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CONDENSATION EFFECTS AND AIR DRYING SYSTEMS FOR SUPERSONIC WIND TUNNELS

by

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SUMMARY

Various aspects of the condensation problem for supersonic wind tunnels are reviewed. Part one is devoted to a study of the phenomenon of condensation effects, the resulting flow disturbances, and an investigation of the possibility of complete removal of condensation effects. A criterion is presented, based on certain experimental results. Conditions under which flow disturbances are sufficiently small are also indicated.

The second part is a study of various drying processes in use and the third part contains a survey of the different measuring devices for the determination of humidity.

SOMMAIRE

Les divers aspects du problème de la suppression des effets de la condensation sont examinés dans le présent travail.

La première partie du rapport est consacrée à l'étude phénoménologique de ces effets de condensation, en vue de l'évaluation des perturbations résultantes de l'écoulement, et de l'examen des possibilités d'élimination totale de la condensation. L'étude est fondée sur plusieurs résultats expérimentaux. L'élimination complete de la condensation n'étant pas toujours réalisable, les conditions sons lesquelles les perturbations peuvent être suffisamment réduites ont également été indiquées.

La seconde partie est consacrée à l'étude des procédés de dessiccation les plus communément utilisés en vue de satisfaire les conditions décrites dans la première partie: la réfrigeration, la compression et l'adsorption.

Enfin, une troiciéme partie est consacrée aux divers appareils de mesure utilisés en vue du contrôle de l'humidité des souffleries.

TERMINOLOGY

a	Speed of Sound
$\bar{a}$	Critical Speed of Sound
argErfc	Inverse Function of the Function Erfc (see Erfc)
A	Heat of Adsorption
c'	Thermal Capacity of an Adsorbent
c^*	Thermal Capacity of H_2SO_4
c_F	Coefficient of Force
c_p	Specific Heat of Air at Constant Pressure
C	Concentration of Water Vapor in Air
C_e	Water Concentration in Adsorbent at Equilibrium
C_L	Concentration of Water Vapor in Air at Section L of Cooling Dryer Surfaces
C_m	Mean Concentration of Water Vapor in Air in a Section of Dryer
C_s	Moisture Concentration Corresponding to Saturation at Temperature Indicated by Wet Bulb Thermometer
C'	Local Instantaneous Water Concentration of an Adsorbent
$\hat{C}$	Angle between Condensation Shock Line and Streamline
$\bar{C}'$	Mean Water Concentration of an Adsorbent
$\bar{C}_b$	Breakpoint Concentration of an Adsorbent
dQ	Heat of Condensation Released per Unit Mass of Air in Condensation Shock
D	Coefficient of Diffusion of Water Vapor through Air
Erfc	Complementary Error Function, $\mathrm{Erfc}(z)=\int_z^{\infty} e^{-z'^2}\,dz'$
f	Friction Coefficient in Solid Granular Adsorbent

TERMINOLOGY
(Continued)

$\mathcal{F}$	Force on Model
F	Mass Flow of Air
F^*	Mass Flow of Acid (sulphuric acid hygrometer)
F_B	Mass Flow of Air through a Bypass
F_ℓ	Mass Flow of Air through Tunnel Leakage
F_m	Mass Flow of Water to Adsorbent
G	Function Giving the Dependence of Water Concentration in a Desiccant on Layer Depth and Humidity Ratio
h	Absolute Humidity of Moist Air
h'	Absolute Humidity after Compression and Cooling
h_a	Absolute Humidity of Atmospheric Air
h_e	Absolute Humidity Corresponding to Equilibrium with Adsorbent
h_E	Inlet Absolute Humidity
h_L	Absolute Humidity in Section L of Cooling Dryer
h_m	Mean Absolute Humidity in a Section
$h_{s,a}$	Absolute Humidity Corresponding to Atmospheric Saturation
H_a	Heat of Adsorption
H_c	Heat of Condensation
H	Heat of Vaporization
i	Enthalpy
i_c	Enthalpy at Condensation Shock
J	Rate of Formation of Condensation Nuclei

TERMINOLOGY
(Continued)

k	Overall Coefficient of Mass Exchange in a Solid Granular Adsorbent
k_ℓ	Overall Coefficient of Heat Exchange in a Solid Granular Adsorbent
K	Heat Conduction Coefficient of Air
K'	Heat Conduction Coefficient of Adsorbent
K_E	$\sigma(\mathcal{M}/\rho_\ell)^{2/3}$
$\log_e$	Neperian Logarithm
ℓ	Length of Lower End of Glass Tube (sulphuric acid hygrometer)
L	Coordinate of a Section of a Cooling Dryer
m	Mass of Water Condensed or Adsorbed
m'	Amount of Desiccant (in mass)
m_m	Mass of Water Captured by a Surface in Contact with Mist
m_t	Total Mass of Air to be Dried
$\mathcal{M}$	Molecular Mass
M	Mach Number
M_c	Mach Number at Condensation Shock
Nu	Nusselt Number
Nu_d	Nusselt Number for Diffusion
p	Air Pressure
p_a	Atmospheric Pressure
p_c	Pressure at Condensation Shock
$\bar{p}$	Pressure at Throat of Wind Tunnel Nozzle
p_2	Pressure Immediately behind Condensation Shock

TERMINOLOGY
(Continued)

p^*	Vapor Pressure
p^*_∞	Vapor Pressure at Equilibrium with an Infinite, Plane Liquid Surface
p^*_E	Inlet Vapor Pressure
p^*_f	Outlet Vapor Pressure
p^*_L	Vapor Pressure in Section L of Cooling Dryer
p^*_m	Mean Vapor Pressure in a Cooler Section
$p^*_{m'}$	Mean Vapor Pressure in the Section at which Mist Disappears
$p^*_s(T)$	Saturation Vapor Pressure at Temperature T
Pr	Prandtl Number, $\mu c_p / K$
q	Flow of Heat
Q_{max}	Maximum Amount of Heat which can be Added to Unit Mass of Air at Condensation Conditions
R	Perfect Gas Constant
Sch	Schmidt Number, $\mu/(D\rho)$
S'	Cross Sectional Area of Adsorption Dryer
$\bar{S}$	Cross Sectional Area of Wind Tunnel Nozzle Throat
t	Time
t_c	Contact Time of Air in Dryer
t_T	Total Dryer Operation Time between Successive Reactivations
t'	$= kt$
T	Absolute Temperature
T'	Adsorbent Temperature
T_d	Temperature Indicated by Dry Bulb Thermometer

TERMINOLOGY
(Continued)

T_E	Air Temperature at Dryer Inlet
T_f	Air Temperature at Dryer Outlet
T_L	Wall Temperature in Section L of Cooler
T_m	Mean Air Temperature in a Cooler Section
T_o	Stagnation Temperature in Wind Tunnel
T_t	Cooler Temperature at Time t
T_w	Temperature Indicated by Wet Bulb Thermometer
v'	Volume of Adsorbent
V	Air Velocity
V_c	Air Velocity at Condensation Shock
V_{cn}	Air Velocity Normal to Condensation Shock
V_{ct}	Air Velocity Component Tangential to Condensation Shock
V_m	Mean Velocity
V_n	Normal Air Velocity
V_{2n}	Air Velocity Normal to Shock and Immediately behind Shock
w	Total Mass of Water to be Adsorbed between Successive Reactivations
W	Power Required to Condense Unit Mass of Water by Compression
W_r	Power Required to Condense Unit Mass of Water by Cooling
x	Length Coordinate in One-Dimensional Adsorber
x'	$= kx/V$
X	$= (K/c_pF) \int Nu_L \, dL$
X'	Value of X for Cooler Section at which Mist Disappears

TERMINOLOGY
(Continued)

α	Temperature Coefficient in Approximate Exponential Representation of Water Saturation Pressure: $p_S^*(T) \cong \text{const.}\, e^{\alpha T}$
β	Coefficient for the Rate of Water Evaporation from Wet Bulb Thermometer
γ	Ratio of Air Specific Heat
δ	Coefficient for the Rate of Heat Exchange from Wet Bulb Thermometer
Δp	Pressure Loss through a Dryer
ΔT	Temperature Rise in an Adsorption Dryer
ΔT_i	Difference of Temperature Indicated by Thermocouples in Sulphuric Acid Hygrometer
∇^2	Laplacian Operator, div. grad
θ	Thickness Ratio of Model
λ	Coefficient in Linear Relationship between Concentration and Temperature in Exchanger
μ	Dynamic Viscosity of Air
ν	Kinematic Viscosity of Air
ρ	Air Density
ρ_C	Air Density at Condensation Conditions
ρ_ℓ	Density of Liquid Condensate
ρ'	Density of Adsorbent
$\bar{\rho}$	Air Density at Wind Tunnel Nozzle Throat
σ	Surface Tension of Liquid
Φ	Function Giving Outlet Humidity for an Adsorption Dryer (in terms of inlet humidity and temperature, mean concentration and layer depth)
ψ	Disturbance Stream Function
ω	Constant in Linear Relationship between Concentration and Temperature in Exchanger

I. EFFECTS OF CONDENSATION IN SUPERSONIC FLOW

(a) Introduction

Condensation shocks were first described by Prandtl (Ref. 1), who observed them in a high-speed tunnel at Aachen, Germany. Because a similar phenomenon had been observed in steam nozzles, extensive experimental material is available on this subject (see Refs. 2, 3, 4, 5, and 6).

A complete set of equations describing the quasi-unidimensional flow of humid air or saturated steam in a de Laval nozzle, together with some experimental corroboration, has been published by Oswatitsch (Refs. 7, 8, 9, 10, and 11).

Since the partial pressure of a vapor expanded through a nozzle decreases with the temperature according to the adiabatic law, while the saturation vapor pressure decreases exponentially with the temperature, it is possible to associate with each value of initial humidity a certain amount of expansion at which saturation will be reached. Expansion beyond this point will result in either condensation or supersaturation of the air-water vapor mixture.

As a result of the rapid expansions and short transit times involved in high-speed nozzle flow, large amounts of supersaturation may be expected to occur without condensation. However, beyond a certain degree of supersaturation, which depends on the initial conditions of the air, a strong condensation, often called a condensation shock, will occur.

For condensation to occur, a large number of nuclei must be available. These nuclei must be large enough so that the equilibrium pressure associated with the nuclei is smaller than the existing vapor pressure. The greater the degree of supersaturation of the air the smaller the dimensions of the nuclei required to produce condensation. Furthermore, for a small increase in the degree of supersaturation, the number of nuclei which are effective in producing condensation may increase enormously so that the condensation may occur abruptly like a compression shock.

This condensation shock, which in many respects corresponds to an instantaneous addition of heat, has a very detrimental effect on the uniformity of flow in a supersonic nozzle. It is therefore advisable either to prevent the occurrence of such a condensation shock or to reduce its effect to such a point that the accuracy of the measurements to be made in the tunnel is unimpaired.

(b) An Outline of the Oswatitsch Theory

Oswatitsch has shown that the effect which foreign nuclei exert on condensation in supersonic wind tunnels is small because of the short time which these foreign particles spend in the nozzle. Oswatitsch then obtains the equations for one-dimensional, steady flow of a mixture of air, water vapor, droplets, and ice crystals in a de Laval nozzle. These equations, which contain terms describing the appearance and growth of the nuclei or condensation germs, are then used by Oswatitsch to calculate the pressure distribution in a nozzle in which condensation occurs.

(c) Theory of Condensation Germ Formation

The rate of formation of the condensation germs is given theoretically as

$$\log_e J = \log_e 9.589 \times 10^2 + \log_e \left[\frac{p_\infty^*}{T} K_E^{1/2} \frac{\mathcal{M}^{1/2}}{\rho_\ell^{2/3}} \right]$$

$$+ 2 \log_e \frac{p^*}{p_\infty^*} - 17.558 \left[\frac{K_E}{T} \right]^3 \left[\log_e \frac{p^*}{p_\infty^*} \right]^{-2} \tag{1}$$

where

J = rate of germ formation

T = absolute temperature in°K

p^*_∞ = saturation pressure (in dynes/cm^2) in equilibrium with an infinite liquid surface

$\mathcal{M}$ = molecular mass in g./mole

ρ_ℓ = density of the liquid in g./c.c.

p^* = partial pressure of the vapor

K_E = $\sigma(\mathcal{M}/\rho_\ell)^{\frac{2}{3}}$

σ = surface tension of the liquid in dynes/cm.

This expression was given by Volmer (Ref. 12) and the numerical constants are those which have been modified by Burgess and Seashore (Ref. 13).

The rate of formation of the condensation germs, which is a strong function only of the last term on the right hand side of Eq. (1), has been computed by Burgess and Seashore under the assumption that K_e varies linearly with the absolute temperature T. This linear variation is experimentally verified in the range 260°K<T< 320°K, (Ref. 13).

Burgess and Seashore have also experimentally determined the value of J at which condensation occurs. This was accomplished by analyzing static pressure measurements in various sections of several nozzles as a function of the stagnation temperature for constant inlet humidity. The value of J, which corresponds to the occurrence of the condensation shock, may then be computed. A mean value from several tests of $\log_e J = 4$ was found to correspond to the occurrence of the condensation shock. Therefore, by imposing the condition that $\log_e J < 4$ throughout the nozzle, a criterion for condensation free flow is obtained which imposes definite requirements on the stagnation temperature and the initial humidity.

This criterion, however, disregards an essential factor, the temperature gradient along the nozzle, the influence of which was pointed out by Oswatitsch (Ref. 7). The criterion is, therefore, as will be shown, generally pessimistic for small wind tunnels.

A widely used method of characterizing the onset of condensation is the adiabatic supercooling (or supersaturation) attained by the vapor. It is defined as the difference between the dewpoint (temperature of saturation) of the vapor, at the actual vapor pressure, and the actual temperature of the vapor.

The value of this supersaturation for which condensation occurs has been determined in many experiments (Refs. 10, 14, and 18). It has been shown to be a decreasing function of the stagnation, or initial, relative humidity, as might be expected, and an increasing function of the temperature gradient along the nozzle. This dependence is indicated by Fig. I-1, presenting unpublished results of Eber and Gruenewald, and Fig. I-2, summarizing tests by Oswatitsch, Eber and Gruenewald, Lukasiewicz, Wegener and Smelt (Refs. 10, 14, and 18). The latter figure, however, does not take into account the influence of stagnation initial humidity on the various tests. Head (Ref. 20) gives a more explicit dependence of the condensation occurrence upon the amount of humidity and the temperature gradient, which will be used (see (d) below) to obtain a criterion for condensation free flow.

It is seen from Fig. I-1 and Fig. I-2 that supercoolings ranging from 40° up to 100° C can be expected, while the Burgess and Seashore criterion corresponds to a nearly constant supercooling of less than 30° C, which is very close to the static condensation conditions (in the absence of condensation nuclei) obtained by Sander and Damkoeler (Ref. 16).

(d) A Criterion for Condensation Free Flow

We will now indicate a criterion for condensation free flow deduced from the experimental data of Head (Ref. 20), which were the most detailed and best adapted to the purpose at our disposal. Points indicating data from Refs. 18 and 13 were included as a check.

The criterion is given by Fig I-3. The graph involves two sets of curves. The curves of the first (lower) set are to be chosen according to the Mach number, M_c, at which condensation is expected to occur. The curves of the second (upper) set are to be chosen according to the temperature gradient dT/dx (in °C/cm) in the nozzle considered.

The graph allows prediction of the stagnation dewpoint of the tunnel air required to delay the onset of condensation up to a Mach number which, in case a condensation free flow is desired, must be larger than the test section Mach number.

The use of the graph is very simple. Draw a horizontal line through the stagnation temperature (T_o) which is shown on the scale at the left in Fig. I-3. Let A be the point of intersection of this horizontal line with the curve of the lower set which corresponds to the Mach number at which condensation occurs. Now draw a vertical line through point A. Let B be the intersection of this vertical line with the curve of the upper set which corresponds to the temperature gradient of the nozzle under consideration.

The dewpoint scale of Fig. I-3 must either be cut out or reproduced on a transparent paper. If the scale is then placed along AB, with point P of the scale coincident with A, the dewpoint can be read at point B. Incidentally, the temperature at which the shock occurs, T_c, can be read at the intersection of AB with the T_c scale. The dewpoints on the scale are the saturation points for water above 0° C, for ice below 0° C. The results obtained from the graph are independent of stagnation pressure.

Results of tests by Burgess and Seashore (Ref. 13) and at the NOL 2.5 cm by 2.5 cm wind tunnel are indicated on the graph. The former are seen to agree remarkably well with the line dT/dx = 1°C/cm. The latter agree approximately with Head's data in the common range and indicate the trend for high nozzle temperature gradients.

Two remarks must be made in connection with the criterion just mentioned. First of all, the line dT/dx = 1°C/cm can be used as an indication of what is to be expected in the larger wind tunnels (test sections of the order of 1 m by 1 m).

Second, it is not clear which temperature gradient must be chosen. In fact, the temperature gradient in the test section where the occurrence of the condensation must be prevented, is zero. This seems to imply that one should use, at most, the curve $dT/dx = 1°C/cm$ (equivalent to Burgess and Seashore's data on condensation in constant area test sections) irrespective of the nozzle temperature gradient.

The difficulty obviously originates in the fact that the occurrence of condensation depends on the whole time history of the vapor state in the nozzle, not only on some mean temperature gradient. Very little experimental data seems to be available on this question, in which the transit time in the test section must play an important part.

Also, it is not easy to choose M_c safely, because of the further expansions around models. The temperature gradients in these expansions also must be taken into account, when the choice of M_c is made. The number of parameters involved and the lack of information on the subject render any further statement about the choice of M_c impossible at present.

Finally, it must be stressed that the graph in Fig. I-3, which is intended as a criterion for shock free flow, can also be used to predict the shock location, in case it is allowed to occur upstream of the test section. This will be of use in the next section.

(e) Computation of Disturbances Caused in a Supersonic Nozzle by a Weak Condensation Shock

In case it is impractical to eliminate condensation shock completely, it may still be possible to minimize the disturbance by sufficiently lowering the stagnation humidity. In order to establish suitable criteria for drying, in terms of the flow quality required, one has to examine the magnitude and character of the disturbances created by a weak condensation shock.

The requirement that the condensation shock be small necessitates a low stagnation absolute humidity. The influence of the small amount of water vapor present will be limited to the release of the latent heat of condensation of the vapor at the shock. Thus, the condensation shock will be accurately described by the same set of equations as a weak shock wave with the addition of heat (detonation wave).

The disturbances caused by the condensation shock can be divided into three parts. First, the new, disturbed conditions immediately downstream of the shock induce a slight change in the whole downstream flow pattern. As a result the flow in the test section will become slightly nonuniform, and the mean Mach number will be slightly different from the nominal Mach number of the nozzle.

Second, the interaction of the shock with the nozzle blocks generates a reflected disturbance which will be propagated and reflected downstream into the test section.

Third, the ice particles or supercooled water droplets appearing in the condensation shock may exert some influence on the reaction experienced by the model. This effect can be shown to be negligible for usual stagnation humidities. It amounts, roughly, to an increase of drag coefficient of $2h\theta$, where h is the stagnation absolute humidity (in gm water per gm air), θ the thickness ratio. This is obviously negligible for the usual humidities which are a fraction of 0.001.

(1) Equations for a Weak Condensation Shock

The effects on physical parameters of a weak condensation shock are readily evaluated. The discussion in the previous section concerning the conditions for shock removal is pertinent here because it gives information as to the probable location of the shock upstream of the test section. If this location is known, the computation of the effects is quite straightforward. Let the subscript c denote the conditions immediately upstream of the shock. Assuming the shock to be very weak, the heat of condensation released, per unit mass of air, will be very small.

If the shock shape is known, the equations admit two solutions. One solution tends to zero with the condensation heat, the other is obtained simply by adding the discontinuities arising in an ordinary adiabatic shock. Only the first one has a physical meaning. The corresponding deviation from upstream conditions may accordingly be indicated by differentials, which greatly simplifies the equations.

These equations are

$$\rho_c dV_{cn} + V_{cn} d\rho_c = 0 \qquad \text{(continuity)} \quad (2)$$

$$dV_{ct} = 0 \qquad \text{(momentum)}$$

$$dp_c + \rho_c V_{cn} dV_{cn} = 0 \quad (3)$$

$$V_{cn} dV_{cn} + di_c = dQ \qquad \text{(energy)} \quad (4)$$

where

dQ = heat of condensation released per unit mass of air in the condensation shock in mechanical units, ergs/gm

V_n, V_t = components of velocity normal and parallel to the shock in cm/sec

ρ = specific mass gm/cc

i = air enthalpy in mechanical units, ergs/gm

p = pressure in dynes/cm^2.

These equations can be solved with respect to the disturbances, which gives

$$dV_{cn}/V_c = -\sin\hat{C}\,(d\rho_c/\rho_c)$$

$$= -(\gamma M_c^2 \sin\hat{C})^{-1}(dp_c/p_c)$$

$$= -\left[\frac{(\gamma-1)M_c^2}{2} + 1\right](M_c^2 \sin^2\hat{C} - 1)^{-1}(\sin\hat{C})(dQ/c_p T_o) \quad (5)$$

where

M_c = Mach number

$\hat{C}$ = angle between condensation shock line and streamline

T_o = stagnation temperature

c_p = specific heat at constant pressure in mechanical units, erg/ °K, gm.

It is thus seen that the disturbances become infinite at points where the shock line touches a Mach line. The shock intensity therefore strongly depends on the angle of its direction with local Mach lines. This makes the present analysis completely dependent on the knowledge of the shock configuration. The theoretical prediction of this configuration, based, for instance, on Oswatitsch theory, would be a formidable task, if one considers the amount of computation already required in the one-dimensional case.

According to Eq. (5), it is to be expected that disturbances become infinite whenever the slope of the condensation shock coincides with a Mach line, if one remembers that the heat can be added in a normal shock is limited and that it becomes zero at sonic conditions (Ref. 14).

This fact can be of use in obtaining an upper limit to the disturbances, in the absence of general data on shock configurations. Such an upper bound may be derived from the formula for maximum heat addition to a steady flow. This will be done for the case of an oblique shock.

If the complete shock equations are written, without introducing differentials, a quadratic equation describes the normal velocity behind the shock. This quadratic equation has real roots only if the heat added is smaller than a certain value Q_{max}, given by

$$Q_{max}/c_p T_0 = (1/2)(\gamma+1)^{-1}(M_c^2 \sin^2\hat{C} - 1) \left(\frac{\gamma-1}{2} M_c^2 + 1\right)^{-1} M_c^{-2} (\sin\hat{C})^{-2} \tag{6}$$

the corresponding disturbances being given by

$$\frac{(p_2 - p_c)}{p_c} = \left[\gamma/(\gamma+1)\right](M_c^2 \sin^2\hat{C} - 1) \tag{7}$$

$$\frac{(V_{2n} - V_{cn})}{V_{cn}} = \left[-1/(\gamma+1)\right](M_c^2 \sin^2\hat{C} - 1)(M_c^{-2} \sin^{-2}\hat{C}) \tag{8}$$

etc., the subscript 2 referring to the conditions behind the shock.

If we now consider the case where Q_{max} is just equal to the condensation heat dQ, the disturbances will again be small, as indicated by differentials, but the formulas become quite different from those obtained in Eq. (5)

$$\frac{dV_{cn}}{V_c} = -(\sin\hat{C})\,d\rho_c/\rho_c = -(\gamma M_c^2 \sin\hat{C})\,dp_c/p_c = -\left[2\left(\frac{\gamma-1}{2} M_c^2 + 1\right)(dQ/C_p T_0)/(\gamma+1)\right]^{1/2}/M_c. \tag{9}$$

It should be pointed out that the use of these formulas to obtain an order of magnitude of the shock disturbances would tend to impose too severe a drying criterion in cases where the condensation shock has, everywhere, a direction quite different from any characteristic direction. This is, of course, due to the appearance of the root of the ratio $dQ/c_p T_0$. We feel, however, that they are safer, considering the absence of accurate data on weak shock configurations.

(2) Flow Behind the Condensation Shock

The flow downstream of the shock has not the same energy as the unperturbed flow, and is not even isentropic in general, the entropy increases being dQ/T_c. The disturbance field therefore must be built up as follows:

a. An ordinary, irrotational, disturbance field, with zero energy disturbance.

b. A disturbance field due to the energy disturbance, also irrotational.

c. A rotational disturbance field due to the variable entropy increase, and, eventually, from a variable heat addition along the condensation shock.

The potential describing the first field satisfies a homogeneous partial differential equation of the second order, with boundary conditions at the walls and on the shock, where the potential is constant, while its normal derivative is the normal velocity disturbance dV_n.

The potential describing the second field satisfies a nonhomogeneous equation, the potential being constant on the shock and its normal derivative, zero. A particular solution of said equation, however, can be given if the shock occurs at constant temperature; the corresponding potential is dQ/V_c^2 times the unperturbed potential. The problem then reduces to the preceding with different boundary conditions, of course.

The third field is better described by a stream function satisfying a nonhomogeneous partial differential equation of the same type as the preceding ones.

The general problem of finding those fields cannot be solved unless the shock configuration is known in detail, but we will indicate a roughly approximate method whereby general conclusions are to be drawn. It consists in considering the unpertubed flow as nearly uniform. The splitting of disturbance fields alluded to is then irrelevant; the rotational part is neglected. The velocity disturbance stream function ψ approximately satisfies the partial differential system

$$\psi_{ab} = 0. \tag{10}$$

The subscripts denote derivation with respect to characteristic coordinates in the undisturbed flow. The disturbances on the shock are now split into a constant, mean part and a variable part. The first gives rise to a uniform field and represents what would be obtained in a one-dimensional approach. The variable part is strongly localized on the shock, which usually has a V-shape with strongly curved apex near the nozzle axis. Strongly variable disturbances can be expected near the nozzle axis (Fig. I-4a) as well as near the points where the shock direction becomes approximately characteristic (Fig. I-4b).

All disturbances propogate along characteristics of the undisturbed flow. The disturbances originating from the apex can be shown to carry a total nonuniformity in velocity of the order of

$$dV_{apex} = \sin\hat{C}\,\cos\hat{C}\,(M_c^2\sin^2 C-1)^{-1}(1-M_c^{-2})^{1/2}\,V_c$$

$$\left(\frac{\gamma-1}{2}M_c^2+1\right)dQ/c_pT_0 \tag{11}$$

while it is clear that if the shock touches a characteristic, a discontinuity, as described by Eq. (9) will be carried along this characteristic.

Another effect which produces nonuniformities is the shock interference with the nozzle walls. A reflected disturbance is generally emitted, unless the shock happens to be normal to the wall. This reflected disturbance has about the same magnitude as the disturbance issuing from the apex, in the case of a V-shock, but opposite sign.

All these local perturbations are propagated through the nozzle into the test section along Mach waves. In the rather rough approximation just used (uniform undisturbed flow) this propagation would occur without any change in the magnitude of the travelling disturbances. A more accurate picture is outlined in Ref. 22 and indicates a variation of the propagated velocity disturbances proportional to a factor depending on local Mach number (propagation factor). A graph of this factor in terms of the Mach number is given in Fig. I-5.

The most severe of the nonuniformities appearing in the test section as a consequence of the occurrence of a condensation shock upstream is, as already stressed, the effect described by Eq. (9). We will now calculate the flow deviation in the test section induced by this velocity perturbation. Because the velocity perturbation propagated by a Mach wave is perpendicular to the wave direction one obtains

$$d\hat{a} = (dV_n/V)\ (M^2-1)^{\frac{1}{2}}/M = \left[f(M)/f(M_c) \right]$$

$$\left[2(M^2-1)\left(\frac{\gamma-1}{2}M^2+1\right)(dQ/c_p T_0)/(\gamma+1)\right]^{\frac{1}{2}}/M^2 \tag{12}$$

where

$d\hat{a}$ = flow deviation in radians

$f(M)$ = propagation factor

$$= \left(\frac{\gamma-1}{2}M^2+1\right)^{\frac{1}{2}(\gamma-1)} (M^2-1)^{-\frac{1}{4}}.$$

The factor of $\sqrt{dQ/c_p T_0}$ in this formula is plotted in Fig. I-6.

With respect to Eq. (12) it should be remembered that the condensation shock is assumed somewhere to have the local characteristic direction. This seems to be the case, for instance, for the weaker shocks in the schlieren photographs in Ref. 14. Now, if a given nozzle is operated with decreasing stagnation humidity, the shock will travel downstream and change its shape from convex (Fig. I-4a and I-4b) to concave with respect to downstream flow direction. Therefore, some intermediary configuration probably exists for which the shock direction is nowhere characteristic.

It is also highly probable that the configuration is dependent on scale and that the shock would tend to occur at a constant temperature for sufficiently large scales (i. e., large nozzle transit times). In this case, it can have a characteristic direction only in a simple wave flow region (if unusual occurrences such as Mach line inflexion are disregarded). This can happen if the shock is pushed far downstream by efficient drying and penetrates into the simple wave flow regions adjacent to the test rhombus. In this case, it would seem that less drying should improve the situation considerably. Unfortunately, we were unable to find any mention of such an occurrence in the literature, and our deductions must remain conjectural.

The test section values of the constant (average) part of the shock disturbances can be obtained most easily if one observes that the influence of the condensation on the value of test section perturbations results from an increase in entropy of dQ/T and an increase of specific energy of dQ. This remark leads to the following formula for the uniform test section perturbations:

$$dp/p = (\gamma/2)(M^2 + M_c^2 + (\gamma-1)M^2M_c^2)(M^2-1)^{-1}(dQ/c_p T_0) \tag{13}$$

$$d\rho/\rho = (1/2)(\gamma M_c^2 - M^2 + 2)(M^2-1)^{-1}(dQ/c_p T_0) \tag{14}$$

$$dM/M = -(1/2)\left(\frac{\gamma-1}{2}M^2 + 1\right)(\gamma M_c^2 + 1)(M^2-1)^{-1}(dQ/c_p T_0) \tag{15}$$

The coefficient of dQ/c_pT_0 in the latter formula is plotted in Fig. I-7. The errors in force measurements introduced by these deviations, will accordingly be given by

$$d\mathcal{F}/\mathcal{F} = -(1/2)\left[\gamma M_c^2 - M^2 + 2 + Mc_{\mathcal{F}}\left(\frac{\gamma-1}{2}M^2 + 1\right)(\gamma M_c^2 + 1)\left(\frac{dc_{\mathcal{F}}}{dM}\right)\right](M^2-1)^{-1}(dQ/c_p T_0) \tag{16}$$

where $c_{\mathcal{F}}$ is the nondimensional force coefficient, a function of M (if the effect of Reynolds number perturbation is disregarded).

In the linearized Ackeret theory, for instance, $c_{\mathcal{F}}$ is proportional to

$$(M^2-1)^{-\frac{1}{2}}$$

so that

$$(d\mathcal{F}/\mathcal{F})_{\text{Ackeret}} = (1/2)\left[(\gamma M_c^2 + 1)\left(\frac{\gamma-1}{2}M^4 + 1\right) + (M^2-1)^2\right](M^2-1)^{-2}(dQ/c_p T_0) \tag{17}$$

The coefficient of dQ/c_pT_0 in this formula is plotted in Fig. I-8 for various values of M_c and M.

(f) Criterion Based on Flow Quality Requirements

The results of the preceding section will now be used to obtain information on the drying required for a given flow quality in cases where it has been found impossible to eliminate condensation. First of all, the heat added, dQ, must be computed. The detailed knowledge of this parameter involves the complete shock mechanism. The Oswatitsch equations should be solved for the particular nozzle under consideration. Such a detailed solution is not required if only the order of magnitude of the condensation effects is of practical interest.

In view of the large number of condensation germs present, it seems reasonable to assume that the air downstream of the condensation shock is saturated. But this is very nearly equivalent to assuming that all of the moisture will condensate because of the considerable amount of supercooling to be expected in supersonic nozzles. It is therefore possible to write

$$dQ = hH_c \tag{18}$$

where

h = absolute humidity (in gm water vapor per gm air), given as a function of the stagnation dewpoint by the curve in Fig. I-11 (for atmospheric pressure = 760 mm Hg.)

H_c = heat of condensation (for T_c above -63° C, solid condensate) or sublimation (for T_c below -63° C, solid condensate) expressed in units consistent with dQ.

The order of magnitude of both latent heats is similar, and the temperature dependence is small, so that one can use the following estimation

$$H_c/c_p \cong 2900^\circ\ C. \tag{19}$$

For wind tunnels in which there is no preheating of the air, the stagnation temperature is generally about 300° K. Therefore

$$dQ/c_p T_0 \cong 10h. \tag{20}$$

In hypersonic facilities operating from a high stagnation temperature, dQ/c_pT_0 is, of course, a smaller multiple of h.

Two different criteria must be considered, according to the type of test for which the tunnel is intended. If overall properties of complete models (lift, drag, etc.) are investigated, the emphasis will be on Eqs. (13), (14), (15), and (16) to estimate the errors in Mach number, pressure, and force measurements due to uniform disturbances. However, if detailed investigations of model elements (efficiency of control surfaces, or intakes, etc.) are on the program, it becomes essential to reduce all nonuniformities to a minimum, for even small disturbances are known to produce large errors (due, for instance, to the transition they induce in boundary layers (Ref. 38) or to the considerable sensitivity of neutral points on flow deviations). In the absence of detailed data on weak shock configuration, Eq. (9) will have to be used in order to obtain a safe criterion for this case.

For overall effects a rough orientation can be obtained with the help of the graphs in Figs. I-7 and I-8. It is seen that the most severe requirement will be derived from considering the error in force. In Fig. I-9 this error is plotted as a function of h for various Mach numbers for the most severe case of small nozzle temperature gradient (from data represented in Fig. I-3). An accuracy of about one percent in the force is thus to be expected from $M = 1.6$ on, if the drying requirement $h < 0.5\ 10^{-3}$ is adopted (requirements set forth for the Peenemuende wind tunnels).

The second case in which flow deviations are important leads to much more severe requirements. Fig. I-10 (deduced from Figs. I-6 and I-3) gives the angular deviation in function of h for various Mach numbers in the case just quoted. It is seen that this deviation can be kept lower than one degree only if the air is dried down to $h < 0.2\ 10^{-3}$ (which is the usual requirement). But deviations smaller than 0.1 degree are sometimes indicated as a must for neutral point measurements for control surfaces. This would imply humidities of about one part water in a million (dewpoint about -70° C at 760 mm H_g), small enough to prevent shock occurrence at Mach numbers higher than two even for the smallest temperature gradients. Such low humidities probably can be reached only if drying (by adsorption or cooling) is accomplished at high pressure (of the order of 100 atmospheres). This is the case for tunnels operating on the blow-down principle (Ref. 33), or when high pressure storage of dry air is used. In the case of facilities working under very high stagnation pressures, the situation is somewhat different because a given dewpoint corresponds to a much lower absolute humidity so that all criteria based on shock disturbances are comparatively less severe than those based on shock removal.

(g) Conclusions

The complete elimination of condensation effects in supersonic wind tunnels is theoretically possible, either by sufficient drying, or by sufficient preheating, or by a suitable combination of both, as shown by the criterion presented on Fig. I-3. The amount of drying and/or preheating required is seen to increase with tunnel scale.

An important fact about this criterion is that it involves the dewpoint, not the absolute humidity, of the tunnel air. The facilities operating at high stagnation pressures therefore seem to require more drying, but this apparent disadvantage is completely reversed by the automatic lowering of the dewpoint occurring in the compressor aftercooler. On the other hand, tunnels operating with stagnation pressures lower than the atmospheric pressure are in a favorable situation, but the pressure is seldom low enough for the advantage to be significant.

There are cases, however, where there is no point in completely eliminating condensation, because the disturbances it originates can be kept small enough to lie within the limits of precision by less severe requirements than indicated by the criterion for shock free flow. General rules indicating when one should go over to these requirements are difficult to give because of the numerous parameters involved: Mach number, stagnation pressure, tunnel scale, and precision required. The first three parameters, when increasing, tend to make the requirements for limiting disturbances less severe compared to the criterion for shock free flow, while the last parameter obviously has the opposite influence.

Criteria for small disturbances are characterized by the important part played by the absolute humidity. The stagnation dewpoint still has some bearing on the situation, but only because it controls the shock location. It cannot, therefore, alter the order of magnitude of the disturbances.

In most cases less than one percent uniform deviation from undisturbed flow in the test section can be obtained by keeping the absolute humidity below 0.5 (10^{-3}). If nonuniform, localized deviations are taken into account, more severe requirements are obtained. For instance, the flow deflection due to the condensation shock can be kept below one degree, in the most severe case, only if the absolute humidity is lowered to 0.2 (10^{-3}). Deflections smaller than 0.1 degree, as sometimes required in missile stability tests, are only to be obtained by complete shock removal, unless one operates at Mach numbers much higher than two or with very high stagnation pressures.

II. DRYING SYSTEMS FOR WIND TUNNELS

(a) Problems of Desiccation in the Different Types of Supersonic Wind Tunnels

A distinction must be made between open and closed circuit tunnels when discussing the drying problem. Open circuit tunnels are generally intermittent except for a few turbine driven facilities such as those of the Swiss Department of Defense at Emmen. Closed circuit tunnels, on the other hand, are mainly continuous and fitted with return circuits with the exception of the induction type tunnels at NPL in the United Kingdom.

A closed circuit tunnel operates with the same air over and over so that only a relatively small amount of air, namely that for replacement of the air lost through leaks, needs to be dried. For the open circuit tunnel, the drying problem is more serious since different masses of fresh, undried atmospheric air are used for each run.

One method of alleviating the drying problem for intermittent blow-down from an atmospheric tunnel is to pump the dry air out of the vacuum reservoir back into the upstream reservoir. As far as the drying problem is concerned, such an open circuit tunnel is completely equivalent to a closed circuit facility. This type of system does have some disadvantages, however. The necessity of a constant stagnation pressure during tunnel operation entails a variable volume reservoir, generally a kind of balloon which is difficult to render impermeable to atmospheric humidity. Furthermore, the vacuum pumps used always add not only a certain amount of moisture, but also oil which has a very destructive effect on the adsorption agents used for drying. Methods used to remove the oil have not so far proved entirely satisfactory. A better solution might be to combine this type of tunnel circuit with some sort of refrigeration drying.

The high pressure blow-down to atmospheric type of intermittent wind tunnel is relatively free of drying problems due to the large amount of drying obtained from the aftercoolers generally used in conjunction with the compressors. This aftercooling causes a great deal of the atmospheric humidity to condense at a point where it may easily be removed from the tunnel circuit.

A drying unit for a supersonic wind tunnel has two different functions. First, the tunnel or the upstream reservoir must be filled with sufficiently dry air. Second, the humidity must be kept low during tunnel operation in spite of exchanges of air with the atmosphere through leaks. For intermittent tunnels which do not possess a return circuit only the first function must be performed. If the time between runs is short, a large drying system is necessary. But the dryer installation is straightforward and unique since only one function is required of the drying system.

If both functions are required of the drying system, a greater number of possible drying plant types may be considered. The charging function requires the removal of great quantities of moisture from very humid air. The second function, the maintenance of a constant low humidity, requires the removal of small quantities of moisture from very dry air. The essentially different nature of the two functions imposes severe requirements on a single dryer. A fairly large drying system is needed to fulfill the first requirement, while a smaller dryer combined with a bypass pipe may serve to satisfy the second function. The second function might also be satisfied by using a single dryer to supply dry air to a high pressure reservoir. Then if the tunnel air becomes too humid some dry air may be fed into the tunnel circuit at the same time that some of the humid air in the tunnel is bled off.

Plants with several dryers generally have one large charging dryer and several smaller units which are used to change the stagnation conditions. The smaller dryer may also be placed in a bypass and used as a stabilizer.

The limiting case of a bypass dryer is the full flow dryer which is placed in the main circuit of a continuous return tunnel. This system does not permit any change in the flow through the dryer. This type of drying system is used in a tunnel at Johns Hopkins University in order to prevent nonuniformities of temperature which are unavoidable at a bypass outlet. Such nonuniformities would have disturbed the basic tests on turbulence for which the tunnel was designed.

(b) Drying Systems

The many physical and chemical methods of drying air may be roughly classified into two categories. The first category includes all thermodynamical processes in which the air and water vapor mixture is brought to a state of supersaturation so that the moisture will condense and be separated out. The supersaturation may be obtained either by cooling or compressing the air or by a combination of these two. The second category consists of those processes which involve the physical or chemical action of some substance with which the moist air is brought in contact. The physical action, called adsorption, in which the water vapor adheres to the surface of some porous substance such as silica gel or activated alumina, is by far the most widely used method of drying. The chemical adsorption of the water vapor by some agent such as sulfuric acid or phosphorous pentoxide is not generally used for drying in supersonic wind tunnels due to both the low capacity of the chemical adsorbers and the difficulty involved in regenerating them.

(c) Need for Regeneration of Drying Plants

Obviously the drying process cannot go on indefinitely unless the moisture which is separated can be continuously removed. Such a continuous removal is possible in drying plants which use either cooling to a temperature above the freezing point or compression of the stagnation air. In both of these types of plants the water condenses on the liquid phase and can flow continuously out of the dryer. In all other kinds of drying systems the moisture removed from the air will alter the characteristics of the drying plant in such a way that the drying capacity of the plant will be impaired.

Since agents which adsorb the moisture in the air have a limited water capacity, the ease with which the water may be removed from these agents deserves primary consideration. The popularity of the physical adsorbers results from the ease with which they can be regenerated simply by heating.

In the case of a drying plant which condenses the moisture in the solid phase, regeneration may be accomplished simply by melting the ice deposited on the walls of the cooler and allowing the condensate to run off. The moment at which such a regeneration should take place is not easy to define. It is clear that too thick a layer of ice will impair the efficiency of the cooler. Furthermore, after the thickness of the ice reaches a certain point, there is a tendency for the condensate deposit to become snowy. This snow may be swept along with the air to another part of the tunnel circuit where it will re-evaporate and thus raise the humidity in the tunnel. Few data are available on the optimum time for regeneration of this type drying facility since such drying plants are usually regenerated after each run.

Such periodic regenerations can, however, be avoided by spraying a freezing point depressant (such as ethylene glycol) on the cooler surface. The condensate then remains in liquid state and can thus be continuously eliminated. This method is used at the AEDC tunnels in Tulahoma (U. S. A.). The spraying has the inconvenience of adding further free liquid to the airstream, but the drops which then nucleate are generally large and therefore easy to trap. Commercial equipment is available to reclaim the depressant continuously for further use.

At any rate it is apparent that most drying plants will have to include regenerating apparatus. This apparatus will generally consist of some sort of a heater the size of which will depend on the particular case under consideration.

In cases where long runs or many consecutive runs are anticipated, it may be necessary to provide a battery of two or more dryers so that the need for regeneration will not halt tunnel operation. A second fresh dryer will then always be ready to take over from a dryer which needs regeneration.

(d) Drying by Cooling

(1) Introduction

Two different physical phenomena take place during drying by cooling.

First, the condensation occurs in the form of a mist in the supersaturated air-water vapor mixture. Water in either the liquid or the solid phase may be present in the mist.

Second, the water vapor may diffuse through the mixture and collect in regions where the concentration of the moisture is low, such as the walls of the cooler.

These two phenomena may be used either together or separately to dry air for a supersonic wind tunnel. In order to make use of the first effect, it is necessary to provide a glass wool or felt filter or some sort of inertial separator to remove the condensed water vapor from the air. The replacement of filters is, in a sense, a regeneration of the drying circuit.

(2) Heat Transfer and Diffusion Effects in Dryers

A dryer which dries the air by a cooling process is actually a heat and mass exchanger. The exchanger consists of cooled surfaces of various shapes. The air to be dried may flow around the tubes which contain the refrigeration fluid or vice versa. Fins may be used on the tubes to increase the area of contact as the air flows past the tubes.

Although the theory of heat exchangers is well-known, the parallel which exists between heat transfer and mass transfer phenomena has not received too extensive a treatment. It is therefore not out of place to mention some of these similarities.

The diffusion of a constituent of a gaseous mixture through this mixture at rest is described by Fick's Law when the concentration of the diffusion constituent is sufficiently small. Thus

$$m = D \text{ grad } C \tag{21}$$

where

m = mass flow of the diffusing constituent in gm/cm^2 sec

C = local concentration of the diffusion constituent in gm/cm^3

D = diffusion coefficient with respect to the mixture of the diffusion constituent in cm^2/sec.

The analog of Fick's Law for heat conduction phenomena is Newton's Law

$$q = -K \text{ grad } T \tag{22}$$

where

q = flow of heat in cal/cm^2 sec

T = absolute temperature in $^{\circ}$K

K = heat conduction coefficient of the medium under concentration in cal/cm^{2}sec $^{\circ}$K.

The moisture concentration of humid air may be related to the partial pressure of the water vapor by

$$C = h\rho = \frac{M_{H_2O}}{M_{air}} \rho \frac{p^*}{p} = .625 \frac{p^*}{RT} \tag{23}$$

where

h = absolute humidity in gm H_2O/gm air

ρ = specific density of the air

p = pressure of the air-water vapor mixture

T = absolute temperature of the mixture

p^* = partial pressure of the water vapor which is assumed to be small compared to p

M_{H_2O} = molecular weight of the water vapor

M_{air} = apparent molecular weight of the air.

For the laminar flow of a gaseous mixture the diffusion effects and the thermal convection effects are described by the following equations:

$$\rho V \cdot \text{grad } \left(\frac{C}{\rho}\right) = \text{div } \left[D \text{ grad } C\right] \tag{24}$$

$$\rho C_p V \cdot \text{grad } T = \text{div } \left[K \text{ grad } T\right]. \tag{25}$$

Both of these equations involve the distribution of the velocity, V, of the accompanying flow. If ρ, D, h and μ are assumed constant, Eq. (24) and (25) may be rewritten as

$$V \cdot \text{grad } C = \frac{\nu}{Sch} \nabla^2 C \tag{24'}$$

$$V \cdot \text{grad } T = \frac{\nu}{Pr} \nabla^2 T \tag{25'}$$

where

ν = kinematic viscosity in cm^2/sec

Sch and Pr = Schmidt number and Prandtl number respectively, and are defined below.

$$Sch = \frac{\mu}{D\rho} \cong .49$$

$$Pr = \frac{\mu C_p}{K} \cong .71$$

where μ is the viscosity of the fluid under consideration.

The numerical values given above are for air at normal temperatures and pressures. Since the Prandtl and Schmidt numbers are of the same order of magnitude, the temperature and concentration profiles in both boundary layers and wakes which occur in a heat exchanger will be similar. In other words, a relation of the form

$$C = \lambda T + \omega \tag{26}$$

is approximately valid in each cross section of the flow in the heat exchanger when λ and ω are constants in a section.

A linear relationship such as that in Eq. (26) is extremely useful since it relates the diffusion effects to the more extensively studied heat transfer effects.

Consider a heat and mass exchanger whose thermal properties are known. This exchanger can be considered as a succession of elements which are consecutively crossed by the airflow. Each element of the cooler may be considered to be bounded by the planes parallel to the mean flow direction and dL cm apart. Now the heat exchanged per unit time between the surfaces of the cooler and the air passing by is given by

$$dQ = K Nu_L (T_m - T_L) dL \tag{27}$$

where Nu_L is the Nusselt number, T_m is the mean air temperature and T_L is the temperature of the walls of the heat exchanger element.

Making use of Eq. (26) the mass of moisture, dm, deposited on the walls of the exchanger element per unit time is given by

$$\frac{dm}{dQ} = \frac{D(C_m - C_L)}{K(T_m - T_L)} = \frac{D\rho}{K} \frac{(h_m - h_L)}{(T_m - T_L)} \tag{28}$$

where the subscript m refers to mean air conditions and the subscript L refers to conditions at the surface of the exchanger element.

It should be pointed out here that as soon as a deposit of liquid or solid water appears on the surfaces of the heat exchanger element, h_L in Eq. (28) must be taken equal to the saturation humidity corresponding to the temperature, T_L. The modifications of the preceding equations which must be made if mist is present at the walls of the exchanger will be discussed in the next section.

If F is the mass of air flowing through the dryer element then

$$F dh_m = - dm$$

and

$$F c_p dT_m = dQ.$$

It follows from Eq. (28) that

$$\frac{dh_m}{dT_m} = \left(\frac{D\rho c_p}{K}\right)\left(\frac{h_m - h_L}{T_m - T_L}\right) = \left(\frac{Pr}{Sch}\right)\left(\frac{h_m - h_L}{T_m - T_L}\right). \tag{29}$$

(3) Concentration Distribution in the Presence of Mist

If mist appears on the wall of the exchanger element Eqs. (23) and (26) must be replaced by equations which take into account the resulting decrease in local concentration. For instance, if it is assumed that the maximum amount of condensation takes place, i.e. the vapor is nowhere supersaturated, then Eq. (26) must be replaced in the mist regions by the following relation which gives the equilibrium concentration in terms of the local temperature

$$C = .625 \frac{p_s^*(T)}{RT}$$

where p_s^* (T) is the saturation pressure of water vapor at the temperature T.

Now, C as a function of T must have a continuous derivative or the right side of Eq. (23) would become infinite. The slope of the line represented by Eq. (26) must then be equal to

$$\frac{dC}{dT} = .625\left(\frac{\frac{dp_s^*(T)}{dT}}{RT} - \frac{p_s^*(T)}{RT^2}\right) \cong .625 \left[\frac{dp_s^*(T)}{dT}\right] / RT \tag{30}$$

at the boundaries of the mist region.

A surface in contact with mist captures

a mass of moisture which is given in accordance with Eq. (30) by

$$\frac{dm_m}{dQ} = \left(\frac{D}{K}\right)\left(\frac{dC}{dT_L}\right) = .625\left(\frac{D}{K}\right)\left[\frac{dp_s^*(T)}{dT}\right]/RT_L . \tag{31}$$

The foregoing discussion may be clarified by reference to Fig. II-1 which contains a curve C giving the saturation states $P^* = Ps^*(T)$ as a function of T.

Let L be the point which represents the mean vapor state in the exchanger element under consideration. Draw the line LB tangent to curve (C) at point B. Then if the temperature of the surface of the exchanger element is higher than T_B, T_A for instance, the states of the vapor in the element are all represented according to Eq. (26) by points lying on the line AL and there is no mist in the exchanger. In this case the moisture which condenses on the walls is given by Eq. (28) and is proportional to the slope of AL.

If, on the other hand, the temperature of the exchanger walls is lower than T_B, T_D for instance, then the states of the vapor occurring in the exchanger are given by points lying on the line DBL. The moisture which condenses on the wall of the exchanger is now proportional to the slope at D of the saturation curve (C). This slope may be considerably less than the slope LB, especially in the low temperature range.

(4) Monotherm Cooler

A study of the operation conditions of a refrigerating type of dryer leads to rather involved calculations in general. However, these calculations can be considerably simplified if it is assumed that the temperature of the cooler surface is constant throughout the whole dryer. Although this assumption is not fulfilled in any actual dryer, since the heat exchange between the air being dried and the refrigerating fluid must raise the temperature of the refrigerating fluid, still the operating conditions of a monotherm cooler are typical. This similarity is of course more pronounced for dryers in which only small temperature differences appear at the walls.

If T is the temperature of the cooling surfaces, then Eq. (27) which may be written as

$$Fc_p dT_m = -dQ = -KNu_L(T_m - T_L)\,dL \tag{32}$$

may be integrated to give

$$T_m = T + (T_E - T)e^{-X} \tag{33}$$

where T_E is the air temperature at the inlet and X is defined as

$$X = \frac{K}{C_p F}\int Nu_L d_L . \tag{34}$$

If no mist appears in the dryer then

$$\frac{p_E^* - p_s^*(T)}{T_E - T} \leq \frac{dp_s^*(T)}{dT} \tag{35}$$

and Eq. (29) holds so that

$$\frac{dp_m^*}{dT_m} = \frac{Pr}{Sch}\,\frac{(p_m^* - p_L^*)}{(T_m - T_L)} = 1.45\,\frac{p_m^* - p_s^*(T)}{T_m - T} . \tag{36}$$

Eq. (36) may be integrated to give

$$\left[p_m^* - p_s^*(T)\right] = \left[p_E^* - p_s^*(T)\right]e^{-1.45X} . \tag{37}$$

If now Eq. (37) is divided by Eq. (33)

$$\left[p_m^* - p_s^*(T)\right]/(T_m - T) = \left[p_E^* - p_s^*(T)\right]e^{-.45X}/(T_E - T) < dp_s^*(T)/dT \tag{38}$$

so that there is no mist in the exchanger and Eq. (37) is valid throughout the dryer. Furthermore, Eq. (37) permits the calculation of the mean vapor pressure at the outlet of the dryer if the integral of Eq. (34) is extended throughout the whole dryer.

If there is mist present in the dryer, then

$$\left[p_E^* - p_s^*(T)\right]/(T_E - T) > dp_s^*(T)/dT. \tag{39}$$

The treatment of this case is more complicated since it involves the manner in which the condensation occurs. It is difficult to formulate a general rule for the manner in which condensation occurs, since the process depends on both the number of condensation nuclei present and on the contact time, i. e., the time during which a given mass of air remains in contact with the walls of the dryer. Even the assumption that the air-water vapor mixture is never supersaturated does not materially simplify the computations. No general conclusions may be found, since the evaluation of the derivative dp_m^*/dL can be obtained only if the extent of the mist region is known. The extent of the mist region is, of course, dependent on a knowledge of the detailed temperature distribution which must be studied for each particular case.

In order to obtain conclusions of practical value, it is necessary to assume that the condensation occurs only in the immediate vicinity of the walls. This will be the case if the condensation nuclei are few or if the contact time is short. In either of these cases only the slow moving fluid in the boundary layer has time to reach equilibrium.

If we assume condensation occurs only in a boundary layer near the cooler's surface, then only a negligible amount of moisture condenses as mist, but this small amount is sufficient to control the diffusion of the water vapor toward the cooler's walls. The icing flow will be proportional to $\left[dp_s^*(T)\right]/dT$ which is a constant for the entire exchanger. This assumption, therefore, leads to the most unfavorable operating conditions which are expressed by the equation

$$dp_m^*/dT_m = 1.45\, dp_s^*(T)/dT \tag{40}$$

which has as its solution

$$p_m^* = p_E^* - 1.45\,(T_E - T)(1 - e^{-X})\,dp_s^*(T)/dT. \tag{41}$$

However, even though there is mist at the inlet of the exchanger, it is not necessary that the mist should persist down to the outlet of the cooler. That this may be the case is evident if Eq. (41) is combined with Eq. (33) to give

$$\frac{p_m^* - p_s^*(T)}{T_m - T} = 1.45\left[\frac{dp_s^*(T)}{dT}\right](1 - e^X) + \left[p_E^* - p_s^*(T)\right]e^X/(T_E - T). \tag{42}$$

Eq. (42) shows that the slope $\left[p_m^* - p_s^*(T)\right]/T_m - T$ is a decreasing function of the distance from the cooler outlet so that

$\left[p_m^* - p_s^*(T)\right] / T_m - T$ may become smaller than the critical value of $\left[dp_s^*(T)\right] / dT$.

$$p_s^*(T) + (T_E - T)\,\frac{dp_s^*(T)}{dT} < p_E^* < p_s^*(T) + \left[1.45 - .45e^{-X}\right](T_E - T)\,\frac{dp_s^*(T)}{dT}\,. \tag{43}$$

Thus, there exists in the exchanger an element, defined by the value X' of the parameter X, at which the mist disappears. Furthermore, the value of X' is given by

$$e^{-X'} = \left(\frac{1}{45}\right)\left[1.45 - p_E^* - p_s^*(T) / (T_E - T)\left(\frac{dp_s^*(T)}{dT}\right)\right]. \tag{44}$$

From this value of X' through the rest of the exchanger, the vapor pressure will be given by a slightly modified form of Eq. (37)

$$p_m^* = p_s^*(T) + \left[p_{m'}^* - p_s^*(T)\right] e^{-1.45(X - X')} \tag{45}$$

where $p_{m'}^*$ is the mean vapor pressure in the element at which the mist disappears.

This vapor pressure $p_{m'}^*$ may be calculated from Eq. (41) and (43) as

$$p_{m'}^* = p_s^*(T) + (T_E - T)\,e^{-X'}\,dp_s^*(T)/dT\,. \tag{46}$$

Using the results of Eq. (45) and (46) the partial pressure of the water vapor at the outlet is given by

$$p_f^* = p_s^*(T) + .70\,(T_E - T)\left[\frac{dp_s^*(T)}{dT}\right] e^{-1.45X} \left[1.45 - \frac{(p_E^* - p_s^*(T))}{(T_E - T)\frac{dp_s^*(T)}{dT}}\right]^{-.45} \tag{47}$$

However, if

$$p_E^* > p_s^*(T) + (1.45 - .45e^{-X})(T_E - T)\left(\frac{dp_s^*(T)}{dT}\right) \tag{48}$$

then all of the walls of the exchanger are covered with mist and Eq. (41) holds throughout the dryer so that

$$p_f^* = p_E^* - 1.45\,(T_E - T)(1 - e^{-X})\left[\frac{dp_s^*(T)}{dT}\right] \tag{49}$$

where X is computed for the entire dryer.

All of the above results are shown on the graph of Fig. II-2. The various regions of no mist, part mist, and all mist in the exchanger as expressed in Eq. (43) are clearly shown. In the first region, i. e., no mist in the exchanger, Eq. (27) gives p_f^* as a function of p_E^*. In the region where there is some mist but the mist does not persist to the dryer outlet, Eq. (47) expresses the relationship between p_f^* and p_E^*. For the case where the mist persists to the dryer outlet, p_f^* is the function of p_E^* which is given in Eq. (49). The entire curve, of course, is based on the assumption that when mist occurs it occurs only in the boundary layer near the wall of the dryer.

Two additional curves are given which give the estimated characteristics for a dryer fitted with a mist trap at the outlet. The dashed curve corresponds to a perfect mist trap, i. e., one with a sufficiently high concentration of condensation nuclei so that the partial pressure at the exit approaches the saturation partial pressure for the outlet temperature. The dotted curve corresponds to the case for which there are fewer condensation nuclei so that the air leaving the dryer may be supersaturated.

(5) The Parameter X

The parameter X which is defined in Eq. (34) is obviously dependent on the mass flow through the dryer. However, it should be noted that X is independent of the pressure level in the dryer because the Nusselt number is a function of the Reynolds number only, and this Reynolds number is itself simply proportional to the mass flow F.

Furthermore, since the Nusselt number is generally proportional to a fractional power of the Reynolds number, X is a decreasing function of F, with X decreasing less rapidly than 1/F. Without reference to a particular heat and mass exchanger, it is impossible to state specifically the X-F dependence.

Since X is independent of the pressure p, the graph of Fig. II-2 may be used at any pressure level. If, however, the absolute humidities rather than the vapor pressures are of chief interest, the pressure in the dryer enters through the formula

$$h = .625 \frac{p^*}{p}.$$

(6) Dryer Operation at Different Cooler Temperatures

From Fig. II-2 it is apparent that for a constant value of the cooler temperature, it is impossible to operate indefinitely without mist as the inlet humidity increases. It is, therefore, interesting to study the behavior of a monotherm cooling dryer at various wall temperatures. Fig. II-3 shows the outlet partial vapor pressure as a function of the inlet partial vapor pressure for a given cooler, i.e., a cooler characterized by a definite value of X as the temperature increases from T_1 to T_2. The auxiliary saturation curve (C) is shown to clarify the situation.

Fig. II-3 clearly shows that much lower humidities at the dryer outlet are obtained at higher cooler temperatures for a given range of inlet humidities. This rather surprising result is due to the fact that at the higher cooler temperature, operation without mist is possible for much higher inlet humidities. This important result may explain why cooling dryers have not always met the expectations of their designers.

A cooler which does not operate as a monotherm cooler might give more satisfactory operation over a wider range of inlet humidities.

(7) Pressure Losses in Cooling Dryers

The pressure losses encountered in a cooling dryer are much smaller than those encountered in dryers which use granular adsorbents. The pressure losses in drying plants which utilize adsorbents will be discussed in detail later.

The pressure losses for cooling dryers are of the same order of magnitude as the losses present in the dryer circuit plumbing. Therefore, there is no need for great accuracy in computing the pressure losses; a knowledge of the order of magnitude of the pressure losses is sufficient.

The computations may be further simplified by assuming a Prandtl number of unity so that the Reynolds analogy holds for both turbulent and laminar boundary layers and wakes, and the temperature and velocity profiles are similar.

Thus, in each element of the dryer

$$\frac{dQ}{K(T_m - T_L)} = \frac{S' dp}{\mu V_m} \tag{50}$$

where

S' = cross sectional area

V_m = mean velocity

dp = the pressure loss in the element.

Now if Eq. (27) is used in conjunction with Eq. (50)

$$dp = \rho V_m^2 \mu F^{-1} Nu_L \, dL = \rho V_m^2 dX. \tag{51}$$

Eq. (51) may be integrated if V_m is assumed constant to give

$$\Delta p = \rho V_m^2 X. \tag{52}$$

Eq. (52) is useful as a first approximation for pressure losses.

(8) Drying by Cooling for Various Requirements

In dryer design the following plant types must be considered: First, dryer in bypass used to stabilize the tunnel humidity in spite of given air leaks; second, charging dryers (a) in series with the tunnel or (b) in bypass used to predry the tunnel air; and third, initial dryer used to eliminate the bulk of the atmospheric humidity.

The third type would generally be used in conjunction with a second dryer which would give the final low humidity necessary for the tunnel operation. An example of this third type of installation would be the after-cooling of the compressors used for a high pressure tunnel. The third type of dryer plant is subject to requirements analogous to those for the second type of plant, except that the desired outlet humidity need not be as low.

(9) Bypass Dryer; Compensation for Air Leaks

Let F_B be the mass flow of air through the bypass and let F_ℓ be the mass flow of humidity into the wind tunnel through leaks in gm/sec. Then, if the maximum tolerable humidity is h and the humidity at the dryer outlet is h_f the following relation must hold

$$F_\ell = F_B (h - h_f). \tag{53}$$

It is obvious from Eq. (53) that the maximum practicable bypass flow must be used in order to minimize the dryer requirements. General practice is to bypass a maximum of about ten percent of the tunnel flow in continuously operating supersonic wind tunnels which use the driving compressor to force the flow through the bypass dryer.

If the dryer is to operate without mist, Eq. (43) shows that

$$p_S^*(T) + (T_E - T)\left[\frac{dp_S^*(T)}{dT}\right] > p_E^* = 1.6\,ph \tag{54}$$

where p is the air pressure entering the dryer.

Hence, Eq. (37) applies in the form

$$p_S^*(T) = 1.6\,p\left[\frac{h - (F_\ell / F_B)}{(1 - e^{-1.45X})}\right] \tag{55}$$

if use is made of Eqs. (53) and (54). Eq. (55) gives a relation between $P_S^*(T)$ which is a function of the temperature T of the cooler walls, and the parameter X, which is a function of the amount of cooling surface needed.

For each specific case a choice must be made for one of these quantities after which the other will be determined by Eq. (55). It is furthermore clear that a higher cooler temperature will correspond to a larger cooler surface and conversely, cooling to a lower temperature will lead to a smaller dryer. If, however, the condition of no mist in the dryer is to be maintained, there will be a lower bound for the parameter X since the temperature, T, cannot be lowered too much.

The lowest temperature to which the air may be cooled corresponds to the condition for which mist is about to appear at the dryer inlet, namely from Eq. (54),

$$p_s^*(T) + (T_E - T)\left[\frac{dp_s^*(T)}{dT}\right] = 1.6\, ph. \tag{56}$$

Hence from Eq. (55) it is apparent that X must be greater than

$$X_{min} = -.69\,\ell n\left[1 - \frac{F_\ell / F_B}{(h - .625\, p_s^*/p)}\right]. \tag{57}$$

If the dryer is an ideal mist dryer, i.e., one for which there is no supersaturation at the dryer outlet and which is followed by a perfect mist trap, the design condition of Eq. (53) may then be written as

$$p_s^*(T_f) = 1.6\, p\left[h - F_\ell / F_B\right] \tag{58}$$

where T_f is the outlet temperature.

The temperature of the cooling walls, T, must be less than the outlet temperature, T_f. A lower bound for X may be found from Eq. (33) by putting X = 0 to get,

$$X_{min} = \ell n\,(T_E / T_f). \tag{59}$$

It is, of course, possible to obtain a smaller value of X_{min} in this case, i.e. a dryer with mist may be made smaller than a dryer without mist and still meet the design specifications. However, the operation of the dryer with mist will have to be at a substantially lower temperature than the operation of the dryer without mist.

(10) Charging Dryer

This type of operation is characterized by a high inlet humidity, usually atmospheric humidity. However, the flow rate is not subject to restrictive requirements.

Due to the high inlet humidity, it is generally impossible to design a monotherm cooler which will give the desired outlet humidity and operate without mist. Two possible alternatives are to use a mist trap or to use a cooler with nonconstant wall temperature in which the walls near the inlet are warmer than the walls near the outlet.

The design criterion for this type of dryer system is

$$p_s^*(T_f) = 1.6\, ph \tag{60}$$

where h is the humidity which is tolerable in the tunnel. The problem now reduces itself to designing a heat exchanger which will give an outlet temperature, T_f, which is calculated from Eq. (60).

(11) Dryer in Bypass Used for the Initial Drying of a Tunnel

The bypass dryer is generally intended to handle two functions, initial drying and stabilization. A monotherm cooler designed as a stabilizer will be quite inefficient for initial drying in view of the high inlet humidity, unless the cooler temperature is raised. This temperature should then be gradually lowered during operation, reaching the lowest design value at the end when the humidity has dropped to the required level. Such a technique has not been carried out in practice to our knowledge, but it is expected to be the most efficient.

Here again, an installation which allows a wide variation of wall temperatures can be expected to perform better. The requirements in this case generally include a limit for the duration of the initial drying period.

Let us first compute this duration for an ideal mist dryer, with trap (air saturated at outlet temperature T_f). If h_a is the atmospheric humidity existing initially in the tunnel, h the humidity required after drying, h_f the outlet humidity downstream of the dryer, which is constant in this case, then the humidity evolution during the drying operation is subjected to the obvious differential equation:

$$d(m_t h_t)/dt = -F_B(h_t - h_f) \tag{61}$$

m_t being the total tunnel air mass to be dried, h_t the humidity at a time, t.

The solution of Eq. (61) is

$$h_t = h_f + (h_a - h_f)\, e^{-(F_B/m_t)t} \tag{62}$$

and the duration to be computed, is

$$t = \left(\frac{m_t}{F_B}\right) \ln\left[\frac{(h_a - h_f)}{(h - h_f)}\right] \tag{63}$$

or, approximately,

$$t \cong \frac{m_t}{F_B} \ln\left[\frac{h_a}{h - h_f}\right]. \tag{64}$$

A limitation to this duration imposes a definite value to h_f and therefore to the cooler temperature.

We will now outline the calculation in the ideal case of a monotherm dryer with the temperature varied during the drying in the theoretically most efficient way, i.e. so that at each stage the mist just fails to appear at the inlet. The differential equation is as above, except that h_f now varies with time so as to maintain the no-mist conditions

$$p_t^* = p_s^*(T_t) + (T_E - T_t)\left[\frac{dp_s^*(T_t)}{dT_t}\right] \tag{65}$$

T_t being the cooler temperature at a time t.

The duration is found to be

$$t = \left[\frac{(m_t/F_B)}{(1 - e^{-1.45X})}\right] \ln\left[\frac{(dp_s^*/dT_0)}{(dp_s^*/dT)}\right]. \tag{66}$$

If an exponential law is used for the saturation pressures, which is accurate enough if the considered temperature range is not too wide, this duration is seen to be of the order of

$$t \cong \left[\frac{(m_t/F_B)}{(1 - e^{-1.45X})}\right] \ln\left[\left(\frac{h_a}{h}\right) \frac{1 + \alpha(T_E - T)}{1 + \alpha(T_E - T_0)}\right] \tag{67}$$

which can be much less than the corresponding duration in the case of an ideal mist dryer, if

$$h - h_f \ll h. \tag{68}$$

In Eq. (67) α is the temperature coefficient in the exponential law assumed for the saturation pressures.

In the case of a dryer designed for humidity stabilization by cooling with given wall temperature, the comparison between the two methods of operation is impossible when, as happens in a few instances, one is unable to reach the working conditions if mist is present. This arises if the saturation pressure corresponding to the outlet temperature is higher than the vapor pressure corresponding to the final humidity that can be achieved in absence of mist. In real cases of imperfect capture of moisture, it can even happen when this saturation pressure is lower than the vapor pressure ultimately wanted. Users of cooling dryers should ascertain whether or not they are working under conditions in which a rise in the temperature of the cooler will help the drying, and allow the humidity to be further lowered.

(e) Drying by Compression

(1) Introduction

Compression followed by cooling down to initial temperature is a natural way of drying, if compressed air is needed for tunnel operation. This is the case for blowdown type intermittent wind tunnels, or for continuously operated, closed wind tunnels with high stagnation temperature. Flexible operation of closed wind tunnels, in which many changes of pressure level are made, often requires dry air supply, which can best be preserved in high pressure reservoirs. Predrying by compression is also obtained.

A further dryer is generally needed to satisfy the requirements but it will operate under most favorable conditions, because of the very low inlet humidity. In the case of the adsorption dryer, it appears likely that the high operating pressure adds to the performance. It is by such a combination of compression and aftercooling followed by drying in a complementary adsorption dryer that the lowest humidities can be obtained.

The only drawback of the system is its power requirement, which is very much higher than the power required to operate or regenerate any other type of drying plant. In case compressed air is not used for tunnel driving, this might seem to be an important economic argument against the use of high pressure storage, aside from the cost of the vessels themselves. Actually, this might not be the case if the installed power were kept low and the compressors operating continuously.

(2) Humidity After Compression

If a compression is carried out to a pressure of P, the partial vapor pressure of moisture is raised for a given value h of the absolute humidity to

$$p^* = 1.6\, ph. \tag{69}$$

If a heat exchanger is placed downstream from the compressor, lowering the air temperature to its initial value T_0, the air may become supersaturated, and condensation may occur, the vapor pressure decreasing to the value $p_s^*(T_0)$.

The absolute humidity is consequently lowered to the value

$$h' = 0.625\, p_s^*(T_0)\, p^{-1}. \tag{70}$$

This can be written in terms of the absolute humidity $h_{s,a}$ corresponding to atmospheric saturation:

$$h' = h_{s,a}\,(p_a/p) \tag{71}$$

p_a being the atmospheric pressure.

$h_{s,a}$ is of the order of 10^{-2} gm H_2O/gm air. To obtain by sheer compression the usually required 0.2 X 10^{-3} gm/gm, a compression to 50 atm is thus necessary.

If a lower humidity is required for tunnel operation, an additional dryer will have to be fitted, but it will generally be much smaller than the dryers considered so far.

(3) Power Required for Compression

If an ideal isothermal compression to a pressure p is considered, the work done per unit mass is given by

$$W = RT_0 \int_{p_a}^{p} p\, d\left(\frac{1}{p}\right) = RT_0 \ln\left(\frac{p}{p_a}\right) = RT_0 \ln\left(\frac{h_{s,a}}{h'}\right). \tag{72}$$

Cooling by a perfect refrigerator to a temperature, T, requires a work

$$W_r = c_p (T_0 - T)\left(1 - \frac{T}{T_0}\right). \tag{73}$$

If an exponential law, e^{aT}, is chosen to represent the saturation pressures, the two works may be compared on the basis of same final humidity, their ratio being

$$W/W_r = (R/c_p)\, a\, T_0^2\, (T_0 - T)^{-1}. \tag{74}$$

R/c_p is about 3, T_0 is usually of the order of 300 ° K, $T_0 - T$ of the order of 60° K, so that

$$W/W_r \cong 500\, a \tag{75}$$

a being about 0.10. The ratio is seen to be of the order of 50. This validates the inference of the introduction.

(4) The Aftercoolers

These coolers may be considered as dryers by cooling, so that the considerations of the preceding chapter may be applied.

As already stated, this is the only type of dryer not in practice that does not need regeneration, the moisture condensing as liquid and flowing away into a small reservoir that has to be drained from time to time.

The same problems arise, however, about the mist which has to be separated.

(f) Drying by Adsorption

(1) Introduction

Solid granular adsorbents, widely used in wind tunnel drying plants, are believed to derive their adsorbing potential from some sort of intermolecular attraction between gas molecules and the porous surfaces of the adsorbent. Theories for the steady state or equilibrium case of adsorption have been derived by many physicists and chemical engineers. For details, the reader is referred to the books by Mantell (Ref. 23) and Brunauer (Ref. 24) each of which has an extensive bibliography.

In wind tunnel drying plants, the adsorption process, however, is nonsteady or dynamic. Except for performance data of manufacturers of drying equipment, almost no data are available for the case of dynamic adsorption. Recent work by Dehler (Ref. 25) and others (Refs. 30 and 32) in the U. S. and Bateman (Ref. 26) and others (Ref. 35) in England has started to give some data about the dynamic case. Their work, however, is still in progress and only the initial results are available.

The lack of a theory for the dynamic adsorption process makes it impossible to evaluate the available empirical data systematically.

(2) Equilibrium Adsorption Process

An adsorbent in contact with a gas will adsorb an amount of gas proportional to the quantity of adsorbent. The ratio of mass of gas adsorbed to mass of product is called (equilibrium) concentration or percentage adsorption (if expressed in percent). It is a function of the partial pressure of the gas and the temperature of the adsorbent. The gas of interest to us is a mixture of air and water vapor. Because the adsorption of air and its constituents is almost negligible, the partial water vapor pressure alone will enter into the equilibrium equation. For a given total pressure, this can be expressed in terms of the absolute humidity (or the relative humidity, if the temperature is known).

An adsorbent can be described by the functional dependence among three parameters: (a) its concentration at a given temperature and partial pressure, (b) its temperature, and (c) the absolute humidity or the partial pressure.

The concentration at constant temperature is, of course, a rising function of the partial pressure or humidity. The curves representing this function are the adsorption isotherms of the adsorbent. Figs. II-4 and II-5 give examples of such isotherms for the most widely used adsorbents, silica gel and activated alumina.

If the partial pressure or absolute humidity is kept constant, the concentration is a decreasing function of the temperature. The corresponding curves are the so-called isopiestics. Figs. II-6 and II-7 represent isopiestics for silica gel and activated alumina.

It is seen from these curves that the adsorbing capacity, or concentration, of silica gel is somewhat greater than that of activated alumina under all conditions of pressure and temperature.

(3) Heat of Adsorption

The adsorption of a gas by a granular substance is an exothermic process. The heat liberated per unit mass adsorbed is called heat of adsorption.

It is generally assumed that the heat liberated is proportional to the concentration. Although no general rule can be deduced, the heat of adsorption of practical drying materials is of the same order of magnitude as the heat of condensation but numerically greater. This implies that the adsorption isotherm is concave to the pressure axis for small partial pressures, as shown in Fig. II-4 and II-5.

Numerically, the heat of adsorption is approximately 600 cal/gm water adsorbed, in normal conditions. It is, however, slightly but intricately dependent on temperature and nature of adsorbent and on the amount of water already adsorbed.

The heat of adsorption increases the adsorbent temperature of drying plants and consequently lowers its useful concentration or drying efficiency for given inlet humidity. The removal of this heat of adsorption is one of the main difficulties of adsortion dryers, especially in the case of large plants operating under high inlet humidities. Precooling of the air to be dried has been used in some instances but internal cooling of the plant may also be required.

(4) Dynamic Adsorption Process, General

Nearly all wind tunnel drying plants using the principle of adsorption operate under dynamic conditions. This involves the flow of humid air through the adsorbent, and the time required to lower the humidity to a specified level. Since this is a relatively recent technique of adsorption, sufficient information about it is rather scarce and sometimes misleading.

It has been stated, for instance, that the time required to reach equilibrium conditions is large, varying from fractions of a second to several minutes.

It is believed that the question of reaching low humidity equilibrium must be answered in two parts. First, equilibrium between the humid air in contact with the adsorbent and the active adsorbent itself is obviously reached instantaneously since a molecular process is involved. Second, the time consuming part consists of bringing the entire quantity of air to be dried in contact with the adsorbent so that a certain minimum mass of air attains the low humidity equilibrium. This is essentially a gradual diffusion process analogous to the time-consuming process of reaching thermal equilibrium when a cold gas comes in contact with a hot wall.

A set of partial differential equations can be derived to describe this mechanism of dynamic adsorption. Before presenting this proposed theory, however, the available empirical data will be summarized.

(5) Dynamic Adsorption Process, Empirical Data

First of all, one should recognize that experimental work with adsorption dryers is complicated by the presence of the very important thermal effects. These are generally not considered unless careful temperature measurements of the adsorbent are made.

Manufacturers of adsorbent dryers usually present performance data as a curve of outlet humidity versus mean water concentration (i.e. ratio of total mass of water adsorbed to mass of adsorbent in the dryer) with inlet humidity and airflow as parameters.

An example of such a curve is given in Fig. II-8. It is seen that a steep rise follows a range in which h_f is essentially zero. The steepness of the rise is such as to justify the conclusion that the plant efficiency vanishes at this point. This is called the break point, and the corresponding concentration is the maximum useful concentration of the dryer.

More accurate measurements, such as the ones carried out by Bateman (Ref. 26), show the outlet humidity variation to be not as simple as this. The discrepancy is attributable to the inaccuracies or slow response of hygrometers. However, the curve of Fig. II-8 can be obtained with very slow airflow or with very deep adsorbent layers, both giving a long contact time between any air particle and the adsorbent. (This can also be shown by the theory given in the next section.) Bateman's tests, however, were made under conditions similar to those encountered in current wind tunnel practice, and lead to curves of the type indicated by Figs. II-9 and II-10.

The initial portion of low outlet humidity is followed by a gradual increase of the slope. After the maximum slope is attained, the curve becomes approximately straight. Although a break point no longer exists, the starting point of the region of increasing slope gives an indication of the maximum useful concentration. As seen in Figs. II-9 and II-10, this starting point depends strongly on both the inlet humidity and depth of the adsorbent layer.

The experiments of Bateman included accurate measurements of temperature distribution in the plant.

The tests were made for various values of the three following fundamental parameters: inlet humidity, grain size and contact time. Contact time is defined as the ratio of adsorbent depth to velocity of the airflow, or the ratio of adsorbent volume to air volume flow.

The mean concentration of the adsorbent was deduced from inlet and outlet humidity measurements from the relation

$$\text{mean conc.} = \left(\frac{F}{m'}\right) \int_0^t (h_f - h_E)\, dt \tag{76}$$

where

F = air mass flow (in gm/sec.)

m' = total mass of adsorbent used in the plant

h_E = inlet absolute humidity

h_f = outlet absolute humidity.

The ratio F/m' can also be written

$$\frac{F}{m'} = \left(\frac{\rho}{\rho'}\right) \frac{1}{t_c} \tag{77}$$

where ρ' and ρ are the adsorbent and air specific masses, t the contact time.

We now review the five conclusions Bateman drew from his tests.

First, the maximum temperature rise in the plant is remarkably independent of grain size, depth of adsorbent layer, and contact time, as already observed by Dehler (Ref. 25). It is a nearly linear function of the difference of inlet and outlet humidities as given by

$$\Delta T = \text{const.}\ (h_E - h_f) \tag{78}$$

where

const. = 30° C x 10^2 per gm/gm for silica gel

const. = 34° C x 10^2 per gm/gm for activated alumina.

Second, the useful concentration decreases with increasing inlet humidity, which is contrary to expectations. The discrepancy is certainly due to the increase of adsorbent temperature.

Third, the useful concentration is not proportional to the product of depth and constant contact time, but increases much more sharply with an increase in depth. A deep layer is thus more efficient than a shallow one, except from the point of view of pressure losses. (A tentative explanation of this phenomenon is given in the next section.)

Fourth, the grain size also influences dynamic adsorption efficiency. Smaller grains seem to be more efficient for inlet humidities higher than 0.3 to 0.5 x 10^{-2} gm/gm.

And fifth, there is comparatively little difference between the performance of the adsorbents tested, silica gel and activated alumina, especially if the depths are adjusted to give equal pressure losses.

A more recent experimental study was made by Gruenewald (Ref. 27). The adsorbents, which were under consideration for the NOL 40 cm by 40 cm wind tunnel, were tested in a tube under short contact times of 0.4 sec. No temperature measurements were made. However, since the tests extended to very low humidities (0.5 x 10^{-2} gm/gm) the thermal effects should have been less important.

Fig. II-11 shows the results of these tests. It indicates that silica gel is superior to alumina and sova bead, a product of the Socony Vacuum Oil Company of New York. The previously stated effect of grain size is confirmed. The trend of the curves agrees with what one might expect at these low concontact times even though the humidity measurements were not accurate below $h_o = 2 \times 10^{-2}$ gm/gm. The extensive data on pressure losses of this reference are summarized in a later section.

(6) Proposed Theory of Dynamic Adsorption

A theory of dynamic adsorption, which has recently received some direct experimental substantiation, will be outlined in this section. Preliminary experimental data on an exchange coefficient, which is fundamental if the theory is to be applied, is included. A tentative design method, derived from this theory, under suitable simplifying assumptions, will be explained in the next section.

As stated in section (f) (4) of this chapter, one assumes that equilibrium of adsorption exists between the humid air in direct contact with the adsorbent and the adsorbent itself. The transfer of humidity from the bulk of the airflow to the surface of the adsorbent is a diffusion process which superposes itself to the main airflow through the drying bed.

Consider a section of the adsorbing bed, perpendicular to the airflow, in which a uniform distribution of mean humidity and temperature is assumed. Let h be the mean humidity and h_e the equilibrium humidity corresponding to the instantaneous water concentration and temperature of the adsorbent in this section. By analogy to other diffusion phenomena (see, for example, Section II (d) (2)), it seems reasonable to assume that the transfer of humidity in a section can be expressed as a diffusion mass flow, F_m, as follows:

$$dF_m = \rho k (h - h_e)\, dx \tag{79}$$

where

F_m = mass flow to adsorbent, in gm/sec and cm^2 cross sectional area

k = overall coefficient of mass exchange (analog to Nusselt number) in sec^{-1}

ρ = specific mass of air.

This flux of humidity must equal the rate of change of humidity of the air flowing with a velocity V through the dryer bed. Hence, one has, per unit cross sectional area

$$h_t + V h_x = -k (h - h_e). \tag{80}$$

Subscripts denote partial derivatives.

The local time rate of change of the instantaneous water concentration of the adsorbent, C', is:

$$C'_t = \frac{dF_m}{\rho' dx} = k(\rho/\rho')(h - h_e). \tag{81}$$

The primes indicate absorbent values.

The equilibrium isotherm gives an additional relation of the form

$$h_e = h_e(C', T'). \tag{82}$$

Thermal effects are described by a further set of equations, established by similar reasoning. There results:

$$\rho C_p (T_t + V T_x) = k_1 (T' - T) \tag{83}$$

and

$$\rho' c' T'_t - K' T'_{xx} = A \rho k \ (h - h_e) - k_1 (T' - T) \tag{84}$$

where

c_p = specific heat of air at constant pressure (in cal/°C and gm)

k_1 = overall coefficient of thermal exchange (analog to Nusselt number), in cal/°C, cm^3, and sec.

c' = absorbent thermal capacity (same units as c_p)

K' = adsorbent thermal conductivity cal/cm^2,/(°C/cm)

A = heat of adsorption.

The set of Eqs. (80) to (84) describes the dynamic operation of an adsorption dryer. It is complicated, mathematically, because of the nonlinear nature of Eq. (82).

The coefficients k and k_1 may be expected to be closely related, and it seems natural to assume

$$k_1 = \rho C_p k \tag{85}$$

as in the case of turbulent heat and mass transfer (Lewis relation), which also holds for the overall coefficients of transfer in other cases. There is also indirect experimental support for this relation.

If thermal effects are neglected, the problem is simplified, because only Eqs. (80), (81), and (82) need be considered. This is allowed provided the inlet humidity remains very low (say, about $h_{inlet} = 10^{-3}$), as when a preliminary drier is fitted (compression plant followed by a cooler) or when the dryer operates as a stabilizer (in bypass), removing only leakage humidity from an already dry atmosphere. The neglect of thermal effects is also justified in the case of an internally cooled adsorbent bed, even if it is heavily loaded.

In the cases where thermal effects are important, drastic simplifying assumptions must be made to obtain information from our partial differential system. A first simplification is always justified and consists in dropping the term representing thermal conductivity through the adsorbent in Eq. (84). A more daring one would be also to drop the other term in the first member of the same equation. This can be shown to be reasonable rather generally and rough estimates can be expected.

When those simplifying assumptions are made, the problem becomes equivalent, mathematically, to the isothermal problem. The reason for this lies in the remarkable relations

$$h(A/c_p) + T = h_e(A/c_p) + T' = \text{const.} \tag{86}$$

which are easy to derive from the modified set of equations. An encouraging feature of these relations is the fact that they immediately yield the empirical Eq. (78).

The coefficients k and k_1 which we assumed to be connected by the simple Eq. (85), are both functions of the velocity of the airflow in the dryer. Our first tests, based on purely thermal exchanges, seem to indicate a roughly linear dependence of k and k_1 on V, as indicated by Fig. II-12, the accuracy of the method being rather limited.

It is therefore to be expected that the performance of adsorbing layers will be dependent on the air velocity (or layer depth) even if the contact time is kept constant. This can be grasped at once if the following nondimensional variables are introduced in the Eqs. (80) to (84):

$$x' = \frac{kx}{V}$$

$$t' = kt . \tag{87}$$

The outlet humidity, for instance, will then be given by a formula of the type

$$h_f = \phi(h_E, T_E, \bar{C}', x') . \tag{88}$$

$\bar{C}'$ being the mean water concentration in the plant. The function ϕ depends on the equilibrium curves of the adsorbent. But x' is related to the total depth L of the adsorbent bed by

$$x' = kL/V = kt_c . \tag{89}$$

The important conclusion to be drawn from Eqs. (88) and (89) is the following: The factor governing dryer performance for given inlet conditions is not, as universally admitted (before the Bateman tests), the contact time t_c, but rather the nondimensional product kt_c. According to our first tests concerning the dependence of k_1 on V, this could be converted into a "rule of thumb": the performance of a dryer depends only on its depth, irrespective of the contact time. It must, however, be added that Eq. (88) is valid only if thermal conduction in the adsorbent is negligible.

The functional dependence of k on adsorbent grain size, the existence of which is indicated by Bateman's results, has not yet been tested, neither has its dependence on the pressure. It is likely, however, that k should increase with pressure, which would account for the somewhat higher efficiency obtained when adsorbers are used under high pressures.

(7) Design Method based on Results from Linearized Theory

Provided the equilibrium characteristic of the adsorbent, Eq. (82), can be linearized in the range of T' and h_e obtained during operation, the Eqs. (80) to (84) can be reduced to the so-called Telegraph Equation (always neglecting the thermal conduction through the adsorbent). This leads to graphs describing the dryer operation, which are qualitatively equivalent to the experimental curves given by Bateman.

Let us first consider the case of the isothermal operating dryer. A useful approximate formula can be obtained in practical cases, where x' as given by Eq. (89) is much greater than unity:

$$\bar{C}_b / C_e(h_E) = \left[1 - \left\{ \arg \mathrm{Erfc}\left(\sqrt{\pi}\, h_m / h_E\right) \right\} / \sqrt{kt_c} \right]^2 \tag{90}$$

where

h_m = maximum outlet humidity tolerated

$C_e(h_E)$ = equilibrium water concentration of the adsorbent in the presence of the inlet air (static capacity)

$\bar{C}_b$ = mean concentration obtained when h_{outlet} finally reaches the critical value h_m ("break point" dynamic capacity).

Erfc = complementary error function:

$$\mathrm{Erfc}(z) = \int_z^{\infty} e^{-z'^2}\, dz'$$

argErfc = inverse function to Erfc.

A graph indicating the relation Eq. (90) between the ratio of dynamic to static capacities, the ratio of critical and inlet humidities and the characteristic parameter kt_c is given in Fig. II-13.

A design method derives from this formula. The requirements are generally the following: First, the dryer must keep the outlet humidity below a given level h_m for a given inlet humidity. Second, a total amount of water no less than w must be adsorbed between successive reactivation periods. Eventually, a limitation is imposed on the pressure losses, for instance, in the case of full flow dryers for suction wind tunnels. This leads to the third requirement that the total pressure losses must be kept below a certain level, for a given airflow, F.

The designer now has to choose his adsorbing substance and to calculate the amount he is going to need. Let v' be the volume of adsorbent required. Obviously, one has

$$\bar{C}_b = \frac{w}{\rho' v'} .$$

Eq. (90) then relates v' to the parameter kt_c, or, if k is assumed to be proportional to the airflow velocity V, to the layer depth L ("rule of thumb" in preceding section).

In the next section, an approximate formula for the pressure losses will be indicated, Eq. (92), from which we conclude that

$$\frac{L^3}{v'^2} = (\text{pressure losses}) / (f F^2 / \rho) . \tag{91}$$

This equation, combined with Eq. (90) permits the determination of both L and V, the dryer being thus completely characterized.

Unfortunately, we were unable to obtain a useful general formula, equivalent to Eq. (90), for the cases where thermal effects are important, because the problem then becomes essentially nonlinear, (the main term in the equilibrium capacity $C_e(h_e, T')$ being proportional to the product $h_e T'$.

One has to revert to step by step treatment of the partial differential system Eqs. (80) to (84). To gain a general insight in the cases of practical importance by such methods seems a formidable task indeed, each set of initial conditions involving a separate computation of the solution. This would not be worthwhile without a preliminary experimental study of the equilibrium curves for small humidities (h_e less than 0.001), the published data being rather incomplete in this respect.

(8) Pressure Losses in Solid Granular Adsorbents

Pressure losses in airflows are usually expressed in terms of a friction coefficient, f, as follows:

$$\Delta p = p_E - p_f = f \rho L V^2 \tag{92}$$

where

$p_E - p_f$ = pressure loss in mm of water through a layer of thickness L in cm measured along the flow direction

ρ = air specific mass in gm cm^{-3} (1.3×10^{-3}) under standard conditions

V = velocity in cm sec.$^{-1}$

The friction coefficient, f, depends on the nature of the substance in the layer (chemical composition, grain size, packing, etc.), and the velocity.

Existing experimental data do not indicate a strong dependence of f on the velocity, at least in the velocity range of practical importance. This dependence is accordingly difficult to establish with accuracy, because of the dispersion of measurements. Its practical importance seems to be small in view of the strong dependence of f on grain size, for instance, the definition of which is unavoidably rather vague.

We therefore analyzed the available data under the assumption that f depends only on the desiccant nature and grain size.

A summary of available test data is given in terms of ρf in Table I on page 34.

Wind tunnel operators sometimes complain about large discrepancies between manufacturers' data and measured pressure drops, the first being considered by them as over-optimistic. Table I, which was computed from both manufacturers' data and careful testing by wind tunnel authorities, does not show such large discrepancies.

However, in view of the large influence of grain size and packing, etc., it is recommended that preliminary pressure loss measurements be made on samples of the desiccants considered in the likely range of air velocities and layer depths.

The data contained in the table should be useful, however, to obtain an order of magnitude of the pressure losses to be expected at a predesign stage.

(9) An Outline of the Problem of Designing an Adsorption Dryer

The design of a wind tunnel drying plant using solid granular adsorbent involves the basic problem of dessicant reactivation. Current wind tunnel dryness requirements are such that any sensible amount of desiccant, which a reasonable airflow crosses, will give acceptable outlet humidity. The problem, however, is to comply with requirements on tunnel running hours or number of runs between successive reactivation periods and to minimize pressure losses. Reactivation does not imply interruption of tunnel operation if two or more dryers are installed, but it is a costly operation in terms of heating power.

TABLE I. PRESSURE LOSS THROUGH GRANULAR ADSORBENTS

Substance	Source of Data (Ref.)	Grain Size	Velocity Range cm/sec	Value of f_ρ mm H_2O per (cm^3/sec^2)
Silica gel	26	6/10	20 - 130	3.55×10^{-3}
	27	Coarse[1]	35 - 95	1.1×10^{-3}
		Fine[2]	35 - 95	3.0×10^{-3}
	25	6/16	0 - 50	4.5×10^{-3}
Activated alumina	26	2/4	25 - 150	4.0×10^{-4}
		4/8	40 - 50	8.1×10^{-4}
	27	sec[3]	35 - 95	6.5×10^{-4}
	28,29	2/4	0 - 50	4.85×10^{-4}
		4/8	0 - 50	1.15×10^{-4}
		8/14	0 - 50	2.8×10^{-4}
Sova bead	27	Normal[4]	35 - 95	8.3×10^{-4}
		Split	35 - 95	2.4×10^{-4}

$$\Delta p = (f_\rho) L V^2$$

1. Coarse silica gel: 5/6 + 6/7 + 7/8
2. Fine silica gel: 8/10
3. Activated alumina used in tests of Ref. 27 has grain size comparable to coarse silica gel
4. Sova bead: Normal and split, 9/10 (see Ref. 27)

The pressure losses are particularly important in the case of full flow dryers for suction type intermittent wind tunnels.

The frequency of reactivation periods is inversely proportional to the maximum useful concentration of the desiccant consistent with the maximum outlet humidity which can be tolerated.

The solution of this problem requires knowledge of the relations between useful concentration, inlet humidity, maximum tolerated outlet humidity, operating conditions, and dryer parameters, such as desiccant composition, grain size, contact time, velocity of airflow, cross-sectional area of the desiccant layer, and pressure loss as a function of these parameters.

Unfortunately, a relation of this type is known only in a few particular cases, and even then the influence of various parameters is not completely clear.

The case of a heavy duty or charging dryer with high inlet humidity (atmospheric inlet conditions, for instance) is, by far, the most difficult to handle, because of the strong thermal effects. It is also the most interesting case, because the reactivations are more frequent.

Lack of necessary data compels one to make simplifying assumptions which are sometimes of a very rough nature. Equilibrium conditions are often assumed, although the operating conditions are obviously dynamic, thermal effects being either disregarded or roughly estimated.

The most advanced design method suggested so far seems to be the one by Dehler (Ref. 25). A linear humidity and temperature distribution is assumed. The relation between temperature rise and humidity decrease is given by Eq. (78) which is used to calculate the outlet temperature. The equilibrium curves are then used and the concentration is assumed to correspond to mean humidity and mean temperature.

This theory does not take into account such factors as contact time, flow velocity, grain size, etc., and is, therefore, not satisfactory in view of recent tests (Refs. 26 and 27) which show the importance of such factors and disprove the author's assumptions about temperature distribution.

The case of a dryer operating on low inlet humidity is easier to discuss because thermal effects are of less importance. This may be a bypass stabilizing dryer or dryer situated downstream of some kind of predrying device (compressor plus cooler, for instance).

In this case, thermal effects can be neglected and the only important parameter will be the nondimensional layer depth x' (Eqs. (87) and (89)) which appears in the theoretical relation between humidities and water concentrations given in Eq. (78) and in the explicit Eq. (90), which is valid for isothermal operation.

If a linear isotherm is assumed for the desiccant, the characteristic relation Eq. (89) becomes

$$C = h_E \, G(\chi', h_f / h_E) \tag{93}$$

G is known to be an increasing function of x' and the ratio h_f/h_E, but tests are required to give more detailed information about this function.

Knowledge of the maximum useful water concentration is required for the analysis of specific dryer operating conditions which are discussed below.

According to tests, such concentrations vary considerably. Depending on operating conditions, they range from a few percent to more than 10 percent, which corresponds to the outlet humidities of about 0.2×10^{-3} which are usually required. A predesign value of about 10 percent useful concentration is suggested for light duty plants with contact times of the order of one second.

Other possibilities of improving the efficiency of drying plants should not be overlooked, such as precooling of the air, desiccant cooling, use of a predryer (a refrigerating type, for instance) and lowering the inlet moisture to a value with which sufficiently greater concentrations can be reached in the main dryer. At present almost no performance data are available for these methods of increasing drying efficiency. Ref. 26, however, indicates that tests along these lines are contemplated in England.

(10) Charging Dryer Operation

The requirements for a charging dryer using solid granular adsorbent specify the total air mass m_t one needs to dry between successive reactivation periods. The amount of desiccant, m', needed can be computed from the relation

$$m_t (h_a - h_m) = m' C (h, h_a \ldots) \tag{94}$$

where h_a is the highest atmospheric humidity encountered in the establishment, h_m is the maximum humidity tolerated in the tunnel (generally about 0.2×10^{-3}), m_t is the mass of air to be dried, and $C(h, h_a \ldots)$ the concentration it can reach, which is an intricate function of h, h_a, contact time, etc.

The time t_T required to dry a volume v of air is given by

$$t_T = v/(SL/t_c) = (v/v')t_c \tag{95}$$

where

S = dryer cross section

L = dryer depth

t_c = contact time

v' = volume of desiccant used.

It must be remembered that t_c should be of the order of one second, so that the charging time is easily obtained.

(11) Bypass Dryer Operation

A bypass stabilizing dryer is used in a return circuit tunnel with air leakage of F_l gm air/sec. In this case the dryer inlet humidity will be very low (about 0.2×10^{-3}). No test results are available in this low humidity range relating the outlet humidity with the (low) inlet humidity and the concentration.

The condition for stabilizing the tunnel humidity against leakage can be expressed as

$$F_l h_a = F_B (h_m - h_f) \tag{96}$$

where

h_a = atmospheric humidity

h_m = humidity wanted in the tunnel

h_f = outlet humidity after the dryer

F_B = mass airflow in the bypass.

A further relation between the amount of desiccant used and the mass airflow through the bypass is

$$F_B = \rho v'/t_c \tag{97}$$

where

ρ = air specific mass in the dryer

v' = volume of desiccant used

t_c = contact time.

Solving for the desiccant volume, one gets

$$v' = t_c (F_l/\rho)\, h_a/(h_m - h_f) \cdot \tag{98}$$

For specific conditions of $h_a = 10^2$, $h_m = 0.2 \times 10^{-3}$, $h_f \cong 0$, $t_c \cong 1$ sec., one gets

$$v' = 50 (F_l/\rho). \tag{99}$$

The time t_T between successive regenerations, is given, as before, by

$$F_\ell h_a t_T = m' C \tag{100}$$

where C is the concentration for which the outlet humidity h_f becomes too high to satisfy the condition of stable tunnel humidity.

Using Eqs. (96) and (97) this can be expressed as

$$t_T = t_c (\rho'/\rho) C/(h_m - h_f) \tag{101}$$

where ρ' is the desiccant specific mass. The order of magnitude of t_T is about 5 hours if C is taken to be about 5 percent. The amount of desiccant has to be computed either from contact time conditions or reactivation period conditions, depending on whether the reactivation period is less or greater than five hours.

(12) Bypass Dryer for Initial Drying

A bypass dryer can also be used for initial drying of the tunnel. In this case, the air is circulated in the bypass before the tunnel is started. The duration of such an operation is limited as shown by the following discussion.

The outlet humidity h_f must obviously be less than the required tunnel humidity h_m during the process, if a finite time is to be obtained.

Let h_t be the humidity in the tunnel enclosure at the time t. A differential equation for the decrease of humidity in the enclosure can be written as

$$dh_t = -F_B dt (h_t - h_f)/m_t \tag{102}$$

where F_B is the bypass air-mass flow and m_t the total mass of air enclosed in the tunnel.

Solution of this equation gives

$$h_t = h_a e^{-F_B t/m_t} + (F_B/m_t) \int_0^t h_f(t') e^{-(F_B/m_t)(t-t')} dt' \tag{103}$$

where h_a is the initial (atmospheric) humidity.

It is rather difficult to estimate the integral, because h_f is a function of both h_t and the instantaneous water concentration of the dryer. If h_f is assumed zero during the whole process, then

$$h_t = h_a e^{-F_B t/m_t} . \tag{104}$$

The process of initial drying will be completed when $h_t = h_m$. The duration of this process is

$$t = (m_t / F_B) \log_e (h_a / h_m) . \tag{105}$$

The term (m_t/F_B) can be expressed in more practical terms, namely, the total volume v_t of the tunnel enclosure, the volume v' of the desiccant used, and the contact time in the dryer, t_c:

$$m_t / F_B = t_c (v_t / v') \tag{106}$$

so that

$$t = t_c (v_t/v') \log_e (h_a/h_m) . \tag{107}$$

With the usual values for h_a and h_m, the duration is of the order of $4(v_t/v')$.

If the dryer is also intended to stabilize the tunnel humidity against air leakage, it must be designed in accordance with the equations in the preceding section. A comparison of the time required for initial drying and the time required to stabilize against leakage shows that the initial drying time is about one-tenth of the time required to fill the tunnel enclosure with an airflow equal to the leakage flow. For initial drying times less than this amount the amount of desiccant required must be calculated from

$$\upsilon' = \upsilon_t (t_c/t) \log_e (h_a/h_m) \tag{108}$$

where t is the desired duration of initial drying.

For duration of the order of one hour, the amount of desiccant, υ', needed is about one-thousandth of the tunnel enclosure volume υ_t:

Another design condition has to be taken into account; the water concentration of the desiccant shall not exceed a value giving an outlet humidity comparable to the inlet humidity during the process. This concentration is approximately $(m_t h_a)/m'$ at the end of the process. Let C be the maximum useful concentration, then

$$\upsilon' / \upsilon_t > (\rho / \rho') (h_a/C) . \tag{109}$$

Using Eqs. (34) and (33) the following inequality for the duration results

$$t < t_c (\rho'/\rho)(C/h_a) \log_e (h_a/h_m) . \tag{110}$$

For C = 5 percent the right hand side is approximately five hours. This duration corresponds to the smallest possible dryer for this purpose.

Finally, it should be noted that the duration required for initial drying by a bypass plant is $\ln(h_a/h_m)$ times, or about four times, the drying time of a charging dryer with the same amount of desiccant.

(13) Design of a Full Flow Dryer

In the case of a charging or full flow dryer, the maximum allowable pressure loss through the dryer must be taken into account.

From Eq. (92) the pressure loss, in terms of the dryer geometry, is equal to

$$\Delta p = f \rho L^3/t_c^2 \tag{111}$$

and the mass flow of air to be dried, F , is

$$F = \rho \upsilon'/t_c . \tag{112}$$

The time, t_T, between successive reactivations can be expressed in terms of the maximum useful concentration of the desiccant from the equation

$$t_T F h_a = C \upsilon' \rho' \tag{113}$$

where C is a function of both the atmospheric humidity h_a and the desired tunnel humidity h_m.

Elimination of υ' and t_c from the above relations leads to the following expression for the pressure loss

$$\Delta p = (f \rho L^3/t_T^2)(C/h_a)^2 (\rho'/\rho)^2 . \tag{114}$$

With $f = 10^{-3}$, $C = 5\%$, and the usual values for h_a, we get

$$\Delta p \cong 10^4 (L^3/t_T^2) \text{ mm } H_2O \, . \tag{115}$$

A relation between contact time and total drying time is given by Eqs. (112) and (113) and is

$$t_T = (\rho'/\rho)(C/h_a)\, t_c \, . \tag{116}$$

The ratio of t_T/t_c is therefore about 3×10^3 using the usual values for the other parameters. Thus, for a contact time of one second, the drying time is about one hour.

The important dryer parameters can also be related to the nozzle throat size of a supersonic tunnel. Since sonic conditions exist at the throat, the mass flow through the tunnel can be given as

$$F = \bar{\rho} \bar{S} \bar{a} = \rho \bar{S} a (\bar{p}/p)^{(\gamma+1)/(2\gamma)} . \tag{117}$$

The quantities at the throat are indicated by bars, a is the speed of sound, the critical pressure ratio $\bar{p}/p$ is 0.528 for air, and $\gamma = 1.4$.

The desiccant volume is then from Eqs. (112) and (117).

$$S'L = 0.58 \, \bar{S} \, a \, t_c \tag{118}$$

where S' is the cross-sectional area of the dryer bed.

Solving for L and substituting into Eq. (114) gives the following relation for the pressure loss:

$$\Delta p = 0.8 \cdot 10^{13} f \rho t_c (\bar{S}/S')^3 \tag{119}$$

in mm H_2O. The standard atmospheric value of $a = 3.4 \times 10^4$ cm/sec has been used in the last expression.

If $f\rho$ is taken to be 10^{-3} and $t_c = 1$ sec, the pressure loss is approximately

$$\Delta p = 0.8 \cdot 10^{10} (\bar{S}/S')^3 \text{ mm } H_2O \cdot \tag{120}$$

A pressure loss of the order of 0.01 atm, for instance, will require, therefore, a dryer cross section of around four hundred times the throat cross section.

The depth of the dryer bed in cm is, from Eq. (120),

$$L = 2 \cdot 10^4 (\bar{S}/S') \, t_c \tag{121}$$

or about 50 cm for the case considered above.

Very large dryers are thus required to meet this type of specification. The cross-sectional dimensions are generally much greater than the depth, and the desiccant is therefore placed either in a horizontal layer covering a large surface or in some kind of vertical honeycomb-like structure.

(14) Reactivation of Desiccants

The principle of the reactivation process lies in the reversibility of the adsorption phenomenon. The equilibrium curves (Figs. II-6 and II-7) show that heating the desiccants results in a decrease of the water concentration for given ambient humidity.

A temperature rise of about 150°C will reduce this concentration to a residual 5 or 6 percent, which is negligible for subsequent adsorption cycles. This residual concentration is always disregarded when dynamic operations are considered. The useful concentration is simply computed from the amount of moisture adsorbed from the air during the drying operation.

Since it would be impractical to remove the desiccant from the dryer, the reactivation is generally accomplished by circulating heated air through the dryer itself.

Progress of the reactivation process can be observed either by so-called indicator desiccants, or by measuring the outlet temperature or humidity of the air. An indicator is an adsorbent the color of which changes with its water concentration. To be useful, its reactivation characteristics must be similar to those of the desiccant used ("Indicator Silica Gel" in the case of silica gel, for instance). Measurement of temperature of the air downstream from the dryer gives reliable information because this temperature remains constant during the reactivation period, after which it increases to a value near the inlet temperature. The process can then be considered completed. The outlet humidity is subject to a similar variation, but its value becomes very small near the end of the reactivation cycle and is therefore difficult to measure.

Drying plants with completely automatic reactivation can be designed by letting the outlet temperature control the circuit vanes and the heater units through servos.

(15) Miscellaneous Problems of Adsorbent Dryers

a. Deterioration by Oil Vapors

It is well-known that the presence of oil vapor or droplets has a deleterious effect on solid granular adsorbents. The oil forms a permanent deposit which deprives the desiccant of all its adsorbing capacities.

In wind tunnel installations where the drying has to be performed downstream of compressors, precautions must be taken to prevent the oil vapors from contaminating the dryer.

This requires installation of a suitable oil adsorbing device such as fiber glass filters, a layer of sacrificed desiccant, or special oil vapor adsorber (such as Vap-Oil-Sorb, etc.).

Blow-down type intermittent wind tunnels often have a complementary dryer installed downstream of both compressors and aftercoolers. In this case, the situation is somewhat better because the water condensing in the aftercoolers tends to collect the oil and other impurities. Ferri and Bogdonoff (Ref. 33) estimate that in this case a layer of fiber glass between settling tank and dryer will sufficiently protect the latter.

Oil-less compressors with carbon piston rings have been suggested in cases where the oil separation is critical. This solution, however, is very expensive indeed.

b. Dust from the Desiccant

Solid granular adsorbents continually produce fine dust, which must be kept out of the tunnel circuit. Fiber glass filters are currently used for this purpose.

Nearly all desiccants seem to produce about the same amount of dust, but the amount increases with the air velocity in the dryer and with the number of strong pressure fluctuations to which the dryer is subjected (variable density wind tunnels).

c. Maturing

This term is used to describe the fact that the first eight to twelve drying cycles, between successive reactivations, appear to be better in performance than the subsequent cycles. This was observed by Bateman (Ref. 26) for activated alumina. Hence, a dryer should be judged by its constant matured performance, which is less than its initial performance.

III. HYGROMETRY

(a) Introduction

The many different devices used to measure humidity fall into two classifications, those measuring relative humidity and those intended to measure absolute humidity. The relative humidity is indicated as the percentage ratio of water vapor pressure (or water vapor density) to the saturation value for the actual air temperature. It is significant in subsonic and transonic wind tunnels where condensation can be avoided altogether by the use of stagnation relative humidities below a limit depending on the Mach number.

At higher Mach numbers, it is not always practical to lower the relative humidity sufficiently to prevent condensation (Ref. 14); consequently, in supersonic wind tunnels the absolute humidity, which controls the strength of the condensation shock, is of chief interest. It will be seen, in the subsequent analysis, that few of the devices developed to date for measuring relative humidities have sufficient sensitivity to measure the small humidities required in supersonic wind tunnels.

(b) Hygrometers Giving Relative Humidity

The most widely used hygrometers of this type are dry and wet bulb psychrometer (Assmann psychrometer); mechanical hygrometer (hair hygrometer, gold beater's skin hygrometer); electrical conduction hygrometer (Gregory humidiometer); and the porous diffusion hygrometer (TESA type). It will be observed that they all give direct readings, which do not involve complex operations, a considerable practical advantage.

Each of these devices will now be considered and the possibility of its use in supersonic wind tunnel humidity measurement discussed.

(1) Dry and Wet Bulb Psychrometer

This instrument consists of two thermometers, one of which has its bulb in contact with a wet substance. Its principle of operation can be approximately explained by considering two effects; first, diffusion of moisture from the wet bulb into the atmosphere; and second, heating of the wet bulb through convection.

The first of these effects is described by the formula

$$F_W = \beta (C_S - C) \tag{122}$$

where

F_W = the rate of evaporation of water, in gm/sec

C = moisture concentration in the air tested, gm H_2O/gm air

C_S = moisture concentration corresponding to saturation at the temperature indicated by the wet bulb thermometer, gm H_2O/gm air

β = constant of the apparatus.

The heat transmission effects are described approximately by the equation:

$$Q \cong \delta (T_d - T_w) \tag{123}$$

Q being the heat flow towards the wet bulb thermometer in cal/sec; and T_d and T_w the temperatures read respectively on the dry and wet bulb thermometers.

Steady conditions imply that the heat of evaporation is compensated by the heat flow, so that the following equation can be written

$$C_s - C \cong (\delta / H\beta)(T_d - T_w) \tag{124}$$

H being the heat of evaporation for water.

The reading is not a direct function of the relative humidity, but it does involve saturation conditions (at wet bulb temperature), and this instrument is therefore classified with the relative humidity meters.

The limitations of this device for measuring small humidities can be deduced from Eq. (124). As the value of the humidity becomes small, it is obtained as the difference of two quantities of much larger magnitude, C_s and $(\delta/H\beta)(T_d - T_w)$, both of which depend on the measured temperatures. Only extremely accurate measurement of temperatures can make this device useful for supersonic wind tunnel work. No attempts have been made in this direction, to our knowledge.

(2) Mechanical Hygrometers

Hair hygrometers or hygrometers using other organic elements have been used in meteorology. It seems probable, however, that their undesirable characteristics such as hysteresis, time lag, and change of calibration with time prevent their being of much use in the measurement of small humidities.

(3) Electrical Conduction Hygrometers

The conductivity of superficial layers of some substances (lithium chloride, for instance) has been found to be variable with the relative humidity. The lowest reading obtainable with most instruments of this type is of the order of 30 percent relative humidity. A new electrolytic hygrometer reported by Gregory and Rourke (Ref. 34), however, is said to measure humidities of less than 1 percent.

(4) Porous Diffusion Hygrometer

The apparatus consists of a vessel containing water and covered with a porous plug. If the atmospheric humidity is smaller than the saturation humidity of the water in the vessel, a pressure difference is observed between the vessel and the atmosphere (see Fig. III-1). As in the case of the dry and wet bulb psychrometer, the vapor pressure is given as the difference of two large numbers, for small values of the humidity. Therefore, sufficient accuracy for supersonic wind tunnel use cannot be expected with this type of instrument unless the temperature and pressure measurements are made with extreme precision.

(5) Compressors

Most of the hygrometers described thus far were shown to be unsuitable for measuring low humidities. It has been suggested to use them in conjunction with a compressor which would raise the relative humidity at a constant (low) absolute humidity. The problem of building a leakproof compressor which does not contaminate the air with oil vapor does not seem to have received a solution so far, but this seems to be an interesting field for research.

(c) Absolute Humidity Measurement

The three most widely used methods for determining absolute humidity directly are dewpoint meters, gravimetric (or gas analysis) methods, and the sulphuric acid hygrometer.

(1) Dewpoint Meters

These devices can be divided into two classes according to whether a cooled surface is used, or the whole sample of air is cooled. Most wind tunnel hygrometers are of the first type. The principle on which they are based is extremely simple. The idea is to cool a small metallic surface to a temperature below the dewpoint (or frostpoint) of the moist air being sampled, and then to let the temperature rise slowly through heat losses while watching for the disappearance of the dew or frost layer on the surface. The temperature at which this occurs is then read.

Some details of a typical dewpoint meter are shown in Fig. III-2. The dew or frost deposit is produced on a metal thimble which is cooled either by liquid air or expanding carbon dioxide. The observation chamber, in which tunnel air is made to circulate, is fitted with a glass window, and a magnifier is often used. The temperature of the thimble is measured electrically either by a platinum resistance element or a thermocouple.

The ingenious method of illuminating the observation chamber shown in Fig. III-2 was reported by Brewer, Cwilong, and Dobson in Ref. 35. It consists of a transpex ellipse at one focus of which is placed an illuminating lamp. The other focus is situated in the air chamber; the light concentrated there illuminates the thimble.

The dewpoint meter can be converted into an automatic, continuous reading or recording instrument by the use of a photoelectric cell to pick up the light scattered by a dew or frost deposit and to act on a temperature controlling device, so as to follow the dewpoint. With this arrangement the thimble is usually cooled continuously, and its temperature controlled by an electric resistance or induction heater which is regulated by the amplified photoelectric cell signal.

An instrument of this type described in Ref. 36 measures frostpoints as low as -85°C. The response times at these low frostpoints are about 10 to 20 seconds. This particular instrument uses two photo cells, one to pick up the decrease in specular reflection as frost forms and the other to pick up the increase in scattered light due to frost. (Humidities as low as to correspond to frostpoints of -80°C are generally not required in supersonic wind tunnels, even if operating at low pressures.)

Certain effects, which are avoided in the automatic instruments just mentioned, cause inaccuracies or tend to make measurements difficult with the manually operated dewpoint meter. First of all, it can be very difficult to decide whether the deposit observed is dew or frost. This is an important point, because the ratio of vapor pressure over ice to the vapor pressure over water decreases from unity, at 0°C, to about 50 percent at -70°C. It is accordingly seen that this uncertainty regarding the deposit state can cause considerable inaccuracies in the humidity measurement.

Dew and frost deposits are easily distinguishable in the higher temperature range, from 0° down to about -30° C. In this range, the ice forms in large white crystals, while dew appears as small droplets, uniformly covering the surface. Around -30° C to -40° C, however, frost deposits in the form of microcrystals, which look very much like the corresponding dew deposit of very fine droplets.

The nature of the surface on which the deposit is to form plays a fundamental role in determining its state. It has been observed that dew can appear at temperatures as low as -40° C or in some instances, at -100° C on exceedingly clean, polished metal surfaces. On the other hand, ice can be formed at temperatures well above -40° C if the surface is not very clean.

A second difficulty lies in the fact that, in the case of a frost deposit, the condensation on the surface does not start when the temperature is lowered to the frostpoint (which is higher than the dewpoint, because of the lower vapor pressure on ice). In fact, the deposit appears only when the temperature is lowered nearly to the dewpoint. This is a consequence of the Oswald rule alluded to in the section dealing with condensation shocks, and according to which initial condensation always occurs in the form of supercooled liquid. A good indication of the frostpoint, therefore, can be gained only if the temperature for which the deposit neither grows nor evaporates is found.

The above-mentioned difficulties are not encountered in the case of the automatic frostpoint instruments, since these maintain a constant thickness frost deposit (Ref. 37). Any dew that may be formed initially must change to frost after a certain length of time, because the supercooled liquid is unstable. The main limitation to the sensitivity of the automatic instruments is the fact that, below about -85° C the frost becomes an invisible, glassy film.

(2) Gravimetric Method

This method consists in weighing the moisture trapped in an absorbent or adsorbent of humidity contained in a vessel through which a known amount of the air to be tested has been passed. P_2O_5 on pumice stone of $CaCO_3$ can be used as absorbent, and placed in a U-shaped tube. The tube must be fitted at both ends with glass wool to trap any solid matter, otherwise the weighing would not accurately represent the amount of moisture deposited in the tube. A flowmeter and a sensitive analytic balance must be used.

This measurement of air humidities has a great drawback: each operation is long and requires skill. The result will always yield the mean humidity over a long period. The use of this method in wind tunnels seems, therefore, to be very limited.

(3) Sulphuric Acid Hygrometer

This very interesting continuous reading hygrometer has been developed by Heyser, from an idea of Ackermann, of the Danzig Polytechnicum. It operates on a principle which is somewhat analogous to that of the dry and wet bulb psychrometer. An equilibrium is established between the heat liberated by the absorption of moisture by H_2SO_4 and the heat carried away by convection. The equilibrium temperature difference is read as in the case of the dry and wet bulb psychrometer.

Fig. III-3 gives a schematic view of the apparatus. The pressure and the temperature of the air to be analyzed are measured by a manometer, B, and a thermometer, C. The airflow is controlled by a valve, A, the flow being adjusted to a standard value for which the instrument was calibrated. A venturi, D, coupled to a differential manometer, E, indicates the flow velocity.

The air is led into the vertical tube, F, which is coaxial with an inner tube, G. Sulphuric acid flows from the reservoir, H, through tube G. The H_2SO_4 flow, controlled by a valve, I, must be equal to the calibration value. It is measured by counting the drops falling into the lower reservoir, J.

At the lower part of tube, G, the acid leaks to a central glass tube, K, on which it forms a film, the uniformity of which is controlled by a glass ring, L. A bypass, 0, fitted with a P_2O_5 dryer, is used to equalize pressures in the reservoirs, J and H, so that the acid flow can be kept constant.

Humid air and the acid are brought into contact at the lower end of the central tube, K, where the absorption takes place. This being a most exothermic reaction, a local heating is produced. As the acid flows from the glass ring downward along the tube, its temperature rises continuously. The temperature near the lower end of the tube is measured by the thermocouple, M, and compared with the temperature of the acid before absorption of water, measured by thermocouple, N.

It will be seen, from the theoretical derivation below, that the indications of this meter are remarkably independent of most parameters difficult to measure, such as air and acid flows, air pressure and temperature.

It can be shown, by considering the thermal equilibrium involving the heating by absorption of the sulphuric acid and its cooling by the airflow, that the temperature difference indicated by the thermocouples will be

$$\Delta T = \left[(H_c + H_a)/c_p\right](Pr/Sch)(Nu_d/Nu)\left[1 - e^{-(KNu/c^*F^*)l}\right]_h \tag{125}$$

where

H_c = heat of condensation of water at room temperature, in cal/gm

H_a = heat of absorption of water by H_2SO_4, in cal/gm

c_p = air specific heat at constant pressure, in cal/gm

Pr = Prandtl number

Sch = Schmidt number

K = thermal conduction coefficient for air, in cal/sec, cm and °C

Nu, Nu_d = Nusselt numbers for thermal conduction and diffusion

F^* = mass flow of acid in gm/sec

c^* = specific heat of acid in cal/gm.

In the case considered, Nu is proportional to $(Pr)^{.4}$, so that

$$(Nu/Nu_d) = (Pr/Sch)^{.4} = 1.16 .$$

We thus have:

$$\Delta T = 1.25\left[(H_c + H_a)/c_p\right]\left[1 - e^{-(KNu/c^*F^*)l}\right]_h . \tag{126}$$

The various constructional parameters will now be chosen so as to make the exponential appearing in Eq. (126) small compared with unity, so that the temperature difference measured will be nearly independent of the flows of air and acid.

According to tests by Heyser, one can assume

$$Nu = .08(Re)^{.8}\left[d_1^2/(d_2^2 - d_1^2)\right] \tag{127}$$

where

d_1 = diameter of internal tube

d_2 = diameter of external tube

$Re = (V/\nu)(d_2^2 - d_1^2)/d_1$ provided, $5000 < Re < 30{,}000$.

The hygrometers manufactured at the French armament aeroballistics research center at Vernon, according to this principle, have the following dimensions:

d_1 = 0.5 mm.

d_2 = 9 mm.

ℓ = 20 mm.

The air velocity in the tube is about 7 m/sec, and the acid flow is 1.85 gm/hr.

This is seen to correspond to a Nusselt number of about 15, so that the neglected exponential is of the order of 0.055.

The hygrometer can be calibrated by means of a hygrostat. The Vernon hygrostat uses a mixture of known quantities of air of constant humidities. A part of the mixture is completely dried by flowing through sulphuric acid, the other being saturated at a given temperature because it went through water kept at constant temperature. The proportions of the mixture are measured by flowmeters.

We believe this hygrometer to be very well-suited to wind tunnel humidity control, because of its reliability and the simplicity with which it can be made to record the tunnel humidity continuously. It is inexpensive to build and the calibration does not seem to present special difficulties.

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APPENDIX

Derivation of Eq. (9) of Section I, Assuming Steady Flow Conditions

The complete shock equations with heat addition are

$$\rho_1 V_{1n} = \rho_2 V_{2n} \quad \text{(Continuity)}$$

$$\rho_1 V_{1n}^2 + p_1 = \rho_2 V_{2n}^2 + p_2 \quad \text{(Momentum)}$$

$$V_{1t} = V_{2t}$$

$$\left[\gamma/(\gamma-1)\right](p_1/\rho_1) + (V_{1n}^2/2) + Q = \left[\gamma/(\gamma-1)\right](p_2/\rho_2) + (V_{2n}^2/2) \quad \text{(Energy)}$$

Introducing the velocity of sound $\sqrt{\gamma p/\rho}$ and the normal Mach number M_n,

$$M_n = V_n/\sqrt{\gamma p/\rho}$$

we can write the normal momentum equation as follows, after dividing both members by $\rho_1 V_{1n} = \rho_2 V_{2n}$:

$$V_{1n} + (1/\gamma)(V_{1n}/M_{1n}^2) = V_{2n} + (1/\gamma)(V_{2n}/M_{2n}^2)$$

or,

$$V_{1n}\left(1 + \frac{1}{\gamma M_{1n}^2}\right) = V_{2n}\left(1 + \frac{1}{\gamma M_{2n}^2}\right) . \qquad \text{(A)}$$

The energy relation can be transformed, in analog fashion, into the following:

$$\left(\frac{1}{(\gamma-1)M_{1n}^2} + \frac{1}{2}\right) V_{1n}^2 + Q = \left(\frac{1}{(\gamma-1)M_{2n}^2} + \frac{1}{2}\right) V_{2n}^2 . \qquad \text{(B)}$$

The stagnation texperature can be introduced by recalling that

$$c_p T_o = \left(\frac{1}{(\gamma-1)M_1^2} + \frac{1}{2}\right) V_1^2$$

but,

$$V_{1n} = V_1 \sin C$$

so that Eq. (B) can be written

$$1 + (Q/c_pT_0)\left(\frac{1}{(\gamma-1)M_1^2} + \frac{1}{2}\right)\left(\frac{1}{(\gamma-1)M_{1n}^2} + \frac{1}{2}\right)^{-1}(\sin C)^{-2} =$$

$$(V_{2n}^2/V_{1n}^2)\left(\frac{1}{(\gamma-1)M_{2n}^2} + \frac{1}{2}\right)/\left(\frac{1}{(\gamma-1)M_{1n}^2} + \frac{1}{2}\right). \qquad (B')$$

Elimination of the normal velocity ratio V_{2n}/V_{1n} between Eqs. (A) and (B') yields

$$1 + (Q/c_pT_0)\left(\frac{1}{(\gamma-1)M_1^2} + \frac{1}{2}\right)\left(\frac{1}{(\gamma-1)M_{1n}^2} + \frac{1}{2}\right)^{-1}(\sin C)^{-2} =$$

$$\frac{\dfrac{1}{(\gamma-1)M_{2n}^2} + \dfrac{1}{2}}{\left(1 + \dfrac{1}{\gamma M_{2n}^2}\right)^2} : \frac{\dfrac{1}{(\gamma-1)M_{1n}^2} + \dfrac{1}{2}}{\left(1 + \dfrac{1}{\gamma M_{1n}^2}\right)^2} = \phi(M_{1n})/\phi(M_{2n}).$$

The function

$$\phi(M) = \left(1 + \frac{1}{\gamma M^2}\right)^2 / \left(\frac{1}{(\gamma-1)M^2} + \frac{1}{2}\right)$$

being introduced. This function decreases from infinity to a minimum of $2(\gamma^2 - 1)/\gamma^2 \cong 0.98$ when M increases from 0 to 1 (subsonic conditions), and increases then to 2 when M is greater than unity (supersonic conditions).

It is thus seen that there is, for each value of M_1, a definite upper limit Q_{max} for the heat added, given by:

$$1 + (Q_{max}/c_pT_0)\left(\frac{1}{(\gamma-1)M_1^2} + \frac{1}{2}\right)\left(\frac{1}{(\gamma-1)M_{1n}^2} + \frac{1}{2}\right)^{-1}(\sin C)^{-2} = \phi(M_{1n})/\left[2(\gamma^2-1)/\gamma^2\right].$$

This leads, after some simple algebraic manipulations, to the tormula

$$Q_{max}/(c_p T_0) = (1 - M_{1n}^2)(\sin C)^2/\left[2(\gamma^2-1)\left(\frac{1}{2} + \frac{1}{(\gamma-1)M_1^2}\right)\right] =$$

$$(M_1^2\sin^2 C - 1)^2/\left[2(\gamma+1)\left(\frac{\gamma-1}{2}M_1^2 + 1\right)M_1^2(\sin C)^2\right]$$

which is easily seen to be equivalent to Eq. (6) of Section I (where the index 1 has been replaced by c).

It is also seen that the maximum heat addition leads to sonic final conditions ($M_{2n} = 1$).

The pressure and velocity deviations through the shock are easily derived, in the case of maximal heat addition, from Eq. (A):

$$V_{1n}\left(1 + \frac{1}{\gamma M_{1n}^2}\right) = V_{2n}(\gamma+1)/\gamma$$

or

$$(V_{2n} - V_{1n})/V_{1n} = (\gamma+1)^{-1}(1 - M_{1n}^2)/M_{1n}^2$$

which is clearly equivalent to Eq. (8) of Section I.

The relative pressure disturbance results from the normal momentum equation and the continuity relation

$$p_2 - p_1 = \rho_1 V_{1n}^2 - \rho_2 V_{2n}^2 = \rho_1 V_{1n}(V_{1n} - V_{2n}) = p_1(\rho_1/p_1)V_{1n}(V_{1n} - V_{2n})$$

or

$$(p_2 - p_1)/p_1 = \gamma M_{1n}^2 (V_{1n} - V_{2n})/V_{1n}.$$

This formula is equivalent to Eq. (7), Section I, taking into account Eq. (8).

We are now especially interested in conditions near a point P where the condensation shock touches a characteristic of the unperturbed flow. This means that M_{1n} will be close to unity in the neighborhood of P. From the preceding calculations it is seen that the disturbances are not infinite nor even maximum at P. They are, on the contrary, zero.

But a maximum will occur in this neighborhood, for Eqs. (7) and (8) hold wherever $dQ > Q_{max}$ and show that the deviations then increase with $M_{1n} - 1$, while for $dQ < Q_{max}$ the general equations apply, and show a decrease for increasing $M_{1n} - 1$. The maximum disturbance will accordingly be found at points where $Q_{max} = dQ$.

dQ being small, M_{1n} is still close to unity at such points, and Eq. (6) can be approximated by:

$$Q_{max}/c_p T_0 = 2(M_1 \sin C - 1)^2 / \left[(\gamma+1)\left(\frac{\gamma-1}{2} M_1^2 + 1\right)\right]$$

from which we get, for $Q_{max} = dQ$:

$$M_1 \sin C - 1 = \sqrt{(\gamma+1)(\frac{\gamma-1}{2} M_1^2 + 1)(dQ/c_p T_0)/2}\,.$$

Eq. (8) gives for small values of $M_1 \sin C - 1$:

$$dV_n/(V_c \sin C) = -2(\gamma+1)^{-1}(M_1 \sin C - 1)$$

$$= -\sqrt{2/(\gamma+1)}\sqrt{\left(\frac{\gamma-1}{2} M_c^2 + 1\right) dQ/c_p T_0}$$

which is equivalent to Eq. (9); when recalling that $M_c^{-1} = \sin C$.

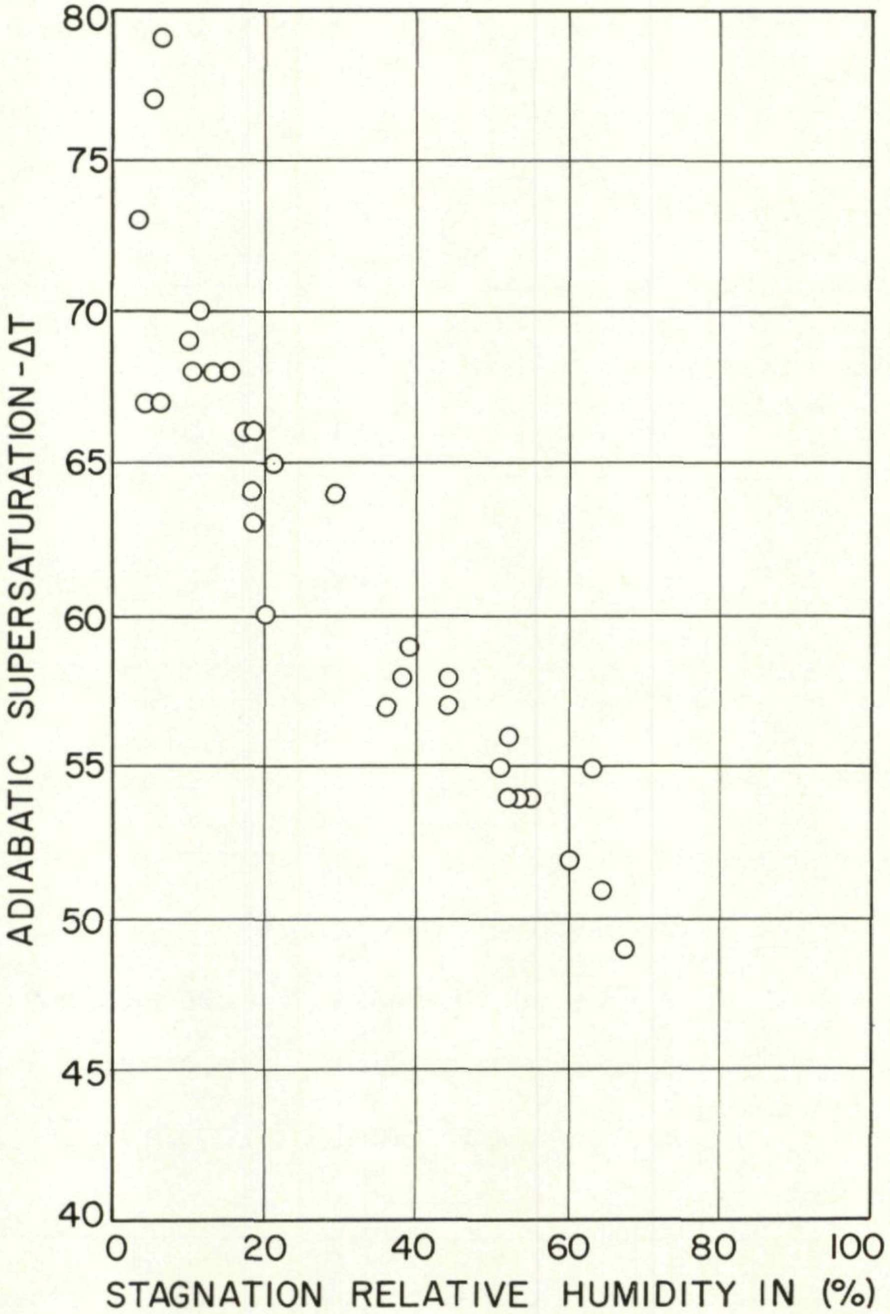

Fig. I-1. Supercooling as a function of stagnation relative humidity (Eber and Gruenewald, unpublished data-Peenemunde 40 x 40 cm tunnel, nozzle for M = 1.86).

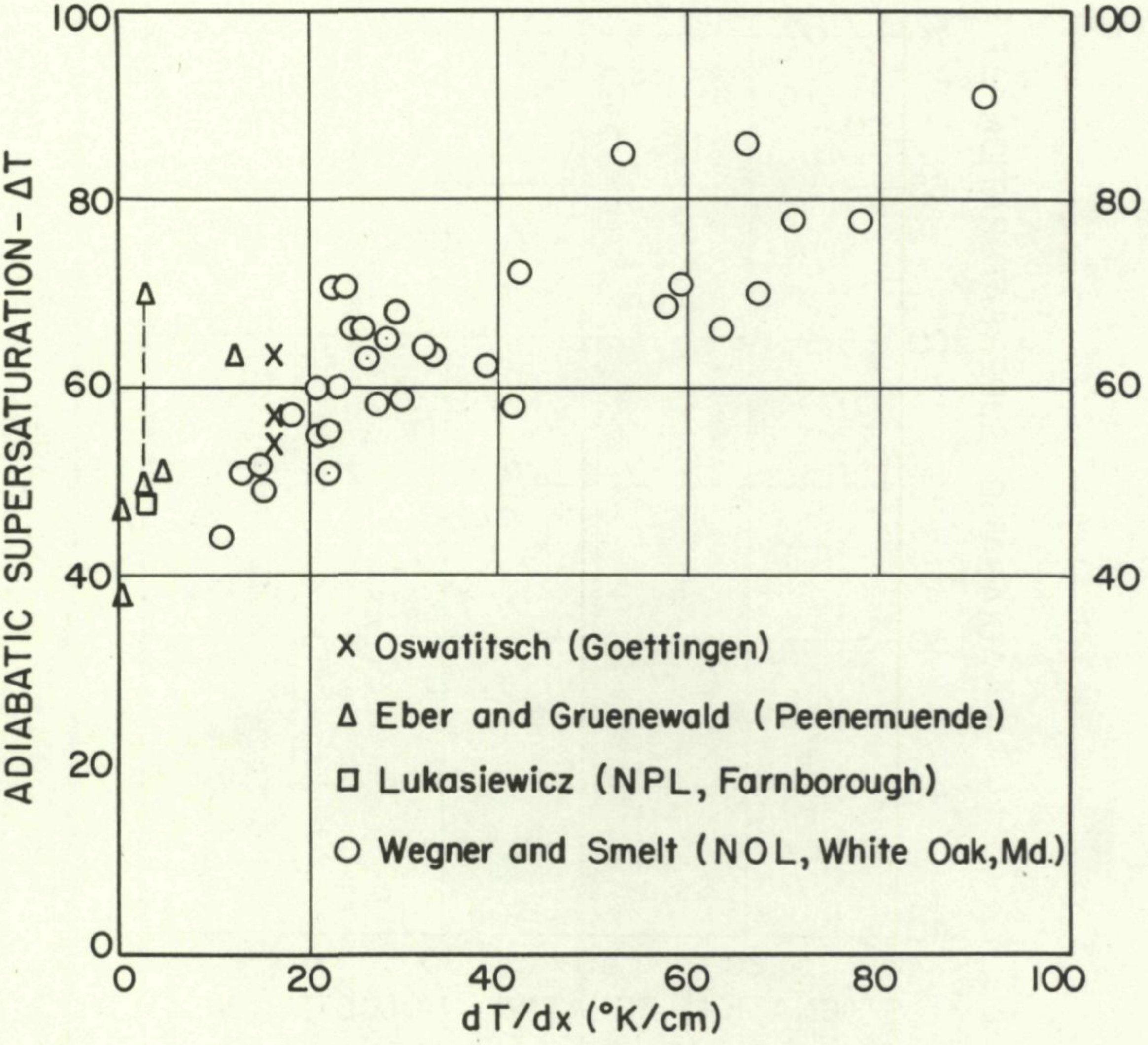

Fig. I-2. Supercooling as a function of nozzle temperature gradient (from Ref. 18).

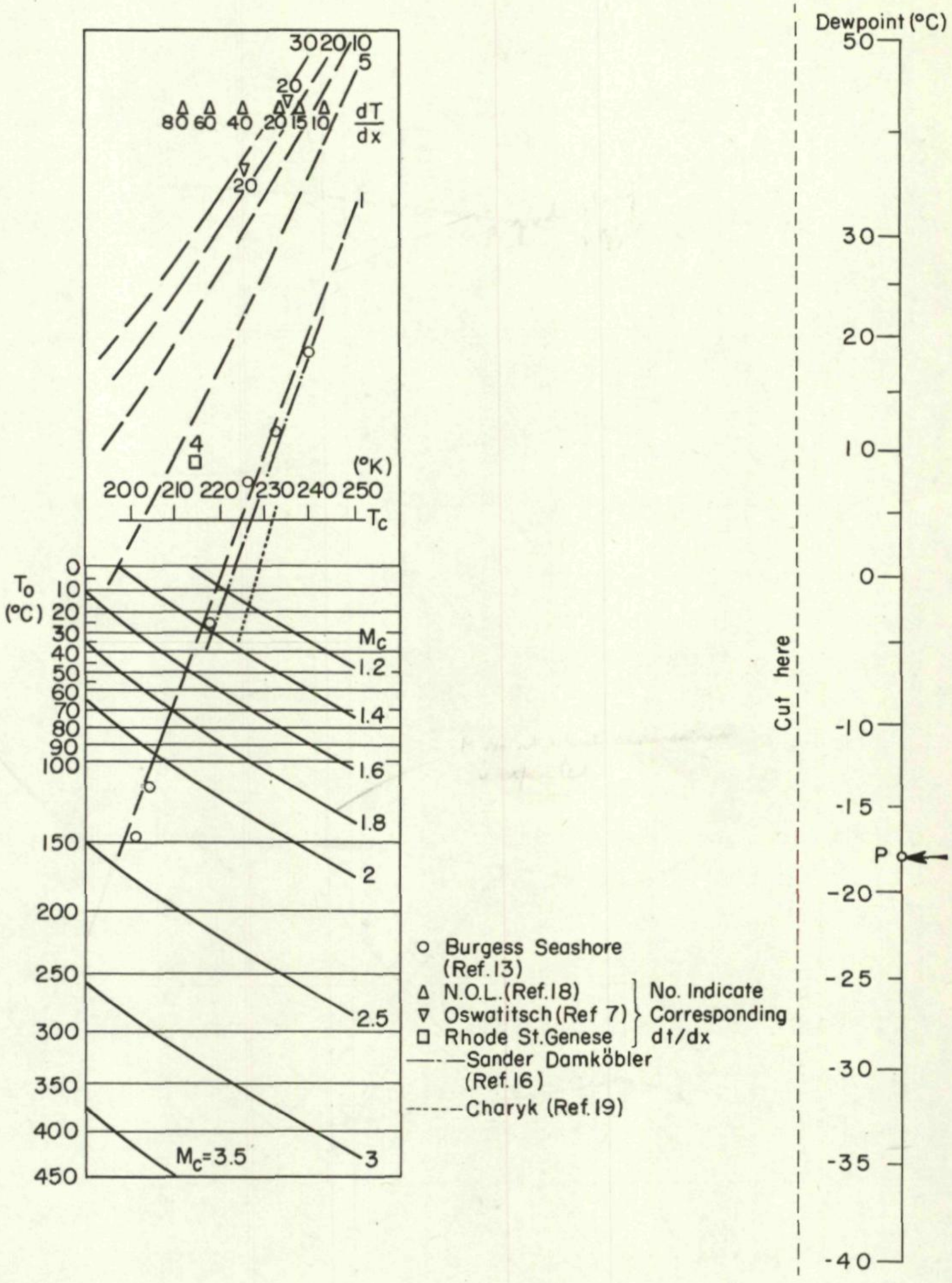

Fig. I-3. Criterion for condensation free flow.

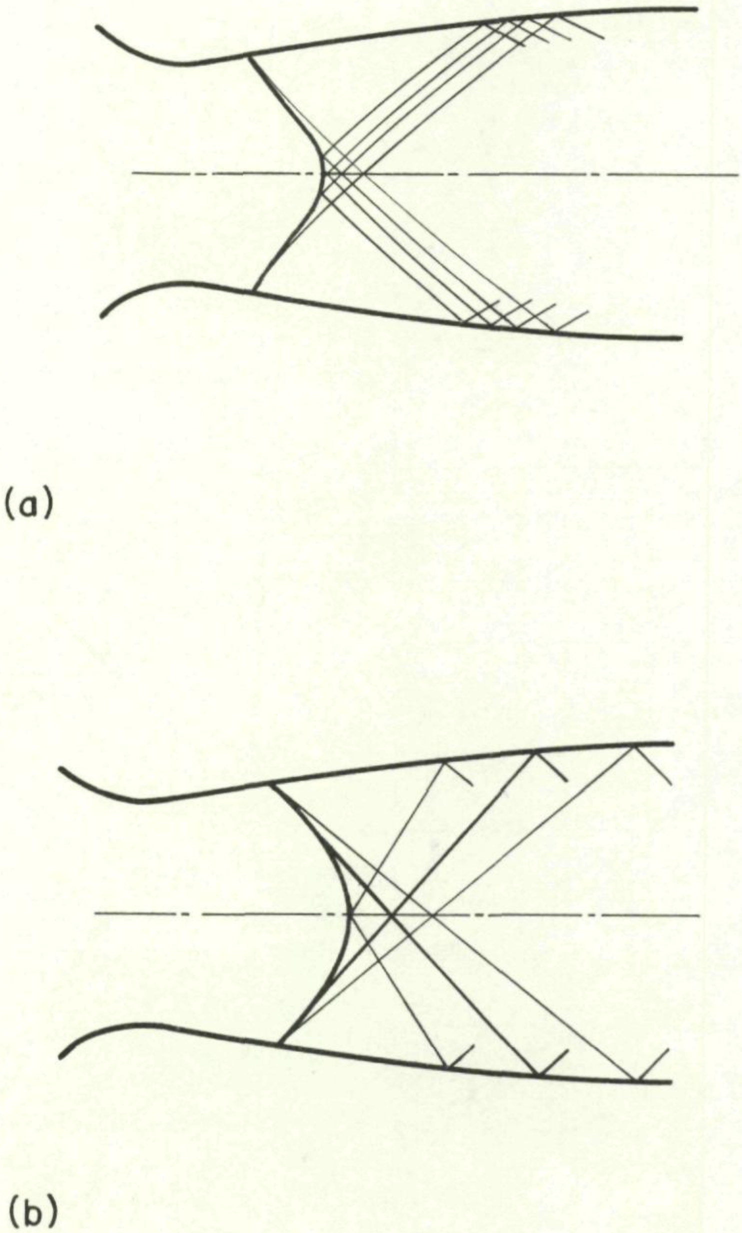

Fig. 1-4. Shock configurations.

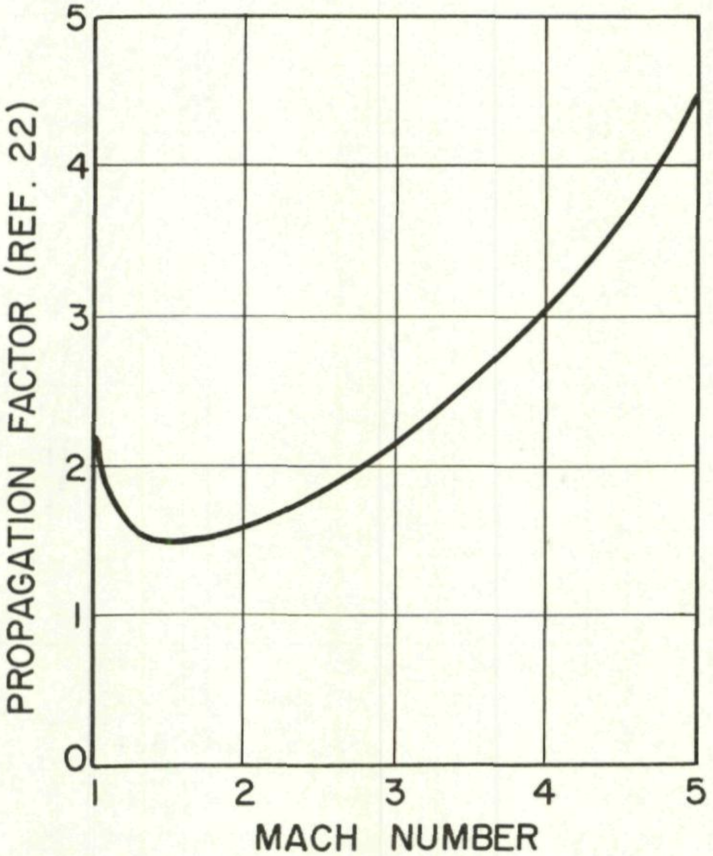

Fig. I-5. Propagation factor (from Ref. 22).

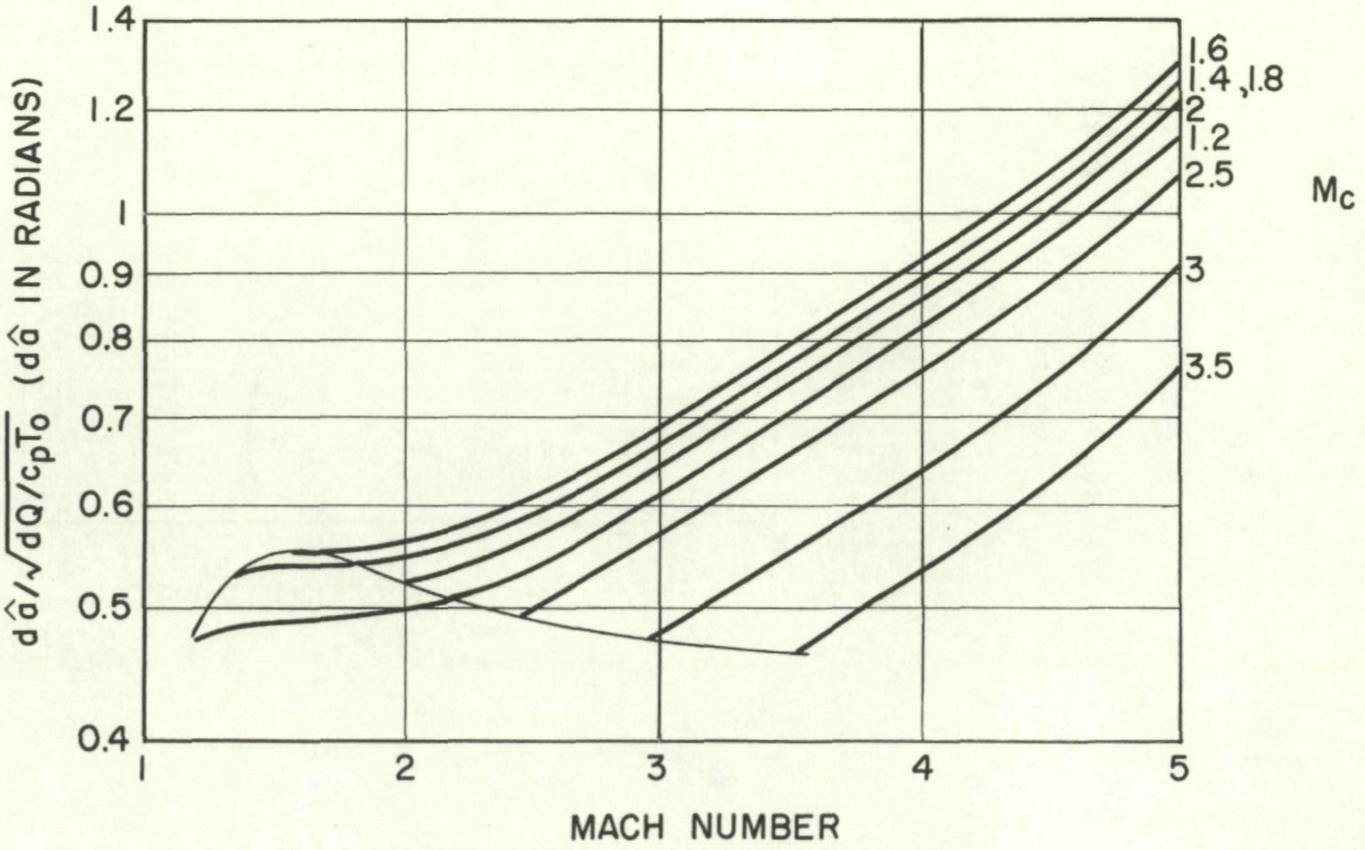

Fig. I-6. $d\hat{a}/\sqrt{dQ/c_pT_0}$ as a function of M and M_c.

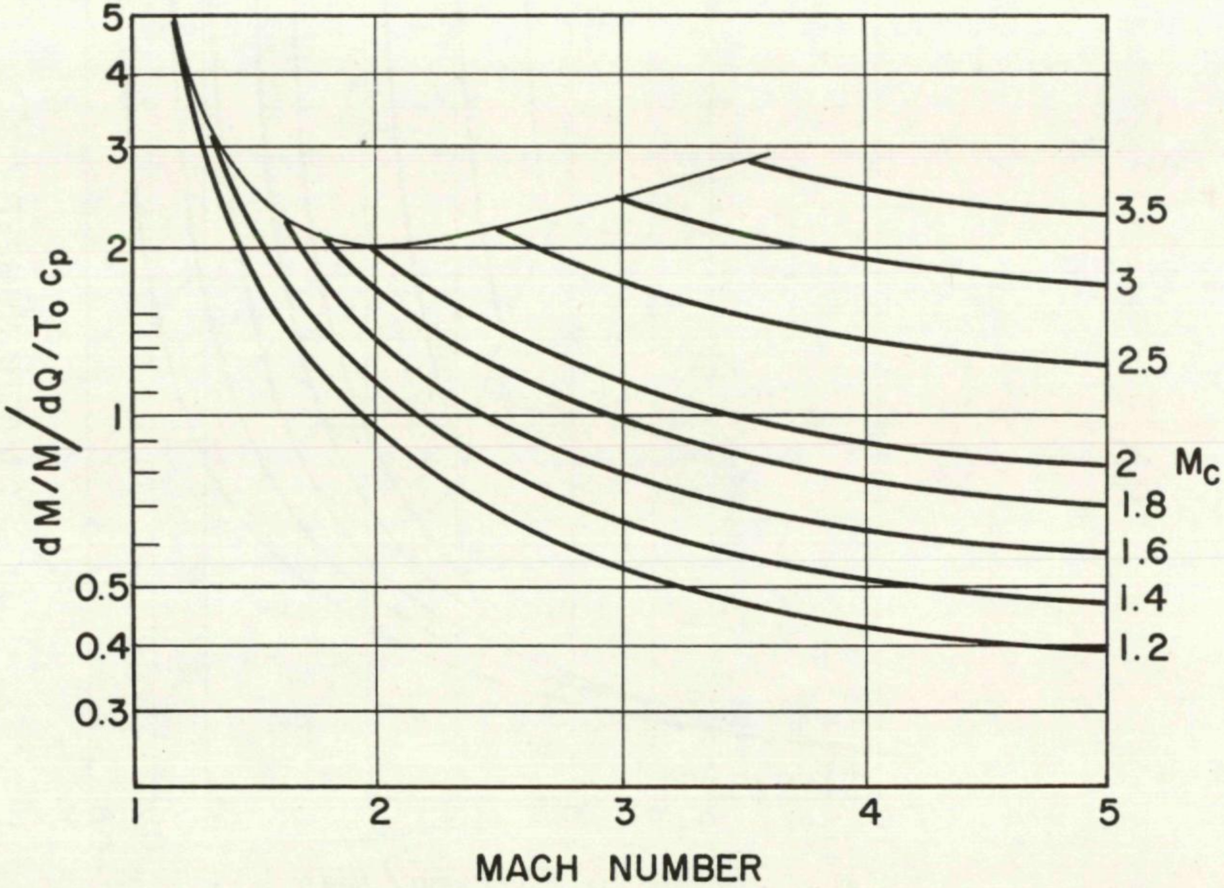

Fig. 1-7. $(dM/M)/(dQ/c_pT_o)$ as a function of M and M_c.

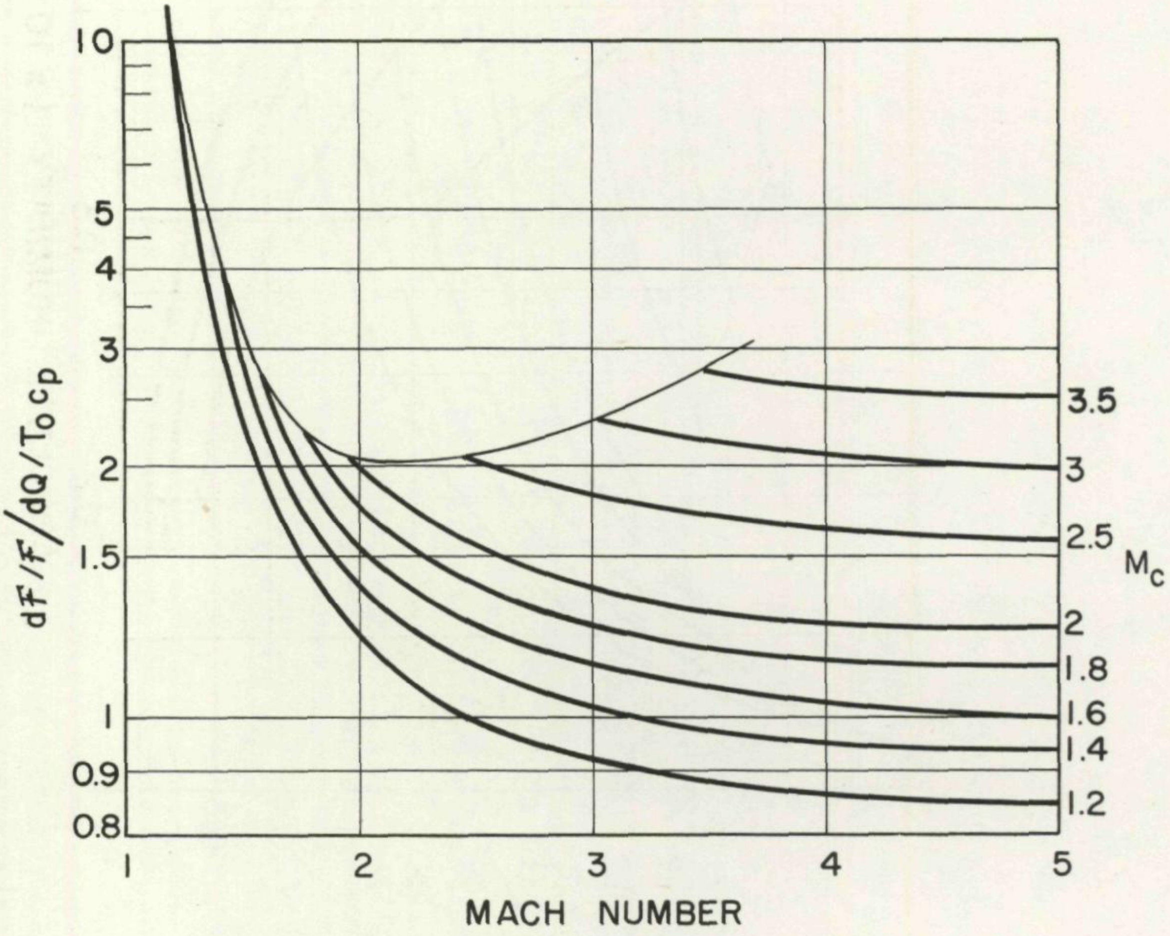

Fig. I-8. $(dF'/F')/(dQ/c_pT_o)$ as a function of M and M_c.

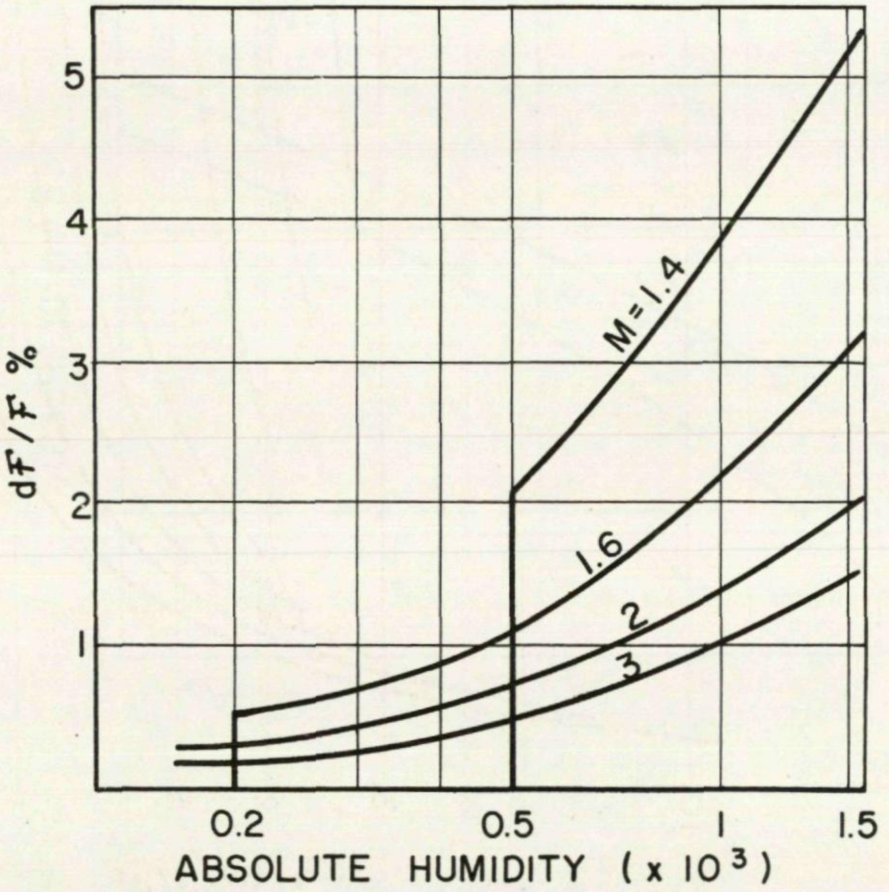

Fig. I-9. (d$\mathcal{F}$/$\mathcal{F}$) as a function of h, for various Mach numbers, and for T_0 = 20°C, dT/dx = 1°C/cm.

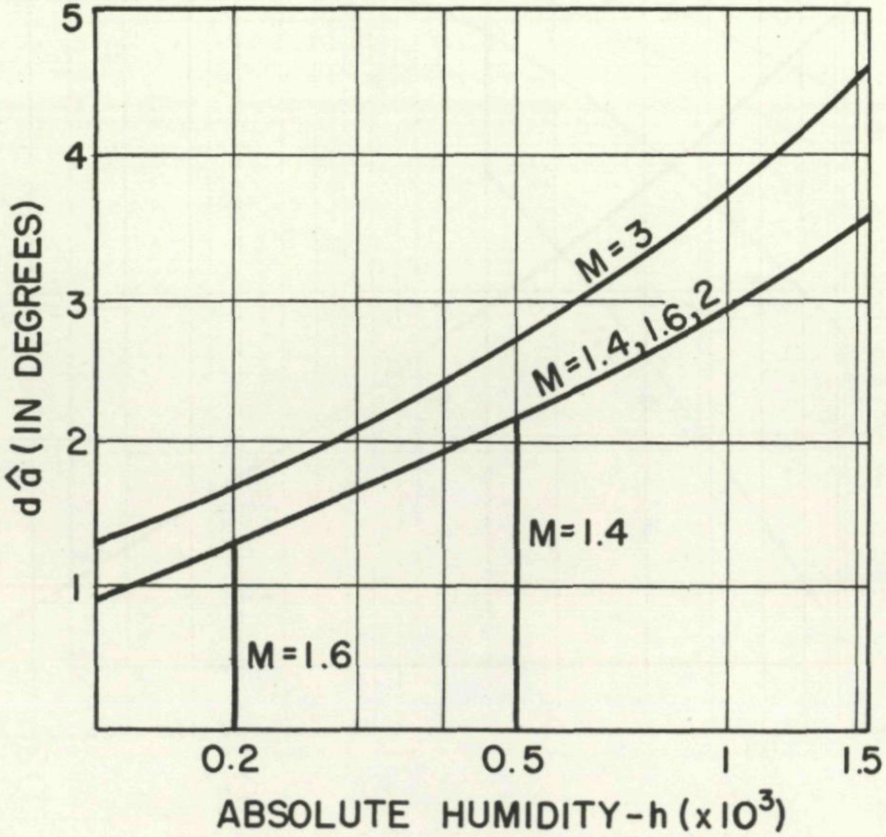

Fig. I-10. dâ (degrees) as a function of h, for various Mach numbers, and for T_0 = 20°C, dT/dx = 1°C/cm.

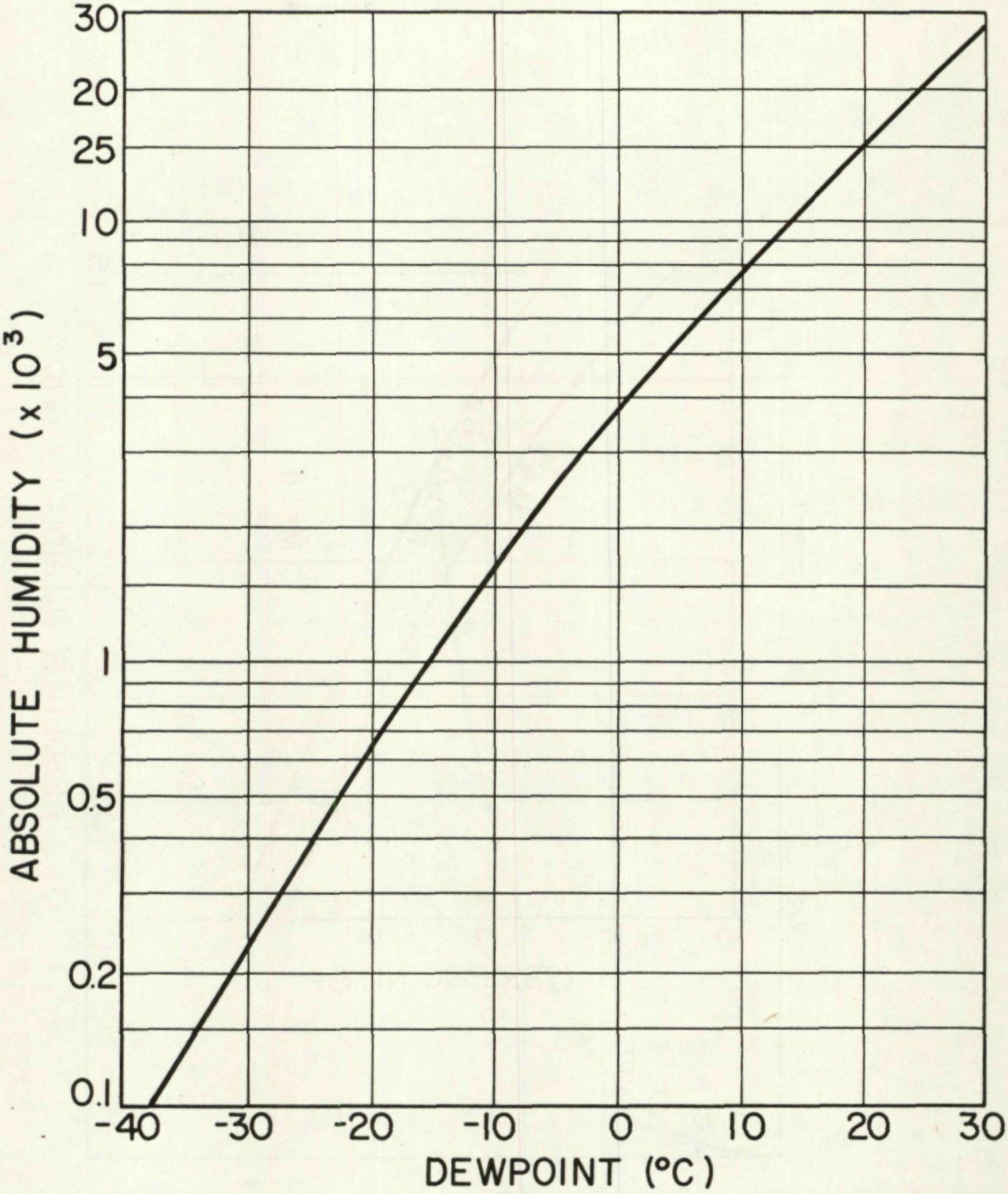

Fig. I-11. Absolute humidity as a function of dewpoint for 760 mm Hg. air pressure.

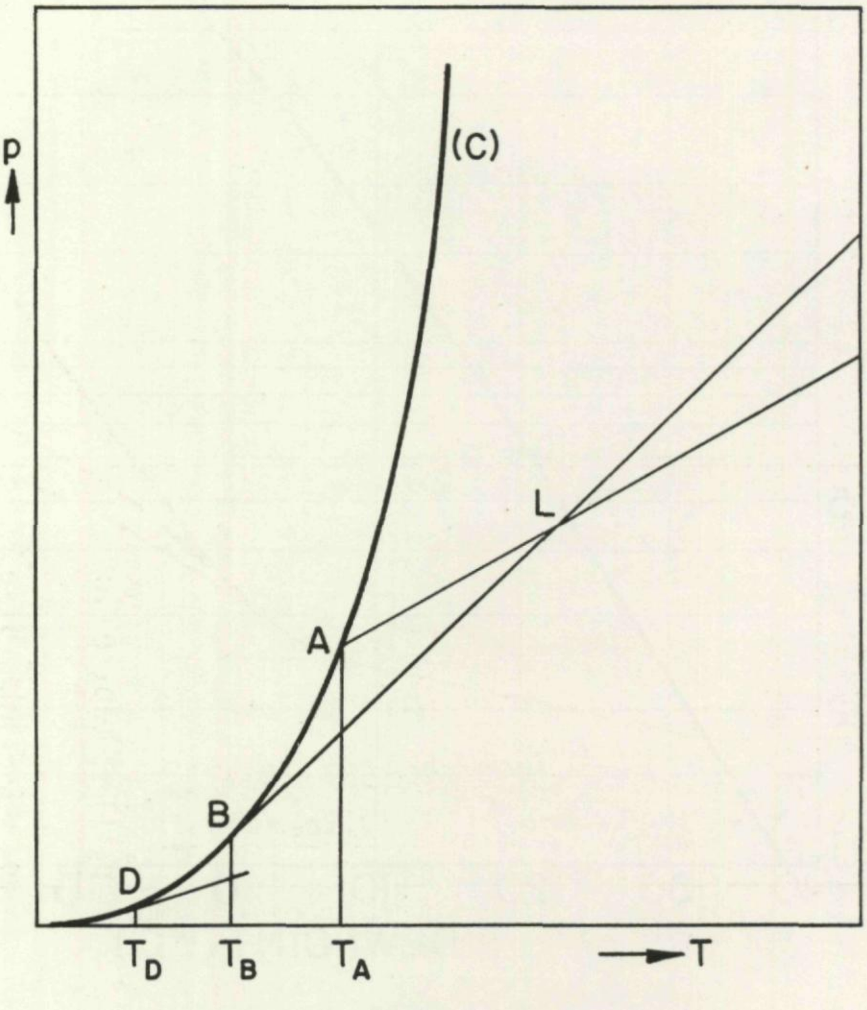

Fig. II-1. Vapor state in an exchanger.

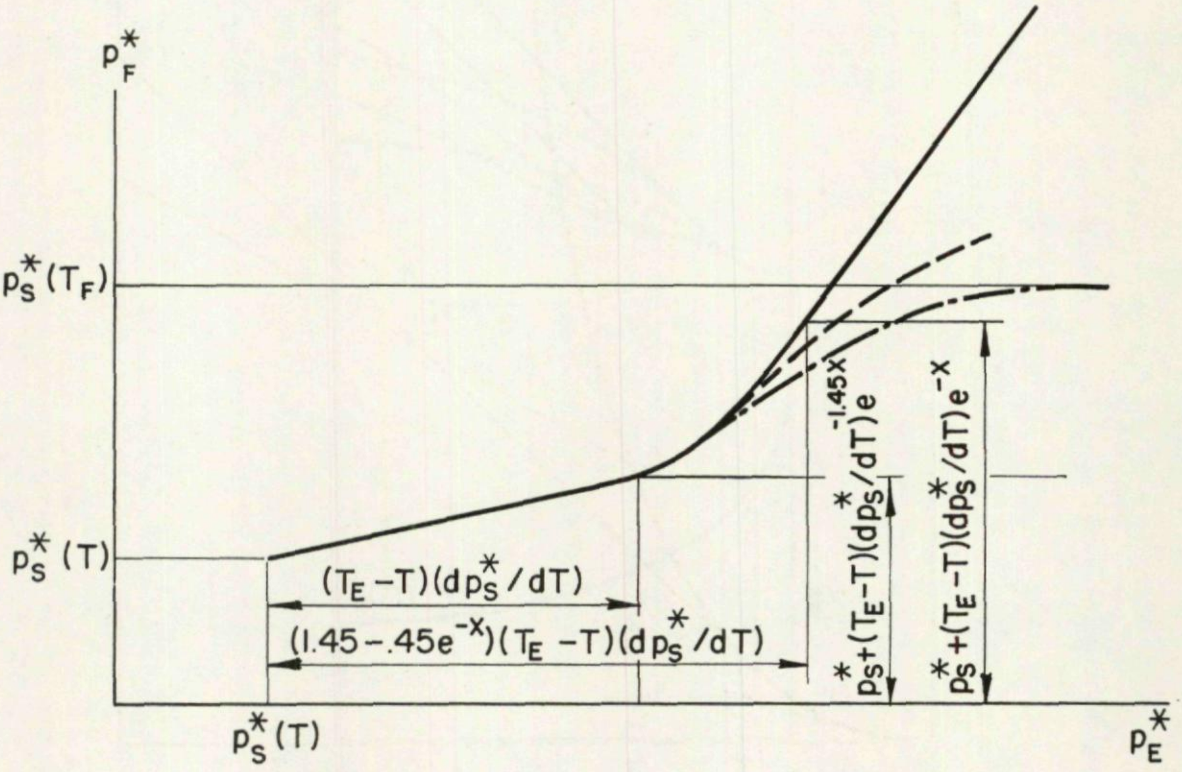

Fig. 11-2. Estimated performance curves for a monotherm cooling dryer.

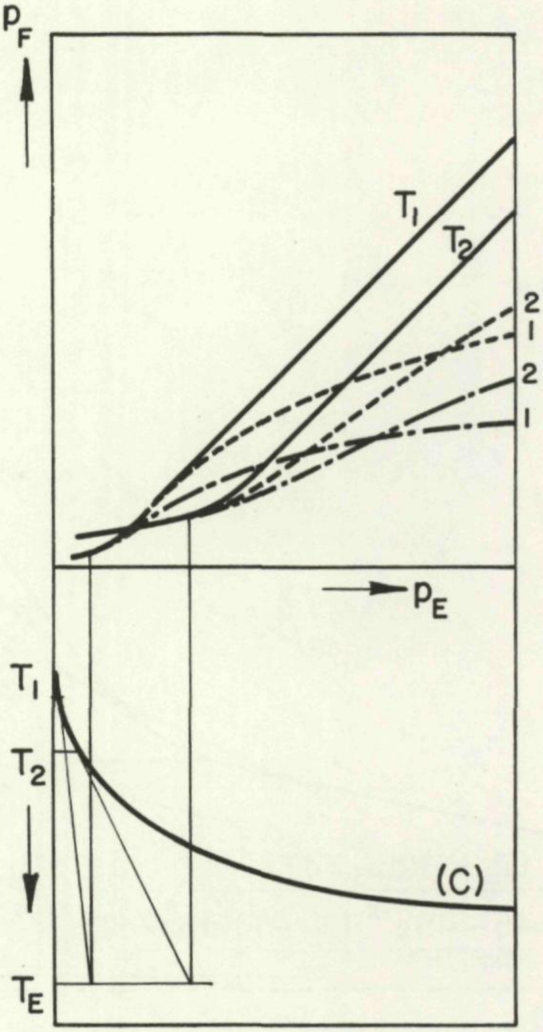

Fig. II-3. Performance of a cooler operated at different temperatures.

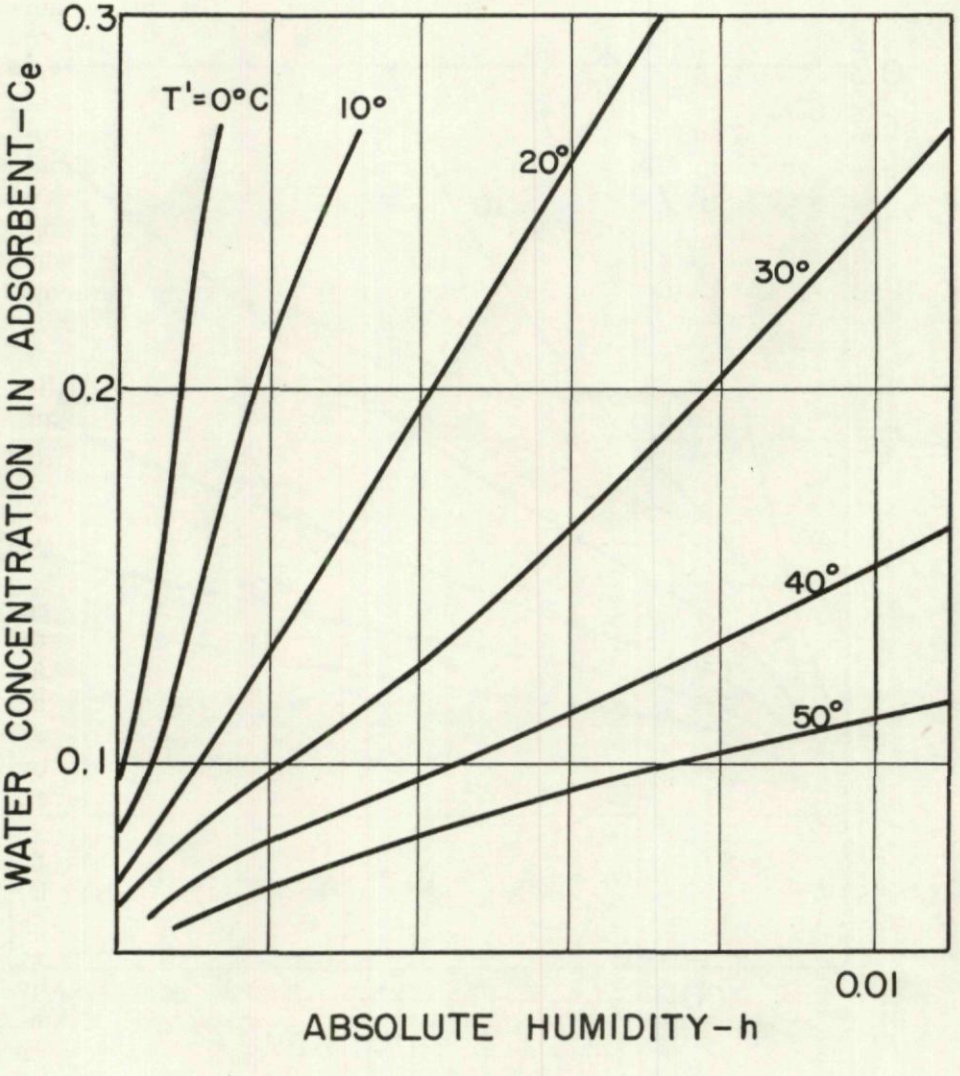

Fig. II-4. Adsorption isotherms for silica gel.

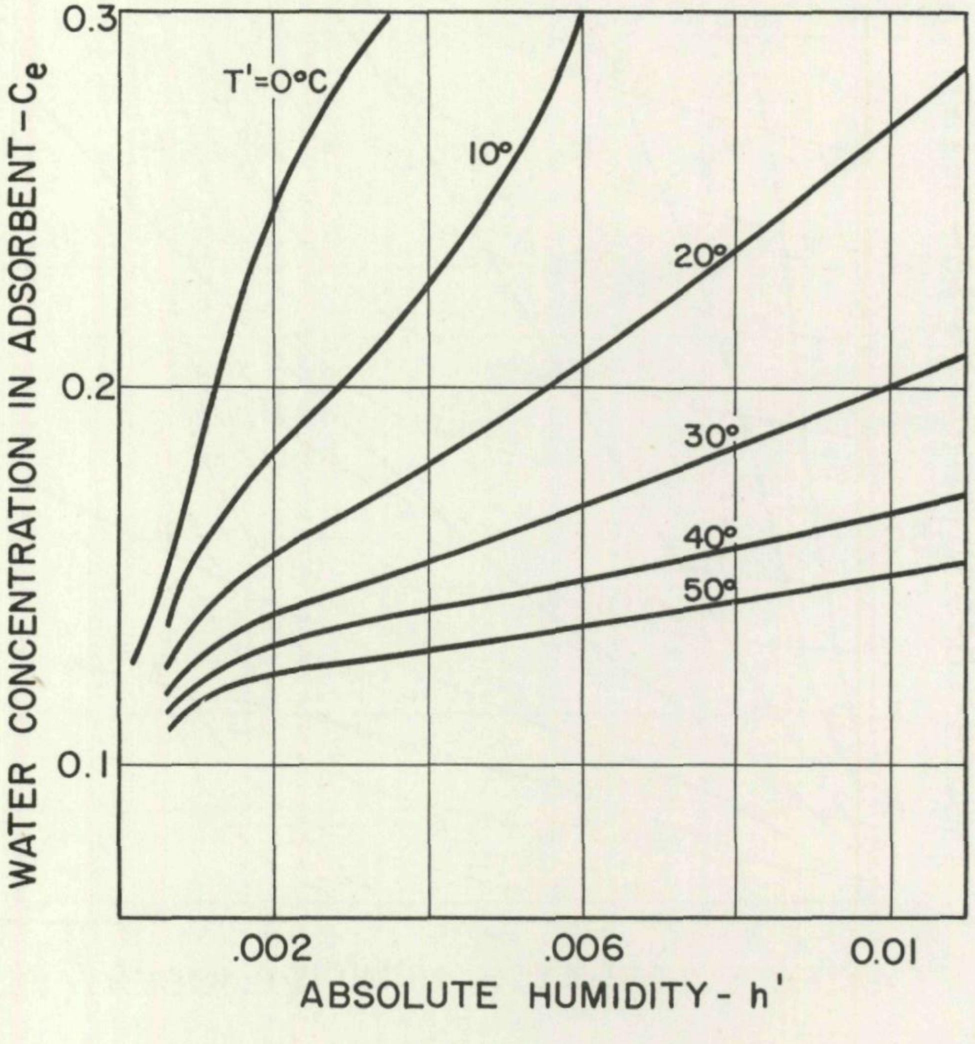

Fig. II-5. Adsorption isotherms for activated alumina.

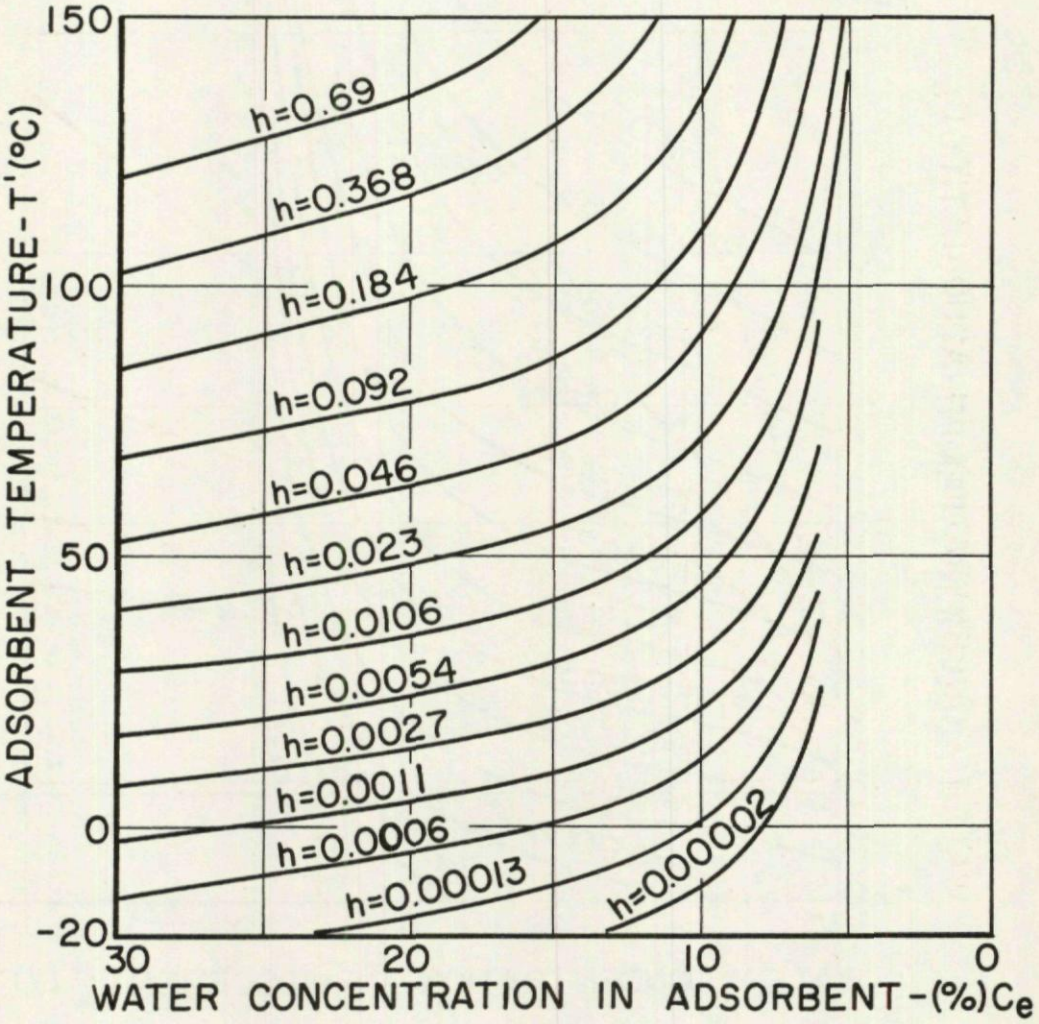

Fig. II-6. Adsorption isopiestics for silica gel (from Ref. 26).

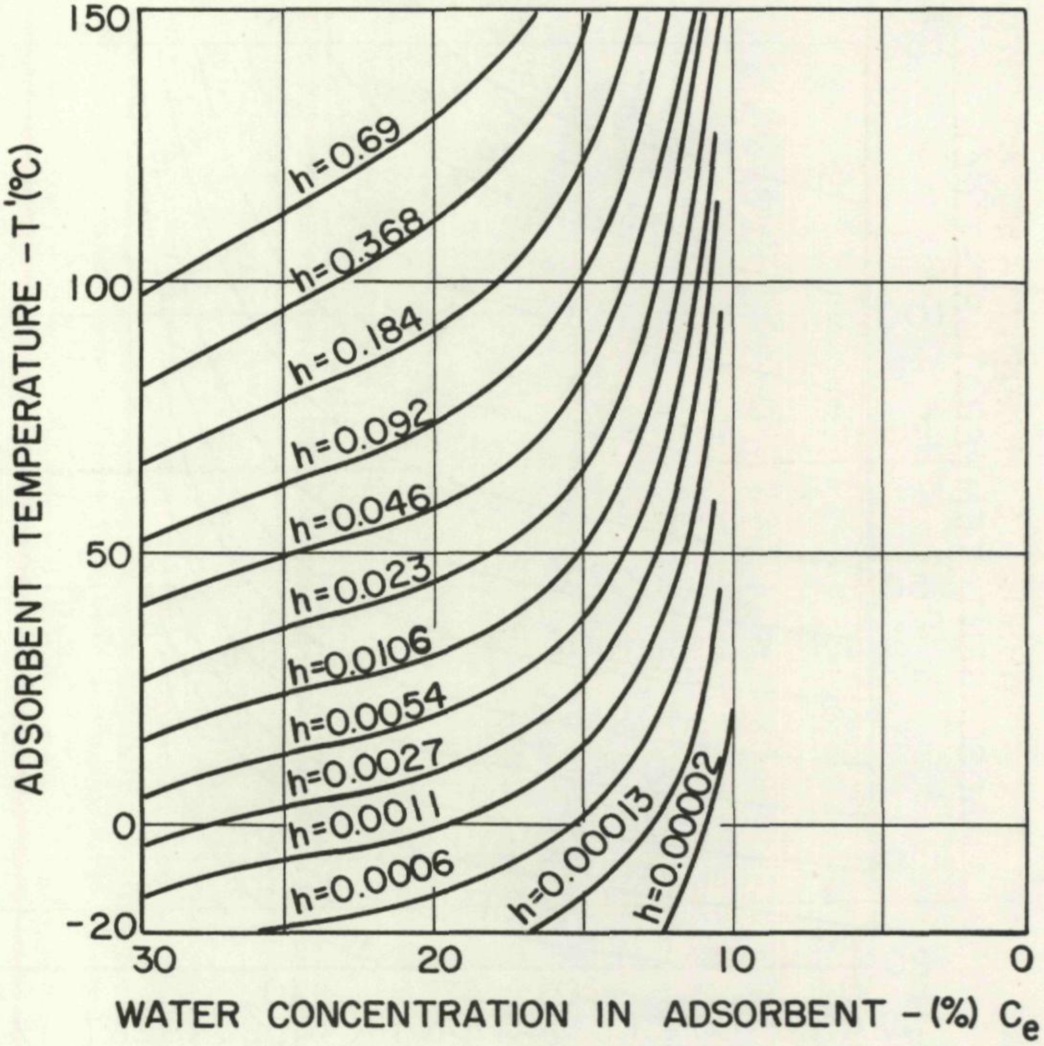

Fig. II-7. Adsorption isopiestics for activated alumina (from Ref. 26).

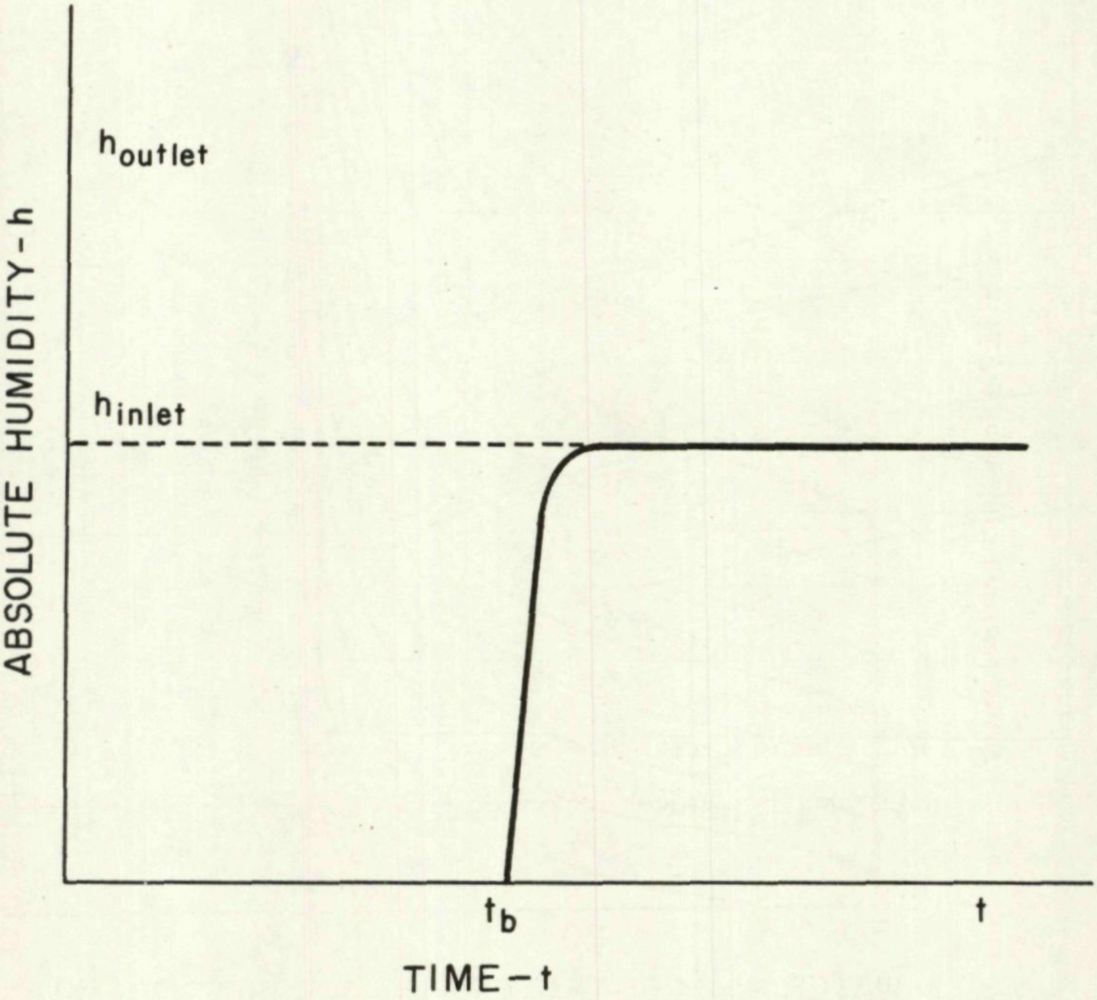

Fig. II-8. Performance curve for adsorption dryer, with break point.

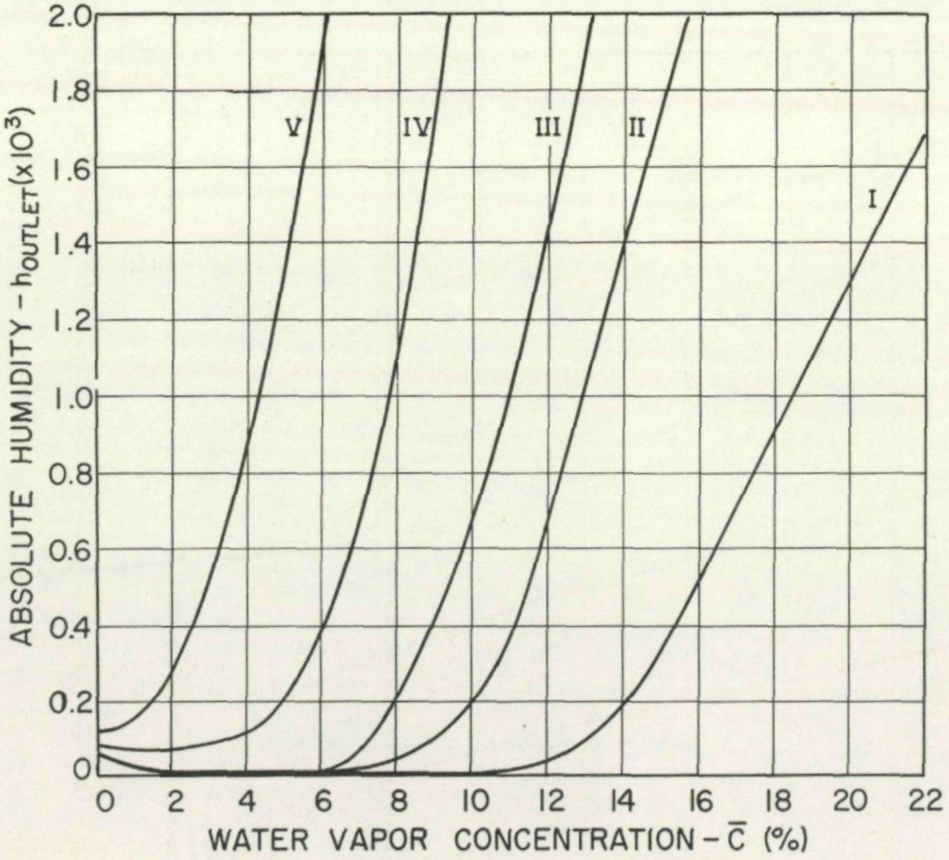

Fig. II-9. Typical performance curves for silica gel (from ref. 26) 6-10 grade.

Curve	Inlet Humidity	Contact Time	Layer Depth
I	5.95 10^{-3}	1.25 sec.	63.5 cm.
II	7.3 10^{-3}	1.25 sec.	63.5 cm.
III	8.8 10^{-3}	1.25 sec.	63.5 cm.
IV	8.4 10^{-3}	1.20 sec.	30.5 cm.
V	10.2 10^{-3}	0.40 sec.	30.5 cm.

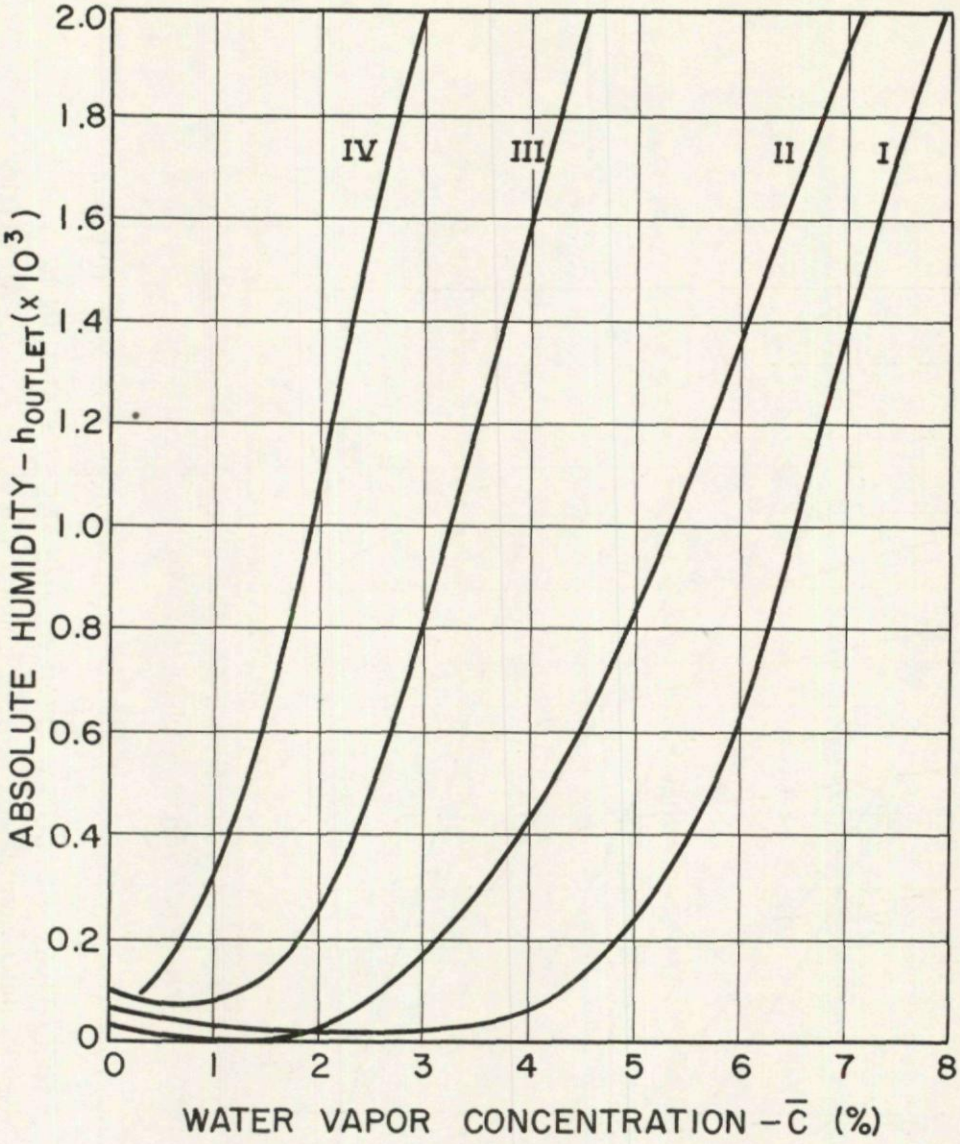

Fig. II-10. Typical performance curves for activated alumina 2-4 grade (from Ref. 26).

Curve	Inlet Humidity	Contact Time	Layer Depth
I	6.3 10^{-3}	2.50 sec.	63.5 cm.
II	7.3 10^{-3}	1.25 sec.	63.5 cm.
III	8.9 10^{-3}	1.25 sec.	63.5 cm.
IV	10. 10^{-3}	0.42 sec.	63.5 cm.

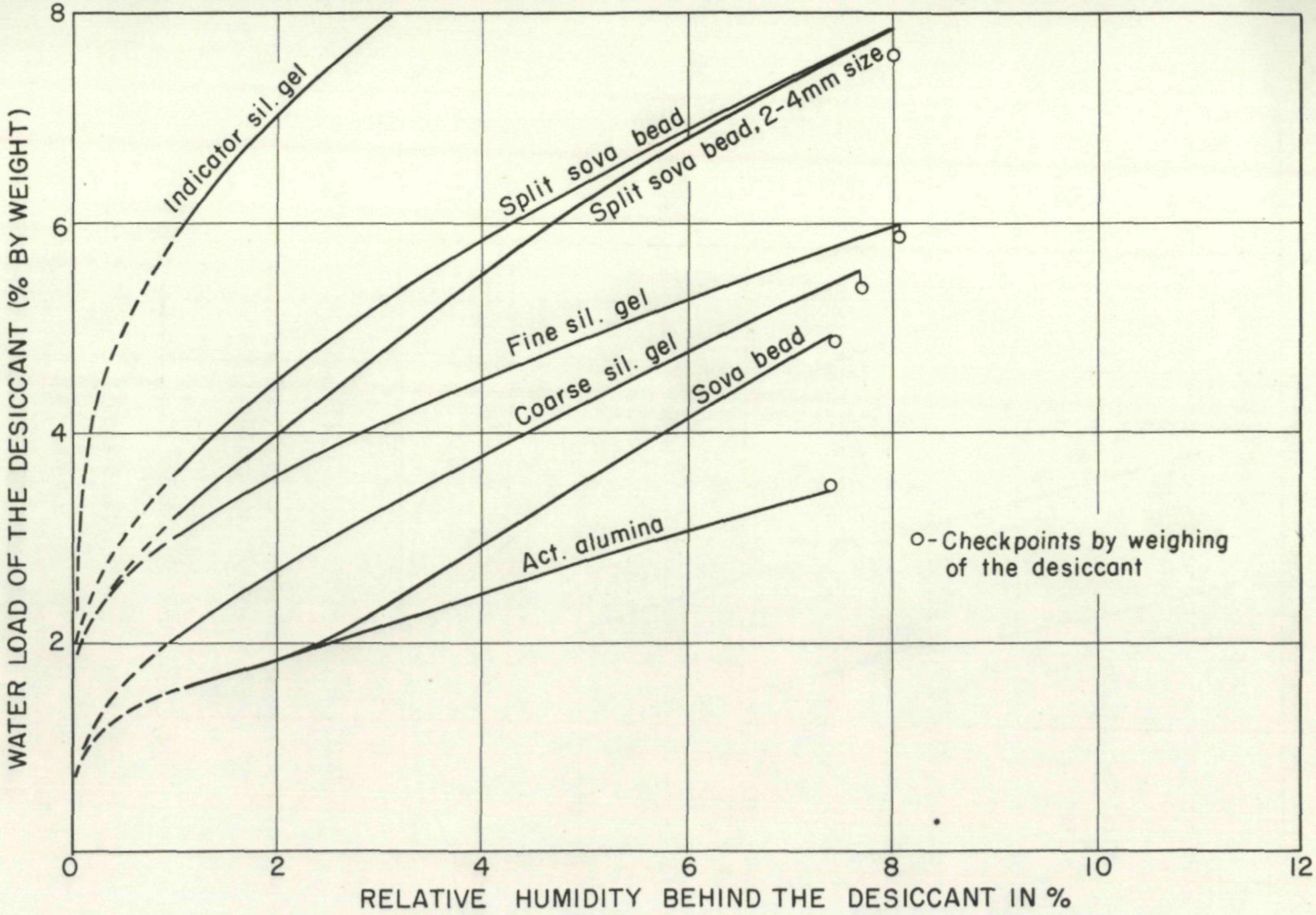

Fig. II-11. Water load of the desiccant vs. relative humidity behind the desiccant. (NOLM 10518).
Average outside air conditions: Relative humidity approximately 42-48%
Absolute humidity approximately 9-10 g/Kg
Temperature approximately 26°C
Barometric pressure approximately 757 mm Hg.

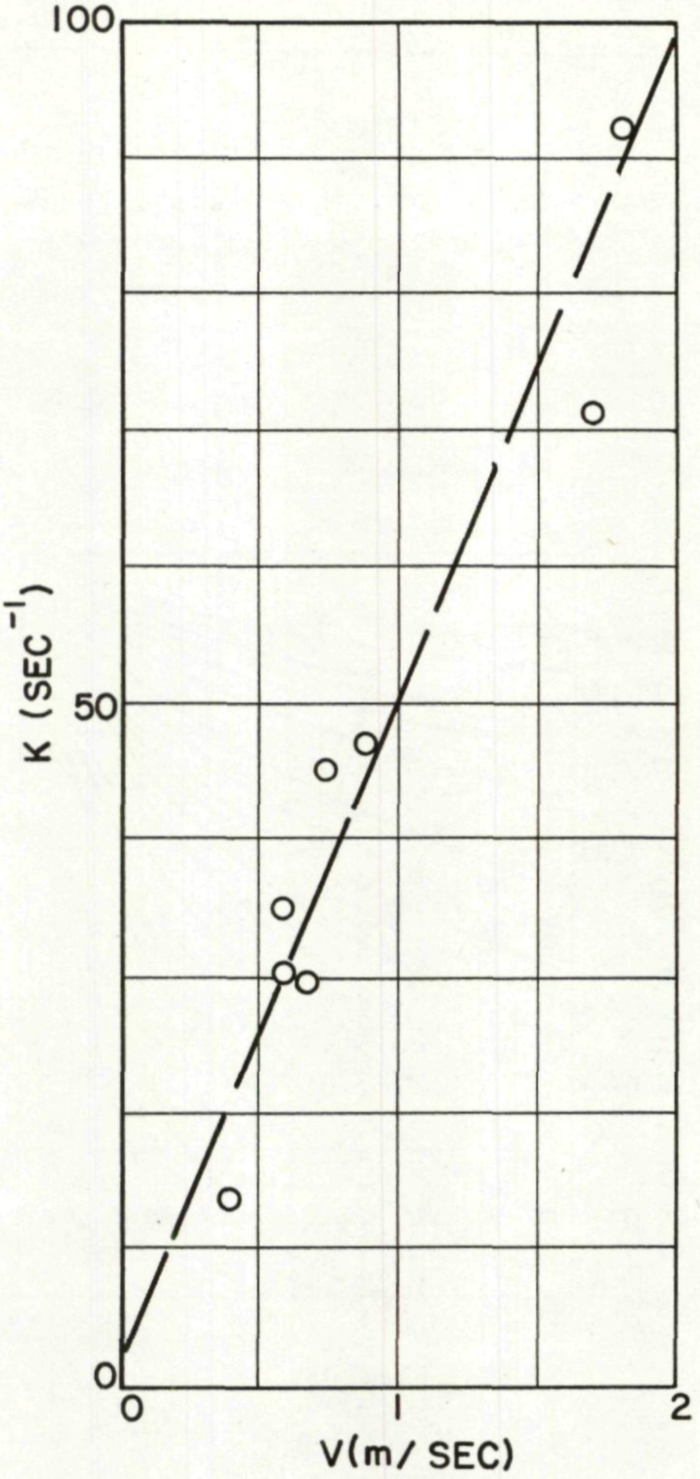

Fig. II-12. Dependence of k on V (author's tests).

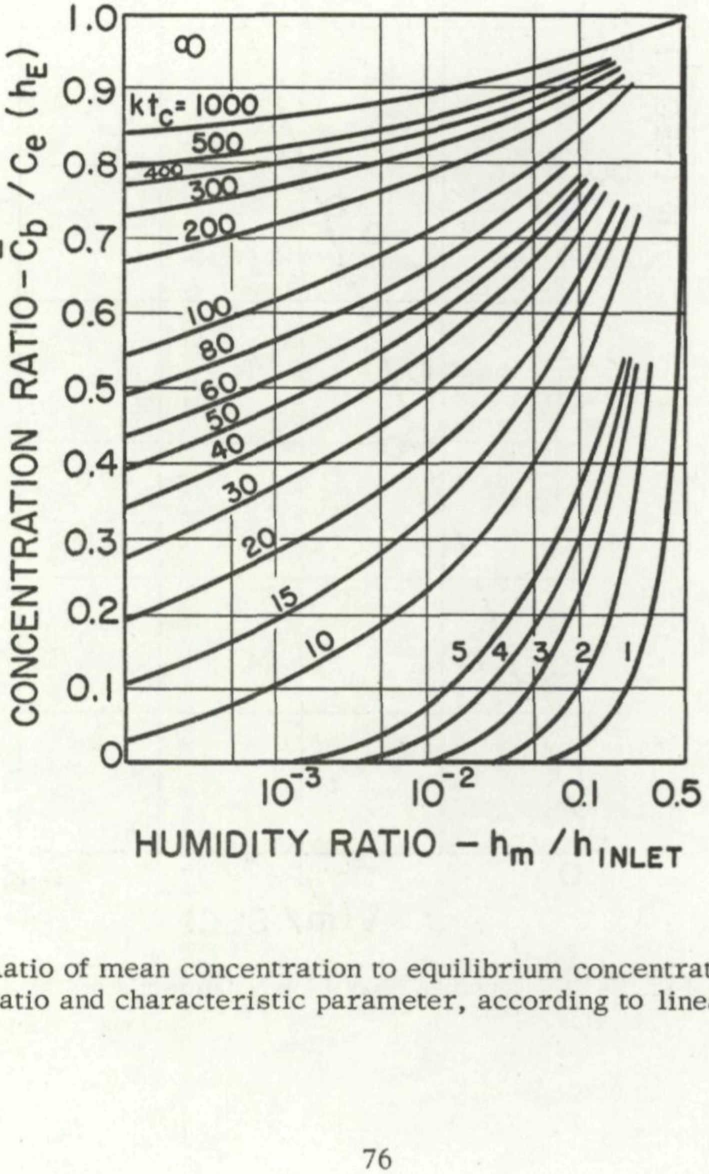

Fig. II-13. Ratio of mean concentration to equilibrium concentration, as a function of humidity ratio and characteristic parameter, according to linearized theory.

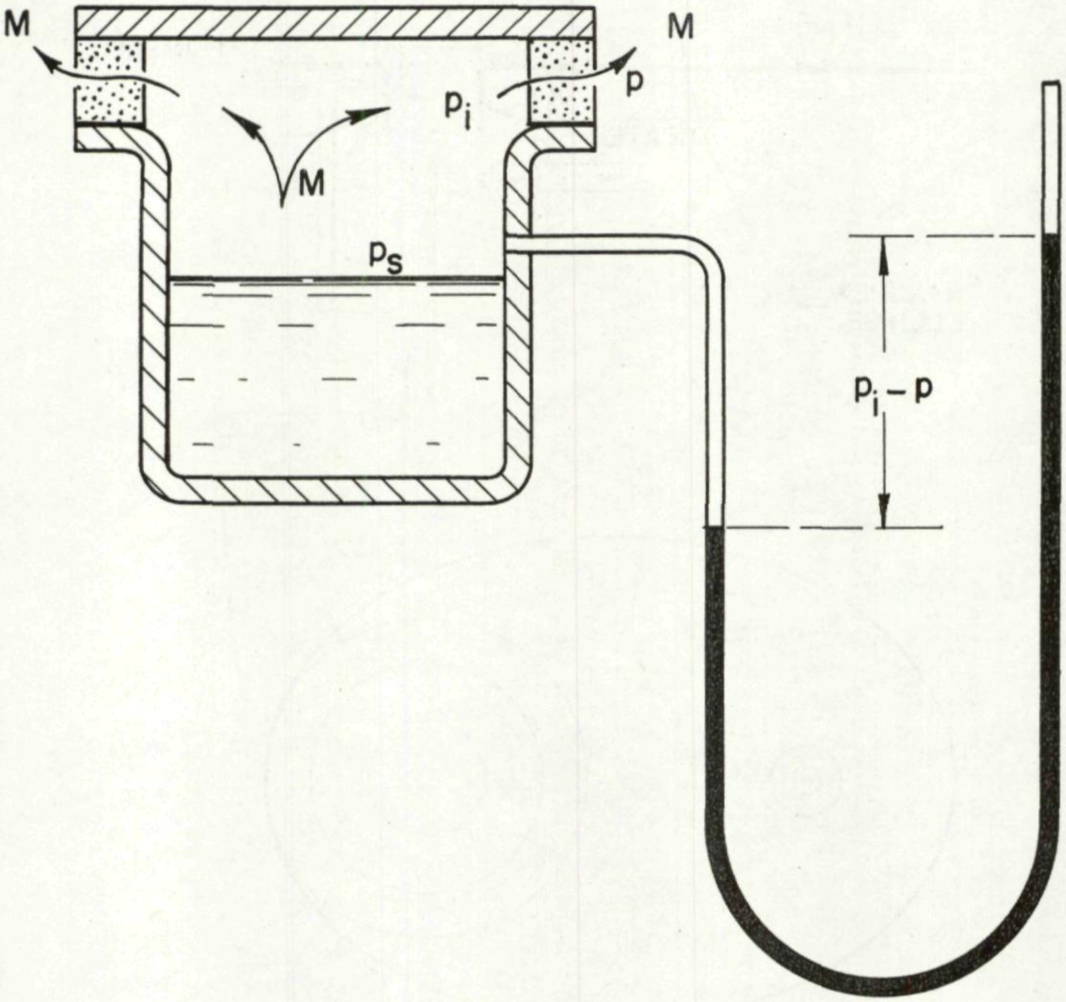

Fig. III-1. Porous diffusion hygrometer.

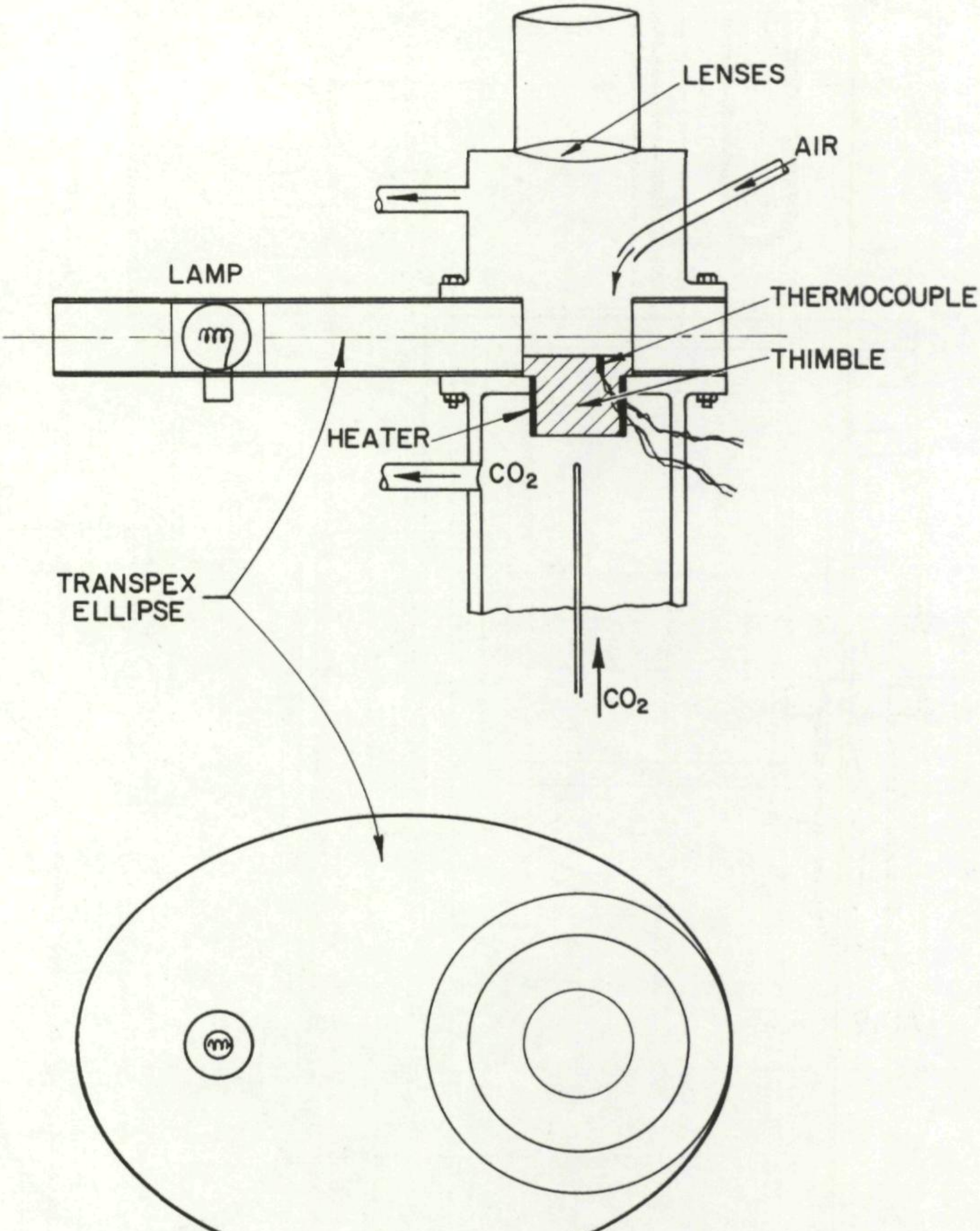

Fig. III-2. Typical dewpoint meter (from Ref. 35).

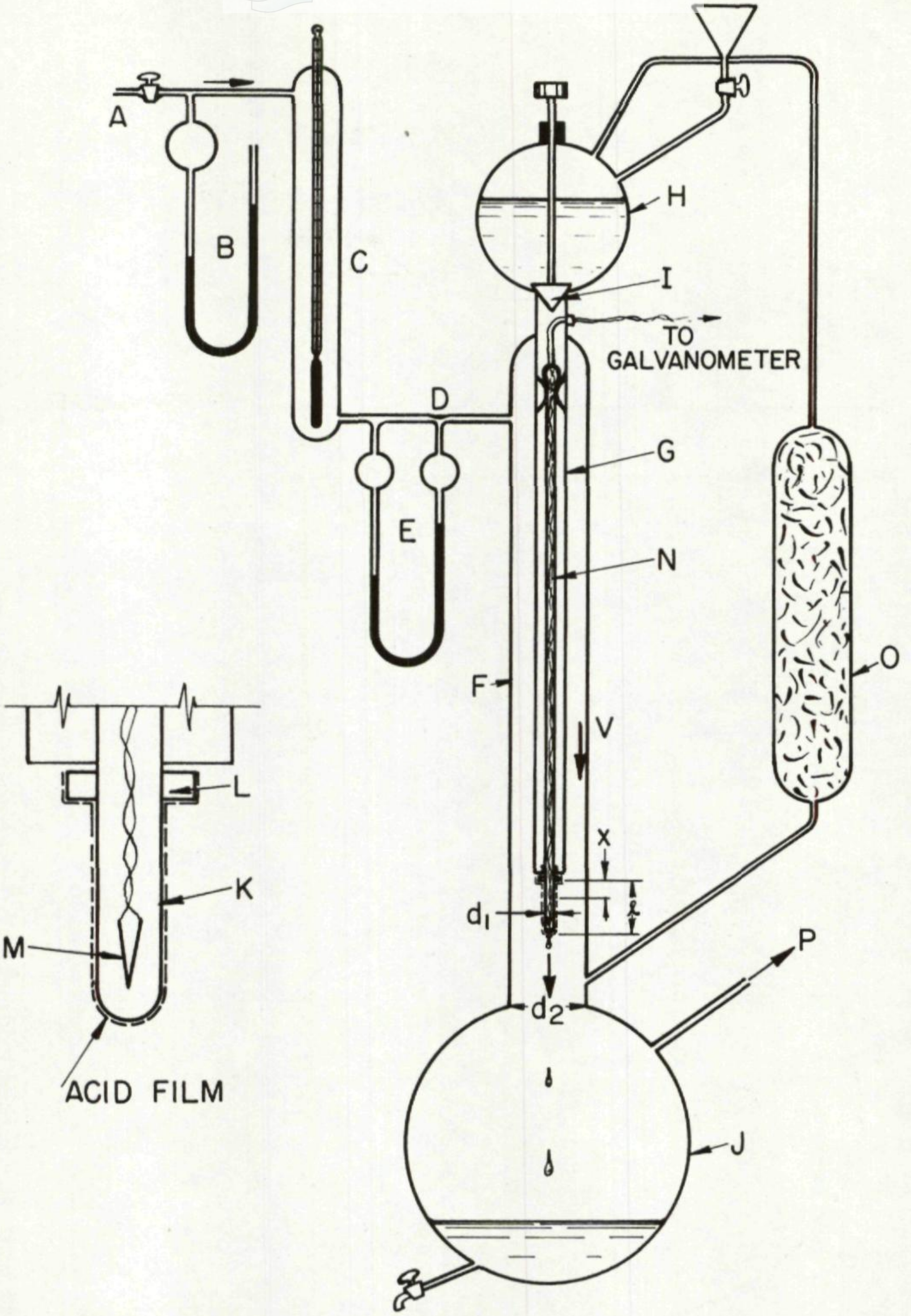

Fig. III-3. Sulphuric acid hygrometer (Laboratoire de Recherches Balistiques et Aerodynamiques, Vernon, France).

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North Atlantic Treaty Organization
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DRYING SYSTEMS FOR SUPERSONIC
WIND TUNNELS
J. J. Smolderen, July 1956
(Wind Tunnel AGARDograph Series)

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