ADVISORY REPORT 1

NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

A SURVEY ON PANEL FLUTTER

by

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Presented at the 21st Meeting of the AGARD Structures and Materials Panel
in Nancy, France, November 1965

This Report is the result of a series of visits to laboratories in the NATO Nations. It reviews the best theory and experiment in the field of panel flutter and reports research work under way in the nations. A selective bibliography is included.

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FOREWORD

This report presents the results of a survey on panel flutter initiated by the Structures and Materials Panel of AGARD.

At the 17th Meeting of the Panel in September 1963 it was decided to initiate a research co-ordination programme on panel flutter and the first result - a comprehensive literature survey - was reported upon at the 19th Meeting of the Panel in October 1964 and later published as AGARD Report 484.

It was apparent to the Panel that a considerable amount of research on the various aspects of panel flutter was being pursued throughout various NATO countries and that it was timely for a detailed survey to be made in an attempt to summarise and present all the relevant information available on the many topics pertinent to panel flutter.

This study was commenced in January 1965 and concluded in August 1965 and is based on visits made during this time to research workers in France, Germany, Italy, United Kingdom and United States of America. This report also contains the results of the discussions, correspondence and literature survey undertaken during 1964. As a result of this study it has been possible to delineate areas in which further emphasis is required.

ACKNOWLEDGEMENTS

Grateful thanks are expressed to all the establishments and individuals visited, for their willingness to participate in this survey and for their ready co-operation in providing relevant literature and opportunity for technical discussions.

Acknowledgement is made to the Editors of the AIAA Journal and the International Journal of Solids and Structures; to the Directors of NASA (Langley) and NASA (Edwards) and to the Manager of the Aerospace Industries Association of America, Inc., who have all given permission for figures to be reproduced from their technical publications.

Particular thanks are due to Mr G.Cooper, Mr B.Heliot and Mr R.A.Willaume of AGARD (Paris) for general administrative assistance and advice, and to Mr W.Miller and Mr W.S.Mykytow of Wright-Patterson AFB, USA for their assistance in connection with the visits to US establishments.

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Part I of this report presents a survey of panel flutter (and related) research in the NATO countries as determined by visits during 1965. The part concludes with a discussion on panel flutter criteria, draws certain conclusions and makes recommendations for future work. Part II is devoted to the results of the visits to individual research establishments and workers. An extensive (288 references) bibliography on panel flutter is given in Appendix A; selected additional references on static panel aeroelastic behaviour and related references are included as Appendices B and C.

RESUME

Dans la Partie I de ce rapport on présente une synthèse des recherches sur la flottement des panneaux (et des recherches connexes) en cours dans les pays OTAN, ainsi qu'elles ressortent d'une série de visites effectuées à ces pays en 1965. On étudie en conclusion les critères relatifs au flottement des panneaux, en tire certaines conclusions et formule des recommandations concernant des travaux ultérieurs. La Partie II se consacre aux résultats des visites effectuées à des centres de recherche individuels, et des entretiens tenus avec des rechercheurs. Une bibliographie importante (288 références) sur le flottement des panneaux se trouve en Annexe A; un complément de références choisies sur le comportement aéro-élastique des panneaux statiques et des références connexes figurent en Annexes B et C.

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NOTATION

a, b		maximum and minimum dimensions of panel (see Figure 15)
c, c_s		speed of sound in air; speed of sound in material $(E/\rho_s)^{\frac{1}{2}}$
D		flexural rigidity of thin isotropic element, $Eh^3/12(1-\nu^2)$
$D_s; D_Q$		flexural rigidity of thin isotropic sandwich; transverse shear rigidity
E		Young's modulus
F		tensile stress resultant
g, g_s		structural damping coefficient
g_A		aerodynamic damping coefficient (see Figures 28 and 40)
g_T	=	$g_s + g_A$
h		panel thickness
k		reduced frequency, $\omega L/U$; spring constant
l		wavelength
L		streamwise length between supports
M		Mach number
m, n		streamwise and spanwise wave numbers
N		number of streamwise bays; number of modes
N_{xx}, N_{yy}, N_{xy}		stress resultants
$\bar{N}_{xx}, \bar{N}_{xy}$	=	$N_{xx}L^2/\pi^2 D$, $N_{xy}L^2/\pi^4 D$ respectively
Δp		pressure differential
q		dynamic pressure
Q		sandwich core stiffness parameter
R		shell radius; aerodynamic damping parameter, $\rho c/\rho_s \Omega_1$
S		spring stiffness parameter, $kL^4/\pi^4 D$
t		time

ΔT		uniform temperature rise
u, v, w		displacements in x , y , z directions
w_0		initial lateral displacement
U		velocity
V		Liapunov functional
W		spanwise length between supports
x, y, z		co-ordinates
α	=	$h^2/12R^2$
β	=	$\sqrt{(M^2-1)}$
δ		boundary layer thickness
ϵ		panel edge restraint parameter in torsion
$\lambda, \tilde{\lambda}, \bar{\lambda}$		flutter parameters: $2qL^3/\beta D$, $2qR^3/MD$, $2qW^3/4\pi^2D$ respectively
ϕ		orientation of stiffening to axes of rectangular panel
Λ		wavelength
ψ		sweep angle
$\sigma_x, \sigma_y, \sigma_{xy}$		stresses
ω, Ω		frequency
Ω_1		first modal frequency
ρ; ρ_s		density of air; density of material of panel.
ν		Poisson's ratio; axial mode parameter

A SURVEY ON PANEL FLUTTER

D.J.Johns

INTRODUCTION

The outer surfaces of all flight vehicles are generally supported on internal structural members which divide the surface into individual panels forming part of an array. These panels are subject to in-plane loads and normal aerodynamic loads and it is well-known that unstable oscillatory motions of the panels (either singly or as an array) can be caused by the couplings set up between the elastic, inertia and aerodynamic forces due to the panel motion. This phenomenon is known as "panel flutter" and since 1945 a very large literature on this subject has developed (see Bibliography in Appendix A).

Static aeroelastic problems (e.g. panel divergence) have received much less attention (see Bibliography in Appendix B) since the occurrence of such incidents is less likely and their effects are generally of less concern.

Dynamic aeroelastic problems (i.e. panel flutter) are of greater concern since either immediate failure or long-term fatigue may be the result. For those outer surfaces of supersonic aircraft and missiles which are not designed to carry primary structural loads, e.g. on fairings to control surfaces and at wing-fuselage intersections etc. the panel skin thicknesses are often so low from static loading considerations that panel flutter may become the primary design criterion for such panels. A further consequence of supersonic flight viz. aerodynamic heating, complicates this problem since large thermal stresses may arise and if the panel experiences high compressive in-plane loads its susceptibility to panel flutter may be increased.

Appendix A contains many references on the theoretical and experimental aspects of panel flutter but unfortunately many of them are now of limited usefulness since at the time they were written many of the important parameters were not recognised or considered. A critical assessment of most of the earlier papers will not be attempted here for that reason. It should be mentioned that many comprehensive surveys of the panel flutter literature have now been published which should be consulted for further detailed descriptions of these earlier papers. These survey papers are referred to at the appropriate points in this report.

A summary of all the establishments visited during the present survey is given in Table I, beginning on page 50 and the results of these visits are reported in Part II. In some cases security regulations inhibited a fuller discussion of the results of research so that only a superficial treatment can be presented.

PART I

SUMMARY REPORT

1. THE PRESENT POSITION

1.1 Methods of Analysis

In many problems of panel flutter the most obvious methods of analysis have been to apply the Galerkin method, using the governing equation(s) of the problem, or the Rayleigh-Ritz method in which one minimises the total energy of the system. In both cases it is necessary to assume the form and number of modes used and care must be taken that sufficient modes are included to ensure convergence of the flutter solution.

The equivalence of the Galerkin method and the Rayleigh-Ritz method has been discussed in Reference C37 and Singer states therein that such equivalence is ensured only *"when the Galerkin method is applied to the equilibrium equation of the problem that results from the variation of the total potential energy, which is in the form of a generalised force or moment"*. Previous studies of the static instability of circular cylindrical shells with clamped edges using Donnell's eighth-order equation, have been suspect for this reason. This is discussed further in Reference C38. One concludes therefore that if the above condition is satisfied the Galerkin method should be applicable.

However, doubt has been cast on the use of the Galerkin method since spurious flutter results were obtained for membrane flutter at high supersonic speeds when "exact" analyses showed no flutter. Subsequently it has been shown that the Galerkin method also leads to the exact result if a sufficiently large number of modes is used: in other words the previous Galerkin analyses had not proved convergence. The applicability of the Galerkin method to the supersonic membrane flutter problem has been studied recently by Ellen[A267] and found to give results in agreement with exact solutions when they are interpreted correctly. The dynamic equation is first made self-adjoint and a two-term analysis gives the correct answer (i.e. no instability) without it being necessary to investigate the problem more deeply.

Of the other available methods of analysis mention must be made of the collocation method, using aerodynamic and structural influence coefficients, developed by Rodden[A56, A251, A252]. This method (see Part II, Section 5.15) is advantageous in cases of multi-bay panels, and similar, where modal symmetry between bays cannot be assumed.

Despite its initial lack of success[A273] one hopes that Liapunov's method will be explored further. Its potential advantages are (a) no modal assumptions are made and (b) non-linear effects are included with little extra complication.

Other methods listed by Fung[A107] are (i) a Laplace transformation method and (ii) integral equation method. The latter has been used, essentially, in the collocation or finite-difference method by Rodden (see above); the former does not appear to have been developed.

Having obtained the governing equations for the panel flutter problem, the subsequent analysis can be either digital or analogue. Both approaches have had their successes but it is clear that the application of an analogue computer to the finite-difference formulation of the problem has not been too successful (see Part II, Section 5.2).

An increasing number of analyses have now been made, using an exact solution employed originally by Hedgepeth[A38], Houbolt[A67] and Movchan[A30]. These analyses are, of course, valuable to compare modal solutions against and more recent applications of this approach have gone far to clarify certain points of contention, e.g., Dowell[A230] on multi-bay panels, Krumhaar[A147] on circular cylindrical shells. Both these studies were concerned with flutter at high supersonic Mach numbers and the governing differential equation could readily be used. Attempts to provide exact solutions for low supersonic Mach numbers are being considered, but the governing equation is then integro-differential with consequent difficulties introduced.

1.2 Aerodynamic Assumptions

Considerable efforts have been made in recent studies to develop more refined aerodynamics for plane and cylindrical shell panels.

At low supersonic Mach numbers inviscid studies on plane panels by Cunningham[A194], Zeydel and Kobett[A232] and Lock[A251,A255] have given most comprehensive and accurate solutions. One feels that, theoretically at least, this régime is now well covered and with further obvious improvements, e.g., to prove convergence of the flutter solutions, it only remains to apply the techniques to the wider range of panels shapes, boundary conditions etc. already considered in high Mach number studies.

At high supersonic Mach numbers the plane panel aerodynamics are adequately provided for by Piston Theory. Ackeret theory is not recommended, since spurious flutter results have been found for flat isotropic and orthotropic panels which could be removed by the inclusion of the aerodynamic damping term. Obviously, for very low aspect ratio panels piston theory is suspect, since three-dimensional effects become important; also piston theory does not apply to travelling wave motions to which such panels are subject.

For cylindrical shells the aerodynamic studies of Krumhaar[A181], Brown and Holt[A206,A235], Stearman[A229], Anderson[A271], and Platzer (Part II, Section 5.17 - unpublished) are all significant in their contributions. The doubtful validity of piston theory in such analyses has been emphasised and the significant aerodynamic pressure attenuations and phase changes which have been reported should be noted.

Boundary layer effects can be important theoretically, as has been shown (see Figures 7, 11-13), e.g., Anderson[A271], but no reliable model yet exists. Anderson's recent success in correlating experimental results for cylindrical shells with inviscid theory (see Part II, Section 5.3) is most encouraging; also, Cunningham's results for a plane panel[A194] at a low supersonic Mach number have shown that in that case three-dimensional inviscid effects might have been present, leading to better correlation with experiment than previous "two-dimensional" studies and thus indicating that boundary layer effects might not have been significant.

In almost all previous panel flutter investigations there has been no attempt to confirm by experiment the actual unsteady aerodynamic pressure acting on the flexible surface. However, certain investigators are now measuring the aerodynamic pressures on rigid wavy surfaces and this will enable some check on the validity of various aerodynamic theories to be made (see Part II, Sections 5.3, 5.17 and 1.1).

Most workers are conscious of the possible effects of the still-air acoustic coupling in the space beneath the panels to be analysed or tested. Results have shown little such effect when one or more of the panel edges is free, or when the cavity depth is large.

1.3 Structural Assumptions

It is clear that for small deflection thin plate or thin shell analyses one should use the most accurate and consistent set of equations available. For plane isotropic panels there is general agreement on the form of these, but for cylindrical and conical shells there is considerable disagreement in view of the large number of formulations available. The author believes that equations based on the work of Novozhilov[C39] are adequate for panel flutter studies of isotropic shells, although the differences resulting from most other formulations are likely to be small.

Orthotropic structures are receiving considerable attention and caution is advised in the use of well established orthotropic plate theory (e.g. Refs.C35, C36). The application of a theory is recommended which recognises the "built-up" nature of most orthotropic structures and the eccentric character of many structural reinforcements (Refs. A268, C31, C34).

Boundary support conditions can have a pronounced effect on the flutter problem and, in the case of complex structural configurations, analytical prediction procedures for the *in vacuo* modes and frequencies can be quite inadequate; recourse should be made to measured values.

The role of structural damping is now fairly well established. It may be destabilising or stabilising, depending on many factors, and it is recommended that theoretical studies should include this as a variable. There is no trend towards the sole use of either viscous or hysteretic damping and both have been extensively used. Since both forms of damping model are suspect in particular circumstances, this question remains unanswered. For both types of damping the effect is greatest at higher altitudes when aerodynamic damping is least.

Multi-bay panels need special consideration, particularly at low supersonic Mach numbers when the aerodynamic coupling between bays is greater. The boundary conditions and the modes must be determined carefully since structural coupling might be important.

1.4 Flat Rectangular Panels

Flat, rectangular isotropic panels have received far more attention than any other configuration, but very few of the studies merit detailed consideration because of inherent deficiencies in these studies.

For high supersonic Mach numbers Hedgepeth's[A55] and Houbolt's[A67] results are still the best available provided small-deflection behaviour is involved. For panels subjected to in-plane direct and shear stresses the results of References A166, A176 and A223 are of interest and if buckling is involved the results of References A176 and A203 are applicable (see Figure 34). Movchan, in a series of papers, has also analysed rectangular panels with (a) all edges simply-supported[A30, A42] (b) leading and trailing edges clamped but sides simply-supported[A102]. In-plane loads were assumed to act in both cases.

For low supersonic Mach numbers earlier analyses of two-dimensional panels showed that the critical panel thickness was very large and that the flutter mode was probably single-degree-of-freedom. More recent studies[A150, A190, A194, A224, A232] have shown that three-dimensional panels are much less susceptible to panel flutter and that as the panel length/width ratio increases the degree of coupling increases. Thus, for $L/W = 10$ twelve modes are required in Reference A194 to obtain a converged flutter solution. Typical results are shown in Figures 16-19 and 29-32. It can also be seen that the Mach number variation of critical thickness in the range $1 < M < 2$ is much smaller as the length/width ratio gets larger.

For multi-bay panels the present view, based on References A79, A98, A230, A251, A252, is that the two-dimensional (infinite span) panel extending over many bays in the streamwise direction is insensitive to bay number at high supersonic Mach numbers. The conclusion for panels of finite aspect ratio is not necessarily the same, but aerodynamic coupling should be small for all but the smallest aspect ratios, e.g. less than 1. In Reference A150 the same infinite span plate is analysed in the range $1 < M < 2$ and it is concluded that an increase in the number of bays N increases the critical panel thickness. For panels of finite aspect ratio, i.e. $L/W = 1/4$ and 1, the critical single-degree-of-freedom modes at $M = 1.35$ are the 2nd and 3rd respectively, whilst for $L/W = 0$ the 1st mode is critical. The results for the square panel show single-degree-of-freedom flutter at $M = 1.25$ and 1.35, but coupled mode flutter at $M = 1.1$, 1.5 and 2.0. Although Reference A230 contains an exact analysis, References A252 and A150 are only approximate and convergence has not been proved. The results in References A79 and A98 confirm those in Reference A230 where comparable (see Figures 28, 29).

The flutter of flat plates subjected to supersonic flow in the presence of a magnetic field has been considered in References A119 and A165. In this phenomenon electromagnetic energy is exchanged with the gas flow in addition to mechanical energy, with consequent effects on stability. Reference A165 concludes that (a) the critical flutter speed is slightly reduced when the magnetic field is in the plane of motion (b) there is no effect if the magnetic field is normal to the panel.

Reference A248 has re-examined the earlier papers above and concluded that aero-magnetic flutter is definitely not possible in practice when the magnetic field is normal to the surface, although critical flow velocities do exist for infinitely high values of magnetic fields. For finite values no closed-form solution exists but a numerical solution of the governing equations has been made.

The flutter of orthotropic panels has been receiving considerable attention in recent years. Using orthotropic plate theory References A114 and A187 attempted to show that an orthotropic plate could be approximated as an isotropic plate of unchanged streamwise length L and steamwise flexural rigidity D_x, but with a width modified to $W(D_x/D_{xy})^{\frac{1}{2}}$ where D_{xy} is the twisting rigidity with respect to the streamwise and spanwise direction.

Reference A209 has shown that the preferred orientation of the stiffeners in an orthotropic panel is not necessarily in alignment with the flow and that this depends on the panel length/width ratio, amount of damping and amount of orthotropicity. However, as pointed out in Section 1.3, most of the available orthotropic panel analyses are suspect, since either eccentricity effects have not been included or convergence has not been proved. In this last respect the findings of Gaspers (Part II, Section 5.11) are particularly significant if proven.

Sweepback has been considered in References A153 and A166 at high supersonic Mach numbers and shown to have a beneficial effect for low aspect ratio panels and a detrimental effect for aspect ratios greater than unity. This result has been found also for clamped elliptic panels in Reference A197. (See also Figures 14, 15).

The studies referred to earlier[A166, A176, A223] have shown the significant effect on the flutter boundary of various combinations of in-plane direct and shear stresses. Thus, the frequency spectrum of flat isotropic panels is very sensitive to such stresses and frequency coalescences between various modes is possible. Such coalescences only cause weak flutter, in general, which is prevented in practice by the inclusion of quite modest amounts of structural damping (See Figures 8-10).

Generally, streamwise compressive loadings are the most important but spanwise loads are also significant; in fact Reference A176 shows that spanwise *tension* added to a state of biaxial uniform *compression* can be *destabilising*.

Calculations of the panel amplitude under flutter conditions using non-linear theory have been made by various authors. Bolotin[A99] analysed the high M case and used 3rd order piston theory and showed that stable oscillations are possible below the linear flutter speed. Extensions of this work by Kotyukov[A253], who added a third mode in the analysis, have shown a considerable decrease in the amplitude amplification ratios at flutter. Reference A150 gave the flutter amplitude for the range $1 < M < \sqrt{2}$ of a two-dimensional panel fluttering in a single degree of freedom. Results for $M > \sqrt{2}$ are given also in Reference A180.

1.5 Flat Non-Rectangular Panels

Studies of elliptic panels are contained in References A66, A108, A133 and A197 and these can, of course, be reduced to the special case of the circular panel. These studies are mainly for panels with clamped edges at high supersonic Mach numbers and are summarised in References A197 and A217. Among the parameters studied were sweepback, edge support stiffness, membrane stresses, structural damping and concentrated masses.

Further results for circular panels are given in References A113 and A182, but unfortunately there are no reliable experimental results with which comparison can be made.

Similarly, oblique panels in the shape of a parallelogram have also been studied[A241] with sweepback of the panel leading edge as prime variable. This is always beneficial for "aspect ratios", (swept leading edge length)/(streamwise length), less than or equal to unity.

Probably the best approach to the analysis of flat panels of arbitrary contour is to use the direct matrix method of Rodden[A56, A151] with the structural influence coefficients evaluated by a finite-difference method (e.g. Ref.C40).

1.6 Cylindrical and Conical Shells

Many authors have analysed the problem of flutter of an infinitely long isotropic cylinder with an external, axial, inviscid flow[A32, A36, A37, A47, A263]. These analyses are based on widely differing assumptions and have showed quite different results. Thus, References A36 and A37 disagree on the choice of circumferential modes, with Reference A36 concluding that the case $n = 2$ is the most critical whilst Reference A37 considers $n = 0$ to yield the minimum flutter speed. References A32 and A47 have employed piston theory which is known to be inadequate for travelling wave motions on infinitely long panels and shells[A110].

Analyses are given by Dowell[A263] which attempt to overcome the limitations of the previous studies and results are given for incompressible and compressible flow. It is shown that, for all $n > 1$, $n = 2$ is indeed the most critical circumferential wave number, but that for a particular shell (aluminium cylinder at sea level), $n = 0$ is the most critical for $M > 1.52$. However the instability for high Mach numbers is very weak and the low M results (i.e. $n = 2$) may prove to have greater practical significance. In the range $1 < M < 1.52$, $(h/R)_{crit} \simeq 0.095$.

The validity of infinite length shell analyses to practical shell structures has not been proven and, since such large values of critical shell thickness result, they must be used with caution. A simple analysis in Reference A111 showed that the form of solution obtained from a travelling wave type of analysis, when the presence of internal structural restraints (rings or bulkheads) is considered, is similar to that obtained by standing wave analyses for finite length shells. Dowell[A263] contends that the infinite length shell results are applicable to finite length shells in a qualitative sense (see Figure 40) especially as g_A and h/R increase. Unfortunately the values of h/R which occur in practice may violate the conditions on which that contention is based.

For cylindrical panels of finite length it has seemed that inviscid analyses were incapable of predicting accurately the flutter observed experimentally. Certain studies have shown correct qualitative trends; e.g. Shulman[A90] showed that the minimum flutter speed corresponded to a finite value of n which is not necessarily close to the value for minimum *in vacuo* frequency, but his analysis neglected aerodynamic damping. Voss[A97] showed the significance of the in-plane inertia forces on axisymmetric flutter. He showed asymmetric flutter to be critical. His analysis however has apparently shown poor correlation with experiment. Results given in Reference A111 for axisymmetric and asymmetric flutter include the effects of aerodynamic damping but not the in-plane inertias. The latter should not be so important at high n ; but the closed-form solution given in Reference A111 is based on a two-mode analysis only, which is probably quite inadequate even for the high-n case (see later).

It has been shown by Voss[A97] that for $n = 0$ the modal frequency spectrum is very dense and many modes are required for convergence, but at high n this is not so. Results for $n \neq 0$ are given in Figure 25 (Ref.A214).

The above four papers used the Galerkin method and piston theory aerodynamics, which is suspect for the problem considered, and there is a need for much more accurate and detailed aerodynamic studies for the flutter analysis of cylindrical shells. In this respect the work of Holt, Anderson etc. (see Section 1.2) is of great interest. The applicability of the Galerkin method to cylindrical shells has been confirmed by Krumhaar[A147] who has developed "exact" results for the axisymmetric case ($n = 0$).

The influence of the boundary layer on the flutter of cylindrical shells has been examined in References A191 and A196. A uniform, parallel, subsonic layer of constant thickness is assumed to exist between a uniform external supersonic flow and the oscillating circular cylindrical shell. The effect of this layer on the unsteady aerodynamic pressures acting on the shell surface has been shown to be significantly stabilising for high-n flutter. These studies have been developed further in Reference A271.

An experimental flutter criterion was suggested by Fung[A196],

$$\left(\frac{q}{E\beta}\right)^{1/3}\left(\frac{R}{h}\right) \simeq 7 \, , \tag{1}$$

which is seen to be independent of the shell length. Since the test shells were of the same length this criterion needs further confirmation. Additional tests have now been reported in Reference A266 which confirm the form of Equation (1) but these were also on shells of the same length. From the tests in Reference A189 and A266 it is seen that the value of n_{crit} is in the range 14 to 25 and for these shells $L/R = 2$ (see Figure 24). It is of interest to note that for $n = 25$ the effective L/W for each shell element between the streamwise nodal lines is about 16, and for $n = 14$, $L/W \simeq 9$. Equation (1) may be rewritten as

$$\left(\frac{\beta E}{q}\right)^{1/3}\left(\frac{h}{L}\right) \simeq \frac{1}{7}\frac{R}{L} \tag{2}$$

and, if Equation (2) with $L/R = 2$ is compared with Figure 14 for unswept panels, it can be seen that the agreement is good for L/W in the range 9-16. Thus it would appear that for high-n flutter of cylindrical shells the individual panels between longitudinal nodal lines behave similarly to flat rectangular panels and Equation (1) could be interpreted as a special case of the general results shown in Figure 14.

For the high-n, case, Voss[A97] has indicated that aerodynamic damping should not modify the flutter boundaries significantly, presumably because the flutter frequencies are then much lower than for the $n = 0$ case where aerodynamic damping is known to be most important. If the two-mode ($m = 2,1$) closed-form solution in References A111, A217, is analysed for the high n case with aerodynamic damping neglected, it can be shown that[A283], if $n^2 \gg \lambda_m^2$ (with $\lambda_m = 2\pi(R/L)$),

$$n_{crit}^6 = 5\left(\frac{1-\nu^2}{\alpha}\right)\left(\frac{\pi R}{L}\right)^2 . \tag{3}$$

This result was derived also in Reference A90. The corresponding expression for the critical dynamic pressure parameter is

$$\left(\frac{q}{E\beta}\right)^{1/3}\left(\frac{R}{h}\right) \simeq 0.912\left[n_{crit}^2\left(\frac{R}{L}\right)\right]^{1/3} . \tag{4}$$

Inserting the shell data of Reference A189 gives the results

$$n_{crit} = 31 \tag{5}$$

$$\left(\frac{q}{E\beta}\right)^{1/3}\left(\frac{R}{h}\right) \simeq 7.2 . \tag{6}$$

Although the estimate of n_{crit} is in error, the agreement of equation (6) with Equation (1) is remarkably good.

Minimum critical flutter velocities for cylindrical shells have also been found by Shveiko[A118], using piston theory in a Galerkin analysis. A more recent paper has considered the effect on the shell flutter of a liquid contained within the shell[A243]. The results show that the presence of the liquid increased the critical velocity but that the value of n_{crit} was not modified, when the shell was completely filled. The hydrodynamic forces associated with the internal liquid were obtained using Reference C41.

The studies of Kobayashi[A214] also reduce to the above results (Equations (3)-(6)) when the various simplifying assumptions are introduced.

Only three theoretical papers have apparently been published on the flutter of conical shells. Reference A90 concludes from a four-mode Galerkin analysis that, compared with an equivalent cylindrical shell, the primary effect of conical taper is to increase the flutter speed and lower the value of n_{crit} (see also Reference A212). No numerical results were given in Reference A179, which presented a method of solution for a truncated conical shell with an internal supersonic flow. Experimental studies of conical shell flutter are contained in References A19 and A144.

1.7 Curved and Buckled Rectangular Panels

Tuovila and Hess[A85] have reported tests (see also Reference A60) made at transonic and supersonic speeds concerned mainly with buckled curved panels with longitudinal stringers. The air flow was along the generators. By comparison with similar flat panels it was shown that curvature is stabilising, as are deep buckles.

For flow normal to the generators of a curved panel there are many flutter studies. Von Kármán's large deflection plate theory was used for two-dimensional panels in References A177 and A178 and the analyses using linear piston theory showed fair agreement with experiment; the initial panel deflections were taken into account. Similar studies for three-dimensional panels are reported in References A176 and A203. Some results are shown in Figure 34. The most significant result obtained is that an increase in the streamwise compression decreases the flutter speed as long as the panel remains flat but that once buckling occurs the flutter speed is increased. The intersection of the flat-panel and buckled-panel boundaries gives the minimum flutter speed which approximates the "transtability" value given in Reference A55. This latter theory is easier to apply than the full, non-linear theory[A221]. This "minimum" flutter speed has been obtained experimentally in the studies of References A146, A169, A186 and A228.

In Reference A92 a linearised form of Marguerre's equation was used to describe the elastic behaviour of buckled plates. Quasi-steady aerodynamic forces were used in analyses at $M = 1.5$ and 2.0 and it was concluded that increases in initial curvature have a destabilising effect on the panel up to a certain critical value at which the frequencies of the first two modes coalesce and produce a large panel thickness for stability. Beyond this critical value of curvature, increased initial curvature is stabilising. These results have also been shown in two more recent studies[A227, A237] using more refined non-linear plate theory. The implication is therefore that, for a panel of fixed thickness, increasing initial curvature will first initiate instability and later will end it.

Results of Reference A186 are discussed in some detail in Part II, Section 5.19. The curved panels have air flow along the generators and correlation of the flutter data with existing flat panel data was obtained by adjusting the actual thickness of the curved panel to give the same axial buckling stress for the equivalent flat panel (Fig.33).

The variation of the critical flutter parameter for buckled panels with panel length/width ratio is shown in Figure 6. A simple theoretical result, for pinned-edge panels buckled by streamwise compression only, is superimposed and is given by the equation,

$$\frac{2qL^3}{\beta D} = \frac{9\pi^4}{16}\left|5 - 2\left(\frac{L}{W}\right)^2\right| \qquad (7)$$

for $L/W \geqslant 1$. From Equation (7), $q_{crit} \to 0$ as $L/W \to (2.5)^{\frac{1}{2}}$ which is of course impractical, but for $L/W > 2$ the curve in Figure 6 appears to give an acceptably conservative result.

1.8 Other Configurations

Supersonic flutter analyses have been performed for two, parallel elastic panels separated by many linear springs with in-plane stresses acting in both panels. These studies (Fig.35, Ref.A233) have relevance to the design of certain types of heat shield and micrometeoroid bumpers, and have shown that, for certain combinations of spring stiffness, panel rigidity and in-plane stresses, flutter might be very critical.

The possibility of significant coupling between an individual panel and its main, parent structure has been considered in Reference A105 but shown to be remote.

Panels of sandwich construction have been analysed and tested. Reference A72 reports some tests on a honeycomb sandwich panel but flutter was not observed. Analyses are given in Reference A239 (See Part II, Section 5.19).

Multi-layered plates have been analysed in Reference A142.

The flutter of membranes has not received much attention. Subsonic flutter was reported in References A74 and A217 but this appears to have no practical significance. Supersonic flutter has recently been considered in References A26 and A267 and results in Reference A267 indicate that flutter is not possible. A Galerkin method was used and the agreement with exact analysis confirmed.

The subsonic behaviour of panels and membranes has previously been reviewed in Reference A217. There is no conclusive evidence that this flight régime affords any significant problems, although subsonic panel flutter has been reported in flight[A175]. Static aeroelastic behaviour might be of more concern at subsonic speeds. (See Bibliography in Appendix B).

1.9 Criteria

Despite the large number of published papers on panel flutter (Appendix A contains 288 references) one is reluctant to state categorically which criteria should be used for design purposes. It is true to say that any criterion is only as good as the test data or analysis on which it is based and therefore it can only be used with confidence when those tests or analyses were completely rigorous! Only when the proposed design has similar conditions of construction, environment etc. as those from which the criterion evolved can that criterion be recommended. It is for this reason that all criteria should be used with caution.

It is of interest that, from all the papers referred to above, only the following general criteria have been proposed.

For flat and buckled, isotropic rectangular panels the form of Figure 6 was first obtained in Reference A114. Subsequent data modified the figure slightly (Fig. 14) which was reassessed in Reference A175 and Figure 15 gives proposed tentative flutter boundaries which allow for the presence of sweepback. These boundaries apply to panels simply-supported or clamped on all edges and have apparently proved reliable to many of the US aircraft companies, so could be used more widely. The degree of conservatism implicit in Figure 15 is not known but is believed to be acceptable from a weight point of view. For the low supersonic régime References A150, A190, A194, A224 and A232 are available but need further experimental confirmation.

There have been few reported experiences of cylindrical shell flutter, so the question of an appropriate criterion might appear academic. However, Equation (1) has been obtained from tests performed by the California Institute of Technology and, since it agrees both with a simple two-mode analysis (Equation (6)) for simply-supported shells and the flat-panel data of Figure 14 (as a special case), it appears to be quite useful. In view of its agreement with Figure 14 for large L/W, it might be suggested that Figures 14 and 15 together form the basis for criteria for plane isotropic panels and isotropic cylindrical shells. The appropriate value of L/W to use for the isotropic cylindrical shell is outlined in Section 1.6, i.e. $W = \pi R/n_{crit}$, and n_{crit} could be found by Equation (3) or as the value corresponding to the minimum *in vacuo* frequency for the shell overtone mode ($m = 2$). This latter value is suggested, based on the test data of Reference A266. Conversely, Equations (3) and (4) might be considered for general use.

A more general theory for asymmetric flutter of cylindrical shells is that given in References A111 and A214. The results of Equations (3) – (6) are special cases of the analyses in those papers.

For panels or shells of orthotropic construction, criteria are not proposed for the reasons given earlier. In such cases due account must be taken of the eccentric stiffening in a particular analysis.

For panels or shells whose edges are free or elastically supported there are no analyses available which offer general criteria, so that special tests or analyses must be performed.

For infinitely long plane, isotropic panels (viz. with $L/W > 10$) an alternative criterion is that given in Figure 38. The agreement therein from a travelling wave analysis for the infinitely long panel with the standing wave analysis of a finite length panel is quite encouraging.

In conclusion it should be said that many of the excellent studies referred to in this review paper may provide reliable criteria in the future. However, the dearth of confirmatory experimental data does not encourage their use at this time. Where conditions are similar, however, it would be advisable to check design studies against the appropriate analytical results as well as against the criteria in Figures 14, 15 or 38.

2. CONCLUSIONS AND RECOMMENDATIONS

On the basis of the papers reviewed and other information received during this survey it is concluded that:

(i) The efficient design of existing and future structural components requires a consideration of panel flutter if the flight vehicle exceeds Mach 1. Minimum weight design could be penalised by panel flutter.

(ii) Whilst the flutter characteristics of flat isotropic panels have been fairly well established by analysis, conclusive correlation with definite and rigorous experiments is awaited.

(iii) The flutter characteristics of all other structural configurations of interest are less well established, either by analysis or, even more so, by test.

To bring panel flutter research to the point where our understanding of the problems is complete and the uncertainties least, various actions are required and thus the following recommendations are made (not necessarily in order of priority):

(i) A reassessment should be made of the various aerodynamic theories being used and confirmatory data obtained on the unsteady surface pressures acting on oscillating panel or shell elements. Where this is impractical the corresponding information for "rigid" deformation modes should be obtained to determine the general applicability of the theories being used. If, simultaneously, information on boundary layer effects can be determined this would be most desirable.

(ii) All experiments on panel flutter should be preceded by a thorough assessment of the test panel configuration for the particular test environment. Thus preliminary static and/or dynamic tests should be made to ascertain precisely the natural modes of the structure, and acoustic effect of any cavity behind the panel, and the edge support conditions. The possible effect of the thermal environment during the test on the material properties and the natural modes and frequencies of the panel should be assessed.

Tests should preferably be undertaken with external forced excitation of the panel, so that control can be exercised over panel amplitude, mode and frequency. The flutter condition would then correspond to zero input from this external source. This could be approached by varying the excitation frequency and some other parameter, e.g. in-plane loading until the in-phase and out-of-phase components of the external force became zero simultaneously.

Preliminary tests should also be made to ascertain the boundary layer state and the possible magnitude of turbulence-induced vibrations in the panel. The interaction of these vibrations with the flow and the interaction of these vibrations with panel flutter needs to be clarified and the combined effects on large-amplitude panel fatigue should be examined.

(iii) Analytical studies should include, as far as possible, experimental data on panel modes, frequencies and damping coefficients, and unsteady aerodynamic force measurements.

Where "exact" analysis is not possible, multi-mode analyses should be undertaken to ensure convergence of the flutter solutions and such convergence should be proved in the published data, by showing the results of analyses using less than the maximum number of modes. To ease the readers appreciation of the published data, information on the various modal frequencies should also be presented, for example.

(iv) Further experimental and analytical studies are obviously required for cylindrical and conical shells and panels of very low aspect ratio at all Mach numbers. Flat isotropic panel tests are required, particularly in the range $1 < M < 2$, and the development of reliable criteria for all anticipated structural forms is essential.

PART II

VISITS

1. FRANCE

1.1 ONERA

The only paper published so far[A158] contains details of experiments conducted on plane, rectangular panels mounted vertically in the sidewall of the transonic tunnel at Melun-Villaroche. The panel (steel) measured 340 mm (chord) × 200 mm (span) and was mounted with its leading and trailing edges clamped but with its streamwise edges free. Tension loads could be applied to the panel without introducing significant bending moments.

The correlation between the experimental and theoretical vibration modes and frequencies was good, with no significant effect due to the acoustic cavity behind the panel.

The instrumentation for the flutter tests consisted of velocity pick-ups for mode measurement, transducers for unsteady pressure measurement, strain gauges and thermocouples. Panels of 0.43 mm and 0.60 mm thickness were tested and the tunnel Mach number range was from 0 to 1.3.

The tests have proved rather inconclusive because:

(a) the onset of flutter was preceded by a large "static" panel deflection. In this connection, analyses using two-dimensional aerodynamics indicated a single-degree-of-freedom type of flutter whereas tests showed coupled flutter. More refined three-dimensional aerodynamics have shown that the possibility of panel divergence exists. This might partly explain the large initial static deflection.

(b) The existence of a temperature gradient in the air flow and over the panel span has had an unknown effect on the panel tension loads.

(c) It was impossible to control the static pressure differential across the panel with free streamwise edges.

A boundary layer of less than 12 mm was experienced in the tests at Melun but in future tests at Modane much thicker boundary layers will be involved.

Further tests have been reported which show much more satisfactory correlation with three-dimensional aerodynamic theory. These tests have also shown that the variation of modal frequencies due to changes in panel test temperature is small.

As a check on the validity of the aerodynamic theory some pressure measurements have been made on *rigid* profiles corresponding to the two modes which are assumed to be coupling. The computed generalised forces obtained compare well with the theoretical values for tests in the range $1.09 < M < 1.2$.

The flutter analyses made so far appear to be rather limited, since only two modes have been used. The introduction of large deflection effects is partly empirical in that vibration tests are first made to determine the natural frequencies as a function of the initial "static" deflections of the panel which were observed in the wind tunnel tests. The analyses have made use of the source method[C5].

2. GERMANY

2.1 DVL

The main contributions of the principal investigator, H.Krumhaar, have been to the subject of cylindrical shell flutter, most of which was initiated during a Senior Research Fellowship held at California Institute of Technology.

The accuracy of applying linear piston theory to cylindrical shell flutter was examined in Reference A181 and it was concluded that, for radial harmonic oscillations of the form

$$w = \bar{w} \cos (n\theta) \sin (\nu x) e^{i\omega t} ,$$

the linear piston theory expression

$$\Delta p = \rho c \left[U \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \right] \tag{1}$$

can be considered as an acceptable first-order approximation for the case

$$M_1 > 1 , \quad M_2 > 1 ,$$

where

$$M_1 = M - (\omega/\nu c)$$

$$M_2 = M + (\omega/\nu c) .$$

However, the replacement of Equation (1) by the following was suggested as a first-order improvement:

$$\Delta p = \rho c \left[U \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} - \frac{cw}{2R} \right] \tag{2}$$

The effect of the inclusion of the additional term is now thought to be not significant in closing the gap between the theoretical and experimental results mentioned in Reference A147. It was further suggested that, for large values of n, neither of Equations (1) or (2) is acceptable. This is also true for the following cases:

$$|M_1| < 1 , \quad |M_2| < 1 ; \qquad |M_1| < 1 , \quad |M_2| > 1 ; \qquad |M_1| > 1 , \quad |M_2| < 1 .$$

An "exact" analysis has been made for axisymmetric flutter, using Equation (1), which is reported in Reference A147 (see Figure 1). As a by-product it could be shown by a comparison of these exact results with those obtained by Galerkin's method that the use of the latter method leads to dependable results if applied to the corresponding non-self-adjoint eigenvalue problem. However, the larger the value of the velocity of the airstream, the more modes one has to employ to obtain reliable results from the Galerkin method.

It should be emphasised that in-plane inertia forces were neglected in these analyses and that axisymmetric flutter is believed to be less critical than asymmetric flutter. Viscous damping was included in the analyses and both this and aerodynamic damping were found to have a considerable stabilising effect (see Figure 1).

It was proposed to proceed further in three distinct steps:

(a) Add longitudinal inertia forces into the existing axisymmetric analysis.

(b) Consider the asymmetric case similarly.

(c) Use a stress function approach to the asymmetric problem so as to reduce the number of governing differential equations and variables.

3. ITALY

3.1 University of Rome

Following a general interest in large-deflection aeroelastic effects, two papers have recently been published on panel flutter.

Reference A227 deals with the flutter stability of a plate which has constraints to prevent any edge displacements, under the action of a known temperature distribution. The differential equations are linearised and the limit value of the stability obtained.

Reference A237 considers panels with pinned edges which were not permitted to have in-plane displacements. Starting from the large-deflection theory for thin plates, and on the basis of Ackeret aerodynamics (two-dimensional, static), the influence of a normal static pressure differential across the plate was examined. The equations were first linearised to separate out the static deflections due to the pressure (i.e. $w_0 \neq 0$, $u_0 = 0$, $v_0 = 0$) from the dynamic (flutter) deflections (i.e. w_1 u_1 v_1). A two-mode approximation to the deflections was made and the Galerkin method used. The following closed-form stability criterion was obtained,

$$\frac{2qL^3}{\beta D} = \frac{9\pi^4}{16}\left[5 + 2\left(\frac{L}{W}\right)^2 - F\left(\frac{w_0}{h}\right)^2\right] . \tag{3}$$

where F is a function of Poisson's ratio and the length/width ratio (L/W). Curves are presented (see Figure 2) and numerical results given which show that the influence of w_0 is initially destabilising, but then progressively stabilising, as w_0 increases. This result has been shown previously by other authors[A92].

Because of the limited number of modes and the use of two-dimensional aerodynamics the results should be used with caution, particularly for high values of L/W .

Further studies are planned to include aerodynamic non-linearities, and it is also hoped to undertake optimisation design studies for corrugated panels in an aerothermal and aeroelastic environment. Work on stiffened cylindrical shells is also proposed.

3.2 University of Pisa

At present no panel flutter studies are in progress but analyses are being made of the vibration characteristics of stiffened circular cylindrical shells[C6]. The shells are stiffened by transverse webs of equal sizes, spaced uniformly, and by longitudinal stringers of equal sizes, also spaced uniformly. The webs were assumed to be rigid in their own plane and four types of shell mode were considered, viz. symmetric or antisymmetric with twisting alone or bending alone of the stringers. The method of analysis given in Reference C6 results in a transcendental equation for which explicit solutions are not yet available. It should be mentioned that in-plane inertia forces of the shell material have not been included but the radial and rotational inertias of the stringers have been considered.

Extensions are planned to this work to include large deflection effects and aerodynamic forces due to linear piston theory will be added for the panel flutter studies.

3.3 Fiat

No published work on panel flutter exists but design studies have been made for a combination of hemisphere and cone-cylinder representative of the upper stage fairing for the ELDO Satellite Launcher. Figure 3 illustrates the form of honeycomb sandwich construction considered.

The studies attempted to find the thickness of an equivalent isotropic thin shell and relate this to the critical shell thickness from existing theoretical studies. The maximum dynamic pressure region occurs at about $M = 1.3$.

Some information on the static and dynamic characteristics of such a form of construction as that shown should be available when planned tests are completed.

4. UNITED KINGDOM

4.1 Cambridge University

Studies have been in progress for several years on the general topic of "fluid flow with flexible boundaries", which, while not specifically concerned with panel flutter, is of sufficient relevance to it to merit more detailed attention. A review paper on this topic was recently given[C7], which considers an infinite plane non-rigid surface with uniform properties to bound a parallel, or approximately parallel, flow. The conditions for the excitation of surface waves by the flow were examined and, conversely, the influence of the surface motion on the stability of a laminar flow; on the later stages in the growth of disturbances if instability occurs; and on the properties of a fully developed turbulent flow were all discussed. It is clear that the influence of the boundary layer on panel flutter (and the converse) are subjects forming a natural part of the general problem posed. The problem of shearing flow over a wavy boundary is treated in detail in Reference C8.

Starting from Miles's theory of surface wave excitation in Reference C9, Benjamin[C7] gives a clear description of the mechanisms involved in various types of instability for flow over flexible surfaces. References are quoted for analyses dealing with membranes, elastic layers, viscoelastic layers, porous boundaries and brush-like surfaces. The following general conclusions are drawn. Firstly, three types of wave disturbance can be distinguished, each having a separate stability criterion and no other type is possible. These disturbances have been named Class A, B and C and the physical principles underlying this threefold classification have been well stated in Reference C10. Class A disturbances (e.g. Tollmien-Schlichting waves) and Class B disturbances (e.g. water waves generated by wind) are essentially oscillations involving conservative energy exchanges between the different parts of the system but their stability is determined by the net effect of irreversible processes including dissipation and energy transfer from the mean flow through the action of non-conservative hydrodynamic forces. Thus dissipation (damping) is destabilising for Class A, but stabilising for Class B. Class C instability (e.g. of Kelvin-Helmholtz type) occurs when conservative hydrodynamic forces cause a unidirectional transfer of energy to the flexible solid. Class C instability is virtually independent of dissipation in the fluid or solid and for many practical cases the condition approximates to a static "divergence" type of instability.

The comments in Reference C7 are very relevant to those studies being made elsewhere on the role of the boundary layer in panel flutter.

Experimental studies are in progress using a water channel which it is hoped will support the theoretical studies on the stability of laminar flows.

4.2 University of Southampton

Concerning the dynamic response of thin panels to boundary layer pressure fluctuations, it is believed that the possibility of panel flutter causing changes in the boundary layer characteristics sufficient to react back on the dynamic behaviour of the panel is remote. It was pointed out that turbulent boundary layers at both subsonic and supersonic speeds have a frequency spectrum which shows the greatest energy content at very high frequencies. The scale of the turbulent eddies is of the order of the boundary layer thickness and should, in general, be much smaller than the flutter wavelength of the panel. Therefore no significant coupling should occur with the lower frequency panel flutter problem.

Work on the response of thin panels to turbulent boundary layer pressure fluctuations – and the effect of the panel motion on the boundary layer – was done with Ram at the College of Aeronautics (See Section 4.3). Earlier theoretical and experimental studies there on pressure fluctuation are given in References C11 and C12.

Experimental work at Southampton on boundary layer excitation of typical aircraft structures has been conducted for some time. Unsteady pressure measurements on rigid walls have been used to calculate the panel dynamic response. Prior to the actual dynamic response tests, knowledge has been gained of each panel's precise modal shapes, frequencies and dampings. Correlation of these results with prediction has been good, with rotary and in-plane inertias included where appropriate. A useful summary of recent work is given in Reference C13 and mention should also be made of the review paper Reference C14.

The second method of Liapunov has proved useful in a number of stability problems involving *ordinary differential equations* but the first application to a panel flutter problem, involving a partial differential equation, is believed to be given in Reference A273. The problem considered is that of a two-dimensional plane panel in supersonic flow, for which piston theory may be used to obtain the aerodynamic forces. This problem had been analysed previously, using a two-mode Galerkin method[A117].

Briefly, the Laipunov method involves finding a functional, V, which is a positive definite function of certain variables, for example displacements and velocities of the panel, which describe the motion of the system when it is disturbed from its equilibrium position. If dV/dt can be shown to be a negative definite function of these variables then $V \to 0$ as $t \to \infty$ and the disturbed motion must die out.

The problem of course is to find a suitable functional and unfortunately the degree of conservatism shown by Parks's results in Reference A273 leads one to assume either that a better functional exists than the one selected or that the method is not applicable. The stability criterion obtained in Reference A273 is

$$U^2 < \left(F + \frac{\pi^2 D}{L^2}\right) \Big/ \rho_s h \ , \tag{4}$$

whereas from Reference A117 one obtains, by a Galerkin approach,

$$U^2 < \frac{81\pi^4}{256\rho^2 L^2 c^2}\left(F + \frac{5\pi^2 D}{L^2}\right)^2 + \frac{45\pi^2}{128}\left(F + \frac{17\pi^2 D}{5L^2}\right) \Big/ \rho_s h \tag{5}$$

The second term in Equation (5) corresponds to aerodynamic damping which is often neglected, but if $L \to \infty$ it becomes most significant. It is then seen that the Liapunov criterion gives a result of the order of a term which is often neglected in the Galerkin approach. The ratio of critical velocity for $L \to \infty$ from Equations (4) and (5) is 1.86 to 1.

Equation (4) has a form similar to that obtained by Miles[A26] for waves travelling on an infinite chord panel (with L replaced by the half-wavelength). Except for very large chord panels Equation (4) is very conservative. However, further examination of the Liapunov method for panel flutter studies is recommended, since it is well suited to dealing with non-linear structural and aerodynamic terms in the governing differential equations.

4.3 College of Aeronautics, Cranfield

At the present time there is no direct interest in panel flutter. All the earlier work conducted by, or under the supervision of, the author has already been reported in detail in Reference A217.

A programme has, however, been under way for some time on boundary layer pressure fluctuations at subsonic speeds which is relevant to the present survey. Tests to date have measured velocity fluctuations in the boundary layer using hot-wire anemometers (i.e. local effect) and, also, pressure fluctuations have been measured on rigid walls (i.e. integrated effect). These latter values have been presented in the doctoral thesis of T.Hodgson and are discussed in Reference C12.

Using the measurements of turbulent boundary layer pressure fluctuations, Ram has attempted to deduce the corresponding deformations and stresses for a flexible two-dimensional membrane of infinite chord. It is concluded that although the membrane is excited the motion produced does not interact back on the boundary layer characteristics. Although these studies are for subsonic speeds it is thought that the conclusions apply in a qualitative sense to supersonic flows, since the in-phase and out-of-phase components of the pressure fluctuations do not themselves vary significantly from subsonic to supersonic flows. These studies are expected to be published shortly.

4.4 NPL

There has been little active interest in panel flutter here since Acum's early papers on the subject[A16, A40, A59] but work is now proceeding on an experimental investigation into the effects of the boundary layer on panel flutter at low supersonic speeds.

The general scheme is for a series of measurements of the flutter critical conditions and flutter modes for carefully controlled experimental conditions at $M \simeq 1.2$ and comparison of these results with theoretical predictions. The rectangular test panel will be clamped at its leading and trailing edges and free along its side edges and an arrangement is to be incorporated to apply (and measure) chordwise tension to the panel. The panel will be 14 in span and 20 in chord and will be mounted flush in the floor of the tunnel and spanning it.

It is intended that the relatively thick turbulent boundary layer will be partially removed in stages by applying suction ahead of the panel, and thus the effect of the boundary layer on the flutter determined. Considerable difficulty is being experienced in engineering a suitable arrangement for the suction system.

Studies are also in progress by Wills to investigate the boundary layer characteristics subsonically on a flexible panel. It is hoped that travelling wave motions will be set up and that correlation will be possible between the unsteady surface pressures and panel displacements.

The panels are of 14 in chord and 18 in span clamped at the front and rear only and of various thicknesses between 0.002 in and 0.010 in in steel. Panel tension can be applied by adjustable springs at the rear.

The maximum free-stream velocity in the tunnel is 160 ft/sec and it is hoped to obtain running waves in the panel as the peak intensity wavelengths in the pressure fluctuations will be about a third of the panel chord. The reflection properties at the ends of the panel will be varied by introducing damping and it is hoped to vary the response from modal to running waves. Capacitor displacement gauges will be used to measure the spectrum of the panel displacement forced vibrations which will be compared with that for the pressure field.

4.5 RAE

Following the early papers by Davies[A59] and Smith[A128] there has been little active interest in panel flutter.

However, work is now in progress by Sobey, who is viewing the problem as a fundamental investigation into the stability of nonconservative systems.

The recent paper by Parks[A273] has been used as a basis and Liapunov's method applied, using a series of assumed modal shapes. With the functional given in Reference A273 it was found that a similar form of criterion was obtained to Equation (4) and that only about 18% improvement could be found to reduce the conservatism of Equation (4). (See Reference A273 and Figure 4).

In fact the criterion obtained, which may be compared with Equation (5), is

$$U^2 < \frac{9\pi^2}{64}\left(F + \frac{\pi^2 D}{L^2}\right)\Big/\rho_s h \ . \tag{6}$$

It is anticipated that "exact" solutions will be found to the governing differential equation for *non-linear* panel flutter by converging on the precise values of M_{cr} and ω_{cr} so as to satisfy all the boundary conditions to the problem. The problem of axial buckling of a circular cylinder with initial imperfections has already been treated on similar lines[C15]. There the total potential energy of the cylinder was minimised by the Fletcher-Powell method[C16].

Sobey has translated certain relevant Russian papers on the subject of non-linear panel flutter[A99, A253]. These translations will be published shortly.

4.6 Loughborough College of Technology

Earlier studies by Johns on the flutter of flat panels and cylindrical shells have been reported in References A66, A111 and A197. Reference A66 showed that, for two-dimensional, isotropic, simply-supported flat panels, aerodynamic damping as given by linear piston theory could be neglected. Reference A66 also contained the first analyses for non-rectangular panels, viz. elliptic, clamped-edge panels. The cylindrical shell studies in Reference A111 gave a closed-form modal solution, using Novozhilov's shell equations, and included aerodynamic damping terms. The solution was applicable to asymmetric and axisymmetric flutter. These various studies and others have been summarised in several review papers, viz. References A217 and A258.

By extending the Galerkin-type analyses of Reference A111, Johns is attempting to generalise rectangular panel and cylindrical shell flutter solutions so as to investigate, in an easy but arbitrary manner, variations in many variables. The philosophy behind this work is that the modal frequencies are first found for any combination of these variables, e.g. in-plane stresses, geometry of panel, thickness etc. and the flutter determinant is solved quite simply in terms of the unknown modal frequencies. This approach is analogous to conventional aircraft wing flutter studies. Figure 5 presents some typical results which are completely general and it only requires the unknown modal frequencies to be determined for any panel or shell configuration to obtain a flutter solution. $R = 0$ corresponds to zero aerodynamic damping, which is normally unjustifiable. The stabilising effect of aerodynamic damping is particularly noticeable for modal frequency coalescence (i.e. as $\Omega_r \rightarrow \Omega_s$) which has been observed elsewhere (e.g. Reference A166).

Associated with this study an experimental and theoretical investigation has been started into the vibration characteristics of stiffened and unstiffened circular cylindrical shells.

Work has just started to investigate the possible interactions of subsonic flutter and turbulence-induced vibration of thin flat panels. Since turbulence can induce forced vibrations but the boundary layer can apparently stabilise self-induced (flutter) oscillations, the role of the boundary needs careful experimental examination. Initial tests will be conducted with very thin boundary layers and the panel displacements and stresses measured at subcritical and critical conditions. Progressive thickening of the boundary layer will be introduced later and the effects measured.

It is well-known that for certain combinations of in-plane stress the critical thickness for flutter can become very high. Reference A258 contains a very simple criterion for a buckled, simply-supported rectangular panel which, when plotted on available experimental data, shows a moderately conservative flutter boundary (Fig.6).

4.7 Other Contributions

Correspondence and/or discussion was also had during 1964 with the following representatives of aircraft companies and research establishments in the UK:

Mr G.C.C.Smith	British Aircraft Corporation	Bristol Division (Aircraft)
Mr S.A.Smith	British Aircraft Corporation	Bristol Division (Guided Weapons)
Mr D.C.L.Francis	British Aircraft Corporation	Bristol Division (Guided Weapons)
Mr H.P.Hitch	British Aircraft Corporation	Weybridge Division (Aircraft)
Mr J.Perry	Hawker Siddeley Dynamics,	Coventry Division (Guided Weapons)
Mr J.C.A.Baldock	Royal Aircraft Establishment,	Farnborough
Mr W.G.Molyneux	Royal Aircraft Establishment,	Farnborough

It transpired that *no* studies to advance the art have been made by the aircraft companies consulted. Of course all their design studies have been examined using existing panel flutter criteria, and considerable reliance is placed on the criteria in NASA TN D-451[A114] and subsequent similar papers.

In one case it was stated that "normally no panel stiffening is put in hand merely because the panel fails to satisfy the criteria, but a careful watch for cracks or damage is kept on these panels particularly during the flight test programme".

Whilst the seriousness of panel flutter was recognised for the various supersonic aircraft projects considered, there was thought to be no real problem for the guided weapon projects discussed.

There was general agreement on the need for reliable prediction methods, as it was felt that most current theoretical analyses cannot be used for design purposes. However, none of the companies consulted apparently intended to sponsor any panel flutter research in the near future, though willingness to undertake externally sponsored work was expressed in some cases.

5. UNITED STATES OF AMERICA

5.1 Air Force Flight Dynamics Laboratory

Several panel flutter research contracts have been placed with various aircraft companies in recent years and these are described fully in Reference A218. The objective behind these contracts is based on the lack of confidence in existing theoretical studies because of the lack of correlation of these studies with available experimental data. Thus, one of the major features of the current programmes is the strict control over *all* important variables in the experimental investigations, such as (a) temperature difference between the panel and its frame (b) the depth of, and pressure within, the underlying acoustic cavity (c) mechanically induced stresses. The ultimate objective is to develop suitable prediction methods, to produce reliable and repeatable experimental data and to develop design criteria for future vehicles.

Of the various programmes described in Reference A218 only that dealing with free-flight research on the project ASSET is not completed. Reports dealing with the other programmes should be published shortly. The contractors are Boeing (flat panels $1 < M < 10$), North American (SST programme) Lockheed - California (re-entry structures etc.) and McDonnell (ASSET).

The emphasis in these programmes has been on the correlation between experimental and theoretical results. Therefore experiments have been performed with strict control over the important variables and care has been taken in analyses to include all known effects of edge support conditions, panel loadings, etc.

Internal research programmes are mainly concerned with experiments into the vibration characteristics of flat panels and shells. In the latter, Bozich has shown good correlation of his experimental results with the theoretical results in Reference C17 for a free-free shell of radius/thickness ratio = 231, length/radius ratio = 2.667 and 9 in radius.

The ASSET programme referred to above is concerned with the flutter of corrugated panels on an advanced performance vehicle in a near re-entry environment. It has been realised[A218] that the first consideration in evaluating the flutter characteristics is the local flow condition over the panel, which may be very much different from the free-stream conditions. A paper by Langley[A254] presents the panel flutter parameters and trends for several typical trajectories of advanced vehicles. A representative radiative corrugation stiffened panel was assumed to be located on the underside of the vehicles considered. The calculations were based on the critical thickness parameter for a reference panel as determined by windtunnel test and extrapolated to allow for higher Mach numbers and the various trajectories. For some vehicle configurations and flight conditions local values of M up to 24 may be experienced by the panels but the most critical speed range was generally in the transonic and low supersonic region of boosted flight. In some cases the thickness to prevent panel flutter was nearly constant for free-stream Mach number from 1.4 to 7.0.

5.2 North American Aviation

Reference A141 contains a detailed summary of most of the available published literature on panel flutter up to May 1961 and concludes with recommendations for future research - one of which is for "the development of improved analytical techniques using modern analogue and digital computers so that solutions can be obtained which are not affected by simplifying assumptions".

Subsequent investigations have followed this recommendation and resulted in two published papers[A213,C18]. In Reference A213 a high Mach number was assumed so that piston theory could be used. The effects of non-linear piston theory terms were negligible. A potential flow analysis was also made but approximations introduced into certain integrals resulted, effectively, in an Ackeret-type solution, i.e. the aerodynamic damping ignored.

The plate was represented by a large number of closely spaced mass points, the deflections of which enabled the inertia forces and aerodynamic forces to be obtained. Vibration and flutter analyses were made on the analogue computer, with an applied oscillatory external force to excite the motion initially.

It was found that, for a given panel aspect ratio, the best mesh size to represent the panel is that whereby each internal box is a square. This gives the least error function in the panel fundamental frequency.

The pressure on the back face of the panel for small cavity depths was included approximately, as was the effect of the boundary layer (Mach number at displacement thickness used instead of free-stream value); see Figure 7.

A standing wave type of flutter was obtained for square and high aspect ratio panels - and a travelling wave type for low aspect ratio. The results agreed with those of Kordes and Noll[A152] for panels with sweepback and showed that the method was quite simple for simply-supported panels. For other edge support conditions this is not so and the main conclusion reached is that it is probably easier in general to use the Galerkin method directly than to set up a finite-difference approach on an analogue computer.

The variation in the critical dynamic pressure for variations in panel amplitude, cavity depth etc., are sufficiently large to justify extreme care in any experimental studies to reduce the scatter due to variations in such parameters.

5.3 University of Michigan

The main publication in recent years has been a paper[A166] in which stability boundaries were calculated for several cases of simply-supported rectangular panels, either buckled or unbuckled, in the presence of in-plane shear and normal edge forces, with or without sweepback. Small deflection thin-plate theory and Galerkin's method were used and convergence of the flutter solutions was proved in most cases by using up to 16 modes (24 in one case) in the analyses (Fig.8). Static aerodynamic theory[A55] appropriate for $M > 2$ (approximately) was used and the neglect of aerodynamic damping was shown to give spurious flutter solutions when the loading was such that frequency coalescence between a pair of modes occurred (Fig.9). Flutter boundaries for swept rectangular plates have also been presented (Fig.10) and show, by comparison with results from Reference A152 for 4th-order analyses, the importance of proving convergence of the solutions.

Analyses for buckled plates show results similar to those in Reference A203.

Future research will be aimed at exploring the non-linear behaviour of a fluttering panel with an attempt to predict the oscillatory stresses set up under amplitude limited conditions in terms of the initial displacement of the panel.

The influence of the boundary layer on panel flutter has been under active study by Anderson. In Reference A191 the boundary layer is idealised as an annular region of parallel uniform subsonic flow, of Mach number M_δ, surrounding a cylinder which may be oscillating in an asymmetric standing wave having the form

$$w(x,\theta,t) \quad = \quad \bar{w} \cos(n\theta) \sin\left(\frac{m\pi x}{L}\right) e^{i\omega t} \; .$$

The corresponding pressure distribution on the cylindrical surface has been evaluated[A271] for various values of n and $l/2\pi R$, and show several interesting results (Figs.11 and 12), where $l = L/2M$, l being the wavelength and L the cylinder length.

(i) For a stationary wavy wall with $n > 0$ and with no boundary layer, the potential solution of Leonard and Hedgepeth differs greatly from piston theory. For this case, linear piston theory is accurate only when the ratio of axial wavelength to circumferential wavelength is much less than $\sqrt{(M^2 - 1)}$, i.e.

$$\frac{nl}{2\pi R} \ll \sqrt{(M^2 - 1)} \; .$$

(ii) The boundary layer on a stationary wave wall attenuates the pressure and tends to shift the pressure distribution upstream. The maximum effect corresponds to the shorter axial wavelengths and the least effect occurs in the axisymmetric case ($n = 0$). For $n \geqslant 7$ a boundary layer thickness ratio $\delta/R = 0.01$ reduces the pressure by an order of magnitude.

Because of the assumptions made in the boundary layer idealisation the results obtained are suspect and it is hoped that wind-tunnel tests on wavy-walled cylinders will justify some of the assumption.

Three cylinders have been machined from solid brass to have 12 axial waves of 1.57 in wavelength and 0, 10 or 20 circumferential waves. The wave height is 0.08 in and the cylinder diameter is 10 in. In retrospect it is thought that the former dimension (0.08 in) is too large, since the experiment requires that $\delta \gg 2\bar{w}$ and $\delta \ll l$ (wavelength), and it is difficult to get a thick boundary layer (δ) or to machine a small $\bar{w}$.

Experiments to date at $M = 3.0$ have not shown the anticipated phase change[A277]. Some pressure attenuation was noted. A firm comparison between this experiment and the idealised boundary layer was not drawn because of the difficulty with the wave height, as mentioned above.

When the above analyses were applied to the problem of cylindrical shell flutter the results shown in Figure 13 were obtained. A four-mode Galerkin procedure was followed, based on the Donnell shell equations, and the considerable stabilising influence of the idealised boundary layer shown. It should be noted, however, that $n = 22$ was assumed throughout and this is not the critical value for the conditions shown.

5.4 Northwestern University

No work is currently in hand on panel flutter. Previous efforts were concerned with the flutter of conical and cylindrical shells[A90] which at that time (1959) represented a considerable improvement on previous theories in that the full set of shell elastic equations were used with no restrictive simplifying assumptions. The studies indicated that minimum flutter speed occurred in an asymmetric mode with n large. Unfortunately aerodynamic damping was neglected and convergence not proved, but it is of interest that few papers have been published since dealing with conical shells.

General studies into the stability of elastic systems subject to non-conservative forces are being conducted by Herrmann[C19,C20]; though not directly applicable to panel flutter research they are of fundamental interest.

5.5 McDonnell Aircraft Corporation

In 1960 a survey was undertaken[A175] in which a questionaire was sent to various US companies requesting details of panel flutter incidents experienced on flight vehicles. Some 82 incidents were reported of which 81 were on aircraft. Only 31 incidents included precise Mach number information as follows: for $0.63 < M < 1.0$, 6 incidents; $1.0 < M < 1.5$, 18 incidents; $1.5 < M < 2.2$, 5 incidents and for $M > 2.0$, 2 incidents. It is believed, however, that most of the remaining 51 incidents were in the range $1 < M < 1.5$. Reference A175 also contains all available wind tunnel data of panel flutter.

The flight test data for flat, uniform, panels (swept and unswept) is shown in Figure 14, together with the corresponding flutter boundary for unswept panels from Reference A114. The additional swept panel data was found to be above this boundary.

An improved presentation of all the data was obtained when it was replotted as in Figure 15. Here, regardless of the flow direction, b represents the minimum panel dimension, a represents the maximum panel dimension and the sweep angle ψ is measured relative to a. It was suggested in Reference A175 that until more refined data becomes available the tentative swept boundary be used for $20^\circ < \psi < 70^\circ$.

It should be emphasised that the data, and hence the boundaries, only refer to "rectangular" panels having all edges supported. The most critical range of Mach number was 1.2 to 1.3.

When the wind tunnel data (mainly on buckled panels) was plotted in the same form as Figure 15, a more conservative flutter boundary was obtained but it was not recommended that this be used in design for panels which are relatively free of high in-plane stresses, because of consequent weight penalties.

The philosophy adopted in aircraft design and test was expounded by Katz and is summarised here.

(a) Every single panel is checked using the criteria described above (Fig.15) with suitable approximation for non-rectangular panels.

(b) Conservative design data is used for vital panels, e.g. in or near engine intakes.

(c) No equipment or system components are attached to external panels in case the panel vibration and/or fatigue should lead to excessive vibration or to failure of the equipment or system. Fracture of a hydraulic pipe to the undercarriage and eventual loss of the aircraft was reported in the list of flutter incidents.

(d) If flutter occurs during flight tests remedial action consists of increasing the panel thickness or decreasing its size. The addition of damping materials to the panels is also available.

(e) For less vital panels, e.g. non-load-carrying panels, the requirements may be relaxed somewhat. Flight-testing with instrumented panels showed flutter of some panels that did not meet the criteria of Figure 15, whereas others did not flutter. Katz believes that, with the above approach and with criteria in Figure 15 proving reliable, the panel flutter problems for unbuckled panels on aircraft are essentially resolved.

With regard to missile designs, Tucker believes that the designs have been very conservative since so few panel flutter incidents have been reported. The criteria of Reference A175 have been used where appropriate, with due allowance for local flow conditions especially at high angles of attack. Panels are first designed for manoeuvre and thermal stresses and then checked for flutter. For heat shield panels the mountings are designed to minimise thermal stresses yet be rigid against flutter.

A theoretical investigation is currently in progress similar to that reported by Johns (Section 4.6) in which the modal frequencies and structural dampings are retained as variables in a two-mode analysis. Tucker believes that panel flutter analyses can be simplified by reducing the number of variables involved. The effects of these variables would be included in preliminary dynamic analyses to determine the *in vacuo* modal frequencies etc. Similar work has also been reported by Postel (Section 5.14).

A major programme is currently underway on Project ASSET, sponsored by the Air Force Flight Dynamics Laboratory, to study aerothermodynamics, high temperature structures and aerothermoelasticity in free flight. The panel configuration chosen is corrugated with clamped leading and trailing edges but with free side edges. The manner whereby the panel is tensioned many times during flight is quite ingenious[A218] and will, it is hoped, enable many flutter points to be obtained during each flight.

5.6 Midwest Research Institute

A theoretical investigation has recently been published[A170] for the flutter of a ring- and longeron-stiffened cylindrical shell subjected to external, axial supersonic flow. The method used is rather tedious and requires the determination of structural influence coefficients for the individual panel radial deformations under concentrated radial loads. The longerons and rings are assumed to be infinitely rigid in bending but infinitely flexible in torsion. Krumhaar's modified piston theory is used[A181] but the additional term in w/R (see Section 2.1) is found to be unimportant.

The results justify treating the various panels around the shell as being independent and should be comparable, in view of the assumptions made, to certain other shell analyses (e.g. References A97, A111).

Reference A229 contains an extended discussion of the aerodynamic pressure on cylindrical shells in unsteady supersonic potential flow. An "exact" method of analysis is outlined and the validity and utility of various approximations considered. No flutter solutions are given but calculations of the aerodynamic pressure coefficient indicate the possible errors resulting from the use of quasi-steady or high M approximations.

Previous shell studies[A189] were mainly concerned with an experimental investigation in the 8ft × 7ft Ames supersonic tunnel. Preliminary vibration tests indicated "nearly clamped" edges and a frequency spectrum having good agreement with the theory of Arnold and Warburton. The shells were 16 in long, 16 in diameter, with thicknesses, in copper, of 0.002 in to 0.004 in. The onset of flutter was readily determined from measurements of dynamic displacement and a criterion was developed, and since published by Fung[A196], in the form

$$\left(\frac{q}{\beta E}\right)^{1/3} \frac{R}{h} \simeq 7 . \qquad (7)$$

The criterion is independent of the shell length and since all the shells tested had the same length it must therefore be suspect as regards its general application.

A new series of experiments is now planned for the low supersonic speed régime with similar cylindrical shells also formed by an electro-deposition process. The instrumentation will permit the measurement of chordwise and circumferential displacements during flutter.

The work of Kobett has mainly been concerned with detailed studies of flat panels in the low supersonic range ($1.1 \leqslant M \leqslant 2.0$) and various papers have been written (Refs. A190, A224, A232).

In Reference A190 an analysis is given for an infinite span plate separated into an array of rectangular panels. Linearised three-dimensional aerodynamic theory is used and numerical results are given showing the effect of Mach number, aspect ratio, number of panels chordwise (N), panel material, altitude and degree of edge restraint. A constant value of 0.01 for g, the structural damping coefficient is taken.

Using four modes in the Ritz-Galerkin method, the results are presented in a comprehensive series of graphs and Figures 16 and 17 are typical. As the panel edge restraint increases from $\epsilon = 0$ (simply-supported) to $\epsilon = 10, 100, 1000$ it is seen (Fig.16) that the critical panel thickness decreases. Results for all $\epsilon \geqslant 100$ are virtually the same, which suggests that $\epsilon = 100$ is approximately fully clamped.

For a "nearly" clamped edge condition ($\epsilon = 100$), Figure 17 shows the effect of varying Mach number for a square single panel. The thickness ratio to prevent flutter in the low supersonic speed region is about twice that required at $M = 2$.

Increasing the number of panels chordwise to $N = 2$ was shown to have a destabilising effect.

Unfortunately, the majority of the results presented are for $M = 1.35$ and subsequent studies have shown that, for the various values of L/W considered (i.e. 0, 1/4, 1), $M = 1.35$ is *not* necessarily the most critical Mach number (see Figure 18). Hence only Figure 17 (i.e., Figure 6 of Reference A190) is of direct use for design. As L/W increases the predominant mode at flutter also increases, e.g. the third mode predominates at $L/W = 1$, so that with only four modes present the need for convergence to be proved is very great.

Also the results obtained for two panels chordwise ($N = 2$) must be suspect, since spatial periodicity was assumed in the analysis although the aerodynamic loading is certainly not periodic.

The above study has been extended in Reference A224 to cover larger variations in L/W (from 1/4 to 4) and Mach number. The configuration analysed consists of a single panel chordwise ($N = 1$), but a spanwise array of rectangular, simply-supported panels extending to infinity in the spanwise direction. The chordwise deformation is again approximated by the first four natural modes of a simply-supported beam.

Figure 18 gives the envelopes of the critical stability boundaries, for various values of L/W, in the range of Mach numbers 1.1 to 2.0, and the predominant flutter modes. Only for a small region on the curve of $L/W = 2$, and for $L/W = 4$, is the flutter mode coupled. These envelopes are only approximately correct since, from analyses at other intermediate Mach numbers, distinct changes in these envelopes can be anticipated. However the pronounced stabilising effect of increasing L/W is seen, and the form of the parameters plotted on the graphs allows a far greater range of material-altitude combinations to be considered.

Two interesting features are seen in the detailed stability boundaries. First, there are cases where the critical flutter vector changes abruptly from one mode of dominance to another. For instance, for $M = \sqrt{2}$ and $L/W = 1/4$ (Fig.19) an aluminium panel will flutter most readily in the third mode at sea level, the fourth mode at higher altitudes and finally in the first and second modes coupled at altitudes above approximately 17,000 ft. On the other hand brass, steel or titanium panels will always flutter in the coupled mode. This switching of flutter modes was observed for all length/width ratios except 4 and was accompanied by significant changes in flutter frequency. Again the limitation to only four modes is unfortunate as the position of the curves for the 5th dominant mode, and higher, on Figure 19 might have proved interesting. Similarly the effect on Figure 18, especially for $L/W = 2$ and 4, which depend on the fourth mode, might have been noteworthy.

Secondly the stability boundaries are only fairly flat in the high, Mach number range, i.e. the parameter (h/L) $[E/\rho c^2(1-\nu^2)]^{1/3}$ at flutter is insensitive to changes in panel material and altitude only for $M > \sqrt{2}$. It should be noted that the critical Mach number increases with increasing L/W .

In the last two papers considered above the Bessel functions entering the aerodynamic pressure terms were approximated by a sum of circular functions in the manner of Luke and St. John[A53]. In more recent unpublished studies alternative approximations were used; analyses were made for $N = 1$, 2 and 3 and comparisons made with other published work including Reference A194.

5.7 University of Washington

No work is being done currently on panel flutter. O'Brien has participated in the past[A106] but his main interest now is in the general field of non-linear structural analysis.

Matrix methods of analysis continue to be developed, including non-linear effects and also analyses in the theory of shells[C21].

5.8 Boeing Company (Seattle)

There have been several major research contracts underway, all sponsored by the Air Force Flight Dynamics Laboratory. The first involved analyses and tests on flat, uniform panels, clamped on all edges, in the Mach number range of 1.0 to 5.0 and has been fully discussed in References A211 and A218. Summarising briefly the conclusions were as follows:

(i) It is difficult, in practice, to simulate clamped edges without inducing extraneous panel stresses.

(ii) Density and depth of the acoustic cavity behind the panel can have a very significant effect on panel frequencies, mode shapes and flutter conditions.

(iii) Panels can be very sensitive to rather small pressure gradients and/or differentials (of order 0.01 lb/in^2) and small temperature gradients and/or differentials.

(iv) For a panel with length/width ratio of 0.5 the minimum flutter dynamic pressure occurs at about $M = 1.25$ (Fig.20). This experimental result agrees with the theoretical predictions of Reference A224.

(v) The theoretical studies showed that the three-dimensional unsteady potential flow theory could be replaced by simple, quasi-steady, two-dimensional aerodynamic theory for $(\beta b/a) > 1$ and $M > 2$ and the agreement with experimental data up to $M = 5.0$ was satisfactory. For low supersonic Mach numbers and small length/width ratios the theories are less accurate. This loss of accuracy is associated with a change in flutter mechanism from "coupled mode" to "single-degree-of-freedom" flutter.

Subsequently, investigations have been continued into the Mach number range 5.0 to 10.0. Improvements were incorporated into the experimental technique to remedy troubles experienced earlier, e.g. at $M = 8$, 10 the panels tested were only clamped on the leading and trailing edges. The tests were conducted at Mach numbers of 5, 6, 8 and 10 on panels with length/width ratios of 0.76, 1.37, 1.76 and 2.176.

Small seismic pick-ups (vibrometers) were attached to the panels in tests at $M = 5$, 6 and the test technique was to fix M, q and vary Δp to obtain flutter. The measured boundary layer thickness was of the order of $\frac{1}{2}$ in and the panel thickness (aluminium) 0.012 in to 0.016 in.

The aluminium panel was bonded to its frame with an epoxy resin, but it is thought that a chemically milled specimen would be better. The panel modes and frequencies were measured and compared with Warburton's theory[C22]. Correlation was only deemed acceptable if panel frequency and nodal lines agreed. The flutter analyses showed that, for a length/width ratio as high as 4, up to 8 modes were required to get a converged solution for q_{crit}, even though the flutter mode was still essentially the first and second modes coupled.

At the temperatures (of the order of 1200 – 1400°F) experienced during $M = 10$ tests, the steel panels could be clamped on all four sides, as mentioned earlier, and the panel sides were vented. Spanwise stiffeners were added to ensure two-dimensional structural behaviour. The major variable was F, the midplane chordwise tension, which was decreased from a high initial value until flutter occurred.

Inductive pick-ups were used because of the temperatures involved and because they did not modify the vibration characteristics of the panels (as did the seismic type).

The correlation of these tests with piston theory analyses was poor, most likely, because of warping, thermal effects, inconsistencies in tunnel behaviour etc. It is felt that piston theory is suitable for the régime $5 < M < 10$ and that boundary layer effects are less significant at these higher Mach numbers. It was concluded that the tests at $M = 8$ and 10 were unsatisfactory.

There was no attempt in these tests to vary boundary layer characteristics.

It was stated that the weight of the panel can be most significant, e.g. the results of horizontal and vertical panel vibrations and flutter tests can be quite different.

A third programme has been under way to study, experimentally, the unsteady aerodynamic forces acting during panel flutter, and to assess the effect of the boundary layer. The programme is described more fully in References A218 and A225.

The force required to excite the panel is measured and the force vector is plotted in the complex plane as a function of frequency. The loci of the force vector are plotted at constant dynamic pressure and Mach number and flutter reached where the force vector is zero. The tests in the Mach number range 1 to 1.4 were conducted in the Ames 2 ft × 2 ft tunnel on a wall-mounted panel. Total-head probes at the leading and trailing edges of the panel were used to measure the boundary layer (thickness over 1 in). There was no discernible effect on the boundary layer during panel flutter. This data appears to be most satisfactory.

The data for $2 < M < 4$ appears to be quite good, but for $M = 10$ is believed to be suspect.

The pressure transducers used to measure the unsteady pressures on the fluttering panel were developed by the Boeing Company.

In connection with the Dynasoar programme, considerable research was undertaken on orthotropic panels[A256, A257], having configurations similar to that shown in Figure 21.

The frequency spectrum of the panels proved to be very close and unpredictable, apart from the first three modes. This was due mainly to the complicated edge support conditions which caused great difficulty in obtaining clear vibration results and flutter results. The panels experienced their minimum q_{crit} in the range $M = 1.3$ to $M = 1.4$. Catastrophic flutter was experienced on the spot welded panels which had no fixity on their leading and trailing edges but only on the side edges. A square panel was tested which showed that the "best" angle for the corrugations was in-line with the air stream - not normal, as in Figure 21. This is shown in the sketch on Figure 22 but the benefit cannot be used in practice, because large variations in yaw angles ($> 15^o$) are possible on flight vehicles. Correlation of experiment with theory (using piston theory aerodynamics) was shown to be good if the experimental frequencies are used in a modal analysis having as few as two modes in the representation. The two modes giving the lowest frequency difference couple most strongly and the good correlation with experiment is found. Of course, if the frequency difference had decreased considerably as the mode number increased, the difference in the squares of the frequencies might be more constant between adjacent pairs of modes so that more than two modes might be needed for a fully-converged solution - and good correlation with experiment. More detailed analyses are now in progress including in-plane stresses, three- and two-dimensional aerodynamics, structural damping, etc.

A considerable amount of analysis and experiment has been performed on extremely low aspect ratio panels in connection with the Saturn programme. The flutter boundary of NASA TN D-451, when extrapolated, indicated the possibility of some critical panels of high length/width ratio on the external face of the longitudinal "top-hat" stringers. These panels are of 3 in span and have lengths from 20 in to 18 in.

Some model panels have been tested at $M < 1.45$ which were thinner (0.040 in) than the full-scale panels (0.125 in) and were subjected to a larger axial compressive load. As this load increased it was found that two different modal frequencies could exist having the same axial wavelength. Under this condition flutter was observed with nodal lines spaced about 4 in apart axially (i.e. about the same as the panel span). Tests on various length panels (0.040 in thick) in the full-scale configurations showed that the boundary layer is stabilising, viz. that panels at the aft end of the "top-hat" section did not flutter whereas those at the forward end did. Actually, the 0.125 in panels did *not* flutter at all and the 0.040 in panels did *not* flutter except when buckled.

Voss has applied his analyses of cylindrical shells[A97] to a study of the shells tested by Fung et al.[A189] at Ames. Flutter boundaries were obtained which were much less conservative than the actual experimental data and he concludes that the stabilising effect of the boundary layer must be the principal cause.

5.9 University of California

The main emphasis here has been on the flutter of cylindrical shells. Reference A83 considered the cylindrical shell of finite length and used the Goldenveiser equations to obtain numerical results. A convenient expression was obtained for the aerodynamic pressure by means of a Laplace transformation which correctly accounted for the shells finite length. A quasi-steady approximation was introduced into the numerical analysis which can only be justified for low frequency oscillations. It was found that the critical Mach number decreases with increasing number of circumferential waves - this is a result of the structural theory used - and the assumption was therefore made that the minimum flutter speed occurs at about the mode of minimum *in vacuo* frequency. By using Donnell's shell equations or more refined theories, the need for this assumption is circumvented.

Subsequent work has aimed at improving the low frequency aerodynamics and including aerodynamic damping terms[A206]. No actual flutter calculations are included but the earlier static approximation is improved and first-order frequency terms included using the functions given in Reference C23. This causes a phase shift between the shell displacement slope and the corresponding pressure or aerodynamic force coefficient which it is expected will have a significant effect on flutter. The phase shift ϵ varies with distance along the cylinder (x) and, for $n \geqslant 2$, the variation may be oscillatory with x. Figure 23 shows the phase angle at various Mach numbers for the axisymmetric mode $n = 0$ (in which case the oscillatory nature is absent).

Tan ϵ is proportional to the product $\beta\omega/RU$ and Figure 23 shows the phase shift from the static approximation to be quite significant for $\omega/RU = 0.10$.

The above work[A206] was reappraised in Reference A235 and approximated for use at high Mach numbers. The resulting expressions are based on the quasi-steady approximation when only first-order terms in frequency are considered. It is anticipated that the revised expression will be simpler to use than the expression given in Reference A206.

Present studies are concerned with flutter analyses of cylindrical shells, using Donnell's shell equations and more exact aerodynamics than were used previously in Reference A83. For axisymmetric flutter ($n = 0$) the resulting fourth-order equation is of an integro - differential type because of an integral term in the expression for the aerodynamic pressure. This equation is now being solved by means of a Galerkin modal approach.

5.10 Lockheed Missile and Space Company (Sunnyvale)

Recent tests have been made in the Ling-Temco-Vought 4 ft × 4 ft tunnel at $M = 1.4$. The tests were essentially only of an exploratory nature in that no attempt was made to design the panel so as to flutter. Also, the pressure differential was kept constant at 1 lb/in^2, which was sufficient to buckle the panel. The panels were of aluminium and magnesium, 24 in square and 0.04 in thick and were curved normal to the air flow (approximate radius = 5 ft). The boundary layer thickness over the panel could be varied from 0.7 in to 3 in but, even at the maximum dynamic pressure of 3000 lb/ft^2, flutter did not occur. The tests were sufficient, however, to prove an actual structural design.

The main design preoccupation is with shell structures and weight optimisation studies are required for cylindrical meteorite bumper shields. Other problem areas are for flat panels on winged re-entry vehicles.

Translations of relevant Russian literature have been provided (e.g. References A243 and C24).

5.11 NASA (Ames)

Recent theoretical studies[A287] have been concerned with flat, highly orthotropic panels at high Mach numbers. The panels are typical of Dynasoar-type structures with relative bending stiffnesses in mutually perpendicular directions of about 5000 to 1. Ackeret aerodynamics were used and, for square panels, it was found that up to 60 modes (N) were required in some cases to obtain a fully converged solution which would agree with an "exact" solution by the method of Reference A55. It was noted in one case that the critical flutter parameter increased significantly throughout the range $1 < N < 60$. Simultaneously the predominant flutter mode couplings varied from first-second for $N \simeq 4$ to eighth-ninth-tenth for $N \simeq 10$ although, at $N = 60$, again the first and second mode couplings predominated. The comparable isotropic square panel converged with only a few modes.

Based on these results, Gaspers believes that previous published work on orthotropic panels may be suspect where limited modal analysis were employed.

Experiments have been conducted in the 2 ft × 2 ft supersonic tunnel on flat, magnesium and invar panels of various length/width ratios clamped on all four edges. The panel vibration characteristics were checked with theory and the effects of cavity depth determined. Displacements were measured and also the boundary layer profiles and unsteady surface pressures.

5.12 California Institute of Technology

The many contributions of Fung and his students over the years have been well chronicled in various publications and particularly in the various review papers by Fung[A49, A107, A196]. Of the recent contributions, Reference A191 is especially interesting (see Section 5.3) because of its bearing on the importance of the boundary layer in the panel flutter problem. It is Fung's view that the basic assumption in Reference A191 regarding a constant Mach number shear layer is not at present easily justified - however, the analysis did show the important stabilising influence of the boundary layer. Whilst McClure's analysis[A168] appears to be more satisfactory it is thought that even more improvement can be gained and Olsen is now attempting an analysis without certain limiting assumptions in either of the above papers. The analysis will be for the three-dimensional problem of a plane panel at high supersonic speeds undergoing forced oscillations in modes of long wavelength. The differential equation which result has non-constant coefficients including terms dependent on the following ratios: (transverse wavelength)/(streamwise wavelength) and (boundary layer thickness)/(streamwise wavelength).

Recently experiments have been made on the flutter of thin-cylindrical shells in the NASA Ames 8 ft × 7 ft wind tunnel, when subjected to internal pressure and axial compression[A266]. Previous tests[A189, A196] showed that the maximum amplitude of flutter is obtained as buckling is approached and that the flutter disappears when the shell is completely buckled. The shell with large buckles apparently acts like a corrugated shell, whose flutter speed is higher than the circular shell. These results were limited to buckling under radial external pressure, and it was expected that the effect of axial compressive loading would be entirely different. Similar shells were used as previously, viz. 0.004 in thickness and 16 in diameter and the rig was designed so that axial compressive loads could be obtained by pressurising rubber tubes adjacent to each of the copper end rings. This device gave extremely uniform circumferential distributions of axial stress. The radial pressure difference across the shell was varied by adjusting the pressure in the sealed annular cavity under the shell. The Mach number was approximately 3.0 to 3.38.

The results emphasised the importance of non-linear structural effects, as only the limit cycle oscillations could be seen in the experiment and their connection with the initial "linear" instability is merely intuitive. In no case did the shells appear in danger of destruction.

All the flutter modes observed contained circumferential travelling waves (probably a consequence of non-linear effects) and longitudinal standing waves. In qualitative agreement with available theory, small amounts of internal pressurisation were found to be very stabilising but more moderate amounts reduced the stability to the un-pressurised level. However, large amounts of internal pressurisation completely stabilised the shells independently of the axial load or previous permanent buckling deformations.

Axial compressive loading was slightly destabilising for moderate amounts of internal pressurisation but stabilising for low or negative amounts.

The unbuckled shell exhibited large amplitude flutter for low positive or negative internal pressures. The largest amplitude occurred in a localised flutter mode due, it is thought, to some non-uniformity in the static pressure distribution over the shell. The most violent flutter occurred just *prior* to buckling under radial external pressure and just *after* buckling under axial compressive loading. Figure 24 gives the measured still-air frequencies for the unstressed shells. The flutter data shows values of n_{crit} from 14 to 24 with 2 or 3 axial half-wavelengths and flutter frequencies in the range 45 c/s to 835 c/s. The values below the curves shown in Figure 24 appear to correspond to flutter modes of a localised nature.

The flutter results confirm qualitatively those obtained in References A189, A196.

Fung believes that all flutter criteria are suspect especially for shells and orthotropic flat panels - many of the latter should in fact be analysed as built-up structures and not as equivalent thin panels. He recommends that, in design, existing criteria should be considered and only if there is a "safety factor" of less than 2 on the critical thickness would more detailed analyses be advisable. These should then be completely general, including all known effects in a multi-mode or exact analysis. Known modal frequencies should be used if possible.

Studies into the flutter of plane and buckled rectangular panels have been published by Kobayashi[A176-A178]. These studies used quasi-steady aerodynamic loading in a two-mode Galerkin analysis.

The flutter boundaries for a simply-supported square panel[A176] have been given for various combinations of in-plane direct stresses. It was shown that if a spanwise tension is added to a state of biaxial uniform compression then a more critical flutter condition results.

The studies on two-dimensional panels[A177, A178] are of interest because, respectively, an analogue computer was used to determine the non-linear flutter motions, and fair agreement is claimed between experiment and analysis for both the flutter parameter and flutter motion.

The analyses of Reference A176 have since been extended to include the effects of internal pressure and initial panel deflections. The results[A261] are similar qualitatively to those of Reference A177.

Reference A223 is mainly concerned with a square panel subjected to in-plane direct and shear stresses. Figures are presented showing results comparable to those of Reference A166.

Reference A214 presents analyses based on Donnell's equation for an unstiffened circular cylindrical shell of finite length. Two-dimensional quasi-steady aerodynamic theory is used in a two-mode Galerkin analysis and the effects of internal pressure and axial force are studied. The closed-form analytic solution shows that aerodynamic damping cannot be neglected in shell flutter as it raises the flutter boundary considerably (this was shown in References A97 and A217). Also, internal pressure raises the flutter boundary but axial compression lowers it.

The results were applied to a design problem and presented graphically. Figure 25 gives the flutter boundaries for steel shells at sea level.

Comparison of these results with those of earlier papers is made and the conclusion drawn that Stepanov's analysis in Reference A47 is sufficiently accurate, despite its use of Goldenveiser's equation, due mainly to the large contribution of aerodynamic damping which is comparable in both References A47 and A223.

5.13 NASA (Edwards AFB)

Several studies in panel flutter have been reported since 1960, e.g. References A114, A152, A153.

Reference A114 is probably best known for its presentation of flutter data as a plot of the critical flutter parameter $(\beta E/q)^{1/3}\ h/L$ versus panel length/width ratio, L/W .

Reference A152 presents a theoretical analysis which shows that changes in flow direction have a marked effect on the critical dynamic pressure parameter for simply-supported flat rectangular panels. The effect is most pronounced for panels with small values of the length/width ratio.

The effect of sweepback on the flutter of flat rectangular panels is analysed in Reference A153 and used to correlate flight test panel flutter data for swept panels with a flutter boundary for unswept panels.

The main current programme is concerned with flight test experiments on panels mounted on a ventral fin attached to the F-104 (see Figure 26) just aft of the aircraft centre of gravity. This fin is approximately 6 in wide, 80 in long and 24 in in vertical span. It consists of three distinct sections:

(i) a detachable leading edge

(ii) a self-contained instrumentation bay and,

(iii) a panel test bay.

Both flight tests and wind tunnel tests can be performed on the same test panel without introducing variables due to changes in panel mounting etc. from one test condition to another. The panel test bay has a clear volume of 45 in × 14.25 in × 6 in which can be pressurised to ±2 lb/in^2 over ambient. The F-104 flight envelope permits tests at dynamic pressures up to 1700 lb/ft^2 and maximum Mach numbers (depending on altitude) from 1.3 to 2.0.

The test panels have been made conventionally, following normal production techniques, so consequently, some have built-in stresses and initial deformations. The panels were dimpled to form the attachments, essentially clamped-clamped, to the supporting frames. The test programme consists of tests on: flat aluminium panels, 41 in × 10.25 in, of 0.063 in and 0.050 in thickness; flat aluminium panels, 41 in × 4.1 in, of 0.040 in and 0.032 in thickness; and eight flat aluminium panels of length/width ratios of less than one. Ground vibration frequencies were recorded prior to flight.

The flutter mode of the panels of length/width ratio 4 was discernible to the pilot of the chase aircraft. The flutter mode was of fundamental or first overtone form as in an oil-canning type of motion. Flutter frequencies were recorded during flight by the telemetered output of strain-gauge bridges oriented to measure the bending strain of the test panel. In addition, panel temperature distributions and transverse pressure differentials were recorded.

Local flow conditions were monitored by static pressure orifices and by a stagnation pressure probe. The pressure measurements over the rigid fin showed that the spanwise pressure distribution was uniform except for a slight decrease at the tip. The corresponding chordwise distribution was also uniform. Measurements of the boundary-layer velocity profile were recorded continuously by means of traversing pressure probe which moved in and out from the test fixture skin within a range between 0.001 in and 3.000 in. In addition, the fin was tufted in order to visually indicate the flow over the test panel. The results of these surveys in the Mach number range from 1.1 to 1.5 showed very little flow angularity.

The flight tests were made at altitudes ranging between 15,000 and 30,000 ft. The flutter phenomena recorded was distinct and a definite growth in amplitude at a particular frequency could be observed.

The flight tests were made at 15,000 ft and showed no flutter until M was increased to 1.3; the flutter was continuous, however, as M decreased down to 1.1. The flutter was quite distinct and a definite growth in amplitude at a particular frequency could be observed.

5.14 Lockheed Aircraft Company

Studies have been under way to determine design criteria to provide optimum skin panel designs for a specific "advanced concept" application viz. for vehicles to undergo boost, orbit and re-entry flight. These studies were sponsored by the Air Force Flight Dynamics Laboratory and have resulted in anisotropic structures in designs and materials to minimise the severe thermal environments.

Initially the emphasis is on the development of methods for the prediction of the frequencies and mode shapes for flat and curved built-up (corrugation-stiffened) panels with a range of edge conditions from simply-supported on two sides to clamped on all four sides. Such variations are achieved by attaching points a , b and c in Figure 27 in various combinations.

The theoretical vibration work has consisted in part of a lumped parameter approximation whereby the 2 ft × 2 ft panels (A) are represented by 12 × 12 masses connected by massless springs. Originally, panels A had the configuration sketched in Figure 27 but subsequently 3 top-hat stringers were attached at the 1/4-chord points across the corrugations, since this was thought to be more representative of the optimum structural form. The original configuration showed the first 11 modes of vibration to consist of one $\frac{1}{2}$ sine wave in the more rigid direction, viz. X , and up to eleven $\frac{1}{2}$ sine waves in the less rigid direction. Wind tunnel tests were made on the A panels in their final form but flutter was *not* experienced.

The calculated modes and frequencies agreed well with experiment. Calculations of M_{crit} and q_{crit} were made, using piston theory and assumed primitive modes, and were found to be outside the range of the planned wind tunnel tests.

Four further panels (B) have been analysed and tested, of 18 in × 18 in planform and with 5/32 in deep corrugations to which had also been added shear lugs at each end to improve the shear stiffness of the panels. Again, the calculated modes and frequencies agreed well with experiment but neither theoretically nor experimentally could flutter be found in the range of tunnel conditions of interest.

The panels were structurally idealised as plates for which D_x took account of the corrugations; D_y was virtually zero except in the regions of the top-hat stiffeners. Shear deformation terms were included but were found to be unimportant. It is hoped that the vibration studies above will be published shortly[C25].

The 12 × 12 vibration analysis gave up to 121 frequencies and modes. The "Fraction Series" method of sweeping out successive modes in order to determine the next highest mode is claimed to give considerable accuracy on predicted frequency. When the analysis was reduced to a 9 × 9 mass mesh, significant differences in the frequencies were noted.

When primitive modes were assumed in an energy-type vibration analysis, 5 terms were considered in each panel direction, i.e.

$$\text{if} \quad w = \bar{w} \sin(mx) \sin(ny) \quad \text{(simply-supported edges)}$$

$$\text{or} \quad w = \bar{w} \sin(x) \sin(mx) \sin(ny) \quad \text{(clamped edges)}$$

then $m \leqslant 5$, $n \leqslant 5$.

Both the 25-mode and even the 9-mode approximations agreed well with experiment and the latter were subsequently used in the flutter calculations.

The flutter analyses used

(a) piston theory – two-dimensional aerodynamics,

(b) quasi-static – three-dimensional aerodynamics, and

(c) "exact" – three-dimensional aerodynamics.

Cases (b) + (c) utilised a Mach box analytical technique and gave conservative results compared with (a). These analyses were made for the range $1.5 < M < 5.0$ which was the range of the 4 ft × 4 ft blowdown tunnel.

The panels were mounted out of the tunnel wall boundary layer and the supports designed to minimise thermal stress and pressure differential effects. Tunnel starting and shut-down shocks damaged all the test panels.

A two-degree-of-freedom flutter study has recently been made, similar to those reported by Johns (Section 4.6) and Tucker (Section 5.5), which shows the relative importance of aerodynamic and structural damping. Again the flutter boundary has been generalised in terms of parameters containing the assumed chordwise mode numbers and assumed natural frequencies.

Interest in unsteady aerodynamic theories and their application to panel flutter problems has been maintained by Revell (see References A151, C26).

5.15 Aerospace Corporation

A considerable effect has been expended on the formulation and use of unsteady aerodynamic influence coefficients (AIC's) for aeroelastic problems in general[C27-C29] and for panel flutter in particular[A251, A252].

The philosophy behind this approach is that the various aeroelastic problems are formulated most generally by a collocation procedure in terms of matrices of structural and aerodynamic influence coefficients. With the present large digital computation facilities available such a procedure has become realisable. The current state of the art of structural and aerodynamic influence coefficients has been reviewed respectively by Gallagher[C30] and by Rodden and Revell[A151].

The AIC's have the merit that if they are used in modal analyses the generalised aerodynamic forces are easily recalculated for various modal shapes, whereas the conventional approach requires a more detailed recalculation.

The flutter problem is formulated quite simply by

$$\{w_f\} = \frac{\omega^2}{1 + ig} [a] \left([M] + \rho b_r^2 s [C_h]\right) \{w_f\} .$$

where $[a]$ is the matrix of structural influence coefficients

$[M]$ is the mass matrix

$[C_h]$ is the matrix of aerodynamic influence coefficients.

The eigenvalues

$$\lambda = \lambda_R + i\lambda_I = (1 + ig)/\omega^2$$

may be found by complex matrix iteration from which we find

$$g = \lambda_I/\lambda_R$$

$$\omega = 1/\sqrt{\lambda_R}$$

and this frequency must be compatible with the value of reduced frequency assumed in formulating $[C_h]$.

Thus

$$U = \omega b_r/k_r$$

and a plot of U versus g is obtained, from which the critical flutter condition is obtained.

The first panel flutter study on these lines[A56] treated a simply-supported flat panel having two equal chordwise bays. Reference A107 reports an extension of this study to four bays and Reference A252 presents some corrections to these two analyses and compares the results with those of Dowell[A230]. Figure 28 summarises the various results and the agreement with Dowell is in general quite good. The results indicate that the first-order unsteady aerodynamic theory employed by Dowell is probably valid for panel flutter calculations down to $M = 1.56$. The 20% discrepancy for the four-bay panels is attributed to lack of convergence of the 16-degree-of-freedom idealisation of the four-bay panel used in the numerical collocation solution of Rodden.

Reference A255 gives computer programmes to evaluate the AIC's for various M , number of chordwise bays ($N \leqslant 5$) and for one of three alternative aerodynamic theories used for the range $1 < M < 2$.

Results of flutter studies for two-bay panels with clamped or simply-supported edges are given in Reference A251. It was found that structural coupling between panel bays was unimportant for both single-degree-of-freedom flutter which is critical at low supersonic Mach numbers and for the critical coupled flutter modes at higher supersonic Mach numbers.

Aerodynamic coupling was only important for the critical single-degree-of-freedom modes at the lower Mach numbers. The importance of the effect was also dependent upon the panel edge boundary conditions.

Structural damping was shown to have a most significant effect (stabilising) at the lower supersonic Mach numbers changing completely, in some cases, the critical flutter mode(s).

Reference A251 contains a useful summary of all existing multi-bay panel flutter analyses and shows comparable results to those obtained by Zeydel[A150]. Figure 29 gives a plot of critical panel thickness for clamped-edge aluminium panels at sea level for single and two bays and for $g = 0$ and 0.01.

It appears that the conventional modal approach (possibly using AIC's) is advantageous when a limited number of high order modes are involved, since a collocation procedure would then require a large number of points to ensure accuracy. The collocation procedure and the direct matrix method of flutter analysis would be more advantageous in problems not having cyclic symmetry, e.g. multi-bay panels.

The AIC's have not yet been derived for cylindrical shells. However, multi-mode analyses using piston theory have been made by Lock for axisymmetric flutter. Rotary inertias, shear deformation and high order terms in $h^2/12R^2$ in the shell theory have been included. 10th to 15th order modal analyses showed considerable change in critical flutter conditions but little difference occurred from 15 to 20. The in-plane inertias, aerodynamic damping and structural damping had large stabilising effects on the flutter.

5.16 Lockheed Missile and Space Company (Huntsville)

There are no studies at present on panel flutter; previous work by Rattaya was concerned with circular panels having variable edge support stiffness[A113].

A previous review of panel flutter literature[A100] for the period up to 1959 has recently been revised and up dated and will shortly be published[A282].

5.17 NASA (Huntsville)

Considerable emphasis is being placed on the aerodynamics of two-dimensional and three-dimensional wavy plane walls and shells. Comparative theoretical and experimental investigations of the stationary flow over wavy-walled surfaces are expected to yield useful information on the influence of boundary layer, Mach number, wave amplitude etc. on panel flutter.

Theoretical studies at Huntsville comprise the analysis of inviscid supersonic flow past wavy walled planar and cylindrical configurations using linearised potential flow and non-linear non-isentropic method of characteristics solutions. No leading edge effect was found for a finite length shell, $n = 0$, $a/l = 0.025$, i.e. Anderson's cylinder (see Section 5.3), in the Mach number range 1.1 to 4.62. However, the non-linear solution gives an asymmetric pressure difference $C_{p\,max\,+} > C_{p\,max\,-}$, i.e. net inward pressure over a wave length. This asymmetry is greater for the higher M. Deviations from linearized theory are fairly small in the above Mach number range provided the asymmetry is allowed for in any analysis.

At $M = 10$ a pronounced leading edge effect and complete dissimilarity with the anticipated sine-wave pressure distribution is observed. Obviously non-linear theory must be used if a realistic assessment is to be made of the pressure loading.

At lower supersonic speeds the method of characteristics breaks down as soon as local subsonic flow occurs in the flow field. Thus, e.g. on the two-dimensional plane wavy wall at $M = 1.6$ and for $a/l = 0.04$, the flow becomes transonic after only one wave length.

For the case when $Rm\sqrt{(M^2 - 1)} \to 0$ (m = axial wave number), slender-body theory has been used in the analyses and a smooth transition was observed with linearised potential flow results for values of C_p and phase shift. Further calculations by non-linear theory at $M = 1.075$ for $R/l = 0.025$, $a/l = 0.001$, have shown an appreciable leading-edge effect after the first half-wave.

Experimental studies are being carried out in co-operation with NASA Ames. Tests of two-dimensional and three-dimensional rigid wavy walls with various wave amplitudes and wave lengths are planned at Ames. The Mach number range to be considered is $0.8 < M < 1.6$.

5.18 Georgia Institute of Technology

Zeydel is associated with work being done at NASA (Huntsville) on the aerodynamics of a wavy wall. It is his belief that the lack of correlation between theory and experiment in panel flutter studies is due to

deficiencies in experimental techniques, or

deficiencies in the potential theory aerodynamics, or

neglect of the boundary layer.

Hence attempts should be made to confirm the validity of any aerodynamic theories used and, to this end, any experimentation to measure the unsteady surface pressures should be based on a method of forced excitation – by this means frequency, mode, and amplitude can be controlled. Conversely, if the pressure measurements are made under fluttering conditions, a change in these conditions will cause further changes in too many important parameters.

Theoretical studies[A150] have been concerned with the flutter of simply-supported flat panels of finite aspect ratio under small and large deflection conditions. These panels form part of an infinite array in the spanwise direction but are finite in number (N) in the chordwise direction. A simple half-sine wave is assumed for the spanwise mode of each panel, and up to six such modes for the chordwise deflections. An "exact", three-dimensional, linearised, supersonic aerodynamic theory is used and results are presented for aluminium panels at sea level.

Very little difference was found between the results for four and six modes at $M = 1.3$ and the majority of the results are for four modes only. Figure 30 presents some typical results for infinite aspect ratio panels, the one-mode analyses showing that the flutter is essentially single-degree-of-freedom in the range $M = 1.1$ to $M = 1.6$, with each successive mode (1st to 4th) becoming progressively the most critical as M is increased to $M = 1.6$. It is unfortunate that the results of six-mode analyses are not available for the range $M < 2.0$, since it could be intuitively inferred from the trends shown in Figure 30 that a 5th and 6th mode single-degree-of-freedom instability might be found between $M = 1.6$ and $M = 2.0$, whereas the results given indicate that coupled mode flutter is more critical.

Structural damping had a considerable stabilising effect below $M = 1.6$. For $g = 0.01$ and $g = 0.02$, the instability boundaries for the 2nd, 3rd, and 4th modes are reduced and lie within the 1st mode boundary entirely, which is itself appreciably reduced.

Results for panel aspect ratios 4 and 2 (N = 1) show a much greater effect on the boundaries for the lower degrees of freedom than on those dependent on the higher degrees of freedom (Fig.31).

Increasing the number of panels in the chordwise direction has a powerful destabilising effect for two-dimensional panels for all degrees of freedom and which extends the single degree type of flutter up to $M = 1.82$ for $N = 2$ ($g = \sigma_x = \sigma_y = 0$) and to $M = 1.92$ for $N = 3$. Arguments were put forward in Reference A150 to justify the limited number of modes for the $N > 1$ cases and to show that the periodicity in the assumed modes in the chordwise direction was negligible. These analyses have since been extended in References A190, A224, A232 (see Section 5.6).

Further investigations are planned which do not rely on the assumption of modal or collocation procedures.

5.19 NASA (Langley)

The output of published work is too extensive for a complete, detailed review so that a selection only is considered here.

Of the earlier papers, Reference A22 was significant for the range of variables considered. Convergence was not proved explicitly in the analyses but, since for $M \leqslant 1.56$ the flutter mode for the two-dimensional panels was of single-degree-of-freedom type, this was not absolutely necessary. Inconclusive results were apparently obtained in coupled mode solutions when odd numbers of modes were included; it is the practice now always to use even numbers of modes.

In Reference A194 some results are presented, using more sophisticated numerical integration procedures to evaluate the aerodynamic forces, for several panel length/width ratios in the range 0 to 10 for $M = 1.3$. The analyses are for simply-supported and clamped panels with zero structural damping and have been extended in Reference A286.

The box size employed to evaluate the aerodynamic forces is kept constant at all Mach numbers, and analyses using 40×10 and 80×5 box arrangements showed no significant differences. The problem, of course, is to get adequate aerodynamic representation for problems with large numbers of chordwise waves. (The computer programmes associated with this work will shortly be published with Mach number, yaw angle, modal shape and frequency as variables.)

These analyses used upto 12 modes in some cases to get a converged flutter boundary. The actual modes used vary, depending on which modes are expected to be more critical. Thus, when the 10th mode is most important (i.e. flutter frequency $\simeq$ 10th modal frequency) the modes 5-16 are included.

Results have been given for a square brass panel with a clamped leading edge, simply-supported trailing edge and free side edges. Experimental data for such a panel is given in Figure 32, together with previous analytical results. It is seen that the discrepancies were considerable although the inclusion of boundary-layer effects by McClure[A168] showed better agreement. However, whereas previous analyses assumed two-dimensional structural *and* aerodynamic behaviour of the panel, Cunningham[A194] assumed a three-dimensional aerodynamic behaviour more representative of the tunnel condition. This, plus the inclusion of the *measured* panel frequencies for the first two modes in a 6-mode analysis, gave the results shown in Figure 32. It is seen that, for $g = 0.01$ and 0.015, the agreement on critical thickness is good. Unfortunately there were considerable differences in the flutter frequencies and so complete correlation is still awaited.

Many of the results obtained for the low supersonic Mach number régime are comparable with those of Zeydel and Kobett[A150, A190, A224, A232]. Further numerical solutions by Erickson have shown that the "exact" solution pioneered by Hedgepeth[A38, A55], for simply-supported panels, can lead to a flutter frequency at the coalescence of a pair of stream-flow-affected eigenfrequencies well outside the range of their *in vacuo* values. Thus, for the flutter point on Figures 1 and 2 of Reference A240, at a length/width ratio of 10, coalescence of the eigenfrequencies, that originate from the natural (vacuum) frequencies of modes 1 and 2, occurs at a frequency between the 4th and 5th natural frequencies.

Experiments have been reported in Reference A186 of tests on both flat and curved panels mechanically loaded in compression. From these a method has been suggested for estimating the effect of curvature (normal to the flow) on the flutter of axially compressed panels.

The adjusted critical panel thickness for the curved panel was found by determining the thickness of an equivalent flat plate with the same length-width ratio and edge restraint as the curved panel and the same critical buckling stress. When the corresponding result was plotted on a similar basis to NASA TN D-451[A114] the agreement was shown to be good but further confirmation advised. Figure 33 shows the adjusted data plotted on a curve representing the upper limit of flat panel data.

More recent work with improved experimental techniques shows that the critical thickness ratio for flat panels should be increased by 40% over the values shown in Figure 33. These later tests have shown that flutter persists below the point at which it initially started, and the predominant flutter frequency is extremely sensitive to variations in compressive load and pressure differential.

Further tests in Reference A228 have shown the delicate nature of panel flutter. In some cases standing wave flutter was observed and in others a travelling wave type depending upon ΔT, the panel skin temperature rise, and Δp, the pressure differential. For changes of only 0.2 lb/in^2 in Δp considerable differences occurred in the critical thickness ratio and type of instability.

Some recent studies of the effect of eccentric stiffening on the static and dynamic behaviour of plates and shells, and hence on panel flutter, are reported by McElman et al. in Reference A268. When applied to the flutter of a particular square panel the critical dynamic pressure is

$$q_{cr} = 472\beta/L^3$$

using the eccentric plate theory, whereas using the classical orthotropic plate theory (e.g. Reference A222) the result is

$$q_{cr} = 247\beta/L^3 .$$

Thus a significant difference is obtained which may in part explain some of the lack of correlation previously noted between theory and experiment for eccentrically stiffened panels. Reference C31 is also most relevant.

The effect of edge loadings on the vibration characteristics of rectangular orthotropic plates is given in Reference C32. These results are tabulated for a wide range of parameters and should be useful in future panel flutter studies.

Similarly studies in Reference C33 are of interest in that they show that slight variations in assumed edge conditions can have a significant effect on the critical axial buckling loads of a pressurised shell. The relevance of this to flutter studies of axially compressed cylindrical shells is obvious.

Reference A240 contains a useful review of current thinking on the flutter of flat panels. It shows, for instance, that the flutter of a particular orthotropic panel is most likely when the flow orientation is normal to the main stiffening direction (see Section 5.20).

The edge support problem for corrugation stiffened panels is considered and the need to use more accurate experimental modes in the analyses is emphasised. Thus, for a typical panel, the clamped edge condition was more critical than with simply-supported edges. Experiments relevant to this topic are discussed in Reference A284. The elastic constants for corrugation stiffened panels are given in Reference C34. Reference A270 contains "exact" and modal analyses (upto 18 modes) for corrugation-stiffened panels elastically supported at the ends of the corrugations.

Reference A288 gives detailed theoretical results for the flutter of a square panel with a parabolic temperature distribution in both x and y directions which give rise to a self-equilibrating direct stress state, the resulting stresses have a destabilising effect on the flutter.

An analysis of large deflection panel flutter is given by Fralich[A203] based on the use of Ackeret (static) aerodynamic forces and von Kármán's large-deflection plate theory. The panel was subjected to in-plane direct loads N_{xx} , N_{yy} , a lateral pressure differential Δp , and a temperature rise ΔT , and its edges were elastically restrained against in-plane motions. The problem analysed was for small oscillations about a position of finite static deflection. A two-mode approximation was made to the total displacement and the non-linear equations were solved by the Galerkin procedure. The results show that

(a) for $N_{yy} = 0$, in general λ_{cr} decreases as the compression N_{xx} increases in the unbuckled range but after buckling λ_{cr} increases with further increases in N_{xx} ,

(b) for $N_{xx} = 0$, λ_{cr} is constant as the compressive loading N_{yy} increases in the unbuckled range but after buckling λ_{cr} increases with further increases in N_{yy} .

(c) zero flutter speeds are found for certain panel aspect ratios and combinations of in-plane loadings - these are "weak" instabilities however.

For the case of infinite edge restraint where the in-plane compressive loads result from an increase in ΔT , Figure 34 shows typical results. A possible panel test time history is superimposed and it is seen that the theory may provide an explanation of the start-stop type of flutter which has been observed experimentally. The studies also showed that the slope of the right-hand portion of the flutter boundary could be increased or decreased by varying the values of N_{xx} , N_{yy} independently. These results have been discussed also in References A145 and A192.

More recent unpublished studies by Fralich and McElman have been based on two-mode and four-mode analyses of the large deflection problem (with terms in w^2 retained), using an analogue computer. The four-mode analyses permitted the inclusion of two spanwise modes simultaneously. The two-mode results agreed with Reference A203 in all cases, but the four-mode results in many cases give lower λ_{cr} than either the analytical or analogue two-mode solutions. The analogue traces were very sensitive to variations in λ .

McElman has published a two-term Galerkin solution of an idealisation of a possible micrometeoroid bumper configuration in which two flat, parallel, panels are connected by an elastic medium[A233]. The results indicate the significant differences between this configuration and the single flat panel, depending upon whether either or both parallel panels are subjected to in-plane loads. Figure 35 gives a typical set of results which show that tensile in-plane loads can "tune" the various modal frequencies to give zero *in vacuo* frequency differences and zero λ_{cr} . Since only static aerodynamic forces were used, one presumes that the values of λ_{cr} near zero in Figure 35 are "weak" coalescences which would be removed if aerodynamic damping were included.

Another two-term Galerkin analysis by McElman is given in Reference A239 for the flutter of flat and curved sandwich panels. The results show

(a) sandwich panel construction can give substantial weight reductions compared with a homogeneous panel,

(b) curvature effects are found to be completely separate from core stiffness effects and can be treated similarly to homogeneous panels,

(c) the flutter boundaries are very sensitive to small values of the core shear stiffness parameter Q (Fig. 36),

(d) curvature tends to stiffen the panel (Fig. 36).

Further results in Reference A285 show that, depending on the sign of the in-plane chordwise loading, λ_{cr} could either increase or decrease with increase in Q . For $Q > 0.2$ ($Q = 0$ means rigid in shear) and $L/W < 1.5$, clamped edge sandwich panels are more critical than simply-supported edge panels.

Further test results recently published[A269] show clearly the stabilising influence of curvature on rectangular curved panels subjected to compressive loading and aerodynamic heating at $M = 3.0$.

5.20 Massachusetts Institute of Technology (MIT)

An investigation is reported in Reference A209 of the supersonic flutter of rectangular orthotropic panels. Results are presented in detail for two particular orthotropic panels simply-supported on all four edges. Orthotropic plate theory was used as given in References C35 and C36 and the flutter investigation was primarily concerned with the effects of angle of orthotropicity (ϕ) on the flutter boundaries. Piston theory was used for the aerodynamic forces and the effects of aerodynamic (and structural) damping considered.

A modal approach was followed using four streamwise and four spanwise modes,

$$\text{i.e.} \qquad w = \sum_{m=1}^{4} \sum_{n=1}^{4} C_{mn} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{W}\right) e^{i\omega t} .$$

No attempt was made to select the values of C_{mn} used on the basis of increasing modal frequency.

The significant conclusion from the studies of both orthotropic plates is that aligning stiffeners parallel to the air-stream direction does not always result in raising the flutter boundary. Depending primarily on the length/width ratio and total damping g_T (aerodynamic plus structural) other orientations may be preferred. Figure 37 illustrates a typical set of results for a particular panel configuration. The low flutter boundaries in the range $0^o < \phi < 45^o$ may be attributed to "weak coalescences" which are removed by the addition of some damping. For the panels considered the modal frequencies were closest at the higher end of the frequency spectrum. Convergence of the solutions, for the higher values of L/W particularly, has not been proved.

The flutter of two-dimensional panels extending over several (N) streamwise bays has been analysed in Reference A230. Piston theory was used and the "exact" method of solution of Hedgepeth[A38] employed. The agreement of an eighth-order modal approach with the "exact" method was also shown for the case of $N = 2$, but for $N = 3$ a twelfth-order analysis was in considerable error. It is concluded that as N increases an increasingly larger number of modes would be required. Similarly, if a collocation method is used (see Section 5.15) a larger number of points would be required in the multi-bay case. The influence of bay number on the flutter boundary was found to be insignificant since, for the configuration and Mach numbers considered, there was minimum aerodynamic or structural couplings between adjacent bays. Obviously for low supersonic Mach numbers the aerodynamic coupling is large (see Section 5.18).

A "travelling wave" analysis of the flutter of very low aspect ratio panels ($L/W = \infty$) is contained in Reference A249 for subsonic and supersonic flow. It is found that critical modes have wavelengths of the order of twice the panel width. Also, comparison of the results with those for panels of moderately low aspect ratio (i.e. $L/W = 10$), as given in Reference A194, shows encouraging agreement despite the fact that the analyses are quite dissimilar (see Figure 38). However, Cunningham (NASA, Langley) has calculated additional results (unpublished) for $L/W = 10$ at $M = 2$ and 3 which are in less satisfactory agreement with Dowell's results. The agreement becomes worse as Mach number increases. It is Dowell's opinion that this may be due to the weaker character of the instability at high Mach numbers, but both he and Cunningham apparently have doubts as to the correct interpretation of the results obtained at the limit $L/W \to \infty$. Dowell has categorised three types of flutter (see Figure 39) for flat, single, rectangular, isotropic panels:

(a) High aspect ratios and high Mach numbers give coupled modal flutter.

(b) High aspect ratios and low supersonic Mach numbers give single-degree-of-freedom flutter. Both (a) and (b) are "standing wave" phenomena.

(c) Very low aspect ratios at all Mach numbers give a travelling wave instability.

Similar studies are given in Reference A263 for an infinitely long cylindrical shell. Goldenvieser's shell equations are used and the in-plane inertia terms are included. Analyses are made for incompressible flow and compressible flow. An important consequence of the asymmetric mode calculations (i.e. $n \geqslant 1$) is the indication that the aerodynamic loading is of the "slender body" type. Dowell concludes that a similar result will apply to finite length cylinders if the axial wavelength is large compared with the circumferential wavelength or

$$\frac{2}{\pi}\frac{L}{\beta R}\frac{n}{m} \gg 1 .$$

A comparison is made between the infinite cylinder results of the paper and previous finite-length cylinder results (e.g., References A147 and A281). The agreement is good and improves as L/R increases and h/R decreases.

More recent studies by Dowell[A281] are concerned with non-linear flutter of flat isotropic panels. Two-dimensional and three-dimensional panels are analysed, using the large-deflection equations of von Kármán, linear piston theory and a six-mode Galerkin method. The effects of pressure differential and in-plane loads are included. The work is essentially an extension of that by Bolotin[A99] and Kobayashi[A177] except that more modes are employed and a digital method of analysis is used. The modal forms in the large deflection region are similar to those initially but, as q increases in the critical régime, w increases more predominantly in one mode. The limiting flutter amplitude was attained after about 50 cycles oscillation.

The difference between four-mode and six-mode results was small for the two-dimensional panels; moderate values of Δp and in-plane tension showed no important qualitative effect. However, rather small values of Δp, say 0.01 – 0.1 lb/in^2, may give substantial quantitative changes in flutter dynamic pressure, say 50%. This work is now being extended by using the full, linearised potential aerodynamic theory for similar calculations.

Detailed studies are published by Dugundji in Reference A278 for rectangular isotropic plane panels at high supersonic Mach numbers using piston theory aerodynamics. The variables considered include length/width ratio, in-plane direct stresses and elastic foundation stiffness, the method of analysis being "exact" with no modal assumptions.

The results show that standing wave solutions and travelling wave solutions coincide for high values of L/R and aerodynamic damping. An identical result was shown by Dowell in Reference A263 for the cylindrical shell (Fig.40).

Landahl believes that boundary-layer effects are more stabilising on panel flutter for the higher panel aspect ratios and less so for lower aspect ratios. The primary effect, as shown by McClure's analysis (Ref.A168), is *not* due to viscosity in the boundary layer but due to the "buffer" effect between the wall and the outer flow. Essentially the boundary layer stabilises mainly the one-degree-of-freedom flutter and modifies the coupled flutter very little, since the latter is dependent more on the aerodynamic stiffness term, which is unaffected by the boundary layer. McClure's analysis of boundary-layer effects was for travelling waves on an infinitely long panel, the results being adapted for use on a finite-length panel. The analysis was probably suspect in the results for the damping term (which in the travelling wave analysis means the term out-of-phase with the displacement).

TABLE I

List of Establishments, Investigators and Programmes

Establishments	*Investigators*	*Programmes*
Europe 1. ONERA (Paris)	F.Mazet S.Chopin R.Dat G.Faure	Transonic wind tunnel tests at Melun-Villaroche completed on flat panels. Two-dimensional aerodynamic theory used - now being extended to three-dimensional. Further more detailed tests being prepared for transonic tunnel at Modane. Development of very small pressure pick-ups for measurement of unsteady pressures.
2. DVL (Freiburg)	H.Krumhaar H.Muller	Theoretical studies of unsteady aerodynamics and flutter of cylindrical shells. A stress function approach is envisaged to reduce the number of equations and variables.
3. University of Rome	P.Santini G.Quozzo U.Ponzi C.Buongiorno S.Giorgi	Theoretical analyses of large deflection flutter of flat panels, including effect of static pressure differential. Structural optimisations for combined panel aero- and thermo-elastic effects are planned.
4. University of Pisa	G.Bartolozzi	Vibration of stringer- and ring-stiffened cylindrical shells. Extensions planned incorporating large deflection effects. Application to panel flutter to follow.
5. FIAT (Turin)	Danieli Fenzo Bevilacqua Antona (Polytechic of Turin) Chiara	Studies have been made of flutter of cone-cylinder-combination typical of honeycomb fairing structure on upper stage of E.L.D.O. satellite launcher. Dynamic tests are programmed.

(Continued)

Establishments	*Investigators*	*Programmes*
6. Cambridge University	T.Brooke Benjamin D.Gibson	Theoretical and experimental studies of fluid flow with flexible boundaries. The behaviour of laminar and turbulent flows in such cases is also under study.
7. University of Southampton	G.M.Lilley P.C.Parks B.L.Clarkson	Response of thin panels to turbulent boundary layer pressure fluctuation and effect of panel motion on the boundary layer characteristics. Application of Liapunovs Method to panel flutter analyses. Experimental work aimed at correlation with methods of prediction for vibration characteristics of stiffened structures.
8. College of Aeronautics (Cranfield)	T.H.Hodgson G.Ram	Measurement of boundary layer pressure fluctuations and analysis of response of thin membranes to these fluctuations.
9. NPL (Teddington)	N.C.Lambourne J.A.B.Wills	Experimental programme to investigate the effects of the boundary layer on panel flutter at low supersonic speeds. Experimental study of boundary layer characteristics on a thin panel. (Subsonic)
10. RAE (Farnborough)	A.J.Sobey	Application of Liapunov's Method to panel flutter analyses. "Exact" analyses using the governing *partial* differential equation.
11. Loughborough College of Technology	D.J.Johns P.W.Taylor	Supersonic flutter analyses of circular cylindrical shells. Vibration characteristics of stiffened and unstiffened circular cylindrical shells. Subsonic flutter of flat panels. Experimental study is underway to investigate boundary layer effects on flutter and the interactions with turbulence - induced vibration.

(Continued)

Establishments	*Investigators*	*Programmes*
USA 1. Air Force Flight Dynamics Laboratory (Dayton).	M.H.Shirk J.J.Olsen R.Langley W.F.Bozich	Sponsored research being conducted elsewhere. These projects emphasize correlation of experimental and theoretical results; goal is establishment of reliable criteria. Vibration work in progress on flat panels and thin shells.
2. North American (Columbus)	J.Murphy G.Cook	Theoretical studies of flutter of plane panels utilizing finite difference techniques and an operational analogue computer. Non-linear analysis included.
3. University of Michigan	J.G.Eisley W.J.Anderson (formerly at Wright Patterson A.F.B.)	Theoretical studies for plane panels with membrane shear and direct stresses present. Extensions to non-linear problems planned. Theoretical and experimental studies of aerodynamics of oscillating cylindrical shells with boundary layer effects included.
4. North Western University	Y.Shulman G.Herrmann	Flutter of cylindrical and conical shells. Stability of elastic systems subjected to non-conservative forces.
5. McDonnell Company (St.Louis)	C.H.Perisho N.H.Zimmermann J.W.Schweiker H.Katz P.Tucker	Correlation studies of reported flutter incidents with theoretical criteria. Theoretical studies showing value of simplified modal analyses with modal frequencies as variables.
6. Midwest Research Institute	R.O.Stearman D.Kobett	Flutter of cylindrical shells. Development of ultra-thin shell structures. Flutter of multi-bay, flat panels.

(Continued)

Establishments	*Investigators*	*Programmes*	
7. University of Washington	R.J.H.Bollard E.H.Dill T.F.O'Brien H.Martin R.Parmerter	Shell theory and shell stability. Experimental studies. Inclusion of edge, shear, and large-deflection effects. Non-linear aeroelastic studies. Numerical methods of structural analysis.	Not specifically concerned with panel flutter
8. Boeing Company (Seattle)	G.W.Asher D.Ketter L.Sherman C.T.Golden W.Weatherill C.Doherty H.M.Voss J.Turner	Wind tunnel tests on flat, isotropic and orthotropic panels in the Mach number range 1 < M < 10 and for varying panel aspect ratios. Corresponding theoretical studies also made.	
9. University of California (Berkeley)	M.Holt T.M.Li	Flutter of cylindrical shells. Developments in aerodynamic theories being used.	
10. Lockheed Missile and Space Company (Sunnyvale)	J.A.H.Baillie	Theoretical developments of aerodynamic force prediction for panel flutter. Experimental studies of curved square panels.	
11. NASA (Ames)	P.Gaspers E.Muhlstein	Theoretical studies of flat, highly-orthotropic panels at high Mach numbers including effect of sweep. Experimental studies of flat, isotropic and orthotropic panels.	
12. California Institute of Technology	Y.C.Fung M.Olson S.Kobayashi (Visiting Professor from University of Tokyo)	Flutter of cylindrical shells subjected to axial loads. Development of appropriate aerodynamic theories having realistic allowance for boundary layer effects. Theoretical studies for cylindrical shells and plane panels (showing influence of shear and shear buckling).	

(Continued)

Establishments	*Investigators*	*Programmes*
13. NASA (Edwards)	E. Kordes M. Groen	Flight test measurements of panel flutter at transonic speeds.
14. Lockheed Aircraft Company (Burbank)	E. Postel J.D. Revell	Theoretical and experimental studies of anisotropic plane panels. Unsteady aerodynamic theory.
15. Aerospace Corporation (El Segundo)	M.H. Lock W.P. Rodden	Theoretical studies utilising aerodynamic and structural influence coefficients in matrix analyses. Modal analyses for plane panels and axisymmetric cylindrical shells.
16. Lockheed Missile and Space Company (Huntsville)	J.V. Rattaya	Analyses for flutter of circular panels. Comprehensive literature reviews.
17. NASA (Huntsville)	M. Platzer R. Beranek J.C. Sims* L. Saunders* * Not visited.	Analytical and experimental study of the aerodynamics of two- and three-dimensional wavy walls representative of very low aspect ratio panels. Linearised and non-linearised potential flow solutions for axisymmetric deformation of circular cylindrical shells. Method of characteristics analyses also.
18. Georgia Institute of Technology	E.F.E. Zeydel (formerly Midwest Research Institute).	Aerodynamics of very low aspect ratio panels. Flutter of multi-bay, flat panels.
19. NASA (Langley)	I.E. Garrick H.J. Cunningham A.G. Rainey* R.W. Hess* H.L. Bohon S.C. Dixon* L.D. Guy R.W. Leonard M. Anderson H.G. Schaeffer*	Theoretical analyses for three-dimensional plane panels at *all* supersonic speeds. Development of machine programmes for yawed panel aerodynamics. Theoretical studies of orthotropic panels with various edge boundary conditions. Effect of temperature distributions and thermal stresses on panel flutter. Large deflection (non-linear) panel

(Continued)

Establishments	*Investigators*	*Programmes*
19. NASA (Langley)	W.L.Heard* L.L.Erickson* C.P.Shore* R.W.Fralich J.A.McElman * Not visited.	flutter utilising an analogue computer.
20. Massachusetts Institute of Technology	J.Dugundji E.H.Dowell M.T.Landahl	Exact analyses for rectangular isotropic panels. Orthotropic panel analysis with variable orientation of stiffening. Non-linear analysis (digital) for isotropic panels. Boundary layer effects on panel flutter. Flutter of very low aspect ratio panels (i.e. infinitely long flat panels and shells). Subsonic flutter of panels on elastic foundations.

APPENDIX A

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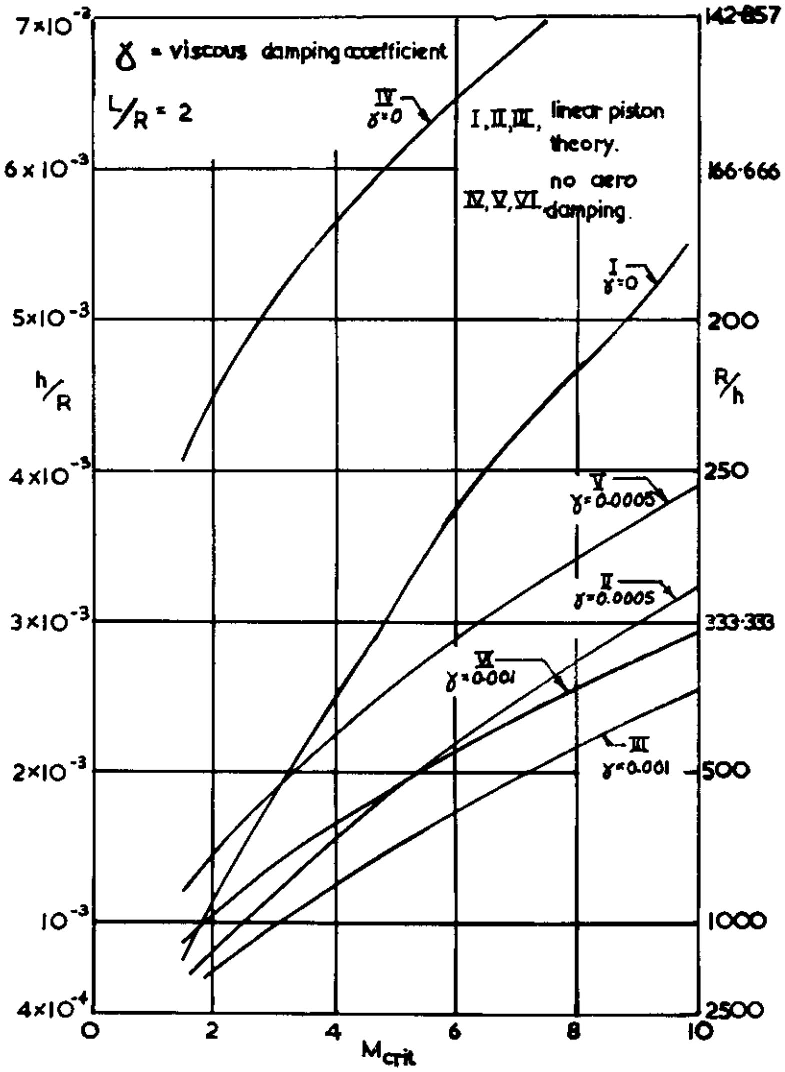

Fig.1 Flutter boundaries for a simply-supported, unpressurised, copper cylindrical shell at 50,000 ft altitude[A147]

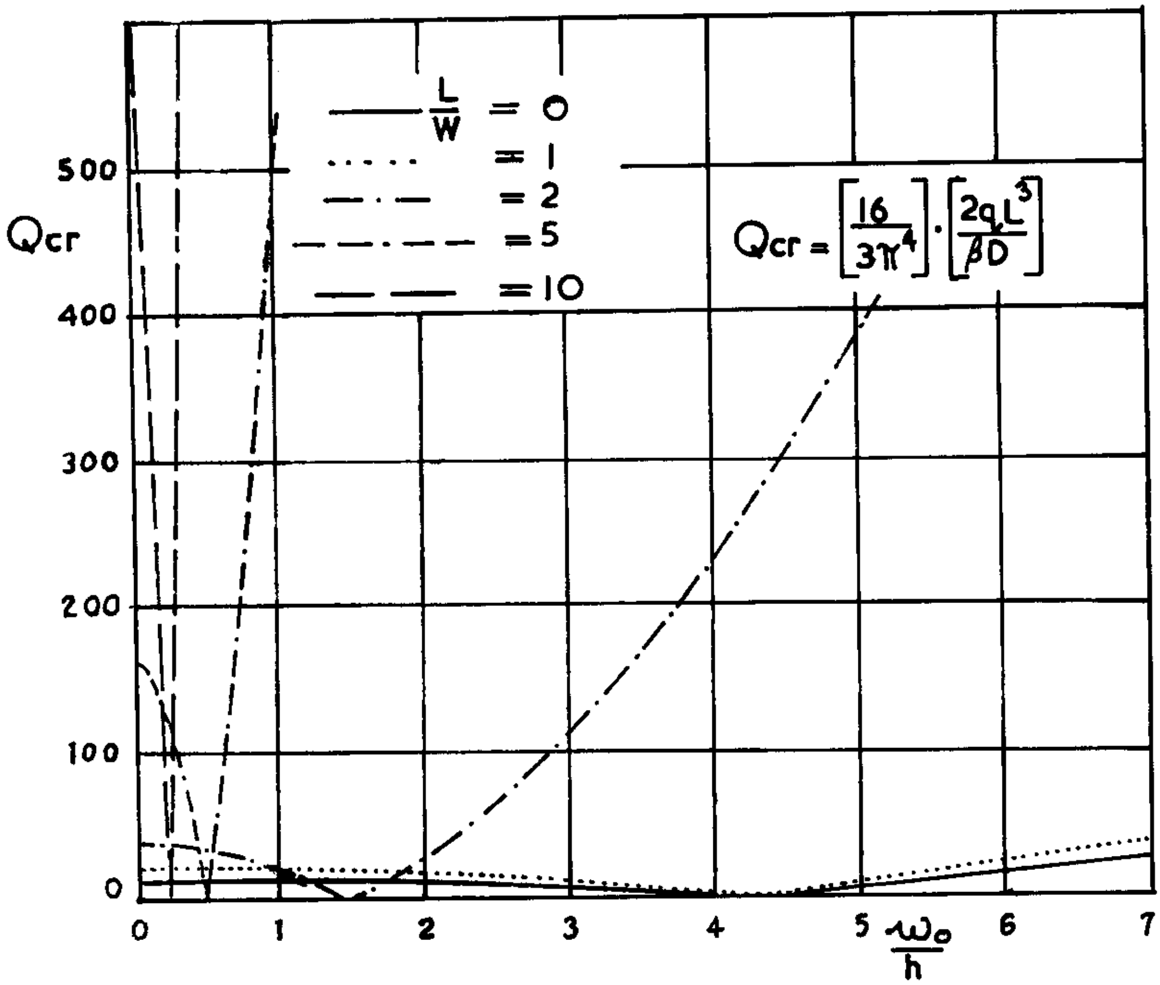

Fig. 2 Flutter boundaries for rectangular panels as a function of L/W and w_0/h (Ref. A237)

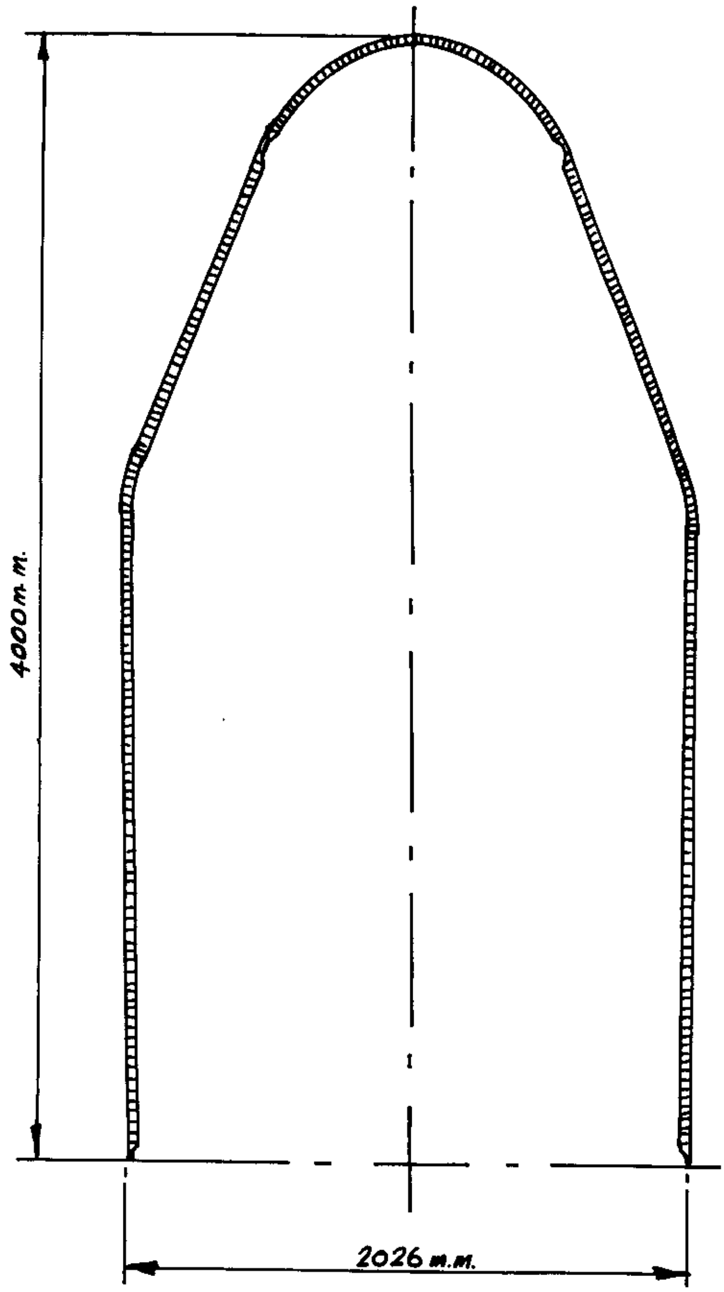

Fig.3 Proposed ELDO launcher upper stage fairing (Ref. Fiat Company)

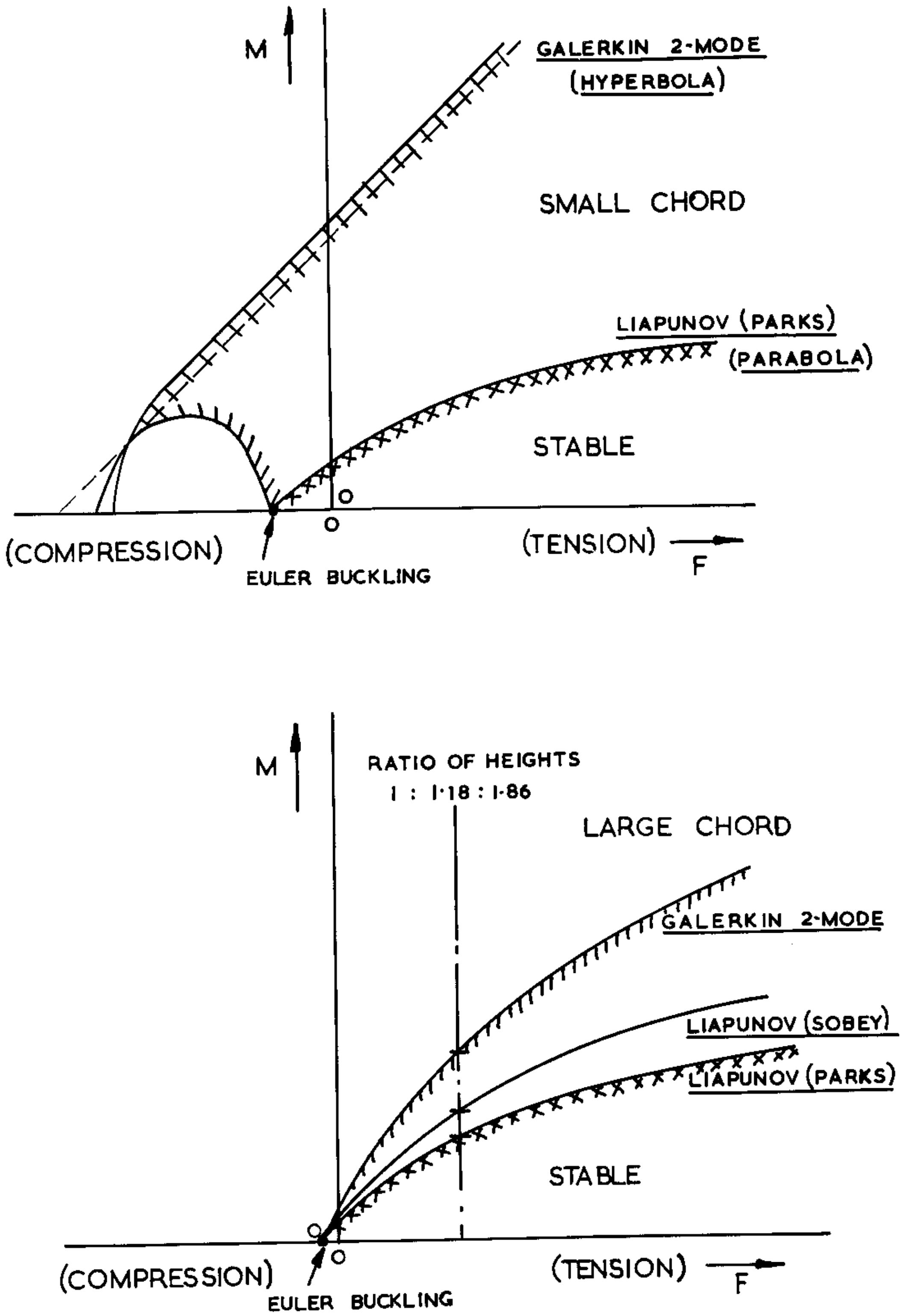

Fig. 4 Flutter boundaries for a simply supported two dimensional panel at high Mach number[A273]

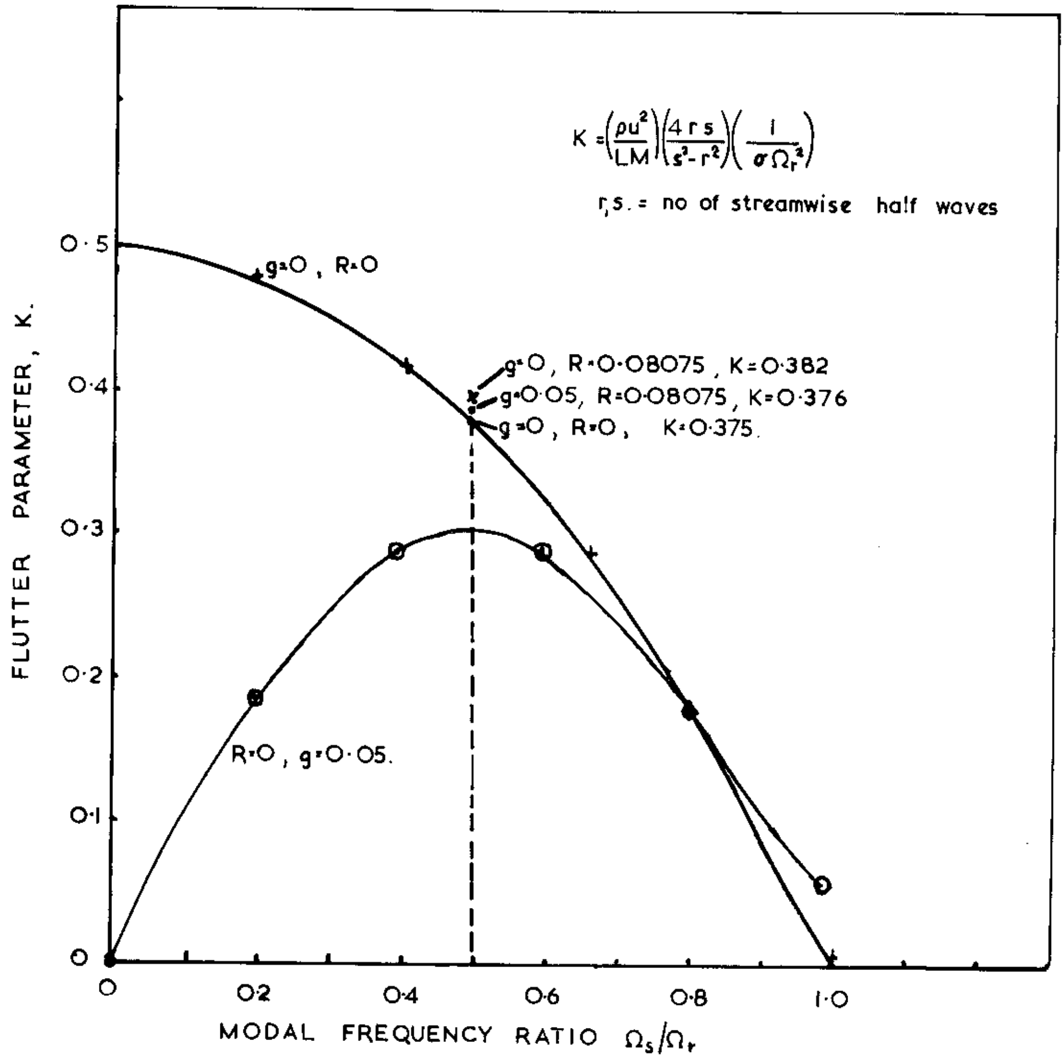

Fig. 5 Variation of flutter parameter with modal frequency ratio.
(Ref. Johns, unpublished)

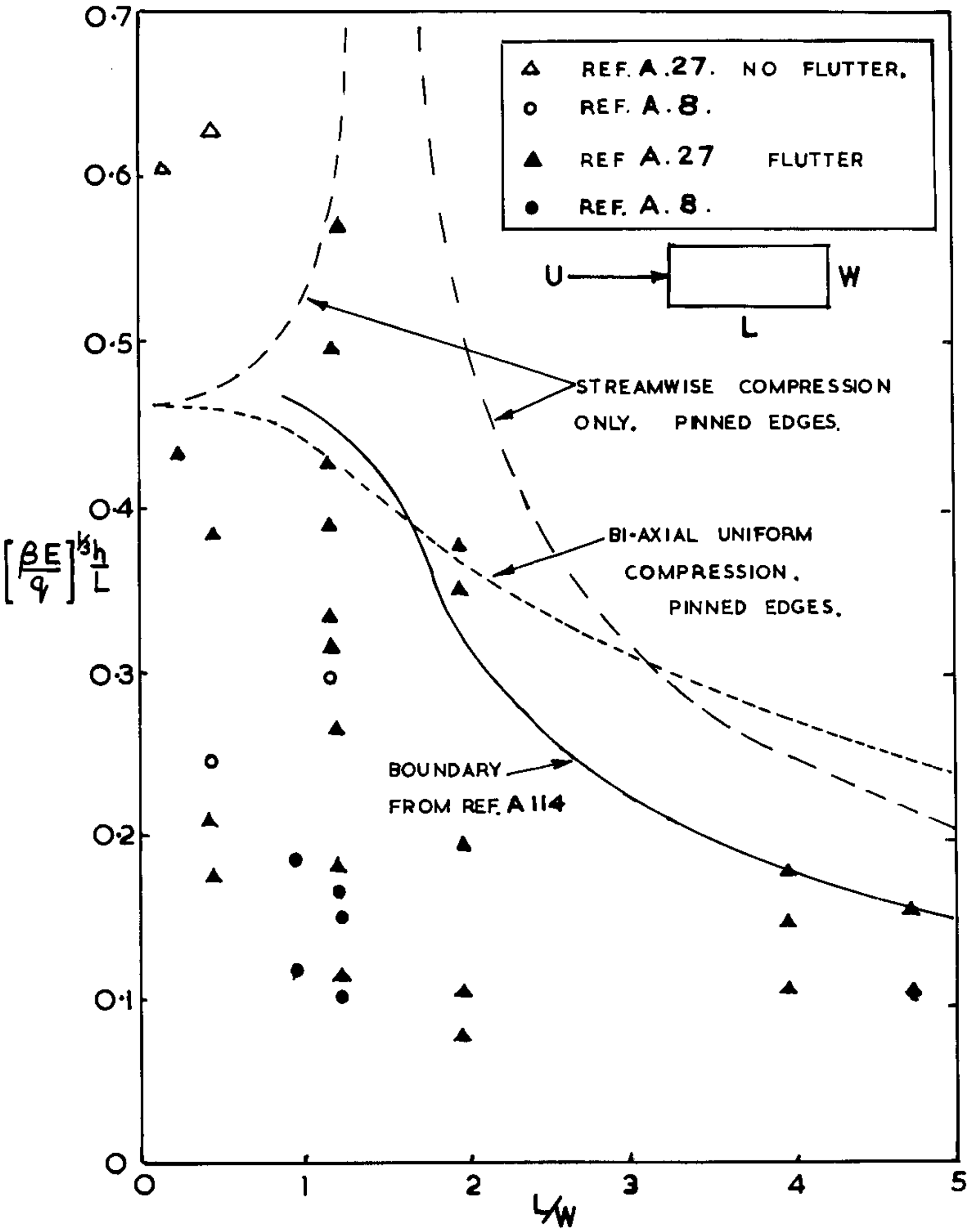

Fig.6 Panel flutter parameter versus inverse of aspect ratio for buckled panels[A258]

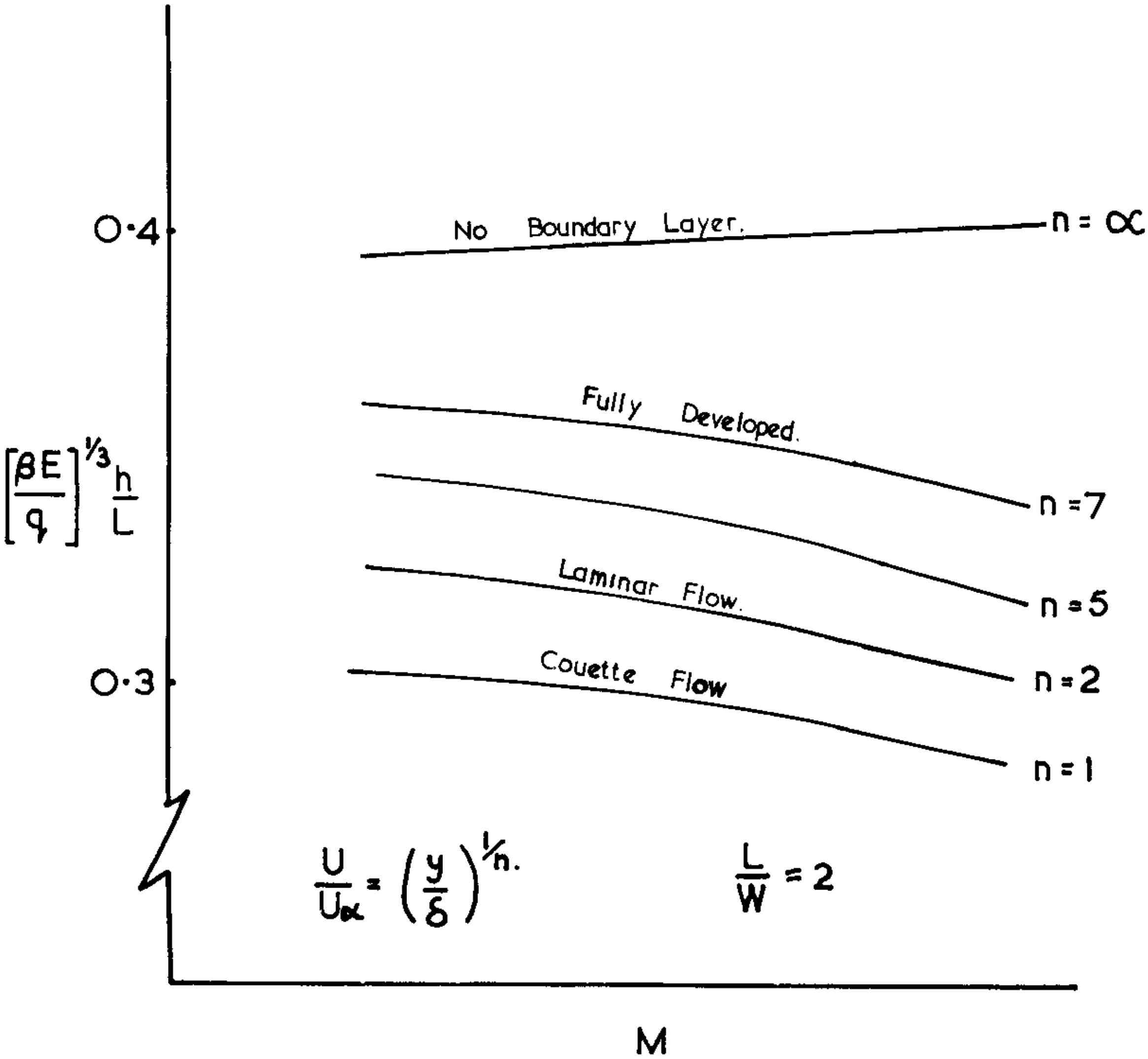

Fig.7 Schematic showing effect of boundary layer profile on critical flutter conditions. (Ref. North American Aviation, unpublished)

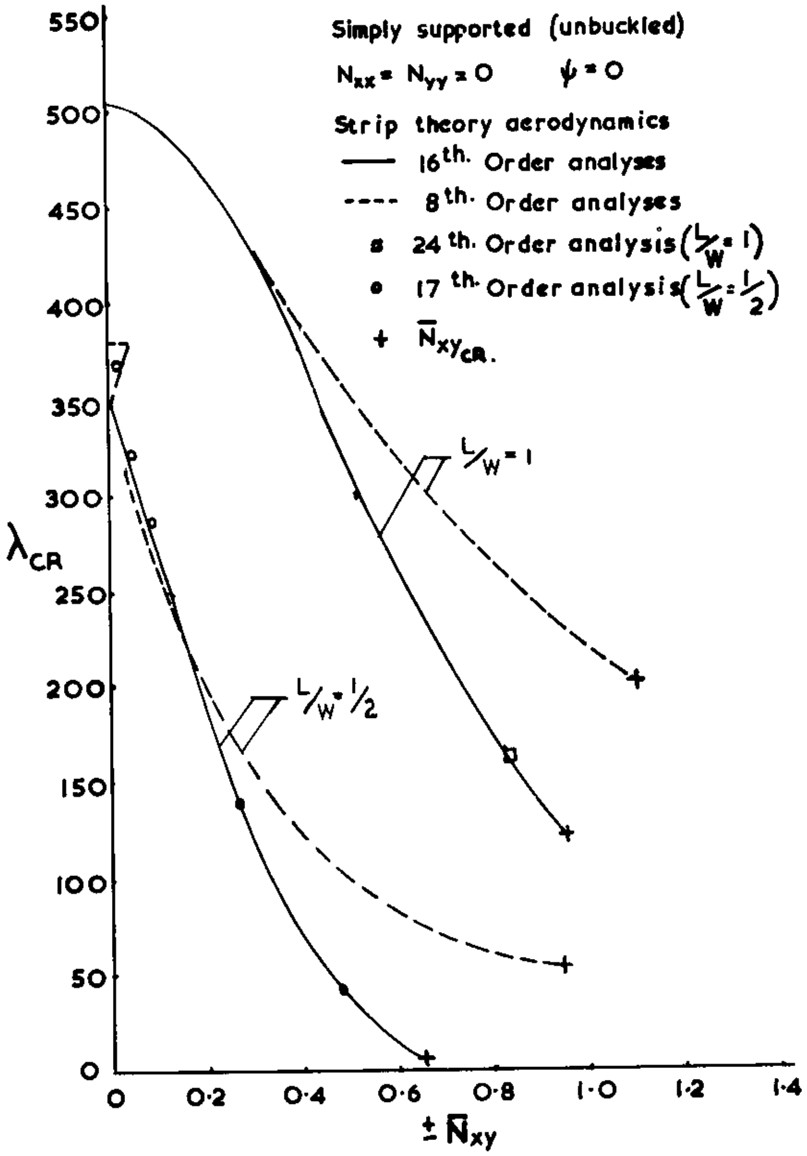

Fig.8 Effect of the number of assumed modes on the flutter boundary of an unswept rectangular plate[A166]

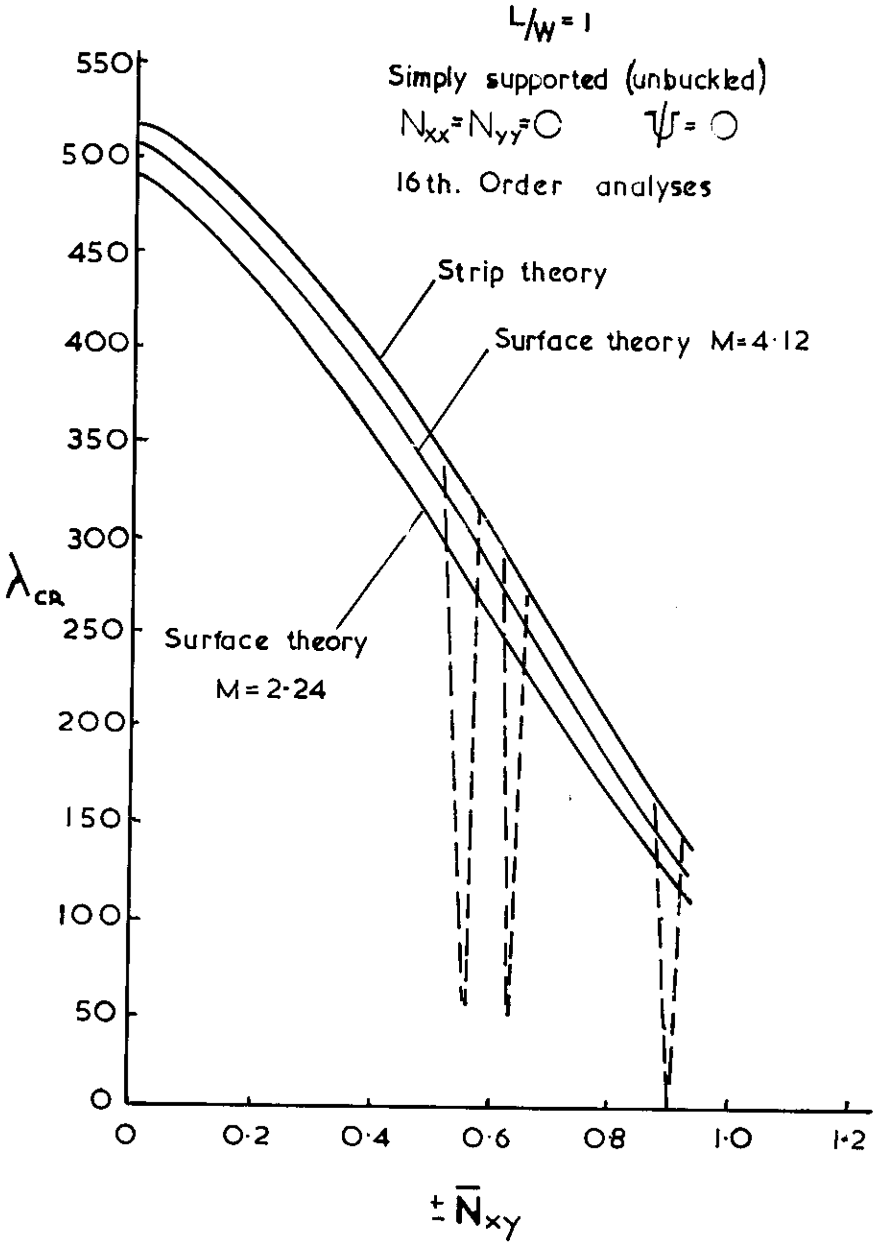

Fig.9 Comparison of flutter boundaries for static strip and static surface theory aerodynamics[A166]

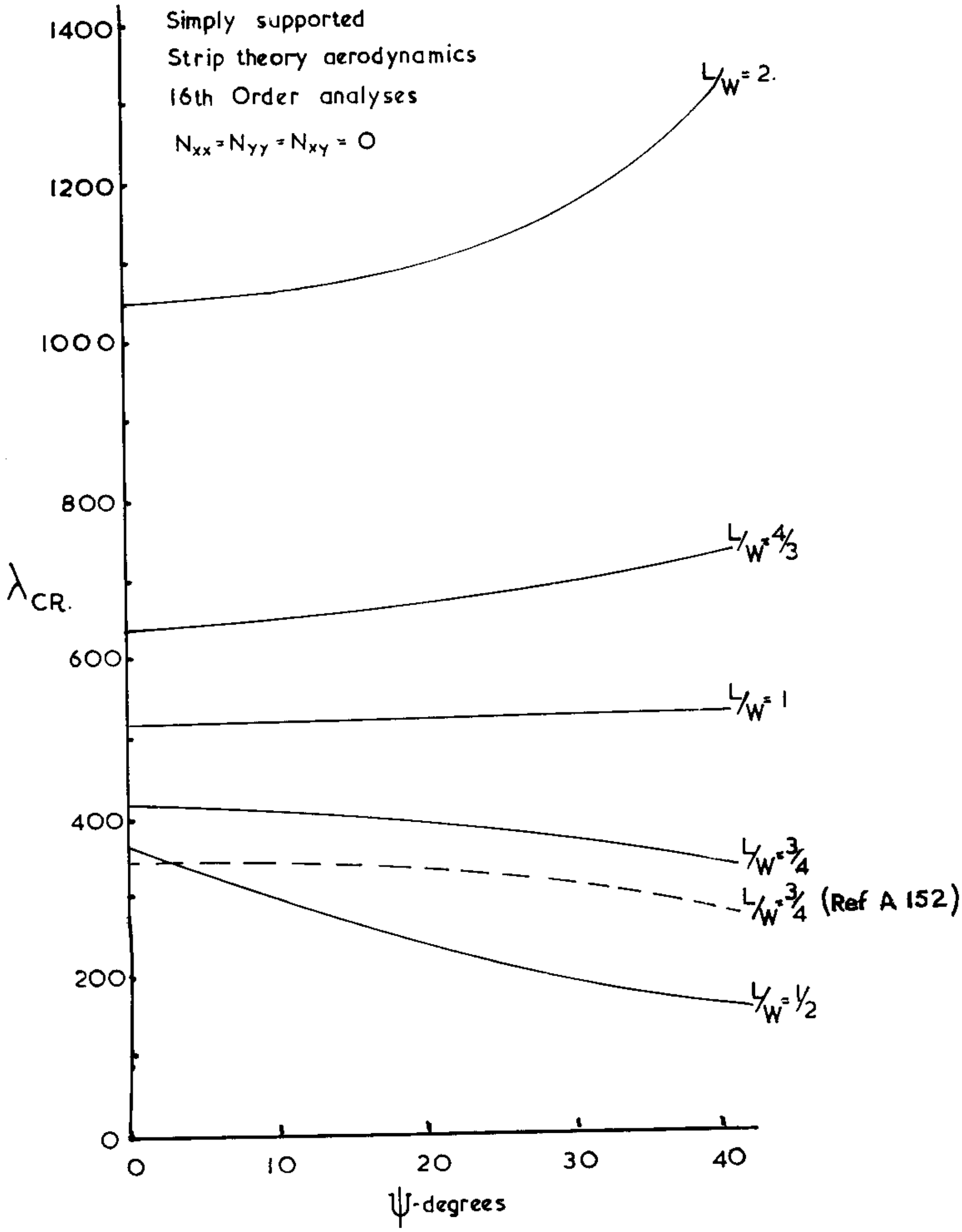

Fig. 10 Flutter boundaries for plates rotated with respect to flow[A166]

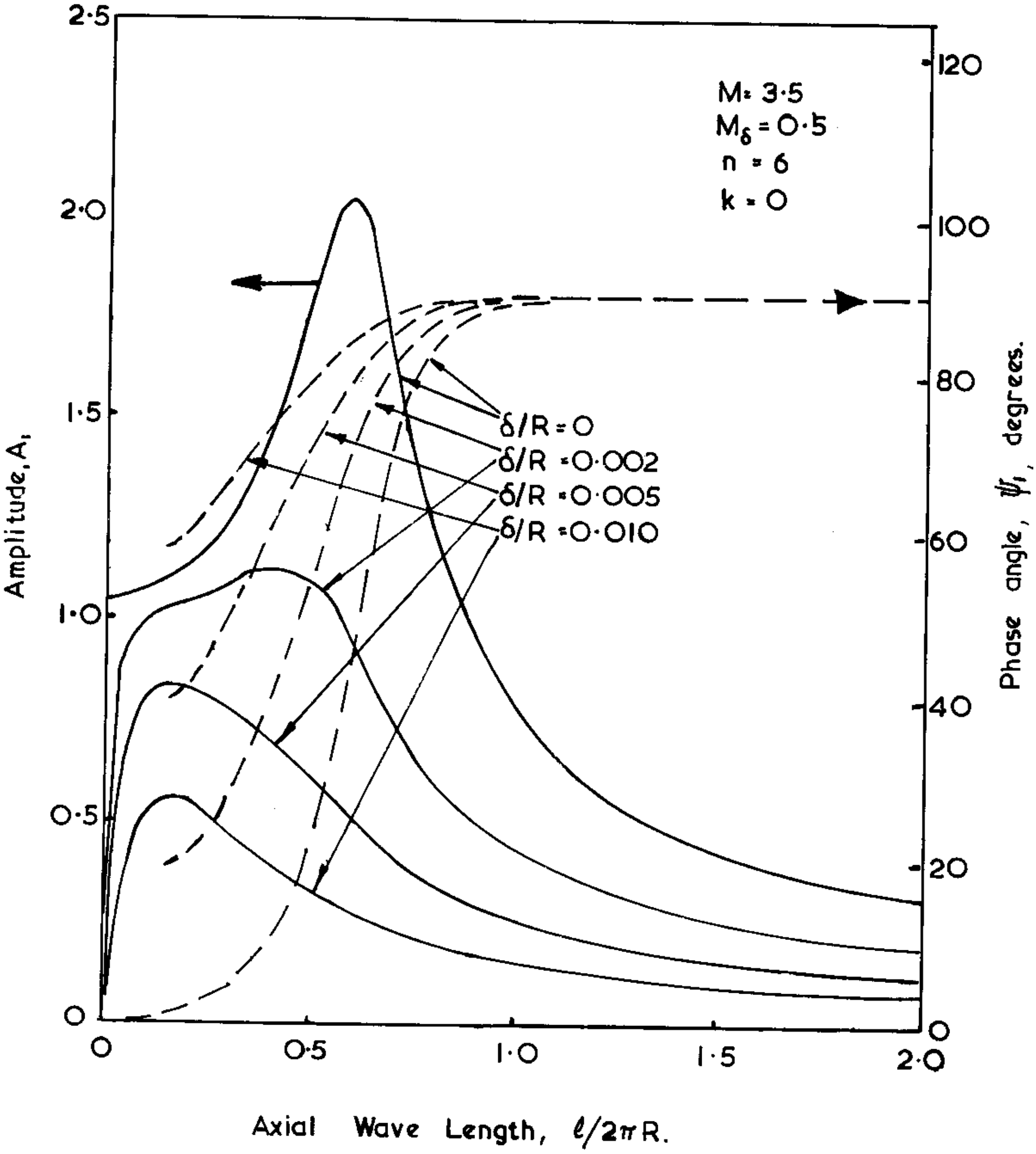

Fig. 11 Pressure amplitude and phase angle on a stationary wavy wall with idealised boundary layer. (Variable axial wavelength) [A271]

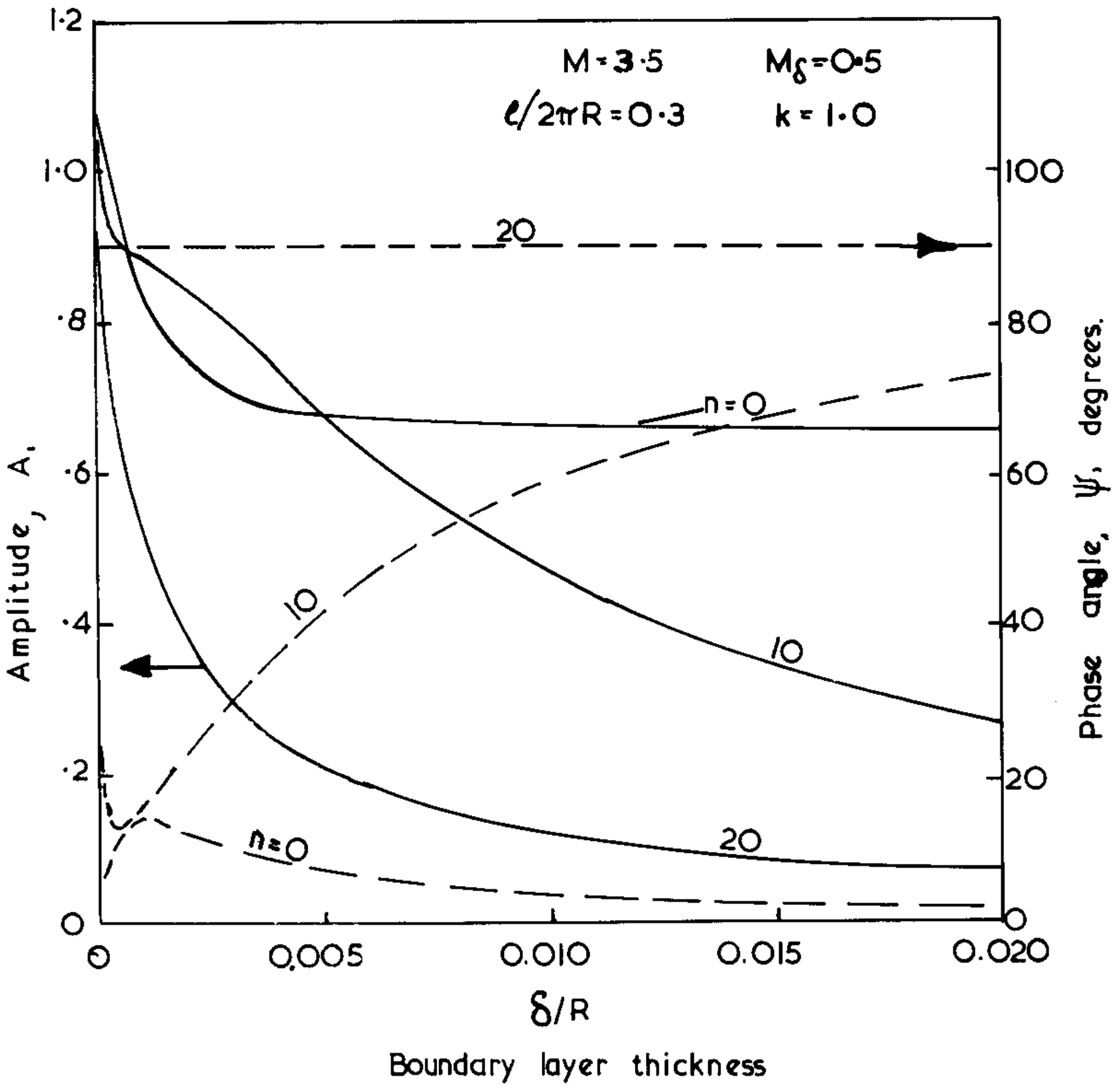

Fig. 12 Pressure amplitude and phase angle on an oscillating wavy wall with an idealised boundary layer. (Variable circumferential wave number)[A271]

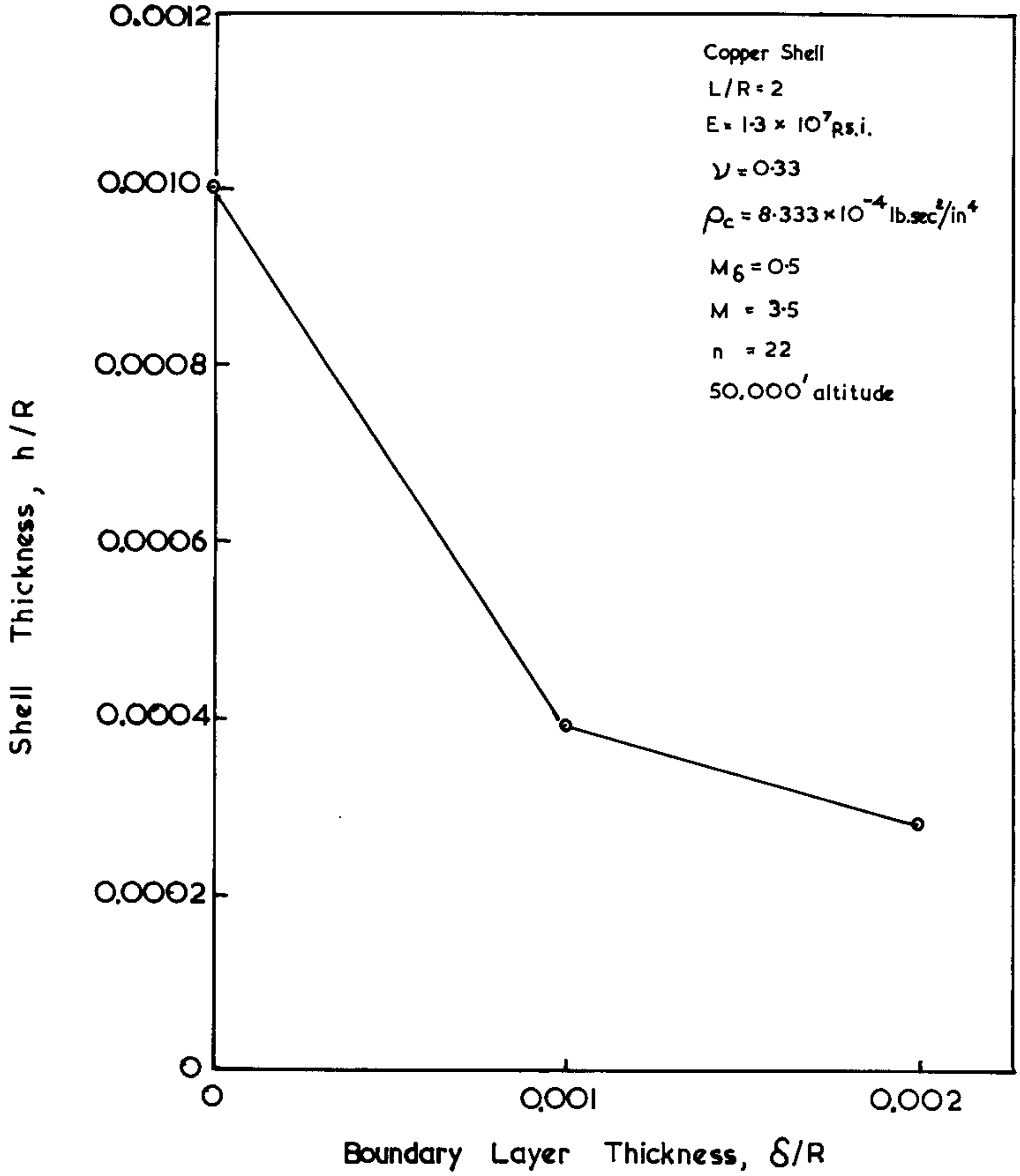

Fig. 13 Asymmetric flutter boundary, assuming an idealised boundary layer[A271]

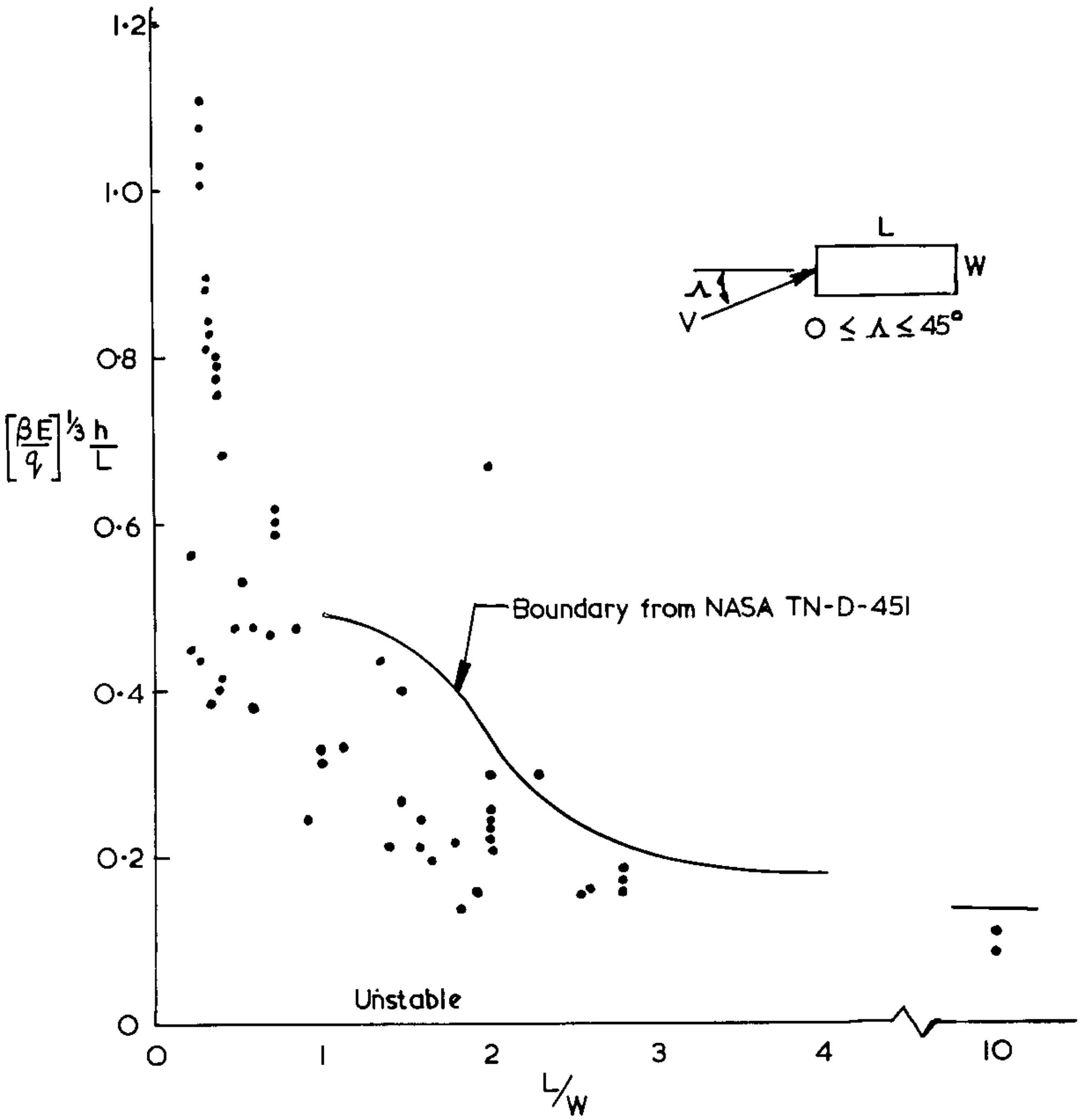

Fig.14 Panel flutter parameter versus length/width ratio[A175]

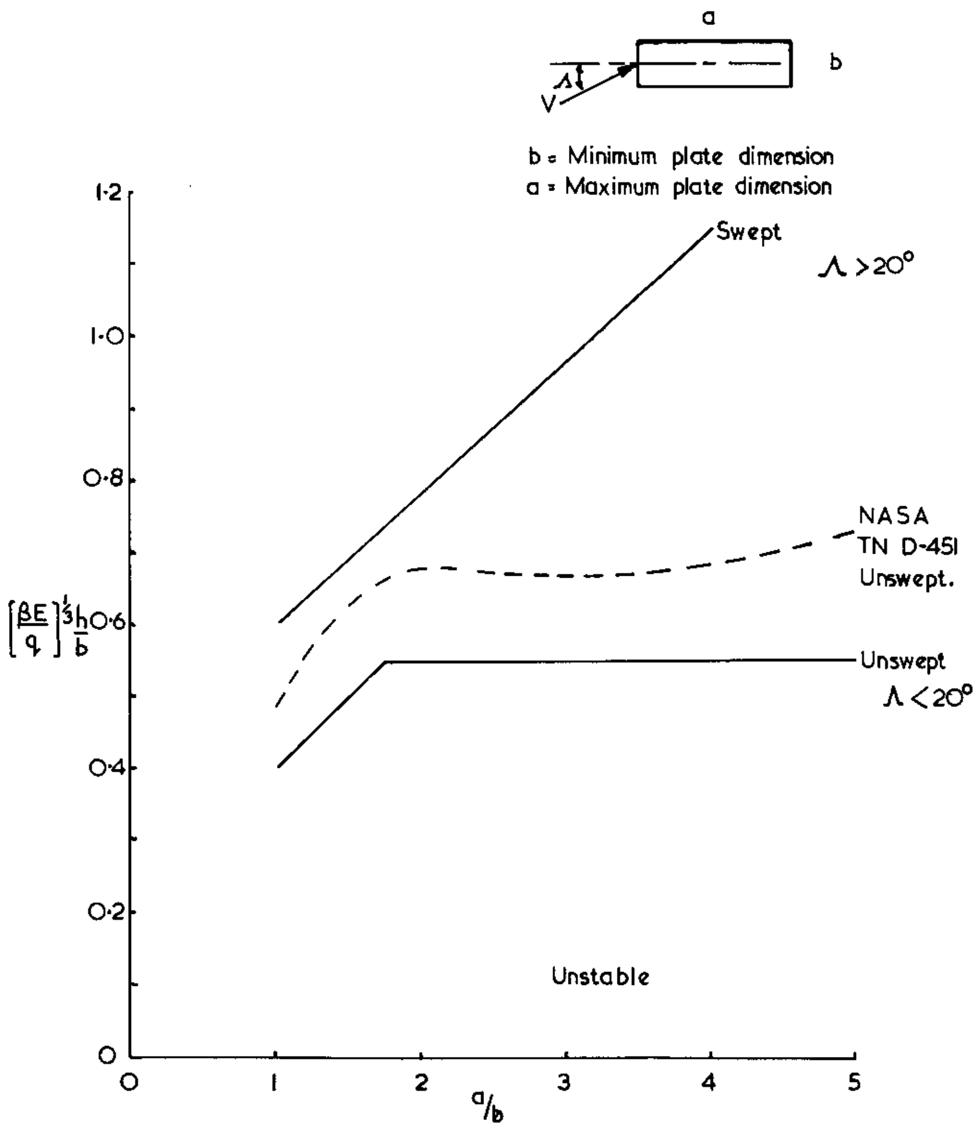

Fig.15 Tentative flutter boundaries for swept and unswept, flat, uniform panels[A175]

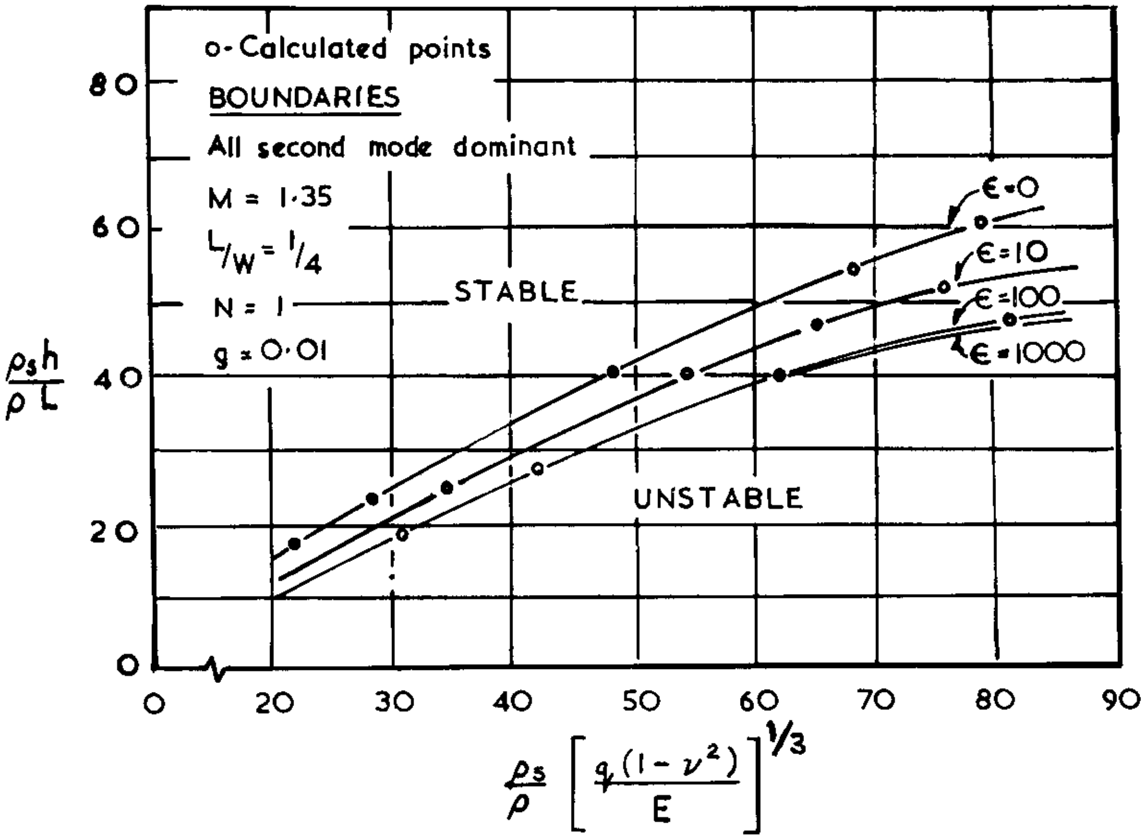

Fig.16 Stability boundaries from four-mode analyses for flat panels with partially clamped edges M = 1.35 , L/W = 1/4 , g = 0.01 , N = 1 . (Ref.A190)

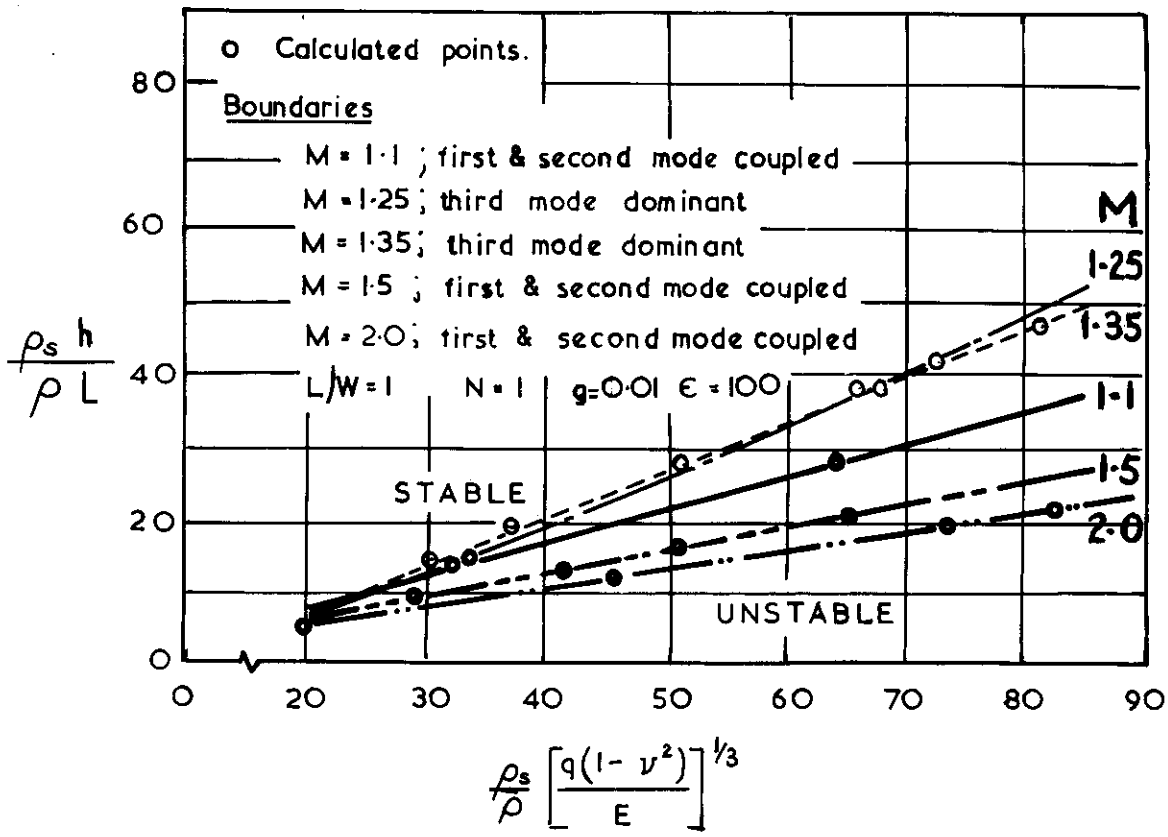

Fig.17 Stability boundaries from four-mode analyses for nearly clamped flat panels at various Mach numbers $L/W = 1$, $N = 1$, $g = 0.01$, $\epsilon = 100$. (Ref.A190)

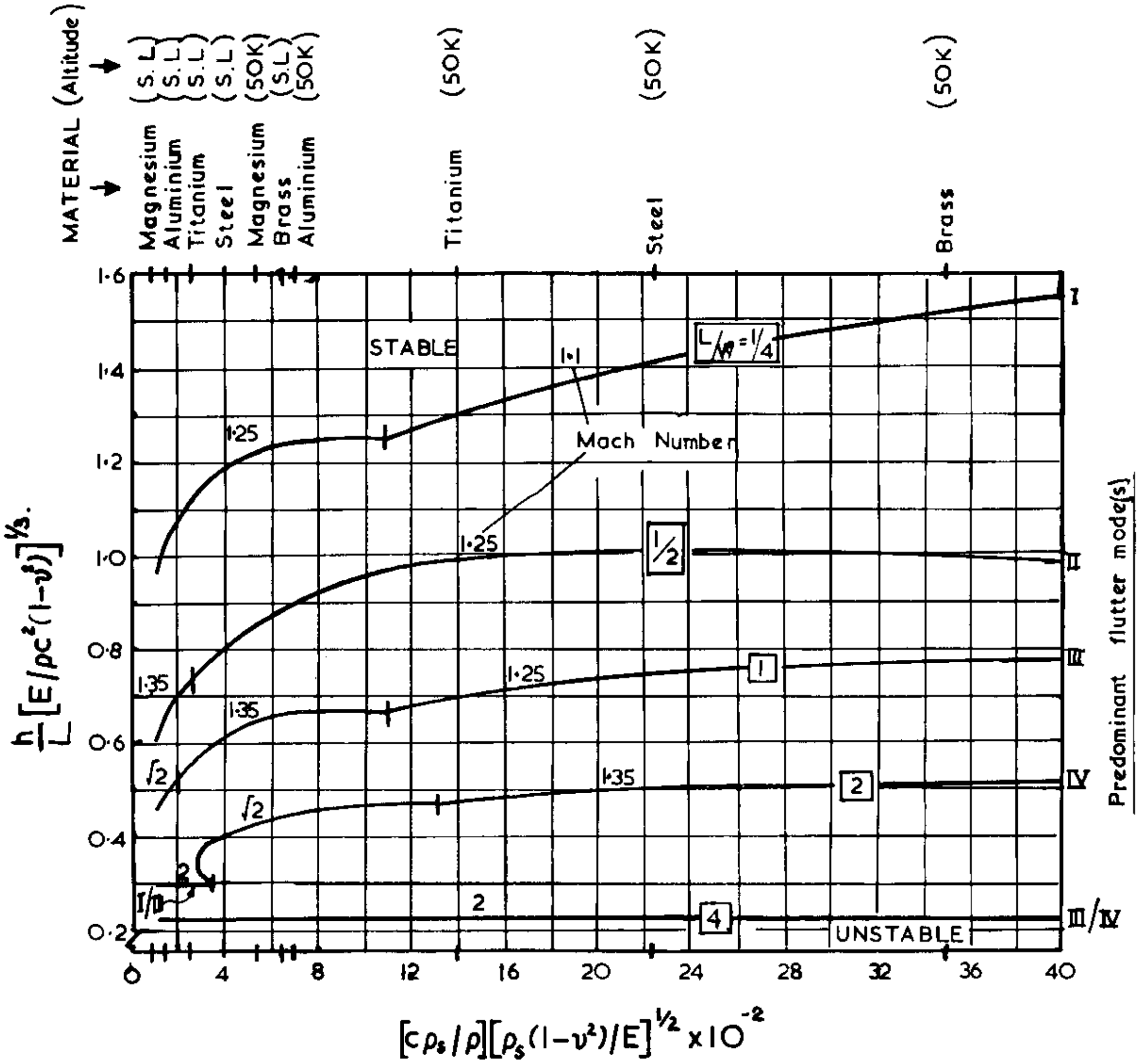

Fig.18 Variation of critical stability boundaries with length/width ratio. (Adapted from Ref.A224)

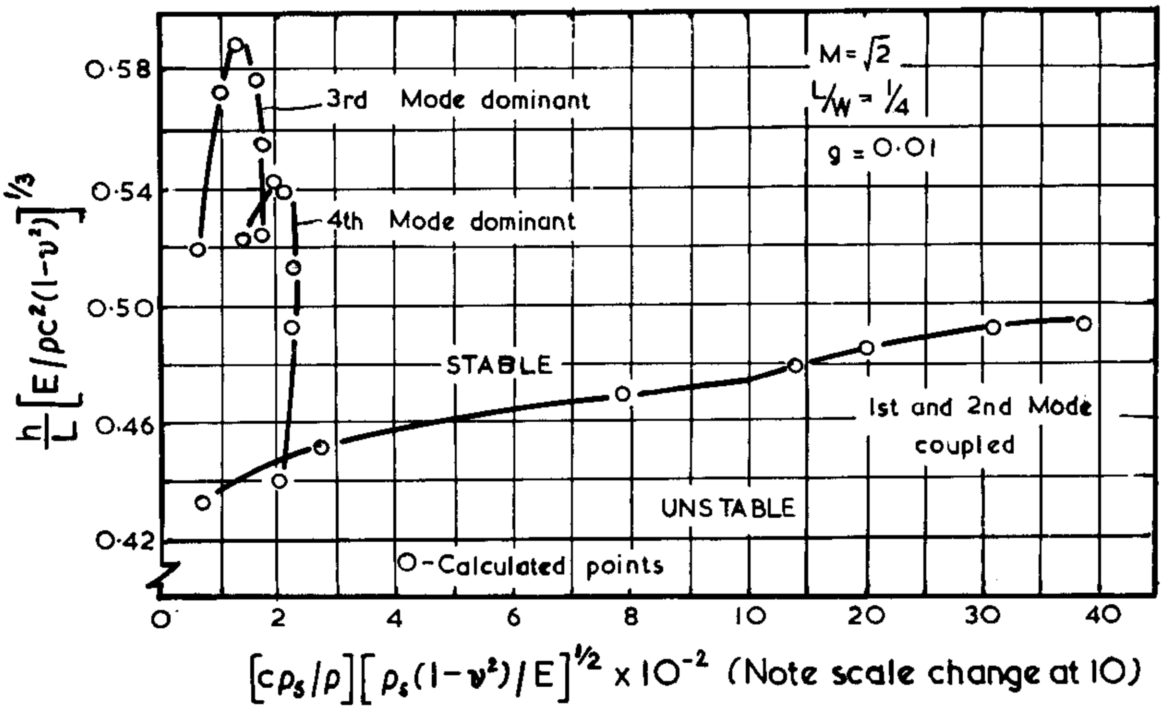

Fig.19 Critical stability boundary from a four mode analysis for simply-supported flat plates[A224]

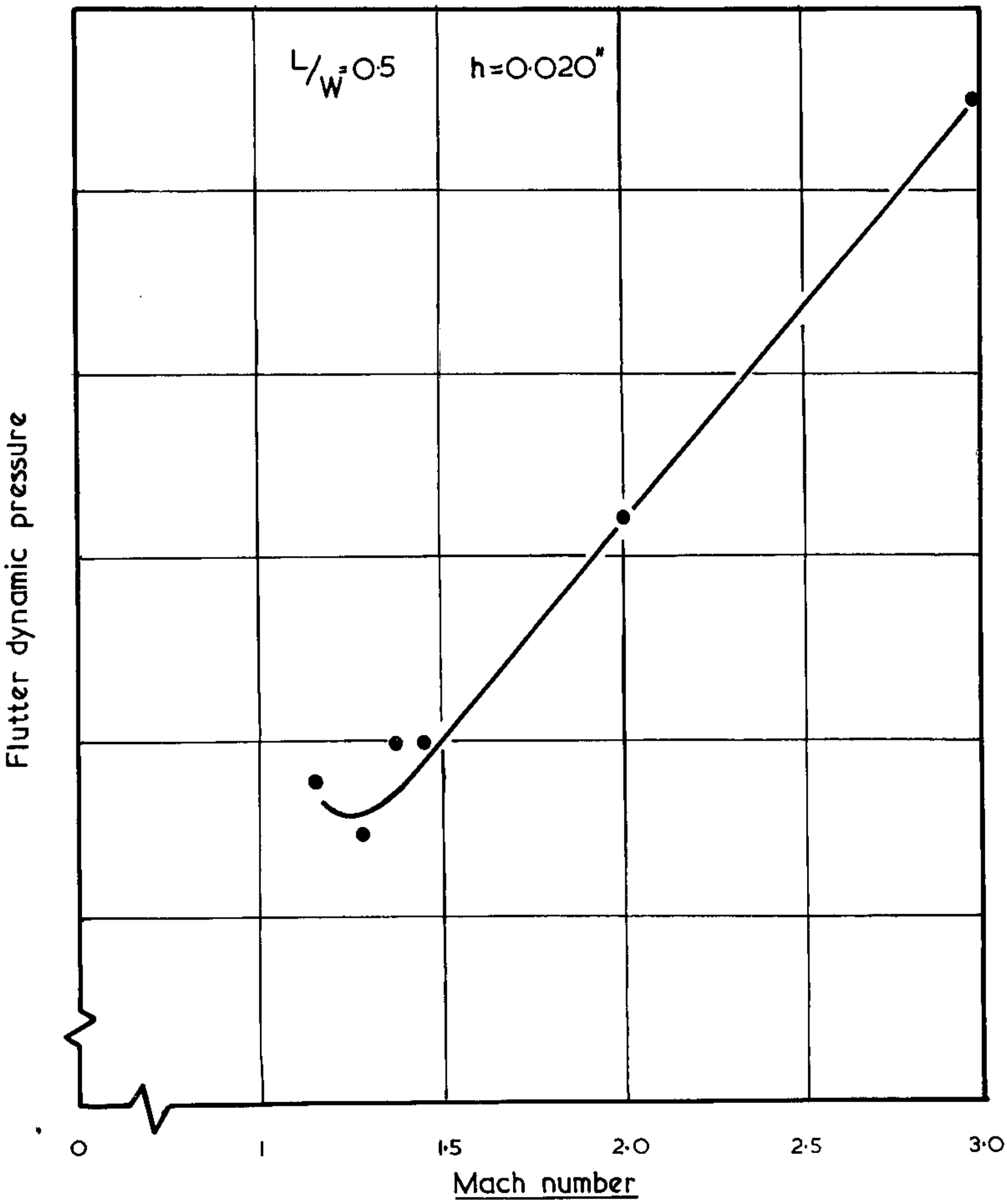

Fig. 20 Experimental trend of flutter dynamic pressure versus Mach number[A218]

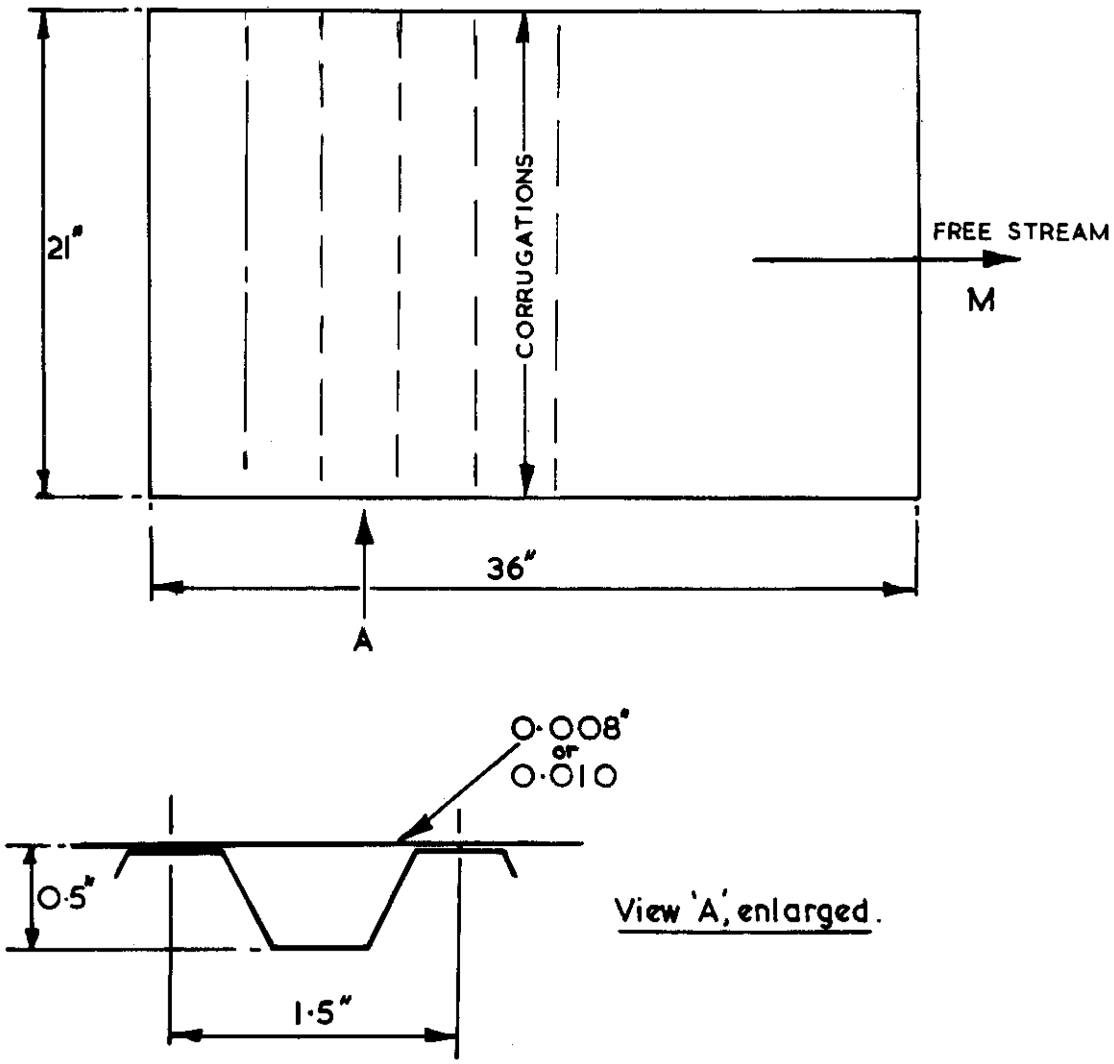

Fig. 21 Schematic of corrugated panel test specimen (Ref. Boeing Company, unpublished)

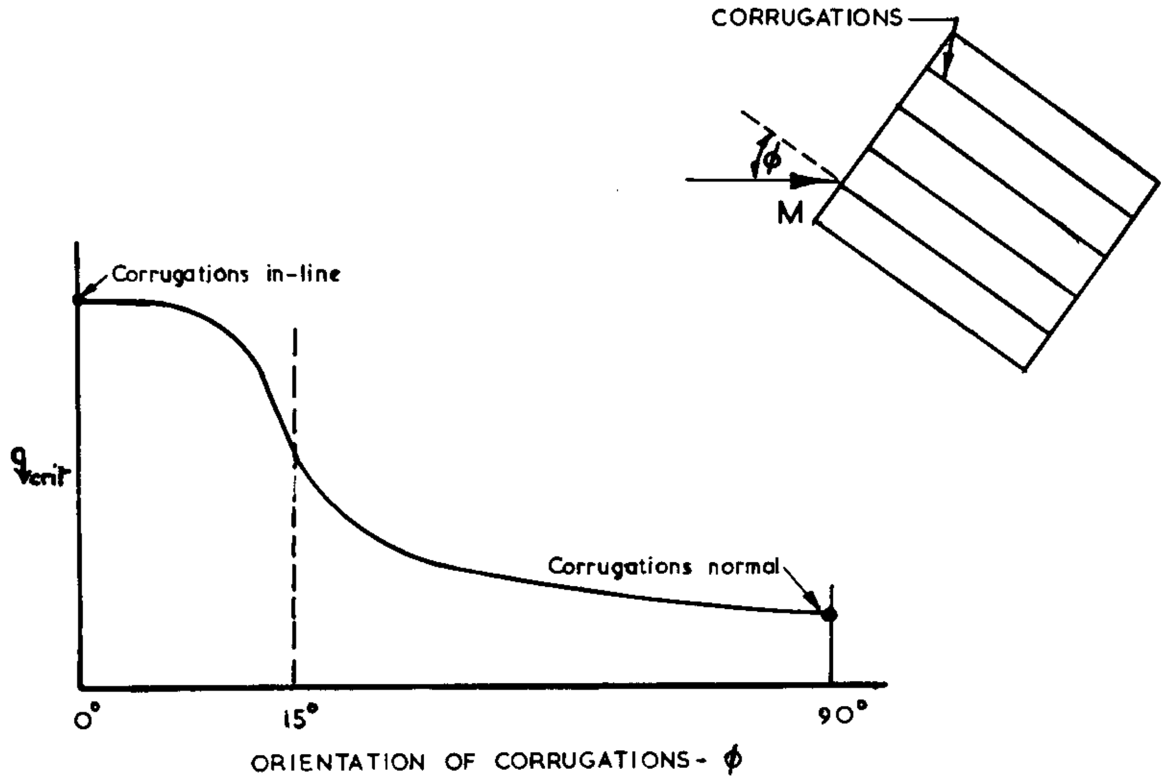

Fig. 22 Typical variation of critical dynamic pressure with orientation of corrugations for a square panel (Ref. Boeing Company, unpublished)

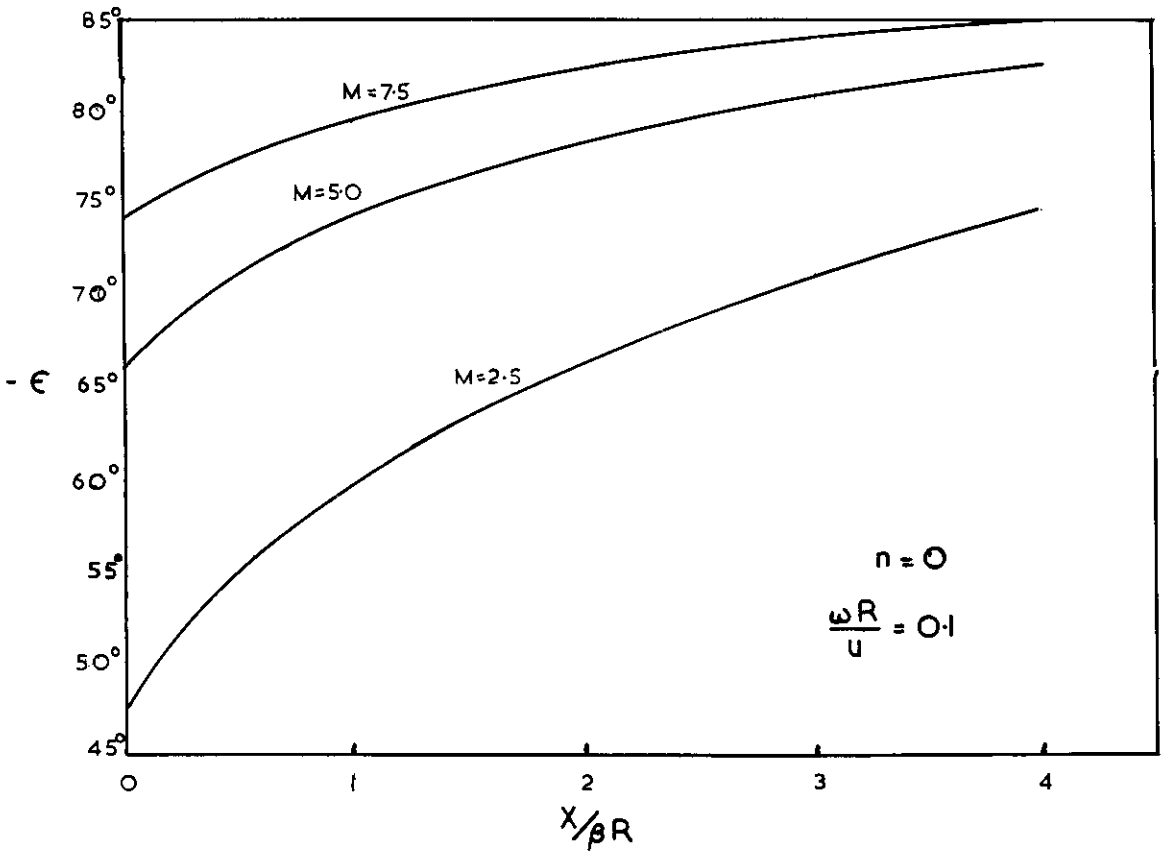

Fig. 23 Variation of pressure distribution phase shift with axial position on an oscillating cylindrical shell[A206]

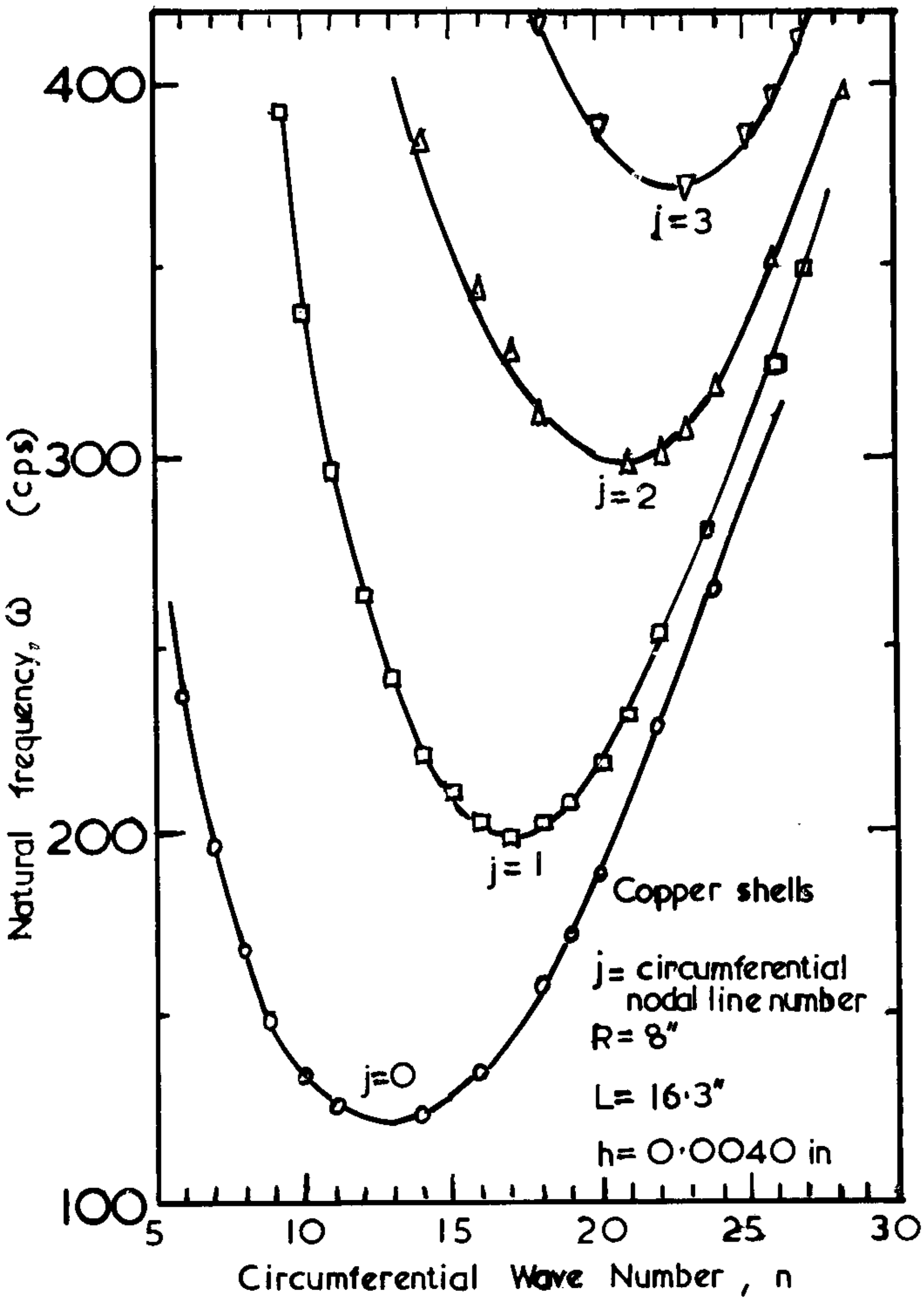

Fig. 24 Natural frequencies of unstressed circular cylindrical shells[A266]

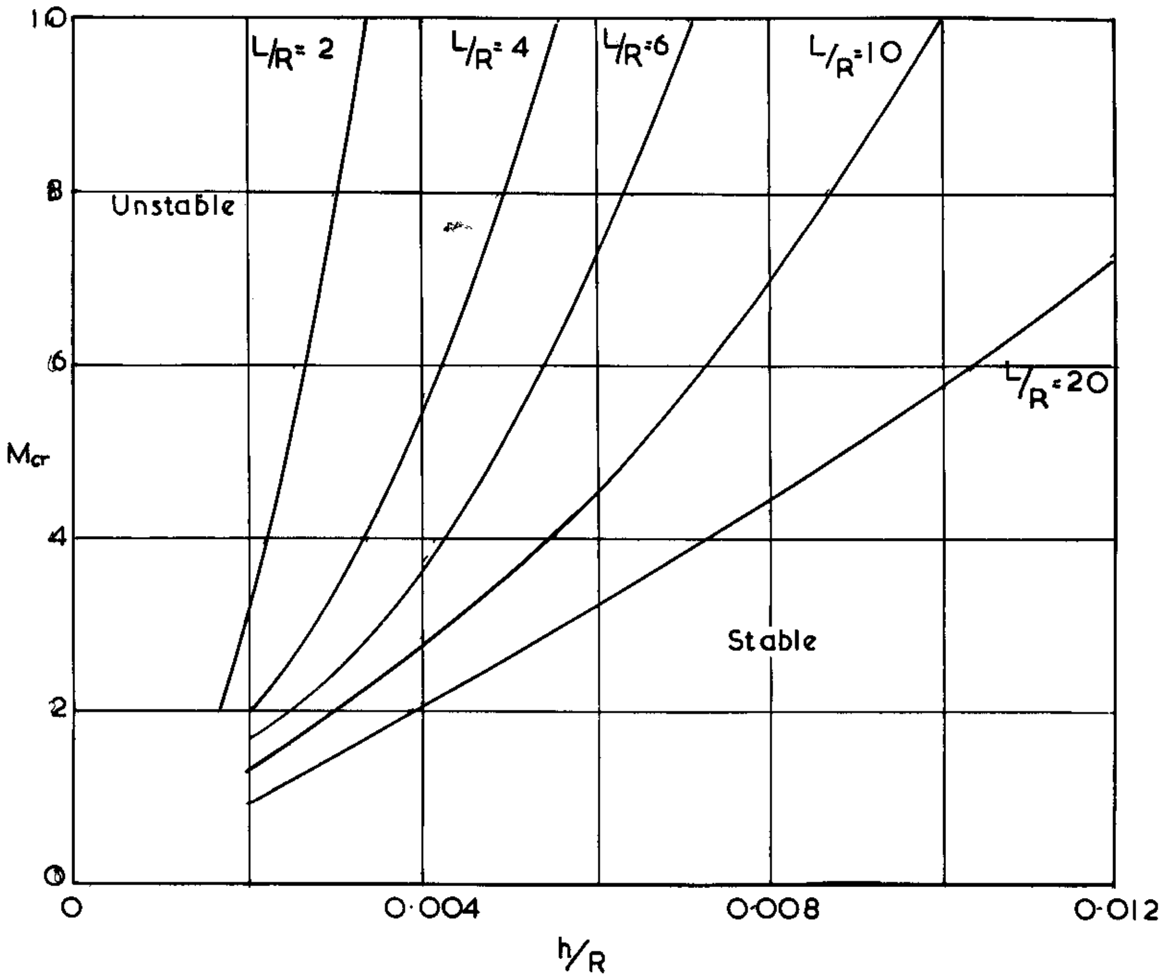

Fig.25 Panel flutter boundary for steel shells at sea level[A214]

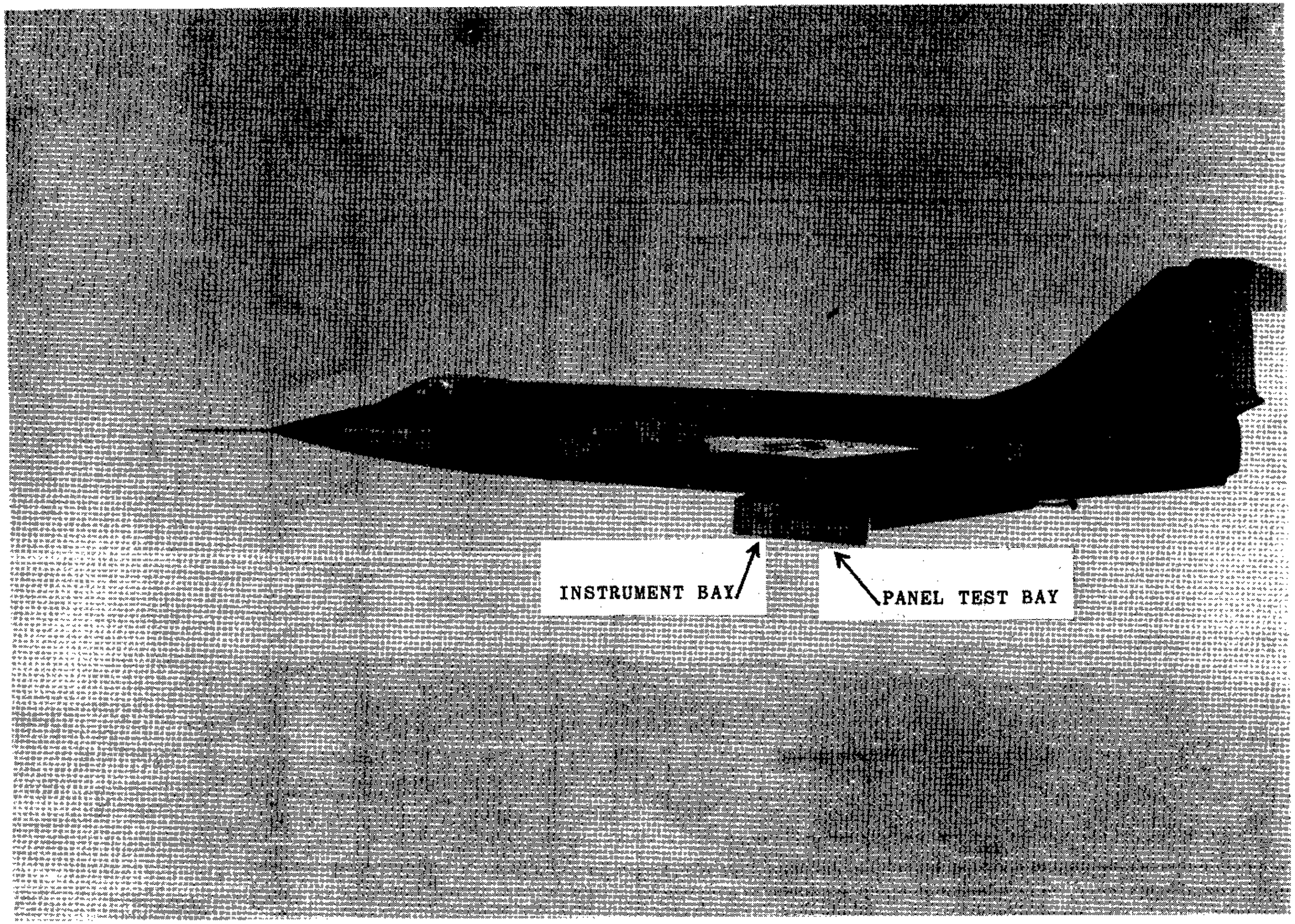

Fig. 26 Photograph showing "Ventral fin panel flutter test fixture on F104."
(Ref. NASA - Edwards AFB)

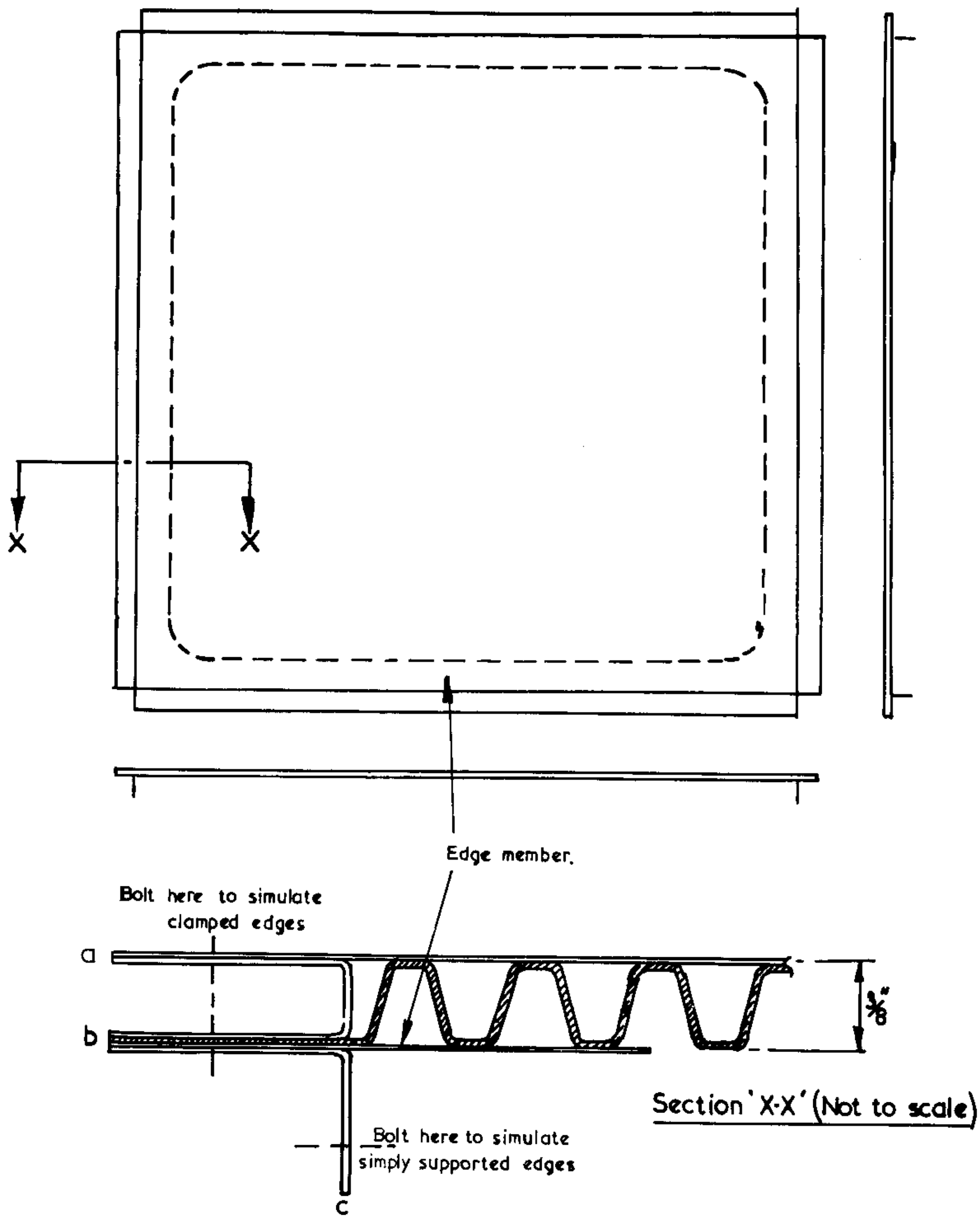

Fig. 27 Typical flat panel of built-up construction A218

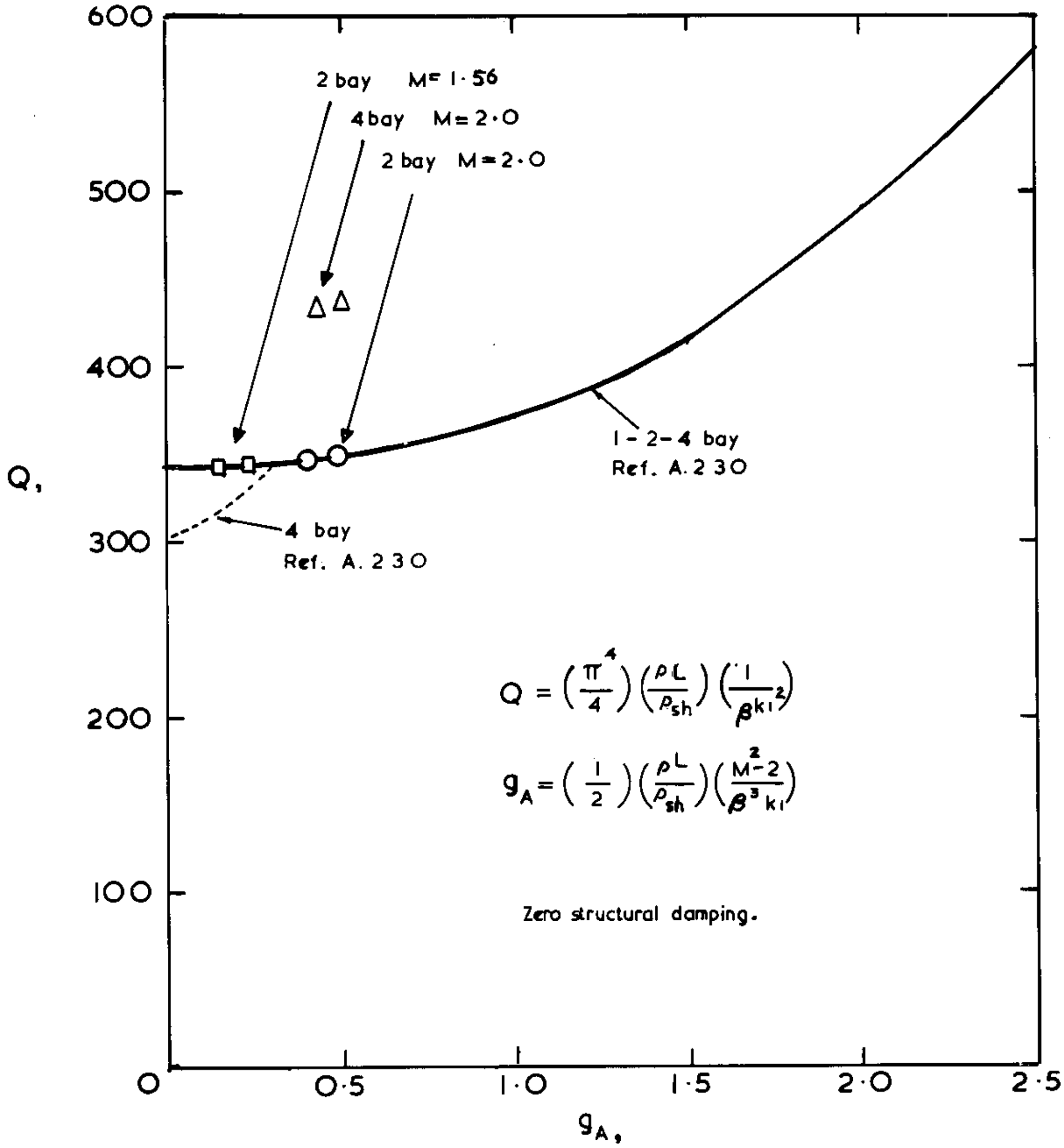

Fig. 28 Comparison of flutter stability boundaries for multi-bay panels[A252]

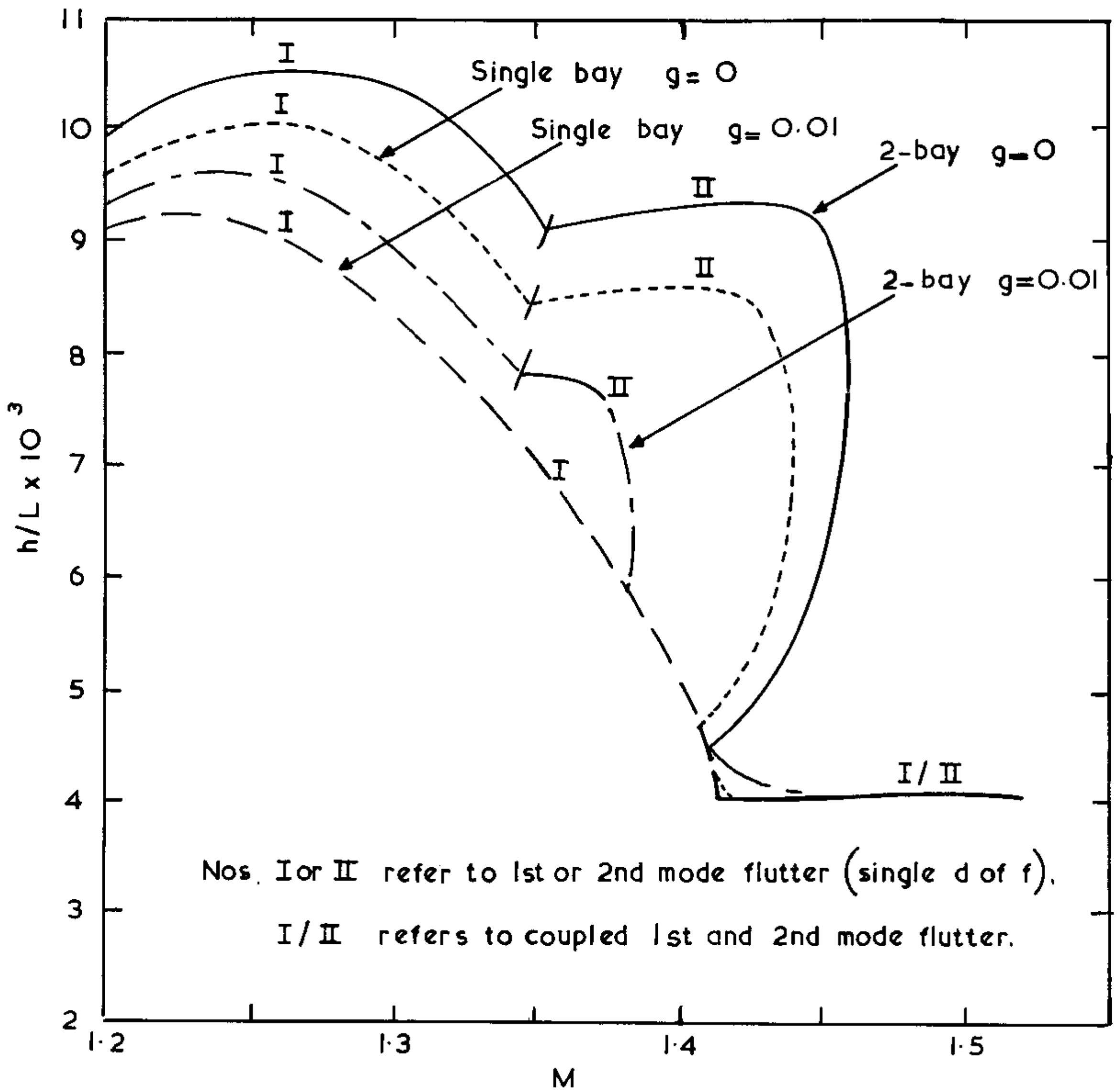

Fig. 29 Thickness ratio versus Mach number for clamped edged aluminium panels at sea level[A251]

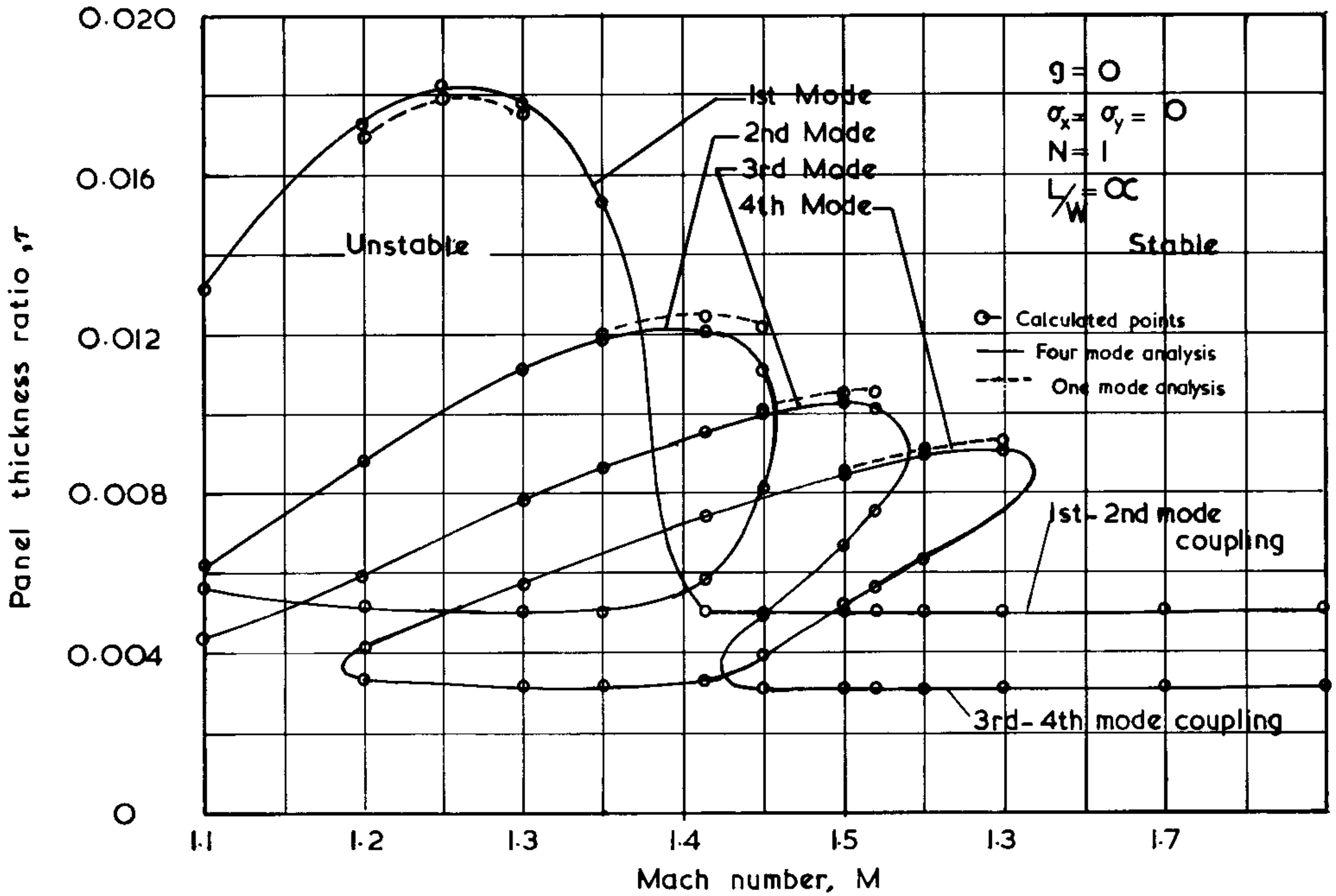

Fig.30 Small deflection stability boundary from four- and one-mode analyses for pinned-edge aluminium panels at sea level[A150]

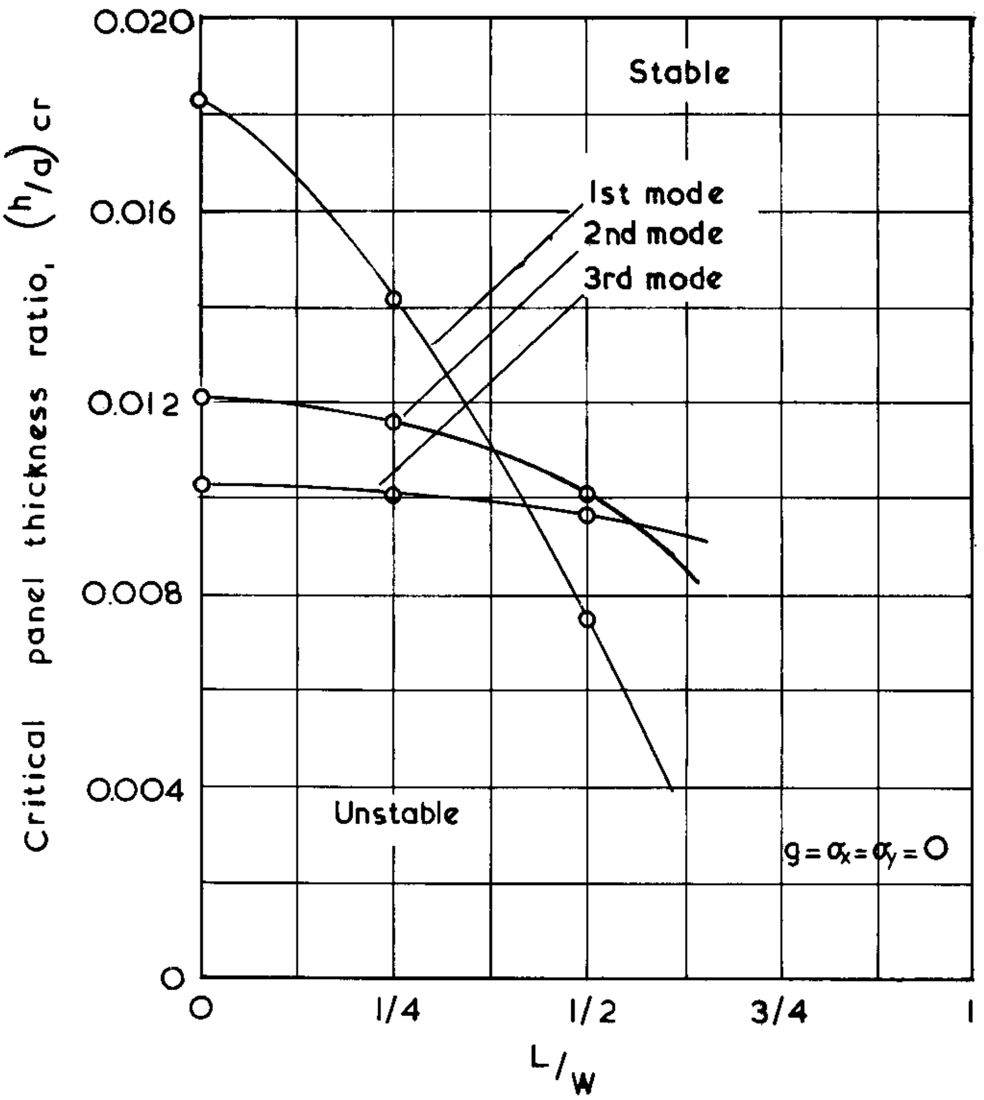

Fig.31 Critical thickness ratio versus length/width ratio for pinned-edge aluminium panels at sea level[A150]

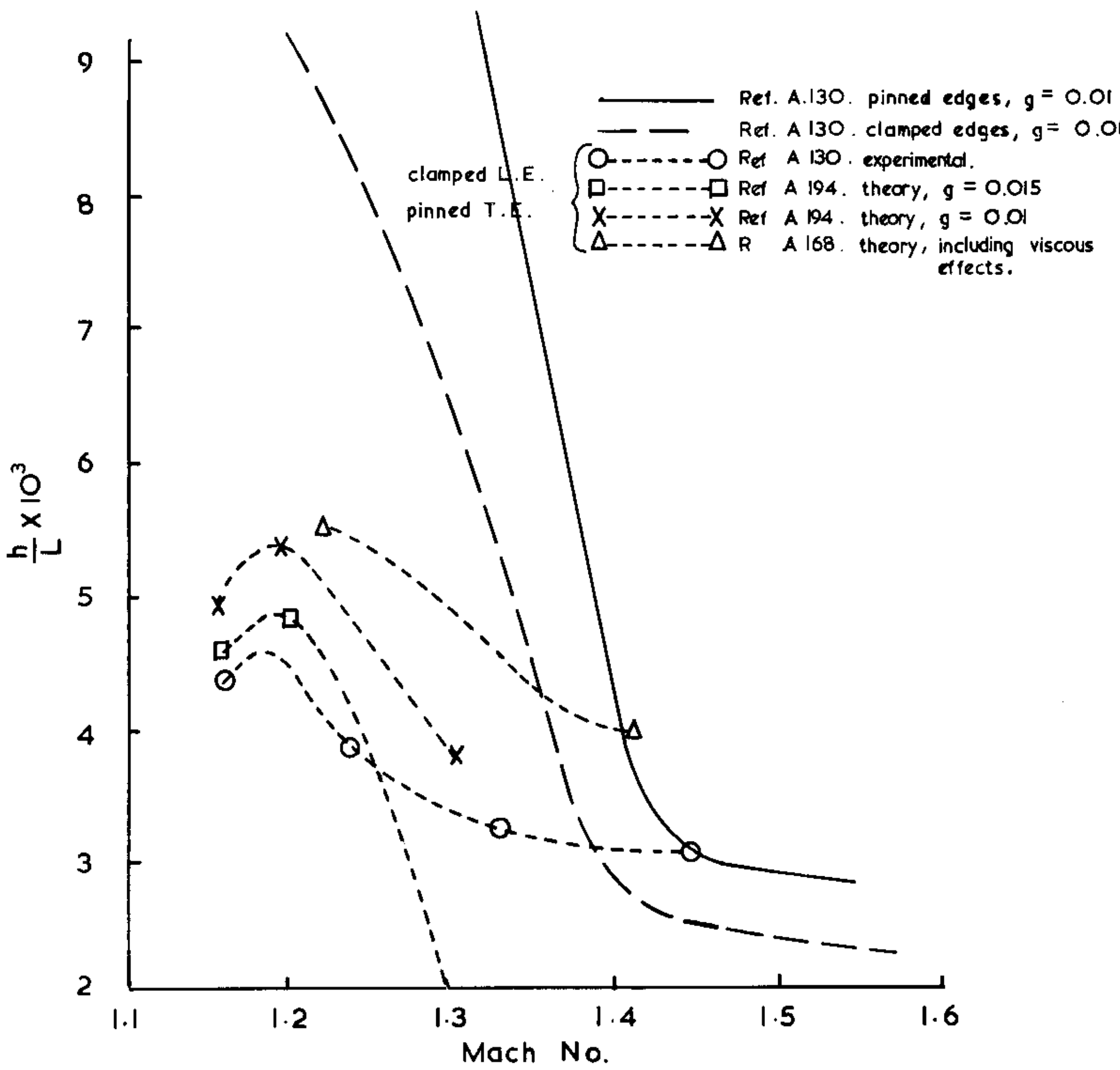

Fig.32 Comparison of experiment with theory for brass panels at sea level A194 MODIFIED

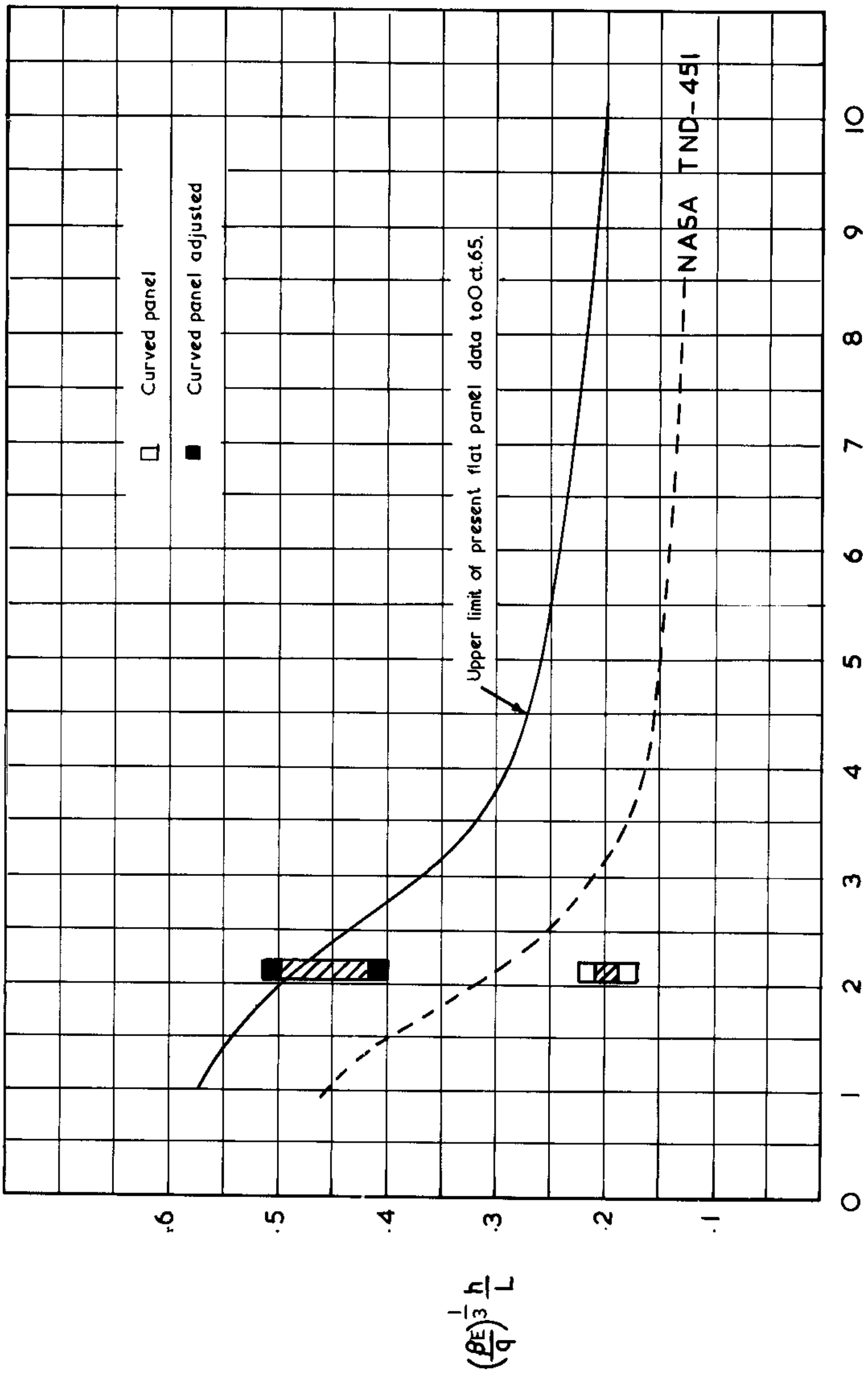

Fig. 33 The "adjusted" critical thickness of a curved panel[A186] MODIFIED

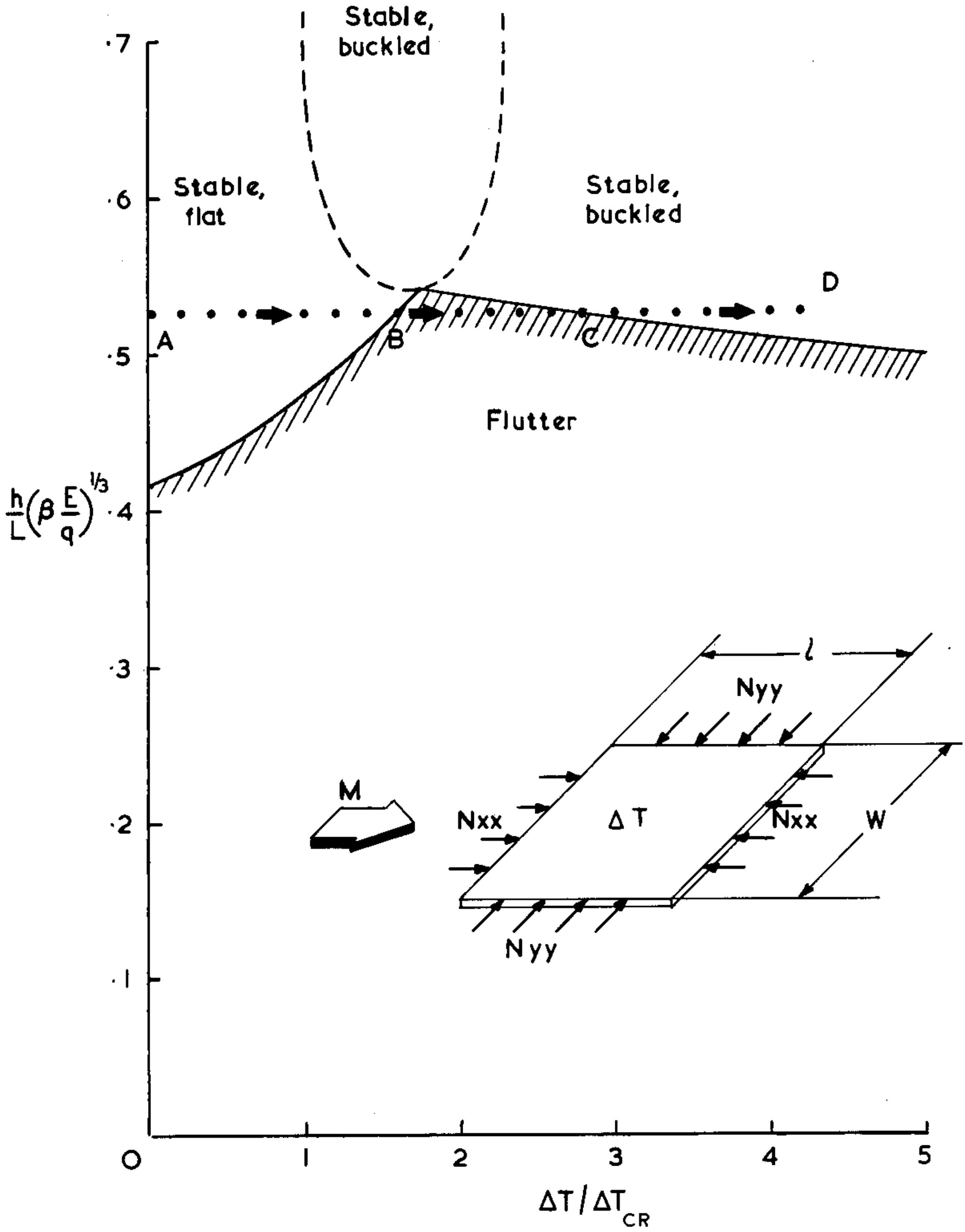

Fig.34 Flutter boundary for a simply-supported square panel with a uniform temperature rise[A192]

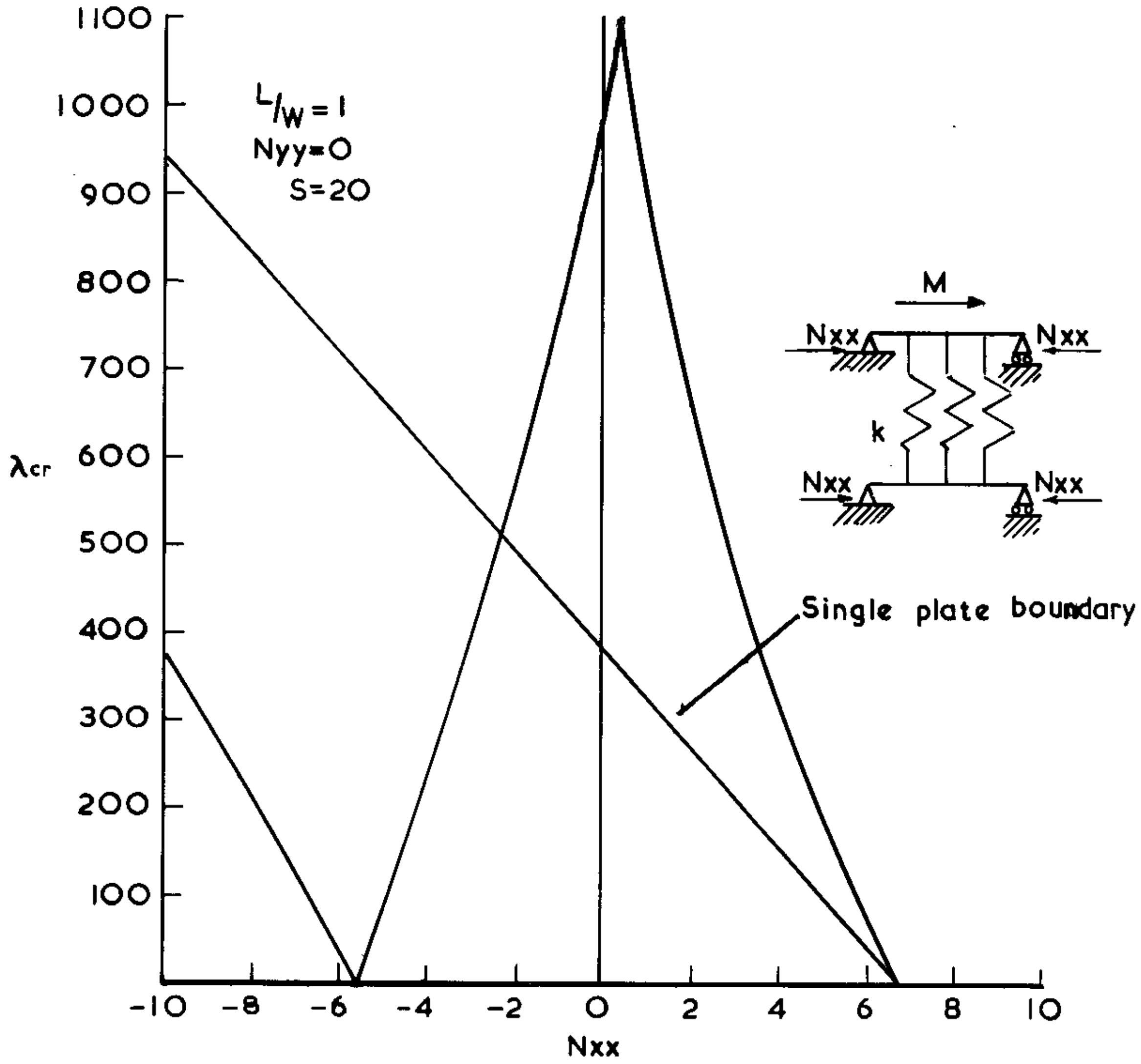

Fig.35 Flutter boundary for parallel flat panels elastically connected[A233]

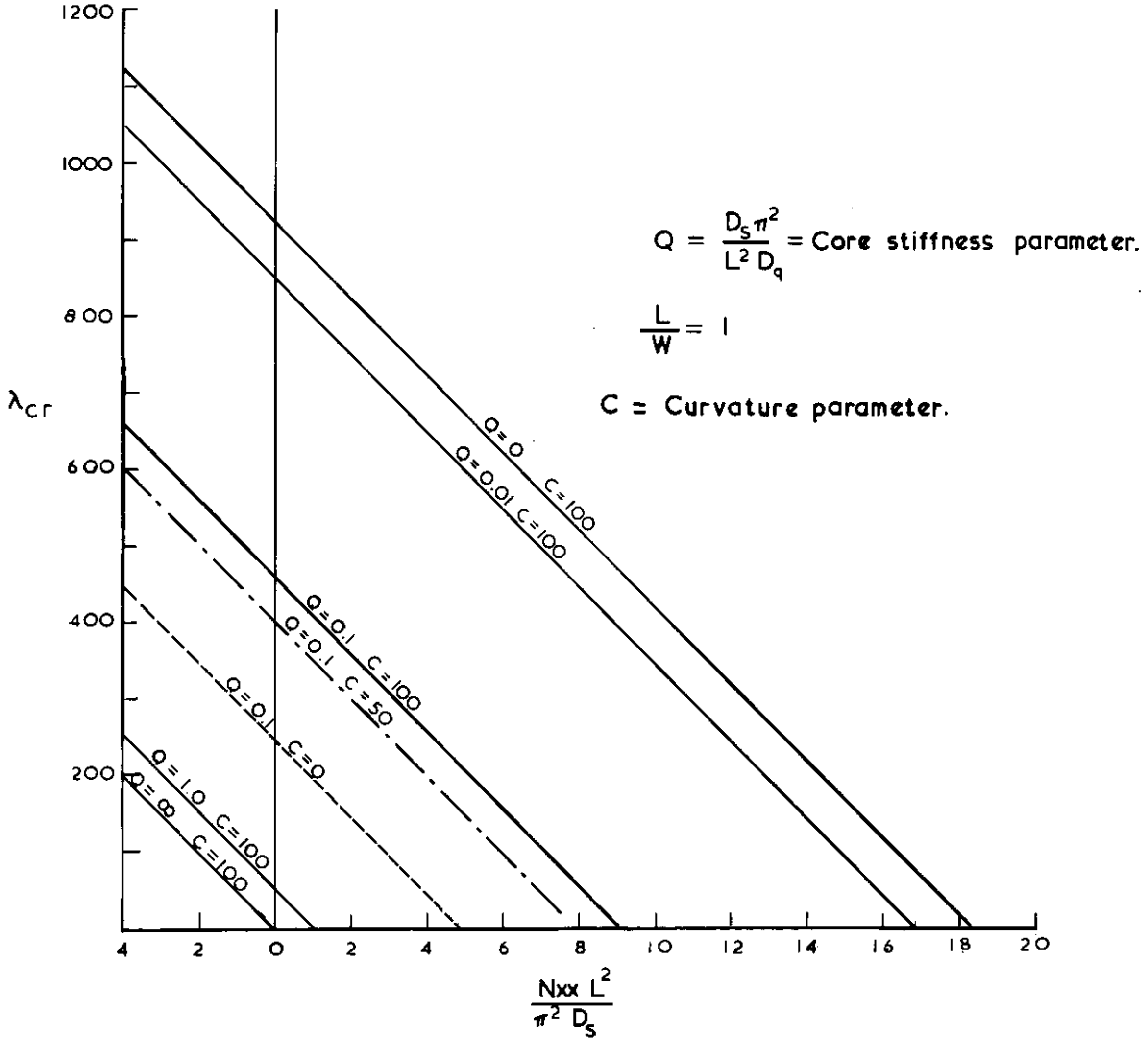

Fig. 36 Effect of transverse shear stiffness and curvature on the flutter of sandwich panels[A239]

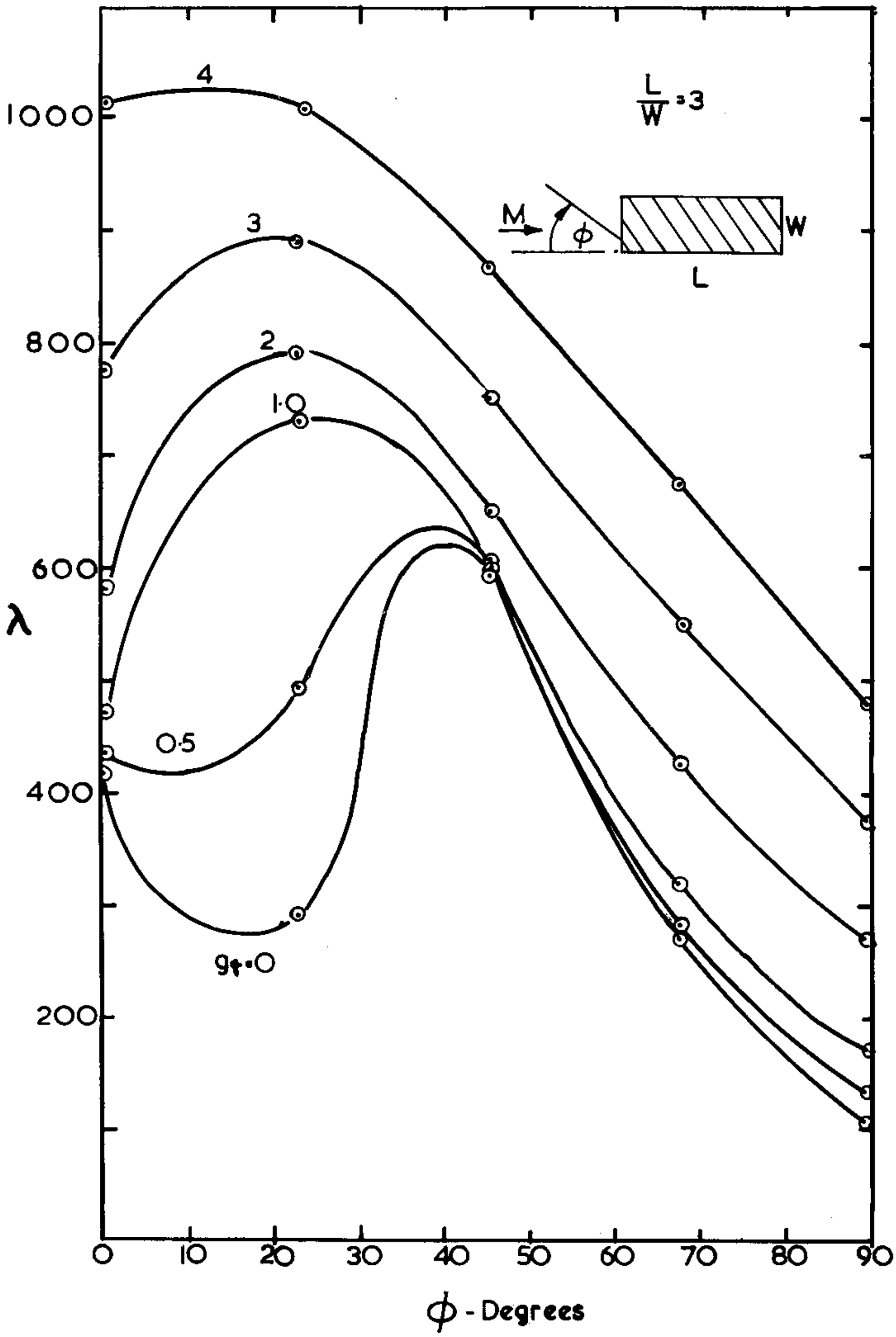

Fig. 37 Effect of angle of orthotropicity on flutter[A209]

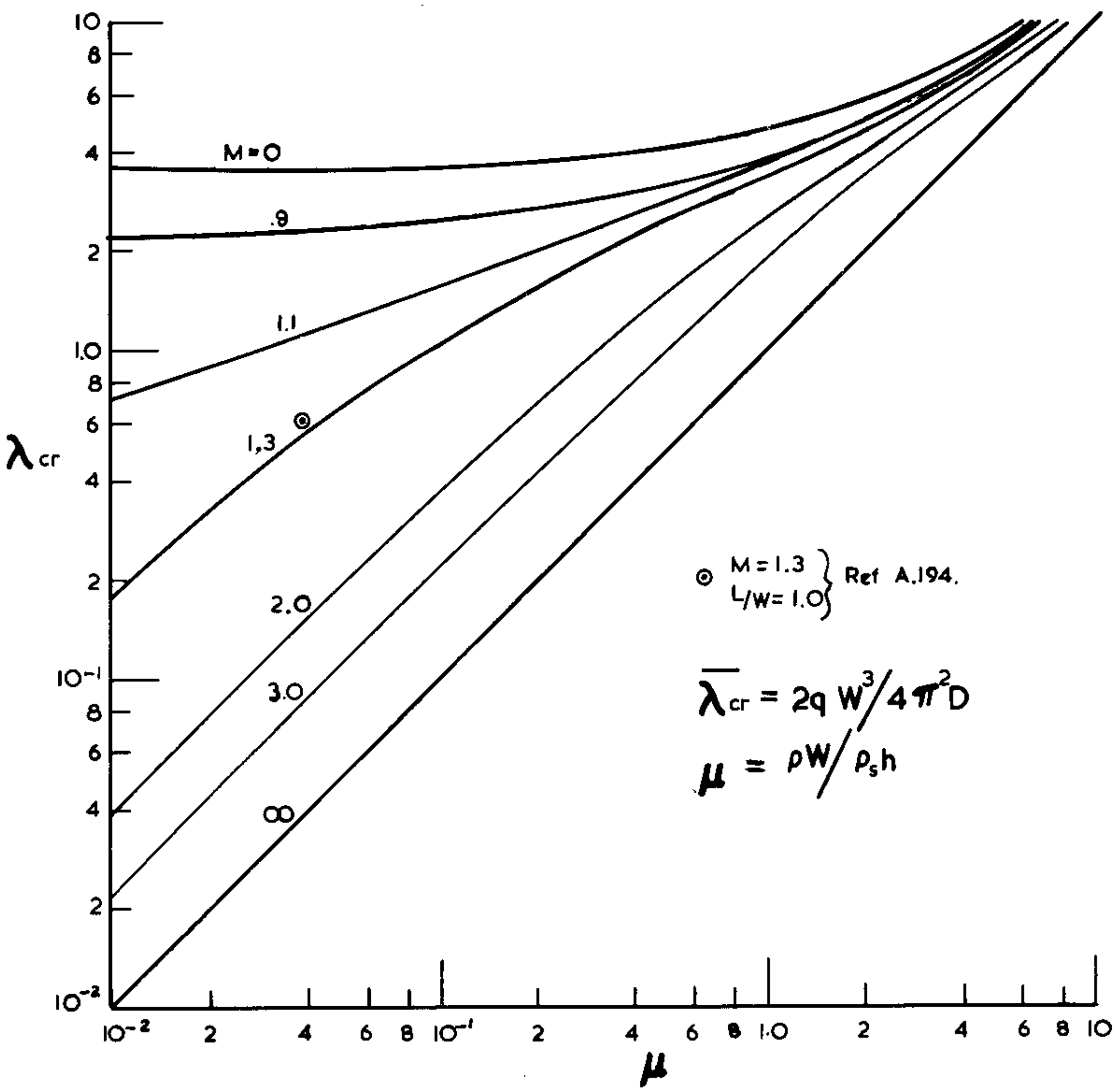

Fig.38 Flutter dynamic pressure versus mass ratio for very low aspect ratio panels ($L/W \to \infty$) (Ref.A249)

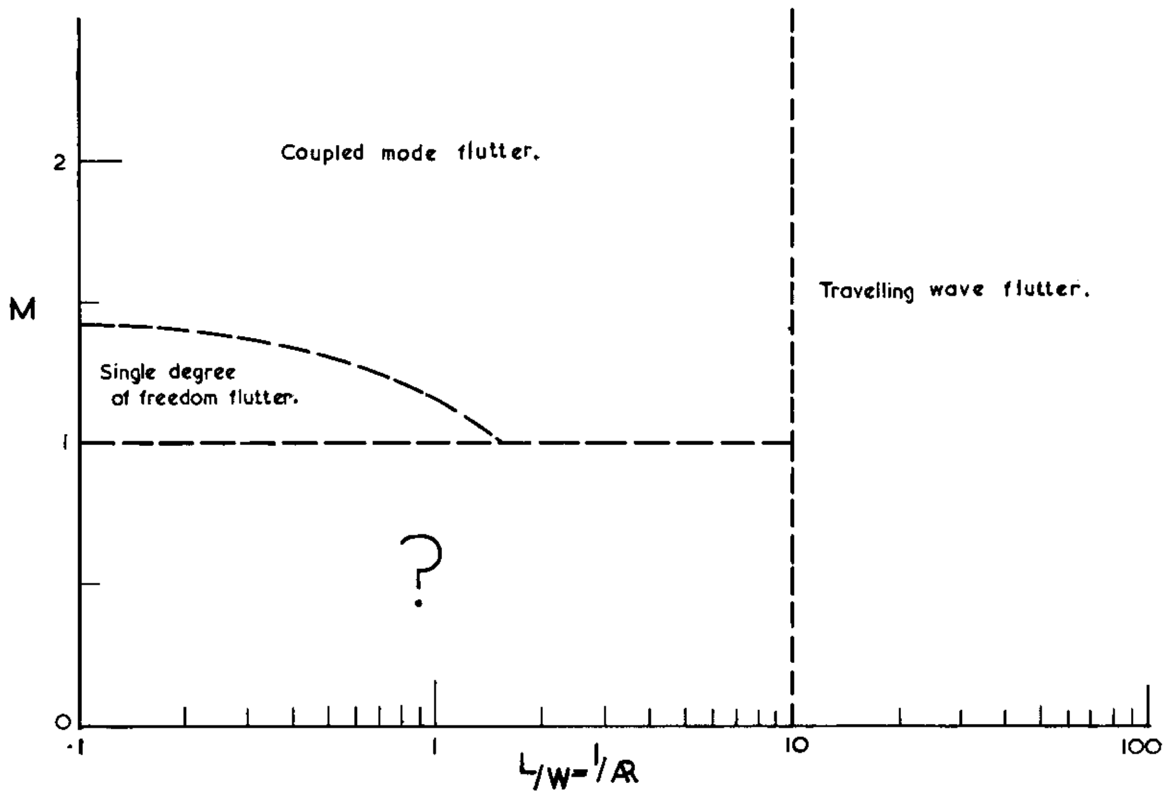

Fig. 39 Plane isotropic panel flutter régimes[A249]

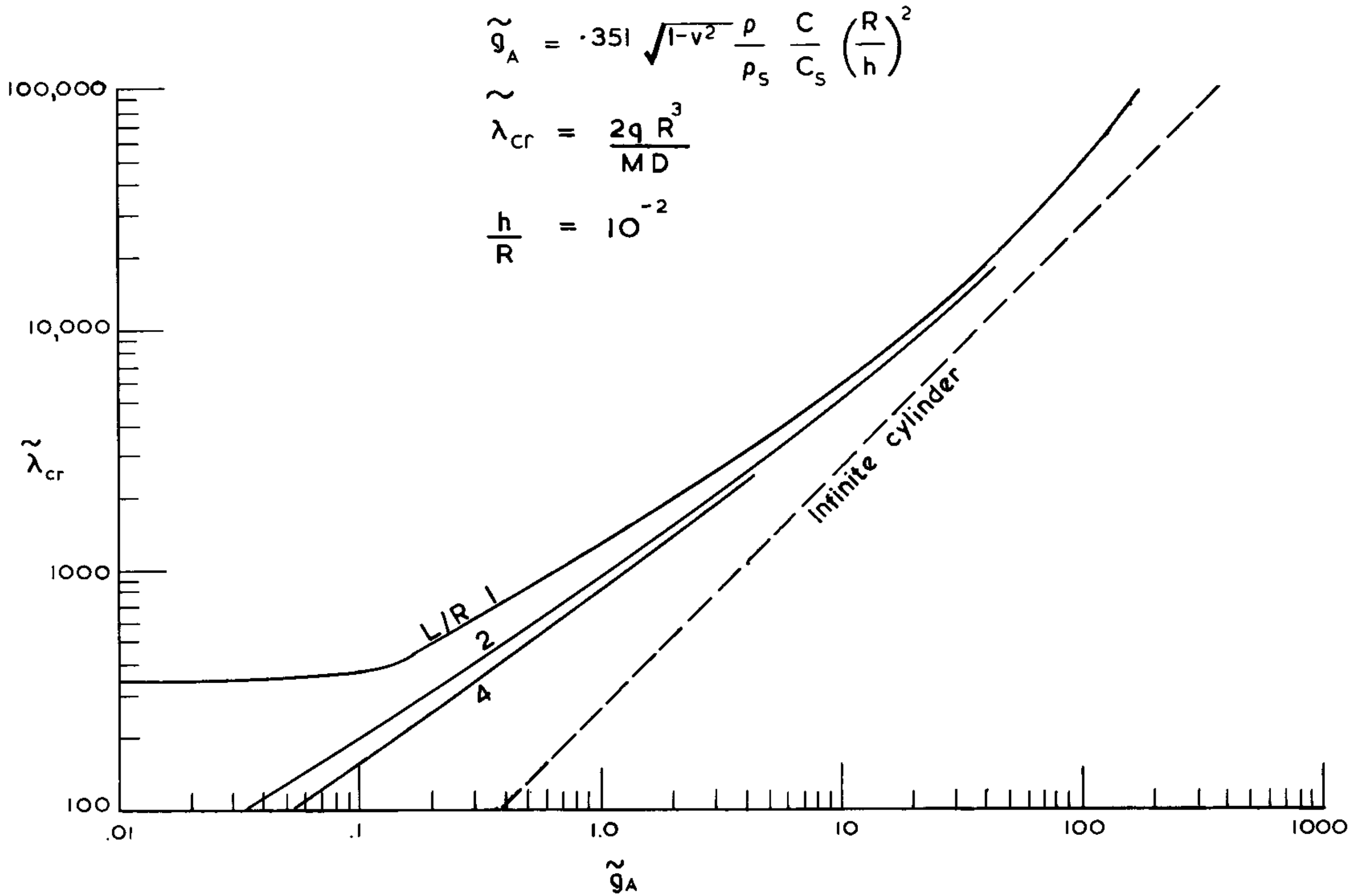

Fig. 40 Flutter dynamic pressure versus aerodynamic damping for cylindrical shells in axisymmetric modes[A263]

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