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# Nonlinear Structural Dynamics Problems in Aeronautics

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# NONLINEAR STRUCTURAL DYNAMICS PROBLEMS IN AERONAUTICS

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## SUMMARY

This paper is concerned with the classification of nonlinear vibration problems which occur in the field of aeronautics - excluding those problems which may be more suitably defined under the heading 'nonlinear problems in control engineering'. It comprises a short catalogue of practical problems, methods of solution and relevant sources of reference.

## (1) INTRODUCTION

In seeking to classify the various problems, the following format has been adopted:-

(i) Problems involving known and unavoidable nonlinearities, in which it is necessary, ab initio, to account for nonlinear effects in calculations at the design stage. For example, response calculations for take-off, taxying and landing in which the oleo nonlinearities play a significant part, fall into this category.

(ii) Problems in which nonlinear effects are known to exist and to play an important part, but in which these effects (in the present state of knowledge) are not amenable to adequate mathematical description. Control surface buzz, for example, falls within this category.

(iii) New problems which, for example, might arise at the flight or resonance test stage - or perhaps even later during operation. These problems themselves fall into two categories:- (a) Problems of known origin such as backlash and Coulomb effects and (b) Problems of unknown origin which may involve extensive basic research in seeking to isolate the 'mechanism'. Parametric instabilities, if not obvious ones, would fall into this category.

With regard to the problems of which the writer has been notified, by far the largest number fall within category (iiia) and an interesting selection of these is included in the catalogue. The problems of category (i) are, by their definition, least interesting, and indeed are treated in a standard manner throughout the aerospace industry.

Prior to the beginning of his quest for practical non-linear problems in the field of aeronautics, the writerhad hoped to present a much more extensive and detailed catalogue of such problems and the methods of solution currently employed. However, it soon became apparent that (apart from problems in category (i)) the approach of industry to these problems was largely ad hoc. The attitude prevailed and indeed still does, that nonlinear problems should be solved by the most

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*Replaces A.R.C.30 821.

†In collaboration with C. H. E. Warren, C. G. Lodge and P. A. T. Christopher.

expedient means available - and this might involve tightening a nut, reducing a tolerance, increasing a stiffness, etc. Once solved in an ad hoc manner, the effects which led to the corrective action are lost to posterity. Thus, while many problems were referred to the writer, there was little in the way of concrete evidence to lend weight to a catalogue of the proportions envisaged originally.

The above paragraph does not constitute a criticism of industry's attitude towards problems involving nonlinear dynamics. On the contrary, the writer observes that industry expends effort proportional to the seriousness of the problem involved - and since the aircraft is designed according to linear laws, it is, substantially, a linear 'beast' and it is therefore hardly surprising that nonlinear effects are, by and large, treated as mere deviations from a linear ideal. In cases in which severe nonlinearities are inevitable, the problems enumerated under category (i) indicate the sound and thoroughgoing approach adopted in industry when dealing with such problems.

Possible dangers which may arise in certain instances as a result of failure to observe the possibility of nonlinear phenomena such as parametric instability are mentioned in the section of the catalogue dealing with problems in category (iiib). So far as the writer is aware, there have been no reports of serious parametric excitation or, indeed, of subharmonic behaviour in aircraft structural dynamics to date. However, such phenomena might have to be given careful attention in new and unconventional designs.

## (2) 'CATALOGUE' OF PROBLEMS

This section is subdivided in accordance with the categories mentioned in the Introduction and thus comprises three sub-sections:-

### 2(i) PROBLEMS INVOLVING KNOWN AND UNAVOIDABLE NONLINEARITIES

#### (a) Response on Take-Off, Landing and Taxying

Problems falling under this heading are dealt with as a matter of course in the aircraft industry. The procedures adopted are quite standard - the majority being digital. The principal nonlinearities arise from the oleos, and care is invariably exercised in obtaining the correct pneumatic and hydraulic characteristics for use in the response calculations. Oleo friction is more often than not approximated by a square law characteristic while tyre stiffness properties are obtained from data measured at the appropriate inflation pressure. Where appropriate, bogie characheristics are incorporated as additional linear degrees of freedon, though bogie pitching damping might be nonlinear and will thus be viewed as an additional nonlinear element. Selected rigid body and structural modes of the aircraft are incorporated in the calculations and linear aerodynamic derivatives for $M = 0$ are usually employed.

With regard to runway roughness, this is usually fed in as a random process whose power spectrum has been assessed by field measurements. In case involving bogies, care is taken to introduce the correct delay time in the random input between leading and lagging wheels on the bogies. Similar delays are also incorporated between inputs to nose and mainwheels or to main and tailwheels, as appropriate. In many cases, the response calculations under stochastic input conditions are supplemented by calculations for discrete ramp and steep inputs. It should be observed that the stochastic approach has, as yet, only been applied in certain <u>analogue</u> studies - there being no known method (based on the Fourier transformation procedure, or otherwise) which facilitates the calculation of the response to random input in cases where $H(i\omega)$ is nonlinear, $H(i\omega)$ being the frequency response function of the aircraft. In digital studies, typical runway profiles are used as input functions, and the calculation takes the form of a multi-variable Runge-Kutta process, the solution being in 'time series' form. Such occurrences as wheel hop are accounted for, both in the digital and analogue procedures.

Responses are calculated for a variety of touchdown and take-off speeds - due account being taken of aircraft weight and, where necessary, braking actions. In the helicopter touchdown problem, the approach is given a 'twist' in that the possibility of low frequency rolling motion of the landing area is also considered.

(b) Undercarriage Effects in Helicopter Impedance Testing

For the purpose of Coleman instability calculations, it is desired to measure the impedance at the rotor head in the plane of the rotor. Since Coleman instability can occur at any time during take-off and landing, the rotor head impedances are required at all lift conditions for which the undercarriage remains in contact with the ground. Lift is simulated by hoisting the aircraft via a cable at the rotor heard. (Rotor head impedance is a function of lift primarily because the undercarriage stiffness varies with deflection). The dependence of undercarriage stiffness and damping on deflexion and velocity gives rise to nonlinearities in the measured impedances. Another source of nonlinearity is wheel hop during high lift simulations.

(c) Nonlinear Damping on Hinged Rotors

Problems have arisen as a result of the nonlinear dampers used at the hinges of certain helicopter rotor blades. These dampers are usually of the orifice type and incorporate a relief valve. Their characheristic curves approximate to the friction characteristic - especially for the smaller orifice sizes: their main purpose is to damp lagging motions of the blades. These damper characteristics present problems in two areas, namely Coleman instability and rotor speed governing calculations. In the former, the calculations are done on a linear basis for various points on the damping characteristic.

(d) Backlash in Missile Nozzle Actuator Mechanism

A particularly troublesome nonlinearity in the field of missile dynamics is associated with the nozzle actuator mechanisms which invariably exhibit backlash, Coulomb friction and other structual nonlinearities. These nonlinearities must be accounted for in calculations involving autopilot, structural response and fuel sloshing mode interactions. The structural modes involved are the first few bending modes of the mossile - are these, of course, are sensibly linear. The sloshing modes also are linearisable in this application[1] and are represented by an equivalent mass-mass/spring model. (Some experimental justification for this idealisation is afforded in Reference 2). In multistage rockets, the procedure is complicated by the necessity of having to incorporate fuel modes for the various stages. It should be noted that damping in the fuel modes is accomplished with baffles, and that this damping is incorporated in the calculations on a linear (viscous) basis. Initial calculations, involving only a few structural and fuel modes, are performed on an analogue computer. Subsequent, more sophisticated, calculations are performed digitally using predictor-corrector or Runge-Kutta techniques to produce time series solutions. The equations of motion here are of the autonomous type[3,4].

A further complicating feature of the above problem arises when the I.G.C. digital computer generates a staircase-like characteristic curve. If the operating point sits on a point of discontinuity, a state of affairs obtains which which is closely related to 'ankylosis'[5]. This complication had been overcome by the expedient of generating a high frequency 'dither' signal which prevents the operating point from resting in any particular spot and yet is of such a high frequency that the overall system response is virtually unaffected.

(e) Fuel Sloshing in Flexible Containers

In the fuel sloshing representations mentioned in (d) above, the container was considered flexible in bending, but was assumed to retain its cross sectional shape. Under these conditions, the assumption of linearity in small motions of the fuel surface is good[1, 2], as it is also when the bottom of the container is flexible[6]. However, when the sectional shape may vary - say in one of the circumferential shell modes - grave nonlinearities of the softening type may arise, even at small emplitudes. Complex hydroelastic modes and jump phenomena have been observed in such cases[7]. These phenomena have also been predicted theoretically[8] from linearised shell equations and potential theory (for the fluid) using Galerkin's method followed by a second order perturbation solution of the resulting set of nonlinear ordinary differential equations.

Such problems have been experienced in the field of missile dynamics, but have been eliminated by the introduction of circumferential stiffening rings.

2(ii) PROBLEMS INVOLVING NONLINEARITIES NOT AMENABLE TO MATHEMATICAL DESCRIPTION

(a) Control Surface Buzz

This class of phenomena is of extreme importance in modern aeronautics and instances in flight are often reported. The various buzz mechanisms have been carefully studied by Lambourne[9, 10, 11] and others, but the question of adequate mathematical description of these mechanisms, which involve shock wave movements and time delays in the propagation of pressure waves between the control surface and the shock wave position, has not been satisfactorily answered. Indeed, aside from purely academic interest, the need for such a description is by no means a priority since, for conventional geometries, experimental knowledge of the buzz phenomena is sufficiently comprehesive to enable predictions and suggestions for avoidance to be made. (Cf. closing paragraph of Reference 12).

In Reference 9, three types of buzz are described in relation to a two-dimensional aerofoil and flap. These involve (i), subsonic flow over the flap with shock waves on the main surface causing separation there, (ii), mixed flow over the flap with shock waves between the flap hinge and trailing edge and (iii), supersonic flow over the whole flap with shock waves at the flap trailing edge. For thick aerofoils, these flow types lead to discrete Mach number ranges for buzz instability - at least for small incidences. For thinner aerofoils, the region of instability associated with flow type (i) shrinks and eventually disappears. While oscillations involving flow types (i) and (ii) involve shock induced separations and shock wave movements as essential phenomena, those occurring in flow type (iii), do not and such oscillations may be associated with the well-known potential flow negative damping[13,14] in the 1.0 M 1.4 range. In regions (i) and (iii) oscillations are self starting at the appropriate critical Mach numbers, whereas in region (ii), the 'hard' oscillator characteristic is encountered[12].

Suggestions for the avoidance of buzz are outlined in Reference 9. These include such measures as increasing flap natural frequency, use of vortex generators, and spoilers on the surface of the wing or flap.

It should be observed that attempts to formulate the buzz problem on a rigourous mathematical basis would involve a detailed knowledge of transonic aerodynamics including information as to how shock waves respond to cyclic pressure fluctuations downstream and as to the delay times between the inception of a pressure pulse and its arrival at the shock wave position. With such features defined, the equation of motion of the flap would, at best, be a linear autonomous difference differential equation (d.d.e.) and at worst, a nonlinear nonautonomous d.d.e.. (If fact, nonautonomous effects are inevitable in 'type (i)' buzz due to buffeting from the shock separated boundary layer. These nonautonomous effects are fairly clearly distinguished from buzz instability in view of random nature of the response in the former case.).

(b) Stall Flutter

This is another problem in which the complex nature of the aerodynamic terms resists rigourous mathematical description. Aerodynamic hysteresis near the stall is responsible for the phenomenon which is well catalogued in standard works of reference[15]. Stall flutter is still very much a problem in turbine work, and local stall flutter behaviour is extensive in the field of Industrial Aerodynamics[16,17,18]. Here again, a rigourous mathematical treatment would involve nonlinear autonomous d.d.e.'s.

(c) Wheel Shimmy

It might be thought inappropriate to include the shimmy phenomenon under the heading of the present section. The reason for so doing centres on the unknown nature of some of the tyre forces - these being essentially nonlinear. Linear treatments are common,(see Reference 19, for example) being based on assumed or experimentally derived tyre characteristics. An interesting mathematical model of a tyre is given in Reference 21 and this is used in a nonlinear shimmy calculation for a motor car wheel. This calculation also takes account of bearing backlash and king-pin Coulomb friction and is performed on a third order system using the method of equivalent linearisation. The results are presented in the form of shimmy limit cycles. Factors stimulating shimmy include (i) pneumatic trail length, which is a function of tyre construction, (ii) contact path length, inflation pressure and wear (which are clearly related to pneumatic trail), (iii) the ratio of mass to lateral stiffness, (iv), the ratio of camber torsional stiffness to camber inertia and (v) wheel bearing play.

An extensive study of the shimmy phenomenon has been undertaken by the British Aircraft Corporation, but unfortunately the final report was not available at the time when the present paper was prepared.

Some useful additional references are given in Reference 20.

(d) Helicopter Problems

An interesting nonlinear problem has been foreseen in the performance calculations for non-articulated flexible rotors which are being undertaken currently at Westland Aircraft Ltd. The modes and frequencies of the blades are known to be functions of the deflected shape and cyclic pitch, both of which vary during the course of each revolution of the rotor. This inevitably leads to differential equations of motion with periodic coefficients, but in obtaining these via the usual normal modes approach, account must be taken of the time dependent nature of the actual mode shapes. Bearing in mind the complexity of the aerodynamics for such rotors, this problem is formidable.

Solutions of linear differential equation with periodic coefficients (Mathieu type in many degrees of freedom) are obtainable on application of Floquet's theorem[4], have set down a relationship defining a 'fundamental matrix'[22]. A solution of a typical helicopter rotor problem is described in Reference 23. When nonlinear equations with periodic coefficients arise, the above method is no longer applicable, and one might be advised to use a multi-freedom version of Minorsky's stroboscopic method[4] or a similar adaption of the method of Krylov and Bogoliubov[4].

(e) Buffeting Problems

In these problems, the major difficulty again resides in obtaining an adequate description of the causative aerodynamics. Assuming the aerodynamics known, the only structural dynamics problem is that of obtaining the response - even thought this might involve complicated procedures. It is deemed, therefore, that problems of this type fall without the original terms of reference of this paper. However, if there is any significant structural feedback, we have 'autobuffeting' which certainly falls within the terms of reference. Unforntunately, the writer has not been notified of any problems in this field, but examples may be found in Reference 15.

2(iiiA) UNFORESEEN PROBLEMS ARISING AT THE TEST STAGE OR IN OPERATION

(a) Backlash in Helicopter Rotor Bearing

A problem arose during a resonance test on a certain helicopter in which it was apparent that some unforeseen nonlinearity was playing a significant part. Examination of typical response curves indicated the participation of a hardening spring effect, and this led to difficulty in the interpretation of test results. Examination of the aircraft under test revealed a significant amount of backlash in the main rotor bearing. The problem was overcome (in so far as obtaining a configuration in which responses were substantially linear and therefore comparable with theoretical results) by tilting the aircraft so as to preload the backlash. The backlash was thought to be of no serious consequence in flight.

(b) Radar Bullet Vibration

Radar bullet vibration occurred during flight tests at approximately 1.9M on a certain fighter aircraft. The frequency of vibration was in the range of the predominant intake buzz frequencies and the mode was that of the bullet on its supports (the first structural mode). An increase of bullet support stiffness was effected - the design value being such as to take the resonance in the first structural mode well above the predominant forcing frequency range. However, due to backlash in the support joints, the actual (effective) stiffness fell below the design value and thus little improvement resulted. When the backlash was eliminated, bullet vibration was reduced to an acceptable level.

(c) Backlash in All-Moving Control Surfaces.

Three instances of backlash effects on resonance test results for all moving tailplanes and rudders were referred to the writer. Impedance curves in each case exhibited typical hardening spring characteristics, while the response modes of the surface (fundamental structural only) remained sensibly unaltered over the nonlinear resonance region. This indicated backlash at the jacks. In flight, the tailplane backlash will usually be preloaded and under these conditions would only play a significant part in the aeroelastics if a disturbance of sufficient magnitude led to a tailplane deflexion into the dead zone. On the other hand, the rudder backlash would not be preloaded to any significant extent in level flight and may thus play an important role in the aeroelastics. With regard to possible calculations in this case, a describing function procedure would appear to be appropriate since, physically, the nonlinearity is confined to a single degree of freedom. (See References 3 and 24). However, to return to the point, the major difficulty arising from these relatively small nonlinearities is that of interpretation of resonance test results - a problem which is usually overcome by artifical preloading.

(d) Transmissibility through Backlash

An interesting problem arose on a particular passenger aircraft (with all-moving tailplane) involving an unpleasant lateral vibration at the pilot position.* The only structural mode having a frequency in this range was a tailplane mode, and it was thus hypothesized that due to an excessive amount of backlash in the tailplane on this particular aircraft, the transmissibility of vibration from the tailplane to the pilot position might have increased. (This occurrence initiated an experimental study of the possible effects of backlash on vibration transmissibility at the University of Bristol, and evidence to date suggests that backlash does not increase transmissibility at the primary frequency. However, with 'free' backlash, some extremely violent subharmonic responses were obtained - the subharmonic order depending upon force amplitude, excitation frequency, dead zone extent and the restitution coefficient between the contact surfaces within the backlash element[25]).

(e) Nonlinear Effects in Modern Resonance Testing Techniques

The seriousness of the presence of backlash and other nonlinearities in a structure under resonance test is highlighted by the experience of Hawkins & Mousley[26] in their GRAMPA test of a Beagle aircraft. In one of the higher modes, it was found to be not possible to lock onto phase resonance because of the nonlinear effect of a small loose component within one of the wings. Similar difficulties would have arisen in the application of any of the modern techniques based on the assumption of structural linearity.

(f) Undercarriage Judder

The phenomenon of undercarriage judder has been the subject of many investigations in industry - these usually involving the analogue simulation of the braking torque characteristic and including the possibility of tyre slip. Several problems were referred to the writer - notably one involving brake judder on a certain fighter aircraft. An analogue investigation had been carried out (following a simple analytical appraisal of the stability problem) in which the braking torque characteristics provided by the brake manufacturer had been simulated using a bilinear approximation. It was thought unnecessary to approximate more closely to the measured characteristics, since these were known to vary considerably from tun to tun. The undercarriage was represented by a cantilever bearing the wheel, and the analysis was performed in three degrees of freedom (viz, fore and aft translation of the aircraft as a whole). The analogue results for this simple model exhibited the typical judder oscillations (bursts of instability at low speed (below 15 kts for the brakes in question) involving appreciable undercarriage leg amplitudes). While these results were convincing - especially in view of the fact that the system is classically unstable at very low rubbing velocities where the slope of the braking torque characteristic is negative - the possibility of full scale judder at low speeds due to 'high spots' was not ruled out.

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*Other aircraft in this range did not exhibit this phenomenon.

(g) Control Surface Jack Nonlinearities

The prediction of flutter speeds in cases where the control jack has stiffness and damping properties which vary with amplitude and frequency is probably one of the most important problems encompassed by the report. The nonlinearities are most acute when a near stalling load is applied to the control surface. While the writer has not been able to obtain concrete details of a particular problem under this heading, it would seem that the describing function method is used fairly extensively in such applications; the necessary basic data being obtained from impedance tests.

(h) Oscillations of Towed Loads

Violent coupled longitudinal and lateral instabilities have been experienced on freight loads carried on pallets suspended beneath helicopters. Linear stability calculations have been performed by Sheldon[27], who has suggested a strop design which confers asymptotic stability at practical forward speeds. Large amplitude stability of suspended pallets is being studied surrently. The major nonlinearities in this case are of aerodynamic origin.

(j) Panel Flutter.

Panel flutter problems have been the subject of many researches over the past two decades and there is now an extensive literature on nonlinear panel flutter problems. For example, limit cycle calculations (based on Galerkin's method, employing up to six modes, and using a time series solution) for flat panels with initial in-plane stresses and static pressure differential, have been performed by Dowell[28]. Buckled plates are also dealt with. (See also References 29 and 30). Reference 31 contains an interesting application of optimization techniques to the nonlinear panel flutter problem. In all of these references, linear aerodynamics is employed - the nonlinear effects being purely structural.

Nonlinear flutter of shells is the subject of many current researches; References 32 and 33 being typical of these.

(k) Vortex excitation of Rockets at Launch

This is a troublesome current problem involving transverse vibrations of a rocket at the launching bay. The excitation is from shed vorticity in natural winds in a certain critical speed range. The 'capture phenomenon' is evident and the problem is thus nonlinear in this respect. Furthermore, conventional 'industrial' solutions are inadmissible - though the idea of 'straking' has been entertained.

(l) Structural Nonlinearities in Aeroelasticity - General

Finally, in this section, we mention some standard works of reference relating to the effects of structural nonlinearities on flutter. For nonlinearity in one degree of freedom only of a multi-freedom system, references 34, 35 and 24 are appropriate, these being based on the describing function idea. Reference 24, in particular, contains comparisons of experimental, analogue and theoretical (describing function) results for a wing/aileron system with backlash and Coulomb friction at the hinge. The describing function is shewn to produce reasonable accurate results even when the nonlinear effects are strong. Reference 35 contains an outline of the types of structural nonlinearity likely to be encountered in aeronautical problems, along with an interesting matrix formulation of the Ritz averaging method as applied to autonomous and non-autonomous systems.

2(iiiB) NEW PROBLEMS

(a) Parametric Excitation

Probably the best known case of a parametric oscillator[4] is the string and tuning fork in Rayleigh's experiment. When the fork is caused to oscillate at frequency 2f, transverse oscillations at frequency f build up in the string. The motion of the string, for small amplitudes, is governed by a linear Mathieu equation - the solutions of which may be represented on the usual stability diagram. The principal solution is represented by a band of instability centred on frequency f.

In engineering applications, there is a wide variety of instances in which parametric excitation may arise due, for example, to periodic variations of inertia or stiffness. In cases where the parametric excitation effects are large, serious consequences may result if these effects are not investigated. A particular example to which the writer's attention was drawn involved the parametric excitation of a large store on a model wing. The wing was excited on its fundamental bending mode at frequency 2f, say. The store responded in a large amplitude lateral mode at frequency, f. This example was somewhat artificial in that the store lateral frequency (wing rigid) had been designed at one half of the wing bending frequency. However, the artificiality is partly justified by the fact that higher order parametric excitation is also possible (i.e. with factors of 3, 4, 5, etc. between the store and wing frequencies: excitation under conditions where the sums of combinations of the wing natural frequencies correspond with integral multiples of the store frequency is also theoretically possible). However, it is easy to show, by the method of equivalent linearization (Krylov and Bogoliubov[36]), that if 'e' is a measure of the smallness of the parametric actions (i.e. of the coefficient of the periodic term in Mathieu's equation), then the first approximation solution (order 'e') contains only those terms involving response at half the parametric frequency. The 'higher order' parametric instabilities are thus of order $e^2$, at least, and would, in practice, be subdued by damping.

It should be notes that the periodically varying forces which arise in rotor dynamics (leading to d.e.'s with periodic coefficients) give rise to parametric type instabilities[22].

(b) Aerodynamic Nonlinearities

These can lead to unforeseen buffeting response problems: for example vortex burst is an obvious case in which unknown forces may be brought to bear on the structure.

An outstanding problem is that involving aerodynamics at interfaces between main and control surfaces. Industry still lacks basic information in this area, and flutter calculations based on theoretically derived control surface derivatives are subject to errors in consequence.

(3) ADDITIONAL NONLINEAR PROBLEMS

In this short section, we deal with problems and areas of study which are not covered by the classification of the previous section. Most of the non-aeronautical problems mentioned involve the use of methods which may find application in the aeronautical sphere.

(a) Inertia Nonlinearities in (Rigid) Flight Dynamics

In deriving the well-known stability equations for a rigid aircraft, small perturbation theory is used and this leads to the separability of the longitudinal and lateral modes. In certain manoeuvres involving high rates of roll, the nonlinear (Euler) inertia terms are no longer ignorable and these lead to coupled lateral and longitudinal modes, with the possibility of instabilities such as divergence in yaw and pitch. Such possibilities have been investigated linearly assuming constant roll rate and constant forward speed and stability boundaries have been plotted[37]. There have also been many investigations in which the full set of nonlinear equations have been solved by digital and analogue means. The digital solutions are via predictor-corrector/Runge-Kutta procedures.

(b) Vibration of a Butterfly Valve

During tests, at Bristol University, on a butterfly valve in a square duct (side 2"), severe vibrations in pitch (accompanied by small translatory motions of the spondle) occurred at $M = 0.6$ about an equilibrium incidence slightly above the stall. These vibrations were obviated when the small amount of backlash in the spindle support bearings was taken out.

(c) Nonlinear Structural Damping

A general formulation of the problem of the steady state response of continuous dissipative systems with small nonlinearities is given in Reference 38. A general method of solution is suggested based on a perturbation procedure. Only first order perturbation solutions are given since the nonlinearities are assumed very small. (For the particularisation to a single degree of freedon, the results are identical to those of Krylov and Bogoliubov). The method is applied to two problems: (i) vibrations of a slender bar in torsion bending and extensional modes under harmonic extraneous loading of a general type and (ii), torsional vibrations of a bar with riveted joints, pinned at its ends and acted on by a simple harmonic couple at its mid-point.

(d) Overhead Transmission Line Instability

Problems related to the galloping phenomenon have received much attention in the literature in the years following the war. These problems involve aerodynamical nonlinearities, but the motions are usually so small cpared with the span of a typical line that the structure (i.e. the catenary) may be assumed linear. To date four types of galloping instability have been recognised: (i) conventional type associated with negative lift-incidence slope[16] and giving rise to vertical motions limited by aerodynamical nonlinearities, (ii), Reynolds number transition type, involving differential separations on upper and lower cable surfaces due to strand effects and not requiring ice deposits, and giving rise to combined lateral and vertical motions in a certain critical windspeed range[39], (iii), flutter type - usually on tandem twin cable arrangements[40], and (iv) sub-conductor type experienced on twin and quad arrangements due to wake effects.

Limit cycle calculations for conventional galloping are usually performed graphically[16], but for coupled types, a multi-freedom (matrix) version of the Krylov-Bogoliubov method has been used[39].

(e) Other Mechanical Engineering Problems

Several problems involving the nonlinear vibrations of crankshafts and governors have been brought to the writer's attention. In certain of the crankshaft problems (where the nonlinearities are of the variable inertia type and the systems are essentially nonautonomous) the solutions are characterised by the fact that the most dominant response mode is a subharmonic of order 2. Such problems have been studied using a multi-freedom variant of the Krylov and Bogoliubov technique[41, 42] which may well find application in the aeronatical field.

## (4) CONCLUSIONS AND SUGGESTIONS

A list of typical nonlinear problems and related areas of research has been drawn up and a long list of works of reference has been provided. For reasons given in the Introduction, the list is neither as extensive or as detailed as the writer had originally envisaged.

The general 'tenor' of the catalogue (section 2) will provide a fair indication of the status of problems involving nonlinearities in the field of aircraft structural dynamics:- The problems dealt with in 2(i) are seen to be of a type which must be 'lived with' and whose effects must therefore be closely studied. Likewise, those in 2(ii) must also be 'lived with', but their detailed study is made difficult by the mathematical intractability of their mechanisms. Here, save for possible advances in the theory, we have to resort to carefully conducted experiments on particular cases and thus build up experience. The problems of 2(iiiA) are of the 'run of the mill' type which require expedient solution if progress is not to be severely impaired. Such is the required expediency that problems, once solved, are often not recorded. (This is unfortunate, since an accumulation of information on trivial and seemingly unrelated problems could become a useful source of reference.). Of the 'new' problems (2(iiiB), we should certainly not lose sight of the possibility of parametric instability - particularly in new and unconventional designs - and especially those involving large inertias mounted on outriggers attached perpendicularly to a flexible surface.

The following should be noted:-

(1) The vast amount of textbook information (and other published literature) on single degree of freedom nonlinear systems and their associated differential equations does not find industrial application in the field of aircraft structural dynamics.

(2) Problems are almost invariably complicated ones involving motions in many degrees of freedom. With modern digital computing facilities, the most popular current approach (as reflected in the catalogue) is to solve by numerical integration and to produce a time series solution. Thus, only in isolated cases do the methods of equivalent linearisation find application (as, for example in 2(iiiA)g). This is unfortunate, since the numerical approach is devoid of 'feel' for the problem in question. This situation is redeemed partly by the fact that analogue studies often accompany the digital sums giving some knowledge of parameter sensitivity, etc.

(3) Subharmonic behaviour did not figure at all in the problems referred to the writer by representatives of the aircraft industry.

With regard to areas where effort is required, the following seem to be of importance:-

(i) The situation appears to be far from satisfactory in the matter of the prediction of flutter speeds in cases involving control jack nonlinearities. Some additional effort is called for here, directed, perhaps, towards a better description of the nonlinearities and to the means by which they are included in the flutter calculations. With regard to the latter, the describing function method might not be adequate in view of the strength of the nonlinearities under certain adverse conditions, and some 'higher order' technique might need to be developed. This situation is aggravated by the fact that

(ii) The state of the art in the control surface derivative field is such that much, in the way of accuracy, is left to be desired.

(iii) The extension of the well-known Fourier transform methods to cases involving nonlinear structural transfer functions. (Some progress has been made in this field, but at the present stage, the nonlinearities appear to be chosen to fit the method).

## (5) ACKNOWLEDGEMENTS

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