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L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

Hailstone Extremes for Design

IRVING I. GRINGORTEN

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AIR FORCE SYSTEMS COMMAND
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13. ABSTRACT

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In any one year, in the severest area and month for hail, the maximum hailstone size, reaching the ground, has been estimated to average slightly better than 1 in. across, and to have a 10 percent chance of exceeding two inches. For a 10-year period, the size with a 10 percent probability is estimated to be more than 3 in. in diameter.

For flights at 10,000 to 20,000 ft altitude, over the area of the United States where hail is most likely, there is a 0.1 percent encounter risk of hailstones as great as 1.9 in. in a 100-mile traverse and 2.4 in. in 200 miles.

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L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

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I

Abstract

To a climatologist, concerned with design criteria, hailstones offer one of the greatest challenges. The risk or probability of occurrence of hailstones is needed for the revision of MIL-STD-210A, "Climatic Extremes for Military Equipment." Eventually some kind of instrumentation, using radar or hailpads, is expected to measure the destructive power of hailstorms, but for this study, the literature has led only to tentative figures for the distribution of diameters.

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For flights at 10,000 to 20,000 ft altitude, over the area of the United States where hail is most likely, there is a 0.1 percent encounter risk of hailstones as great as 1.9 in. in a 100-mile traverse and 2.4 in. in 200 miles.

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1. INTRODUCTION

For revision of MIL-STD-210A, "Climatic Extremes for Military Equipment," sizes of hailstones at the earth's surface that equipment must withstand without irreversible damage over the usual periods of 2 to 25 years must be specified. A 10-percent risk is acceptable. Also, "...when the inoperability of an item of equipment directly endangers human life," as in the case of aircraft in flight, "the design criteria for equipment should be established so as to result in a percentage of inoperability which is as close to zero as possible."* In this study the hail size estimated for a 0.1 percent risk will be used as the value nearest the zero risk because of the crudeness in the data sample.

For either surface or airborne equipment, the hailstone problem is caused by several factors in combination, including hail size, density of the hailstones, terminal speed at the ground, or relative speed upon impact with an aircraft, and the added driving force of the wind for hailstones impacting at the earth's surface. Duration of the hailstorm, the number of stones per unit volume or the number of

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*Memorandum for the Secretary of Defense, Military Standard MIL-STD-210A, "Climatic Extremes for Military Equipment," Joint Chiefs of Staff, Washington, D.C., JCSM-502-69, 12 August 1969.

(Received for publication 23 December 1971)

impinging hailstones per unit area per minute could also be of importance. For the vehicle in flight, the horizontal extent of the hailstorm becomes a factor.

To take into account all factors is difficult or impossible. Perhaps they can be integrated into a single number, the impact power of the motion of the hailstones per square foot. A new kind of instrumentation can be imagined to catch hailstones, to sort and weigh them, measure their size, and to count the numbers by size that collect in a basket of some kind. Recent radar-type instrumentation (Dennis et al, 1971) as well as the hailpad (Changnon, 1971b) appear to come close to this, and may yield observations of hailstorm destructive power. Probability distributions of these observations, the climatology required for design criteria, could then be prepared.

For this study, the criterion of hailstorm intensity is arbitrarily restricted to maximum stone size in the storm. It is tacitly, or even explicity, assumed by several authors to be sufficient. In their reviews, Souter and Emerson (1952), Flora (1956), Hull (1957) and Foster (1961) refer to a hailstone diameter of 3/4-in. or 1 in. as the minimum size to do damage to aircraft, based on tests by the Civil Aeronautics Administration (Harrison and Beckwith, 1951).

Damage to crops is believed by some to occur with hailstones of the size of golf balls, approximately 1.6 in. diameter. These are larger than designated critical for aircraft because the impact speed at the ground – the terminal velocity of the hailstones plus horizontal motion from the wind – is much smaller than speed of impact with a flying aircraft. However, Changnon (1967, 1971b) writes that wheat, corn, and soybean crop damage occur even with 1/4-in. stones, adding the effect of their frequency. With respect to equipment or installations on the ground, it is more reasonable to expect the diameter to be a primary factor. Harrison and Beckwith (1951) wrote that to damage the metal surface of a DC 6 airplane at rest on the ground, the stone would have to have 3 in. diameter.

The decision to restrict the study of hailstorms to the maximum hailstone diameters is strengthened by previous investigations that relate one of the important factors, terminal velocity, to the diameter. At the surface, hailstones of the same size should impact a horizontal surface with equal energy since they would be falling with the same speed.

2. HAILSTONE SIZE MAXIMUM

Estimates of the frequency distribution of maximum hailstone diameter per hailstorm have been published as early as 1899 and in the most recent literature. Eliot's (1899) frequencies were for qualitatively described diameters, to which Gerson (1946) assigned quantitative values. Fawbush and Miller (1953) gave frequencies in 274 cases, which Miller (1967) later revised with 529 cases. Changnon's (1971a) frequencies were based on reports using hailpads at a network of 49 sites in 1967, and 196 sites in 1968 and 1969 distributed throughout a 1600-sq. mile area. Chmela (1960) and Donaldson et al (1960) reported on a survey of some 290 hailstorms in New England. Using a network of stations around Denver, Colorado, Beckwith (1960) obtained 825 reports in 10 years.

The earlier frequencies, as reported by Eliot and by Fawbush and Miller, were admittedly biased toward the larger diameters because observers were likely to ignore hailstones of small size. Battan and Wilson (1969) reported the smallest hailstones of all, on Arizona mountaintops, but they expressed doubts about calling these ice particles "hail."

For this paper, Changnon's (1971a) estimate of frequencies by size, obtained objectively, is initially accepted. His maximum diameters, determined with the hailpads, have the following frequencies:

Diameter (in.)	Relative Frequency
≤0.354	0.48
≤0.618	0.80
≤0.870	0.91
≤1.11	0.95

Interpolating between 0.870 and 1.11 in., one obtains a probability of 0.07 of hailstones greater than 1 inch.

As mentioned, Miller (1967) presented a distribution of hail diameters based on 529 reports. Disregarding his stratification by the height of the wet-bulb-zero level aloft (Miller's Figure 44), the frequency distribution for the 529 cases is as follows:

Diameter (in.)	Frequency
1/8 to 1/2	365
3/4 to 1	91
1.5 to 3.0	69
> 3.0	4

Thus, of Miller's 73 reports greater than 1.0 in., 4 cases occurred of reported sizes greater than 3.0 inches. If the 73 cases are considered as 0.07 of the whole distribution, to fit onto Changnon's distribution, the probability of diameters exceeding 3.0 in. becomes

$$4/73 \times 0.07 = 0.00384.$$

Figure 1 shows a composite of Changnon's and Miller's results. The part of the distribution for diameters less than 0.354 in. is obtained by adding the value of Donaldson et al (1960) for the frequency of hail of 0.125 in. diameter. The symbol for this probability is P(h|H), implying that the maximum hailstone size is h, given that a hailstorm (H) is in progress.

For a number of years, the largest hailstone on record had been 5.4 in. at Potter, Nebraska, 6 July 1928. Extrapolation of the curve of Figure 1 gives this size a probability of one chance in 27,000 hailstorms. Recently, a photograph was published of a hailstone, irregular in shape, with diameter approximately 5.6 in., that fell in Coffeyville, Kansas, 3 Sept 1970 (Weatherwise, 1971).

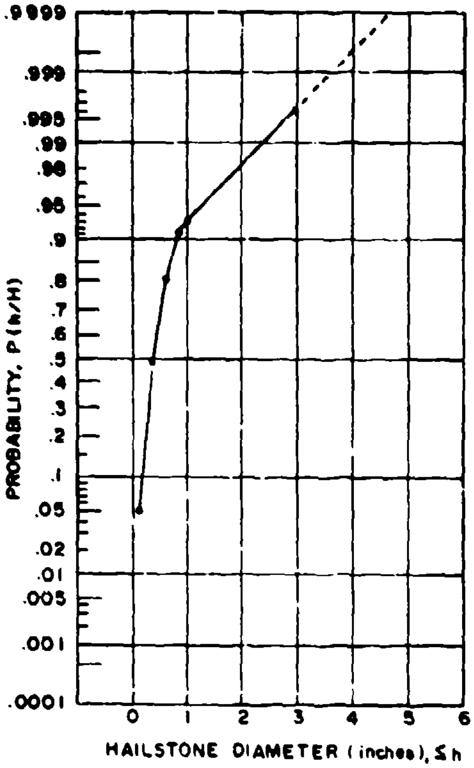

Figure 1. Composite Estimate of P(h|H), the Cumulative Probability of the Maximum Hailstone Diameter h in a Hailstorm, Given That There is a Hailstorm (H)

3. FREQUENCY OF HAILSTORM DAYS: GROUND LEVEL

Following the practice for the MIL-STD-210B studies, one finds it necessary to decide on the most severe location for hailstorms and the most severe month. The United States has the dubious distinction of being a prime center for hail activity. An examination of the climatological maps of the United States (Visher, 1966; Stout and Changnon, 1968; Shands, 1944; Lemons, 1943) shows that the ratio of hailstorms to thunderstorms varies tremendously with geography, so that thunderstorm frequency is no criterion of hail frequency. Also, the month of greatest hailstorm frequency varies with geography. Western Nebraska or southeastern Wyoming is the center of United States activity. The greatest average number of hailstorm days in the United States is at Cheyenne, Wyoming, with 9.4 days per season, based on 40 years of record (Hull, 1957). Nevertheless, such an extreme number is so localized (Harrison and Beckwith, 1951) that the value 7 per year (Visher, 1966) is a more representative estimate, especially when the probability over a flight path of several hundred miles is to be considered. For the most severe month in the most severe location, on the average 2.9 hailstorms occur. A more representative average for a flight path is assumed to be 2 per month.

The hail season in the United States is from April to September, with June and July the most severe months. Most hail occurs between 1200 and 1900 LST. Changnon (1968) shows a peak time for damaging hail in Illinois at 1600 LST.

4. DURATION

The earliest survey that could provide a frequency distribution of the duration of hailstorms is Eliot's (1899). Some 563 storms had an average duration of 19.3 minutes. Subsequent estimates are lower. Donaldson et al (1960) using the reports from cooperative observers in a 3-yr period gave the average hailstorm duration in New England as 3 to 4 minutes. Yet, on several occasions, storms lasted 1-1/2 hours in a swath 30 miles long. Battan and Wilson (1969) present figures on hail in Arizona mountains with a mean duration of 4.8 min, but with at least one observation as high as 54 minutes. Their duration distribution is close to the distribution of Donaldson.

The large differences between the estimate of duration from Eliot's early figures and the more recent estimates are reconcilable if Eliot's durations apply, not to a single location, but to a storm as an entity, or more specifically to "hail-streaks" within a storm. Changnon's (1970) single-point durations averaged 3.2 min in Illinois storms. His Table 2 shows a median duration of 10 min for the hailstreak itself. However, Changnon's figures also show a wide distribution of hailstreak durations, with 27 percent lasting longer than 15 min in Illinois. In a sample of hailstreaks in S. Dakota, the average duration was 27 min, that is, almost 3 times as long as those in Illinois. Changnon infers that storms in S. Dakota generate hail 2.7 times longer in time as well as in distance than in Illinois. This would give the single-point duration in S. Dakota an average of 2.7 × 3.2 min, or 8.6 minutes.

In view of the foregoing data, the assumed average point duration for the "most severe area" in the "most severe month" is rounded herein to 10 minutes.

5. PROBABILITY: RISK OF NOT WITHSTANDING DAMAGE

For the probability of a certain number of hailstorm days per year, Changnon and Schickedanz (1969) found the Poisson distribution applicable more frequently than the negative binomial distribution. If the average is λ hailstorm days per year, assumed independent of each other, then for the probability of n hailstorm days per year, the Poisson distribution gives

$$p(n) = e^{-\lambda} \cdot \frac{\lambda^n}{n!} .$$

An accepted value for λ is 7 per year (see Section 3).

Figure 1 gives the cumulative probability distribution $P(h|H)$ of maximum hailstone size ($\leq$ h) on any one hailstorm day. The cumulative probability distribution of the maximum size in n independent hailstorm days is thus given by $P^n(h|H)$. Therefore the cumulative probability of hailstones equal to or less than size h for a whole year is given by

$$\sum_{n=0}^{\infty} p(n) \cdot P^n(h|H).$$

This cumulative probability is closely approximated by assuming an upper limit of n = 20 storms per year.

Figure 2 shows on extreme probability paper the plot of this probability for size h varying from 0.25 to 4.0 in. Figure 2 automatically gives the transformation of the diameter h into the reduced variate y (Gumbel, 1958). The reduced variate y has a cumulative probability distribution given by

$$P(y;1) = \exp(-e^{-y}).$$

It has a mean of 0.5772, standard deviation 1.28255 and a 10-percent value of 2.25. The probability $P(y;n)$ of the n-year extreme is related to y by the following equation (Gringorten, 1963):

$$y = \ln n - \ln[-\ln P(y;n)].$$

Since for each value of y there is a corresponding value of the hailstone size (Figure 2), one can use the above equation to calculate hailstone extremes for different percentiles of risk, $P(y;n)$, for various periods of years, n. The values in Table 1 are for the averages (57-percentiles) and 10-percentiles for the 2-, 5-, 10- and 25-year hailstone extremes.

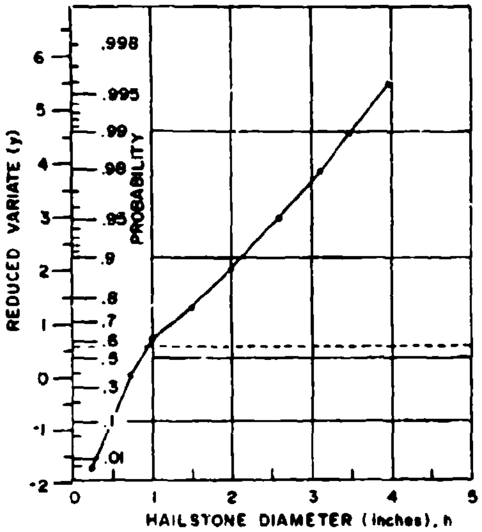

Figure 2. Cumulative Probability of the Annual Largest Hailstone Diameter in the Most Severe Location, Plotted on Extreme Probability Paper

Table 1. The Average and 10-Percentile of the Extreme Hailstone Diameter in the Most Severe Location in 1, 2, 5, 10, and 25 years

Lifetime (Years)	Average		10-percentile	
	y	Diameter (in.)	y	Diameter (in.)
1	0.58	1.0	2.25	2.1
2	1.27	1.5	2.94	2.6
5	2.19	2.1	3.86	3.1
10	2.88	2.5	4.55	3.5
25	3.80	3.1	5.47	4.0

6. PROBABILITY: RISK OF INOPERABILITY

Figure 1 shows estimates of $P(h|H)$, the conditional probability of hailstones equal to or less than size h, given that there is a hailstorm. To find the probability of encountering hailstones of greater size, the conditional probability of exceeding a given size, $1 - P(h|H)$, must be multiplied by the probability, $p(H)$, that at any given instant a hailstorm is in progress.

As previously discussed (Section 3), for the most severe area in the most severe month, a frequency of 2 hailstorms per month, each lasting an average of 10 min, can be assumed. Since there are $(60 \times 24 \times 31)$ min in a month, then 2 hailstorms per month give a probability of hailstorm occurrence at a single point

$$p(H) = (2 \times 10)/(60 \times 24 \times 31) = 0.000448.$$

Thus the probabilities of exceedance, $P(\geq h)$, of hailstone diameters $(\geq h)$ at a single station can now be calculated, using $p(H)$ and $P(h|H)$:

$$P(\geq h) = p(H) \cdot [1 - P(h|H)].$$

These probabilities are listed in Table 2. At the earth's surface, even during the worst month in the worst location, the probability of encountering a significantly large hailstone is extremely small at a randomly selected instant. If all hailstorms are considered to occur in a limited 6-hr period of the afternoon, the probabilities are roughly quadrupled. However, this refinement is not pursued since military operations might occur at any time of the day.

The probabilities of Table 2 also apply to levels up to 5000 ft for reasons presented in Section 10.

Table 2. Estimates of the Probability of Encountering Hailstones of Given Diameter at a Single-point Location Near the Earth's Surface and Up to 5000 ft*

Hail diameter (in.) h	Conditional probability of size ≥h	Single-station probability
Any size	1.000	0.000448
≥0.25	0.790	0.000354
≥0.5	0.360	0.000161
≥0.75	0.135	0.0000605
≥1.0	0.070	0.0000314
≥2.0	0.019	0.00000851
≥3.0	0.0038	0.00000170
≥4.0	0.00055	0.00000025

*See Section 10 for application to the higher levels.

7. HORIZONTAL EXTENT OF A HAILSTORM

Kaster (1944) gave the average width of a swath of hailstone incidence as 2-1/2 miles, with a width as much as 5 miles on a rare occasion. Lemons (1943) presented a distribution of widths based on a survey of 2105 reported hailstorms in the United States in the years 1924 to 1939 inclusive. Commonly the width was 1 to 2 miles, but with variation from a few yards to 75 miles.

The length of the path is considerably greater than the width, but more difficult to define. In his recent study of well-defined hailstreaks, Changnon (1970) presented frequency distributions of the maximum widths and lengths of Illinois and S. Dakota hailstreaks, showing widths in Illinois varying from 1/10 to 4 miles and lengths from 1 to more than 15 miles. The median width was 1.1 miles in Illinois but 2.3 miles in S. Dakota. The median length was 5.9 miles in Illinois, 15.3 miles in S. Dakota, making the ratio of width to length roughly 1:6. All told, as previously mentioned, the S. Dakota hailstreak is 2.7 times greater than the Illinois. The areal extent of the hailstreaks ranged from 1 to nearly 800 sq. miles.

8 AREAL AND LINEAR FREQUENCIES COMPARED WITH POINT FREQUENCY

Beckwith (1960), reporting on observations for 10 years at a network of some 50 stations in a 150-sq. mile area around Denver, gave the average ratio of occurrence of hailstorm-days in the area to the occurrence at a single station as 4.4 to 1.

Thus, if the probability at a single station is 0.000448 (see Section 6) then the probability of one or more occurrences in a 150-sq. mile area would be 4.4 × 0.000448 or 0.00197. Changnon (1971c) examined Beckwith's data along with similar data from a dense network of stations in Illinois and S. Dakota, as well as Colorado. He concludes that the area-to-point ratio (R) of hail-day frequencies is reasonably independent of geography but directly related to area size (A).

Changnon's areas (A) varied from 1 sq. mile in Illinois to 56,400 sq. miles for the whole state. But the representativeness of the station network within each studied area varied from 6 stations per square mile to 0.0027 stations per square mile. It is easy to believe that the area-to-point ratios (R) of hailstone incidence were underestimated for the large areas. As the size of the area becomes greater, approaching the size of the state, the probability of a hailstorm in the area could be much greater than Changnon's observational figures suggest.

As this report is being written, an investigation of spatial variability of meteorological parameters is in progress in the Design Climatology Branch, using random numbers in a simulation technique. The results, so far, have been based upon the assumption that the correlation between two normalized variables decreases with the square of the distance between them. For the single-point hailstorm probability of 0.000448 (Section 6), the probability of hailstorm occurrence in an area of s_m^2 square units, and the probability of hailstorm occurrence along a straight-line segment of length s_m units given by the model are as shown in Table 3.

Table 3. Model Estimates of the Probability of Occurrence of an Event in an Area of s_m^2 Square Units and Along a Line Segment of Length s_m Units When the Single-point Probability is 0.000448*

s_m (units)	Probability for area s_m^2	Probability for line segment s_m
0	0.000448	0.000448
1	0.000500	0.000460
2	0.000750	0.000520
4	0.00150	0.000710
8	0.0033	0.00120
16	0.0085	0.0020
32	0.0250	0.0035
64	0.070	0.0063
128	0.183	0.0128
256	0.400	0.028

*See Section 8.

From Changnon's report (1971c), for the two smallest areas of his study (1 and 10 sq. miles) the following probabilities of one or more hailstone occurrences can be calculated:

Area size, s^2 (sq. miles)	Probability
0	0.000448 (see Section 6)
1	0.000538
10	0.000941

From graphical interpolation of the hailstone probabilities in areas of s_m^2 sq. units (Table 3), it is found that the probability = 0.000538 corresponds to s_m = 1.18 units, and the probability = 0.000941 corresponds to s_m = 2.64 units. This leads to the following mile/unit ratios (s/s_m):

Probability	Area s^2 (sq. miles)	s (miles)	s_m (units)	Ratio (s/s_m)
0.000538	1	$\sqrt{1} = 1$	1.18	0.85
0.000941	10	$\sqrt{10} = 3.16$	2.64	1.20

Changnon had used a 10-year record for the 10-sq. mile area (1960-1970) compared with a 3-year record (1967-1970) for the 1-sq. mile area. On the other hand, 6 stations of the 1-sq. mile area represented the small area much more intensely than 13 stations of the 10-sq. miles. Therefore, an acceptable ratio (s/s_m) of distance (in miles) to the distances (in units) of Table 3 is somewhere near 0.85 or 1.20 or in between. It is expedient to assume $s/s_m = 1.0$ and to treat the units of Table 3 as miles. When this is done the right-hand column of Table 3 gives, by graphical interpolation, the probabilities of hail encounter along path lengths of 100 miles and 200 miles as follows:

Path length (miles)	Probability
0	0.000448 (see Section 6)
100	0.010
200	0.021

These probability estimates of hail encounter, multiplied by the conditional probabilities of maximum hailstone sizes, give the probabilities of encountering such maximum sizes on 100- and 200-mile routes (Table 4).

Table 4 shows, for instance, that there is a 0.07 percent chance of encountering hailstones equal to or greater than 1.0 in. in size on a 100-mile route in the most severe location and most severe month near the earth's surface. For an exact 0.1 percent probability, graphical interpolation of the numbers in Table 4 gives hailstone sizes of ≥ 0.9 in. on a 100-mile route and ≥ 1.2 in. on a 200-mile route.

The probabilities of Table 4 apply to levels up to 5000 ft for reasons presented in Section 10.

Table 4. Estimates of the Probability of Encountering Various Size Hailstones on 100- and 200-mile Routes Near the Earth's Surface and Up to 5000 ft*

Hail diameter (in.) h	Conditional probability of size ≥ h	Probability	
		100-mile route	200-mile route
Any size	1.000	0.010	0.021
≥ 0.25	0.790	0.00790	0.0166
≥ 0.50	0.360	0.00360	0.00756
≥ 1.0	0.070	0.00070	0.00147
≥ 2.0	0.019	0.000190	0.000399
≥ 3.0	0.0038	0.000038	0.000080
≥ 4.0	0.00055	0.0000055	0.0000116

*See Section 10 for application to the higher levels.

9. OTHER FACTORS

9.1 Specific Gravity

The density/specific gravity of hailstones is a variable and documented figures of density are scarce. In a recent paper, Prodi (1970) reported on estimates of the density of large natural hailstones (8 to 21 g) in several storms in the midwest (U.S.A.); he used both an X-ray absorption technique and an immersion technique corrected for the effect of penetration. The resulting density figures ranged from 0.828 to 0.867 g/cc. A single round figure of 0.9 could be acceptable in calculations of impact energy.

9.2 Terminal Velocity

The results of most authors indicate that the terminal velocity (w) of hail can be related directly to the hailstone diameter (d), that is

$$w = K\sqrt{d}.$$

For w in centimeters per second, d in centimeters, Houghton's (1951) figures yield K = 1150. Foster and Bates (1956), assuming spherical hailstones, applied the aerodynamic equation

$$w = \sqrt{\frac{2}{3}\,\frac{\rho_h}{C_D}\,\frac{g}{\rho_a}} \cdot \sqrt{d} \,,$$

where

ρ_h is the hailstone density (see Section 9.1),

g is the acceleration of gravity,

ρ_a is the density of air, and

C_D is the drag coefficient.

Using their figures, one notes that K = 1880.

For smooth spheres simulating hailstones having diameters of 2 to 4 cm, Landry and Hardy (1970) found fallspeeds that suggest K = 1650. However, they also found that terminal velocities increase considerably with the roughness of the hailstone until the diameter exceeds 8 or 9 cm; then the fallspeed is approximately 5800 cm/sec, which yields a K = 1990.

Aloft, for a drag coefficient C_D = 0.45, Battan and Theiss (1968) obtained:

at	900 mb	700 mb	500 mb	400 mb
K	1590	1750	2000	2200

By linear interpolation, a station at 850 mb that is about 5000 ft above sea level (for example, the worst area in western Nebraska) would have K = 1530.

9.3 Hail Melting During Fall

There are "balance" levels (Atlas, 1966) in the upper air where hailstones accumulate through suspension in the thunderstorm updraft. When they fall below freezing level, there must be some melting before reaching the ground, in this way reducing the diameter.

Mason (1956) derived an equation to relate the hailstone diameter initially at the freezing level to the diameter when it falls to a lower level. He has allowed for the density of the hailstone, latent heats of fusion and evaporation, the thermal conductivities of water and air, the diffusion coefficient of water vapor in air, a so-called ventilation coefficient that involves the Reynolds number for a sphere of ice traveling through air, the lapse rate, the vertical distance through which the hailstone is falling and the fallspeed. Mason's model [Eq. (10)] assumes saturation of the air throughout the vertical fall. Thus latent heat of condensation is added to the falling sphere as ambient water vapor condenses on the cold hailstone, thereby hastening its melting.

We use Mason's equation in the area of greatest frequency of hail in the United States, where the ground elevation is approximately 5000 ft and the freezing level approximately 13,500 ft above sea level (about 600 mb). It is assumed that the hail falls from freezing level to the surface, a fall distance of 8500 ft, and in this layer the terminal fallspeed is given, using an average K of 1750 based on estimates of Battan and Theiss (see Section 9.2).

For hailstones that would melt completely by time of arrival at the 5,000-ft ground level, the initial diameter would have been by Mason's equation, no larger than 0.13 in. For hailstones to reach the ground with a diameter of 0.25 in., the initial diameter needs to be only 0.01 in. greater. For hailstones that reach the ground with a 1-in. diameter, the initial diameter needs to be only 0.00004 in. greater. It is concluded that the conditional probability distribution of hailstone diameters aloft must be practically the same as at ground level in the area of worst hailstorms for hailstones of significant diameters.

In the Ohio region where the Thunderstorm Project of 1947 was conducted, the ground level is approximately 1000 ft above sea level, the freezing level in June or July approximately at 13,500 ft. For hailstones melting before reaching the ground, Mason's formula yields an initial diameter of 0.17 in. Hailstones of diameter 0.25 in. begin with diameter 0.024 in. greater. The 1-in. hailstones begin with diameter just 0.004 in. greater. Hence the differences between hail size at the surface and aloft can still be neglected, as in Section 10, when considering the maximum hailstone sizes aloft.

10. PROBABILITY: OPERATIONAL RISK ALOFT

10.1 Hailstone Frequencies Aloft

Beckwith (1960) offered the opinion that nearly every thunderstorm has some hail at one stage or another of its development, at least those that form in the Denver area. Battan and Wilson (1969) similarly held that virtually all cumulonimbus clouds contain snow pellets, ice pellets, or hail at some stage of development.

The U.S. Weather Bureau (1948) Technical Paper No. 7 on the Thunderstorm Project indicates that for the Ohio and Florida areas combined, hail was encountered at approximately 10,000, 15,000, 20,000 and 25,000 ft in thunderstorms about 9, 7, 5, and 3 times as often as it was at 5,000 feet. But hail, at any given time, occurred in a shallow layer. It was encountered in less than 25 percent of the traverses in the Thunderstorm Project. The total sample is probably far from representative.

Above 30,000 ft, hail has been encountered infrequently but cannot be discounted (Foster, 1961). A hailstone of size 5 in. has been reported at 29,500 ft, 4 in. at 31,000 ft and 3 in. at 37,000 feet.

In the absence of sufficient and objective data, a certain amount of inference is in order on the frequency of hail aloft as a function of height. Hailstones must form, and grow in size, above the freezing level in the atmosphere. Over northeastern Colorado, western Nebraska or southeastern Wyoming, the average height of the 0°C isotherm is 13,500 ft during the hail season. Once formed, the hailstones will fall or be buffeted vertically up and down, or become suspended at a "balance" level (Atlas, 1966). This should be a level of high concentration and therefore a level having a high probability of hail occurrence. Balance levels are near, but below, the level of updraft maximum (Srivastana and Atlas, 1969), which places them significantly above the level of freezing.

Atlas (1966) depicts balance levels as roughly at 20,000 ft, the same height described by Donaldson (1961) as having the greatest concentration of large hail in several analyzed thunderstorms. The latter quotes other authors in support of the observation that hailstones tend to have a radar-reflectivity maximum between 15,000 and 30,000 feet.

Future efforts, such as the National Hail Research Experiment, which began in June, 1971 in "Hail Alley" in northeastern Colorado, and which is a joint project of several academic and governmental institutions (Am. Meteorol. Soc., 1971) may provide the enlightenment required. However, with the present information on the relative frequencies of hail encounter aloft, as observed, inferred or conjectured, and without trying exhaustively to resolve this problem, it is expedient to assume that the probability of encountering hail is uniformly the same at any level from 10,000 to 20,000 ft, and that any level in this interval can become a level of hail concentration. Concomitantly it is assumed that the probability of hail encounter decreases downward from 10,000 to 5,000 ft and decreases upward from 20,000 to 45,000 ft.

Since hailstones do not form or grow at the 5,000-ft level but simply fall through that level, and since hailstones do not appreciably melt from that level to the surface, the probability of hail encounter at levels at (and below) 5,000 ft is assumed to be the same as that found at the surface, 0.000448, (see Section 6).

The only available ratios of probabilities of hail aloft to hail at a lower level are those given by the Thunderstorm Project, which are based on 1947, 1948 data in Ohio and Florida thunderstorms. Averaging the reports of hail encounter at 10,000, 15,000 and 20,000 ft, one estimates that any level between 10,000 and 20,000 ft experiences 7 times more hail occurrences than the 5,000-ft level. Since the probability at 5,000 ft is assumed to be the same as the probability at the surface, the probability of hailstone encounter at any level between 10,000 and 20,000 ft is accepted as

7 × 0.000448 = 0.00314.

At 25,000 ft, again from the Thunderstorm Project, the probability is accepted as 3 times greater than at 5,000 ft, or 3 × 0.000448 = 0.00134.

Above 25,000 ft, the probability of hailstone encounter must diminish steadily. Arbitrarily, it is assumed that the probability decreases linearly to a probability of zero at 45,000 feet. A summary of the probability of a hailstorm, is given in Table 5. Between 5,000 and 10,000 ft, 20,000 and 25,000 ft, 25,000 and 45,000 ft, the probability is assumed to change linearly with altitude.

Table 5. Estimates of the Probability of Encountering Hail of Any Size at a Single-point Location by Altitude

Altitude (ft)	Probability
Ground level	0.000448
5,000	0.000448
10,000	0.00314
15,000	0.00314
20,000	0.00314
25,000	0.00134
30,000	0.00100
35,000	0.00067
40,000	0.00034
45,000	0.000

10.2 Single-point Risks Aloft

As pointed out in Section 9.3, the conditional probability distribution of hailstone size at ground level (Figure 1) can be used as the conditional probability distribution for upper levels. Therefore, by multiplying the probability of encountering a hailstorm at a given altitude (Table 5) by the conditional probability of exceeding a given size (Figure 1), the probability of encountering hailstones equal to or greater than the given size for that altitude is obtained (Table 6).

The greater frequency of large stones aloft, compared with ground-level frequency, is attributed to the greater frequency of hail of any kind aloft, especially at the "balance" level somewhere between 10,000 and 20,000 feet.

Table 6. Estimates of the Probability of Encountering Various Size Hailstones at a Single-point Location at Various Levels Aloft

Hail Diameter (in.) h	Level (thousands of feet)					
	10-20	25	30	35	40	45
Any size	0.00314	0.00134	0.0010	0.00067	0.00034	0
≥ 0.25	0.00250	0.00106	0.0008	0.00053	0.00027	0
≥ 0.5	0.00113	0.00048	0.00036	0.00024	0.00012	0
≥ 1.0	0.00022	0.00009	0.00007	0.00005	0.00003	0
≥ 2.0	0.00006	0.00002	0.00002	0.00001	0.000005	0
≥ 3.0	0.00001	0.00001	.....	.....	.....	0

10.3 Enroute Risks Aloft

The same model of spatial correlation (Section 8) that yielded estimates of probability of hail encounter as a function of the length of the route (Table 3) also gives estimates of enroute probabilities aloft. For the 100- and 200-mile routes, these probabilities are as shown in Table 7.

The probability estimates of Table 7, multiplied by the conditional probabilities of maximum hailstone size (Figure 1), give the probabilities of encountering such maximum size on 100- and 200-mile routes, similar to the figures in Table 4. By graphical interpolation, the sizes having a risk of encounter of 0.1 percent were found (see Table 8).

Table 7. Model Estimates of the Probability of Encountering Hail of Any Size on 100- and 200-mile Routes Aloft

Altitude (ft)	Probability	
	100-mile route	200-mile route
5,000	0.010	0.021
10-20,000	0.050	0.095
25,000	0.023	0.044
30,000	0.019	0.037
35,000	0.0145	0.030
40,000	0.0090	0.019

Table 8. Estimate of Hailstone Size Equalled or Exceeded With 0.1 Percent Probability of Encounter While Enroute Aloft

Altitude (ft)	100-mile route (in.)	200-mile route (in.)
5,000	≥ 0.9	≥ 1.2
10-20,000	≥ 1.9	≥ 2.4
25,000	≥ 1.3	≥ 1.9
30,000	≥ 1.2	≥ 1.7
35,000	≥ 1.0	≥ 1.5
40,000	≥ 0.8	≥ 1.1
45,000	.....	.....

11. SUMMARY

11.1 Designing for Operations

For actual operation of military equipment, climatic extremes which have a very low probability of exceedance over the worst geographical area during the severest month are used in design. For high surface temperature, humidity, and wind, a 1 percent probability of occurrence is considered an acceptable risk in guidance provided for revision of MIL-STD-210A by the Joint Chiefs of Staff (JCS). For surface precipitation, an extreme with only 0.5 percent probability was recommended. However, the JCS (1969) also indicated that when inoperability of an item of equipment directly endangers human life, the design criteria for equipment should be established so as to result in a percentage time of inoperability which is as close to zero as possible.

11.1.1 SURFACE OPERATIONS

Since the probability of hail at the surface of even the smallest size is very small, less than 0.05 percent (0.000448 × 100), and since the effect of hail on most surface equipment will not result in the endangerment of human life, there seems no need for specifying a hail size extreme as an operational design criteria for surface military equipment. If hail interferes with the operation, postponement until it ends will be acceptable since this will be so improbable.

11.1.2 OPERATIONS ALOFT

Aloft probabilities of hail are much higher than at the surface being as high as 0.314 percent at altitudes between 10,000 and 20,000 feet (see Table 5). These are still sufficiently low so that there is no need to specify a hail size extreme for equipment whose failure due to hail would not endanger human life. However, designers should consider the results of this paper for those equipments, especially aircraft and/or aircraft components, whose failure due to hail would result in the endangerment of human life. Since the record hailstone sizes range from 5.61 in. at the surface to 3 in. at 37,000 feet, these should be considered as criteria if such extremes are at all possible to accommodate in the design of a particular item. When design for these record sizes is not feasible, a possible extreme is that with a probability of only 0.1 percent. This is about as low a probability for which the corresponding extreme can be estimated with any degree of confidence. Since the most applicable design problem is aircraft travel through hailstorms, extremes have been estimated for two typical traverse distances in geographic areas of hail extremes. Hailstone diameters with 0.1 percent risk of occurrence in the worst month in the worst area are as follows:

Altitude (ft)	Traverse (mi) 100	 200
	Diam (in.)	
5000 and below	0.9	1.2
Maximum activity 10,000 to 20,000	1.9	2.4

11.2 Designing for Withstanding

Withstanding extreme applies to equipment which need not operate during hailstorms but may be continuously exposed for field lives of 2 to 25 years. The equipment should not be irreversibly damaged any time hail falls. A 10 percent risk is acceptable.

This type of design problem applies only to surface-located equipment. For withstanding without irreversible damage, this study has arrived at hailstone diameters, with 10 percent risk in the worst area, as follows:

Exposure (yr.)	Size (in.)	Terminal Velocity* (m/sec)
2	2.6	42
5	3.1	46
10	3.5	49
25	4.0	52

*Computed using $W = k \sqrt{d(cm)}$ with $K = 1630$

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