e - P_\infty)/\frac{1}{2}\rho V_e^2$, the equivalent form of Jones's formula is found to be

$$D_J = \int \rho V_e \left[V_\infty - V_e \left\{ 1 + \frac{P_e - P_\infty}{\rho V_e^2} - \frac{1}{2}\left(\frac{P_e - P_\infty}{\rho V_e^2}\right)^2 + \ldots \right\}\right] dA_e . \qquad \ldots (V.4)$$

Thus/

Thus equations (V.3) and (V.4) differ only in the terms of order $\left(\frac{p_e - p_\infty}{\rho V_e^2}\right)^2$ and higher.

Betz's formula has the apparent advantage over Jones's that it gives a real answer when the total pressure at exit is less than the undisturbed static pressure, for which case Jones's formula gives a complex answer. Betz's formula has, in fact, been introduced into internal drag problems (e.g., in Ref.10) in order to cope with this 'weakness' of Jones's formula, although Squire in 1936 pointed out that Betz's formula should not be used when Jones's would give a complex answer. Betz's formula was in fact developed for determining the profile drag of closed (unducted) bodies, and has never been extended to give a formula for the internal drag of a ducted body.

It is recommended therefore that Jones's formula should be used if an assessment of the internal drag of a ducted body is required, and that Betz's formula should not be used for this purpose.

Last/

List of Symbols used in this Report

The symbols in this list have been used in writing this report, but it is not recommended that they should all necessarily be adopted for general use.

A	station area (normal to the direction of the undisturbed stream)
A_f	effective nozzle area quoted by engine manufacturers
D_B	internal drag according to Betz's formula (see equation (V.2))
D_J	internal drag according to Jones' formula (see equation (V.1))
M	Mach number, given by $V/\sqrt{\gamma p/\rho}$
m	rate of mass-flow
p	static pressure
p_b	local static pressure of the external fluid surrounding the jet at exit
p_∞	pressure of the undisturbed stream
V	resultant velocity
γ	ratio of specific heats of fluid
Θ	thrust, given by equation (3.1)
Θ_{int}	intrinsic thrust (see equation (3.4))
Θ_{Jg}	gross Jones thrust (see equation (4.3))
Θ_{Jn}	net Jones thrust (see equation (4.3))
Θ_{Pg}	gross Pearson thrust (see equations (4.5a) and (b))
Θ_{Pn}	net Pearson thrust (see equations (4.5a) and (b))
Θ_{post}	post-exit thrust (see equation (3.5))
Θ_{pre}	pre-entry thrust (see equation (3.3))
Θ_{sg}	gross standard thrust (see equations (4.1) and (4.2))
Θ_{sn}	net standard thrust (see equations (4.1) and (4.2))
ρ	fluid density
ϕ	angle between local direction of flow and direction of undisturbed stream (positive when streamtube is diverging).

Suffices/

Suffices

c	denotes the exit station (see Fig. 1)
f	denotes the station, corresponding to the "effective area" of the jet, where the flow is parallel to the undisturbed stream (see Appendix III).
g	denotes the station at which the static pressure of the internal fluid attains the local static pressure of the external fluid
i	denotes the intake station (see Fig. 1)
w	denotes a station at infinity downstream, at which the internal static pressure is uniform and equal to the static pressure of the undisturbed stream (see Fig. 1)
∞	denotes a station at infinity upstream (see Fig. 1).

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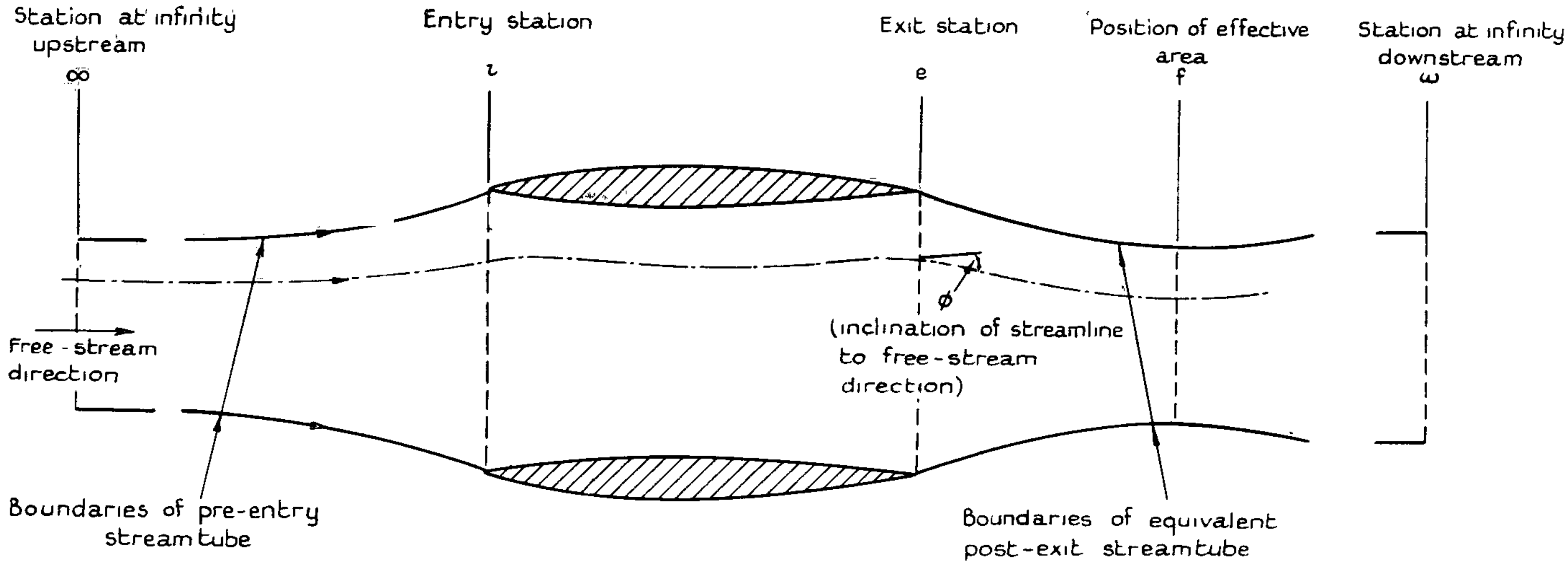
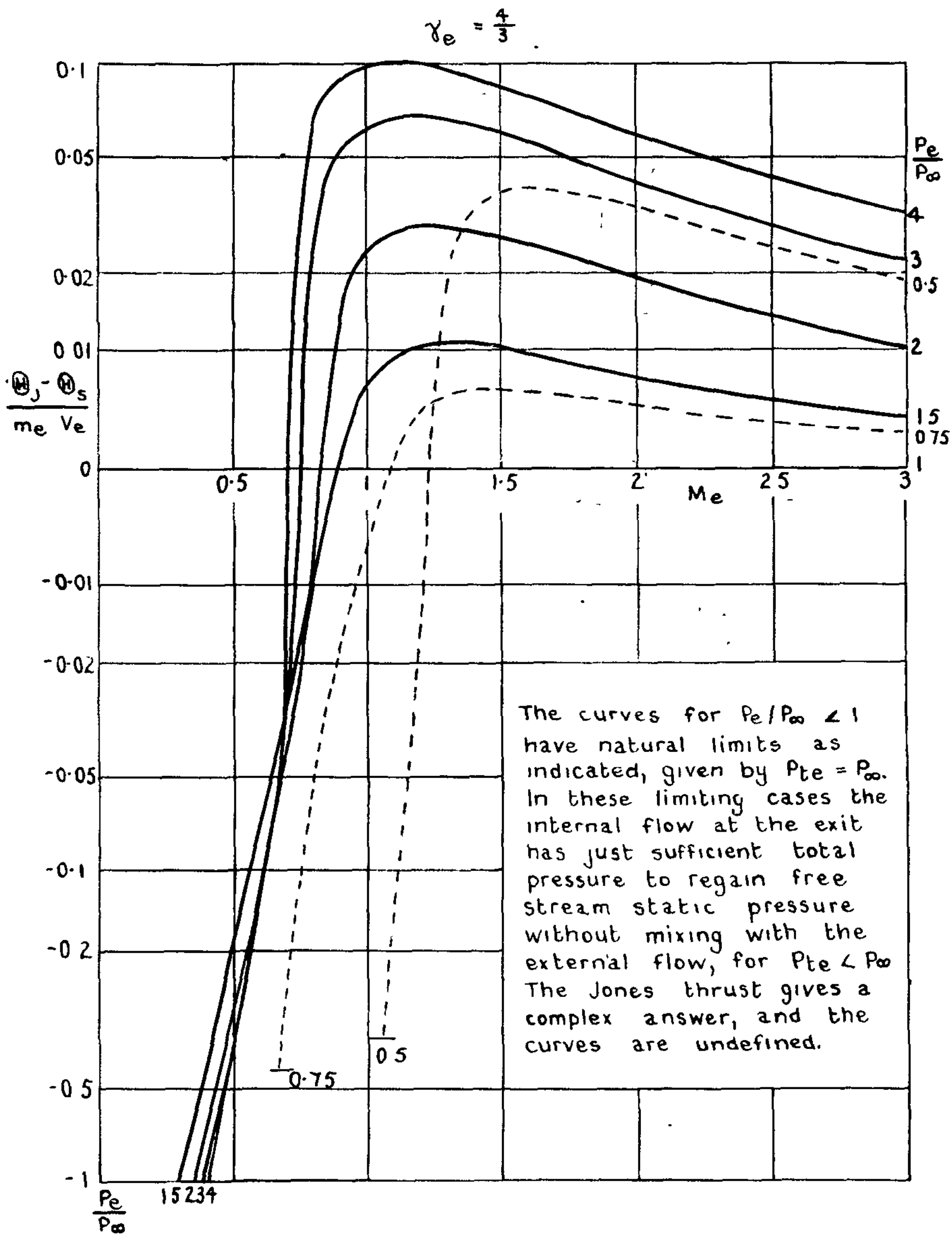
Fig. 1.



Diagrammatic representation of the flow through a ducted body

FIG 2.



Curves of the difference between the Jones thrust and the standard thrust.

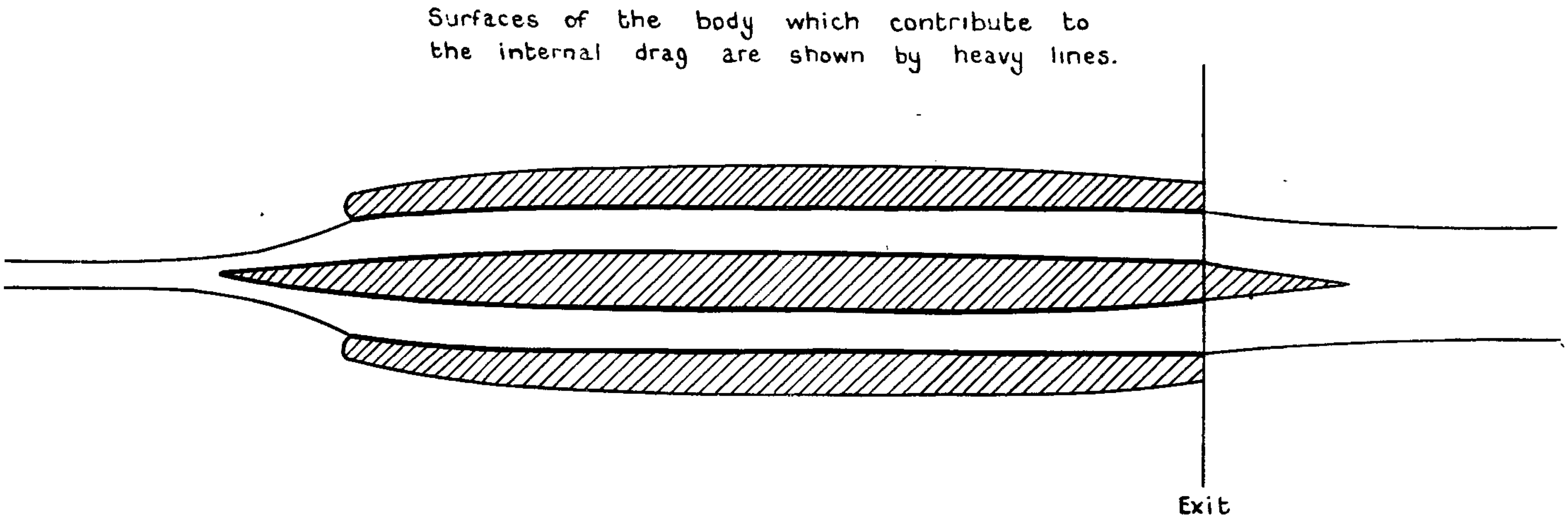


FIG. 3

Diagrammatic representation of a ducted body having a central body within it.

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