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on

**Extreme Value Analysis and its
Application to C. G. Vertical Accelerations
Measured on Transport Airplanes of Type C-130**

by

Otto Buxbaum

NORTH ATLANTIC TREATY ORGANIZATION



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NORTH ATLANTIC TREATY ORGANIZATION
ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT
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EXTREME VALUE ANALYSIS AND ITS APPLICATION
TO C.G. VERTICAL ACCELERATIONS MEASURED ON
TRANSPORT AIRPLANES OF TYPE C-130

(7)

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(8)

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Notes for "airplanes" "airplane"
with "aeroplanes" "aeroplanes" !

SUMMARY

// The interpretation of cumulative frequency distributions of measured flight loads can be increased significantly by an additional extreme value analysis. This method not only leads to a higher reliability in fatigue design but may be used also for a prediction of extreme loading conditions and for a description of the effect of airplane and flight parameters on loads, as is demonstrated for e.g. vertical accelerations and gust velocities measured on airplanes of type C-130 the

address

Hercules transport

FOREWORD

This report is a sequel to a paper presented by Dr.O.Buxbaum on 5th November 1970 at the 31st meeting of the Structures and Materials Panel in Tønsberg, Norway.

In this paper, Dr.Bauxbaum, whose contribution to the study of statistics related to atmospheric turbulence has been particularly fruitful, attacks a problem of considerable importance to both civilian and military aeronautical industry: the prediction of the extreme loads which a structure must be able to withstand.

The application of the model proposed by Dr.Buxbaum to measurements obtained on the C-130 aircraft provides an excellent confirmation of the method.

G.COUPRY
Chairman,
AGARD Structures and Materials Panel
Working Group on Environmental
Statistical Data

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EXTREME VALUE ANALYSIS AND ITS APPLICATION TO C.G. VERTICAL ACCELERATIONS MEASURED ON TRANSPORT AIRPLANES OF TYPE C-130

Otto Buxbaum

1. INTRODUCTION

The static design of an aircraft structure is based on loads, which have been settled either by convention or by experience and which have to be proved with a certain safety factor against ultimate load of the structure.

This design procedure, however, cannot guarantee that no cracks or failures caused by fatigue will occur within the required life time.

Even if these static design loads would in fact occur during service operation, we first have to know how often they will occur, before we can use them for the fatigue design. This has not only to be required for the design loads but also for all smaller loads which occur more frequently.

Information about magnitude and frequency of loads can be obtained only by measurements under service conditions. If it has been decided to count certain characteristic occurrences, e.g. level crossings or peaks (Refs.1,2), the choice of which may be supported by theoretical considerations or to some extent by experience about material behaviour, then modern measurement and data processing techniques allow one to provide the so-called cumulative frequency distribution (CFD). It is derived from the continuous history of the load P , see Figure 1.

For such an analysis we have to distinguish between the so-called basic load P_b , which is constant in the example shown and which corresponds to the load in the undisturbed condition at the load factor $n = 1$, and super-imposed ascending and descending incremental loads ΔP , resulting, e.g., from gusts, maneuvers, or runway roughness.

The CFD as used for fatigue analysis is plotted normally in a semi-logarithmic grid. It says that there occur load increments, which reach or exceed with a certain cumulative frequency H_i the corresponding maximum load $P_{\max,i} = P_b + \Delta P_i$ and minimum load $P_{\min,i} = P_b - \Delta P_i$.

In addition also the basic load can vary with time. As an example Figure 2 shows the stress time history as measured at the lower wing spar boom of a transport airplane. It is necessary and reasonable to distinguish here between so-called loading segments, which are characterized by a constant value of the basic load and which are repeated in similar form during each flight. For each of those loading segments a separate CFD has to be presented.

Such CFDs result in most cases from relatively short measurements if compared with the life of the structure. Therefore they normally do not include those maximum loads, the occurrence of which during life has to be taken into account and the knowledge of which is absolutely necessary for a reliable design.

2. SOME GENERAL REMARKS ABOUT CFD AND ITS EXTREME VALUES

2.1 Parameters of a CFD

The procedure followed so far was to sum up a series of CFDs obtained for a specific loading segment and to divide the resulting cumulative frequencies by the number of loading segments observed or, in other words, to calculate the arithmetic mean value of cumulative frequencies for each load level. The maximum loads then had to be deduced from extrapolation of such an average CFD. The deficiencies of that method are that uncertainties are involved in such an extrapolation and that no information at all is available about the possible scatter of the maximum loads and that of the corresponding CFD.

In order to overcome the deficiencies of current method, the following parameters of a CFD must be known (Refs.3,4):

- shape of distribution (distribution law)
- cumulative number of basic-load crossings, H_t
- maximum load increments, $\pm\Delta\bar{P}$
- scatter of the CFD.

Shape of the CFD:

It is generally known that various types of CFDs occur dependent on the type of loading, be it e.g. gust, maneuver or ground loading (Refs.5,6,7). In order to get comparable, generalized loads for design purposes, these CFDs may be standardized (Ref.8) or approximated by superimposing several standardized distributions (Ref.9). The latter procedure corresponds to the concept of PSD application to non-stationary random loads, where a given CFD is supposed to result from a sequence of stationary Gaussian processes with various RMS values.

Cumulative Number of Crossings of Basic Load:

In view of a subsequent application it is expedient to relate the total number of basic-load crossings, H_t , which have been counted either in ascending or descending direction, to the number of loading segments observed and to designate H_t per loading segment as H_0 .

Experience has shown that for measurements repeated several times under equal conditions, H_0 is not constant, see Figure 3. This example shows the CFD of stress increments measured on the structure of a passenger cabin of an aerial ropeway during six drives between the same two ropeway supports with constant speed, constant payload and in the same direction, carried out within two hours under the same environmental conditions. Nevertheless the difference between the smallest and the largest value of H_0 is about 350:700, i.e. 1:2.

The commonly used method to determine H_0 by calculating the arithmetic mean of the basic-load crossings recorded for the individual loading segments, \bar{H}_0 , will continue to be used in the future.

Maximum Load Increments:

The maximum load increments, $\pm\Delta\bar{P}$, belonging to a CFD generally may be defined as those positive and negative load increments which are expected to occur once within the estimated service life of the structure.

Nevertheless program fatigue tests have shown that for a constant basic load those incremental loads which occur with a relative cumulative frequency less than $1:10^6$ (related to H_t) practically do not influence the fatigue life (Refs.8,9). Under these conditions Gassner suggests to regard those increments as maximum loads which occur with the relative cumulative frequency of $1:10^6$.

Schijve et al. come to a similar conclusion if they recommend to truncate the CFD at a load level which on the average would be reached or exceeded 10 times in the target life of the structure (Ref.10).

This obviously could be done with respect to fatigue analysis. If static loads are considered, then the CFD is not allowed to be truncated, unless there are physical or mechanical reasons that a certain load level cannot be exceeded.

Scatter of the CFD:

As mentioned before, deficiencies are involved in the present method as used for extrapolating the maximum loads of a CFD. With that method in addition it is impossible to assign to the (extrapolated) maximum loads a defined probability with which they are reached or exceeded.

Only when the scatter of the maximum load increment $\Delta\bar{P}$ and its appropriate probability function are known, can a measured CFD be reasonably employed in design problems.

These two requirements, i.e. the reliable extrapolation and the information about the probability function of $\Delta\bar{P}$, can be satisfied by the following method.

2.2 Extreme Value Distributions

For many years already a method using extreme value distributions (EVD) is known (Refs.11, 12, 13), which allows to predict the probability of a maximum value being equalled or exceeded within a determined period of observation.

It is intended to study the applicability of EVDs to incremental loads occurring once within a specific loading segment in order to assign to the corresponding CFD a probability with which it is reached or exceeded.

To the theory, which will be used here and which has been developed by Gumbel (Ref.14), three requirements are tied. How far these requirements can be satisfied in the intended application, is briefly outlined in the following:

- 1) It is presumed, that the CFD of the sequence of events, out of which the extreme values are taken, follows a distribution law of the exponential type. Usually this requirement is satisfied, since the standardized distributions as mentioned before, by which the CFDs observed in service can be approximated, are exponential distributions.
- 2) According to Gumbel the sample size, which compares here with the number of basic-load crossings per loading segment, H_0 , must be a constant. As has been shown in Figure 3, this is not true, even if the conditions, under which CFD and EVD have been observed, remain the same. The variations of H_0 , however, are negligible from the point of view of the strength under variable amplitude loading, particularly, if the arithmetic mean value of the individual basic-load crossings, \bar{H}_0 , is chosen.
- 3) A further requirement for the validity of Gumbel's theory says that the extreme values shall be taken under completely the same conditions. There will, however, be cases where systematic measurements under uniform conditions for the individual loading segments can be achieved only with large expenditure in time and cost, e.g. when measuring on airplanes. If systematic measurements are relinquished, then the conditions vary from one flight to the other and also for the loading segments. The result is that in this case the theory of extremes is no longer rigorously satisfied.

A particularly suitable example for checking whether the above mentioned goal is still attainable under these conditions seems to be the measurement of the loads encountered by the horizontal tail of a transport airplane during normal operation. These measurements were continued over 95 flights. Apart from the cruising time, which varied between 15 minutes and 4 hours 20 minutes, also flight altitude, speed, weight, c.g. position, and environmental conditions were changing according to the individual missions flown. The loading segment chosen for extreme value analysis was the cruising condition.

Figure 4 shows the extreme values of the positive load increments, ΔP^* , plotted on Gaussian probability paper with logarithmic scale for the variate. Practically the same results are obtained when using a logarithmic normal distribution, which shows up as a straight line in Figure 4. Analyses of numerous other measurements on airplanes and motor vehicles have shown that the individual extreme values can be approximated by the log. normal distribution with an accuracy satisfactory for practical engineering purposes, even if the service conditions are subject to variations.

Some of the reasons why the log. normal distribution has been chosen are the following:

- (a) The requirements for the validity of Gumbel's theory cannot be satisfied in the intended application.
- (b) The asymptotic behaviour of the two distribution functions for large values is the same.
- (c) The Gumbel distribution is a three parameter distribution. It has to be taken into account, when using it, that each additional parameter (beside mean value and standard deviation) can be derived only with increasing uncertainty from the usually limited sample of measurements.
- (d) The log. normal distribution can be treated much more easily in connexion with the probability function for the natural scatter of structural components as will be shown later.

* Here, obviously, exist two extreme values for each loading segment, one for the positive and one for the negative incremental loads. Similarly a CFD has a positive and a negative leg, see Figure 1.

In conclusion it may be said, that the EVD – approximated by a log. normal distribution – allows to determine the probability W_E , with which an extreme value of defined magnitude is equalled or exceeded within a particular loading segment. Furthermore it allows a prediction of the scatter of the extreme values, expressed by the standard deviation s_f , which at the same time represents the scatter of the CFD for the loading segment considered.

The application will be demonstrated on hand of an example:

Given are the CFDs of level crossings of c.g. vertical accelerations as measured on a transport airplane during the loading segment “low level flight”. Figure 5 shows seven of those CFDs, from which the ranges of variation as well for the basic-load crossings (at $\Delta n_z = 0$) between 850 and 7,000 as those for the extreme values varying between $\pm \Delta n_z^* = 0.35$ g and 1.10 g can be seen.

For this investigation 57 CFDs for the mentioned loading segment were available.

As presumed earlier, the scatter of the extreme values occurring once per loading segment is presumed to be identical with that of the CFDs. That may be proved by the following analysis:

If in addition to the distribution of extreme values, which occur once per loading segment, also the distributions of those values are established, which e.g. occur 10, 100, or 1000 times, then in the probability paper a family of straight lines appears, which approximate the individual values, see Figure 6. The deviation from the log. normal distribution for those extreme values which have been reached or exceeded 1000 times per loading segment results from the fact that some of the loading segments had a total number of basic-load crossings less than 1000, as mentioned before. But here only the small values with large probabilities deviate from the log. normal distribution.

If now those vertical acceleration increments which are reached or exceeded e.g. with 10, 50, and 90 per cent, are taken from the EVDs in Figure 6 and plotted into the semi-log. grid at the corresponding cumulative frequencies, see Figure 7, then the result is a family of CFDs with defined probabilities for the specific loading segment.

For purposes of comparison also that CFD has been plotted, which resulted from the method used till now, i.e. by averaging the individual CFDs. This CFD is accidentally reached or exceeded with a probability of about 30 percent.

This example demonstrates also that extreme values other than those which occur once per loading segment can be relinquished. It is rather sufficient if the extreme values, which occur once per loading segment, are plotted into the semi-log. grid at the cumulative frequency $H = 1$ and if they are connected with the arithmetic mean of basic-load crossings \bar{H}_0 by means of the CFD as found by counting, a procedure which corresponds to a geometric magnification.

2.3 Derivation of Maximum Loads

Each extreme value within a series of measurements is related to the same relative frequency referred to \bar{H}_0 . Dependent on the value of \bar{H}_0 observed for the loading segment concerned, however, the frequency of an extreme value differs more or less from that relative frequency associated with the maximum load increment of the CFD, which occurs e.g. once within the estimated service life, see Paragraph 2.1.

When using the EVD, this problem can be solved by means of the so-called return-period, which is defined as the reciprocal of the probability W_E , being equal to the number “a” of observations necessary to obtain an extreme value of a given magnitude once (Ref.14).

$$a = \frac{1}{W_E}$$

For the example shown in Figure 4 an additional scale for the return period has been plotted beside the scale for the probability W_E , which says that the extreme value being reached or exceeded e.g. with $W_E = 50$ percent occurs (in the average) in every two loading segments, that with $W_E = 10$ percent in every 10, etc. The extreme value, which is reached or exceeded in every loading segment ($a=1$), has the value of zero and occurs with the probability $W_E = 100$ percent, because the return period is related by its definition to the probability $W_E = 100$ percent.

For the problem considered here it is not useful, however, to determine the number of loading segments necessary for a defined extreme value to be equalled or exceeded once with a probability of 100 percent, rather to determine at a given number “a” of loading segments that extreme value which occurs once in “a” loading segments and is reached or exceeded – in view of safety – with less probability, e.g. $W_E = 90, 50, \text{ or } 10$ percent.

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If the probability \bar{W}_E of a reference point of the EVD is introduced into the above mentioned equation for the return period, we can draw similar scales, see Figure 8. The equation for this generalized return period is

$$a = \frac{\bar{W}_E}{W_E},$$

where the reference probability \bar{W}_E can vary between 0 and 100 percent.

Attention must be given only to the fact that all points taken from the EVD by means of such a scale have the probability of the reference point.

With that information it is possible to extrapolate from an extreme value with defined probability W_E , occurring once per loading segment, to the maximum load, which has the same probability. For a given number of loading segments "a" the cumulative frequency H_t of the maximum load increment can be obtained by the relation

$$H_t = a\bar{H}_0.$$

An other example may demonstrate this application.

Let us assume:

- 1) The distribution function of the load spectrum to be of type e^{-x} (being in a semi-log. grid a straight line),
- 2) the average number of basic-load crossings per loading segment

$$\bar{H}_0 = 2 \times 10^3,$$
- 3) the number of loading segments required $a = 500$,
- 4) the EVD of load increments ΔP^* , see Figure 8.

The total number of basic-load crossings belonging to the maximum load increment $\Delta \bar{P}$ then we get from the above mentioned equation as

$$H_t = a\bar{H}_0 = 500 \times 2 \times 10^3 = 10^6.$$

Starting from the extreme value $\Delta P^*_{90} = 1.9 \text{ Mp}$, see Figure 8, which is equalled or exceeded once in each loading segment with the probability $W_E = 90$ percent, the maximum load $\Delta \bar{P}_{90} = 5.0 \text{ Mp}$ may be read from the EVD by using the scale for the reference probability $\bar{W}_E = 90$ percent at the given number of loading segments $a = 500$.

In a similar manner the maximum loads may be determined occurring e.g. with $\bar{W}_E = 50$ and 10 percent. These values and their associated cumulative frequencies H_t represent points in the distribution function of the CFD, which has been assumed to be known, see Figure 9. Plotting on semi-log. paper thus results in a family of CFDs characterized by defined probabilities W_E or \bar{W}_E , respectively. As one may anticipate, the scatter of the extreme values (at a cumulative frequency $H = 1$ per loading segment) is larger than that of the maximum loads (at a cumulative frequency $H = 2 \times 10^{-3}$ per loading segment).

This schematic example has been used to explain the procedure for getting the maximum loads and their CFDs. An application of that procedure to a practical problem is shown briefly in the following.

After measurements have been carried out on an airplane of type Boeing 720-B the question arose about the CFD of c.g. vertical accelerations for 10,000 flights. Data were available for only 571 flights (corresponding to a flight time of about 1,500 hr) (Ref.15). The maximum c.g. accelerations have been derived for these 571 flights by means of the EVD, see Figure 10. In Figure 11 the CFD for 571 flights is being compared with those CFDs which have been predicted for 10,000 flights and which are reached or exceeded with a probability of $\bar{W}_E = 10$ and 90 percent.

3. EXTREME VALUES OF C.G. VERTICAL ACCELERATIONS MEASURED ON AIRPLANE OF TYPE C-130

A further application of the EVD demonstrates the possibility to separate the effect of certain airplane and flight parameters on incremental gust loads. The data for this investigation were provided by C.G. Peckham, and that for the maximum and minimum values of c.g. vertical acceleration increments of airplanes of type C-130, which have been observed once per flight and which have been caused by gusts (Ref. 16).

These extreme value data have been presented in tabular form for about 4,000 flights. There was, in addition to each of the extreme values, information given about the airplane model, type of mission, season, base of the airplane, airborne time per flight, gross weight of the airplane, flight altitude, and air speed.

Since the c.g. vertical acceleration increments in the present case have been provided in classes and not as absolute peak values, as happened e.g. for the 720-B data, see Figure 10, the individual points occur, when plotted in a Gaussian probability paper with a log. scale for the variable, not more or less steady about a line but as groups of equal values, see the example of Figure 12. The appropriate distribution function, however, has to be a straight line in that paper, as explained in the foregoing chapters; it, therefore, has been calculated from the data, see the Appendix. In the following Figures 13, 14, 16, 18, 20 the individual extreme values have been omitted and only the calculated EVDs have been plotted.

Figure 13 shows the EVDs for five models. As in this plot all speeds, altitudes, missions, weights, and seasons have been included, it does not seem very suitable for a detailed interpretation.

The analysis has been restricted in the following to data observed on airplanes of model E. Figure 14 shows the effect of season. From a plot of the two parameters of the EVD, i.e. mean value and standard deviation, see Figure 15, it can be seen that there is an increase of both mean value and scatter during winter.

Generally an increase in mean value is equal to an increase in the average loading intensity, and an increase in scatter, as expressed by the standard deviation, can be explained as an increase in variation of mission profile characteristics. In order to explain the significance of the difference between both mean value and standard deviation more accurately, statistical tests, as e.g. T- or F-tests, can be employed.

The extreme value distributions for the different missions, which were designated as short and long range logistics, training, and aerial delivery, see Figure 16, have almost the same standard deviation but different mean values, see Figure 17. This also happens if different airspeeds, see Figures 18 and 19, or different airplane gross weights, see Figures 20 and 21, are analyzed. The latter results can be used e.g. to check the effect of airspeed and gross weight on the intensity of gust loading.

The true scatter of gusts encountered once per flight can be investigated only exactly if other effects are eliminated. In order to do that for flight altitude, air speed, and gross weight of the airplane, the measured c.g. vertical acceleration increments have been converted into corresponding gust velocities by means of so-called \bar{A} -values, which were provided also by C.G. Peckham and for which Sewell has set up a formula for interpolation (Ref. 17).

Such an EVD of gust velocities is shown in Figure 22. It says that during each flight an upward and downward gust of about 15.5 ft/s has been exceeded with 50 percent probability. The maximum gust encountered once in a total of 318 flights was a positive one of 43 ft/s and a negative one of 52 ft/s.

4. CONCLUSION

The numerical description of the effect of specific parameters like season, mission, airspeed, etc. on loads has to be regarded as an addition compared with the advantage of EVDs, if applied in order to get a criterion for a reliable design of structural elements, as the following considerations will demonstrate (Ref. 18).

It is known from fatigue tests carried out with specimens, joints, or components, that the fatigue life under defined loading conditions is subject also to scatter resulting from effects like material, stress concentration, and production. In order to demonstrate this, let us assume that the loading for a series of specimens of the same kind is given by the shape of the CFD, by the total number of basic-load crossings, H_t and by the maximum stress amplitudes $\pm \bar{S}_a$. If this CFD has been simulated properly in tests then the scatter shows itself in a differently lasting life to failure of the individual specimens, see Figure 23. In this example the logarithm of the maximum stress amplitude, \bar{S}_a , of the given CFD with a known number of basic-load crossings has been plotted as a value characterizing the load versus logarithm of fatigue life expressed by the total number of basic-load crossings having been endured to failure.

With a maximum stress amplitude $\bar{S}_a = 40 \text{ kp/mm}^2$ nine tests have been made, for which, after an appropriate statistical analysis, defined probabilities of survival W_S , e.g. 90, 50, and 10 percent, can be specified. If similar tests are repeated with other maximum stress amplitudes, e.g. with $\bar{S}_a = 34 \text{ kp/mm}^2$, and if the points of equal

probabilities of survival are being connected, then the relation between endurable stress and fatigue life to failure has been found. This relation appears usually in a double-log. grid as a straight line (Ref.19). Since the difference in the slopes of such lines for common materials, stress concentrations, and various types of CFDs is not significant (Ref.20), we can derive the approximative rule that an error in maximum load or stress amplitude, respectively, by 10 percent results in an error of life estimation by about 100 percent.

From the relation between endurable stress and corresponding fatigue life it is further evident that the probabilities of survival, W_S , characterize not only the scatter of fatigue life but likewise also the scatter of maximum stress amplitudes \bar{S}_a , which are being endured by the specimen with a given total number of cycles, e.g. $\bar{S}_a, 10^6$.

A reliable life estimation has to be based on the fact that, besides the natural scatter of endurable stresses, defined by the probability W_S , also the maximum stress of the CFD is subject to scatter, expressed by the probability \bar{W}_E . If the scatter band of maximum service stress is schematically plotted into the grid for the relation between endurable stresses and fatigue life, see Figure 24, then this horizontal scatter band intersects that of the endurable stresses, which decreases with increasing fatigue life. Since both probability functions can be approximated with satisfactory accuracy by log. normal distributions, the total probability of failure, W_F , can be easily calculated from the difference between the two mean values and from the two standard deviations (Ref.18).

It is recommended, therefore, to gather information about extreme values of measured flight loads in the manner described in order to get an effective tool, which enables us to calculate reliable safety margins instead of estimating the fatigue life by means of empirical safety factors. Beside the description of the effect of specific airplane or flight parameters on the loads we shall also be able to predict from EVDs e.g. upset conditions, which are interesting for the static design of a structure.

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APPENDIX

Log. Normal Distribution

The probability W_A that a logarithmically distributed variate x is less than or equal to a given value of X is

$$W_A(\log x) = \frac{1}{\sqrt{(2\pi)\log s_l}} \int_{-\infty}^X \exp \left[-\frac{(\log x - \log \bar{x})^2}{2 \log s_l} \right] d(\log x),$$

where

$$\log \bar{x} = \text{mean value,}$$

$$\log s_l = \text{standard deviation.}$$

The probability W_E that x exceeds a given value of X is then

$$W_E(\log x) = 1 - W_A(\log x).$$

Since

$$\frac{d(\log x)}{dx} = \frac{1}{x} \log e,$$

the density function is given by

$$w(\log x) = \frac{dW(\log x)}{dx} = \frac{\log e}{x\sqrt{(2\pi)\log s_l}} \exp \left[-\frac{(\log x - \log \bar{x})^2}{2 \log s_l} \right].$$

Mean value and standard deviation of a sample of values x_i , where $i = 1 \dots n$, and $n =$ sample size, can be calculated as follows:

$$\log \bar{x} = \frac{\sum_{i=1}^n \log x_i}{n}$$

and

$$\log s_l = \sqrt{\frac{\sum_{i=1}^n (\log x_i - \log \bar{x})^2}{n - 1}}.$$

The individual observations x_i may be plotted directly on a Gaussian probability paper with log. scale for the variate, if they are ordered in decreasing magnitude and a rank $m = 1 \dots n$ is ascribed to each of them.

The plotting position then can be calculated from the relation

$$W_E = \frac{3m - 1}{3n + 1} \times 100 \text{ in per cent.}$$

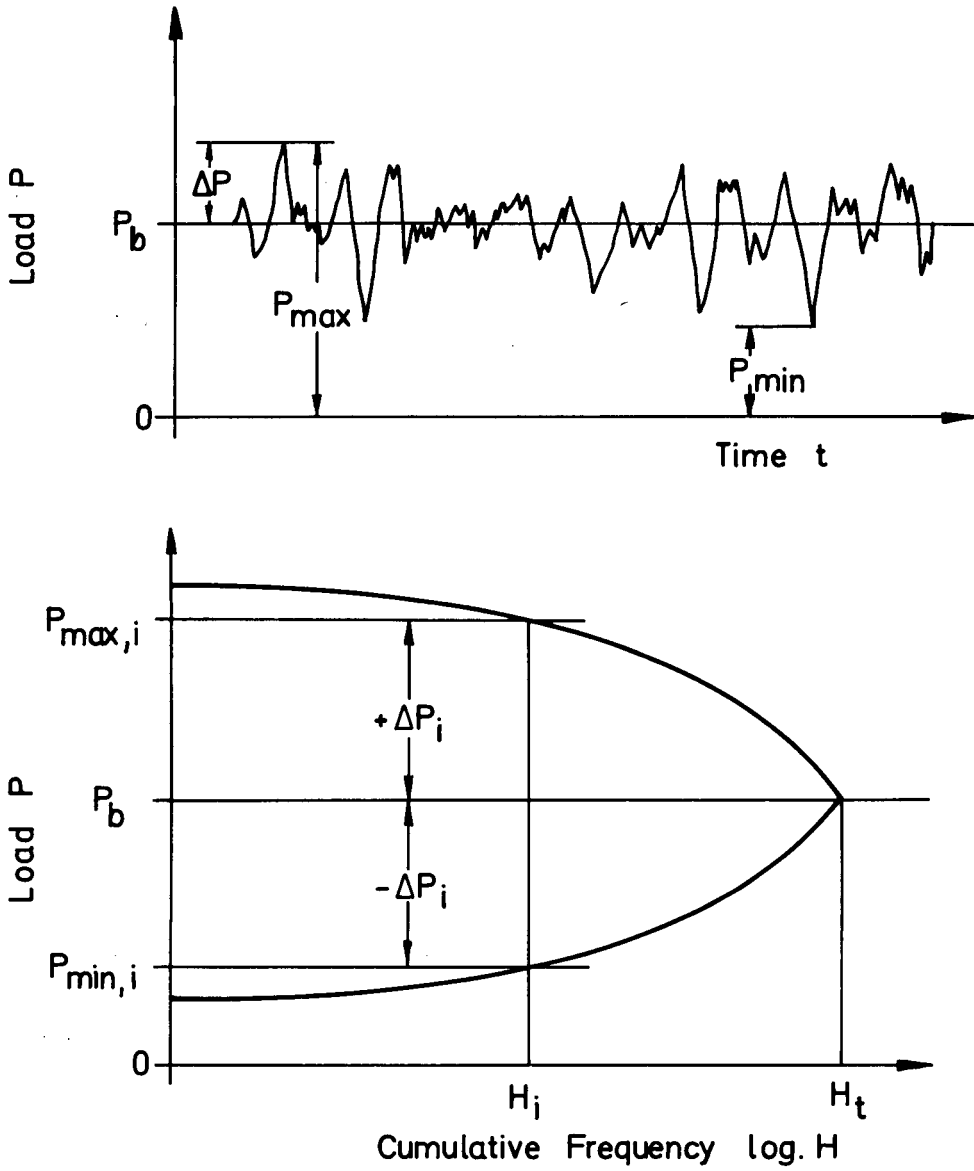


Fig.1 Random load-time history and cumulative frequency distribution

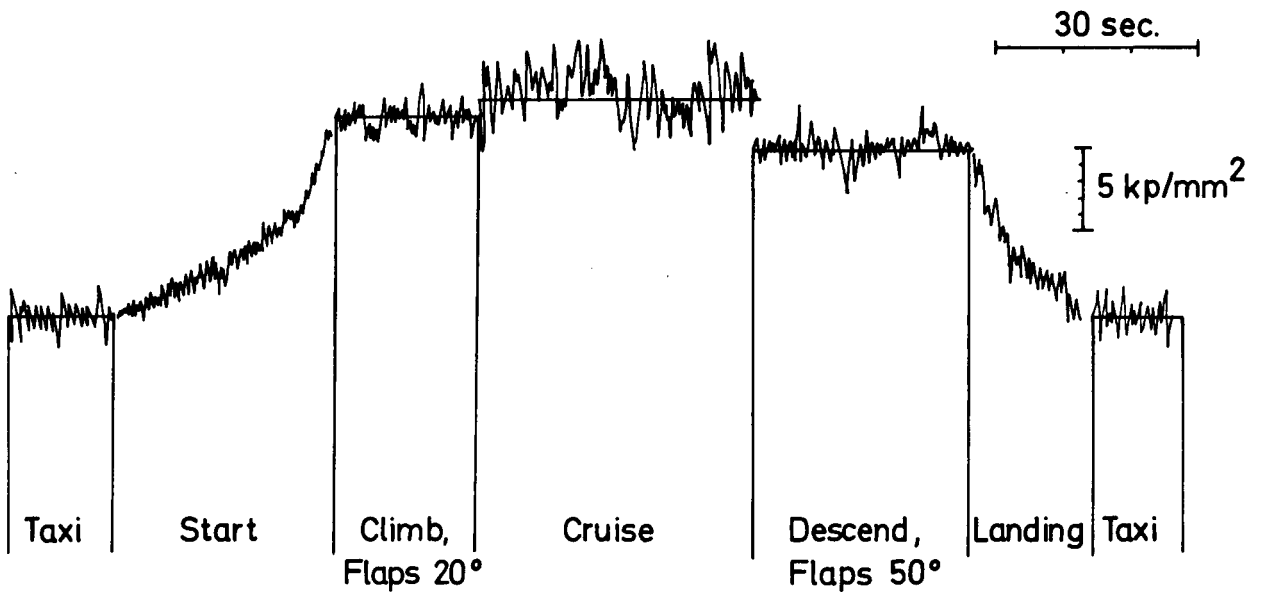


Fig.2 Stress-time history of lower wing spar boom and loading segments

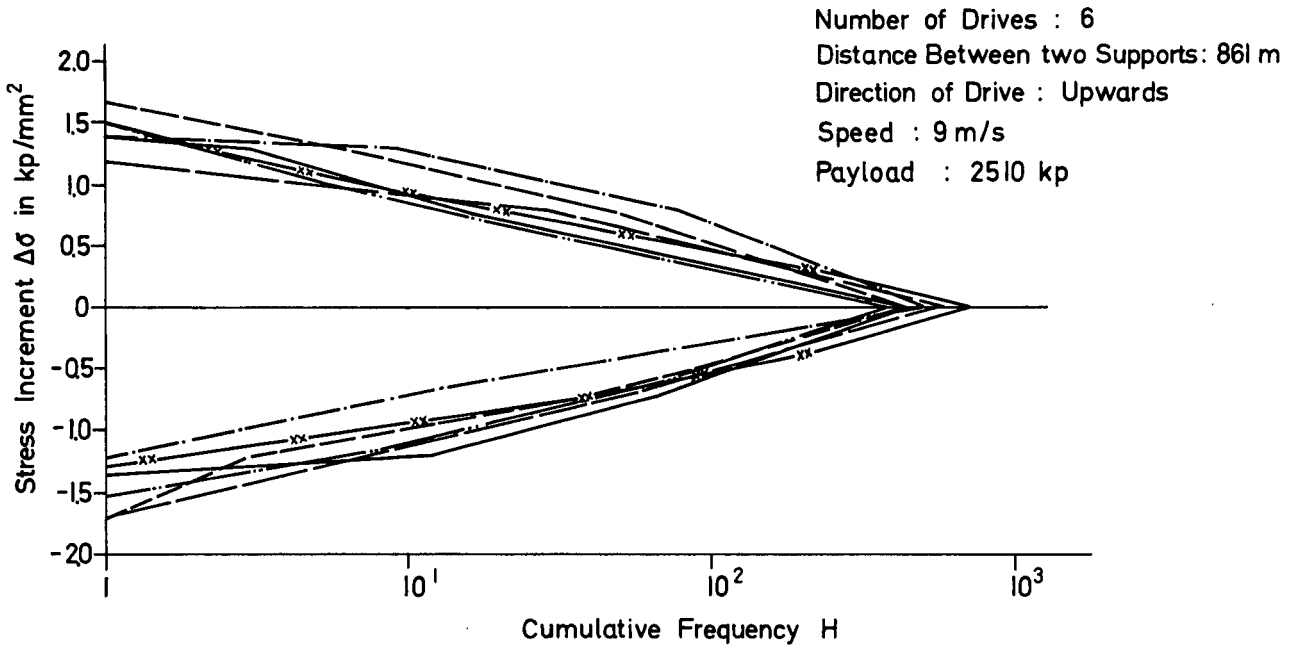


Fig.3 Scatter of cumulative frequency distributions measured on a cable railway under equal conditions

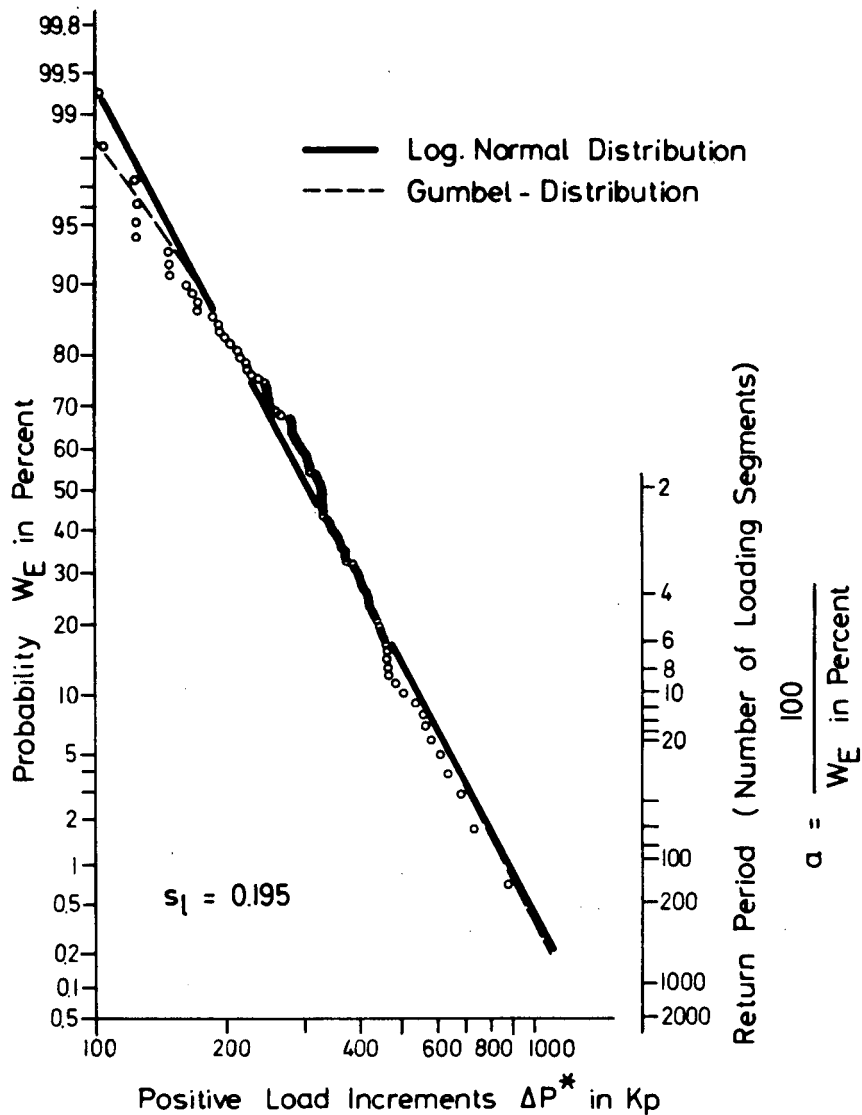


Fig.4 Example of an Extreme Value Distribution

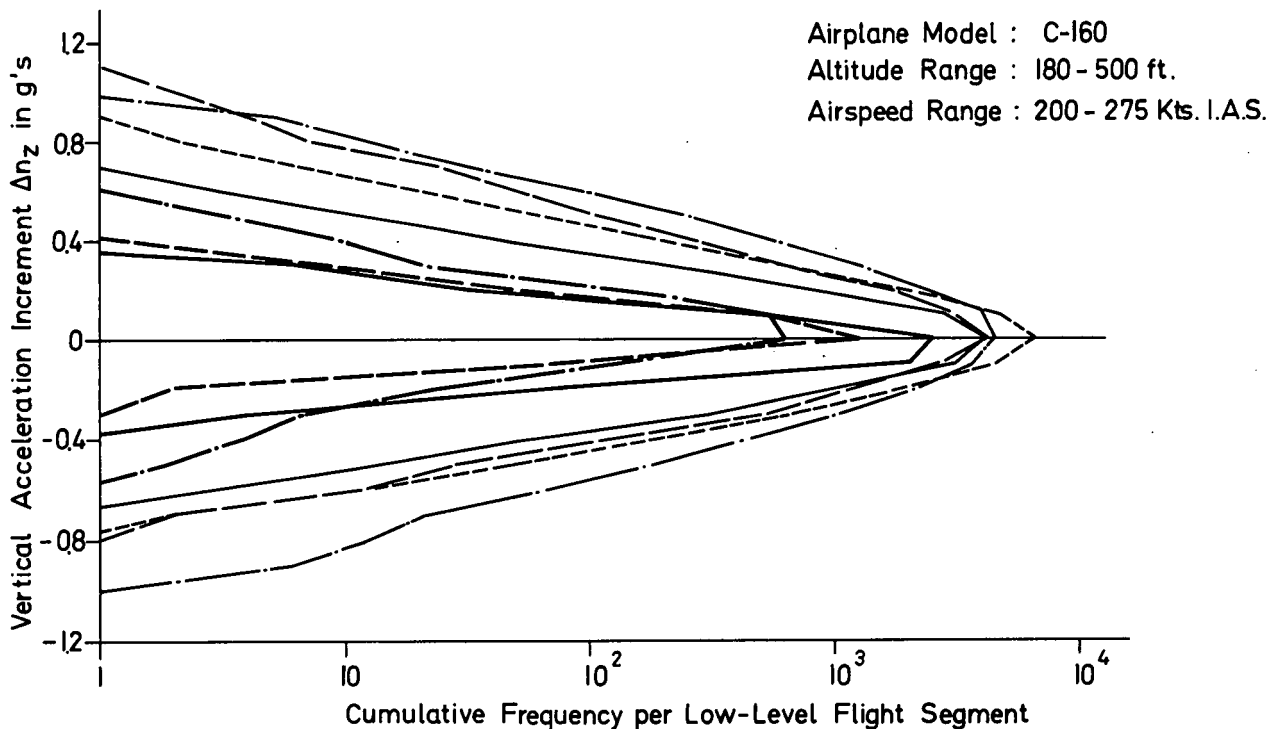


Fig.5 Sample of CFDs of c.g. vertical acceleration for the loading segment low-level flight

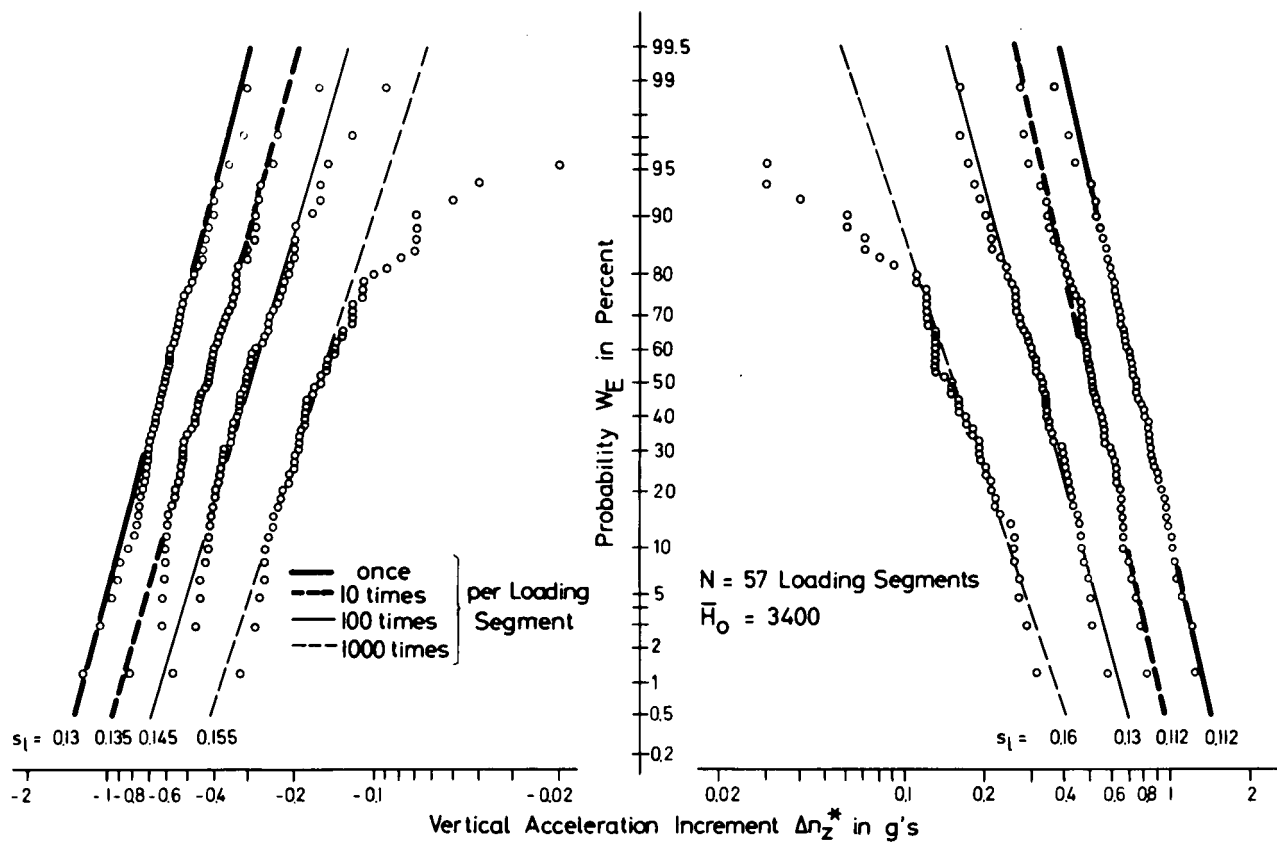


Fig.6 Extreme Value Distributions for the loading segment low-level flight

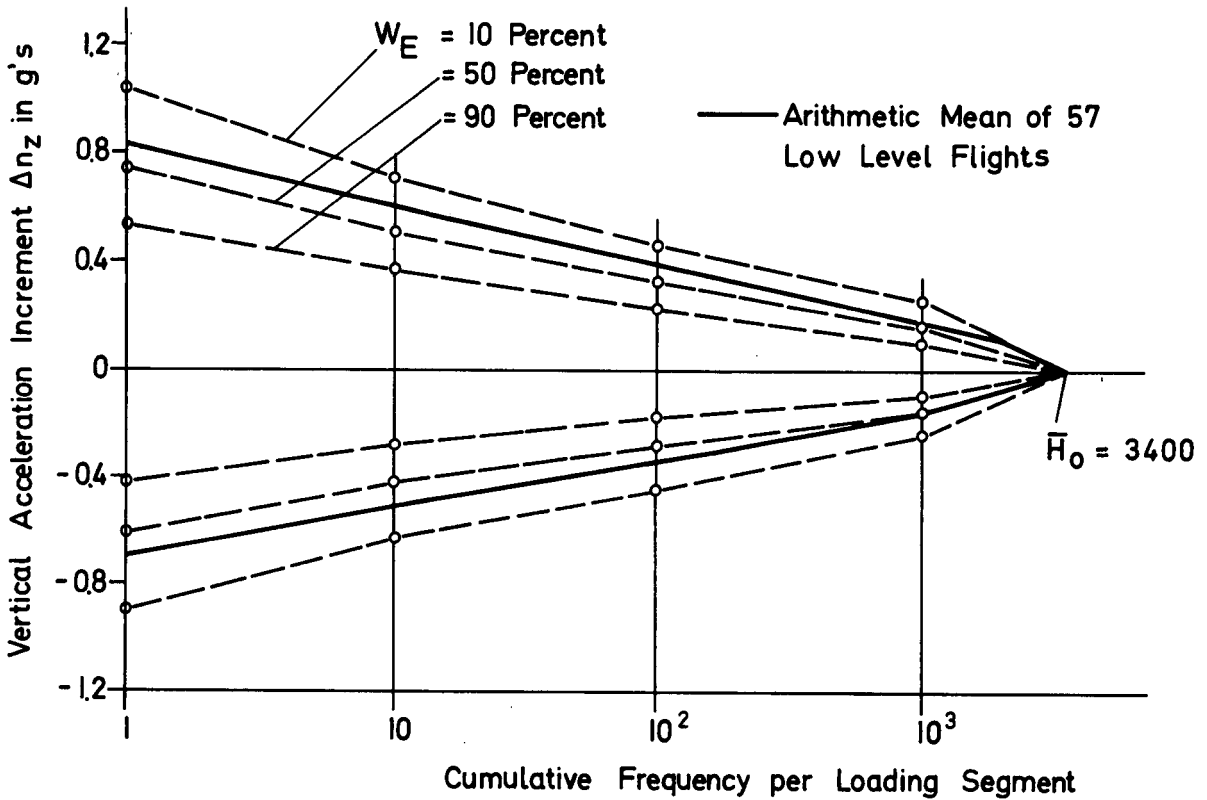


Fig.7 Family of CFDs for the loading segment low-level flight as derived by use of Extreme Value Analysis

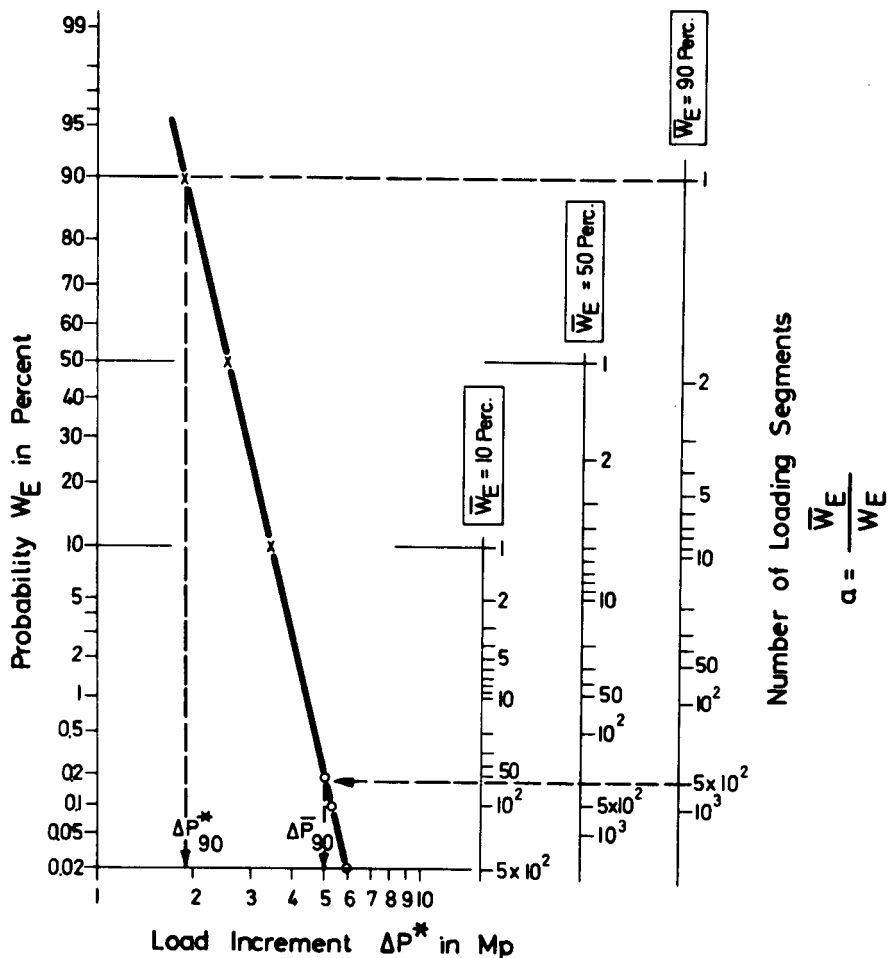


Fig.8 Schematic example for definition of probabilities of maximum load increments

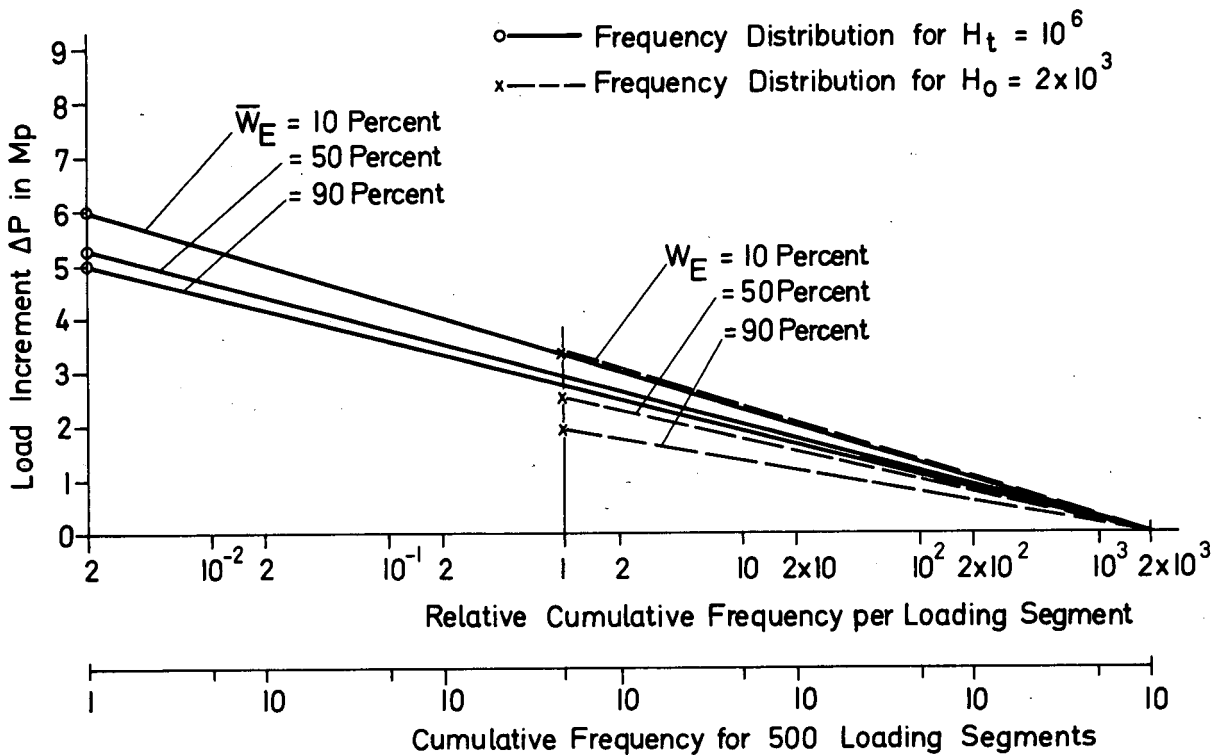


Fig.9 Example for scatter of extreme values and of maximum loads as derived from Extreme Value Distribution in Figure 8.

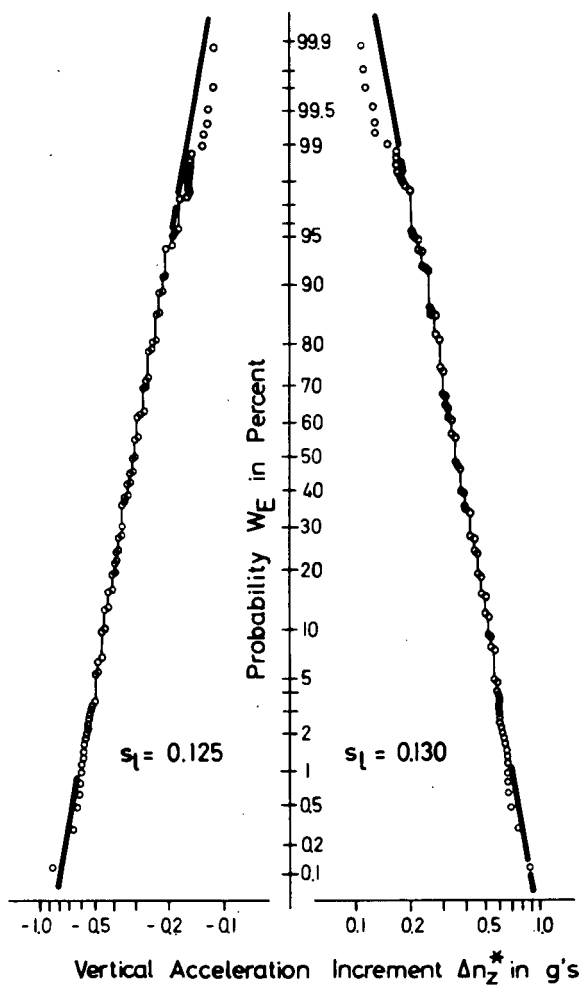


Fig.10 Extreme values of c.g. vertical acceleration during cruise measured at an airplane of Type 720-B

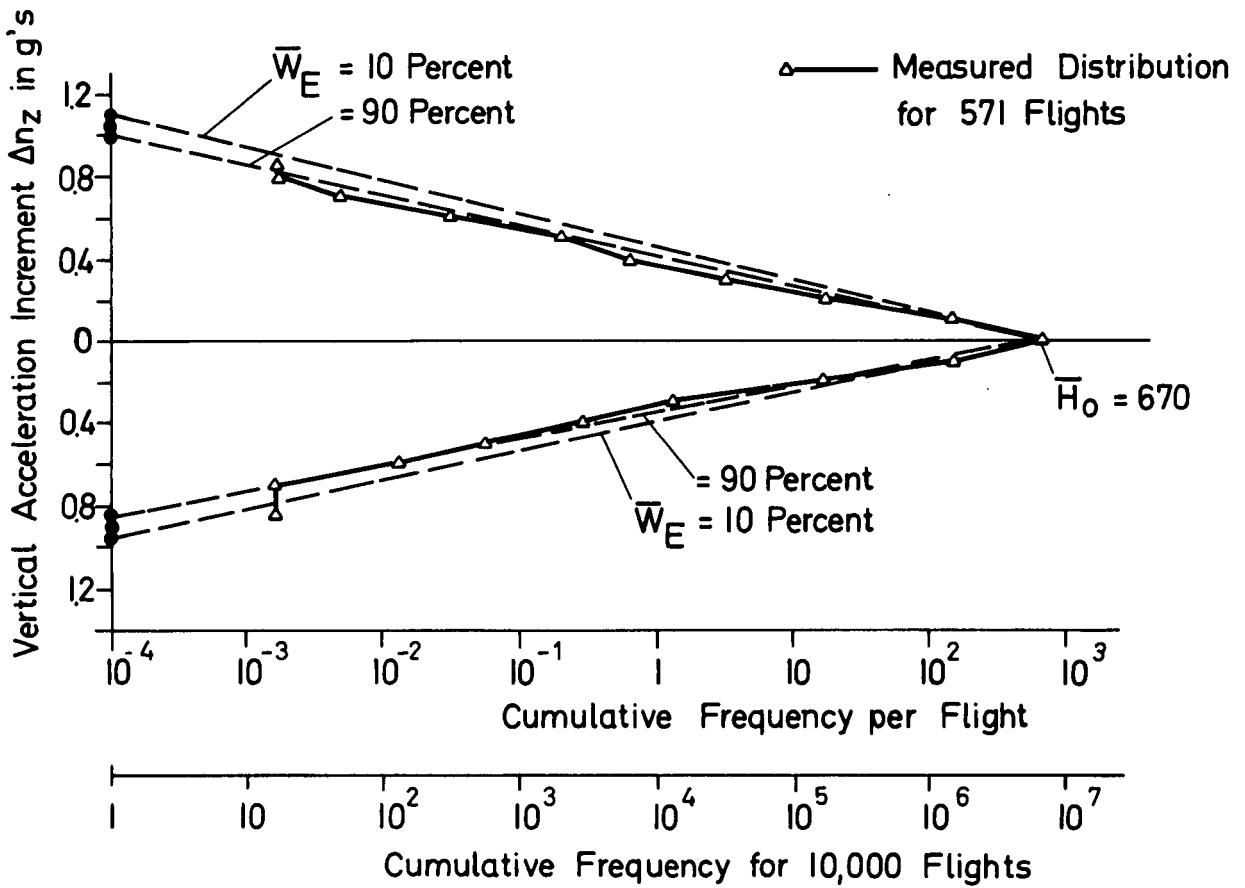


Fig.11 CFD of c.g. vertical acceleration during flight for an airplane of Type 720-B

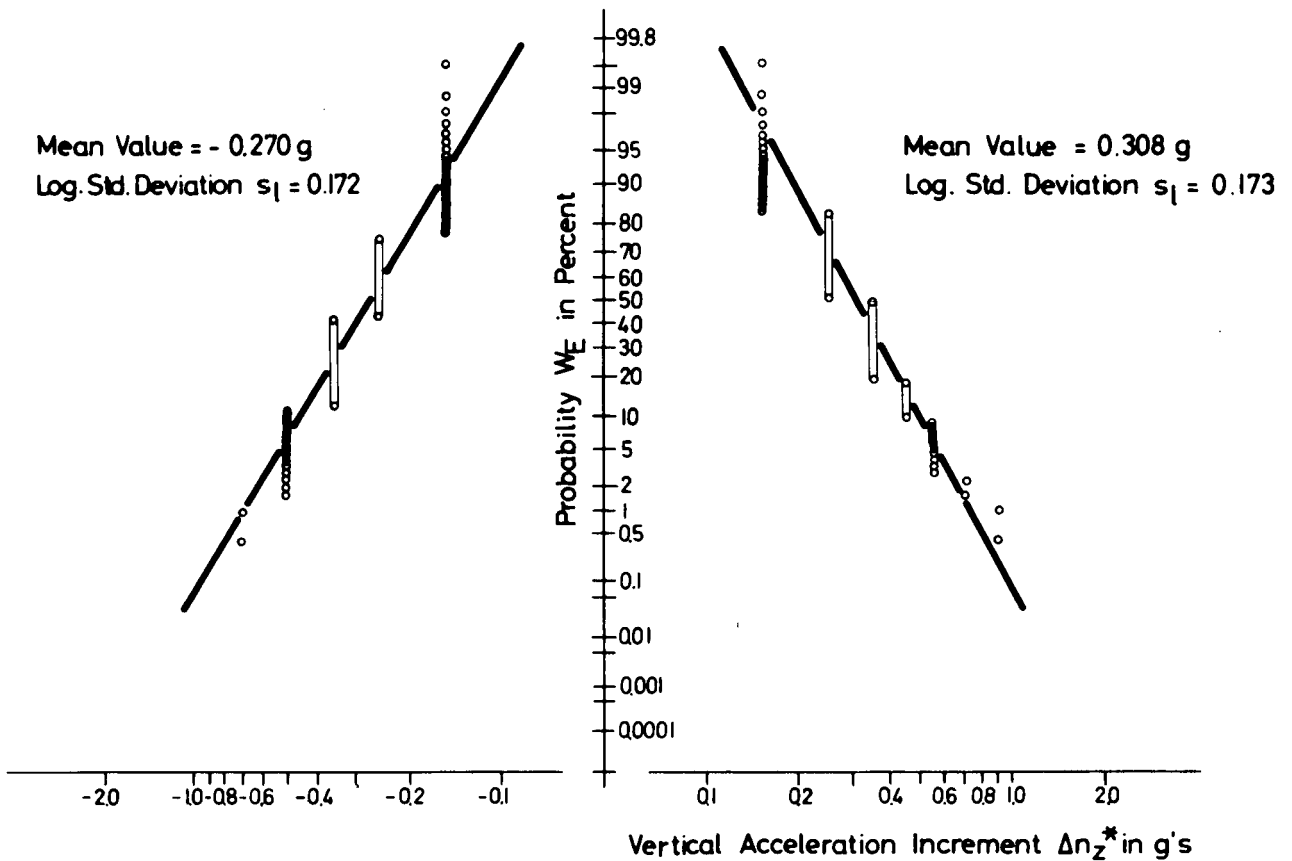


Fig.12 Extreme Value Distribution of c.g. vertical accelerations for 155 flights of model C-130 HC

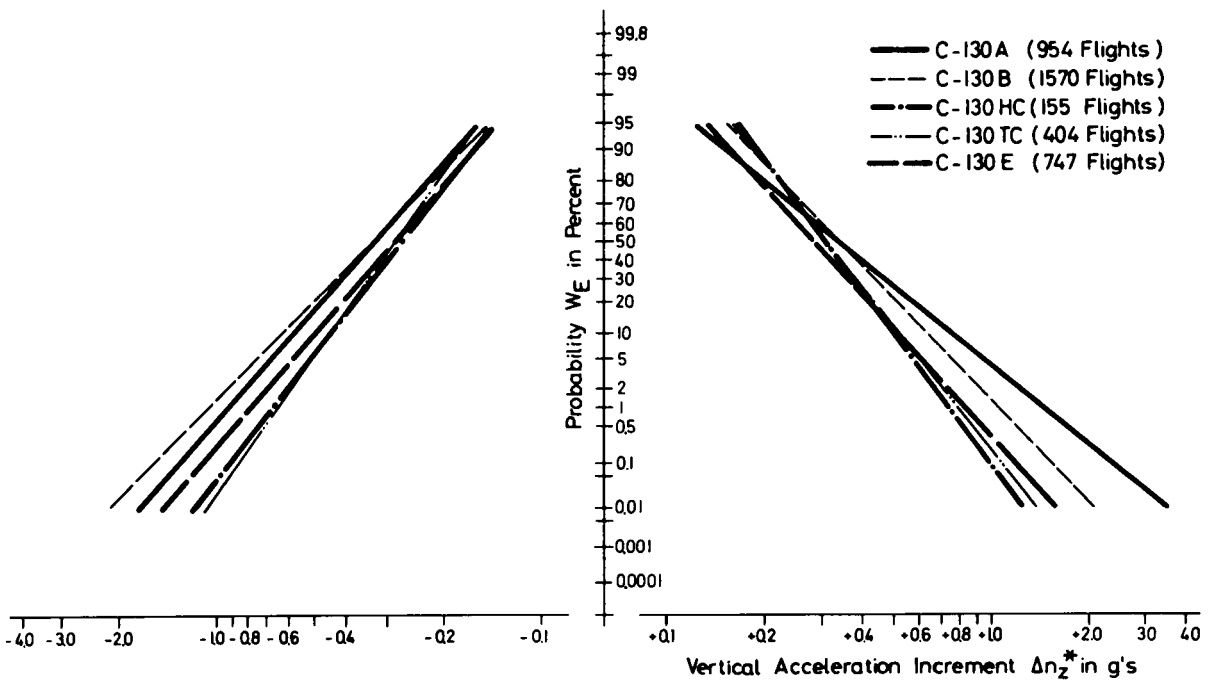


Fig.13 Effect of model on the EVD of c.g. vertical accelerations

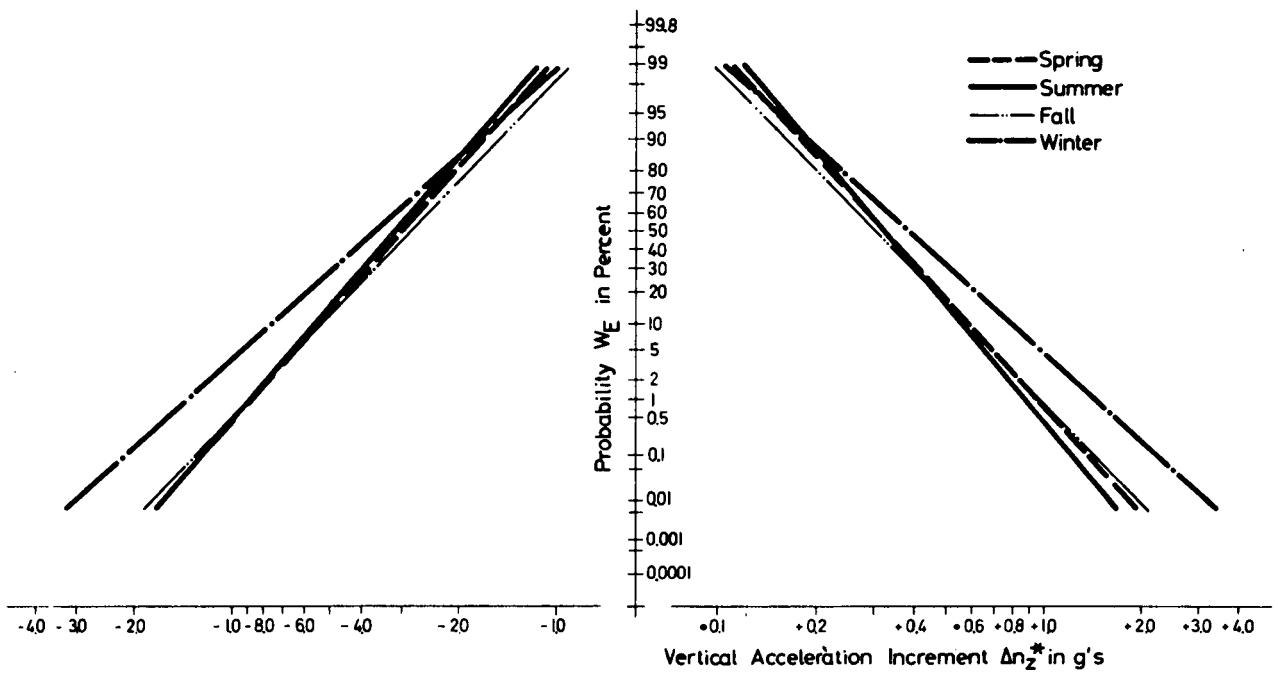


Fig.14 Effect of season on the EVD of c.g. vertical accelerations of model C-130 E

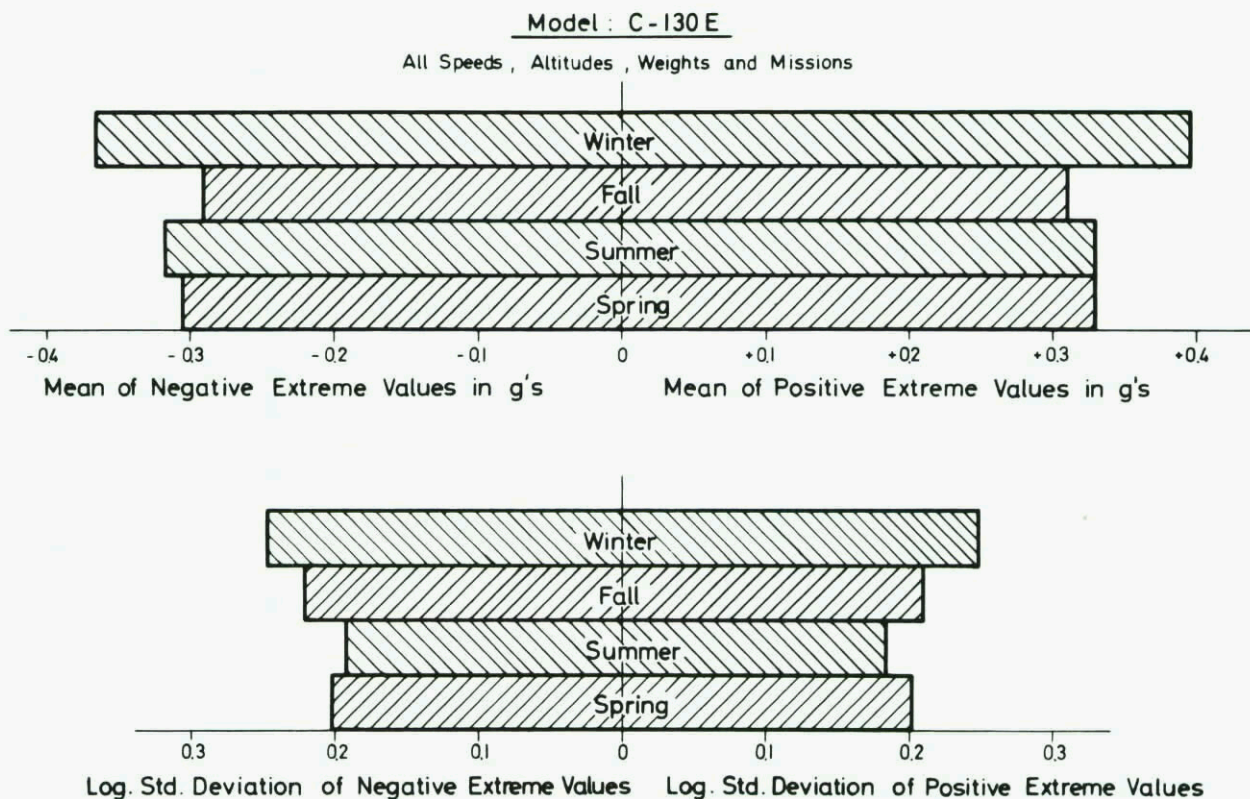


Fig.15 Effect of season on mean value and log. standard deviation

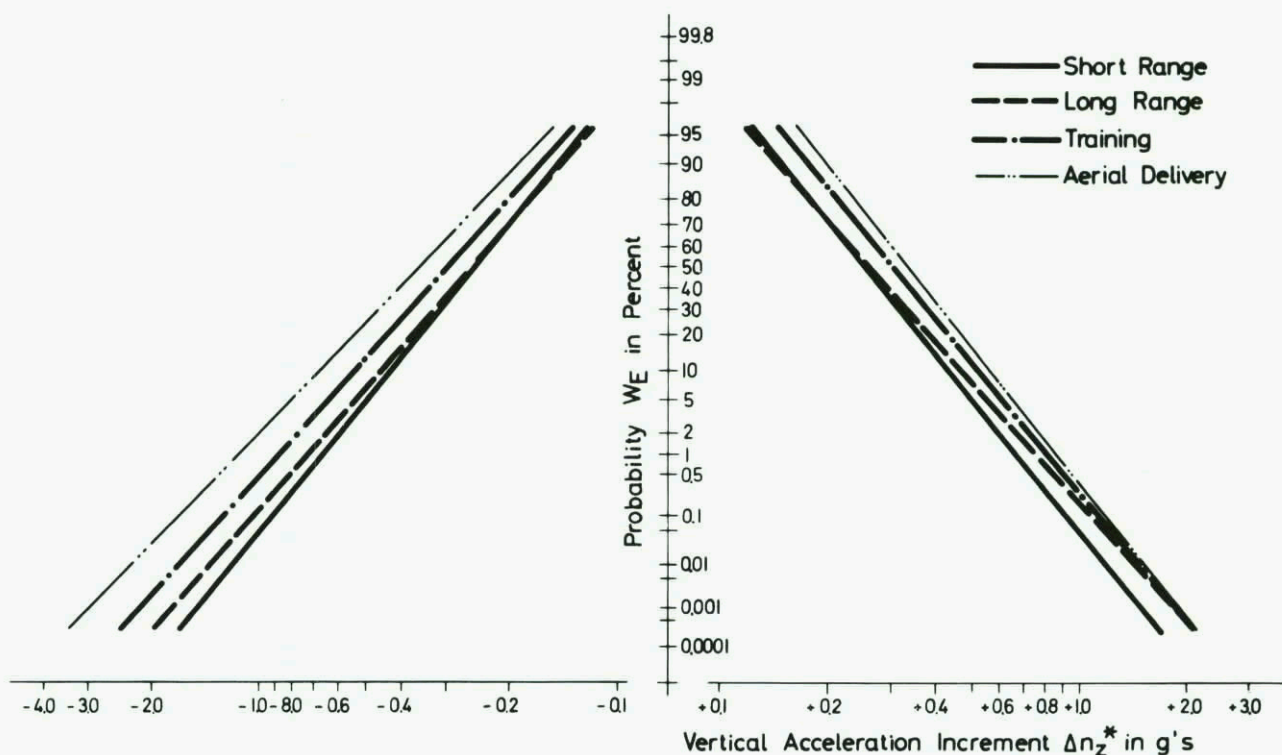


Fig.16 Effect of mission on EVD of c.g. vertical accelerations of model C-130 E

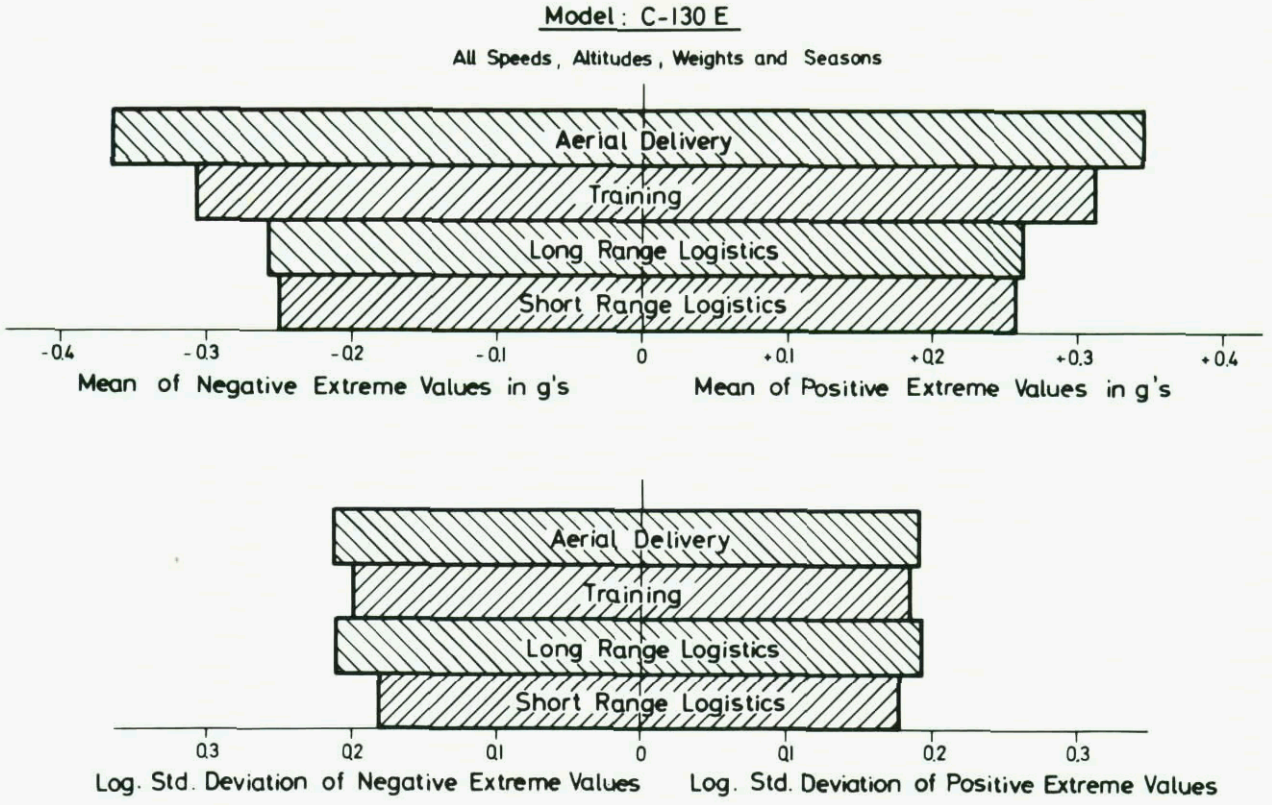


Fig.17 Effect of mission on mean value and log. standard deviation

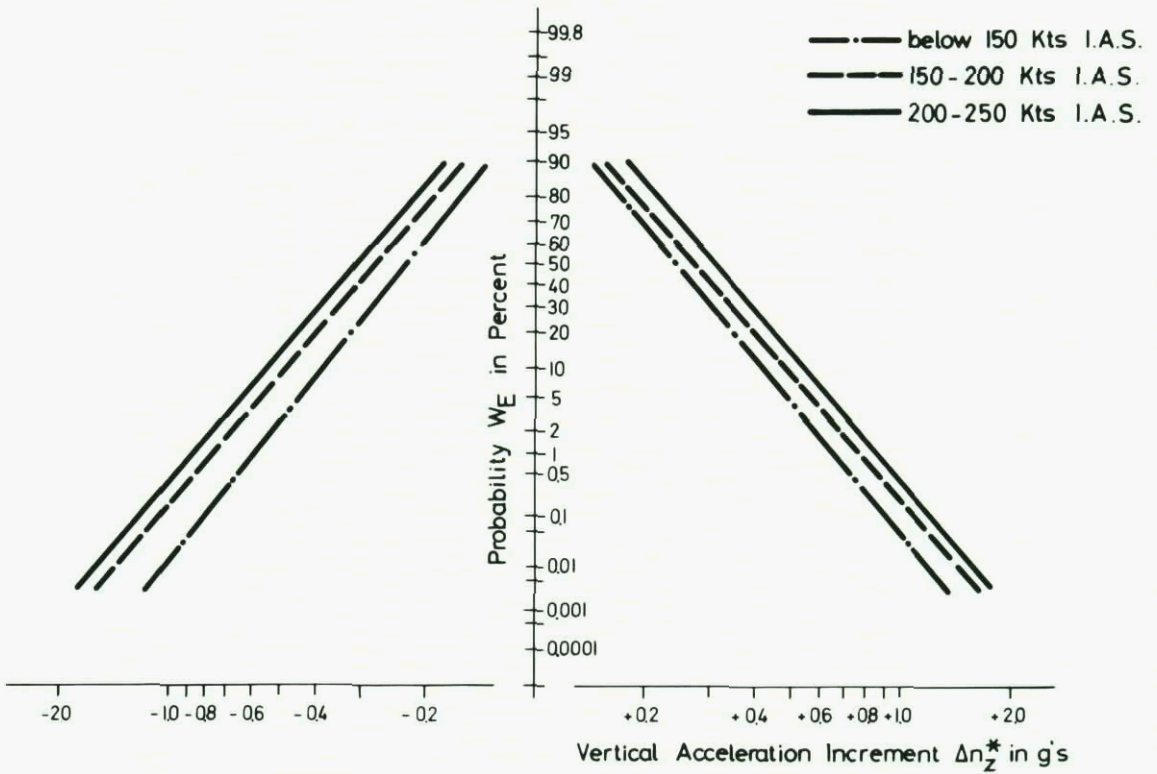


Fig.18 Effect of airspeed on EVD of c.g. vertical accelerations of model C-130 E

Model : 130 E

All Altitudes, Weights, Seasons and Missions

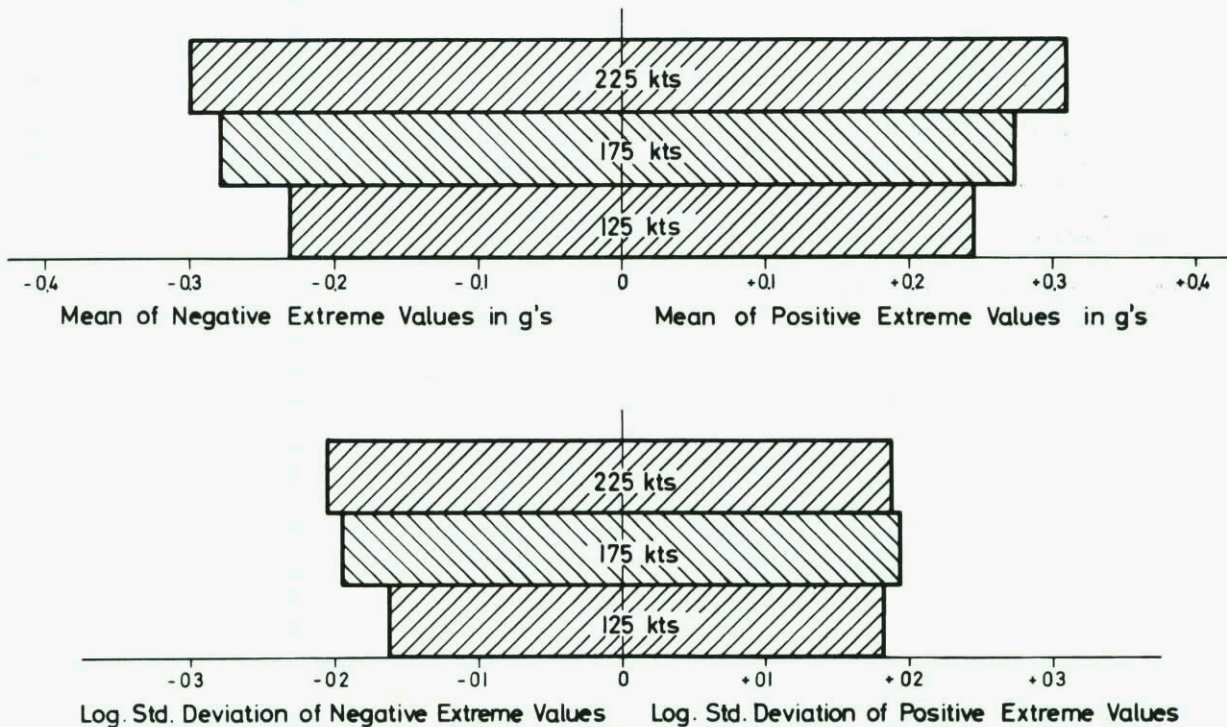


Fig.19 Effect of airspeed on mean value and log. standard deviation

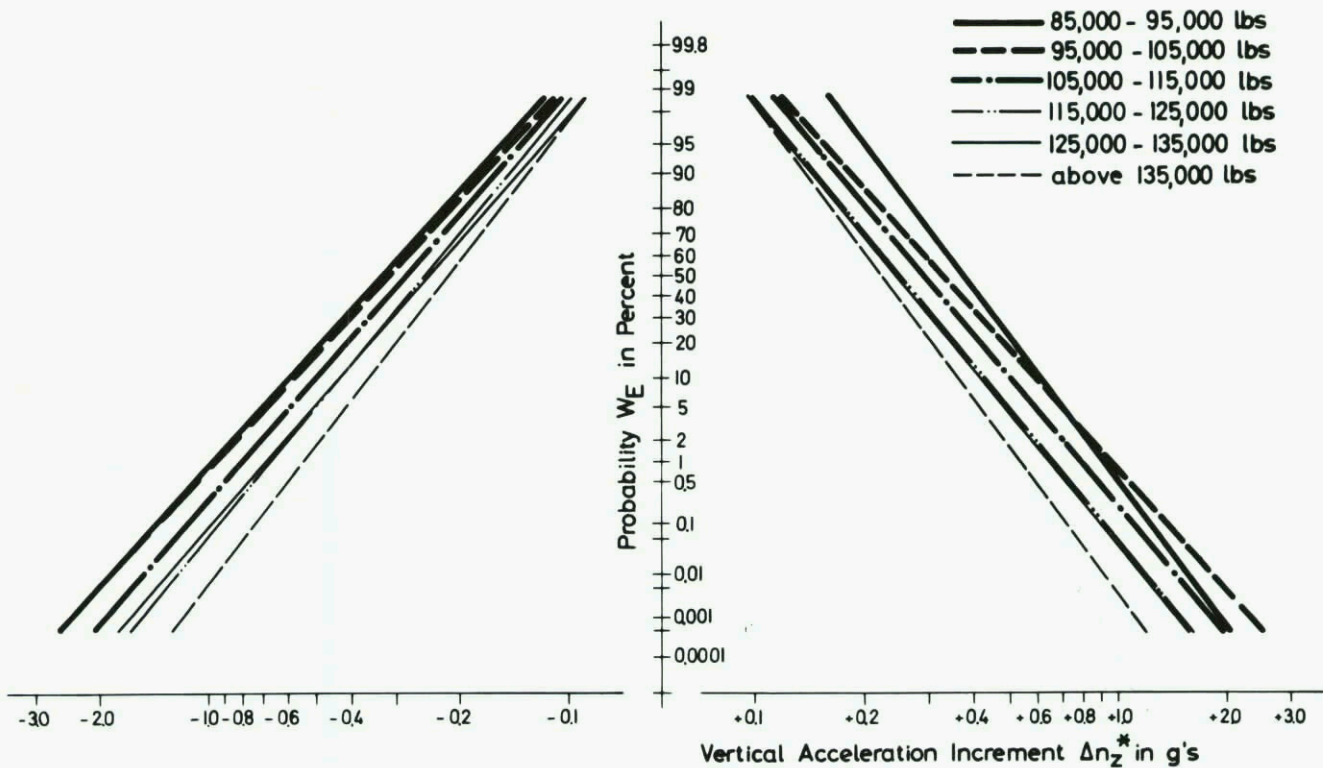


Fig.20 Effect of airplane gross weight on the EVD of c.g. vertical acceleration of model C-130 E

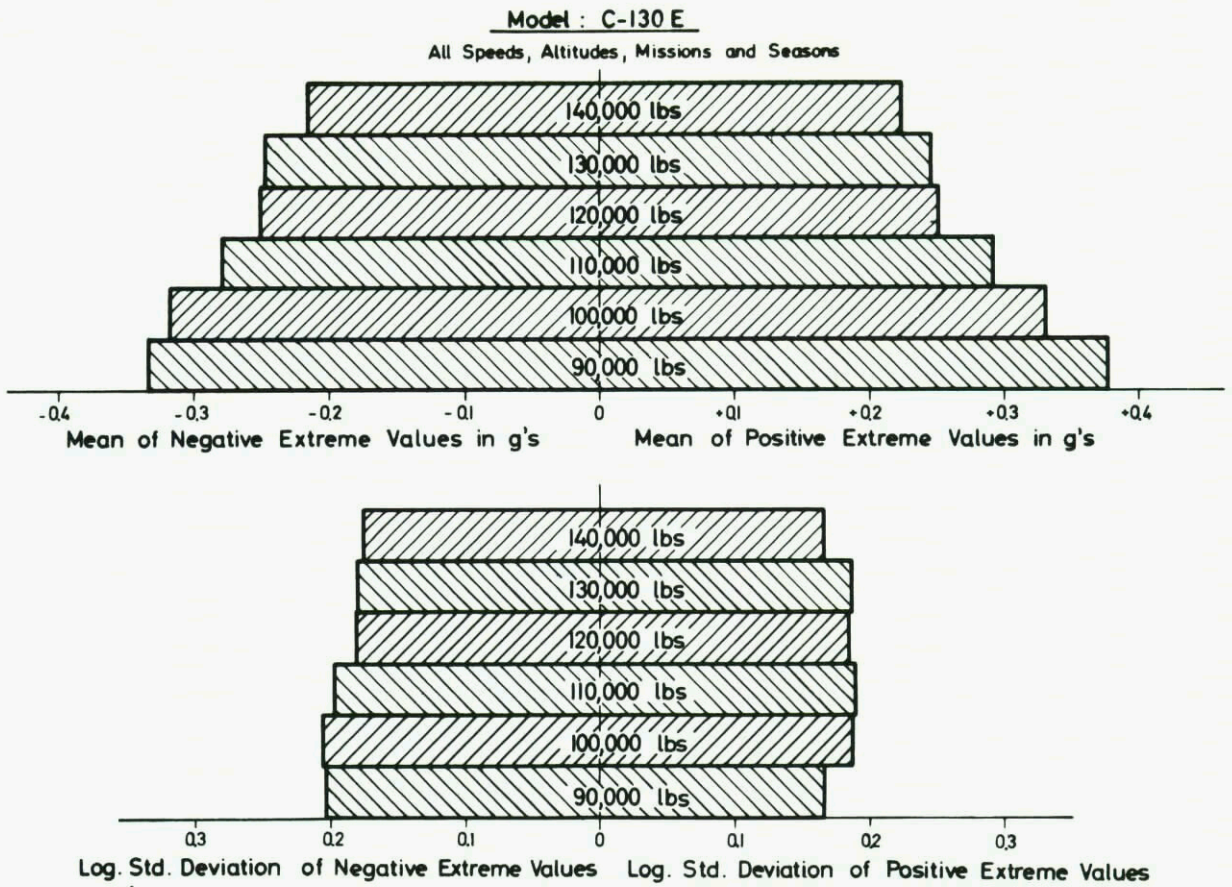


Fig.21 Effect of airplane gross weight on mean value and log. standard deviation

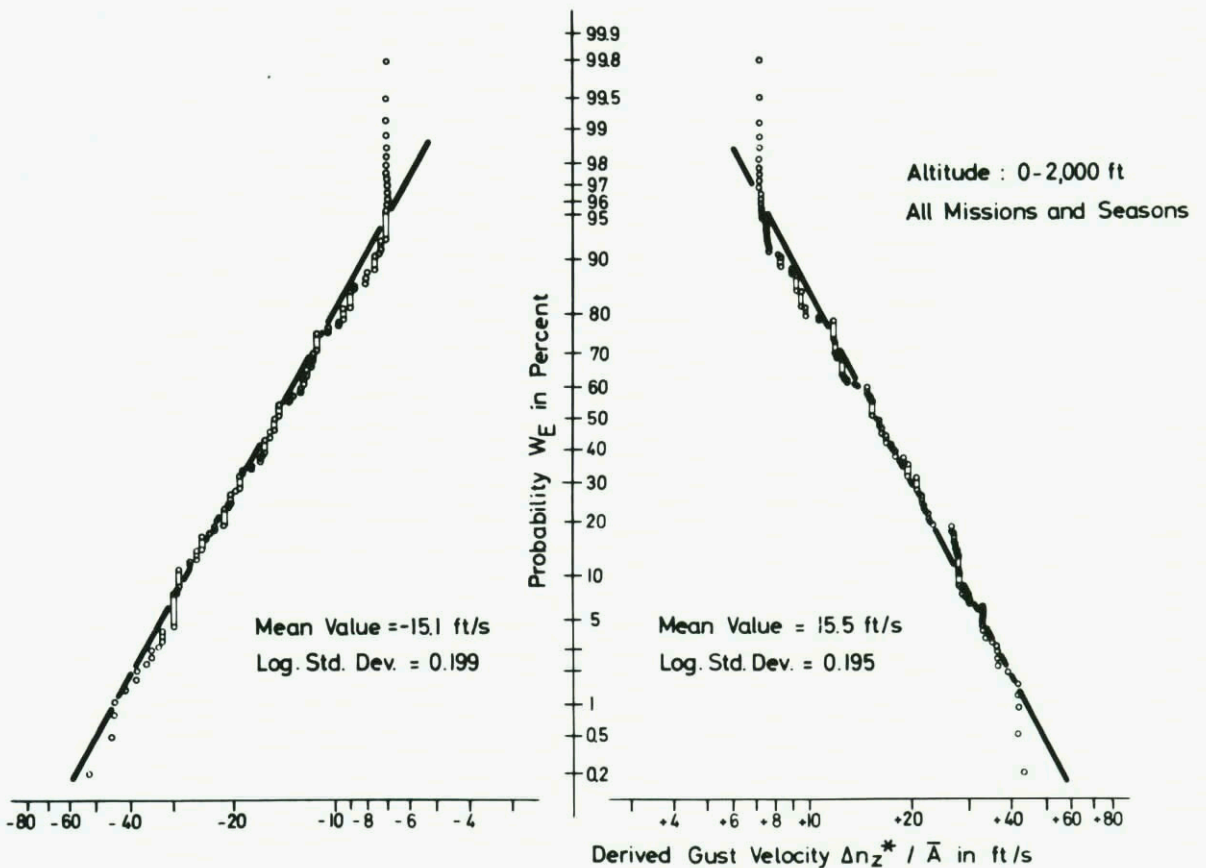


Fig.22 EVD of derived gust velocities from 318 flights of model C-130 E

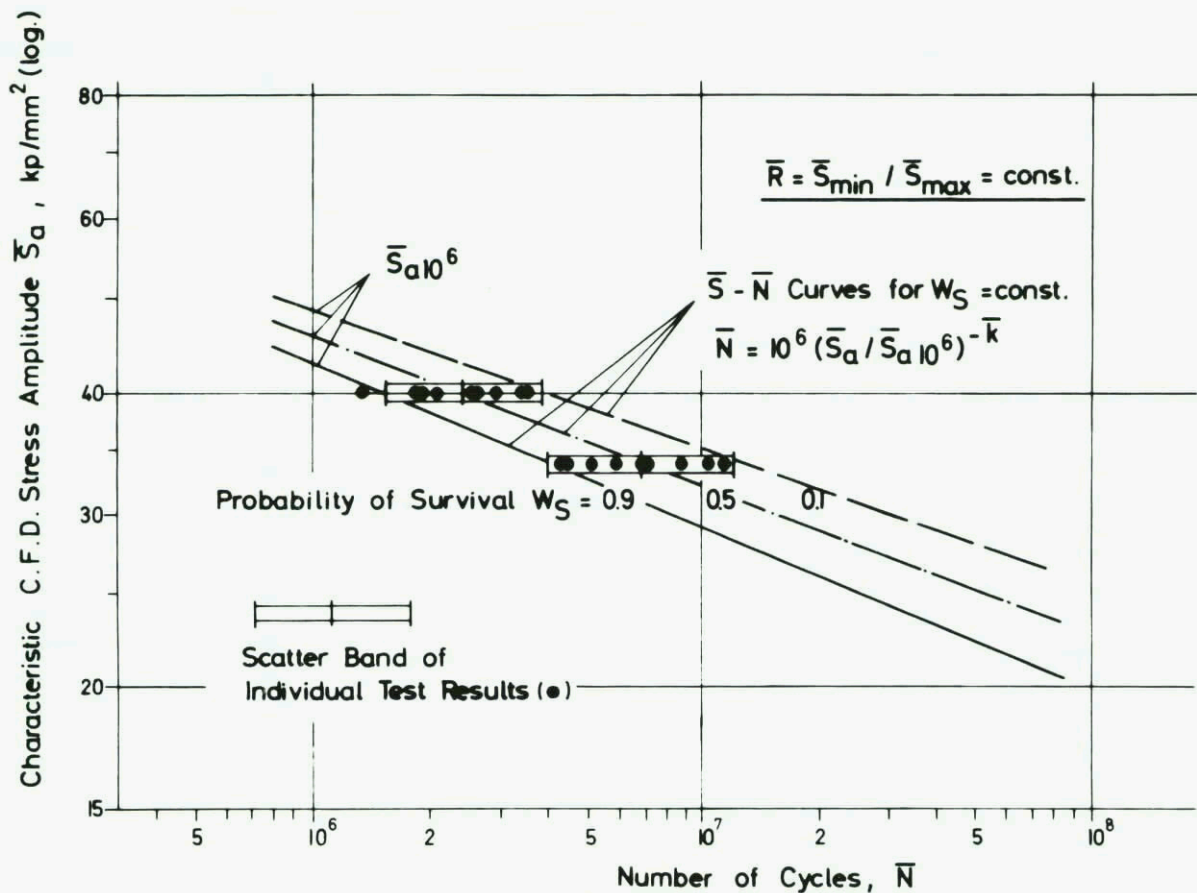


Fig.23 Example for the relation between endurable stress and corresponding fatigue life under variable amplitude loading for a given type of CFD

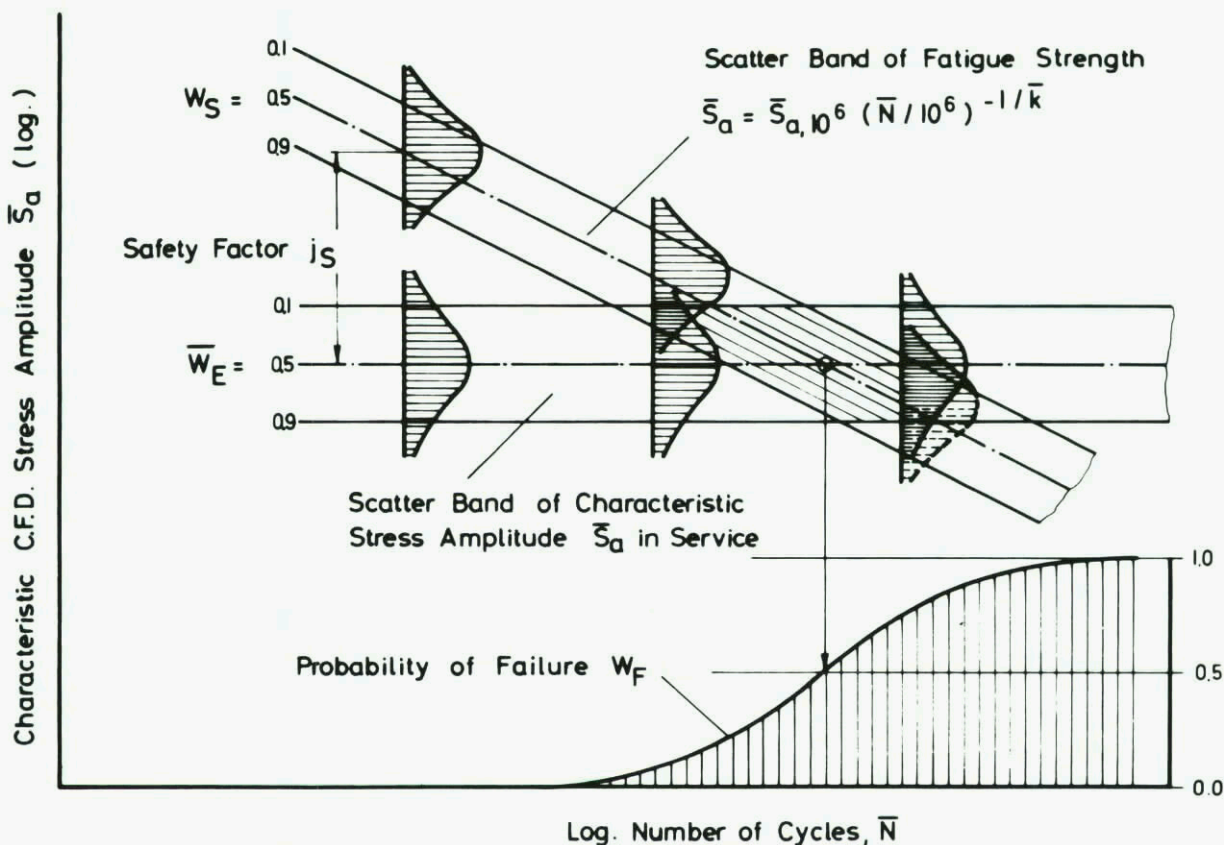


Fig.24 Relation between scatter of CFD in service and natural scatter of fatigue life

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