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Normal Shock-Wave Tables for Air, Argon,
Carbon Dioxide, Carbon Monoxide, Hydrogen,
Nitrogen, Nitrous Oxide and Oxygen

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1. Introduction

Equilibrium conditions behind the incident and reflected shock wave are evaluated using the method of Ref. 1. In this method the enthalpy of the gas is regarded as the sum of two parts: (i) the enthalpy evaluated for a gas consisting of rigid molecules having no internal excitation and (ii) a correction term to be added to (i), to take account of internal excitation. The specific enthalpy, h , is given by the following expression:

$$h = \left(1 + \frac{n}{2} \right) RT + \phi(T) \quad \dots (1)$$

where/

where $\left(1 + \frac{n}{2}\right) RT$ is the specific enthalpy for rigid molecules of n degrees of freedom ($n = 3, 5$ or 6 according as the gas particles are monatomic, linear molecules or non-linear molecules respectively). R is the gas constant per gramme. The term $\phi(T)$ represents the contribution to the enthalpy due to internal excitation of the molecules. There is also a pressure-dependent part to the enthalpy but, under many shock-tube conditions, this is very small and is neglected here. The method of calculation is briefly outlined below.

2. Calculation of Conditions behind the Incident Shock Wave

From the equations of state and continuity and the equations of conservation of momentum and energy the following two equations may be derived¹:

$$T_2 = \left(\frac{i}{m} - u_2 \right) \frac{u_2}{R} \quad \dots (2)$$

$$u_2 = \frac{\left(1 + \frac{n}{2}\right) \frac{i}{m} - \left[\left\{ \left(1 + \frac{n}{2}\right) \left(\frac{i}{m}\right) \right\}^{1/2} - 2(n+1) \{h_0 - \phi(T_2)\} \right]^{1/2}}{n+1} \quad \dots (3)$$

where i , m and h_0 are constants of the flow given by

$$i = p_1 + \rho_1 u_1^2$$

$$m = \rho_1 u_1$$

$$h_0 = h_1 + \frac{1}{2} u_1^2$$

p_1 , ρ_1 and h_1 are the initial pressure, density and specific enthalpy and u_1 is the speed of the incident shock wave. T_2 is the temperature behind the incident shock and u_2 is the speed of the gas downstream of the shock in a frame of reference in which the shock is stationary.

Equations (2) and (3) are simultaneous equations for u_2 and T_2 . As a first approximation, we put $\phi(T_2) = 0$ in (3) (i.e. we assume no internal excitation of the molecules) and the first approximation to u_2 is calculated. This is then used in (2) to derive T_2 . With a knowledge of T_2 , $\phi(T_2)$ can be determined from enthalpy tables. This new $\phi(T_2)$ is substituted in (3) and the cycle repeated. It is found that successive solutions oscillate on either side of the true solution and convergence is greatly assisted if each new solution is taken as the arithmetic mean of the previous two solutions.

With the solutions for u_2 and T_2 the remaining conditions behind the incident shock wave are readily calculated.

3. Calculation of Conditions behind the Reflected Shock Wave

The equations of state, continuity, momentum and energy lead to the following two equations¹:

$$T_3 = \left(\frac{I}{M} + u_3 \right) \frac{U u_3}{R(u_3 + U)} \quad \dots (4)$$

$$u_3 = - \frac{1}{n} \left\{ \left(1 + \frac{n}{2} \right) \frac{I}{M} - U - \frac{H_0 - \phi(T_3)}{U} \right\}$$

$$+ \frac{1}{n} \left[\left\{ \left(1 + \frac{n}{2} \right) \frac{I}{M} - U - \frac{H_0 - \phi(T_3)}{U} \right\}^2 + 2n \left\{ H_0 - \phi(T_3) \right\} \right]^{1/2} \quad \dots (5)$$

where

$$U = u_1 - u_2$$

$$I = p_2 + \rho_2 U^2$$

$$M = \rho_2 U$$

$$H_0 = h_2 + \frac{1}{2} U^2$$

all of which are known since conditions 2 are obtained from the solutions of the equations for the incident shock wave. u_3 is the speed of the reflected shock wave and T_3 is the temperature behind the reflected shock.

The calculation proceeds as for the incident shock wave: a first approximation to u_3 is found by putting $\phi(T_3) = 0$ and this is used as the starting point for the iteration.

The calculations were carried out for equal intervals of 0.1 in incident shock Mach number and the range of shock speed was adjusted for each gas so that the complete range of tabulated enthalpy was covered.

It was found convenient for the computer calculation to represent the enthalpy of each gas by a sum of Chebyshev polynomials as outlined in Section 4 below.

The results of these calculations are given in Tables 5.1-5.8.

4. Representation of the Enthalpy of Chebyshev Polynomials

The tables of Hilsenrath et al² were used to provide enthalpy data for air, argon, carbon dioxide, carbon monoxide, hydrogen, nitrogen and oxygen. The tables of McBride et al⁴ provided enthalpy data for nitrous oxide. All the data used are for ideal gases in which no account is taken of the dependence of enthalpy on pressure and the effects of dissociation and ionisation are neglected.

The enthalpy data for each gas are fitted by a polynomial, which is a "best fit" in the least squares sense, of the form:

$$h = \frac{1}{2} C_0 + C_1 x_1(x_r) + C_2 x_2(x_r) + \dots + C_k x_k(x_r) + \dots + C_j x_j(x_r) \quad \dots (6)$$

where j is the degree of the polynomial and C_k is the k th Chebyshev coefficient of the Chebyshev polynomial $x_k(x_r)$, h is the enthalpy and x_r is given by

$$\left. \begin{aligned} x_r &= \lambda \theta_r + \mu \\ \lambda &= \frac{2}{\theta_m - \theta_1} \\ \mu &= -\frac{\theta_m + \theta_1}{\theta_m - \theta_1} \end{aligned} \right\} \quad \dots (7)$$

where θ_r is the temperature corresponding to the r th entry in the enthalpy table and $r = 1, 2, \dots, m$. θ_r is in ascending order of magnitude.

A computer programme, based on the method of Davis and Robertson⁴ was used to evaluate the Chebyshev coefficients for the various sets of enthalpy data. The number of terms in (6) was chosen to be sufficient to fit the tabulated enthalpy data to better than 3 significant figures over the whole range of temperature. The Chebyshev coefficients for the various gases are given in Table 6.

5. Shock Tables

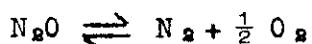
Nomenclature

M	=	Mach number of incident shock wave based on the speed of sound ahead of the shock.
U1	=	Speed of incident shock wave in cms per second.
P21	=	Pressure ratio across incident shock wave.
R21	=	Density ratio across incident shock wave.
T2	=	Temperature behind incident shock wave in degrees Kelvin. (In all cases, the temperature T_1 ahead of the shock is 293°K).
U2	=	Speed of gas behind incident shock in cms per second.
P31	=	Pressure ratio across the reflected shock wave.
R31	=	Density ratio across the reflected shock wave.
T3	=	Temperature behind the reflected shock wave in degrees Kelvin.
U3	=	Speed of reflected shock wave in cms per second.

The results for the various gases are given in Tables 5.1-5.8.

The tables do not take into account the effects of dissociation since they rely on enthalpy data in which dissociation was ignored. Therefore, the tables must be used with caution at high temperatures and low pressures. Fig. 1 shows boundaries for 6 of the 8 gases in the p,T plane along which there is 1 per cent increase in compressibility due to dissociation (and chemical reaction in the case of air). These boundaries were calculated using the "JANAF Thermochemical Data"5. Moving to the left and upwards from the boundary for any gas the tables can be used with increasing confidence; moving to the right and downwards from a boundary the tables become less accurate.

A boundary for argon does not appear in the p,T plane as nowhere within the bounds of the plane shown does it achieve 1 per cent ionisation. Nitrous oxide (N_2O) is omitted too since the reaction



is exothermic and a boundary does not exist in the p,T plane.

No account has been taken of relaxation time. In regions behind shock waves where there is full equilibrium of internal excitation but before there is appreciable dissociation or chemical reaction, the tables will be accurate.

Tables 5.1-5.8/

6. Table of Chebyshev Coefficients

The coefficients in Table 6 may be used to derive enthalpy over the specified temperature range by use of equation (6). The enthalpy generated is in the dimensionless form H/RT_0 where R is the gas constant and $T_0 = 273.16^{\circ}\text{K}$. The temperature, in degrees Kelvin, must be transformed according to equation (7).

References

<u>No.</u>	<u>Author(s)</u>	<u>Title, etc.</u>
1	K. C. Lapworth	A simple method for calculation of conditions behind shock waves. J.Phys.A (Gen. Phys.), 1969, Ser.2, Vol.2. Letters to the Editor, p.735.
2	J. Hilsenrath et al.	Tables of thermodynamic and transport properties of air, argon, carbon dioxide, carbon monoxide, hydrogen, nitrogen, oxygen and steam. Pergamon Press, 1960.
3	B. J. McBride et al.	Thermodynamic properties to 6000°K for 210 substances involving the first 18 elements. NASA SP-3001, 1963.
4	J. D. Davis and H. H. Robertson	Programme-550, Curve Fitting. Mercury Auto-Code Library Specification. Computer Section, Central Instrument Laboratory, I.C.I. Limited, Wilton.
5		JANAF Thermochemical Data. The Dow Chemical Company, Midland, Michigan.

Table 6

Gas	Temperature Range	Chebyshev Coefficients
Air	250 - 3000°K	$C_0 = 4 \cdot 68677807, +1$ $C_1 = 2 \cdot 10226383, +1$ $C_2 = 6 \cdot 47638388, -1$ $C_3 = -1 \cdot 35679492, -1$ $C_4 = 1 \cdot 70648402, -2$ $C_5 = 7 \cdot 83222069, -3$ $C_6 = -7 \cdot 67614091, -3$ $C_7 = 3 \cdot 58417033, -3$ $C_8 = -9 \cdot 89829902, -4$ $C_9 = 5 \cdot 99110415, -6$ $C_{10} = 2 \cdot 08067305, -4$
Argon	250 - 5000°K	$C_0 = 4 \cdot 80487646, +1$ $C_1 = 2 \cdot 17363437, +1$
Carbon Dioxide	250 - 5000°K	$C_0 = 1 \cdot 25746776, +2$ $C_1 = 6 \cdot 27763909, +1$ $C_2 = 1 \cdot 96793967, +0$ $C_3 = -7 \cdot 62005141, -1$ $C_4 = 3 \cdot 31053452, -1$ $C_5 = -1 \cdot 31419391, -1$ $C_6 = 4 \cdot 77501858, -2$ $C_7 = -1 \cdot 56437941, -2$ $C_8 = 4 \cdot 61895414, -3$ $C_9 = -1 \cdot 39852246, -3$ $C_{10} = 6 \cdot 40561648, -4$ $C_{11} = -4 \cdot 21105504, -4$ $C_{12} = 1 \cdot 91901735, -4$
Carbon Monoxide	250 - 5000°K	$C_0 = 7 \cdot 93788901, +1$ $C_1 = 3 \cdot 78265814, +1$ $C_2 = 9 \cdot 14801170, -1$ $C_3 = -3 \cdot 18792491, -1$ $C_4 = 1 \cdot 07762824, -1$ $C_5 = -2 \cdot 25687609, -2$ $C_6 = -4 \cdot 74870239, -3$ $C_7 = 9 \cdot 10501648, -3$ $C_8 = -6 \cdot 98797773, -3$ $C_9 = 3 \cdot 67347931, -3$ $C_{10} = -1 \cdot 79443795, -3$

Table 6 (contd.)/

Table 6 (Contd.)

Gas	Temperature Range	Chebyshev Coefficients
Hydrogen	250 - 5000°K	$c_0 = 7.69316603, +1$ $c_1 = 3.71667578, +1$ $c_2 = 1.71605946, +0$ $c_3 = -1.70349186, -1$ $c_4 = -1.90907122, -2$ $c_5 = 3.63587547, -2$ $c_6 = -1.70183463, -2$ $c_7 = 2.50764223, -3$ $c_8 = 3.74125707, -3$ $c_9 = -3.70886691, -3$ $c_{10} = 1.60824431, -3$ $c_{11} = -9.08727266, -4$ $c_{12} = 6.05646460, -4$
Nitrogen	250 - 5000°K	$c_0 = 7.88164660, +1$ $c_1 = 3.75698741, +1$ $c_2 = 9.58209877, -1$ $c_3 = -3.23315411, -1$ $c_4 = 9.79042425, -2$ $c_5 = -1.36442033, -2$ $c_6 = -9.61825963, -3$ $c_7 = 1.11124251, -2$ $c_8 = -7.14712647, -3$ $c_9 = 3.37438633, -3$ $c_{10} = -1.17595304, -3$
Nitrous Oxide	100 - 6000°K	$c_0 = 1.53631304, +2$ $c_1 = 8.06172075, +1$ $c_2 = 3.09618315, +0$ $c_3 = -1.01421975, +0$ $c_4 = 5.47298192, -1$ $c_5 = -2.52710661, -1$ $c_6 = 1.08582218, -1$ $c_7 = -4.27628604, -2$ $c_8 = 1.52831142, -2$ $c_9 = -4.98668045, -3$ $c_{10} = 1.54193724, -3$ $c_{11} = -5.01327698, -4$ $c_{12} = 1.37830560, -4$

Table 6 (Contd.)/

Table 6 (Contd.)

Gas	Temperature Range	Chebyshev Coefficients
Oxygen	250 - 5000°K	$C_0 = 8.37078463, +1$ $C_1 = 4.04393305, +1$ $C_2 = 1.42092080, +0$ $C_3 = -2.41145725, -1$ $C_4 = 6.80734190, -2$ $C_5 = -4.47649273, -2$ $C_6 = 2.01324076, -2$ $C_7 = -3.29978871, -3$ $C_8 = -3.12089396, -3$ $C_9 = 3.58014684, -3$ $C_{10} = -2.50900883, -3$ $C_{11} = 1.35522549, -3$ $C_{12} = -6.71371962, -4$

Acknowledgements

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AIR

T₁ 293.0

INCIDENT SHOCK

M	U1	P21	R21	T2	U2
6.30	2.164, +5	47.70	6.20	2255	1.815, +5
6.40	2.198, +5	49.26	6.24	2314	1.846, +5
6.50	2.233, +5	50.84	6.28	2373	1.877, +5
6.60	2.267, +5	52.44	6.32	2432	1.908, +5
6.70	2.301, +5	54.07	6.36	2493	1.939, +5
6.80	2.336, +5	55.73	6.39	2554	1.970, +5
6.90	2.370, +5	57.41	6.43	2617	2.001, +5
7.00	2.405, +5	59.11	6.46	2680	2.032, +5
7.10	2.439, +5	60.85	6.50	2743	2.064, +5
7.20	2.473, +5	62.60	6.53	2808	2.095, +5
7.30	2.508, +5	64.38	6.57	2873	2.126, +5
7.40	2.542, +5	66.19	6.60	2940	2.157, +5

ARGON

T₁ 293.0

INCIDENT SHOCK

M	U1	P21	R21	T2	U2
6.30	2.014, +5	49.63	3.72	3909	1.473, +5
6.40	2.046, +5	51.23	3.73	4026	1.497, +5
6.50	2.078, +5	52.85	3.74	4145	1.522, +5
6.60	2.110, +5	54.50	3.74	4265	1.546, +5
6.70	2.142, +5	56.17	3.75	4388	1.571, +5
6.80	2.174, +5	57.87	3.76	4512	1.595, +5
6.90	2.206, +5	59.59	3.76	4638	1.620, +5
7.00	2.238, +5	61.34	3.77	4766	1.644, +5
7.10	2.270, +5	63.11	3.78	4896	1.669, +5

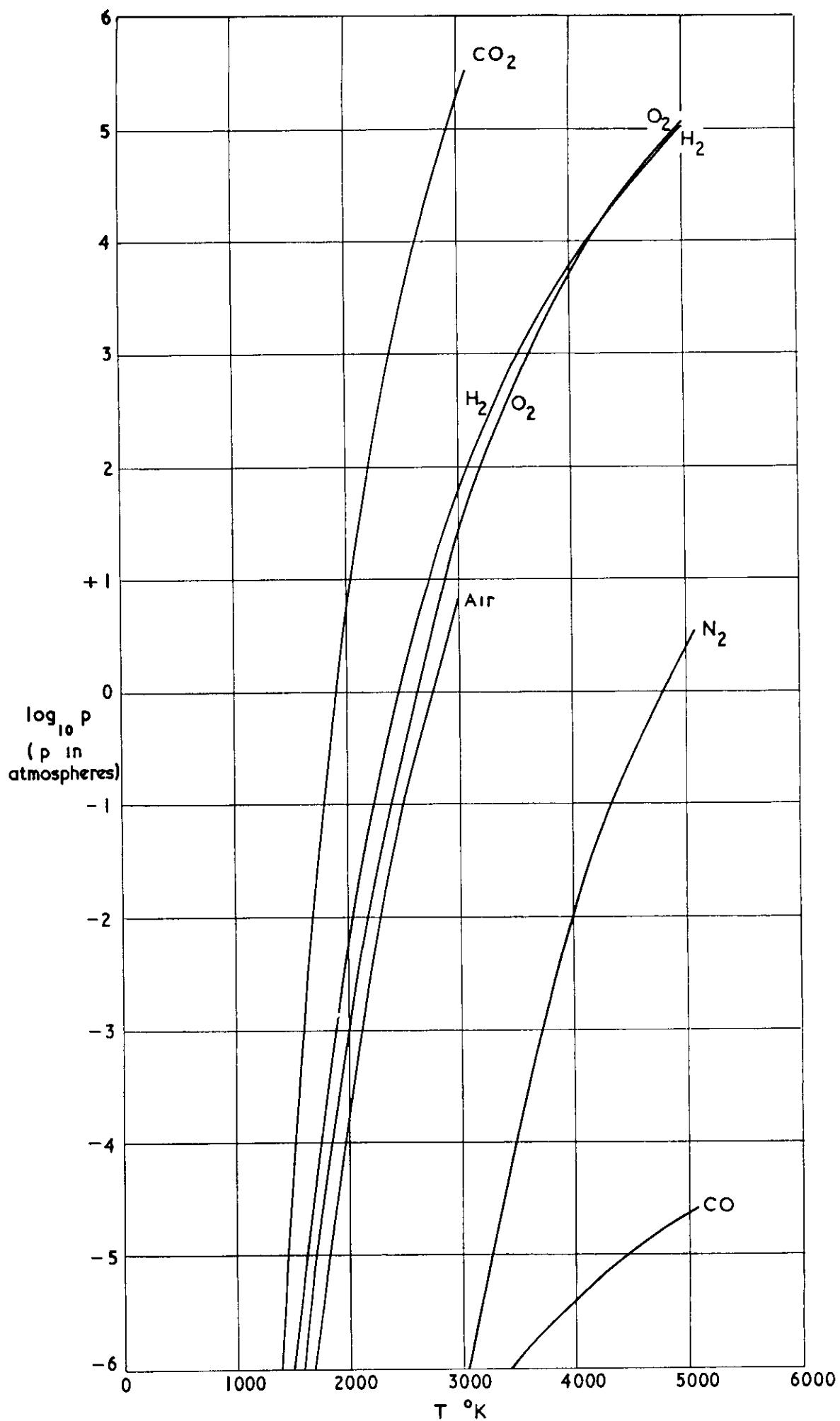
CARBON DIOXIDE

T_I 293.0

INCIDENT SHOCK

M	U1	P21	R21	T2	U2
11.50	3.078, +5	157.85	11.98	3861	2.821, +5
11.60	3.105, +5	160.63	12.01	3918	2.846, +5
11.70	3.131, +5	163.43	12.04	3976	2.871, +5
11.80	3.158, +5	166.26	12.08	4034	2.897, +5
11.90	3.185, +5	169.11	12.11	4093	2.922, +5
12.00	3.212, +5	171.99	12.14	4152	2.947, +5
12.10	3.238, +5	174.89	12.17	4212	2.972, +5
12.20	3.265, +5	177.81	12.20	4272	2.997, +5
12.30	3.292, +5	180.76	12.23	4332	3.023, +5
12.40	3.319, +5	183.73	12.25	4393	3.048, +5
12.50	3.345, +5	186.73	12.28	4455	3.073, +5
12.60	3.372, +5	189.75	12.31	4517	3.098, +5
12.70	3.399, +5	192.80	12.34	4579	3.123, +5
12.80	3.426, +5	195.87	12.36	4642	3.149, +5
12.90	3.452, +5	198.96	12.39	4705	3.174, +5
13.00	3.479, +5	202.08	12.42	4768	3.199, +5
13.10	3.506, +5	205.22	12.44	4833	3.224, +5
13.20	3.533, +5	208.39	12.47	4897	3.249, +5
13.30	3.560, +5	211.56	12.49	4962	3.275, +5

FIG I



Boundaries for 1 per cent increase in compressibility

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OXIDE AND OXYGEN

A method for calculating equilibrium conditions behind
incident and reflected shock waves in a shock tube is
briefly outlined. Tables are presented giving conditions
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In addition, the tables give the speed of the gas behind
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