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Analysis of Flight Measurements on the Airborne Path during Take-off

By

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SUMMARY

Analysis of a series of systematic take-off tests with a Meteor IV aircraft has shown that to a good approximation the minimum airborne path to 50 ft may be treated as an arc of a circle. With this assumption, it is a simple process to derive the mean equivalent lift coefficient used during this part of the take-off.

It has been found that the total equivalent lift coefficient used during the airborne phase decreases with increase in the ratio of the airspeed to the stalling speed in a simple manner, which is independent of the thrust/weight ratio when the shortest possible distance is required.

Using the results of a similar analysis applied to three other aircraft, an empirical rule has been developed, from which the mean equivalent lift coefficient increment, and hence the minimum airborne distance to 50 ft, can be estimated simply and with reasonable accuracy.

The airborne distances thus obtained must be regarded as the minimum possible values. A factor of 1.5 may be required to allow for normal take-off techniques, particularly when the thrust/weight ratio is low.

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1 Introduction

The major uncertainty in the estimation of take-off distance to 50 ft arises from the assumptions that have to be made regarding the piloting technique during the airborne part of this manoeuvre. Earlier methods of estimating take-off distance may be modified, as in Section 2 below, to allow for variations in piloting technique. The theory serves to emphasize the importance of the technique on the distance involved, but the accuracy of the estimation could not be improved until more quantitative data were available on piloting technique during actual take-offs.

The uncertainty of the estimates has increased in recent years, since these earlier methods generally assumed that steady climb conditions would be achieved before the standard 50 ft height was reached. With modern aircraft, this is often not the case, and estimation methods need modification accordingly.

To obtain quantitative information on piloting technique, and to test the accuracy of proposed methods of estimation, a series of recorded take-offs has been made with a Meteor IV aircraft, having a static thrust/weight ratio of around. 0.5.

Test conditions were slightly artifiaial in that the pilot was asked to achieve the shortest practicable take-off distance, consistent with safety. The results must therefore be interpreted. as minimum distances.

To check such conclusions as were obtained from analysis of the Meteor results, use was made of the results of a large number of recorded take-offs made by the A & A.E.E. on two propeller-driven and one jet-propelled transport aircraft. This large volume of information has proved invaluable.

2 Information Required from Flight Tests

The take-off manoeuvre may be considered to be divided into three phases:-

(1) the ground run up to the take-off speed;

(2) the transition phase, during which the speed and. climbing angle are changed to the steady climb values,

and (3) the steady climb.

The airborne path from the point of take-off to the point where the standard 50 ft height is reached may involve both phases (2) and (3), or it may lie entirely in phase (2). It is in phase (2) that the main assumptions have to be made regarding piloting technique.

In an early method[1] of deriving the equation to the path followed in phase (2), the main assumption was that the lift coefficient was held constant at the initial value C_{L_o} (appropriate to steady flight at the take-off equivalent airspeed V_g, ft/sec) until the aircraft reached a speed V_a ft/sec and climbing angle γ radians equal to the steady angle of climb at V_a. The lift coefficient was then supposed to drop instantaneously to the value $C_{L_o} \cdot (V_g/V_a)^2$ and the steady climb followed.

The take-off technique thus defined is one in which the aircraft is allowed to fly itself off. The theory can, however, be modified to allow for the known ability of the pilot to increase the lift ooeffioient at take-off', producing a finite normal acceleration from the start. The authors are indebted to C.H. Naylor for this suggestion, which leads to a relation between height gamed, h, and forward distance travelled, s, both in feet, of the form:-

$$h = \gamma_o \left(s - \frac{V_g^2}{\sqrt{2}.g\sigma} \cdot \sin \frac{\sqrt{2}.g\sigma s}{V_g^2} \right) + \frac{\Delta C_L}{C_{L_o}} \cdot \frac{V_g^2}{2g\sigma} \left(1 - \cos \frac{\sqrt{2}.g\sigma s}{V_g^2} \right) \qquad (1)$$

where ACL is the increment in lift coefficient applied at take-off in excess of that required for flight at the take-off speed. The total lift coefficient is assumed to remain constant throughout the transition, and the increase in drag associated with the lift inorement is assumed to be small. The take-off longitudinal acceleration (in g-units) and the steady angle of olimb (in radians) at the take-off equivalent airspeed V_g are both equal to γ_o. Over the range of speed involved, variation of γ_o with airspeed is ignored. The remaining symbols have their usual meaning.

The airspeed, V_a at any point during this manoeuvre is related to the take-off airspeed V_g by the equation

$$V_a^2 = V_g^2 \left(1 + \sqrt{2}.\gamma_o \sin \frac{\sqrt{2}g\sigma s}{V_g^2} - \frac{\Delta C_L}{C_{L_o}} \left[1 - \cos \frac{\sqrt{2}g\sigma s}{V_g^2} \right] \right) \qquad (2)$$

and the instantaneous angle of climb, y , is given by

$$\gamma = \gamma_o \left(1 - \cos \frac{\sqrt{2}g\sigma s}{V_g^2} \right) + \frac{\Delta C_L}{\sqrt{2}.C_{L_o}} \sin \frac{\sqrt{2}g\sigma s}{V_g^2} \qquad (3)$$

The transition ends when $\gamma = \gamma_o$, i.e. when

$$s = \frac{V_g^2}{\sqrt{2}g\sigma} \cdot \tan^{-1} \gamma_o \sqrt{2}\, C_{L_o}/\Delta C_L \qquad (4)$$

If the height gained (given by equation (1)) at the end of the transition exceed 50 ft, then, clearly, conditions are not steady at 50 ft, and equation (1) would be used to estimate the airborne distance to 50 ft with the substitution h = 50 ft.

If, however, steady conditions are reached. before the 50 ft point is passed, we then define the transition distance (Fig.1) as the difference between the actual distance to some point on the steady olimb path and the distance in which this point would have been reached had the aircraft boon able to olimb straight off the ground at the steady olimbing angle γ_o. The transition distance, s_T, (which is <u>not</u> the same as that given by equation (4)) may then be written

$$s_T = f \cdot \frac{V_g^{\,2}}{\sqrt{2g\sigma}} \qquad (5)$$

where the factor f is given by

$$\left.\begin{aligned} f &= \sin\theta \;\frac{\Delta C_L}{C_{L_o}}\;(1 - \cos\theta)/\sqrt{2}\,\gamma_o \\ \text{and} \qquad \theta &= \tan^{-1} \sqrt{2}\,\gamma_o\, C_{L_o}/\Delta C_L \end{aligned}\right\} \qquad (6)$$

The steady climb distance to 50 ft is then simply $50/\gamma_o$ feet, and the total airborne distance to the 50 ft point is:-

$$s_A = s_T + 50/\gamma_o \qquad (7)$$

Fig.2 shows the variation of the factor f with γ_o for a range of values of $\Delta C_L/C_{L_o}$ from 0.1 to 0.8. When $\Delta C_L = 0$, we have $f = 1$. It is clear that the value of $\Delta C_L/C_{L_o}$ chosen has a marked effect on the transition distance, as it has also, of course, on the total distance to 50 ft derived from equation (1).

It is worth noting that equation (1) and (2) may be combined to give, with h = 50 ft,

$$s = \frac{1}{\gamma_o}\left(\frac{V_a^{\,2} - V_g^{\,2}}{2g\sigma} + 50\right) = s_A \qquad (8)$$

where s_A is now the total airborne distance to 50 ft. This equation does not involve ACL, but requires instead a knowledge of the variation of the airspeed during this phase.

Equation (8) is, of course, one that could have been obtained quite simply by consideration of the changes in energy occurring during the airborne path, with the assumption that the drag remains sensibly constant.

We have, now, basically, two methods available for the estimation of the airborne distance to 50 ft. We may use either of equations (1) or (7) (according to whether the steady climb state is reached after or before the 50 ft point is passed), requiring a knowledge of $\Delta C_L/C_L$, but with variations in speed appearing only as a dependent variable? Alternatively, equation (6) may be used, not directly dependent on ACL, but requiring a knowledge of the variations in speed occurring during the airborne phase.

The flight tests described below might therefore have been directed towards providing data as a basis for estimating either the value of $\Delta C_L/C_{L_o}$ used in practice, or the changes in airspeed that occur, during the airborne phase.

The recording technique was such that changes in airspeed could not be measured with sufficient accuracy, and, further, it is seen from equation (2) that these speed changes are a complex function of γ_0 and ACL, i.e. of the aircraft characteristics and piloting technique. Attention has therefore been concentrated on deriving the mean equivalent ACL used during the airborne phase. This information could then be used in developing a method for predicting the value of ACL to be used in estimating the airborne distance for other aircraft, using equations (1) or (7) as appropriate.

Equations (1) or (7) result in very cumbersome methods for evaluating the mean equivalent ACL for a particular take-off' and, in addition, a knowledge of γ_0 is required. On the Meteor IV, which was the principal subject of this investigation, γ_0 has been measured by partial. climb tests, but it could not be obtained accurately for other aircraft on which take-off measurements were available.

Fortunately, it was found that, in the case of the Meteor, the mean equivalent lift coefficient increment, ACL, could be derived with sufficient accuracy by assuming that the flight path up to the 50 ft point is an arc of a circle, provided that this mean equivalent increment is defined so as to include the increase in lift arising from any increase In airspeed during the airborne path.

If V_m is the R.M.S. equivalent airspeed during the airborne path, whose constant radius of curvature *in the vertical* plane is R, and C_{L_m} is the lift coefficient corresponding to steady flight at V_m, then, if the speed changes are small,

$$V_m^2/Rg\sigma = \Delta C_L'/C_{L_m} \qquad (9)$$

$\Delta C_L'$ is the mean equivalent lift coefficient increment and, by simple geometry, the airborne distance, s_A, to 50 ft is:-

$$s_A = \sqrt{\frac{200\ w_s}{\rho g\ \Delta C_L'} - 2500} \qquad (10)$$

This expression for s_A, which is independent of both V_g and γ_0 is plotted in Fig.3 as a function of $\Delta C_L'$ for a range of values of the wing loading w_s.

The *mean* equivalent lift coefficient increment $\Delta C_L'$ is defined by equation (10), and includes any increase in lift arising from the increase in airspeed occurring during the airborne phase. The increment thus defined can therefore remain positive for the airborne path as a whole, even though there may be no increase in the actual lift coefficient at take-off, and is generally larger than the increment, ACL, used in equation (1) et seq.

Lift ooeffioient increments quoted in this Note were obtained by this method and include the effect of the increase in airspeed. The problem is therefore to ~~devise a method of predicting~~ $\Delta C_L'$ for any given aircraft condition.

3 Test Procedure - Meteor IV

Take-offs were made from a concrete runway and photographed with the F.47 take-off camera. Having established the absolute minimum airspeed at which the aircraft could be pulled off, the pilot was asked to do a series of take-offs in which the aircraft left the ground at airspeeds 10, 20, 30 and 40 knots above this minimum. Preliminary tests showed that the piloting technique which could be repeated most consistently was that in which a rearward. pressure on the stick was applied at about 10 knots below the desired take-off speed, with the pilot attempting to keep the increase in airspeed after take-off as small as possible. This technique was expected to produce the shortest practicable airborne distance.

For each nominal take-off sped, each of three engine powers was used, corresponding to 14,600 R.P.M. (full throttle), 13,800 R.P.M. and 13,000 R.P.M. At maximum thrust, the pilot was able, on the average, to keep the speed increase between take-off and 50 ft down to 10 knots. At 13,800 R.P.M., the increase averaged 3 knots, while at 13,000 R.P.M. there was on the average a 3 knots reduction in airspeed, suggesting that in this case, the climbing angle was too high.

The take-off weight was varied only by oonsumption of fuel. The flap setting was 25 degrees throughout.

A two-axis accelerometer mounted in the aircraft was used to record the normal acceleration during most of the take-offs.

A minimum of 3 take-offs was made at each combination of take-off speed and engine R.P.M.

In addition to the take-off tests, partial climb tests were made, covering the whole range of airspeeds and engine powers used for the take-offs. When corrected to the atmospheric conditions appropriate to each take-off, the longitudinal acceleration, γ_o, at take-off could be obtained (neglecting ground effects).

4 Corrections

The airborne distance, from the point at which the wheels left the ground to the point 50 ft above the take-off point, was corrected to zero headwind by the method of Ref.2. No further corrections to the distance were necessary, since for each take-off the mean equivalent lift coefficient increment $\Delta C_L'$ could be calculated from equation (10), using the appropriate values for wing loading (allowing for fuel consumption), air density (from Meteorological Office records) and the airborne distance to 50 ft in zero headwind.

5 Results and Discussion

Table I presents the results of the measurements of airborne distance to 50 ft and the corresponding airspeeds and climb angles, together with the longitudinal acceleration at take-off, derived from the results of the partial climb tests.

In Fig.4, $V_g^2/\sqrt{2}g\sigma$ has been plotted. against $(s_A - 50/\gamma_o)$. The straight lines through the origin correspond to various values of the correction factor, "f", defined in equation (6), and if the transition had ended before the 50 ft point had been reached, this diagram could be used to determine the-value of $\Delta C_L/C_{L_o}$ used during the transition, since the acceleration γ_o is known. Though this process does, in fact, give values of $\Delta C_L/C_{L_0}$ (using Fig.2) which are comparable with those determined from equation (10) in some-cases, it is theoretically unsound since the records show that, generally, conditions are not steady at the 50 ft point, although, at the lowest engine thrust, the steady climb angle was reached and exceeded before the 50 ft point was passed.

Fig.4 shows that the factor "f" decreases as the take-off speed is increased (at constant weight) and as the longitudinal acceleration γ_0 decreases. This is in line with the reduction in transition distance indicated in Fig.2. At the higher take-off speeds, larger values of $\Delta C_L/C_{L_0}$ can be applied without danger of stalling.

It will be noted that some of the take-offs, particularly those at the lowest engine R.P.M., have produced very low values of the factor "f", in some cases less than 0.1. In these cases, the climbing angle at 50 ft was greater than that appropriate to a steady climb at the same speed, producing an exceptionally short airborne distance.

With the assumption that the airborne path is a continuous manoeuvre, with a constant radius of curvature, the mean equivalent lift coefficient increments have been evaluated, from equation (10), and the values are given in Table II, together with the lift coefficient C_{L_0} corresponding to steady flight at the take-off speed Vg. The calculated value of $\Delta C_L/C_{L_0}$ is compared with that derived from the accelerometer records.

In Fig.5, the calculated values of $\Delta C_L'$ are shown graphically as a function of $(V_m/V_S)^2$, where V_m is the root mean square equivalent airspeed during the airborne path to 50 ft and VS is the engine-on stalling speed. On each of the three graphs (one for each take-off R.P.M.) the lower set of points gives the calculated increment, while the upper set of points shows the variation of total lift coefficient $(C_{L_m} + \Delta C_L')$ with $(V_m/V_S)^2$, where C_{L_m} is the lift coefficient corresponding to steady flight at the R.M.S. airspeed Vm. The intermediate curve indicates the lift coefficient C_{L_m} as a function of $(V_m/V_S)^2$.

It will be seen that the total lift coefficient $(C_{L_m} + \Delta C_L')$ is approximately a linear function of $(V_m/V_S)^2$ and that the increment $\Delta C_L'$ vanishes at a value of C_{L_m} equal to the maximum available lift coefficient, power on. This maximum, which includes the effect of engine thrust, varies slightly with take-off R.P.M.

At the upper end of the speed scale, the curves of $(C_{L_m} + \Delta C_L')$ and C_{L_m} will, if extrapolated, intersect again at some lower lift coefficient C_L'. It is, of course, not implied that take-off would be impossible at lift coefficients less than this value, and the significance of this second point of intersection lies mainly in the use of C_L' in fixing the position of the $(C_{L_m} + \Delta C_L')$ line for the purpose of predicting the $\Delta C_L'$ likely to be used at any particular value of $(V_m/V_S)^2$.

6 Use of Additional Data

The Meteor take-off measurements have shown that a mean equivalent lift coefficient increment $\Delta C_L'$ may be derived from equation (10) with satisfactory accuracy and that $\Delta C_L'$ could be estimated for any take-off speed if we could predict where the $(C_{L_m} + \Delta C_L')$ line would re-intersect the C_{L_m} curve, as in Fig.5.

Tho above analysis has therefore been applied to the results of take-off measurements made by the A & A.E.E. on the Nene-Viking, Dakota and Hermes.' The increments of lift coefficient $\Delta C_L'$, and tho total lift ooefficients $(C_{L_m} + \Delta C_L')$ are plotted. in Figs.6, 7 and 8 as functions of $(V_m/V_S)^2$, for these 3 additional airoraft.

The scattor of the points in Figs. 6, 7 and 8 is larger than that in Fig.5, as we should expect. The Meteor pilot was attempting, in every oase, to produce the shortest practioable airborne distance, whereas the A & A.E.E. results, obtained on civil aircraft, were more strongly influenced by safety considorations.

In each case, the $(C_{L_m} + \Delta C_L')$ line has been drawn to intersect the C_{L_m} curve at a lift ooefficient equal to the maximum lift ooefficient in the take-off condition, power on. The line passes mainly through the points oorresponding to the larger values of ΔC_L, since these were presumably obtained under conditions more closely resembling those for the Meteor take-offs. For normal take-off conditions, it is suggested that the increment $\Delta C_L'$ might be taken as half the maximum practical value, thereby increasing the airborne distance by a factor of about 1.5.

The $(C_{L_m} + \Delta C_L')$ and the C_{L_m} lines have been extrapolated as necessary to re-intersect at the lower value of lift coefficient, C_L', referred to in Section 5. In Fig.9, this lift coefficient C_L' is plotted against $C_{L\ max.}$ in the take-off configuration. The 3 points for the Meteor, plus these 3 extra points, are seen to define reasonably well a straight line.

No attempt is made here to justify theoretically this linear relation between C_L' and $C_{L\ max.}$, nor that between $(C_{L_m} + \Delta C_L')$ and $(V_m/V_S)^2$. However, a reduction in total lift ooofficient $(C_{L_m} + \Delta C_L')$ with increasing airspeed is to be expected. A finite time is required to apply the increment $\Delta C_L'$ after take-off (especially if this is done by an increase in inoidonoo) and at the higher take-off speeds, the time during which the lift coefficient is increasing is a proportionately larger fraction of the total time to reach the 50 ft point. The mean effective lift coofficient increment therefore may be expected to decrease although the final value may be the same. The basic lift coefficient C_{L_m} decreases with increase in speed and so the total ooeffioient $(C_{L_m} + \Delta C_L')$ does likewise.

7 Methods of Prediction

The proposed method is based mainly on the empirical relationships established in the previous sections for the estimation of the lift ooefficient increments used during the airborne phase. The increments thus derived must be regarded as the maximum practicable values, and their use may, in some oases, lead to excessively steep angles of climb at the 50 ft point, or an undesirable loss in airspeed. The first prediction method described below takes no account of the condition of the aircraft at the 50 ft point and gives the minimum practicable airborne distance.

The result illustrated in Fig.9 may be expressed in the form

$$C_L' = 0.53\ C_{L\ max} - 0.38 \qquad (11)$$

from which it may be shown that

$$\Delta C_L' = \left[\left(\frac{V_m}{V_S}\right)^2 - 1\right]\left[C_{L\,max}\left(\left(\frac{V_S}{V_m}\right)^2 - 0.53\right) + 0.38\right] \qquad (12)$$

and, finally, using the approximate expression for the airborne distance s_A from equation (10), in standard atmospheric conditions, we have

$$s_A = 51\sqrt{\frac{w_s}{\left[\left(\frac{V_m}{V_S}\right)^2 - 1\right]\left[C_{L\,max.}\left(\left(\frac{V_S}{V_m}\right)^2 - 0.53\right) + 0.38\right]}} \qquad (13)$$

It should be noted that to use the lift coefficient increment $\Delta C_L'$ estimated from equation (12) in the more exact expression for the airborne path given in equation (1) would be to include the effect of the increase in lift due to increase in airspeed twice. In the case of the Meteor this processs gae distances up to 10% less than those derived f&m equation (13). In Fig.10, the distance estimated for each individual take-off by equation (13) is compared with that actually measured. It will be seen that 75% of the results are within 10% of the measured values. In view of the marked dependence of the distance upon piloting technique, this agreement is considered to be satisfactory.

'It is considered that a more normal piloting technique will result from the use of a lift coefficient increment of about half that predicted by equation (12). The use of this reduced increment would increase the distance to 50 ft (using equation (13)) by a factor of 1.5.

In some cases, particularly for civil aircraft, close attention must be paid to the airspeed and angle of climb at the 50 ft point. The above prediction method is based on a technique which may only be regarded as safe when the thrust/weight ratio is adequate.

Equation (4) gives the distance from the take-off point to the point at which the instantaneous angle of climb is equal to the steady angle of climb at the take-off speed. This value of the distance may be substituted in equation (1) and the solution of the resulting equation for the case when h = 50 ft will give the value of the longitudinal acceleration γ_o for which the climbing angle at 50 ft is equal to the steady climb angle. This solution is shown graphically in Fig.11. If the longitudinal acceleration at take-off is less than that given by this diagram (at the appropriate values of Vg and $\Delta C_L/C_{L_o}$) then the technique which forms the basis of the first prediction method will result in a climbing angle at the 50 ft point in excess of the steady climb angle. Similarly, Fig.12, which is derived from equation (8), shows the value of γ_o which will ensure that the speed at the 50 ft point is not less than the take-off speed.

When it is inadmissible for the climb angle at the 50 ft point to exceed the steady climb angle, or for the speed at that point to be less than the take-off speed (as predicted by Figs.11 and 12), then steady climb conditions must be assumed before the 50 ft point is reached, and an alternative estimation method used.

The lift coefficient increment used during the initial transition phase is estimated by the process already described. This increment is used to determine the factor "f" from Fig.2, and hence the transition distance may be calculated from equation (5). The error introduced by the use of a lift coefficient increment which includes the effect of an increase in airspeed will be small, since we are concerned here mainly with relatively low thrust/weight ratios, and the transition distance is, in any case, only part of the total distance to 50 ft. To this transition distance is added the steady climb distance $50/\gamma_o$, where γ_o is the steady climb angle at the take-off speed (in radians), or the longitudinal acceleration at take-off (in g-units).

Two examples will serve to illustrate the application of the various methods.

Example I.. (Fighter)

Take-off speed = 140 knots (= 1.15 x engine-on stalling speed.)

Wing loading = 60 lb/sq.ft. Longitudinal acceleration at take-off = 0.3g.

$C_{L\,max.}$ = 1.2 (engine on, but not including ground effect)

$\Delta C_L'$ = 0.21 (from equation 12), and $\Delta C_L/C_{L_o}$ = 0.23

∴ Minimum airborne distance to 50 ft = 870 ft (from equation 13)

Using half the above lift coefficient increment (i.e. $\Delta C_L'$ = 0.105)

Normal airborne distance = 1230 ft.

From Fig.11, the minimum acceleration, γ_o, required to ensure that the climb angle at 50 ft is not greater than the steady climb angle is 0.125 at the maximum ΔC_L or 0.095 at half the maximum. The value of γ_o required to ensure that the speed does not fall below the take-off speed is 0.058 at the maximum ΔC_L, or 0.040 at half the maximum. The available γ_o (0.3) is well above these limits, so that the quoted distances do not involve an exceptional technique.

Example 2. (Overloaded bomber)

Take-off speed = 180 knots (= 1.20 x engine-on stalling speed)

Wing loading = 80 lb/sq.ft. Longitudinal acceleration at take-off = 0.05g

$C_{L\,max.}$ = 1.05 (engine-on)

$\Delta C_L'$ = 0.24 (from equation 12) and $\Delta C_L/C_{L_o}$ = 0.33

∴ Minimum possible airborne distance to 50 ft (irrespective of climb angle or speed at that point) = 930 ft.

Fig.11 shows that the available γ_o (0.05) is insufficient to ensure that the steady climb angle has not been exceeded, even at half the stated value of ACL, and Fig.12 shows that the speed would have fallen below the take-off speed, when using the full value of ΔC_L. The alternative estimation method is therefore used.

The factor "f" = 0.11 (from Fig.2)

∴ Transition distance = 220 ft (from eqn.5)

Steady climb distance = 1000 ft

∴ Total airborne distance = 1220 ft.

Using half the above value of $\Delta C_L'$, i.e. 0.12, we have

f = 0.21 (from Fig.2)

∴ Transition distance = 430 ft

and total airborne distance = 1430 ft.

8 Choice of Method

The choice between the first and second methods, and between the use of the full or half lift coefficient increment depends on what safety factors are to be applied in deciding on safe runway lengths. Two alternatives are apparent - either to calculate the absolute minimum distance and to apply a generous safety margin, or to calculate the distance which the average pilot might reasonably be expected to achieve, and to apply a reduced safety margin.

The absolute minimum distance is obtained by using the full lift coefficient increment and ignoring the speed and angle of climb at the 50 ft point. At the other extreme we should use half the lift coefficient increment and pay strict attention to conditions at the 50 ft point. The difference in the two estimates of the distance depends on the longitudinal acceleration at take-off, but is of the order of $\frac{1}{2}$ of the shorter distance, i.e. as a rough estimate, the comfortable, safe distance may be taken as 150% of the minimum possible distance.

9 Choice of speed margin at Take-off

Speed margins (V_g/V_S) of 1.15 and 1.20 have been used in the two examples. The shortest airborne distances will be obtained when the speed margin is such as to make $\Delta C_L'$ a maximum (equation 12). Typical values of the optimum ratios are 1.3 when $C_{L\,max.}$ is 2.0, or 1.6 when $C_{L\,max.}$ is 1.0. The ground run, however, increases roughly as the square of the take-off speed, so that the shortest overall distance, from the start of the ground run to the 50 ft point, is obtained at relatively small values of this speed margin. The margin must give adequate protection against inadvertent stalling, and should allow the application of the desired lift coefficient increment in safety. For this reason, a speed margin of at least 1.15 is recommended.

10 Conclusions

Analysis of a series of systematic take-off tests with a Meteor IV aircraft has shown that to a good approximation the minimum airborne path to 50 ft may be treated as an arc of a circle. With this assumption it is a simple process to derive a mean equivalent lift coefficient for this part of the take-off.

It has been found that the total equivalent lift coefficient Used during the airborne phase decreases with increase in the ratio of air-speed to stalling speed, for a particular aircraft, in a simple manner which is independent of the thrust/weight ratio.

Using the results of a similar analysis applied to three other aircraft, an empirical rule has been developed, from which the mean equivalent. lift coefficient increment, and hence the airborne distance to 50 ft, can be estimated simply and with reasonable accuracy. Allowance can be made for the effect of low thrust/weight ratios.

The airborne distances thus obtained must be regarded as minimum possible values. A factor of 1.5 may be required to allow for normal take-off techniques.

11 Further Work

To enable this empirical method of estimation to be used with greater confidence, it is desirable to compare estimated airborne distances to 50 ft with measured values on as many aircraft as possible.

LIST OF REFERENCES

No.	Author	Title, etc.
1	Ewans and Hufton	Note on a method of calculating take-off distances. RAE B.A. Dept. Note No.20. August 1940.
2	Jackson	The reduction to standard conditions of take-off measurements on a turbo-jet aircraft. R & M.2890. June 1951.

Attached:

Tables I and II
Drg.Nos. 27612.S to 27621.5

Wt.2078.CP.156.K3. Printed in Great Britain.

TABLE I

Measured Take-off Data - Meteor IV EE.597

Take-Off Weight lb.	Nominal Take-off Speed knots	Engine R.P.M.	Take-off Airspeed $V_g/\sqrt{\sigma}$ ft/sec. (see note 1)	Take-off Accelera-tion γ_0 (g-units) (see note 2)	Airborne Distance to 50 ft s_A feet (see note 3)	Airspeed at 50 ft $V_A/\sqrt{\sigma}$ ft/sec. (see note 4)	Climb Angle at 50 ft. γ_{50} rads.
1	2	3	4	5	6	7	8
13,375	110	14,600	175.8	0.294	565.0	197.5	0.195
13,090	"	"	176.2	0.300	565.0		0.206
12,767	"	"	181.0	0.310	561.5	193.1 200.2	0.201
13,650	120	14,600	185.0	0.297	582.5	204.7	0.200
13,375	"	"	184.3	0.301	585.0	208.6	0.218
13,132	"	"	183.0	0.305	507.0	197.8	0.227
14,426	130	14,600	201.2	0.288	590.0	215.6	0.186
14,184	"	"	197.8	0.294	538.5	210.5	0.200
14,064	"	"	197.2	0.296	561.0	210.1	0.205
14,362	140	14,600	246.8	0.259	526.5	253.8	0.221
14,022	"		241.0	0.268	505.0	256.9	0.215
13,767	"	"	252.8	0.264	530.0	258.7	0.207
14,556	150	14,600	271.8	0.240	586.5	301.0	0.182
13,941	"	"	273.0	0.249	587.0	297.1	0.188
13,333	"	"	287.0	0.250	595.0	312.2	0.182
14,508	110	13,800	176.8	0.177	936.0	190.8	0.145
13,900	"	"	177.7	0.186	700.0	192.4	0.171
13,619	"	"	176.4	0.191	643.0	192.7	0.171
13,375	"	"	186.0	0.197	631.5	207.7	0.185
14,508	120	13,800	193.8	0.187	648.5	216.8	0.186
14,103	"	"	211.3	0,200	615.5	223.4	0.196
13,642	"	"	196.3	0.200	551.0	200.6	0.218
13,294	"	"	198.7	0.205	540.0	202.8	0.223
14,022	130	13,800	211.5	0.202	593.0	232.5	0.193
13,698	"	"	215.6	0.207	585.0	217.8	0.213
13,456	"	"	220.9	0.206	576.5	219.3	0.221
14,184	140	13,800 "	231.3	0.189	549.0	236.9	0.226
13,860	"		244.8	0.185	576.5	uC9.2	0.233
13,198	"	"	240.0	0.194	514.5	254.1	0.231
14,022	150	13,800	288.5	0.158	591.5	263.1	0.203
13,537	"	"	271.5	0.170	543.0	252.4	0.214
13,300	"	"	267.3	0.173	514.9	243.3	0.209
14,556	110	13,000	187.2	0.112	1020.7	185.2	0.113
14,103	"	"	192.0	0.116	799.6	199.0	0.117
12,930	"	"	183.3	0.122	726.0	200.3	0.150
14,540	120	13,000	200.3	0.112	666.4	187.1	0.133
14,184	"	"	195.1	0.115	647.2	174.8	0.157
13,860	"	"	194.5	0.117	595.7	180.0	0.163

/Continued

Table I (Contd.)

Take-off Weight lb.	Nominal Take-off Speed knots	Engine R.P.M.	Take-off Airspeed $V_g/\sqrt{\sigma}$ ft/sec. (see note 1)	Take-off' Accelera-tion γ_o (g-units) (see note 2)	Airborne Distance to 50 ft s_A feet (see note 3)	Airspeed at 50 ft $V_A/\sqrt{\sigma}$ ft/sec. (see note 4)	Climb Angl at 50 ft. γ_{50} rads.
1	2	3	4	5	6	7	a
14,508	130	13,000	225.2	0.112	687.5	237.5	0.163
14,070	"	"	22Itc.3	0.116	593.0	227.8	0.182
13,698	"	"	220.3	0.119	609.0	232.9	0.184
13,294	"	"	221.4	0.122	573.5	225.4	0.201
14,589	140	13,000	232.8	0.110	528.1	216.2	0.209
14,265	"	"	239.5	0.110	518.1	224.5	0.206
13,860	"	"	244.0	0.112	559.5	227.2	0.203
14,589	150	13,000	254.5	0.102	561.0	249.5	0.211
14,184	"	"	254.5	0.105	537.3	249.0	0.224
13,860	"	"	255.2	0.107	537.8	242.1	0.224

Note 1 $V_g/\sqrt{\sigma}$ = Measured ground speed from P.47 film + wind speed.

2 γ_o is derived from partial climb tests, converted to test atmospheric conditions.

3 s_A has been corrected to zero headwind.

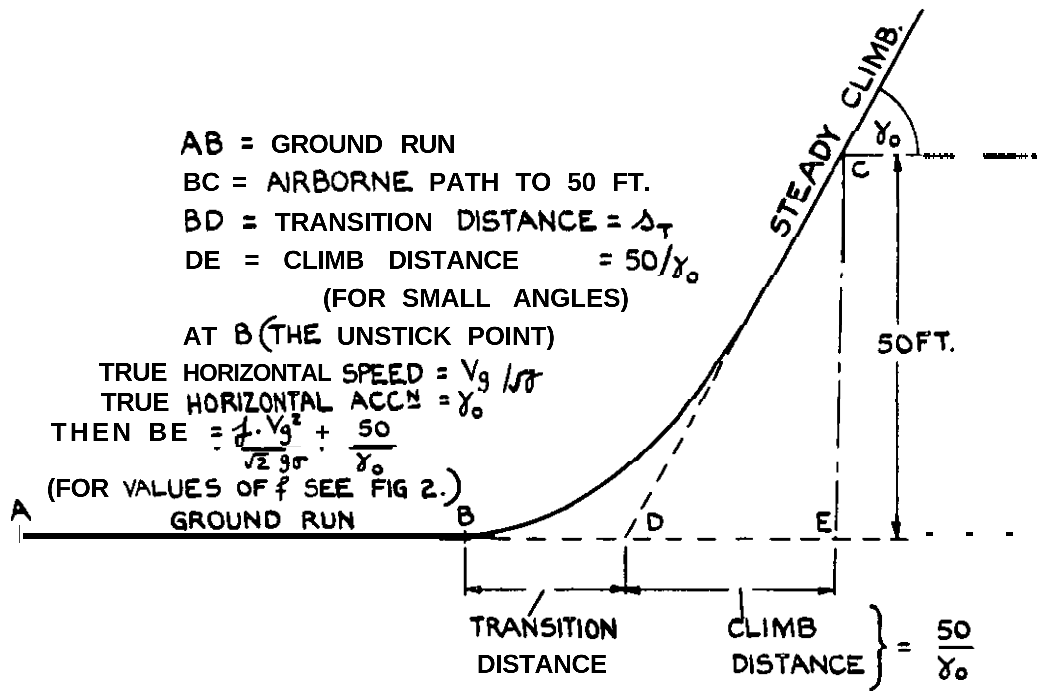
4 $V_A/\sqrt{\sigma}$ = Measured airspeed along flight path (includes vertical component and wind speed).

TABLE II

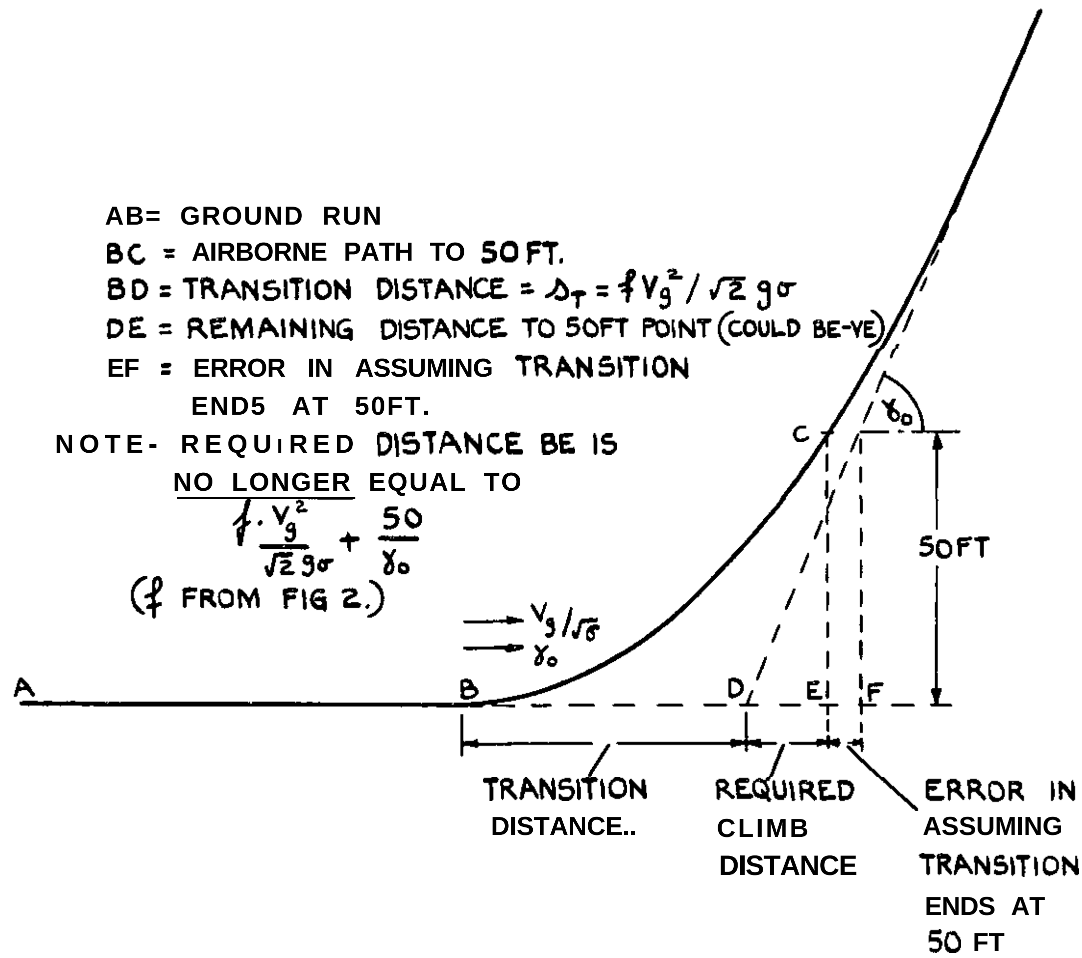
Information derived from take-off measurements

Meteor IV EE597

Nominal Take-off Speed kts.	Engine R.P.M.	Take-off lift coeff: C_{L_0}	Mean lift coeff: increment $\Delta C_L'$	$\frac{\Delta C_L'}{C_{L_0}}$	Mean excess normal accn. g-units (accelerometer)
110	14,600	1.039	0.310	0.298	0.306
"	"	1.010	0.303	0.300	0.307
"	"	0.935	0.299	0.320	0.343
120	"	0.959	0.298	0.311	
"	"	0.947	0.290	0.306	
"	"	0.943	0.378	0.401	
130	"	0.858	0.307	0.358	
"	"	0.872	0.362	0.416	
"	"		0.331	0.381	
140	"	0.566 / 0.870	0.383	0.676	0.672
"	"	0.579	0.406	0.701	0.618
"	"	0.518	0.362	0.699	0.741
150	"	0.475	0.314	0.662	0.592
"	"	0.451	0.301	0.667	0.631
"	"	0.390	0.280	0.718	0.625
110	13,800	1.111	0.123	0.111	0.135
"	"	1.054	0.210	0.200	0.230
"	"	1.050	0.244	0.232	0.195
"	"	0.922	0.248	0.269	0.249
120	"	0.926	0.256	0.276	0.258
"	"	0.757	0.276	0.363	0.282
"	"	0.851	0.332	0.390	0.340
"	"	0.808	0.337	0.417	0.347
130	"	0.751	0.295	0.393	0.387
"	"	0.707	0.296	0.418	0.403
"	"	0.660	0.299	0.453	0.463
140	"	0.636	0.348	0.547	0.547
"	"	0.556	0.309	0.557	0.688
"	"	0.550	0.364	0.664	0.630
150	"	0.411	0.298	0.725	
"	"	0.442	0.341	0.771	
"	"	0.449	0.372	0.828	0.757
110	13,000	0.955	0.104	0.109	
"	"	0.916	0.164	0.179	
"	"	0.924	0.182	0.197	0.225
120	"	0.872	0.243	0.279	
"	"	G.896	0.251	0.280	
"	"	0.896	0.295	0.329	
130	"	0.688	0.228	0.331	0.282
"	"	0.673	0.297	0.441	0.389
"	"	0.679	0.274	0.403	0.348
"	"	0.652	0.300	0.459	0.361
140	"	0.647	0.388	0.599	
"	"	0.599	0.394	0.658	
"	"	0.561	0.328	0.586	-
150	"	0.542	0.343	0.634	
"	"	0.527	0.364	0.690	
"	"	0.521	0.361	0.692	



CASE (A) STEADY CLIMB STARTING BELOW 50 FT.



CASE (B) STEADY CLIMB STARTING ABOVE 50FT.

FIG. 1. ALTERNATIVE FORMS OF FLIGHT PATH TO 50 FEET.

FIG.2.

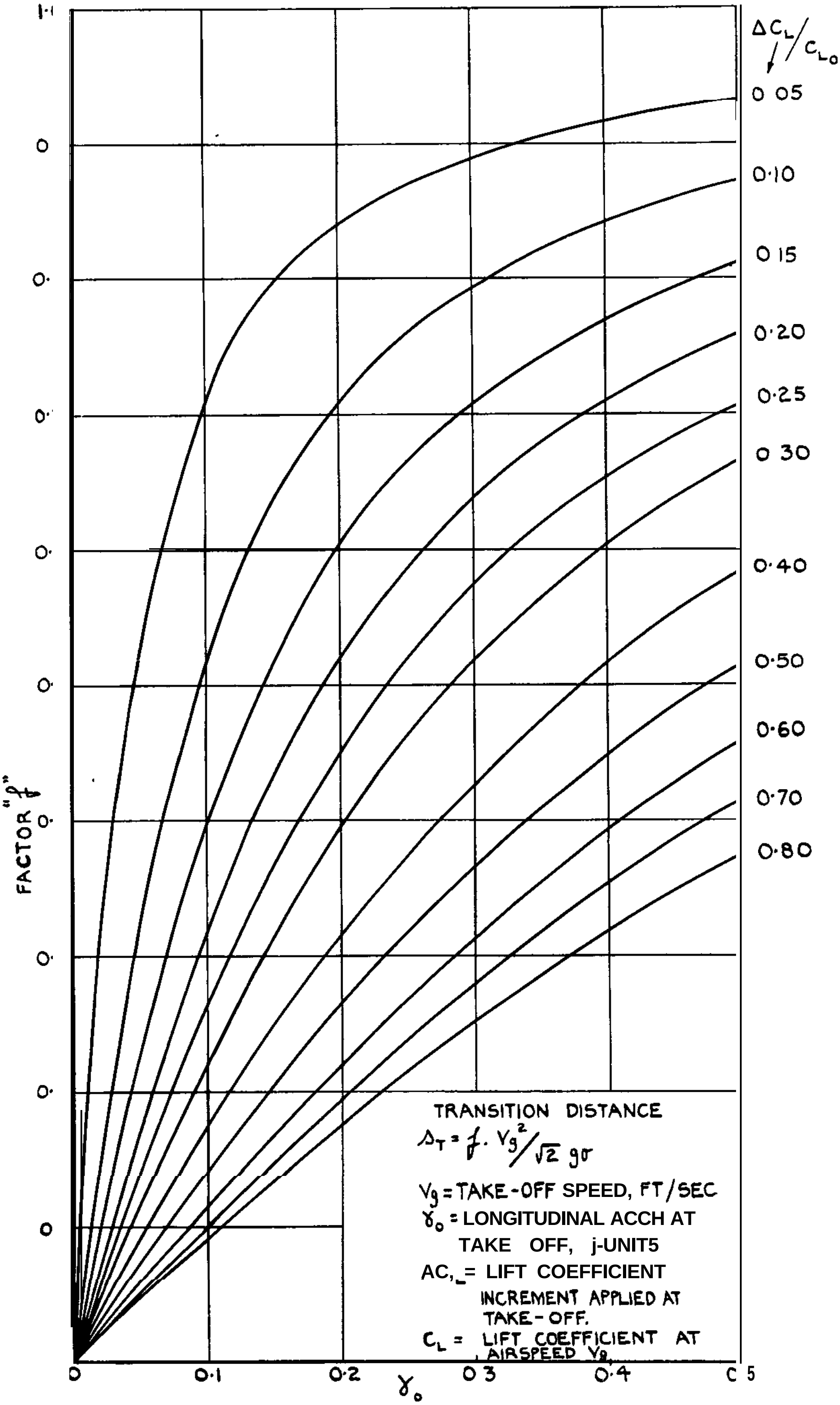


FIG.2. EFFECT OF NORMAL ACCELERATION ON TRANSITION DISTANCE.

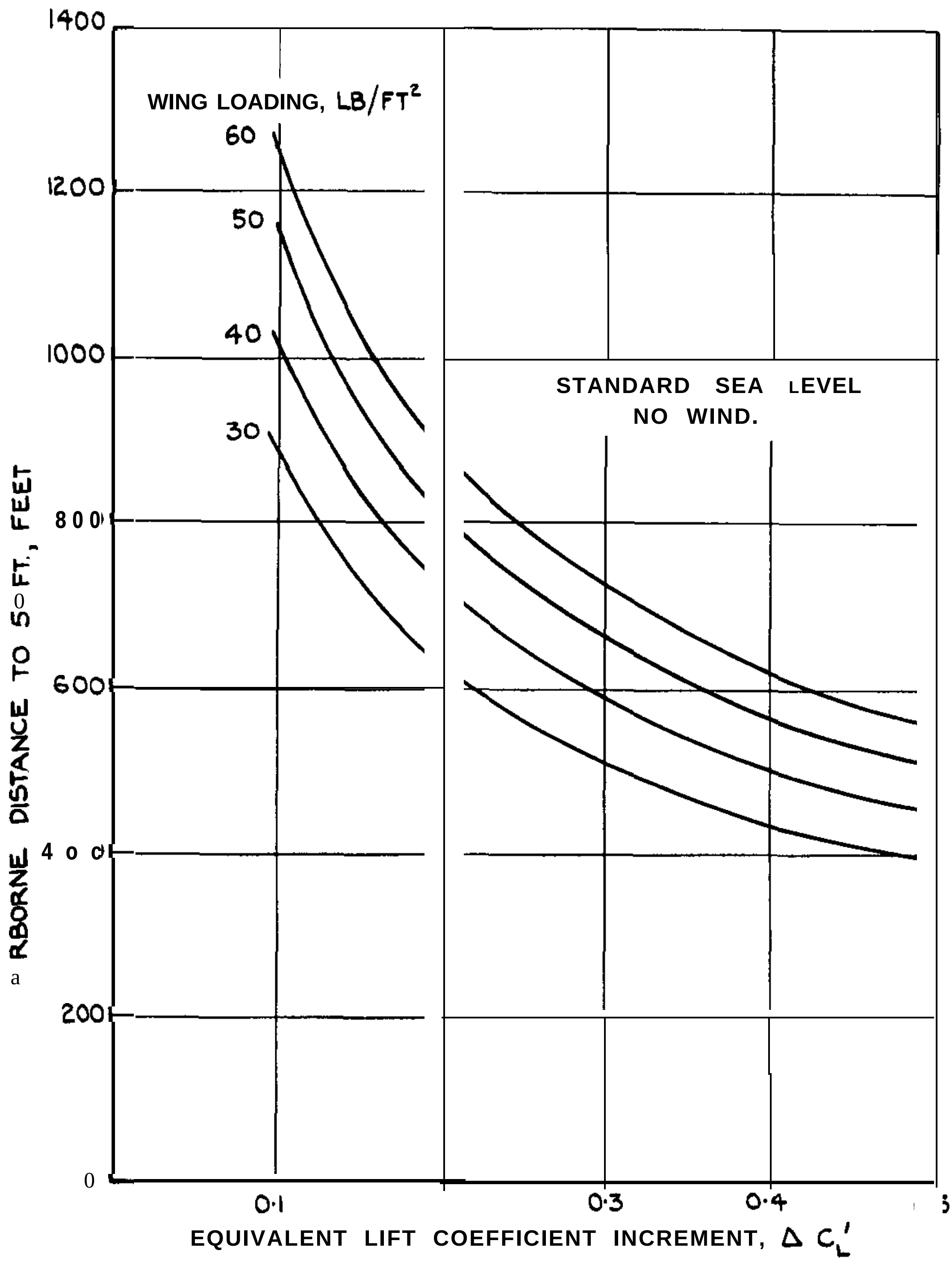


FIG.3. AIRBORNE DISTANCES TO 50 FT., FOR CIRCULAR FLIGHT PATH.

FIG.4.

NOTATION

S_A = DISTANCE FROM TAKE-OFF POINT TO 50 FT, FEET

$\gamma_0 g$ = ACCELERATION AT TAKE-OFF, FT/SEC^2

$V_g/\sqrt{\sigma}$ = TAKE-OFF SPEED, FT./SEC. (T.A.S)

x 14,600 RPM, MEAN γ_0 = 0·281

⊙ 13,000 RPM, MEAN γ_0 = 0·190

t 13,000 RPM, MEAN γ_0 = 0·113

MEAN TAKE-OFF WEIGHT = 13,860 LB

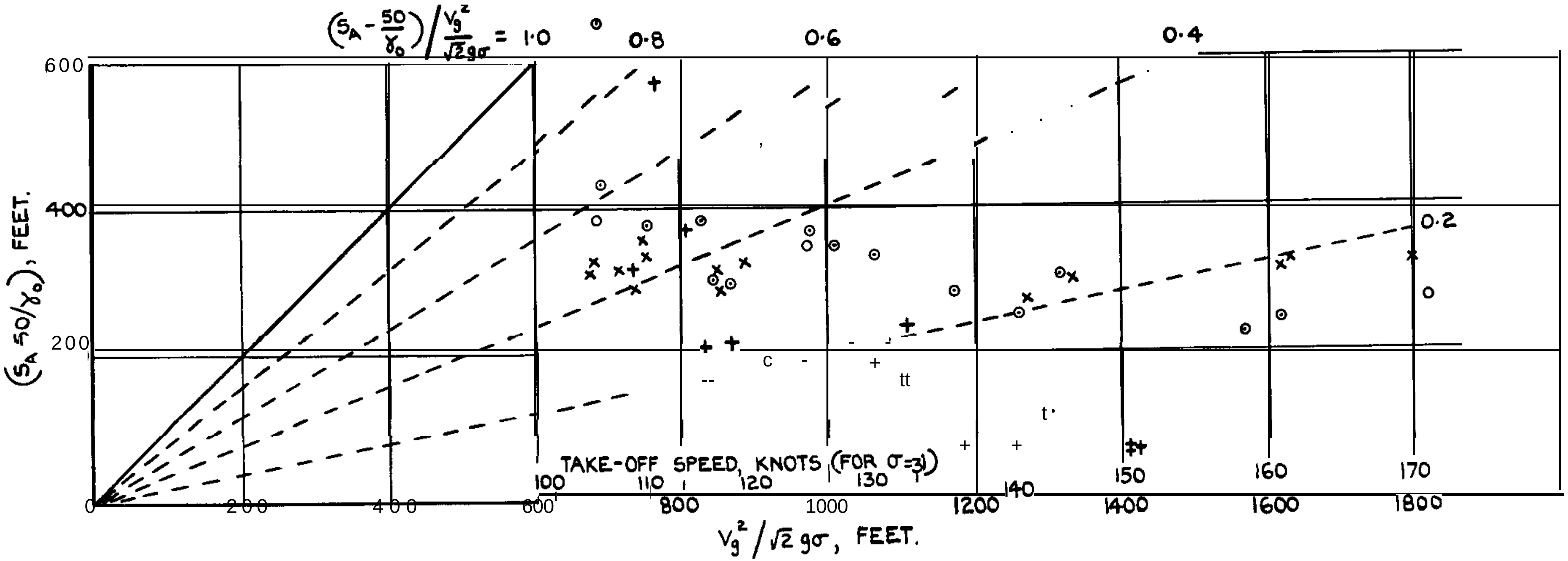


FIG.4. EFFECT OF NORMAL AND. LONGITUDINAL ACCELERATION ON AIRBORNE DISTANCES TO SOFT.

FIG.5.

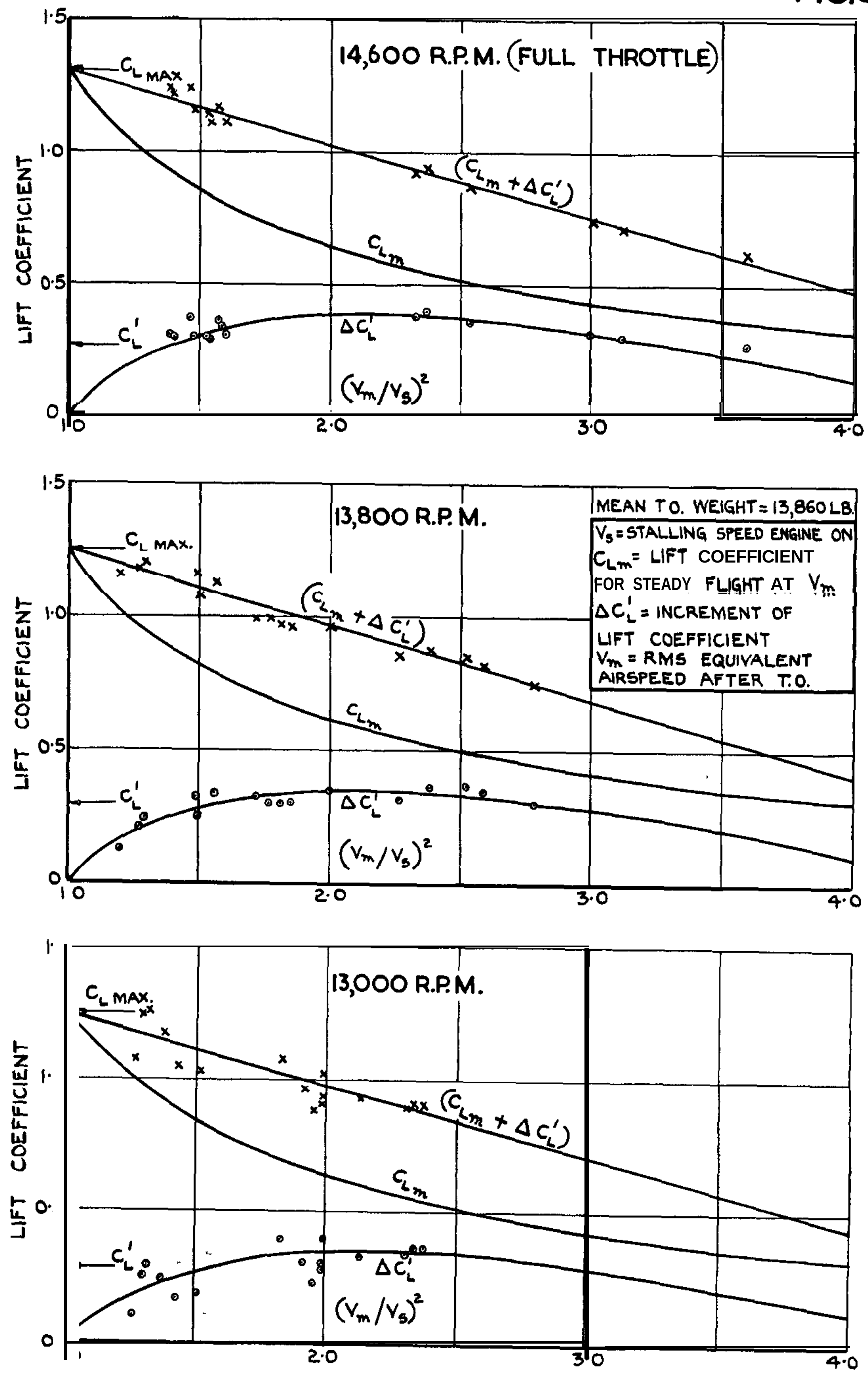


FIG. 5. LIFT COEFFICIENTS USED DURING TAKE OFF — METEOR IV

FIG.6 & 7.

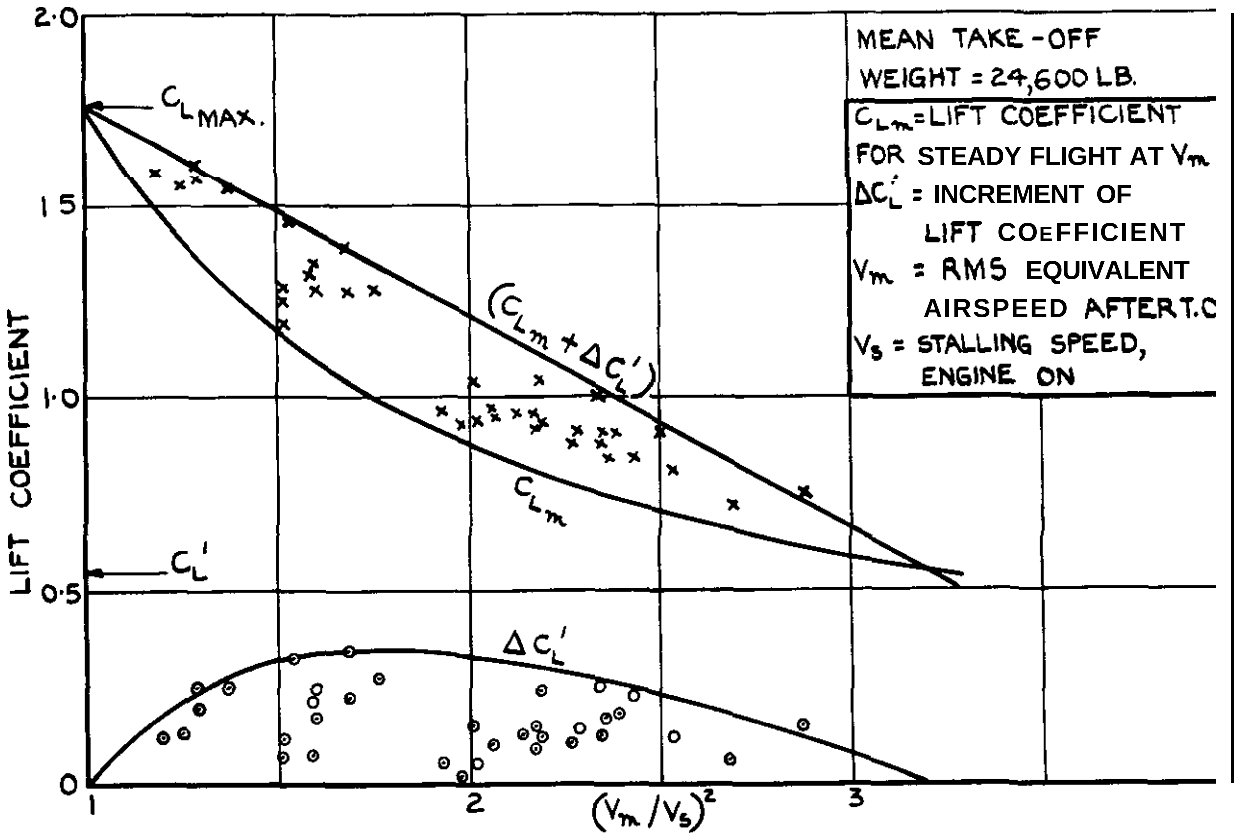


FIG.6. LIFT COEFFICIENTS USED DURING TAKE- OFF — DAKOTA.

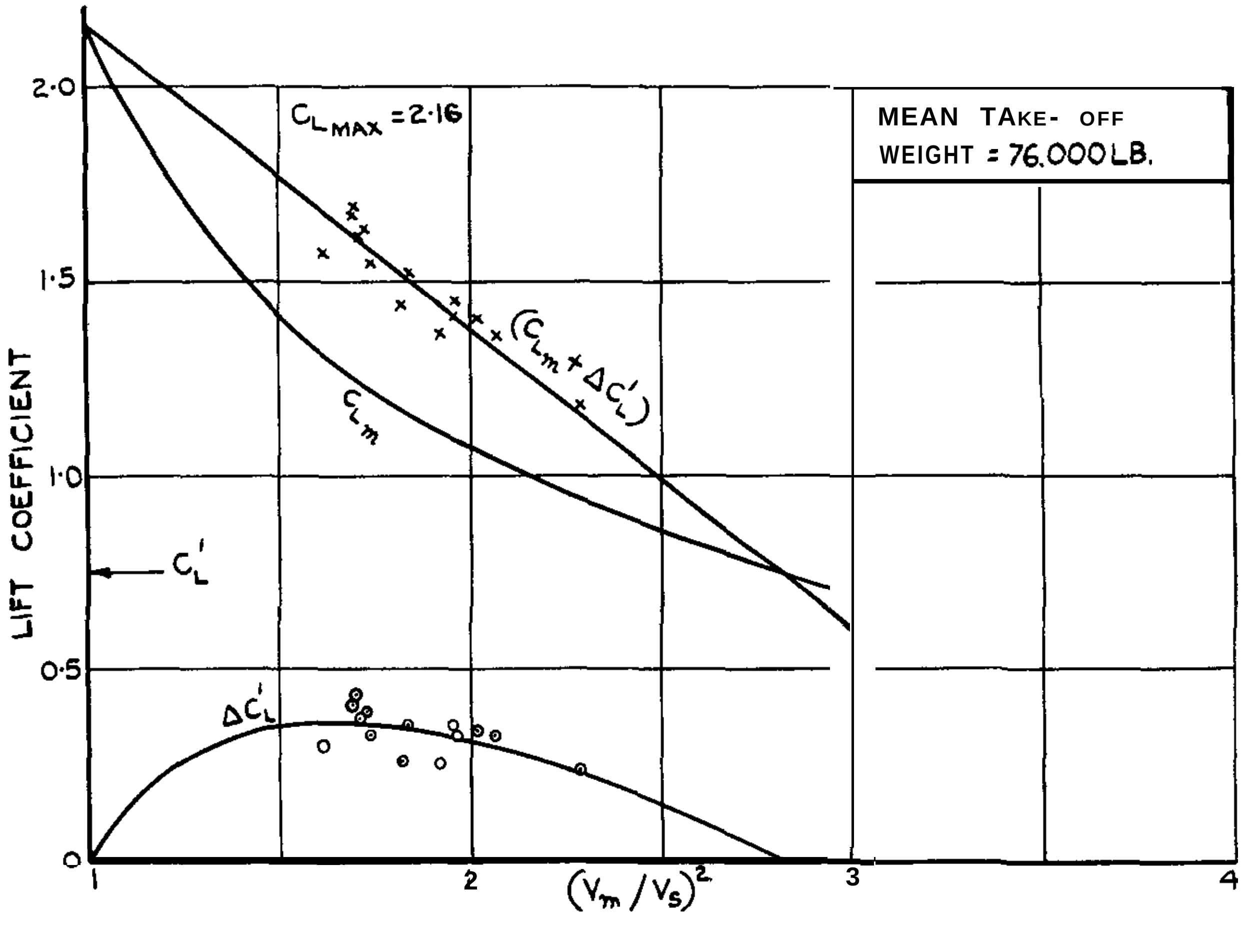


FIG.7. LIFT COEFFICIENTS USED DURING TAKE - OFF — HERMES.

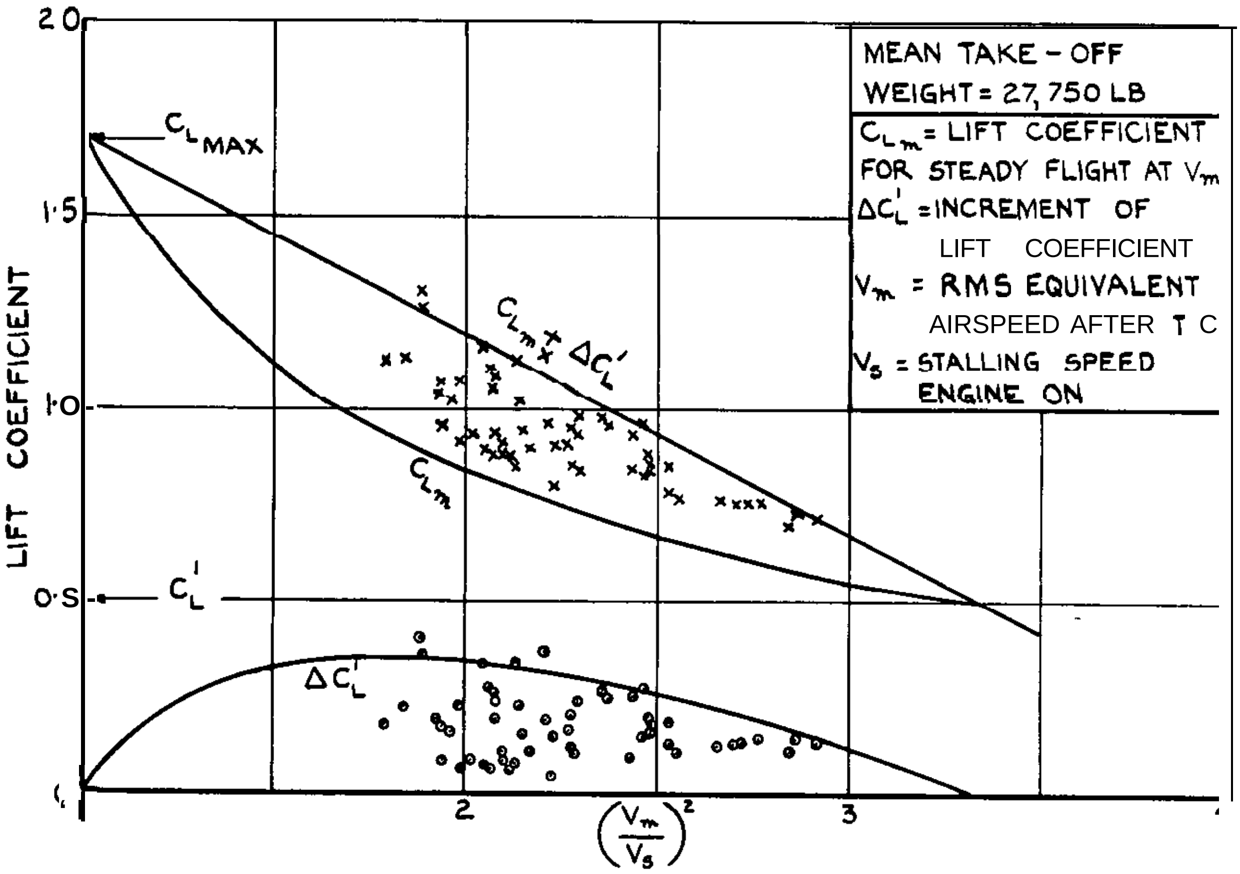


FIG. 8. LIFT COEFFICIENT-S USED DURING TAKE - OFF — NENE - VIKING.

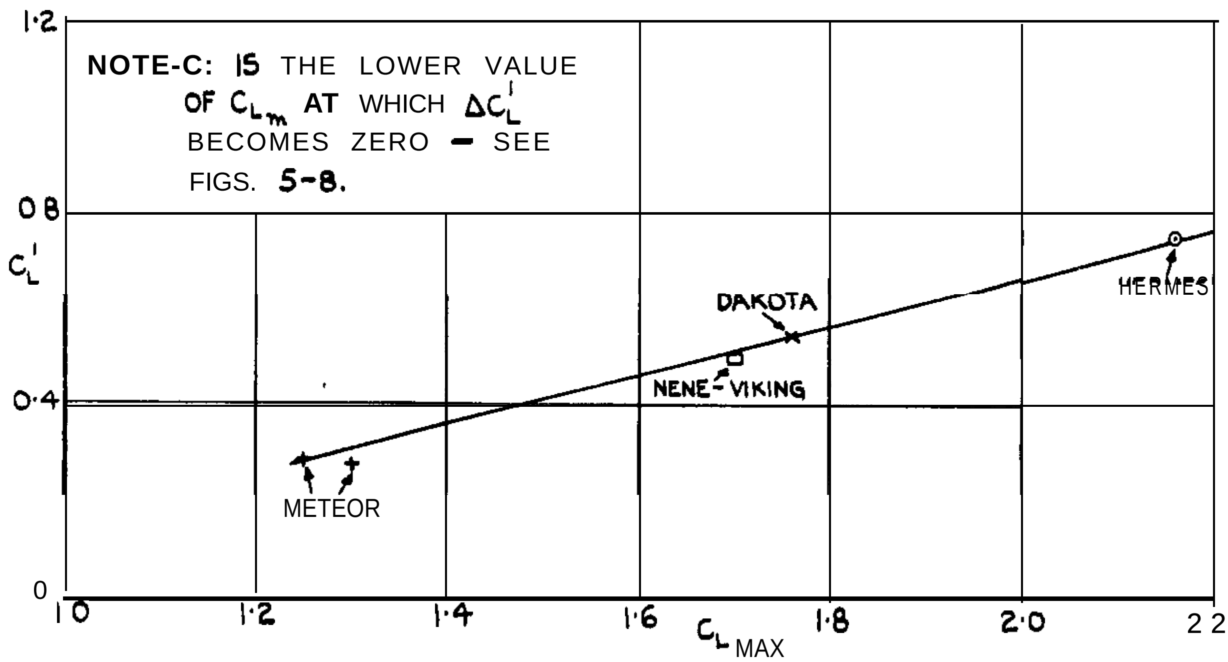


FIG.9. ESTIMATION OF C_L' FROM $C_{L_{MAX}}$.

FIG. 10.

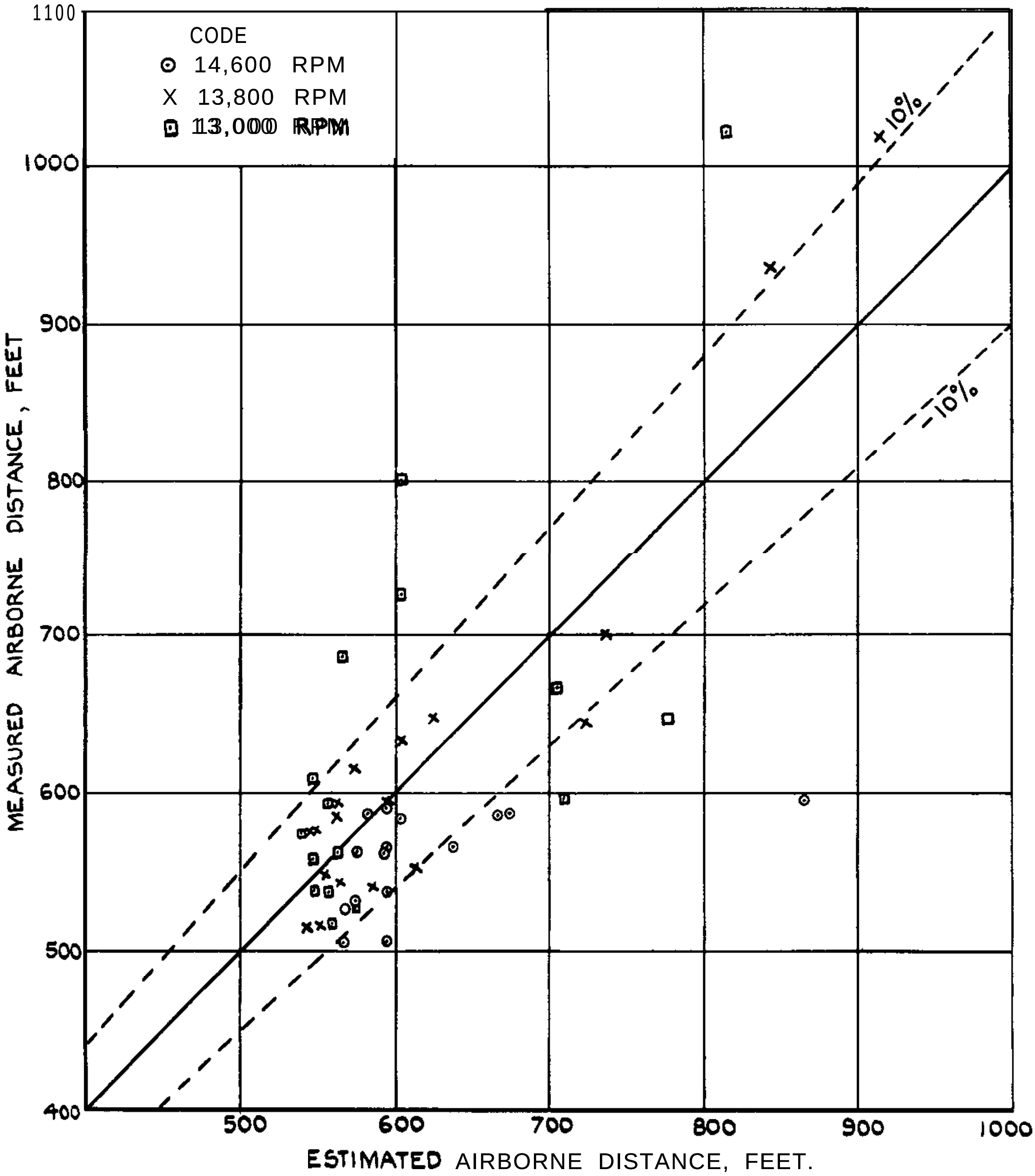


FIG. 10. APPLICATION OF PREDICTION PROCESS TO METEOR IV.

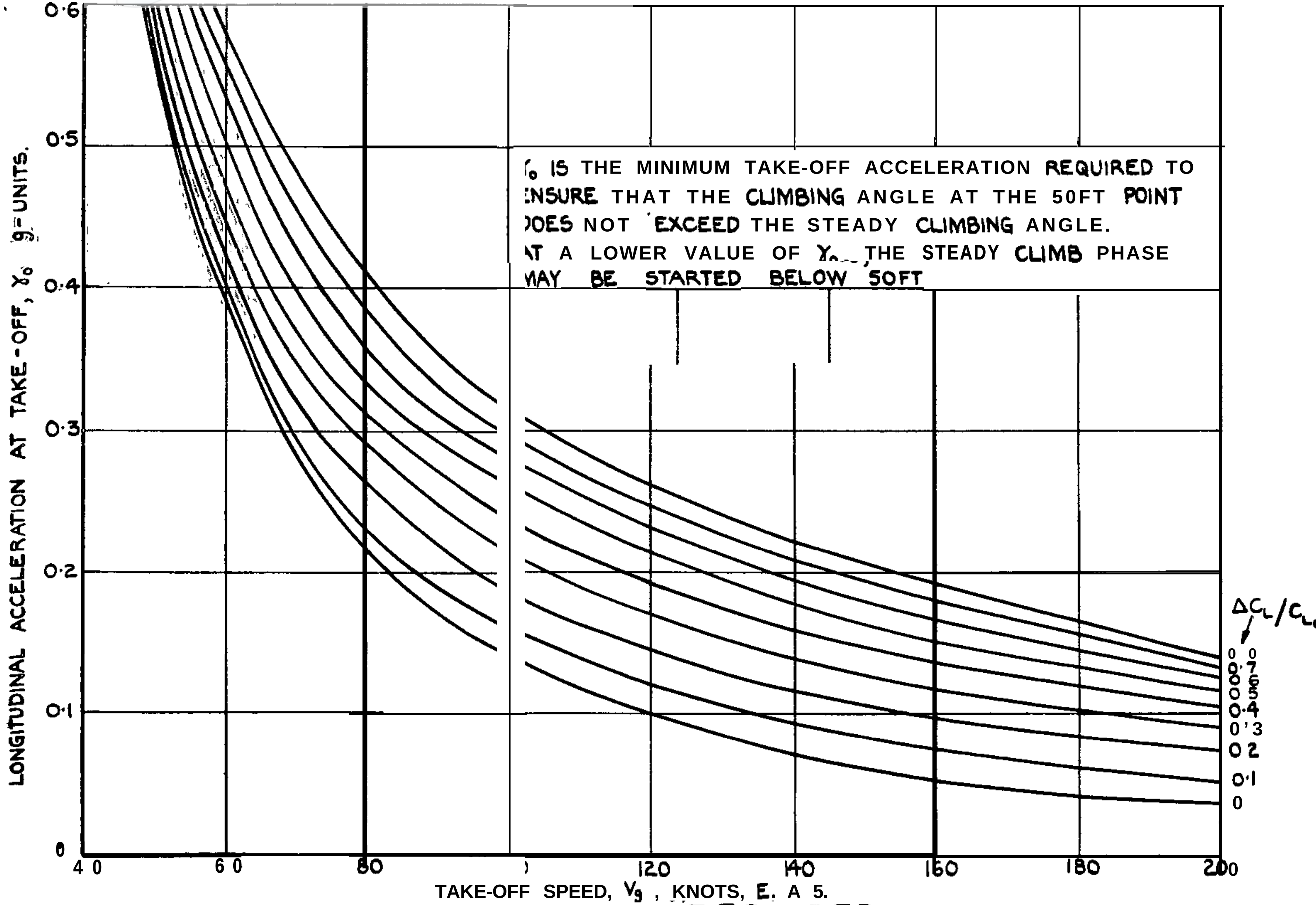


FIG.11. LONGITUDINAL ACCELERATION REQUIRED FOR THE STEADY CLIMB ANGLE TO BE REACHED AT 50FT.

FIG.11.

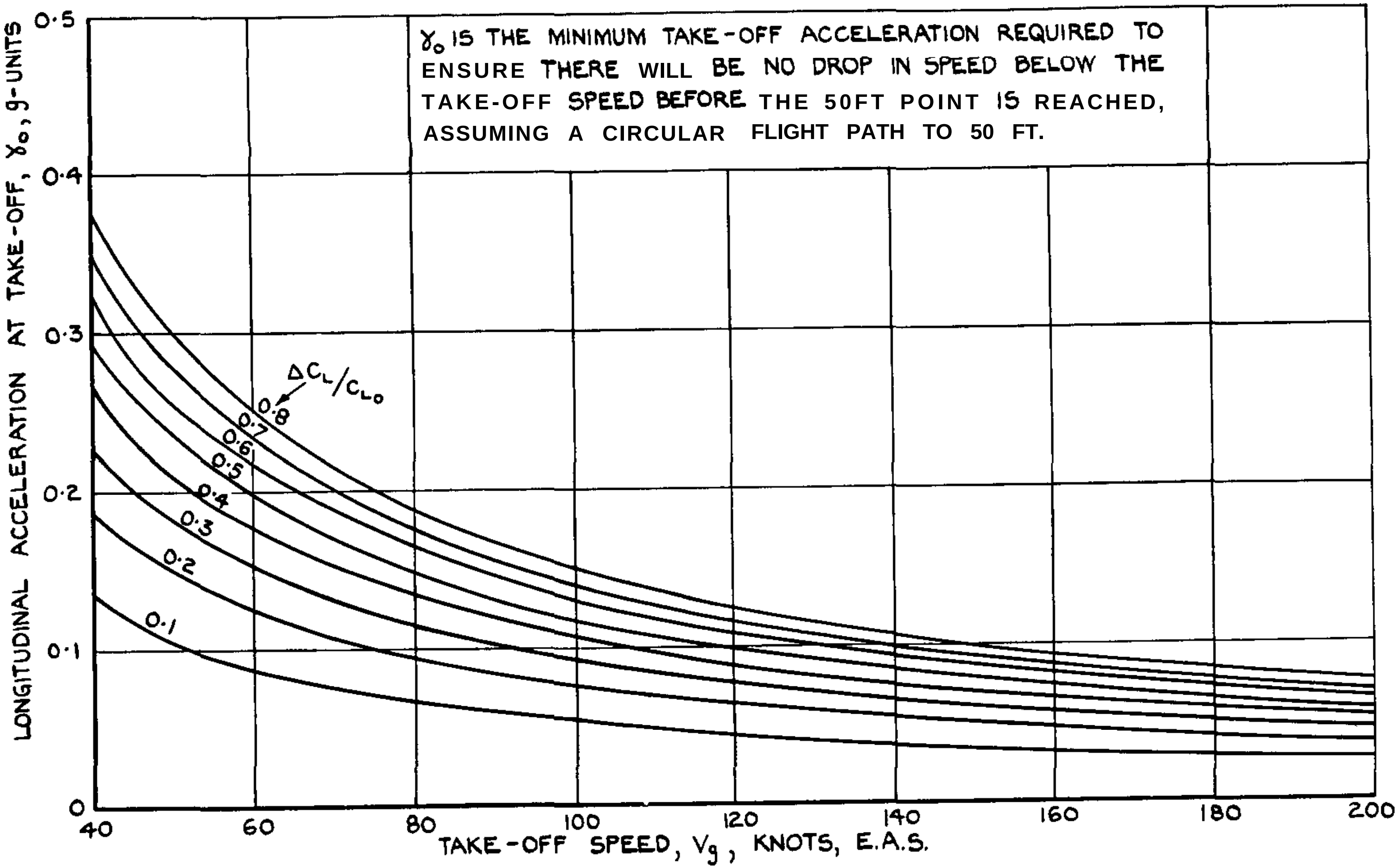


FIG.12. LONGITUDINAL ACCELERATION REQUIRED FOR NO DROP IN AIRSPEED BEFORE THE 50 FT. POINT.

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